Spatial Regression Models for Demographic Analysis

Guangqing $Chi \cdot Jun$ Zhu

Published online: 27 September 2007 Springer Science+Business Media B.V. 2007

Abstract While spatial data analysis has received increasing attention in demographic studies, it remains a difficult subject to learn for practitioners due to its complexity and various unresolved issues. Here we give a practical guide to spatial demographic analysis, with a focus on the use of spatial regression models. We first summarize spatially explicit and implicit theories of population dynamics. We then describe basic concepts in exploratory spatial data analysis and spatial regression modeling through an illustration of population change in the 1990s at the minor civil division level in the state of Wisconsin. We also review spatial regression models including spatial lag models, spatial error models, and spatial autoregressive moving average models and use these models for analyzing the data example. We finally suggest opportunities and directions for future research on spatial demographic theories and practice.

Keywords Spatial regression \cdot Spatial data analysis \cdot Spatial weight matrix \cdot Spatial autocorrelation and heterogeneity \cdot Spatial demographic analysis

Introduction

Although spatial statistics has been applied to numerous fields in the last few decades, it has drawn demographers' attention only recently. While demography has

J. Zhu

J. Zhu

G. Chi (\boxtimes)

Social Science Research Center, Mississippi State University, P O Box 173, 41 Bulldog Circle, Mississippi State, MS 39762-0173, USA e-mail: gchi@ssrc.msstate.edu

Department of Statistics, University of Wisconsin-Madison, Madison, WI, USA

Department of Soil Science, University of Wisconsin-Madison, Madison, WI, USA

a rich body of methodologies, many current demographic studies lack a spatial perspective (Tiefelsdorf [2000\)](#page-25-0). Most existing sociological demographic models treat a geographical unit, such as a census tract, a small city, or a county, as an independent isolated entity rather than as an entity surrounded by other geographic units with which it may interact (e.g., through commuting and shopping patterns). Spatial effects in population dynamics have been theorized in several disciplines of social sciences such as geography and regional science, including the spatial diffusion theory, growth pole theory, central place theory, and new economic geography theory. Spatial effects in demographic dynamics, on the other hand, have been implicitly considered in various demographic and sociological theories, as well as empirical studies, in disciplines such as human ecology, urban sociology, and rural demography. For example, rural demographers are interested in the spatial dimension of population and their studies of ''turnaround migration'' and residential preference are often spatially oriented. However, neither rural demographers nor other sociological demographers have fully taken advantage of recent developments in spatial statistics and econometrics for data analysis in empirical studies. In particular, spatial effects are often not formally incorporated into population modeling in most demographic and sociological research. It is important to consider spatial effects in demographic modeling because from a methodological viewpoint, if spatial effects exist but are not accounted for in a model, estimation and statistical inference may be unreliable (e.g., effects of explanatory variables may be overstated or understated).

In the spatial statistics and spatial econometrics literature, spatial data analysis is often categorized into three types, namely point data analysis, lattice data analysis, and geostatistics, each of which has its own set of objectives and approaches (e.g., Cressie [1993;](#page-22-0) Schabenberger and Gotway [2005\)](#page-24-0). Very briefly, point data analysis concerns the spatial pattern of locations of events and is often aimed at determining or quantifying spatial patterns in the form of, for example, regularity or clustering as deviation from complete randomness. In contrast, lattice data analysis concerns the spatial pattern of an attribute on a regular or irregular spatial lattice, which is observed either at the grid points or aggregated over a grid cell. The objective there is usually to quantify the spatial pattern through a pre-specified neighborhood structure and examine relations between the attribute of interest and potential explanatory variables while accounting for any spatial effect. Furthermore, geostatistical data refer to spatial data sampled at point locations that are continuous in space. Geostatistics has objectives similar to lattice data analysis, with an additional goal of predicting values of the attribute at unsampled locations (Anselin [2002;](#page-21-0) Cressie [1993;](#page-22-0) Goodchild [1992\)](#page-23-0). A key difference that distinguishes geostatistics from lattice data analysis is that geostatistics uses distance-based functions rather than neighborhood structures to represent spatial autocorrelation. In addition, spatial interaction modeling is sometimes viewed as the fourth category of spatial data analysis and is aimed at quantifying the arrangement of flows and building models for the interactions occurring between origins and destinations (Bailey and Gatrell [1995\)](#page-21-0).

Lattice data analysis is currently the most used spatial data analysis approach in demography for various reasons. Aggregated data are one of the two types of data used in most demographic studies with the other being individual-based data. Spatial regression builds upon standard regression, the latter of which has been a popular statistical tool in demographic studies. Moreover, powerful and userfriendly computer software packages like SpaceStat and GeoDa have become readily available for practitioners. We note that point data analysis, geostatistics, and spatial interaction modeling are still useful for demographic studies. For example, geographers often use geostatistics for demographic studies (e.g., Cowen and Jensen [1998](#page-22-0); Jensen et al. [1994](#page-23-0); Langford et al. [1991;](#page-24-0) Langford and Unwin [1994;](#page-24-0) Mennis [2003](#page-24-0)), whereas point data analysis and spatial interaction models are suitable for epidemiological research and social network studies, respectively. However, here we restrict our attention to lattice data analysis.

The purpose of this article is to review spatial regression models and related statistical techniques for analyzing geographically referenced demographic data. We illustrate the ideas by an example of population change from 1990 to 2000 at the minor civil division (MCD) level in Wisconsin. In the sections to follow, we first briefly summarize spatially explicit and implicit theories of population dynamics in the disciplines of geography, regional science, human ecology, urban sociology, and demography. We then describe some of the basic concepts and related issues in spatial demographic analysis, including spatial autocorrelation and heterogeneity, spatial neighborhood structure, and the modifiable areal unit problem. We also outline the key steps in spatial regression analysis, with subsections describing standard linear regression, spatial linear regression, model evaluation, conditional autoregressive regression, and further extensions from spatial regression to spatialtemporal regression and spatial logistic regression. Finally in the discussion section, we suggest opportunities and directions for future research in spatial demographic theories and practice.

Spatial Demographic Theories

Spatial autocorrelation in population dynamics is suggested and considered implicitly in several demographic and sociological theories and empirical studies of human ecology, urban sociology, and rural demography, although spatial effects are not formally incorporated into their population modeling. Human ecology plays an important role in informing sociologists of spatial distribution of population (Berry and Kasarda [1977](#page-21-0); Frisbie and Kasarda [1988](#page-23-0)).¹ McKenzie ([1924\)](#page-24-0) defines human ecology as the study of the spatial-temporal relations of human beings affected by the environment. Hawley [\(1950](#page-23-0)) views spatial differentiation within urban systems as one of the main topics of human ecology, whereas Robinson [\(1950](#page-24-0)) considers human ecology to be studies using spatial information rather than individual units. Logan and Molotch [\(1987](#page-24-0)) see spatial relations as the analytical basis for understanding urban systems in human ecology.

¹ However, some human ecologists (e.g., Poston and Frisbie [2005\)](#page-24-0) see these definitions as misunderstanding human ecology.

Studies of segregation, which have been one of the largest bodies of urban sociological research, suggest spatial effects in population distribution (Charles [2003;](#page-22-0) Fossett [2005\)](#page-22-0). There are various theoretical approaches to explaining segregation. The spatial assimilation approach claims that segregation is caused by differences in socioeconomic status and the associated differences in lifestyle (Clark [1996;](#page-22-0) Galster [1988\)](#page-23-0). The place stratification approach states that segregation is caused by discrimination (Alba and Logan [1993;](#page-21-0) Massey and Denton [1993](#page-24-0)), while the suburbanization explanation argues that the process of suburbanization leads to segregation (Parisi et al. [2007\)](#page-24-0).

Neo-Marxists study the spatial dimension of population dynamics mainly by focusing on population redistribution. They see the structure of cities, land use, and population change as the result of capitalism in pursuit of profit (Hall [1988](#page-23-0); Jaret [1983\)](#page-23-0) and argue that capital accumulation is the basis of urban development in the U.S. (Gordon [1978;](#page-23-0) Hill [1977](#page-23-0); Mollenkopf [1978,](#page-24-0) [1981\)](#page-24-0). "Since the process of capital accumulation unfolds in a spatially structured environment, urbanism may be viewed provisionally as the particular geographical form and spatial patterning of relationships taken by the process of capital accumulation'' (Hill [1977,](#page-23-0) p. 41).

Rural and applied demographers are also interested in the spatial dimension of population and conduct research on migration, population distribution, and population estimation and forecasting (Voss et al. [2006\)](#page-25-0). Rural demographers study population redistribution through residential preferences and find that migrants prefer locations somewhat rural or truly ''sub''-urban within commutable distance of large cities (Brown et al. [1997;](#page-22-0) Fuguitt and Brown [1990;](#page-23-0) Fuguitt and Zuiches [1975;](#page-23-0) Zuiches and Rieger [1978](#page-25-0)). They also attribute the post-1970 ''turnaround migration'' in part to the attraction of natural amenities in rural regions (Brown et al. [1997;](#page-22-0) Fuguitt and Brown [1990](#page-23-0); Fuguitt et al. [1989](#page-23-0); Fuguitt and Zuiches [1975](#page-23-0); Humphrey [1980](#page-23-0); Johnson [1982,](#page-23-0) [1989](#page-23-0); Johnson and Beale [1994;](#page-23-0) Johnson and Purdy [1980;](#page-23-0) Zuiches and Rieger [1978](#page-25-0)). Applied demographers often use the information of neighbors in small-area population estimation and forecasting. For instance, the populations projected by extrapolation at the municipal level are often adjusted to agree with their sum to their parent county population projections. However, this neighborhood context is different from the spatial population effects that we will address in this article.

Spatial distribution and differentiation of population have long been studied by researchers in other disciplines such as regional science, population geography, and environmental planning. These fields have well-established theories and methodologies for spatial demographic analysis, which can be adopted by demographers and sociologists. Regional economists are good at explaining and modeling the change of land use patterns, which are almost always associated with population change (Boarnet [1997,](#page-22-0) [1998](#page-22-0); Cervero [2002,](#page-22-0) [2003](#page-22-0); Cervero and Hansen [2002](#page-22-0)). For example, the growth pole theory applies the notion of spread and backwash to explain mutual geographic dependence of economic growth and development, which in turn leads to population change (Perroux [1955\)](#page-24-0). The central place theory places population in a hierarchy of urban places where the movement of population, firms, and goods is determined by the associated costs and city sizes (Christaller [1966\)](#page-22-0). More recently, Krugman ([1991\)](#page-23-0) adds space to the endogenous growth in the ''new'' economic geography theory and studies the formation process of city network over time.

Population geographers are interested in spatial variation of population distribution, growth, composition, and migration, and seek to explain population patterns caused by spatial regularities and processes (Beaujeu-Garnier [1966](#page-21-0); James [1954;](#page-23-0) Jones [1990;](#page-23-0) Trewartha [1953;](#page-25-0) Zelinsky [1966](#page-25-0)). Spatial diffusion theory argues that population growth tends to spread to surrounding areas (Hudson [1972\)](#page-23-0), which implies that population growth is spatially autocorrelated.

Environmental planners focus on how the physical environment and socioeconomic conditions encourage or discourage land use change, which in turn leads to population change. The approach is generally empirical, by typically using geographic information system (GIS) overlay methods such as those supported through the ModelBuilder function $(^{R}ESRI)$ in ArcGIS to answer "what-if" questions. Similar work includes developable lands (Cowen and Jensen [1998\)](#page-22-0), qualitative environmental corridors (Lewis [1996](#page-24-0)), quantitative environmental corridors (Cardille et al. [2001\)](#page-22-0), and growth management factors (Land Information and Computer Graphics Facility [2000,](#page-24-0) [2002](#page-24-0)).

Put short, some demographic and sociological theories and empirical studies implicitly suggest spatial process in population dynamics, which, however, has not been stated as explicitly as in regional science, geography, and environmental planning.

Exploratory Spatial Data Analysis

Regression analysis often begins with exploratory data analysis, $²$ the importance of</sup> which should not be overlooked. Exploratory spatial data analysis (ESDA) is an additional crucial step in spatial regression modeling, focusing on the spatial feature of data. ESDA often involves visualizing spatial patterns exhibited in the data, identifying spatial clusters and spatial outliers, and diagnosing possible misspecification of spatial aspects of the statistical models, all of which can help better specify regression models (Anselin [1996;](#page-21-0) Baller et al. [2001](#page-21-0)). In the following we discuss basic concepts and related issues in the context of ESDA. In particular, we review spatial autocorrelation, spatial heterogeneity, spatial weight matrix based on spatial neighborhood structures, and discuss the modifiable areal unit problem. These concepts and issues are essential in spatial regression modeling.

² Exploratory data analysis summarizes and displays data without formal statistical inference. For the purpose of regression, it is common practice to examine the distributions of the response variable and the explanatory variables as well as the correlation among all the variables. Expectations may include normal distributions of the variables, a linear relation between the response variable and individual explanatory variables, and a reasonably low correlation among the explanatory variables. If the data do not appear to follow normal distributions or the relations among the variables are not linear, we could consider transforming the variables. However, the transformation may not reduce spatial dependence if it exists (Bailey and Gatrell [1995](#page-21-0)). Alternatively additional variables such as higher-order terms and interaction terms can be incorporated (Fox [1997\)](#page-23-0). In addition, a high correlation among the explanatory variables may make estimation and statistical inference unreliable, which is known as the problem of multicollinearity (Baller et al. [2001\)](#page-21-0). Principal component or factor analysis may be used to create new explanatory variables from the highly correlated explanatory variables.

Spatial Autocorrelation

Spatial autocorrelation³ (also known as spatial dependence, spatial interaction, or local interaction) can be loosely defined as a similarity (or dissimilarity) measure between two values of an attribute that are nearby spatially. In other words, with positive spatial autocorrelation, high or low values of an attribute tend to cluster in space whereas with negative spatial autocorrelation, locations tend to be surrounded by neighbors with very different values. Spatial autocorrelation can be measured by various indexes, of which probably the most well-known is Moran's I statistic (Moran [1948\)](#page-24-0). Moran's I statistic measures the degree of linear association between an attribute (y) at a given location and the weighted average of the attribute at its neighboring locations (Wy), and can be interpreted as the slope of the regression of (y) on (Wy) (Pacheco and Tyrrell [2002\)](#page-24-0). Spatial autocorrelation can be visually illustrated in a Moran scatter plot, in which (Wy) on the vertical axis is plotted against (y) on the horizontal axis (Anselin [1995](#page-21-0)).

Statistics like Moran's I describe spatial autocorrelation in the data across the entire study area and are often viewed as a global diagnostics tool. While useful for analyzing data sets in a relatively homogeneous region, it may not be as informative to compute the Moran's I value for data across a region that could have several spatial regimes (Anselin [1996](#page-21-0)). For example, a Moran scatter plot may show a mix of two types of spatial autocorrelation (e.g., positive and negative spatial autocorrelation), which indicates the presence of different spatial regimes and thus local instability. In this case, the global indicator of spatial autocorrelation may be too crude a measure of the actual spatial autocorrelation (Anselin [1996\)](#page-21-0). One solution is to develop a set of local indicators of spatial association (LISA), such as local Moran's I (Anselin [1995](#page-21-0); Cliff and Ord [1973](#page-22-0), [1981](#page-22-0)), G and G^{*} statistics (Ord and Getis [1995](#page-24-0)), and K statistic (Getis [1984](#page-23-0); Ord and Getis [1995\)](#page-24-0). LISA can be used to assess assumptions of spatial homogeneity, determine the distance beyond which there is no more discernable spatial autocorrelation, allow for a decomposition of a global measure into contributions from individual observations, and identify outliners or different spatial regimes.

Spatial Heterogeneity

Spatial heterogeneity (also known as spatial structure, nonstationarity, or large-scale global trends of the data) refers to differences in the mean, and/or variance, and/or covariance structures including spatial autocorrelation within a spatial region (LeSage [1999](#page-24-0)). In contrast, spatial homogeneity (also known as stationarity)

³ Apparently scholars from different fields understand these terms differently. For example, some demographers distinguish spatial autocorrelation from spatial dependence, and argue that the former simply is one indicator of the latter and, possibly, of spatial heterogeneity. Geographers view spatial autocorrelation as being composed of large-scale spatial irregularities and local-scale spatial interaction effects. Here we use the terms of spatial autocorrelation and spatial dependence as synonymous, explain the conceptual difference between spatial autocorrelation and spatial heterogeneity, and focus on spatial autocorrelation in the data analysis.

requires that the mean and the variance of an attribute be constant across space, and that spatial autocorrelation of the attribute at any two locations depends on the lag distance between the two locations, but not the actual locations (Bailey and Gatrell [1995\)](#page-21-0).

While spatial autocorrelation is in line with the First Law of Geography (Tobler [1970\)](#page-25-0), spatial heterogeneity is related to spatial differentiation (Anselin [1996\)](#page-21-0). However, it is not always easy to distinguish between spatial autocorrelation and spatial heterogeneity (Bailey and Gatrell [1995;](#page-21-0) Graaff et al. [2001](#page-23-0)). For instance, clustering may induce spatial autocorrelation among neighbors but may also signal the possibility of different spatial regimes (Anselin [2001](#page-21-0)). Also, tests for determining spatial autocorrelation or heteroscedasticity (i.e., unequal variance) may yield inconclusive results. For example, Anselin [\(1990](#page-21-0)) and Anselin and Griffith [\(1988](#page-21-0)) found that tests for spatial autocorrelation can detect heteroscedasticity and conversely, tests for heteroscedasticity may also signal spatial autocorrelation.

Neighborhood Structure and Spatial Weight Matrix

To account for spatial autocorrelation in lattice data analysis, it is necessary to establish a neighborhood structure for each location by specifying those locations on the lattice that are considered as its neighbors (Anselin [1988\)](#page-21-0). In particular, we need to specify a spatial weight matrix corresponding to the neighborhood structure such that the resulting variance-covariance matrix can be expressed as a function of a small number of estimable parameters relative to the sample size (Anselin [2002\)](#page-21-0). Popular spatial weight matrices in spatial econometrics include the so-called "rook's case" and "queen's case" contiguity weight matrices of order one or higher, the k-nearest neighbor weight matrices, the general distance weight matrices, and the inverse distance weight matrices with different powers, the latter three of which are distance-based (Anselin 1992).⁴ More complex spatial weight matrices can be created based on additional theory and assumptions, such as those based on economic distance (Case et al. [1993\)](#page-22-0). While a spatial weight matrix is necessary for lattice data analysis, there is little theory guiding the selection of neighborhood structure in practice. Often a spatial weight matrix is defined exogenously and comparison of several spatial weight matrices is performed before selecting a defensible one (Anselin [2002\)](#page-21-0). For example, we can create and compare several spatial weight matrices, and select the one that achieves a high coefficient of

⁴ The first-order queen contiguity spatial weight matrix defines all observations that share common boundaries or vertices as neighbors. The first-order rook contiguity spatial weight matrix defines the observations that share common boundaries as neighbors. The second-order queen and rook contiguity weight matrices see both the first-order neighbors and their neighbors as neighbors. The k-nearest neighbor weights are constructed to contain the k nearest neighbors for each observation. In the distance weight matrices, all observations that have centroids within the defined distance band from each other are categorized as neighbors. The general weight matrices see all neighbors as equally weighted, and the inverse distance weight matrices assume continuous change of interaction between two observations with distance (e.g., a squared inverse distance spatial weight matrix can be constructed for the gravity model of spatial interaction).

spatial autocorrelation along with a high level of statistical significance (Voss and Chi [2006\)](#page-25-0), although this procedure has little theoretical backing.

There are two potential problems associated with the specification of spatial weights in practice (Anselin [2002\)](#page-21-0). One problem is that the weights structures can be affected by the topological quality of GIS data. For instance, imprecision in the storage of polygons and vertices in GIS may mistakenly yield islands (i.e., locations without any neighbor) or other connection structures. The other problem is that the use of some of the distance-based spatial weight matrix requires a threshold value, which may be difficult to determine especially when there is strong spatial heterogeneity. A small threshold value may yield too many islands, while a large threshold value may yield too large a neighborhood. This is especially the case with census units, because census units at sub-county levels are often delineated according to population size, which makes them irregular. The use of distance-based spatial weight matrix tends to make too many neighbors in urban areas and too few neighbors in rural areas. One solution to this problem is the k-nearest neighbor structure (Anselin [2002](#page-21-0)), which appears to be supported by our data example below. It will be noted that the 4-nearest neighbor weight matrix provides the highest spatial autocorrelation of population change and the 5-nearest neighbor weight matrix offers the highest spatial autocorrelation of the residuals of the standard regression, among forty different types of spatial weight matrices.

The Modifiable Areal Unit Problem

The modifiable areal unit problem (MAUP) occurs when the results of statistical analysis are highly influenced by the scale as well as the shape of the aggregation. The former is known as the scale effect and the latter, the zoning effect (Fotheringham and Wong [1991;](#page-22-0) Langford and Unwin [1994;](#page-24-0) Openshaw [1984;](#page-24-0) Openshaw and Taylor [1981\)](#page-24-0). More specifically, the scale effect refers to the fact that, when the same data are aggregated at different scales, results of statistical analysis are disparate over scales. In analysis of census data, the possible aggregation scales are by state, county, township, blocks, etc. For instance, a seemingly clustered spatial pattern of an attribute may be present at one scale (say, county-level) but not at another scale (say, block-level). Or relations between two attributes at one scale may not hold at another scale. The MAUP is closely related to the notion of ecological fallacy in that relations between attributes at aggregated levels may not hold at the individual level (Green and Flowerdew [1996](#page-23-0); Robinson [1950;](#page-24-0) Wrigley et al. [1996\)](#page-25-0).

The zoning effect, on the other hand, refers to the fact that, when data are grouped in different manners within the same scale, statistical analysis gives different results. Adjustment of boundary changes at a small-area scale (e.g., MCD) often results in the zoning effect (Tolnay et al. [1996;](#page-25-0) Voss and Chi [2006](#page-25-0)), as different approaches for adjustments might change the data analysis results dramatically. The data example below is aggregated at the MCD level, which consists of nonnested, mutually exclusive and exhaustive political territories. An alternative would be census tracts. In many states of the U.S., census tracts have

Fig. 1 The response variable and residuals of the standard regression

sizes similar to MCDs but are geographic units delineated by the Census Bureau for the purpose of counting population. Obviously, both the scale effect and the zoning effect are important issues in spatial data analysis, especially lattice data analysis (Paelinck [2000](#page-24-0)).

Data Example

Here we use an actual data set to illustrate exploratory spatial data analysis (this section) and spatial regression modeling (next section). The study concerns the effect of highway expansion from 1980 to 1990 (the explanatory variable) on population change from 1990 to 2000 (the response variable; see the left panel of Fig. 1) at the MCD level⁵ in Wisconsin. A review of the literature reveals that the findings on the effects of highway construction on population change are sometimes conflicting or equivocal, partly due to the different approaches taken for data analysis (Chi et al. [2006](#page-22-0)). While some studies account for spatial autocorrelation, others ignore it in the statistical models. These studies are also conducted at quite different geographic scales ranging from communities, municipalities, counties, to regions, and thus highway effects on population change may vary greatly at different scales as a result of the MAUP. We argue that the MCD is an appropriate scale to match the population-highway dynamics, because traffic analysis and

 $⁵$ The boundaries, and even the names, of MCDs in Wisconsin are not fixed over time. Boundaries</sup> change, new MCDs emerge, old MCDs disappear, names change, and status in the geographic hierarchy shifts, e.g., towns become villages, villages become cities. In order to adjust the data for these changes, we have set up three rules: new MCDs must be merged into the original MCDs from which they emerge; disappearing MCD problems can be solved by dissolving the original MCDs into their current ''home'' MCDs; and occasionally, several distinct MCDs must be dissolved into one super-MCD in order to establish a consistent data set over time. In the end, 1,837 MCD-like units (cities, villages, and towns) constitute this analytical dataset.

forecast at the city, village, and town level is often necessary to transportation planning.

In this section, we examine spatial autocorrelation of population change, which is the response variable. In the next section, we examine the residuals after fitting a regression model to assess how the incorporation of spatial autocorrelation in the regression model affects the statistical analysis. A visual examination of the left panel of Fig. [1](#page-8-0) suggests that spatial autocorrelation of population change is plausible and that the assumption of independent errors may not be appropriate in a standard linear regression. But, at this point, we do not know whether the spatial autocorrelation is statistically significant, and even if it is significant, we do not know whether it can be ''explained away'' by the spatial autocorrelation in the explanatory variables.

In order to explore spatial autocorrelation in the data, we have considered and tested the significance of Moran's I using forty different spatial weight matrices. The spatial weight matrices include the general distance weight matrices and the inverse-distance weight matrices with power 1 or power 2, from 0 to 100 miles at 10-mile increments based on the distance between the centroids of MCD, the rook's case and queen's case contiguity weight matrices with order 1 and order 2, and the k-nearest neighbor weights matrices with k ranging from 3 to 8 neighbors (Table 1). The magnitudes of the Moran's I statistics are not particularly high, with the 4-nearest neighbor weight matrix having the highest value (0.2136) . A z-score,

Spatial weight matrix	Moran's I (p -value)		
	Population change 1990-2000	OLS residuals	Spatial lag model residuals
Inverse distance WM, 10 miles, power 1	$0.1844***$	$0.0973***$	-0.0262
Inverse distance WM, 10 miles, power 2	$0.1772***$	$0.0846***$	$-0.0487**$
Inverse distance WM, 20 miles, power 1	$0.1361***$	$0.0740***$	0.0029
Inverse distance WM, 20 miles, power 2	$0.1491***$	$0.0726***$	$-0.0278*$
Inverse distance WM, 30 miles, power 1	$0.1073***$	$0.0557***$	0.0033
Inverse distance WM, 30 miles, power 2	$0.1339***$	$0.0617***$	$-0.0268*$
Inverse distance WM, 40 miles, power 1	$0.0868***$	$0.0448***$	0.0037
Inverse distance WM, 40 miles, power 2	$0.1236***$	$0.0555***$	$-0.0259*$
Inverse distance WM, 50 miles, power 1	$0.0735***$	$0.0372***$	0.0036
Inverse distance WM, 50 miles, power 2	$0.1171***$	$0.0513***$	$-0.0253*$
Inverse distance WM, 60 miles, power 1	$0.0639***$	$0.0309***$	0.0025
Inverse distance WM, 60 miles, power 2	$0.1122***$	$0.0481***$	$-0.0252*$
Inverse distance WM, 70 miles, power 1	$0.0564***$	$0.0272***$	0.0022
Inverse distance WM, 70 miles, power 2	$0.1085***$	$0.0459***$	$-0.0251*$
Inverse distance WM, 80 miles, power 1	$0.0503***$	$0.0237***$	0.0014
Inverse distance WM, 80 miles, power 2	$0.1055***$	$0.0440***$	$-0.0251**$
Inverse distance WM, 90 miles, power 1	$0.0452***$	$0.0212***$	0.0011

Table 1 Spatial autocorrelation of the response variable and the residuals

Notes: * Significant at $p \le 0.05$ for a two-tail test

** Significant at $p \le 0.01$ for a two-tail test

*** Significant at $p \le 0.001$ for a two-tail test

which is the test statistic for the significance of the Moran's I statistic, is computed as the ratio of Moran's I and the corresponding standard error. The p -values are computed using a normal approximation and, as shown in the first panel of Table [1,](#page-9-0) are all less than 0.001 indicating that there is strong evidence of spatial autocorrelation in population change across the MCDs of Wisconsin in the 1990s based on all the forty spatial weight matrices.

The Moran scatter plot in Fig. [2](#page-11-0) illustrates population growth rate from 1990 to 2000 for each MCD $(x\text{-axis})$ in relation to the average population growth rate of each MCD's neighbors weighted by the 4-nearest neighbor weight matrix (y-axis). The upper-right quadrant of the scatter plot corresponds to MCDs with population growth that are surrounded by MCDs with population growth. There are far fewer MCDs with population decline surrounded by MCDs with population decline

Fig. 2 Moran scatter plot of population growth rate from 1990 to 2000

(lower-left quadrant), MCDs with population growth surrounded by MCDs with population decline (lower-right quadrant), and MCDs with population decline surrounded by MCDs with population growth (upper-left quadrant). The slope of the regression line in the Moran scatterplot is 0.2136, which indicates considerable spatial autocorrelation of population growth rate from 1990 to 2000.

Figure [3](#page-12-0) displays spatial autocorrelation of population change by the combinations of high–high (i.e., high growth MCDs surrounded by high growth MCDs), low–low (i.e., low growth MCDs surrounded by low growth MCDs), low–high (i.e., low growth MCDs surrounded by high growth MCDs), and high–low (i.e., high growth MCDs surrounded by low growth MCDs), showing only those MCDs where the local Moran statistic is significant at the 0.05 level based on a randomization procedure. High–high MCDs are mainly in lake-rich areas and suburbs of major cities (such as Madison, Milwaukee, Green Bay, and Appleton), whereas low–low MCDs are mainly in the southwest and north central Wisconsin, as well as the area between La Crosse and the Minneapolis-St. Paul metropolitan. Most of the low– high MCDs are neighbors of the hot-spot (growth–growth) MCDs, whereas high– low MCDs are neighbors of cold-spot (decline–decline) MCDs and scatter throughout north central Wisconsin.

Regression Analysis

After completing the ESDA, we now specify regression models to examine highway effects on population change. We describe standard linear regression and spatial

Fig. 3 Spatial autocorrelation of population growth rate from 1990 to 2000

linear regression including a spatial lag model, a spatial error model, and a spatial autoregressive moving average model. We then fit these models to the data example for illustration. We also discuss briefly model evaluation, conditional autoregressive regression, and further extensions from spatial regression to spatial-temporal regression and spatial logistic regression.

Standard Linear Regression

A standard linear regression model assumes that the error terms are independently, identically, and normally distributed (e.g., Draper and Smith [1998;](#page-22-0) Fox [1997;](#page-23-0) Greene [2000\)](#page-23-0). After fitting a standard linear regression model, model diagnostics are usually carried out to see whether these model assumptions are satisfied at least approximately. Model diagnostics are often based on residuals, which are the difference between the observed and fitted response variable. Certain distinct patterns in the residuals may suggest violation of model assumptions and indicate nonlinear relations, unequal variances, and/or nonnormality. If any model assumption is violated, standard linear regression may not be appropriate or adequate and the subsequent statistical inference of model parameters may not be reliable.

Here we are particularly interested in the independence assumption of the errors, which is often violated in demographic studies due to spatial autocorrelation. Possible diagnostic tools are Moran's I plot, 6 contour plot, and Moran's I statistic of the residuals (Baller et al. [2001;](#page-21-0) Loftin and Ward [1983](#page-24-0)). If there is spatial autocorrelation in the errors, then standard linear regression fitting may yield unreliable results. For example, standard errors of the regression coefficient estimates tend to be underestimated or overestimated, giving rise to significant or insignificant relations that may be otherwise (Baller et al. [2001](#page-21-0); Doreian [1980;](#page-22-0) Loftin and Ward [1983\)](#page-24-0).

Spatial Linear Regression

Spatial linear regression models may be viewed as generalization of standard linear regression models such that spatial autocorrelation is allowed and accounted for explicitly by spatial models. The model parameters include the usual regression coefficients of the explanatory variables (β) and the variance of the error term (σ^2). In addition, the most commonly used spatial regression models have a spatial autoregressive coefficient (ρ) , which measures the strength of spatial autocorrelation. A spatial weight matrix (W) corresponding to a neighborhood structure and a variance weight matrix (D) are pre-specified. More complicated spatial models are possible, but we restrict our attention to the spatial models with at most two spatial autoregressive coefficients. Spatial linear regression models are usually fitted by maximum likelihood (or equivalently generalized least squares for β) (Anselin [1988\)](#page-21-0). Other approaches are possible including spatial filtering (Getis [1995](#page-23-0); Griffith [2003\)](#page-23-0) and geographically weighted regression focusing on the specification of spatial heterogeneity (Fotheringham et al. [1998\)](#page-23-0).

Spatial Lag Model, Spatial Error Model, and Spatial Autoregressive Moving Average Model

Now, we consider specific spatial linear regression models that vary in the way spatial models for the error terms are specified. Two popular spatial regression models are the spatial error model and the spatial lag model. A spatial lag model is specified as:

⁶ The Moran's I plot of errors can also detect if there are any outliers. Outliers are not necessarily ''bad,'' and further exploration of the outliers might provide interesting findings. Practically, we can use the outliers as one independent variable where the outliers are represented as 1 while others as 0. If these outliers are ''real'' outliers, the coefficient should be statistically significant. In the spatial data analysis, outliers detected by Moran scatter plot may indicate possible problems with the specification of the spatial weights matrix or with the spatial scale at which the observations are recorded (Anselin [1996\)](#page-21-0). Outliers should be studied carefully before being discarded.

$$
Y = X\beta + \rho WY + \varepsilon,\tag{1}
$$

where Y denotes the vector of response variables, X denotes the matrix of explanatory variables, W denotes the spatial weight matrix, and ε denotes the vector of error terms that are independent but not necessarily identically distributed. In contrast, a spatial error model is specified as:

$$
Y = X\beta + u, \ u = \rho Wu + \varepsilon,\tag{2}
$$

where the terms are defined in the same way as the spatial lag model.

For spatial lag models, spatial autocorrelation is modeled by a linear relation between the response variable (y) and the associated spatially lagged variable (Wy) , but for spatial error models, spatial autocorrelation is modeled by an error term (u) and the associated spatially lagged error term (Wu) (Anselin and Bera [1998](#page-21-0)). In either case, interpretation of a significant spatial autoregressive coefficient is not always straightforward. A significant spatial lag term may indicate strong spatial dependence, but may also indicate a mismatch of spatial scales between the phenomenon under study and at which it is measured as a result of the MAUP. A significant spatial error term indicates spatial autocorrelation in errors, which may be due to key explanatory variables that are not included in the model.

However, the taxonomy of spatial lag model and spatial error model may be overly simplistic and may exclude other possible spatial autocorrelation mechanisms, such as the existence of both lag and error autocorrelations (Anselin [1988,](#page-21-0) [2003\)](#page-21-0). A spatial autoregressive moving average (SARMA) model can be constructed to include both the spatial lag and spatial error models. A simple version of the SARMA specification combines a first-order spatial lag model with a first-order spatial error model and can be expressed as a combination of (1) and (2):

$$
Y = \rho_1 W_1 Y + X\beta + u, \ u = \rho_2 W_2 u + \varepsilon. \tag{3}
$$

With some algebraic manipulation, (3) can be rewritten as:

$$
Y = \rho_1 W_1 Y + \rho_2 W_2 Y - \rho_1 \rho_2 W_2 W_1 Y + X\beta - \rho_2 W_2 X\theta + \varepsilon. \tag{4}
$$

In practice, we can treat a spatially weighted response variable as an additional explanatory variable when fitting model (4). A test for lack of spatial autocorrelation can be conducted, although the test does not provide any specific guidance regarding the exact nature of autocorrelation when the null hypothesis is rejected (Anselin and Bera [1998\)](#page-21-0).

Model Evaluation

For model assessment and comparison, there are at least two approaches addressed in current literature. One is a data-driven approach, which tests for lack of spatial error autocorrelation after fitting a spatial lag model, and then tests for lack of spatial lag autocorrelation after fitting a spatial error model. In a study examining population growth, Voss and Chi [\(2006\)](#page-22-0) find that the data-driven approach can help

determine which specification is the better model for accounting for the spatial autocorrelation. The other is a theory-based approach, which suggests that the choice between the spatial lag model and the spatial error model should be based on substantive grounds (Doreian [1980\)](#page-22-0). While both approaches are used in spatial regression, the data-driven method is often preferred, because often it is the data rather than formal theoretical concerns that motivate spatial data analysis (Anselin [2002,](#page-21-0) [2003\)](#page-21-0).

For a given data set, various linear regression models can be specified. Likelihood ratio tests (LRT) can be performed to compare models that are nested (i.e., one simpler model can be reduced from the other more complex model by constraining certain parameters in the complex model). If two models are not nested, Akaike's Information Criterion (AIC) and Schwartz's Bayesian Information Criterion (BIC) are often used, which measure the fit of the model to the data but penalize models that are overly complex. Models having a smaller AIC or a smaller BIC are considered the better models in the sense of model fitting balanced with model parsimony. In addition, several other tests are useful for model diagnostics. For example, heteroscedasticity can be detected by the Breusch-Pagan test and the spatial Breusch-Pagan test, while goodness-of-fit (GOF) of the models can be detected by the Lagrange Multiplier (LM) test. A test for nonlinearity appears to be useful for detecting anisotropy and nonstationarity (Bailey and Gatrell [1995;](#page-21-0) Graaff et al. [2001\)](#page-23-0).

CAR Models

Spatial error models and sometimes spatial lag models are referred to as the simultaneous autoregressive model (SAR). Another popular class of models is the conditional autoregressive (CAR) models. The key distinction between the SAR and the CAR models is in the model specification (Cressie [1993\)](#page-22-0). SAR models explain the relations among response variables at all locations on the lattice simultaneously and the spatial effect is considered to be endogenous. In contrast, CAR models specify the distribution of a response variable at one location by conditioning on the values of its neighbors in the neighborhood and the spatial effect of the neighbors is considered to be exogenous (Anselin [2003](#page-21-0)). While CAR models are popular in the statistics literature and many other disciplines, SAR models are favored in spatial econometrics and spatial demography, possibly because interpretation of the spatial autocorrelation coefficient resembles that of standard linear regression and thus may seem more natural. The relation between the two types of models is close, however, as SAR models (from a spatial error model) may be represented by possibly higher order CAR models (Cressie [1993\)](#page-22-0).

Spatial-temporal Regression

The spatial regression models considered so far account for spatial autocorrelation within a same time period but not across different time periods, as all the variables in (1) to (4) refer to a given cross-sectional point in time. Nevertheless, for all variables, we can add their corresponding time-lagged variables to establish a spatial-temporal regression model, which captures both the spatial and the temporal autocorrelation (Elhorst [2001\)](#page-22-0). For example, we may extend model (2) to:

$$
Y_t = X_t \beta + \rho W Y_t - \rho W X_t \beta + \tau_1 Y_{t-1} + X_{t-1} \tau_2 + \tau_3 W Y_{t-1} + W X_{t-1} \tau_4 + \varepsilon. \tag{5}
$$

The spatial-temporal regression model in (5) provides a practical way for forecasting, if we consider only the time-lagged explanatory variables. That is, by removing explanatory variables from the same time point as the base year of forecasting, we obtain a spatial-temporal regression model suitable for forecasting such as follow:

$$
Y_t = \tau_1 Y_{t-1} + X_{t-1} \tau_2 + \tau_3 W Y_{t-1} + W X_{t-1} \tau_4 + \varepsilon. \tag{6}
$$

Spatial Logistic Regression

In the above models, the response variables are continuous and are assumed to follow normal distributions. When the response variables are binary (0 or 1), logistic regression would be more suitable. However, to account for spatial autocorrelation, spatial logistic regression models would be more appropriate. A number of approaches have been proposed, such as autologistic regression models (Anselin [2002;](#page-21-0) Besag [1974](#page-22-0)), marginal models fitted by generalized estimating equations (GEE) (Diggle et al. [2002\)](#page-22-0), and generalized linear mixed models (GLMM) (Littell et al. [2006](#page-24-0)). With autologistic regression models and GLMM, it is becoming increasingly popular to employ the Markov Chain Monte Carlo (MCMC) approach, a powerful statistical computing technique, for statistical inference (Banerjee et al. [2003](#page-21-0); Fleming [2004](#page-22-0); Gelman et al. [2003\)](#page-23-0). In Zhu et al. [\(2005](#page-25-0)), spatial autologistic regression models are extended to a class of spatial-temporal autologistic regression models for the analysis of spatial-temporal binary data.

Data Example

For illustration, we fit both standard regression model and various spatial regression models to the population change data example here. We first describe the response variable and the explanatory variables. We then fit a standard linear regression to the data and perform model diagnostics to check the model assumptions. Afterwards, we fit a spatial lag model, a spatial error model, and a SARMA model to the data and again perform model diagnostics. Finally we assess and compare these various models using the aforementioned techniques.

The response variable here is a rate of population growth, expressed as the natural log of the ratio of the 2000 census population over the 1990 census population. The explanatory variables are four dummy variables indicating MCDs within 10 miles of highway expansion in 1980–1985, within 10–20 miles from highway expansion in 1980–1985, within 10 miles of highway expansion in 1985– 1990, and within 10–20 miles from highway expansion in 1985–1990, respectively. If a MCD fits into a distance buffer category, it is coded as 1; 0 otherwise. We also attempt to include other explanatory variables, as many variables are potentially related to population changes and omitting relevant explanatory variables may give rise to various problems (Dalenberg and Partridge [1997](#page-22-0)). However, many of these variables may be highly correlated, which is known as multicollinearity. To solve the dilemma, we use principal component analysis and the ModelBuilder function of ArcGIS to generate five indices, namely demographic characteristics, social and economic conditions, transportation accessibility, natural amenities, and land conversion and development in 1990, from 37 key factors of population change.⁷ In total, we have six additional explanatory variables (which we will call controlled variables) including these five indices as well as the rate of population growth from 1980 to 1990.

We now regress the response variable of population change from 1990 to 2000 on the explanatory variables of highway expansion plus the six controlled variables (the first panel of Table [2\)](#page-18-0). The results from fitting a standard linear regression model indicate that highway expansion finished 5–9 years before the population change period has no significant effect on population change, whereas highway expansion finished just before the population change period has a significant positive impact on population change, for MCDs within 10 miles and 10–20 miles from the segments of highway expansion.

The Moran's I test on the residuals after fitting the standard linear regression suggests that there is strong evidence of spatial autocorrelation among the residuals (Moran's $I = 0.122$; *p*-value $\lt 0.001$). Thus the independence assumption of the error term appears to be violated and we proceed to fit spatial linear regression models in order to account for the spatial autocorrelation.

In particular, we use a spatial lag model, a spatial error model, and a SARMA model to reanalyze the data. For the spatial weight matrix, we select the 4-nearest neighbor weight matrix, which provides the highest spatial autocorrelation of the response variable (the left panel of Table [1](#page-9-0)), for the spatial lag model. For the spatial error model, we select the 5-nearest neighbor weight matrix, which provides the highest spatial autocorrelation of the residuals (the middle panel of Table [1\)](#page-9-0). The SARMA model has both a spatial lag term and a spatial error term. Thus we use

 7 An extensive review of the relevant literature results in more than 37 variables that significantly affect population change theoretically or empirically (Chi [2006\)](#page-22-0). These 37 variables are chosen for this research on the basis of a combination of judgment established theoretical or empirical relationships, and the availability of data. The variables that have been used to generate the demographic index are population density, age structure, race, college population, educational attainment, stayers, female-headed households, and seasonal housing. Social and economic conditions include crime rate, school performance, employment, income, public transportation, public water, new housing, buses, county seat status, and real estate value. Transportation accessibility is made up of residential preference, accessibility to airports and highway, highway infrastructure, and journey to work. Natural amenities contain forest, water, the lengths of lakeshore, riverbank, and coastline, golf courses, and slope. Land development and conversion include water, wetlands, slope, tax-exempt lands, and built-up lands.

Notes: * See Table [1](#page-9-0)

the 4-nearest neighbor weight matrix for the spatial lag term, and select the squared inverse distance within 10 miles for the spatial error term, as it provides the highest spatial autocorrelation of the residuals after fitting a spatial lag model (the right panel of Table [1\)](#page-9-0).

Table [2](#page-18-0) summarizes the results from fitting three spatial regression models in addition to standard linear regression. Recall that, in standard linear regression, highway expansion finished just before the population change period for MCDs within 10 miles and 10–20 miles from the segments of highway expansion are both significant. These two explanatory variables remain significant in the spatial lag model and spatial error model, although the regression coefficients are smaller in magnitude and the p -value is not as small. In the SARMA model, however, these two explanatory variables are no longer significant.

Moreover, all the three spatial linear regression models appear to be better fit than the standard linear regression, based on the fact that the AIC and BIC values are smaller for the spatial regression models. Between the spatial error model and the spatial lag model, the former may be preferred, because of slightly smaller AIC and BIC values. Of all the three spatial regression models, the SARMA model has clear advantage over the spatial lag and spatial error models, judging again from the AIC and BIC values. All the spatial autocorrelation coefficients are significant in the three spatial linear regression models.

The residuals from the spatial lag and spatial error models do not exhibit spatial autocorrelation, but the residuals from the SARMA model have significant spatial autocorrelation, all with respect to the corresponding spatial weight matrix. Although the SARMA model has a better fit than the other two spatial models, the remaining spatial autocorrelation in its residuals casts slight doubt about the adequacy of the model. One possible remedy would be to consider other spatial weight matrices and/or somewhat more complex spatial models to account for the additional spatial autocorrelation. In future research, all forty spatial weight matrices can be applied to the four regression models. According to each spatial weight matrix, one set of the AIC and BIC values and the spatial autocorrelation of residuals will be examined for each model, as it may be fairer to compare the four regression models based on the same spatial weight matrix. AIC and BIC may also help select the optimal spatial weight matrix within and across different regression models, although the legitimacy of such an approach needs to be further established.

The scale effect may have played a role in this example because the five controlled variables influence population change at different scales but are still examined only at the MCD level. For example, natural amenities tend to attract migrants at the regional level and thus, even though the natural amenity index is expected to have important effects on population change, the analysis gives opposite results (Table [2\)](#page-18-0). To mitigate the scale effect, we could consider constructing a hierarchical model at various spatial scales (e.g., from regional, local, household, down to individual levels). Possible approaches are a two-stage procedure (Sampson et al. [1999](#page-24-0)), a Bayesian hierarchical modeling approach (Parent and Riou [2005\)](#page-24-0), and simultaneous estimation of spatial dependence within a hierarchical context (Chi and Voss [2005](#page-22-0)), although analysis can be challenging due to lack of suitable data and limitation of software support.

Discussion

Spatial demographic analysis is emerging as an important and interesting topic for demographers to explore, as evidenced by an increasing number of publications and conference presentations that apply spatial econometrics and analysis techniques for demographic studies. In this article, spatial theories of population dynamics are summarized, and spatial regression models and the associated basic concepts and issues are discussed through an illustration of highway effects on population change in the 1990s at the MCD level in Wisconsin.

Despite its complexity, spatial demographic analysis has in recent years become more accessible for demographers to explore, due to the upsurge in the availability of geographically referenced data, the development of user-friendly spatial data analysis software packages, and the computing power combined with affordable computers. Three types of demographic products are especially useful for spatial demographic analysis: the topologically integrated geographic encoding and reference (TIGER) system products, census summary files of 1980, 1990, and 2000, and the sociological and demographic survey database when companioned with the geocoding technique. Related geophysical information from rich GIS data sources and remote sensing images are often useful for demographic studies and can be easily added into the geographically referenced demographic database. In addition, the last decade has seen rapid development of spatial statistical software packages including GeoDa, SpaceStat, R, S-plus, GWR, and others. Extensions have also been developed for spatial statistical analysis in traditional software such as SAS, SPSS, Stata, and others. GIS software also has powerful functions of spatial statistics. Moreover, the increasing computing power facilitated in inexpensive computers makes it affordable for demographers to conduct spatial demographic analysis. Furthermore, recent years have seen a dramatically growing number of textbooks, journal articles, and conference presentations advancing or using spatial data analysis (Florax and Van der Vlist [2003\)](#page-22-0), which creates abundant opportunities for demographers to study this technique.

Looking forward, spatial demography may advance in two perspectives. First, explicitly spatial demographic theories may be proposed. There are some demographic and sociological theories and empirical studies suggesting spatial effects in population dynamics as discussed in the section of spatial demographic theories. Nevertheless, spatial effects have not yet been explicitly stated in current demographic theories. It is obvious that advances in spatial techniques and availability of spatial data are allowing us to ask new demographic questions and develop new demographic theories. The field of demography can borrow the strong spatial components from the spatially explicit theories in geography and regional science to strengthen itself.

Second, besides spatial regression models, other spatial analysis techniques may be applicable to demographic studies. Spatial point data analysis, which has been used widely in diverse disciplines such as epidemiology and forestry, could become a potentially useful technique for formal demographic studies, especially with the development of geocoding techniques and individual demographic survey database. Geostatistics, which has been applied frequently in physical and biological sciences, can be used potentially as an interpolation technique for demographic estimation. Spatial interaction modeling, which becomes a fourth type of spatial data analysis techniques, can be very useful for studying migration and demographic network. Put short, numerous spatial data analysis methods are available in other fields and can be well employed for demographic studies.

We believe that spatial demography is moving to a new and exciting stage along with the rapid advances in spatial analysis techniques and the increasing availability of geographically referenced data. The time appears to be ripe for demographers to explore and enrich the field of spatial demography.

Acknowledgments We are indebted to Paul R. Voss for his guidance with this research and for providing us with insightful suggestions on earlier drafts. Appreciation is extended to three anonymous reviewers for their many helpful comments. We also acknowledge support from the Social Science Research Center at Mississippi State University and Department of Statistics and Department of Soil Science at University of Wisconsin-Madison. Funding has been provided for this research by the USDA Cooperative State Research, Education and Extension Service (CSREES) Hatch project WIS04536 and the Wisconsin Alumni Research Foundation.

References

- Alba, R. D., & Logan, J. R. (1993). Minority proximity to whites in suburbs: An individual-level analysis of segregation. American Journal of Sociology, 98(6), 1388–1427.
- Anselin, L. (1988). Spatial econometrics: Methods and models. Dordrecht, Netherlands: Kluwer Academic Publishers.
- Anselin, L. (1990). Spatial dependence and spatial structural instability in applied regression analysis. Journal of Regional Science, 30, 185–207.
- Anselin, L. (1992). SpaceStat tutorial: A workbook for using SpaceStat in the analysis of spatial data. National Center for Geographic Information and Analysis, University of California, Santa Barbara CA.
- Anselin, L. (1995). Local indicators of spatial autocorrelation—LISA. Geographical Analysis, 27, 93-115.
- Anselin, L. (1996). The Moran scatterplot as an ESDA tool to assess local instability in spatial association. In M. Fischer, H. J. Scholten, & D. Unwin (Eds.), Spatial analytical perspectives on GIS (pp. 111–125). London, England: Taylor & Francis.
- Anselin, L. (2001). Spatial econometrics. In B. Baltagi (Ed.), A companion to theoretical econometrics (pp. 310–330). Oxford, England: Blackwell.
- Anselin, L. (2002). Under the hood: Issues in the specification and interpretation of spatial regression models. Agricultural Economics, 27, 247–267.
- Anselin, L. (2003). Spatial externalities, spatial multipliers, and spatial econometrics. International Regional Science Review, 26, 153–166.
- Anselin, L., & Griffith, D. A. (1988). Do spatial effects really matter in regression analysis? Papers in Regional Science, 65, 11–34.
- Anselin, L., & Bera, A. (1998). Spatial dependence in linear regression models with an introduction to spatial econometrics. In A. Ullah & D. Giles (Eds.), Handbook of applied economic statistics (pp. 237–289). New York: Marcel Dekker.
- Bailey, T. C., & Gatrell, A. C. (1995). Interactive spatial data analysis. Harlow, England: Longman Scientific & Technical.
- Baller, R. D., Anselin, L., Messner, S. F., Deane, G., & Hawkins, D. F. (2001). Structural covariates of U. S. county homicide rates: Incorporating spatial effects. Criminology, 39, 561–590.
- Banerjee, S., Carlin, B. P., & Gelfand, A. E. (2003). Hierarchical modeling and analysis for spatial data. Boca Raton, FL: Chapman & Hall/CRC.
- Beaujeu-Garnier, J. (1966). Geography of population. London, England: Longman.
- Berry, B. J. L., & Kasarda, J. D. (1977). Contemporary urban ecology. New York: Macmillan.
- Besag, J. (1974). Spatial interaction and the statistical analysis of lattice systems. Journal of the Royal Statistical Society, Series B, 36, 192–236.
- Boarnet, M. G. (1997). Highways and economic productivity: Interpreting recent evidence. Journal of Planning Literature, 11(4), 476–486.
- Boarnet, M. G. (1998). Spillovers and the locational effects of public infrastructure. Journal of Regional Science, 38(3), 381–400.
- Brown, D. L., Fuguitt, G. V., Heaton, T. B., & Waseem, S. (1997). Continuities in size of place preferences in the United States, 1972–1992. Rural Sociology, 62(4), 408–428.
- Cardille, J. A., Ventura, S. J., & Turner, M. G. (2001). Environmental and social factors influencing wildfires in the Upper Midwest, USA. Ecological Applications, 11, 111–127.
- Case, A., Rosen, H. S., & Hines, J. R. (1993). Budget spillovers and fiscal policy interdependence: Evidence from the states. Journal of Public Economics, 52, 285–307.
- Cervero, R. (2002). Induced travel demand: Research design, empirical evidence, and normative policies. Journal of Planning Literature, 17(1), 3–20.
- Cervero, R. (2003). Road expansion, urban growth, and induced travel: A path analysis. Journal of the American Planning Association, 69(2), 145–163.
- Cervero, R., & Hansen, M. (2002). Induced travel demand and induced road investment: A simultaneousequation analysis. Journal of Transport Economics and Policy, 36(3), 469–490.
- Charles, C. Z. (2003). The dynamics of racial residential segregation. Annual Review of Sociology, 29, 167–207.
- Chi, G. (2006). Environmental demography, small-area population forecasting, and spatio-temporal econometric modeling: Demographics, accessibility, developability, desirability, and livability. Dissertation, Department of Urban and Regional Planning, University of Wisconsin-Madison, Madison WI.
- Chi, G., & Voss, P. R. (2005). Migration decision-making: A hierarchical regression approach. Journal of Regional Analysis and Policy, 35(2), 11–22.
- Chi, G., Voss, P. R., & Deller, S. C. (2006). Rethinking highway effects on population change. Public Works Management and Policy, 11, 18–32.
- Christaller, W. (1966). Central places in southern Germany (Die zentralen Orte in Süddeutschland, Baskin CW, 1933, Trans). Englewood Cliffs, NJ: Prentice-Hall.
- Clark, W. (1996). Understanding residential segregation in American cities: Interpreting the evidence. Population Research and Policy Review, 5, 95–127.
- Cliff, A., & Ord, J. K. (1973). Spatial autocorrelation. London, England: Pion Limited.
- Cliff, A., & Ord, J. K. (1981). Spatial processes, models and applications. London, England: Pion Limited.
- Cowen, D. J., & Jensen, J. R. (1998). Extraction and modeling of urban attributes using remote sensing technology. In D. Liverman, E. F. Moran, R. R. Rindfuss, & P. C. Stern (Eds.), People and pixels: Linking remote sensing and social science (pp. 164–188). Washington, DC: National Academy Press.
- Cressie, N. (1993). Statistics for spatial data. New York: Wiley.
- Dalenberg, D. R., & Partridge, M. D. (1997). Public infrastructure and wages: Public capital's role as a productive input and household amenity. Land Economics, 73, 268–284.
- Diggle, P., Heagerty, P., Liang, K. Y., & Zeger, S. (2002). Analysis of longitudinal data. Oxford, England: Oxford University Press.
- Doreian, P. (1980). Linear models with spatial distributed data: Spatial disturbances or spatial effects? Sociological Methods and Research, 9, 29–60.
- Draper, N. R., & Smith, H. (1998). Applied regression analysis. New York: John Wiley & Sons.
- Elhorst, P. J. (2001). Dynamic models in space and time. Geographical Analysis, 33, 119–140.
- Fleming, M. M. (2004). Techniques for estimating spatially dependent discrete choice models. In L. Anselin, R. J. G. M. Florax, & S. J. Rey (Eds.), Advances in spatial econometrics (pp. 145–168). Berlin, Germany: Springer.
- Florax, R. J. G. M., & Van der Vlist, A. J. (2003). Spatial econometric data analysis: Moving beyond traditional models. International Regional Science Review, 26(3), 223–243.
- Fossett, M. (2005). Urban and spatial demography. In D. L. Poston, & M. Micklin (Eds.), Handbook of Population (pp. 479–524). New York: Springer.
- Fotheringham, A. S., & Wong, D. W. S. (1991). The modifiable areal unit problem in multivariate statistical analysis. Environment and Planning A, 23, 1025–1034.
- Fotheringham, A. S., Brunsdon, M., & Charlton, M. (1998). Geographically weighted regression: A natural evolution of the expansion method for spatial data analysis. Environment and Planning A, 30, 1905–1927.
- Fox, J. (1997). Applied regression analysis, linear models, and related methods. Thousand Oaks, CA: Sage Publications.
- Frisbie, W. P., Kasarda, J. D. (1988). Spatial processes. In N. J. Smelser (Ed.), Handbook of sociology (pp. 629–666). Newbury Park, CA: Sage Publications.
- Fuguitt, G. V., & Brown, D. (1990). Residential preferences and population redistribution. *Demography*, 27, 589–600.
- Fuguitt, G. V., & Zuiches, J. J. (1975). Residential preferences and population distribution. Demography, 12(3), 491–504.
- Fuguitt, G. V., Brown, D. L., & Beale, C. L. (1989). Rural and small town America: The population of the United States in the 1980s. New York: Russell Sage Foundation.
- Galster, G. C. (1988). Residential segregation in American cities: A contrary review. Population Research and Policy Review, 7, 93–112.
- Gelman, A., Carlin, J. B., Stern, H. S., & Rubin, D. B. (2003). Bayesian data analysis. Boca Raton, FL: Chapman & Hall/CRC.
- Getis, A. (1984). Interaction modeling using second-order analysis. *Environment and Planning A, 16(2)*, 173–183.
- Getis, A. (1995). Spatial filtering in a regression framework: Examples using data on urban crime, regional inequality, government expenditures. In L. Anselin & R. J. G. M. Florax (Eds.), New directions in spatial econometrics (pp. 172–185). Berlin, Germany: Springer Verlag.
- Goodchild, M. F. (1992). Geographical data modeling. Computer and Geoscience, 18, 401–408.
- Gordon, D. (1978). Capitalist development and the history of American cities. In W. Tabb & L. Sawers (Eds.), Marxism and the metropolis (pp. 25–63). Oxford, England: Oxford University Press.
- Graaff, T., Florax, R. J. G. M., Nijkamp, P., & Reggiani, A. (2001). A general misspecification test for spatial regression models: Dependence, heterogeneity, and nonlinearity. Journal of Regional Science, 41, 255–276.
- Green, M., & Flowerdew, R. (1996). New evidence on the modifiable areal unit problem. In P. Longley & M. Batty (Eds.), Spatial analysis: Modelling in a GIS environment (pp. 41–54). Cambridge, MA: GeoInformation International.
- Greene, W. H. (2000). Econometric analysis. Upper Saddle River, NJ: Prentice-Hall, Inc.
- Griffith, D. A. (2003). Spatial autocorrelation and spatial filtering: Gaining understanding through theory and scientific visualization. New York: Springer Verlag.
- Hall, P. (1988). The city of theory. In R. LeGates & F. Stout (Eds.), The city reader (pp. 391–393). New York: Routledge.
- Hawley, A. H. (1950). Human ecology: A theory of community structure. New York: Ronald Press.
- Hill, R. C. (1977). Capital accumulation and urbanization in the United States. Comparative Urban Research, 4, 39–60.
- Hudson, J. C. (1972). Geographical diffusion theory. Evanston, IL: Northwestern University.
- Humphrey, C. R. (1980). The promotion of growth in small urban places and its impact on population change. Social Science Quarterly, 61, 581–594.

James, P. (1954). The geographic study of population. In P. James & C. Jones (Eds.), American geography: Inventory and prospect (pp. 106–122). Syracuse, NY: Syracuse University Press.

Jaret, C. (1983). Recent neo-Marxist urban analysis. Annual Review of Sociology, 9, 499–525.

- Jensen, J. R., Cowen, D. J., Halls, J., Narumalani, S., Schmidt, N. J., Davis, B. A., & Burgess, B. (1994). Improved urban infrastructure mapping and forecasting for BellSouth using remote sensing and GIS technology. Photogrammetric Engineering and Remote Sensing, 60, 339–346.
- Johnson, K. M., & Beale, C. L. (1994). The recent revival of widespread population growth in nonmetropolitan areas of the United States. Rural Sociology, 59(4), 655–667.
- Johnson, K. M., & Purdy, R. L. (1980). Recent nonmetropolitan population change in fifty-year perspective. Demography, 17(1), 57–70.
- Johnson, K. M. (1982). Organization adjustment to population change in nonmetropolitan America: A longitudinal analysis of retail trade. Social Forces, 60(4), 1123–1139.
- Johnson, K. M. (1989). Recent population redistribution trends in nonmetropolitan America. Rural Sociology, 54(3), 301–326.
- Jones, H. R. (1990). Population geography. New York: The Guilford Press.
- Krugman, P. (1991). Geography and trade. Cambridge, MA: MIT Press.
- Land Information and Computer Graphics Facility. (2000). Mapping growth management factors: A practical guide for land use planning. Madison, WI: University of Wisconsin-Madison.
- Land Information and Computer Graphics Facility. (2002). Population and land allocation: Evolution of geospatial tools helps citizens engage in land-planning process. Madison, WI: University of Wisconsin-Madison.
- Langford, M., & Unwin, D. J. (1994). Generating and mapping population density surfaces within a geographical information system. The Cartographic Journal, 31, 21–26.
- Langford, M., Maguire, D. J., & Unwin, D. J. (1991). The areal interpolation problem: Estimating population using remote sensing. in a GIS framework. In I. Masser & M. Blakemore (Eds.), Handling geographical information: Methodology and potential applications (pp. 55–77). London, England: Longman Scientific & Technical.
- LeSage, J. P. (1999). A spatial econometric examination of China's economic growth. Geographic Information Sciences, 5, 143–153.
- Lewis, P. H. (1996). Tomorrow by design: A regional design process for sustainability. New York: John Wiley & Sons.
- Littell, R., Milliken, G., Stroup, W., Wolfinger, R., & Schabenberger, O. (2006). SAS for mixed models. Cary, NC: SAS Institute Inc.
- Loftin, C., & Ward, S. K. (1983). A spatial autocorrelation model of the effects of population density on fertility. American Sociological Review, 48, 121–128.
- Logan, J. R., & Molotch, H. L. (1987). Urban fortunes: The political economy of place. Berkeley, CA: University of California Press.
- Massey, D. S., & Denton, N. A. (1993). American apartheid: Segregation and the making of the underclass. Cambridge, MA: Harvard University Press.
- McKenzie, R. D. (1924). The ecological approach to the study of the human community. American Journal of Sociology, 30, 287–301.
- Mennis, J. (2003). Generating surface models of population using dasymetric mapping. The Professional Geographer, 55, 31–42.
- Moran, P. (1948). The interpolation of statistical maps. Journal of the Royal Statistical Society B, 10, 243–251.
- Mollenkopf, J. (1978). The postwar politics of urban development. In W. Tabb & L. Sawers (Eds.), Marxism and the metropolis (pp. 117–152). Oxford, England: Oxford University Press.
- Mollenkopf, J. (1981). Neighborhood political development and the politics of urban growth: Boston and San Francisco 1958–1978. International Journal of Urban and Regional Research, 5, 15–39.
- Openshaw. S. (1984). The modifiable areal unit problem. In Concepts and techniques in modern geography (Vol. 38). London, England: Geobooks.
- Openshaw, S., & Taylor, P. J. (1981). The modifiable areal unit problem. In N. Wrigley, & R. J. Bennett (Eds.), Quantitative geography: A British view (pp. 60–70). London, England: Routledge.
- Ord, J. K., & Getis, A. (1995). Local spatial autocorrelation statistics: Distributional issues and an application. Geographical Analysis, 27, 286–306.
- Pacheco, A. I., & Tyrrell, T. J. (2002). Testing spatial patterns and growth spillover effects in clusters of cities. Journal of Geographical Systems, 4, 275–285.
- Paelinck, J. H. P. (2000). On aggregation in spatial econometric modeling. Journal of Geographical Systems, 2, 157–165.
- Parent, O., & Riou, S. (2005). Bayesian analysis of knowledge spillovers in European regions. Journal of Regional Science, 45(4), 747–775.
- Parisi, D., Lichter, D. T., Grice, S. M., & Taquino, M. (2007). Disaggregating trends in racial residential segregation: Metropolitan, micropolitan, and non-core counties compared. Presented at the annual meeting of the Population Association of America, March 29, 2007, New York NY.
- Perroux, F. (1955). Note sur la Notion de pole de croissance. Economie Appliquée, 8, 307-320.
- Poston, D. L., & Frisbie, W. P. (2005). Ecological demography. In D. L. Poston & M. Micklin (Eds.), Handbook of population (pp. 601–623). New York: Springer.
- Robinson W. S. (1950). Ecological correlations and the behavior of individuals. American Sociological Review, 15, 351–357.
- Sampson R. J., Morenoff J. D., & Earls, F. (1999). Beyond social capital: Spatial dynamics of collective efficacy for children. American Sociological Review, 64(5), 633–660.
- Schabenberger, O., & Gotway, C. A. (2005). Statistical methods for spatial data analysis. Boca Raton, FL: Chapman & Hall/CRC Press.
- Tiefelsdorf, M. (2000). Modelling spatial processes—The identification and analysis of spatial relationships in regression residuals by means of Moran's I. Berlin, Germany: Springer Verlag.
- Tobler, W. (1970). A computer movie simulating urban growth in the Detroit region. Economic Geography, 46, 234–240.
- Tolnay, S. E., Deane, G., & Beck, E. M. (1996). Vicarious violence: Spatial effects on southern lynchings, 1890–1919. American Journal of Sociology, 102, 788–815.
- Trewartha, G. (1953). A case for population geography. Annals of the Association of American Geographers, 43, 71–97.
- Voss, P. R., & Chi, G. (2006). Highways and population change. Rural Sociology, 71(1), 33–58.
- Voss, P. R., White, K. J. C., & Hammer, R. B. (2006). Explorations in spatial demography. In W. A. Kandel, &D. L. Brown (Eds.), Population change and rural society (pp. 407–429). Dordrecht: Springer.
- Wrigley, N., Holt, T., Steel, D., & Tranmer, M. (1996). Analysing, modelling, and resolving the ecological fallacy. In P. Longley & M. Batty (Eds.), Spatial analysis: Modelling in a GIS environment (pp. 23–40). Cambridge, MA: GeoInformation International.
- Zelinsky, W. (1966). A prologue to population geography. Englewood Cliffs, NJ: Prentice-Hall.
- Zhu, J., Huang, H.-C., & Wu, J. (2005). Modeling spatial-temporal binary data using Markov random field models. Journal of Agricultural, Biological, and Environmental Statistics, 10, 212–225.
- Zuiches, J. J., & Rieger, J. H. (1978). Size of place preferences and life cycle migration: A cohort comparison. Rural Sociology, 43(4), 618–633.