

Indifference bands for boundedly rational route switching

Xuan Di^{1,2} · Henry X. Liu^{1,2} · Shanjiang Zhu³ · David M. Levinson⁴

Published online: 5 May 2016
© Springer Science+Business Media New York 2016

Abstract The replacement I-35W bridge in Minneapolis saw less traffic than the original bridge though it provided substantial travel time saving for many travelers. This observation cannot be explained by the classical route choice assumption that travelers always take the shortest path. Accordingly, a boundedly rational route switching model is proposed assuming that travelers will not switch to the new bridge unless travel time saving goes beyond a threshold or “indifference band”. Indifference bands are assumed to follow lognormal distribution and are estimated in two specifications: the first one assumes every driver’s indifference band is drawn from a population indifference band and the second one assumes that the mean of drivers’ indifference bands is a function of their own characteristics. Route choices of 78 subjects from a GPS travel behavior study were analyzed before and after the addition of the new I-35W bridge to estimate parameters. This study provides insights into empirical analysis of bounded rationality and sheds light on indifference band estimation using empirical data.

Keywords Bounded rationality · Indifference band · Empirical estimation · GPS study · Route choice

✉ Xuan Di
sharondi@umich.edu

¹ Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, USA

² University of Michigan Transportation Research Institute, Ann Arbor, USA

³ Department of Civil Environmental and Infrastructure Engineering, George Mason University, Fairfax, USA

⁴ Department of Civil, Environmental, and Geo- Engineering, University of Minnesota, Twin Cities, Minneapolis, USA

Introduction

The I-35W Mississippi River bridge plays a critical role in transporting commuters to downtown Minneapolis and the University of Minnesota. Its collapse in 2007 forced 140,000 daily users (Guo and Liu 2011; Zhu et al. 2010; Di and Xuan 2014a) to switch to other parallel bridges or to cancel their trips. Accordingly, the I-94 Bridge was restriped with one more lane in each direction to relieve traffic pressure across the river. A year later, a replacement I-35W bridge was rebuilt over the same location and the extra lanes on I-94 were closed. The addition of the bridge offered commuters another option to cross the river. Surprisingly, only 100,000 daily trips on average were observed on the new bridge (Danczyk et al. 2015; Zhu 2011). According to Zhu (2011), the total travel demand in the Minneapolis-St. Paul metropolitan area dropped slightly in 2008 due to the economic crisis, but not enough to explain this fall-off. In contrast, daily trips on the I-94 Bridge returned to the original level before the I-35W bridge collapsed. Therefore, in this paper, we assume that variation in travel demands is not the main reason for the significant traffic reduction on the replacement bridge.

To further understand the aforementioned phenomenon, two major GPS-based experiments conducted by University of Minnesota researchers (Carrion et al. 2012; Zhu 2011; Zhu et al. 2012) will be used in this study. In these experiments, 143 commuters were selected whose route choices might be affected by the addition of the replacement bridge. Their trips were tracked by GPS two to three weeks before and eight to ten weeks after the reopening of the new I-35W bridge. By comparing route choices before and after, it is posited that commuters' "stickiness of driving habit" (Zhu 2011) prevented them from taking the new bridge and thus resulted in a traffic flow drop on the new bridge, even using the new bridge can save their travel time.

The disruption and rebuilding of the I-35W Bridge provides us a rare opportunity to use GPS travel survey data to study route choices in response to the change in road network's topology. This paper aims to examine route switching behavior using GPS travel data collected in Minneapolis in 2008.

The rest of the paper is organized as follows: In "Route switching behavior analysis" section, we discuss the details of the GPS experiments and present two categories of commuters of interest, i.e., switchers and stayers, as well as worriers and no-worriers. In "Route switching behavior modeling" section, two models with different indifference band specifications are presented. Accordingly, indifference bands are estimated using GPS travel survey data with probit regression. We will show that subjects' switching behavior primarily depends on time saving, their bridge usage history, and whether they are worried about driving on the bridge. In "Discussions" section, limitations of the paper and the generalization of the model is discussed. Conclusions and future research directions follow in "Conclusions and future research directions" section.

Route switching behavior analysis

GPS-based tracking and travel survey

Figure 1 shows eleven bridges used by subjects across the Mississippi River before and after the new bridge's reopening. The background is the TLG network (generated and maintained by Metropolitan Council and The Lawrence Group) which encompasses the entire seven-county Minneapolis-St. Paul Metropolitan Area.

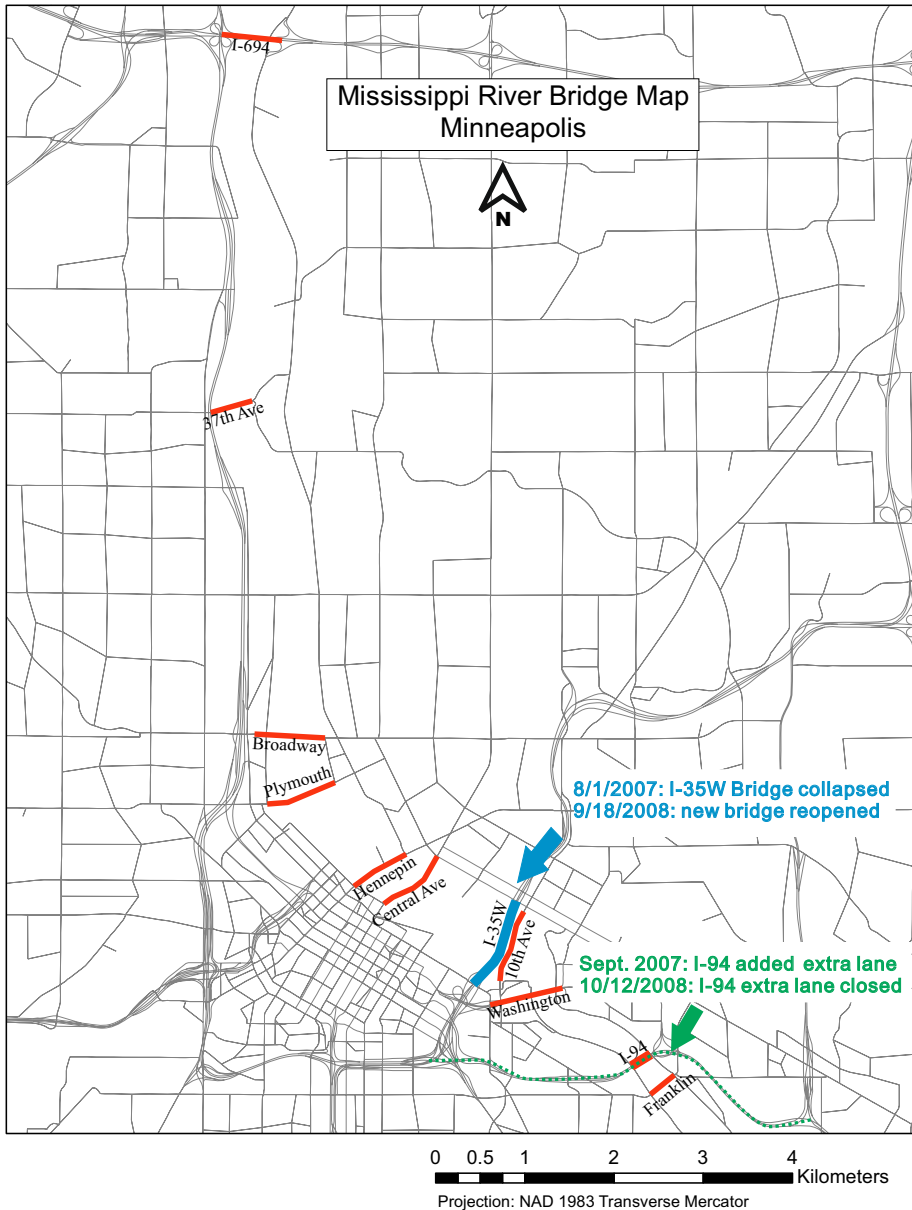


Fig. 1 Bridges over the Mississippi River

To capture commuters’ travel behavioral change through longitudinal observations of travel choices before and after the replacement I-35W Bridge opens, two types of GPS studies were performed: (1) real-time tracking GPS led by Georgia Institute of Technology and conducted by Vehicle Monitoring Technologies (VMTINC) and (2) logging GPS funded by the Oregon Transportation Research and Education Consortium (OTREC2). There are total 143 subjects (VMTINC recruited 46 subjects while OTREC2 had 97

subjects). These subjects were: commuters in the Twin Cities, between the age of 25-65, legal drivers, have a full-time job and follow a “common” work schedule, drive alone to work, and are likely to be affected by the opening of the replacement I-35W Bridge (as most of them should cross the Mississippi River for commuting according to their home and work locations). Interested readers can refer to Zhu et al. (2010) for more details on participant selection. These two GPS studies provide the following data for 143 subjects two to three weeks before September 18, 2008 (the day when the replacement I-35W bridge was opened) and one or two months afterwards:

- Each subject’s home and work locations;
- Each subject’s day-to-day commuting routes;
- Each subject’s day-to-day travel time.

Figure 2 illustrates home and work locations of 143 subjects participating in the study.

Subjects’ sociodemographic information, including age, income, gender, race, and education degree, was also collected via one-time online survey, as well as their familiarity with Twin Cities’ road network (e.g., years living in Twin Cities, the current home and the work place, years living in the current home, and years working in the current workplace). To understand whether occurrence of the disaster affected subjects’ choices, questions were asked as to whether they are worried about driving on or under bridges. Each subject’s travel mode to work was also explored. To understand subjects’ bridge choice before its collapse, we also asked them whether they used the old I-35W Bridge to commute, among them 88.4 % claimed they used the I-35W Bridge as their usual commute

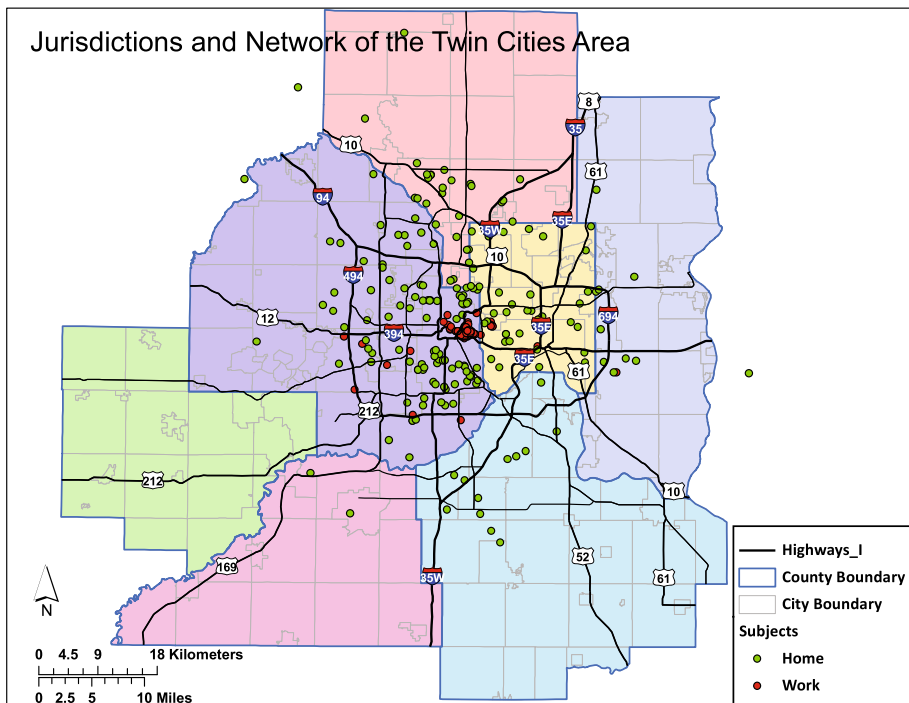


Fig. 2 Home and work locations of 143 subjects [cited from Levinson et al. (2012)]

route before its collapse and 70 % claimed that they planned to use the new I-35W bridge after its opening.

Subject classification

A “crosser” is the commuter whose home and work locations are on different sides of the river. Specifically, a crosser’s commuting route options may be enlarged by the addition of the new bridge. Otherwise the subject is a “non-crosser”. In general a non-crosser’s route options are not enlarged by the existence of the new bridge.

When the addition of the new bridge saves crossers’ commuting time, a crosser chooses either to change to the new bridge or not. Thus we further divide those crossers into the following two categories:

- A crosser who switches to the new I-35W bridge when the new travel time is shorter is a “Switcher”;
- A crosser who does not switch to the new I-35W bridge when the new travel time is shorter is a “Stayer”.

Remark

1. Switchers in our definition refer to those who switch to I-35W Bridge, excluding those who switch routes but to a different bridge or different road segments. We did observe two subjects who switched from one existing bridge to another after the new bridge was built and their travel time got improved, but choosing the new bridge can actually save them more commuting travel time. To avoid confusion, we will not include these two subjects in this study.
2. To model route switching behavior, in this paper, we are only interested in those subjects whose travel time can be saved by taking the new bridge. In the scenarios when the reopening of the I-35W Bridge cannot save travelers travel time: if they stayed, that is because the new route is not appealing, which will not help understand behavior; if they switched, other factors than travel time may dominate their choices and those factors may not be easily observed. Therefore only those whose travel time saving is positive are taken into consideration.

Switchers and stayers

Most subjects use more than one path and switch routes from time to time. To facilitate route switching modeling, we define a “commonly chosen route” as the route a commuter uses most frequently during the study period. The commonly chosen route from the beginning of the study period up to September 18, 2008 is the “before-route”, and the one from September 18, 2008 until the end of the study is the “after-route”.

Remark

1. Even after drivers stabilize their route choices, drivers may still switch en-route at junction points or intersections, but commuters do not switch commuting routes as frequently as they do for other travel purposes. Accordingly, the travel time along all possibly chosen commuting routes should be quite similar, otherwise commuters will not stay on those longer routes. To include these possible alternative routes, we define

the same routes as those which overlap at least 95 % in length and start within a 600 m (approximately 4 city blocks) radius from home and end in a 600 m radius from the work location.

- Each commuter's experienced travel time on a path varies from day to day due to uncertainty in traffic conditions. Therefore "average travel time" on a before-route or on an after-route for a commuter is computed as the mean of day-to-day GPS measured travel time on that route when he or she uses it.

In this study, 47 switchers and 31 stayers are identified. So there are 78 subjects of interest.

To obtain the travel time savings brought by taking the new I-35W bridge, we need to identify routes via the new bridge. For switchers, the after-route is the route via the new bridge. Stayers, on the other hand, never use the new bridge and therefore a route via I-35W bridge for each stayer should be first identified. To identify stayers' hypothetical travel time on the new bridge, a tool generated by Zhu (2011), i.e., speed map, will be used. The speed map aggregates GPS data from the larger sample of 140 drivers including the sub-sample analyzed here. It was pooled from 6,059 commuting trips out of 25,157 total trips. Only links with more than 5 observations before and after the new bridge's reopening were included. The average link speed was estimated from GPS data of all probe vehicles passing this link during the experiment period. This map covers a high portion of the freeway system and a fairly high portion of arterial roads, especially trunk highways and downtown streets. Interested readers can refer to Zhu (2011) for details. Based on the speed map, each link's average travel time can be computed. Then estimated travel times along a route is the sum of average travel times of all links along that route. A shortest path is generated using the speed map between a stayer's home and work location as his or her new route assuming as if he or she uses the new bridge.

Remark Subjects' commuting times are not the same, to make sure the following comparison is performed under the same benchmark, travel time saving proportion instead of absolute travel time saving will be used. Denote $\Delta^{(n)}$ as the travel time saving proportion by taking the new bridge for commuter n . It is computed as $\Delta^{(n)} = \frac{C_a^{(n)} - C_b^{(n)}}{C_b^{(n)}}$, where $C_b^{(n)}$, $C_a^{(n)}$ are estimated travel times experienced by commuter n before and after the reopening of the new bridge.

Table 1 summarizes the statistics of travel time saving:

We can see clearly that switchers' travel time saving by taking the new bridge is larger than that for stayers, showing that travel time saving is a major reason for people's switching to the new bridge.

Old-users and non-users

Among 78 subjects, 44 used the old I-35W bridge regularly before it collapsed and 34 were not the regular old bridge users. Old bridge users (i.e., old-users) are pre-disposed to use the new bridge while non-users may not use it even it could save substantial travel time. Therefore, old-users and non-users should display different route choice behavior in response to the addition of the new bridge. The frequency of stayers and switchers for non-users and old-users are summarized in Table 2.

Figure 3 illustrates the boxplot of travel time saving proportion statistics for two groups (non-user and old-user). The dots are data which are outside third quartile and represent

Table 1 Estimated time saving statistics

Statistics	Switcher	Stayer	Total
Distribution			
Counts	47	31	78
Percentage	60.3	39.7	100.0
Average travel time (minute)			
Before	16.4	19.2	17.5
After	14.5	18.2	16.0
Difference	1.9	1.0	1.5
Average travel time saving percentage (%)			
Average	13.0	5.4	10.0
Minimum	2.6	0.4	0.4
Maximum	10.5	25.2	34.4
Median	34.4	3.5	7.9

Table 2 Contingency table of subjects' categories

	Non-user (U = 0)	Old-user (U = 1)	Total
Stayer (y = 0)	23	8	31
Switcher (y = 1)	11	36	47
Total	34	44	78

outliers. Overall the mean and the median travel time savings for switchers are higher than those of stayers. The mean travel time savings for non-users are slightly higher than those for old-users.

We further divide time saving ranges into 11 bins respectively. The total number of switchers and stayers for each bin can be calculated. Figure 4 illustrates the distribution of switchers and stayers for each bin for old-users. In Fig. 4a, bars illustrate the frequencies of switchers (in blue) and stayers (in red) within each bin. Lines with markers represent cumulative percentages of switchers (in blue) and stayers (in red) among all old-users. As the time saving increases, generally speaking, the proportion of stayers gradually decreases to zero (with only one outlier for the bin (26, 30 %). It suggests that all drivers switched to the new bridge, apart from those with negligible time savings. Similar analysis can be applied to non-users shown in Fig. 4b with one outlier for the bin (24, 27 %). Both patterns reflect bounded rationality, but non-users may have higher thresholds to switch, because they are not aware of the new alternative or they do not consider the new bridge a valid one.

The proportion of switchers and stayers displays opposite patterns for old-users and non-users. For older users, there are more switchers than stayers when time saving is greater than 5 %. When the time saving is less than 35 %, only 20 % of subjects stay while the remaining 80 % switch to the new bridge. On the other hand, for non-users, when the time saving is within 27, 70 % of subjects stay while only 30 % switch to the new bridge.

Worriers and no-worriers

After bridge collapse, people tend to avoid this bridge because of psychological reasons. Therefore we also want to analyze people’s fear of using the bridge versus their route switching behavior. Table 3 summarizes the number of observations for each category.

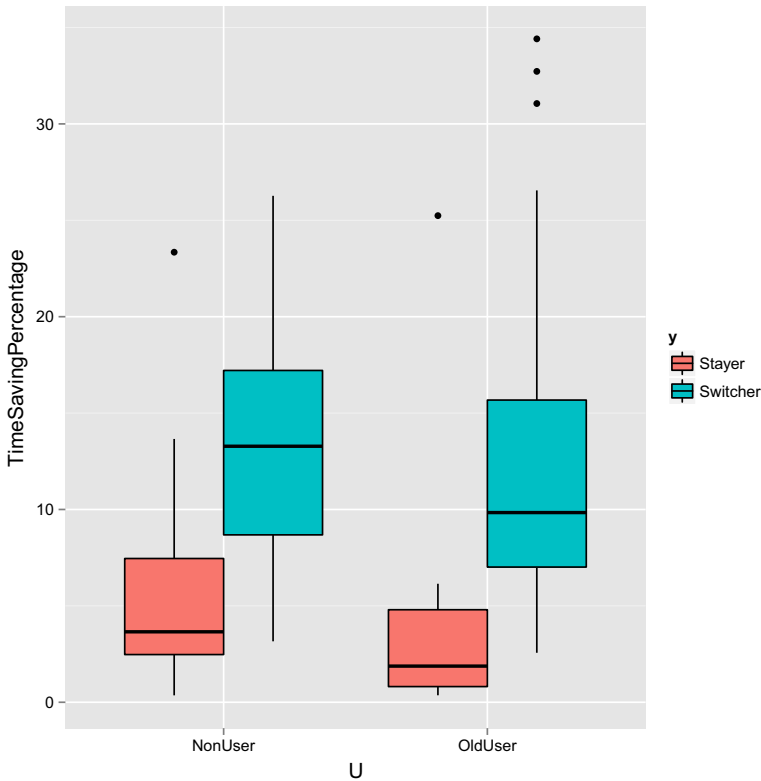


Fig. 3 Boxplot of travel time saving proportions versus usage history

Figure 5 illustrates time saving versus bridge usage history and fear. We can see that people who are worried about driving on the bridge tend to experience higher time saving for both switchers and stayers. In other words, they only switch when the time saving is really significant, otherwise they will stay. Worried switchers is the only exception.

Based on the above analysis, we can see that route switching behavior also depends on other factors than travel time, such as bridge usage history and psychological fear. As a result, people do not necessarily switch to the new bridge even though it provides a shorter path. On the other hand, there does exist a positive threshold only beyond which people will switch routes, which indicates bounded rationality. In the next section, thus, we will develop methodologies for indifference band estimation.

Route switching behavior modeling

Literature review

A route contains a set of attributes: travel distance, travel time, number of intersections, scenery, and so on. In the existing literature, three decision strategies are proposed to describe one's route choice decision-making process:

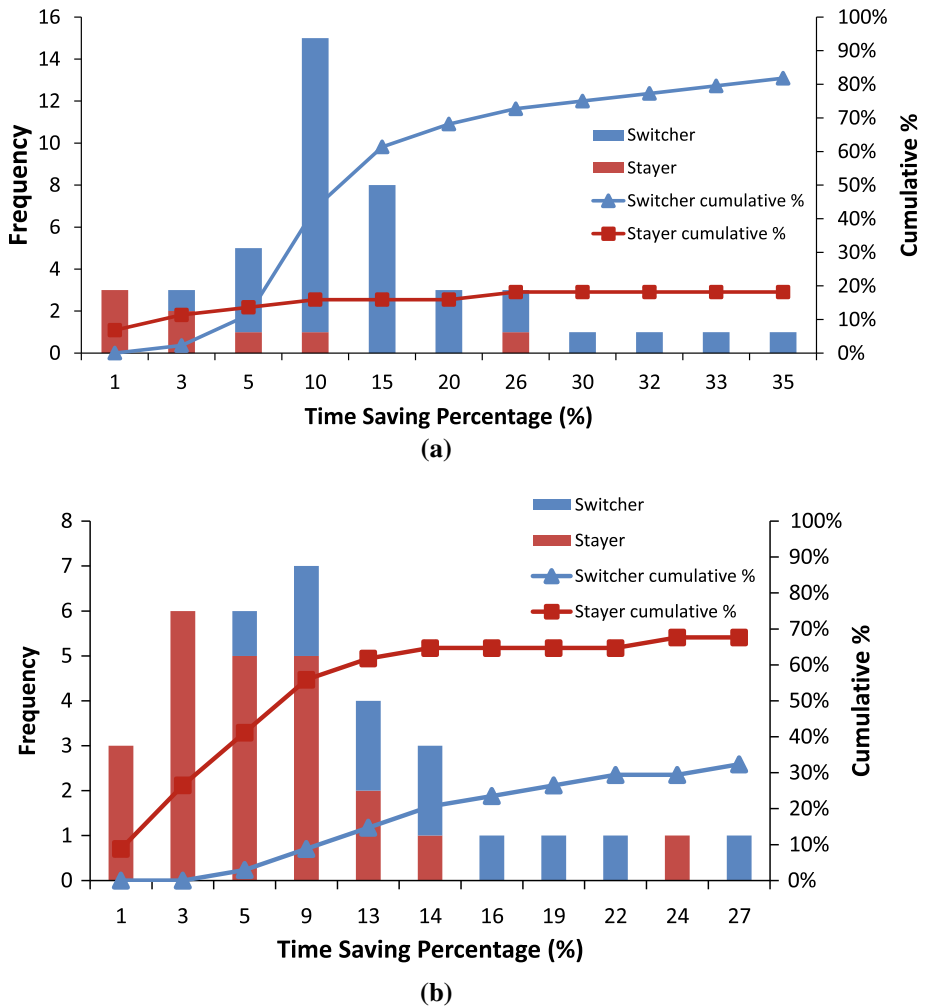


Fig. 4 Travel time saving distribution for both switchers and stayers among, **a** old-users and **b** non-users respectively

Table 3 Contingency table of subjects' categories

Stayers (y = 0)	No-worry (W = 0)	Worry (W = 1)	Total
Non-user (U = 0)	20	3	23
Old-user (U = 1)	6	2	8
Total	26	5	31
Switchers (y = 1)	No-worry (W = 0)	Worry (W = 1)	Total
Non-user (U = 0)	9	2	11
Old-user (U = 1)	31	5	36
Total	40	7	47

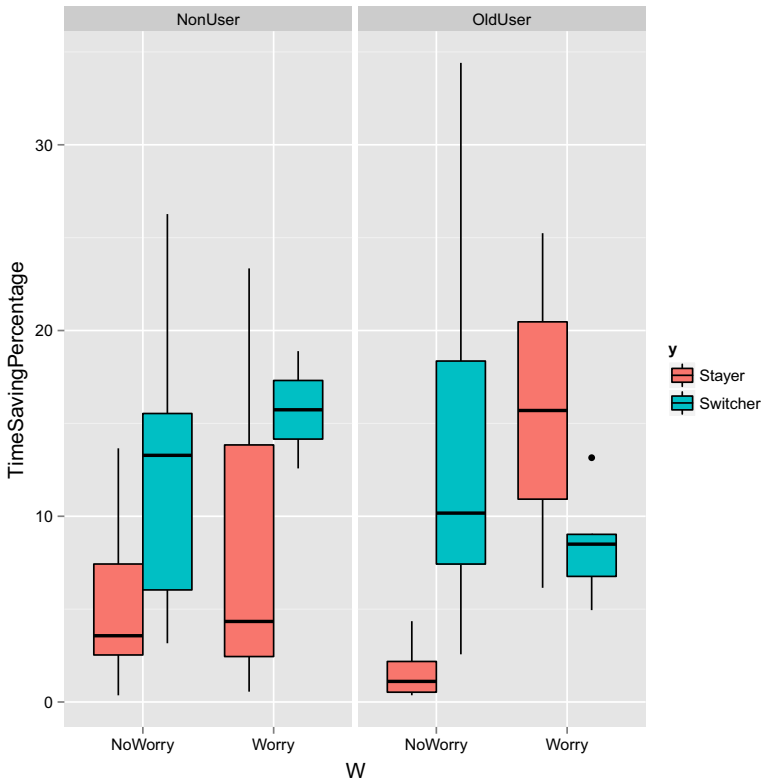


Fig. 5 Boxplot of travel time saving proportions versus both usage history and worry

1. Compensatory strategy: there exist trade-offs among attributes. In other words, the attribute of one alternative can be compensated by another attribute. The traditional random utility maximization framework utilizes the compensatory strategy;
2. Non-compensatory strategy: each alternative is treated as a set of attributes (i.e., aspects). Alternatives are selected attribute-by-attribute. For one alternative, its superior attribute cannot compensate its inferior attribute. The non-compensatory strategy can be heuristic as only a partial set of alternatives are compared to obtain an optimal one. It includes two strategies (Rasouli et al. 2015): conjunctive (i.e., an alternative is selected only when all attributes meet requirements) and disjunctive (i.e., an alternative is selected if at least one attribute meets requirements). Satisfactory heuristic (Simon 1955) is one example of conjunctive strategy as an alternative is selected if it meets the minimum aspiration level on all attributes (Simon 1955).
3. Semi-compensatory strategy: the non-compensatory strategy is employed for choice set generation (Ben-Akiva et al. 1995; Swait et al. 1987; Kaplan et al. 2010, 2012; Swait and Joffre 2001b; Martínez et al. 2009) and the compensatory strategy is used to find an optimal choice.

In all three strategies, the discrete choice model is a common tool to model people's choice behavior. Within the framework of discrete choice modeling, the random utility maximization model (RUM) is adopted in modeling and predicting drivers' route choice

behavior among a set of finite paths. Provided perception errors are Gumbel distribution, RUM can be expressed in the form of a multinomial logit model. The logit model assumes paths are independent of each other and has independence from irrelevant alternatives (IIA) property. In reality, however, many paths overlap with each other and are thus not independent. To overcome this limitation, various RUM models were proposed: C-logit (Cascetta et al. 1996), path-size logit (Ben-Akiva et al. 1998a), nested logit (Jha et al. 1998), cross-nested logit (Vovsha et al. 1998b), multinomial probit (Cascetta 1989; Daganzo et al. 1977; Jotiskansa et al. 2006), and mixed logit or multinomial probit with logit kernel (Ben-Akiva et al. 1996). However, all these models assume decision-makers are fully rational and fully informed, and most importantly, they are utility-maximizers (Swait and Joffre 2001b). In other words, people only take the shortest paths in route choice. Such an assumption is called “perfect rationality”.

As opposed to ‘rationality as optimization’, Herbert Simon, in 1957, proposed that people are boundedly rational in their decision-making processes (Simon 1957). This is either because people lack accurate information, or they are incapable of obtaining an optimized decision due to complexity of the situation. They tend to seek a satisfactory choice solution instead. Since then, bounded rationality has been studied extensively in economics and psychology.

People do not choose routes irrationally. Yet, people do not always choose the shortest paths. Evidence from revealed route choice behavior finds after evaluating habitual routes, only 59 % of respondents from Cambridge, Massachusetts (Bekhor et al. 2006), 30 % from Boston (Ramming and Michael Scott 2001a), and 87 % from Turin, Italy (Prato et al. 2006) chose paths with the shortest distance or shortest travel time. In the Minneapolis-St. Paul region, 90 % of subjects took paths one-fifth longer than average commute time (Zhu 2011; Zhu et al. 2015) based on actual GPS data while more than half commute trips were at least 5 minutes longer than the shortest path based on TomTom GPS data (Tang et al. 2015), and a high percentage of commuting routes were found to differ considerably from the shortest paths in Nagoya, Japan (Morikawa et al. 2005 and Lexington, Kentucky (Jan et al. 2000). All findings above revealed that people do not usually take the shortest paths and the utilized paths generally have higher costs than shortest ones.

The empirical evidence also argues that though travelers do not always select the shortest travel time paths, the chosen routes are within some threshold from the shortest ones. Zhu (2011) calculated travel time differences between the routes actually taken and the shortest time routes from GPS data. It was found that fewer than 40 % of commuters took the shortest paths, while 90 % of subjects took routes which were within 5 min of the shortest paths and almost no commuter chose a route 20 minutes longer than the shortest one.

Boundedly rational behavior may result from habit and inertia, cognitive costs, and individual preference. As these factors can hardly be obtained from the data that is available to researchers, a threshold parameter, named “indifference band”, is introduced to characterize these unobservable attributes. It can be embedded into both static and dynamic discrete choice model to represent bounded rationality.

Static discrete choice model

Reference-dependent model Reference-dependent models play an important role in boundedly rational behavioral modeling. People generally evaluate attributes/utilities compared to the best value or some reference point instead of using absolute values. Such models can be applied to both route choice and choice set generation. Accordingly the BR

principle is reflected as elimination of alternatives whose attributes or utilities are below certain thresholds. The behavioral mechanism includes maximization of relative advantage, maximization of relative utility, minimization of regret (such as maximin, maximax, minimax regret models). Interested readers can refer to Rasouli et al. (2015) for detailed description of each model. Thresholds can be deterministic or stochastic. Thresholds can also be fixed or dynamic. If there does not exist an optimal solution, thresholds can be adjusted dynamically till an optimal solution is attained.

Indifference relation Discrete choice models assume implicitly the existence of a preferable ordering over alternatives even if the difference in utilities is negligible. However, psychological experiments (Guilford and Joy Paul 1954) showed that people may be indifferent to two alternatives with similar utilities. Ridwan (2004) defined three fuzzy preference relations for two alternatives: strict preference; indifference; incomparability, and a fuzzy choice function was proposed capturing the fuzziness feature of choices to calculate their rankings. Krishnan (1977) proposed a minimum perceivable difference (MPD) model describing travelers' mode choices among two alternatives with two relations: strict preference and indifference. The MPD model can only accommodate two alternatives. Lioukas (1984) extended it to the multinomial logit model with more than two choices. In route choice, travelers may be indifferent to small differences in utility values or small changes in utilities when some change happens. The utility function needs to be modified to account for similarity of choice alternatives. Also, "alternative utility threshold" is introduced to capture such indifference (Swait and Joffre 2001b).

Indifference to small changes Due to existence of inertia, people may place higher weight on the alternative he or she regularly uses, which introduces state dependence and serial correlation in dynamic choices. This dependence can be captured by an inertial variable depending on the utility difference between chosen alternative and others. Cantillo et al. (2006, 2007) further specified the threshold by introducing an inertial variable with state dependence (due to inertias, Cantillo et al. 2006) and serial correlation (due to persistence of unobservable attributes across a sequence of choices, Cantillo et al. (2007)). This model was used to describe travelers' mode switching choice after a new mode is added. Two sets of data were collected in Cagliari, Italy: the RP data in terms of people's mode choices among car, bus, and train; the SP data inquiring the choice between a new train service and the current mode choice. Estimation results showed that a misspecified model without inertia and serial correlation may lead to biases and errors when a newly implemented policy has a substantial impact.

Dynamic route switching model

Mahmassani et al. (Hu and Mahmassani 1997; Jayakrishnan et al. 1994; Mahmassani and Chang 1987; Mahmassani and Jayakrishnan 1991; Mahmassani and Liu 1999; Srinivasan and Mahmassani 1999) have conducted a sequence of laboratory experiments where subjects receive real-time traffic information and make pre-trip and en-route choices. These experiments were run on an interactive simulator—DYNASMART, incorporating pre-trip departure time, route choices and en-route path switching decisions. Subjects, as travelers, picked departure time pre-trip based on previous days' travel experiences and chose paths en-route at each node based on available information. Given specifications for utility errors of switching departure-time and routes, repeated observations of departure time and route switching decisions can be modeled as a multinomial logit or a multinomial probit function.

Indifference band

The existence of a threshold in choice behavior has also long been explored in other fields. Biologists are interested in dose-response, which explores the impact of toxic levels on an organ or a tissue. Psychologists and economists focus on the change in decision-makers' preferences or choices in response to the change in utilities of alternatives. The stimulus-response model, popular in biology, psychology, and economics, provides an efficient method of quantifying behavioral response by varying stimulus of specific intensities. The occurrence of a response depended on the intensity of a stimulus and there existed a threshold under which no response was manifest (Cox and Christopher 1987; Krishnan 1977). This threshold was named “just noticeable difference” by Weber or “minimum perceivable difference” by Krishnan (1977). Numerous biological experiments (Clark et al. 1933; Hemmingsen 1933) verified that no response occurred unless the logarithm of stimulus exceeded some threshold. Several models, such as threshold dose-response model (Cox and Christopher 1987), minimum perceivable difference model (Krishnan 1977), and biological probit or logit model (Krishnan 1977), were proposed to estimate the threshold. The indifference band is specified by either a constant or a random variable. For example, (Recker et al. 1979) first assumed the tolerance is a constant and then generalized it to a Weibull random variable.

Indifference band estimation

We believe bounded rationality is one reason for stayers not to switch to the new bridge even it provides a shorter path. However, we should note that bounded rationality is one just option to capture the observation. Though literature on threshold estimation is popular in other fields, boundedly rational route choice modeling is still understudied, this paper contributes to the state-of-arts of empirical analysis of bounded rationality by proposing methodologies of estimating indifference band using GPS travel survey data.

When the road network is perturbed due to real-time traffic information, construction, or disaster, peoples normal route choices are impacted or even disrupted and they may or are forced to switch routes. Indifference bands are able to be observed from their switching behavior. Day-to-day route switching model requires consecutive days' observations of departure-time, pre-route, and en-route choices. The static discrete choice model only requires one's route switching choices before and after a change happens. In this paper, we develop a discrete choice model to capture people's route switching behavior. Ideally a day-to-day route switching model should be built to capture daily commuting route choice variations. Then drivers' daily commuting trajectories collected from GPS devices can be used to calibrate the model. Our dataset, however, is insufficient to support calibration and validation of any type of day-to-day traffic dynamical models due to the following reasons:

1. Many drivers' day-to-day commuting trips are missing, especially during the transition period when they were trying new routes or switching back and forth. This was mainly because data was downloaded from subjects' vehicles periodically and it was difficult to monitor data quality between two data collection periods;
2. GPS data only provides us the travel time along the routes they have used. To calculate the travel time saving by taking the new bridge for those who have never used it, we have to estimate travel time. To achieve that goal, average travel time for each link will be computed by averaging all observations along that link. After averaging, every link has one static travel time without day-to-day variation.

Therefore, instead of proposing a day-to-day route switching model, we will develop a static discrete choice model. We assume that at the end of the study period, all drivers settle down on their frequently used routes. Note that even after drivers stabilize their route choices, drivers may still switch en-route at junction points or intersections, but we mainly focus on their most frequently used route choice. We believe this is a fair assumption as commuters generally do not switch commuting routes as frequently as they do for other travel purpose.

Note. In reality, drivers can also adjust their departure-time in response to the addition of the bridge. However, as we employ a static route choice behavior model, departure-time choice is not taken into consideration. Interested readers can refer to Zhu (2011) for the impact of rebuilding the new bridge on people's departure-time choice.

The indifference band may vary among different origin-destination pairs and alternative routes each traveler takes. To remove the impact of these factors, we use proportion, so that every traveler share the same value regardless of origin-destination pairs and routes. The indifference band can be specified in two ways:

1. Each driver's indifference band is drawn from the same population distribution.
2. Each driver has a different indifference band, depending on travel time saving, its perception error, bridge usage history, and other socio-demographic characteristics.

In the following, we will estimate indifference bands with these two specifications respectively.

A population indifference band

In route choice, the indifference band represents the deviation of the actual utilized path cost from the minimum path cost. It captures the fact that people are indifferent to a shorter route if its time saving is within a critical value. To estimate the indifference band, we need to find those subjects whose travel time can be saved. When the reopening of the I-35W Bridge cannot save travelers travel time: if they stayed, that is because the new route is not appealing, which cannot help estimate the indifference band; if they switched, other factors than travel time (which may not be easily observed) may dominate their choices. Therefore to estimate indifference bands, only those whose travel time saving is positive are selected. Accordingly, indifference band should be positive as well.

Denote ϵ^* as a population indifference band. As $\epsilon^* > 0$ based on the aforementioned analysis, we assume it follows lognormal distribution, whose associated normal distribution has mean $\bar{\epsilon} \geq 0$ (i.e., a deterministic constant) and variance σ^2 where $\sigma > 0$. It can be specified as:

$$\epsilon^* = e^{\epsilon + \eta_2} \sim \text{lognormal}(\bar{\epsilon}, \sigma^2), \quad (3.1)$$

where η_2 is a normal distribution with mean $\mu = 0$ and standard deviation $\sigma > 0$.

According to the principle of bounded rationality, commuters will not switch routes unless the logarithm of the perceived travel time saving is greater than the logarithm of the indifference band. Denote $\hat{\Delta}^{(n)}$ as the logarithm of commuter n 's perceived travel time saving, i.e., $\hat{\Delta}^{(n)} = \log(\Delta^{(n)}) + \eta_1$, where η_1 is a normal random variable with mean $\mu = 0$ and standard deviation $\sigma > 0$. Also, η_2 and η_1 are independent. In other words, they are independent and identically distributed normal. Then the hypothesis is:

$$y^{(n)} = \begin{cases} 1, & \text{if } \hat{\Delta}^{(n)} > \log(\epsilon^*), \\ 0, & \text{if } \hat{\Delta}^{(n)} \leq \log(\epsilon^*). \end{cases} \tag{3.2}$$

The probability of switching for commuter n is then computed as:

$$\begin{aligned} P(y^{(n)} = 1 | \bar{\epsilon}, \sigma) &= P(\hat{\Delta}^{(n)} > \log(\epsilon^*) | \bar{\epsilon}, \sigma) = P(\log(\Delta^{(n)}) + \eta_1 > \bar{\epsilon} + \eta_2 | \bar{\epsilon}, \sigma) \\ &= P(\eta_2 - \eta_1 < -\bar{\epsilon} + \log(\Delta^{(n)}) | \bar{\epsilon}, \sigma) \\ &= \Phi_{\eta_2 - \eta_1} \left(\frac{-\bar{\epsilon} + \log(\Delta^{(n)})}{\sqrt{2}\sigma} \right) = \Phi_{\eta_2 - \eta_1} \left(\beta_0 + \beta_1 \log(\Delta^{(n)}) \right), \end{aligned} \tag{3.3}$$

where $\eta_2 - \eta_1 \sim N(0, 2\sigma^2)$: $\eta_2 - \eta_1$ follows a normal distribution with mean of zero and standard deviation of $2\sigma^2$. $\beta_0 = -\frac{\bar{\epsilon}}{\sqrt{2}\sigma}$, $\beta_1 = \frac{1}{\sqrt{2}\sigma}$. Therefore $\bar{\epsilon} = -\frac{\beta_0}{\beta_1}$, $\sigma = \frac{1}{\sqrt{2}\beta_1}$.

The probit regression is performed to estimate β_0 and β_1 (Table 4):

To show whether the model fits the data, Hosmer and Lemeshow goodness of fit (GOF) test is employed. Hosmer and Lemeshow GOF is used for binary outcomes regression when sample size is small and when continuous explanatory variables are included in the model (Hosmer et al. 2013). A high p -value indicates a good fit. We use “hoslem.test” function contained in the “ResourceSelection” package in R to conduct this test. It gives a p -value of 0.61, meaning that we lack of evidence against a null hypothesis that the model does not fit the data.

Then the estimates are: $\hat{\epsilon} = -\frac{\hat{\beta}_0}{\hat{\beta}_1} = -\frac{2.90}{0.97} = -2.99$, $\hat{\sigma} = \frac{1}{\sqrt{2}\hat{\beta}_1} = \frac{1}{\sqrt{2} \times 0.97} = 0.78$.

Given $\hat{\epsilon}$ and $\hat{\sigma}$, its mean and variance are computed as follows:

$$E[\epsilon^*] = e^{\hat{\epsilon} + \frac{\sigma^2}{2}}, \tag{3.4a}$$

$$Var[\epsilon^*] = \left(e^{\sigma^2} - 1 \right) e^{2\hat{\epsilon} + \sigma^2}. \tag{3.4b}$$

In summary, the population indifference band is a lognormal random variable with mean 6.56 % and variance 0.03 %. Figure 6 plots its probabilistic distribution function.

Table 4 Probit regression coefficients estimation for population indifference band

	Estimate	SE	t value	Pr(> t)
(Intercept)	2.90	0.62	4.67	0.00***
log(Δ)	0.97	0.20	4.35	0.00***

*** Statistically significant at 1 % level

Table 5 Probit regression coefficients estimation for individual indifference bands

	Estimate	SE	t value	Pr(> t)
(Intercept)	2.77	0.78	3.54	0.00***
log(Δ)	1.16	0.28	4.12	0.00***
$U = 1$	1.67	0.42	3.97	0.00***
Worry	-0.89	0.52	-1.72	0.09*

* Statistically significant at 10 % level

*** Statistically significant at 1 % level

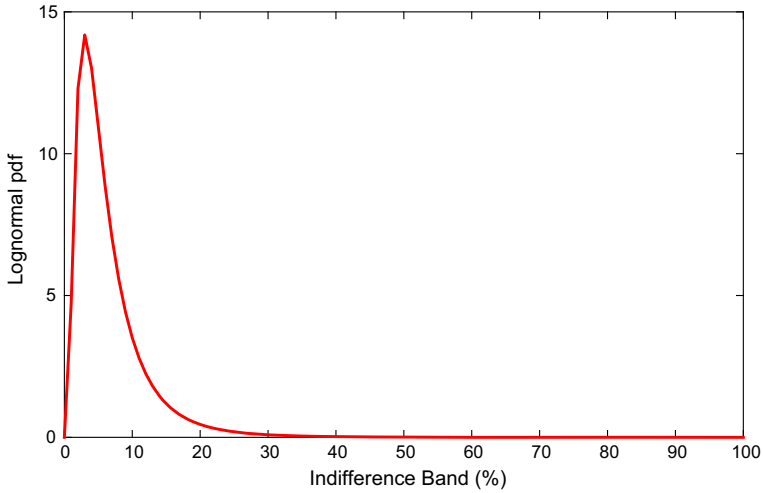


Fig. 6 Lognormal distribution of population indifference band

Individual indifference bands

Indifference band can vary from person to person due to different bridge usage experiences and sociodemographic characteristics. In Appendix, Table 6 illustrate all 78 subjects’ sociodemographic characteristics and other statistics, which will be used to characterize individual indifference band.

Denote $\epsilon^{(n)}$ as commuter n ’s indifference band, which is a random variable depending on travel time saving, its perception error, commuter n ’s bridge usage history, and other socio-demographic information. Denote η_2 as a normal distribution with mean $\mu = 0$ and standard deviation $\sigma > 0$.

$$\epsilon^{(n)} = e^{\sum_i \theta_i X_i^{(n)} + \eta_2} \sim \text{lognormal} \left(\sum_i \theta_i X_i^{(n)}, \sigma^2 \right), \tag{3.5}$$

where $X_i^{(n)}$ is commuter n ’s i th explanatory variable and θ_i is the coefficient before the i th variable.

Take logarithm on both sides, the logarithm of commuter n ’s indifference band is:

$$\begin{aligned} \log(\epsilon^{(n)}) &= \theta_0 + \theta_1 \log(\Delta^{(n)}) + \theta_2 U_n + \theta_3 \text{Worry} + \theta_4 \text{Age} + \theta_5 \text{Income} + \theta_6 \text{Mode} \\ &\quad + \theta_7 \text{Race} + \theta_8 \text{Gender} \\ &= \sum_{i=0}^8 \theta_i X_i^{(n)} + \eta_2. \end{aligned} \tag{3.6}$$

The hypothesis then becomes:

$$y^{(n)} = \begin{cases} 1, & \text{if } \hat{\Delta}^{(n)} > \log(\epsilon^{(n)}), \\ 0, & \text{if } \hat{\Delta}^{(n)} \leq \log(\epsilon^{(n)}). \end{cases} \tag{3.7}$$

The probability of switching for commuter n is then computed as:

$$\begin{aligned}
 P(y^{(n)} = 1|\Theta) &= P(\hat{\Delta}^{(n)} > \log(\epsilon^*)|\Theta) = P(\log(\Delta^{(n)}) + \eta_1 > \sum_{i=0}^8 \theta_i X_i^{(n)} + \eta_2|\Theta) \\
 &= P(\eta_2 - \eta_1 < \log(\Delta^{(n)}) - \sum_{i=0}^8 \theta_i X_i^{(n)}|\Theta) \\
 &= \Phi_{\eta_2 - \eta_1} \left(\frac{-\theta_0 + (1 - \theta_1) \log(\Delta^{(n)}) - \sum_{i=2}^8 \theta_i X_i^{(n)}}{\sqrt{2}\sigma} \right) = \Phi_{\eta_2 - \eta_1} \left(-\sum_{i=0}^8 \theta'_i X_i \right),
 \end{aligned} \tag{3.8}$$

where $\eta_2 - \eta_1 \sim N(0, 2\sigma^2)$ is the same as before. The parameter vector $\Theta = \{\theta_0, \theta_1, \dots, \theta_8, \sigma\}^T$ where $\theta'_1 = -\frac{1-\theta_1}{\sqrt{2}\sigma}, \theta'_i = -\frac{\theta_i}{\sqrt{2}\sigma}, i = 0, 2, \dots, 8$. Accordingly $\theta_1 = 1 - \sqrt{2}\sigma\theta'_1, \theta_i = -\sqrt{2}\sigma\theta'_i, i = 0, 2, \dots, 8$.

Probit regression is conducted to estimate parameters $\theta_i, i = 0, \dots, 9$. The estimation results show that the coefficients before $\log(\Delta)$ and U are significantly different from zero at 5 and 0.1 % significance level respectively. However, other factors turn out not to be significant. Therefore variable selection is needed before the correct probit regression is conducted. An automatic method, “step” function in R, is used to perform variable selection according to a more commonly used criterion, “Akaike information criterion” (AIC, a measure of the quality of a statistical model for a given set of data). The null model (i.e., the one without explanatory variables) and the full model (the one with all explanatory variables) need to be defined first. Then the forward or backward selection will begin to examine each model. The one with the lowest AIC value will be chosen as the final model. After variable selection, only $\log(\Delta), U, Worry$ are significant contributing factors. Accordingly the indifference band can be rewritten in a reduced form as:

$$\log(\epsilon^{(n)}) = \theta_0 + \theta_1 \log(\Delta^{(n)}) + \theta_2 U_n + \theta_3 Worry_n. \tag{3.9}$$

Then the reduced probit regression is:

$$P(y = 1|\Theta) = \Phi_{\eta_2 - \eta_1} \left(\theta'_0 + \theta'_1 \log(\Delta^{(n)}) + \theta'_2 U_n + \theta'_3 Worry_n \right), \tag{3.10}$$

where $\theta'_1 = \frac{1-\theta_1}{\sqrt{2}\sigma}, \theta'_i = -\frac{\theta_i}{\sqrt{2}\sigma}, i = 0, 2, 3$. Then $\theta_1 = 1 - \sqrt{2}\sigma\theta'_1$ and $\theta_i = -\sqrt{2}\sigma\theta'_i, i = 0, 2, 3$. As σ is not identifiable in this case, we set $\hat{\sigma} = 0.73$ as the same as the previous population indifference band so that no extra variability is introduced.

The estimation coefficients using probit regression are listed in Table 5:

Similarly, a Hosmer and Lemeshow goodness of fit (GOF) test is conducted and it gives a p -value of 0.50. Again the model is properly specified. Then $\hat{\theta}_1 = 1 - \sqrt{2}\hat{\sigma}\hat{\theta}'_1 = 1 - \sqrt{2} \times 0.73 \times 1.16 = -0.20$. For $i = 0, \hat{\theta}_0 = -\sqrt{2}\hat{\sigma}\hat{\theta}'_0 = -\sqrt{2} \times 0.73 \times 2.77 = -2.86$. Similar, $\hat{\theta}_2 = -1.72, \hat{\theta}_3 = 0.92$.

As the indifference band is a random variable depending on saved travel time, bridge usage history, and being worried about driving on the bridge, its mean and variance are computed as follows:

$$E \left[\epsilon^*(\Delta^{(n)}, U, Worry) \right] = \exp \left(\hat{\theta}_0 + \hat{\theta}_1 \log(\Delta^{(n)}) + \hat{\theta}_2 U_n + \hat{\theta}_3 Worry_n + \frac{\hat{\sigma}^2}{2} \right), \tag{3.11a}$$

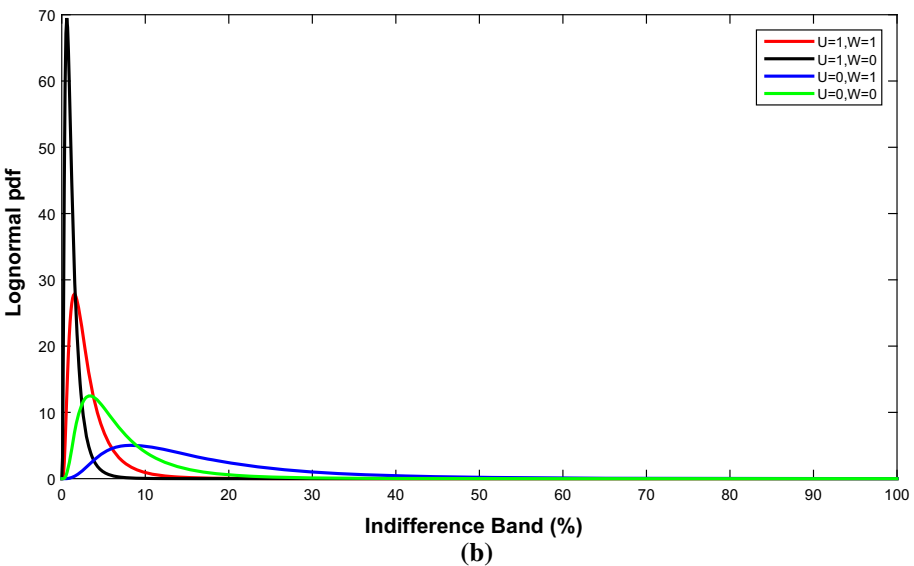
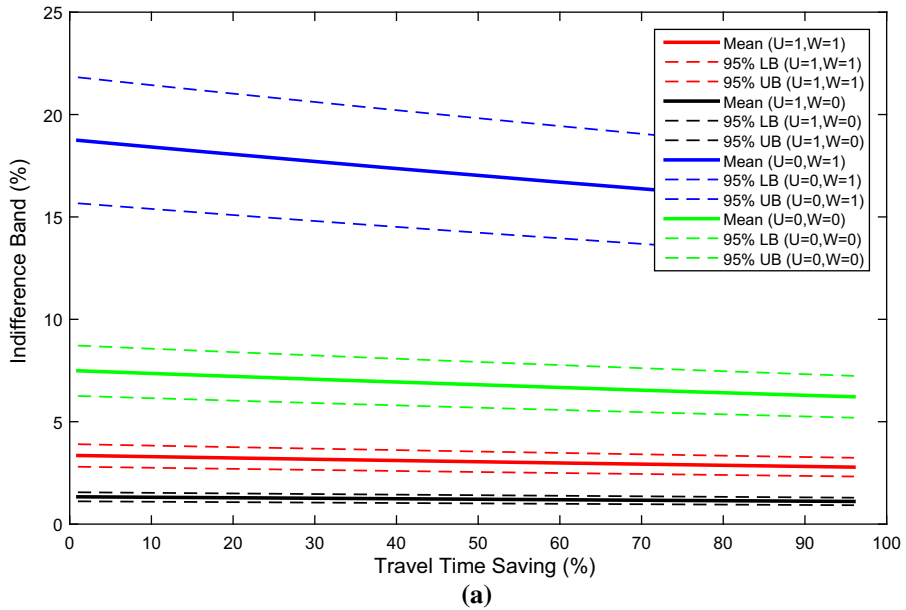


Fig. 7 Population indifference band. **a** Mean and confidence interval. **b** Lognormal distribution

$$Var[\epsilon^*(\Delta^{(n)}, U, Worry)] = (e^{\hat{\sigma}^2} - 1) \exp\left(2(\hat{\theta}_0 + \hat{\theta}_1 \Delta^{(n)}) + \hat{\theta}_2 U_n + \hat{\theta}_3 Worry_n\right) + \hat{\sigma}^2. \tag{3.11b}$$

Figure 7 plots the distribution of individual indifference bands.

In Fig. 7a, given a fixed travel time saving, the indifference band for drivers who used the bridge before and were not worried of driving on this bridge will have the lowest

indifference band (indicated in black), while those who never used the bridge before and were afraid of being on it have the highest indifference band (indicated in blue). For those who either never experienced this bridge or were not afraid of using it, their indifference band are in between. However, being afraid plays a more significant role than having the experience, therefore those who never experienced this bridge but were afraid of using it have higher indifference band than the other way around. As travel time saving increases, everybody's indifference band decreases, indicating that travel time saving on the new route is a critical contribution factor to the value of the indifference band. The value of the indifference band ranges from as high as 19 % to as low as 1 % approximately. The variance of the indifference band is higher when the mean is higher. Figure 7b displays similar patterns of probability density functions of indifference bands for each category given the travel time saving is fixed at 20 %.

Discussions

Indifference band values

In the existing literature, the values of indifference bands vary among different studies using distinct dataset. The absolute indifference band is 23 for mode switch (from car to high speed line between Lindenwold and Philadelphia) (Krishnan 1977) or 0.67 for inertial mean (Cantillo et al. 2006, 2007); the absolute indifference band for departure-time switching is 1 min (Mahmassani et al. 1999). In fractions, the relative indifference band ranges is 19 % for the pre-trip route decision and 18 % for the en-route decision (Mahmassani et al. 1999).

In this paper, we assume indifference band follows lognormal distribution and present two ways for parameter specification. The first model assumes everyone's indifference band is drawn from a population indifference band with mean of 6.56 % and standard deviation of 0.17 %, while the second one specifies each individual's indifference band depending on their bridge related experiences. These two specifications do not differ substantially except that the first one has mean as a constant while the second one has mean as a function of individual parameters. The variance of the indifference band in these two specifications remain the same. In other words, the second specification does not introduce any extra parameter regarding variability within individual drivers.

In the context of the collapse and reopening of the I-35W Bridge, people's bridge related experiences matter a lot to their route switching choices. Thus we recommend the second specification. However, we do not want to judge which model fits better and it really depends on the context and the purpose of a study. For example, if an urban network planner is mainly interested in predicting future traffic distribution considering boundedly rational behavior, then the first specification will be sufficient to estimate a population indifference band.

Note Limited by the sample size, the indifference band in this paper does not show any dependence on people's demographic information. However, readers should keep in mind that it can be individual-specific for different socio-demographic (e.g., age, gender, income) groups.

Indifference band applications

The indifference band plays a critical role in boundedly rational route choice models. Its introduction describes people's behavioral deviation from traditional perfectly rational model. The traffic assignment using boundedly rational assumption can exhibit significantly different results from the regular traffic assignment. Several studies have shown that transportation planning models with indifference bands manifest different properties in equilibrium traffic flow set (Di et al. 2013), Braess paradox occurrence (Di et al. 2014b), dynamic evolution of traffic flow distributions (Guo and Liu 2011; Di et al. 2015), and network design (Di et al. 2016). Therefore indifference band should be incorporated into four-stage transportation demand modeling for more realistic network performance. Interested readers can refer to Di et al. (2016) for a comprehensive review of boundedly rational route choice models and their impact on transportation planning.

Rather than simply inspecting its values, it will be more meaningful to discuss under which conditions an indifference band can be used. Due to inclusion of the extra parameter, i.e., indifference band, boundedly rational models are quite sensitive to relevant parameters. The misspecified models can result in even worse prediction than perfect rational models. Thus the calibration of indifference bands or cognitive processes is critical in determining the prediction accuracy of models. They can be individual-specific or the same across the entire population. Individual specific parameters require more data and more complicated models to estimate. By far there do not exist sufficient empirical studies on estimation processes due to lack of large amounts of individual route choice data. Therefore we should be cautious when using boundedly rational models for the policy-making purpose.

Even a well-specified boundedly rational model is calibrated by data collected from one area, a more critical question is, *whether such a model can easily be transferred to another area, context, or time period*. So far there exists only one study which touched upon the issue of transferability from a laboratory experiment to the real-world scenario in route choice study. By comparing commuter departure time and route choice switch behavior in laboratory experiments with field surveys in Dallas and Austin, Texas, Mahmassani and Jou (2000) showed that boundedly rational route choice modeling observed from experiments provided a valid description of actual commuter daily behavior. Such a claim is quite conservative and whether laboratory experimental experiences can truly represent actual commuter daily behavior still remains unclear. Overall, this parameter can be individual-specific or context-dependent and may not be able to transfer to a different population or a new context.

Contributions

People's response to changes in road networks will determine congestion levels. A network design or transportation planning problem with inaccurate travel behavior models will lead to traffic prediction deviating from reality and consequently wrong planning policies. Thus, understanding such behavioral change will facilitate optimal network design and network performance evaluation. People's boundedly rational route choice behavior provides a more realistic model of route choice behavioral change in response to changes in networks. This paper models boundedly rational route choice behavior after the I-35W Bridge was

rebuilt and provides implication for network design. Transportation engineers, network planners, and decision-makers will all benefit from this study.

Previous studies mainly focused on estimating indifference bands from laboratory experiment data. For example, Mahmassani and Chang (1987) estimated indifference bands by utilizing laboratory experiment data. By comparing commuter departure time and route choice switch behavior in laboratory experiments with field surveys in Dallas and Austin, Texas, Mahmassani and Jou (2000) showed that boundedly rational route choice modeling observed from experiments provided a valid description of actual commuter daily behavior. However, whether laboratory experimental experiences can represent actual commuter daily behavior still remains unclear. Other studies incorporated given values of the thresholds into their route choice models, such as 20 % of the mean travel time (Zhu et al. 2012), 0.5 times the mean travel time of certain previous trips (Carrion et al. 2012), or 10 % of shortest travel time (Guo and Liu 2011). These thresholds were obtained from experience or assumptions and served as inputs of specific route choice behavior models (Carrion et al. 2012; Yanmaz-Tuzel et al. 2009; Zhu et al. 2012). These thresholds were assumed or obtained using engineering judgment, which may not be valid and may provide misleading results when served as inputs of specific route choice behavior models. Our study offers empirical insights into bounded rationality parameters estimation from real-world GPS data and provide one explanation for the underutilization of the I-35W replacement bridge.

Limitations of this paper

Though this paper provides one approach of calibrating bounded rationality using GPS studies, it is subject to several limitations and shortcomings:

1. The number of drivers used for this study (i.e., 78) is relatively small. Accordingly, there are fewer observations for each category (especially for worry or no-worry) and it may affect the goodness-of-fit of the proposed models;
2. The consecutive longitudinal observations are missing, which forces us to use the static discrete choice approach to model route switching instead of day-to-day learning models. Accordingly, departure-time choice cannot be modeled.

To overcome these shortcomings, we propose the following remedies which will facilitate research along this direction in future:

1. A precautions experimental design is needed to collect effective and sufficient panel data. As people manifest heterogeneous behavior, a large sample size covering a wide range of socio-demographic characteristics for a continuous period will likely support more complicated models, such as day-to-day route switching or even latent variable models.
2. A comparative study across multiple cities or metropolitan areas should be conducted to make boundedly rational travel behavior more comparable across geographically different areas.

Though there exist several studies which utilize travel survey data to estimate bounded rationality parameters, they are mainly restricted to laboratory data. Nowadays, not only aggregated detector data at fixed locations, but also mobile sensor data from GPS or smartphones for individual travelers are available. With travel behavioral data from various sources in place, empirical verification of bounded rationality should continue and bounded

rationality parameters need to be estimated for various scales of regions. For example, New York taxi data can be one source to analyze taxi drivers' route choices before and after Hurricane Sandy in 2012.

Conclusions and future research directions

This paper proposes a boundedly rational route switching model to explain the observation that fewer commuters use the new I-35W bridge in Minneapolis. The boundedly rational route switching model assumes that commuters will not switch to the new bridge unless the time saving by taking the new bridge is higher than an indifference band. The route choice behavioral data collected from a GPS travel survey is used to estimate the indifference band parameter. Two specifications of indifference bands are proposed. The first one assumes everybody's indifference band is drawn from a population indifference band with lognormal distribution. The second one assumes individuals' indifference bands are a function of their bridge related experiences and demographic characteristics. Probit regression is used to estimate parameters. In face of a restored bridge, the estimation results show that travelers' route switching behavior depends on time saving by taking the new bridge, bridge usage history, and being worried for driving on the bridge. Demographic information is not significant in route switching.

This study provides the insight into the route choice behavior in the Minneapolis-St. Paul region and is the first empirical study using GPS field data to estimate bounded rationality parameter. However, it needs also to be generalized as follows. Travelers behave heterogeneously in route choice. Travel time, usage history, and fear may not be the only factors affecting route choices and other factors play important roles. To capture such heterogeneity, a larger-scale experiment with more samples are needed. Moreover, each driver's indifference band may also vary as time progresses, which may also depends on their departure-time. To capture drivers' day-to-day route switching behavior, repeated consecutive observations of each driver are needed. Then a hierarchical model with random effects will be more useful to capture their day-to-day variability. Last, the collapse and reopening of bridges are fortunately rare events and travelers' route switching behavior may depend on some factors such as fear which are not observed in other scenarios. To further study route changes in response to the change in road networks, road closures for construction may be good substitutes because they are more frequent and common. Route choice data should be collected from these events to extend the findings of this paper.

Acknowledgments The authors would like to thank the three anonymous referees for their insightful comments and constructive suggestions on the earlier version of the paper.

Appendix

See Table 6

Table 6 Descriptive statistics of pre-survey results

Variable	Level	Frequency	Percentage
Age	[0, 35)	13	17.8
	[35, 45)	15	20.5
	[45, 55)	28	38.4
	[55, 65)	15	20.5
	>65	2	2.7
Income	[0, 50,000)	17	23.3
	[50,000, 75,000)	20	27.4
	[75,000, 100,000)	18	24.7
	[100,000, 125,000)	13	17.8
	[125,000, 150,000)	2	2.7
Race	>150,000	3	4.1
	White	60	83.3
	Hispanic	2	2.8
	African	4	5.6
	Asian	5	6.9
Mode	Mixed	1	1.4
	Drive alone	68	91.9
Gender	Carpool	6	8.1
	Male	26	35.1
	Female	48	64.9

References

- Bekhor, S., Ben-Akiva, M.E., Ramming, M.S.: Evaluation of choice set generation algorithms for route choice models. *Ann. Oper. Res.* **144**(1), 235–247 (2006)
- Ben-Akiva, M., Boccara, B.: Discrete choice models with latent choice sets. *Int. J. Res. Mark.* **12**(1), 9–24 (1995)
- Ben-Akiva, M., Bolduc, D.: Multinomial probit with a logit kernel and a general parametric specification of the covariance structure. D'epartement d'economique, Universit'e laval with Department of Civil and Environmental Engineering, Massachusetts Institute of Technology (1996)
- Ben-Akiva, M., Ramming, S.: Lecture notes: discrete choice models of traveler behavior in networks. Prepared for Advanced Methods for Planning and Management of Transportation Networks, Capri, Italy, 25 (1998a)
- Cantillo, V., Heydecker, B., de Dios Ortúzar, J.: A discrete choice model incorporating thresholds for perception in attribute values. *Transp. Res. Part B* **40**(9), 807–825 (2006)
- Cantillo, V., Ortúzar, J.D.D., Williams, J.: Modeling discrete choices in the presence of inertia and serial correlation. *Transp. Sci.* **41**(2), 195–205 (2007)
- Carrion, C., Levinson, D. Route choice dynamics after a link restoration. In: Presented at Transportation Research Board Annual Meeting, Washington (2012)
- Cascetta, E.: A stochastic process approach to the analysis of temporal dynamics in transportation networks. *Transp. Res. Part B* **23**(1), 1–17 (1989)
- Cascetta, E., Nuzzolo, A., Russo, F., Vitetta, A.: A modified logit route choice model overcoming path overlapping problems: specification and some calibration results for interurban networks. In: Proceedings of the 13th international symposium on transportation and traffic theory. pp. 697–711 (1996)
- Clark, A.J.: The mode of action of drugs on cells (1933)
- Cox, C.: Threshold dose-response models in toxicology. *Biometrics* **43**, 511–523 (1987)
- Daganzo, C.F., Bouthelie, F., Sheffi, Y.: Multinomial probit and qualitative choice: a computationally efficient algorithm. *Trans. Sci.* **11**(4), 338–358 (1977)

- Danczyk, A., Di, X., Liu, H., Levinson, D.: Unexpected cause, unexpected effect: Empirical observations of twin cities traffic behavior after the I-35W Bridge collapse and reopening. *Transp. Policy* (2015) (submitted)
- Di, X., He, X., Guo, X., Liu, H.X.: Braess paradox under the boundedly rational user equilibria. *Transp. Res. Part B* **67**, 86–108 (2014b)
- Di, X., Liu, H.X.: Boundedly rational route choice behavior: a review of models and methodologies. *Transp. Res. Part B* **85**, 142–179 (2016)
- Di, X., Liu, H.X., Ban, X.J.: Second best toll pricing within the framework of bounded rationality. *Transp. Res. Part B* **83**, 74–90 (2016)
- Di, X., Liu, H.X., Pang, J.-S., Ban, X.J.: Boundedly rational user equilibria (BRUE): mathematical formulation and solution sets. *Transp. Res. Part B* **57**, 300–313 (2013)
- Di, X., Liu, H.X., Pang, B., Jeff, X., Yu, Jeong, Whon.: On the stability of a boundedly rational day-to-day dynamic. *Netw. Spat. Econ.* **15**(3), 537–557 (2015)
- Di, X.: Boundedly rational user equilibrium: models and applications. Ph.D. thesis, University of Minnesota (2014a)
- Guilford, J.P.: *Psychometric Methods* (1954)
- Guo, X., Liu, H.X.: Bounded rationality and irreversible network change. *Transp. Res. Part B* **45**(10), 1606–1618 (2011)
- Hemmingsen, A.M.: The accuracy of insulin assay on white mice. *Q. J. Pharm. Pharmacol.* **6**, 39–80 (1933)
- Hosmer, Jr., David, W., Lemeshow, S., Sturdivant, R.X., vol. 398. Wiley, New York
- Hu, T.Y., Mahmassani, H.S.: Day-to-day evolution of network flows under real-time information and reactive signal control. *Transp. Res. Part C* **5**(1), 51–69 (1997)
- Jan, O., Horowitz, A.J., Peng, Z.-R.: Using global positioning system data to understand variations in path choice. *Transp. Res. Rec.* **1725**(1), 37–44 (2000)
- Jayakrishnan, R., Mahmassani, H.S., Hu, T.Y.: An evaluation tool for advanced traffic information and management systems in urban networks. *Transp. Res. Part C* **2**(3), 129–147 (1994)
- Jha, M., Madanat, S., Peeta, S.: Perception updating and day-to-day travel choice dynamics in traffic networks with information provision. *Transp. Res. Part C* **6**(3), 189–212 (1998)
- Jotisankasa, A., Polak, J.W.: Framework for travel time learning and behavioral adaptation in route and departure time choice. *Transp. Res. Rec.* **1985**(1), 231–240 (2006)
- Kaplan, S., Shifftan, Y., Bekhor, S.: Development and estimation of a semi-compensatory model with a flexible error structure. *Transp. Res. Part B: Methodol.* **46**(2), 291–304 (2012)
- Kaplan, S., Shifftan, Y., Bekhor, S.: Development and estimation of a semi-compensatory model incorporating multinomial and ordered-response thresholds. pp. 11–15 (2010)
- Krishnan, K.S.: Incorporating thresholds of indifference in probabilistic choice models. *Manag. Sci.* **23**(11), 1224–1233 (1977)
- Levinson, D., Zhu, S.: The hierarchy of roads, the locality of traffic, and governance. *Transp. Policy* **19**(1), 147–154 (2012)
- Lioukas, S.K.: Thresholds and transitivity in stochastic consumer choice: a multinomial logit analysis. *Manag. Sci.* **30**(1), 110–122 (1984)
- Mahmassani, H.S., Chang, G.L.: On boundedly rational user equilibrium in transportation systems. *Transp. Sci.* **21**(2), 89–99 (1987)
- Mahmassani, H.S., Jayakrishnan, R.: System performance and user response under real-time information in a congested traffic corridor. *Transp. Res. Part A* **25**(5), 293–307 (1991)
- Mahmassani, H.S., Jou, R.C.: Transferring insights into commuter behavior dynamics from laboratory experiments to field surveys. *Transp. Res. Part A* **34**(4), 243–260 (2000)
- Mahmassani, H.S., Liu, Y.H.: Dynamics of commuting decision behaviour under advanced traveller information systems. *Transp. Res. Part C* **7**(2–3), 91–107 (1999)
- Martínez, F., Aguila, F., Hurtubia, R.: The constrained multinomial logit: a semi-compensatory choice model. *Transp. Res. Part B* **43**(3), 365–377 (2009)
- Morikawa, T., Miwa, T., Kurauchi, S., Yamamoto, T., Kobayashi, K.: Driver's route choice behavior and its implications on network simulation and traffic assignment. *Simulation Approaches in Transportation Analysis*, pp. 341–369 (2005)
- Prato, C., Bekhor, S.: Applying branch-and-bound technique to route choice set generation. *Transp. Res. Rec.* **1985**(1), 19–28 (2006)
- Ramming, M.S.: Network knowledge and route choice. Ph.D. thesis, Institute of Technology, MA (2001a)
- Rasouli, S., Timmermans, H.: Models of bounded rationality under certainty, p. 1. *Bounded Rational Choice Behaviour, Applications in Transport* (2015)
- Recker, W.W., Golob, T.F.: A non-compensatory model of transportation behavior based on sequential consideration of attributes. *Transp. Res. Part B* **13**(4), 269–280 (1979)

- Ridwan, M.: Fuzzy preference based traffic assignment problem. *Transp. Res. Part C* **12**(3), 209–233 (2004)
- Simon, H.A.: A behavioral model of rational choice. *Q. J. Econ.* **69**(1), 99–118 (1955)
- Simon, H.A.: *Models of man: social and rational; mathematical essays on rational human behavior in a social setting* (1957)
- Srinivasan, K.K., Mahmassani, H.S.: Role of congestion and information in trip-makers' dynamic decision processes: experimental investigation. *Transp. Res. Rec.* **1676**(1), 44–52 (1999)
- Swait, J.: A non-compensatory choice model incorporating attribute cutoffs. *Transp. Res. Part B* **35**(10), 903–928 (2001b)
- Swait, J., Ben-Akiva, M.: Incorporating random constraints in discrete models of choice set generation. *Transp. Res. Part B* **21**(2), 91–102 (1987)
- Tang, W., Levinson, D.: An empirical study of the deviation between actual and shortest travel time paths. In: Presented at the Transportation Research Board Conference, January 2016, Washington (2015)
- Vovsha, P., Bekhor, S.: Link-nested logit model of route choice: overcoming route overlapping problem. *Transp. Res. Rec.* **1645**(1), 133–142 (1998b)
- Yanmaz-Tuzel, O., Ozbay, K.: Modeling learning impacts on day-to-day travel choice. In: *Transportation and Traffic Theory*. pp. 387–401 (2009)
- Zhu, S.: Do people use the shortest path? an empirical test of wardrops first principle. *PLoS One* **10**(8), e0134322 (2015)
- Zhu, S., Levinson, D., Liu, H.X., Harder, K.: The traffic and behavioral effects of the I-35W Mississippi River bridge collapse. *Transp. Res. Part A* **44**(10), 771–784 (2010)
- Zhu, S.: The roads taken: theory and evidence on route choice in the wake of the I-35W Mississippi River bridge collapse and reconstruction. Ph.D. thesis, University of Minnesota (2011)
- Zhu, S., Levinson, D.M., Disruptions to transportation networks: a review. In: *Network Reliability in Practice*. pp. 5–20 (2012)

Xuan Di is a Postdoctoral Research Fellow at the University of Michigan Transportation Research Institute (UMTRI) and will join Columbia University in Fall 2016 as a tenure-track Assistant Professor. She received her Ph.D. degree from the Department of Civil, Environmental, and Geo-Engineering at the University of Minnesota, Twin Cities in 2014, and M.S. and B.S. degrees in Transportation Engineering from Tongji University of China in 2008 and 2005, respectively. Dr. Di's research interests include transportation systems optimization, shared mobility modeling with gametheoretic approach, and big data applications in urban mobility modeling. Dr. Di received a number of awards during her graduate study including the prestigious Matthew J. Huber Graduate Student Award from the University of Minnesota, Best Paper Award and Best Graduate Student Scholarship from North-Central Section ITE, Outstanding Presentation Award from INFORMS, and Outstanding Reviewer of Transportation Research Part C from Elsevier.

Henry X. Liu is currently a Professor at the Civil and Environmental Engineering Department of the University of Michigan, Ann Arbor. He is also a Research Professor at the University of Michigan Transportation Research Institute (UMTRI). Prior joining the University of Michigan, he was an Associate Professor of Civil Engineering at the University of Minnesota, Twin Cities. Dr. Liu received his Ph.D. degree in Civil and Environmental Engineering from the University of Wisconsin at Madison in 2000 and his Bachelor degree in Automotive Engineering from Tsinghua University (China) in 1993. His research interests are in the area of traffic network monitoring, modeling, and control, which includes traffic flow modeling and simulation, traffic signal operations, network traffic assignment, and mobility and safety applications with connected vehicles. Dr. Liu is an associate editor of Transportation Research Part C, the Journal of Intelligent Transportation Systems, Network and Spatial Economics, and Transportmetrica Part B. He is also on the editorial board of Transportation Research Part B and IET Journal of Intelligent Transportation Systems.

Shanjiang Zhu is an Assistant Professor of Transportation Planning and Engineering at George Mason University (GMU). He graduated from Tsinghua University with a B.S degree in 2003 and a M.S in 2005. During 2001–2003, he studied at the Ecole Centrale de Nantes, in France, as a dual-degree student. He obtained his Ph.D. degree at the University of Minnesota, Twin Cities, in 2010 and worked two years as a Research Scientist at the University of Maryland before joining GMU. Dr. Zhu is experienced in travel demand modeling, travel behavior analysis, GPS-based travel survey method, integrated transportation planning and simulation models, traffic incident management, and transportation economics. Dr. Zhu is a Co-PI of the TransInfo UTC that focuses on Big Data studies in transportation. He has also been funded by FHWA, VDOT and Virginia OPT3 office on a series of projects involving travel demand analysis on both

driving and non-motorized modes. He is Virginia Governor's appointee on the Technical Advisory Board of Northern Virginia Transportation Authority and is a fellow of GMU P3 policy center. Dr. Zhu is a champion of multi-disciplinary research approach. He holds three Master's degrees in Civil Engineering, Automatics, and Applied Economics, respectively. Dr. Zhu is the recipient of International Transport Forum's 2014 Young Research of the Year Award.

David M. Levinson was on the faculty of the Department of Civil, Environmental, and Geo- Engineering at the University of Minnesota when the paper was authored and is now at the University of Sydney. Levinson has authored or edited several books, including *The Transportation Experience*, *Planning for Place and Plexus*, and *The End of Traffic and the Future of Transport*, and numerous peer reviewed articles. He is the editor of the *Journal of Transport and Land Use*.