Photonic Network Communications (2020) 40:103–113
https://doi.org/10.1007/s11107-020-00901-z

ORIGINAL PAPER

Threshold optimization for modified switching scheme of hybrid FSO/RF system in the presence of strong atmospheric turbulence

Shreesh Kumar Shrivastava[1] · Sujata Sengar[1] · Shree Prakash Singh[1] · Soyinka Nath[1]

Received: 24 September 2018 / Accepted: 22 July 2020 / Published online: 28 August 2020

Abstract
In hybrid free space optical/radio frequency (FSO/RF) systems, switching between the optical and RF links depends on the optical and RF signal to noise ratio (SNR) threshold levels; therefore, the correct choice of these threshold levels is essential to make best use of the complementary nature of optical and RF links. Optimal selection of these threshold levels is the prime objective of this paper. A hybrid FSO/RF system using modified switching scheme with two optical and one RF threshold is considered. Here, we propose an algorithm to obtain the optimized thresholds, which yield minimum bit error rate (BER) for a given outage probability under a constraint on maximum transmitted power. The optimized thresholds are dependent on the average optical and RF SNR. Hybrid FSO/RF system use both optical as well as RF power. The average system SNR is computed by considering the actual optical and RF power consumption. Also the performance of the hybrid FSO/RF system is studied with respect to average system SNR using optimized optical and RF thresholds. This analysis reveals the existence of a critical operating point of average optical SNR, beyond which the system should be preferably operated to ensure minimum BER.

Keywords Threshold optimization · Negative exponential · Outage probability · Average system SNR · Modified switching scheme · Hybrid FSO/RF system

1 Introduction

The hybrid FSO/RF system exploits the complementary features of FSO (Free Space Optical Communication) and RF (radio frequency) links to form a high speed and more reliable all-weather system as opposed to individual FSO or RF systems. As long as optical channel conditions are favorable, data are transmitted via the optical (FSO) link. In the event of optical link degradation, a 'stand-by' RF link is enabled, and data transmission is switched from the optical to the RF link [1, 2]. This mode of switching, where, either the optical or the RF link is active is known as 'hard switching' [3]. In some variants of the hybrid FSO/RF system, both the optical and RF links may be active simultaneously, which is known as 'soft switching' [4]. Here, data rate of the FSO and RF links are adjusted according to channel conditions. A third variant of switching-'hybrid switching' [5] causes the data to be switched between two modes of transmission. In mode one, the data are transmitted through the optical link only and in mode two, both optical and RF links are active simultaneously. In hybrid switching, outage is determined by an outage threshold which is decided by considering combined SNRs of both the optical and RF links. In this scheme, the BER increases with the decrease in outage probability, and vice versa. Hence, trade-off between BER and outage probability is not possible.

In order to overcome this shortcoming of hybrid switching, a modified switching scheme, with two optical and one RF threshold was proposed in [6]. This switching scheme has the advantage of saving power while providing better overall system performance as compared to the existing switching schemes. In the modified switching scheme,

✉ Sujata Sengar
nsit_sujata@yahoo.com

Shreesh Kumar Shrivastava
shreesh.92@gmail.com

Shree Prakash Singh
sps_nsit@yahoo.co.uk

Soyinka Nath
soyinkanath@gmail.com

[1] Division of Electronics and Communication Engineering, Netaji Subhas Institute of Technology, Sector-3, Dwarka, New Delhi 110078, India

outage is said to occur when the SNRs of the optical and RF links fall below the lower optical threshold and RF threshold, respectively. Performance of the hybrid FSO/RF system is expressed in terms of outage probability, BER, etc. which in turn are dependent on these threshold values.

In the existing literature, several works have considered the impact of optical and RF thresholds on various system parameters. The hard switching scheme for hybrid FSO/RF system has been discussed in [7] with adaptive rate on each link. In [7], different data rates have been adapted before transmission through FSO and RF links on the basis of received channel state information. The transmission of different data rates requires several threshold levels for both the links. However, authors have not discussed any method to compute these thresholds.

The hybrid FSO/RF system with power adaptation strategies based on truncated channel inversion has been investigated in [8]. The authors have considered two threshold levels, one is used to maintain a target data rate and the other is used to compute the outage probability of the system. In [8], a SNR threshold of 10 dB has been used for the purpose of switching ON/OFF the RF link. The combined effect of atmospheric fading and misalignment on the hybrid FSO/RF transmission has been studied in [9]. The authors have considered the SNR thresholds for both optical and RF link as 6 dB. In [10], an optical SNR of 10 dB has been used as threshold to decide whether the FSO link quality meets the desired quality of transmission. Under FSO link degradation, RF link provides a backup to cope up with the hostile weather conditions. Various literatures have dealt with thresholds for switching purposes in hybrid FSO/RF system, however, limited information is provided regarding selection of threshold values.

A hybrid FSO system using FSO/RF-FSO link adaptation with a single FSO threshold scheme and a dual FSO threshold scheme have been investigated in [11]. In the single FSO threshold scheme, a fixed FSO threshold (≈ 5 dB) is used for primary FSO link and another threshold (≈ 2 dB, 5 dB and 10 dB) is used for mixed RF-FSO transmission path. In the dual FSO threshold scheme, two (i.e., upper and lower) thresholds are used for FSO link along with one threshold for backup (mixed RF-FSO) link. The different values for upper (≈ 6 dB, 6.5 dB, 10 dB and 5 dB) and lower FSO thresholds (≈ 4 dB, 3.5 dB, 5 dB and 2 dB) are adopted to obtain the results. In [5], the optical threshold value is obtained from $Q^{-1}\left(P_e^2\right)$, where Q^{-1} (–) is the inverse of complementary error function and P_e is the maximum probability of error as a function of modulation scheme. This method yields a fixed value for threshold which makes it difficult to optimize the performance of the hybrid FSO/RF system. In the aforementioned papers, the fixed thresholds have been chosen to maintain target data rate/maximum permissible BER/maximum outage probability.

The modified switching scheme provides the flexibility to choose the thresholds independently. The advantage of selecting independent thresholds for optical and RF links allows the optimization of the hybrid FSO/RF system performance in terms of BER for a fixed outage probability. Since the optical (i.e., upper and lower) and RF thresholds are varied independently, so the outage probability can be kept constant by various combinations of optical and RF thresholds. This independent selection of thresholds provides an opportunity to choose the thresholds corresponding to the best BER performance.

In this paper, we discuss optimal selection of the optical and RF thresholds for modified switching scheme to yield best possible performance in terms of outage probability, BER and actual power consumption. The optimization is performed to achieve minimum BER under the constraint on maximum average system SNR, for a fixed outage probability.

To the best of the authors' knowledge, optimization of the hybrid FSO/RF system performance through independent control of thresholds has not been proposed in literature so far. Further, the existing works on the performance analysis of the hybrid FSO/RF systems have reported their investigations on the basis of either average optical SNR or average RF SNR. However, in the hybrid FSO/RF system, both optical as well as RF contributes to the actual power. Hence, it appears to be highly relevant to take into consideration the average system SNR which is computed as a function of actual power consumption.

Rest of the paper is organized as follows. System and channel models are described in Sect. 2. In Sect. 3, the performance of modified switching scheme of the hybrid FSO/RF system is investigated in terms of BER and outage probability. In Sect. 4, optical and RF thresholds are optimized using proposed optimization algorithm, and Sect. 5 includes the results and discussion. Finally, concluding remarks are provided in Sect. 6.

2 System and channel model

2.1 Modified switching scheme

The hybrid FSO/RF system with modified hybrid switching scheme [6] can be better understood from Fig. 1. In this switching scheme, data are transmitted via the optical link while the RF link is put in standby mode under favorable optical channel conditions i.e., instantaneous optical SNR is greater than λ_1. In this condition, the received SNR of the system is equal to the received optical SNR (γ_o). When the quality of the optical link degrades and the optical SNR falls below the upper threshold level λ_1, the RF link is activated. Since both links are simultaneously active, the received SNR is the sum of both optical

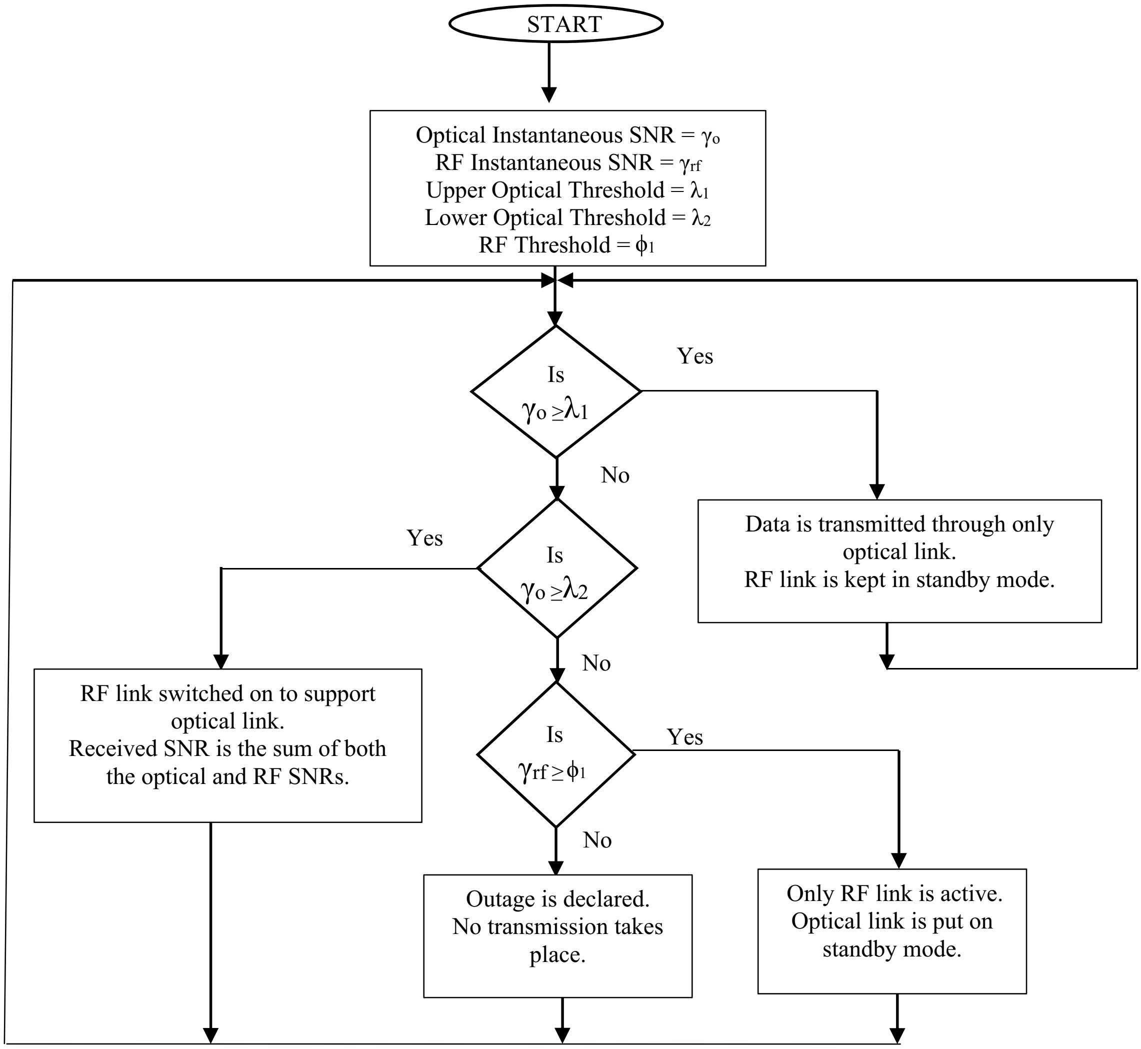

Fig. 1 Flowchart of modified switching scheme for hybrid FSO/RF system

and RF SNR due to maximal ratio combining (MRC). When the FSO signal quality further degrades and the optical SNR falls below the lower threshold level λ_2, the optical link is put in standby mode and only the RF link sustains the required transmission. The received SNR, in this condition, is equal to the RF SNR (γ_{rf}). In this switching scheme, outage is said to occur when the received SNR of the optical link is below its lower threshold λ_2 and the RF received SNR is below the RF threshold ϕ_1. We consider a quasi-static block fading channel model; therefore, the switching is performed on block basis.

2.2 Optical channel model

The optical wireless channel has been modeled as

$$y(t) = RIx(t) + n_o(t) \tag{1}$$

where $y(t)$ is the received optical signal, R the responsivity of photodetector, I the instantaneous optical channel intensity gain, or optical channel state, $x(t)$ the transmitted signal, and $n_o(t)$ the real-valued additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_o^2 \triangleq \mathbb{E}[|n_0(t_i)|^2]$. It is assumed that strong turbulence is present in the FSO channel, hence, negative exponential irradiance model [12, 13] is used to statistically model the turbulence. It has been assumed that there is no misalignment associated with the laser beam. The instantaneous optical SNR is given by $\gamma_o = \bar{\gamma}_o |I|^2$, where $\bar{\gamma}_o$, is average electrical SNR of optical link, and is given as [14]

$$\bar{\gamma}_o = \frac{\mu^2 R^2 P_{\mathrm{To}}^2 G_o^2}{\sigma_o^2}; \quad \bar{\gamma}_o \geq 0 \tag{2}$$

In (2), μ is the modulation index ($0<\mu<1$), P_{To} the average transmitted optical power, G_o the average gain of the FSO link, which depends on the path loss. The probability density function (PDF) of the atmospheric turbulence, in terms of received electrical SNR, γ_o, is given as

$$f_{\gamma_o}\left(\gamma_o\right) = \frac{1}{2I_o\sqrt{\bar{\gamma}_o\gamma_o}} e^{-\frac{1}{I_o}\sqrt{\frac{\gamma_o}{\bar{\gamma}_o}}}, \quad \gamma_o \geq 0 \tag{3}$$

where, the mean of irradiance $E[I] = I_o$, is considered as unity [15].

2.3 RF channel model

The RF wireless communication system can be modeled as:

$$y'(t) = hx'(t) + n_{rf}(t) \tag{4}$$

where $y'(t)$ is the received RF signal, $n_{rf}(t)$ the AWGN with zero mean and variance σ_n^2. The instantaneous RF SNR is given by $\gamma_{rf} = \bar{\gamma}_{rf}|h|^2$, where $\bar{\gamma}_{rf}$ is the average received electrical SNR of RF link, and is given as [14]

$$\bar{\gamma}_{rf} = \frac{P_{T_{rf}}G_{rf}}{\sigma_n^2} \tag{5}$$

here $P_{T_{rf}}$ is the transmitted RF signal power, G_{rf} the average power gain of the RF link. The probability density function (PDF) of RF fading channel in terms of electrical SNR, γ_{rf}, is given as

$$f_{\gamma_{rf}}\left(\gamma_{rf}\right) = \frac{1}{\bar{\gamma}_{rf}} e^{-\left(\frac{\gamma_{rf}}{\bar{\gamma}_{rf}}\right)}, \quad \gamma_{rf} \geq 0 \tag{6}$$

The above channel models along with the described modified switching scheme will now be used for finding the optimized thresholds. In order to achieve this, firstly, we need to find out how some of the commonly used performance metrics such as outage probability and BER are related to the thresholds of modified switching scheme.

3 Outage probability and BER analysis for modified switching scheme

3.1 Outage analysis

Modified switching scheme consists of two optical thresholds and one RF threshold, and outage is said to occur when optical SNR lies below the lower optical threshold (λ_2), and RF SNR falls below the RF threshold level (ϕ_1). Thus, the outage probability $\left(P_{\text{Out}}^{\text{Sys}}\right)$ is given by

$$P_{\text{Out}}^{\text{Sys}} = P\left(\gamma_o < \lambda_2\right)P\left(\gamma_{rf} < \phi_1\right) = \left[\left(1 - e^{-\frac{\sqrt{\lambda_2}}{c}}\right)\right]\left[1 - e^{-\frac{\phi_1}{\bar{\gamma}_{rf}}}\right] \tag{7}$$

where $c = I_o\sqrt{\bar{\gamma}_o}$ and P(.) denotes the probability of (.).

3.2 Average system SNR

In the existing literature, the performance of hybrid FSO/RF system has been evaluated in terms of outage probability and BER with respect to the average optical SNR [6]. However, the system transmission power depends on the mode of operation. In hybrid (when both links are active simultaneously) or RF mode of operation, the RF power contributes significantly in actual power transmission along with optical power, which in turn impacts the overall performance of the hybrid FSO/RF system. Thus, it is important to study the impact of both the optical as well as RF power toward the system performance.

The actual power consumed in the hybrid FSO/RF system depends on the link activation duration. When the hybrid system operates in the optical mode, the average transmitted power is equal to the optical power. In the hybrid mode, when both links are active simultaneously, the average transmitted power is the sum of both optical and RF powers. When the channel is very hostile for the optical link, it is set in standby mode and only the RF link is kept active to maintain the reliability. In this situation, average transmitted power is equal to the RF power. Thus, average transmitted power may be computed by averaging the optical and RF power over all modes of transmission/operation. The actual power consumed will be the sum of the powers consumed in optical mode, hybrid mode and RF mode multiplied by their corresponding operating probabilities. Hence, the average system SNR $\left(\gamma_{\text{Avg}}^{\text{Sys}}\right)$ of the hybrid FSO/RF system is obtained as

$$\gamma_{\text{Avg}}^{\text{Sys}} = P\left(\gamma_o > \lambda_1\right)\bar{\gamma}_o + P\left(\lambda_2 < \gamma_o < \lambda_1\right)\left(\bar{\gamma}_o + \bar{\gamma}_{rf}\right) + P\left(\gamma_o < \lambda_2\right)P\left(\gamma_{rf} > \phi_1\right)\bar{\gamma}_{rf} \tag{8}$$

Based on the above description of the system operation and (8), it can be seen that the average system SNR cannot exceed $\bar{\gamma}_o + \bar{\gamma}_{rf}$, and the minimum value of $\gamma_{\text{Avg}}^{\text{Sys}}$ is equal to the $\min(\bar{\gamma}_o, \bar{\gamma}_{rf})$ for modified switching scheme of the hybrid FSO/RF system.

3.3 Average probability of error

The average probability of error of the system is a combination of the error when only the optical link is active, both the links are active simultaneously and only the RF link is active. Hence, the average BER of the hybrid FSO/RF system $\left(\text{BER}_{\text{Avg}}^{\text{Sys}}\right)$ is derived as [6]

$$\text{BER}_{\text{Avg}}^{\text{Sys}} = P(\gamma_o > \lambda_1)P_e(\bar{\gamma}_o) + P(\lambda_2 < \gamma_o < \lambda_1)P_e(\bar{\gamma}_o + \bar{\gamma}_{rf}) + P(\gamma_o < \lambda_2, \gamma_{rf} > \phi_1)P_e(\bar{\gamma}_{rf}) \tag{9}$$

where $P(.)$ and P_e denote the operating probability and bit error rate of corresponding modes, respectively. Hence, from [6]

$$\mathrm{BER}_{\mathrm{Avg}}^{\mathrm{Sys}} = \left(e^{-\frac{1}{I_o}\sqrt{\frac{\lambda_1}{\bar{\gamma}_o}}}\right)\left[\left\{\frac{e^{\frac{1}{4c^2}}}{4I_o\sqrt{\bar{\gamma}_o}}\right\}\left\{\frac{\sqrt{\pi}}{2} erfc(X_1)\right\}\right] + \frac{1}{4}\left[e^{-\frac{\sqrt{\lambda_2}}{c}} - e^{-\frac{\sqrt{\lambda_1}}{c}}\right]\left[\left(\frac{\sqrt{\pi}}{2c} e^{\frac{1}{4c^2}}\{erfc(X_2) - erfc(X_1)\}\right)\left(\frac{1}{1+\bar{\gamma}_{rf}}\right)\right] + \frac{1}{4}\left[\left(1 - e^{-\frac{\sqrt{\lambda_2}}{c}}\right)\left(e^{-\frac{\phi_1}{\bar{\gamma}_{rf}}}\right)\right]\left[\left(\frac{1}{1+\bar{\gamma}_{rf}}\right)e^{-\phi_1\left(1+\frac{1}{\bar{\gamma}_{rf}}\right)}\right] \tag{10}$$

where $X_1 = \sqrt{\lambda_1} + \frac{1}{2I_o\sqrt{\gamma_o}}$, and $X_2 = \sqrt{\lambda_2} + \frac{1}{2I_o\sqrt{\gamma_o}}$.

4 Threshold optimization

The closed form expressions of outage probability, BER and average system SNR, obtained in the previous section, show that the outage probability is a function of $\bar{\gamma}_o, \bar{\gamma}_{rf}, \lambda_2,$ and ϕ_1 the BER is a function of $\bar{\gamma}_o, \bar{\gamma}_{rf}, \lambda_1, \lambda_2,$ and ϕ_1, and the average system SNR is a function of $\bar{\gamma}_o, \bar{\gamma}_{rf}, \lambda_1, \lambda_2,$ and ϕ_1. In this paper, we propose to minimize the average BER for a given outage probability and total transmitted power by an appropriate selection of the optical thresholds λ_1, λ_2 and RF threshold ϕ_1.

4.1 Problem statement

$$\underset{\lambda_1,\lambda_2,\phi_1}{\mathrm{MIN}}\left(\mathrm{BER}_{\mathrm{Avg}}^{\mathrm{Sys}}\right) \quad \text{w.r.t. } \lambda_1, \lambda_2, \text{ and } \phi_1$$

$$\text{such that: } P_{\mathrm{Out}}^{\mathrm{Sys}} = K_1$$

$$\gamma_{\mathrm{Avg}}^{\mathrm{Sys}} = K_2 \leq \bar{\gamma}_o + \bar{\gamma}_{rf}$$

$$\text{and} \quad \lambda_1 > \lambda_2.$$

where K_1, K_2 are constants. Using Eqs. (7), (8) and (10), the problem may be stated as

$$\mathrm{MIN}\left(\begin{array}{l}\left(e^{-\frac{1}{I_o}\sqrt{\frac{\lambda_1}{\bar{\gamma}_o}}}\right)\left[\left\{\frac{e^{\frac{1}{4c^2}}}{4I_o\sqrt{\bar{\gamma}_o}}\right\}\left\{\frac{\sqrt{\pi}}{2} erfc(X_1)\right\}\right] \\ + \left[e^{-\frac{\sqrt{\lambda_2}}{c}} - e^{-\frac{\sqrt{\lambda_1}}{c}}\right] \\ \times \frac{1}{4}\left[\left(\frac{\sqrt{\pi}}{2c} e^{\frac{1}{4c^2}}\{erfc(X_2) - erfc(X_1)\}\right)\left(\frac{1}{1+\bar{\gamma}_{rf}}\right)\right] \\ + \frac{1}{4}\left[\left(1 - e^{-\frac{\sqrt{\lambda_2}}{c}}\right)\left(e^{-\frac{\phi_1}{\bar{\gamma}_{rf}}}\right)\right] \\ \times\left[\left(\frac{1}{1+\bar{\gamma}_{rf}}\right)e^{-\phi_1\left(1+\frac{1}{\bar{\gamma}_{rf}}\right)}\right]\end{array}\right)$$

such that

$$\left[\left(1 - e^{-\frac{\sqrt{\lambda_2}}{c}}\right)\right]\left[1 - e^{-\frac{\phi_1}{\bar{\gamma}_{rf}}}\right] = K_1$$

$$\left\{\left(e^{-\frac{1}{I_o}\sqrt{\frac{\lambda_1}{\bar{\gamma}_o}}}\right)\bar{\gamma}_o + \left(e^{-\frac{\sqrt{\lambda_2}}{c}} - e^{-\frac{\sqrt{\lambda_1}}{c}}\right)(\bar{\gamma}_o + \bar{\gamma}_{rf}) + \left(1 - e^{-\frac{\sqrt{\lambda_2}}{c}}\right)\left(e^{-\frac{\phi_1}{\bar{\gamma}_{rf}}}\right)\bar{\gamma}_{rf}\right\} = K_2 \leq \bar{\gamma}_o + \bar{\gamma}_{rf}$$

$$\text{and } \lambda_1 > \lambda_2 \tag{11}$$

The closed form expression for BER as a function of λ_1, K_1, and K_2, $\left[\text{i.e., } \mathrm{BER}_{\mathrm{Avg}}^{\mathrm{Sys}}(\lambda_1, K_1, K_2)\right]$ is obtained by applying the variable separation method on (7)–(9). The plot of this BER (i.e., $\mathrm{BER}_{\mathrm{Avg}}^{\mathrm{Sys}}(\lambda_1, K_1, K_2)$) versus λ_1 for a fixed value of K_1 (i.e., outage probability) and different values of K_2 (i.e., average system SNR) is shown in Fig. 2.

It may be observed that the objective function that is to be optimized (i.e., $\mathrm{BER}_{\mathrm{Avg}}^{\mathrm{Sys}}$) is a convex function for some values of K_2 whereas for other values of K_2 the objective function is a non-convex function (i.e., inverted saddle back nature). It is thus clear that the optimization problem under consideration is a non-convex optimization problem. As is well known, there is no general/structured method for solving non-convex constraint optimization problems, and each problem has to be handled individually. Also, standard MATLAB optimization algorithms are unable to obtain global minimum. This scenario motivates the authors to design their own algorithm to minimize the average BER for a given value of outage probability. The primary objective is to obtain precise values of thresholds that give rise to the global minima of average BER. The optimization speed and rate of convergence are secondary issues. It is also observed

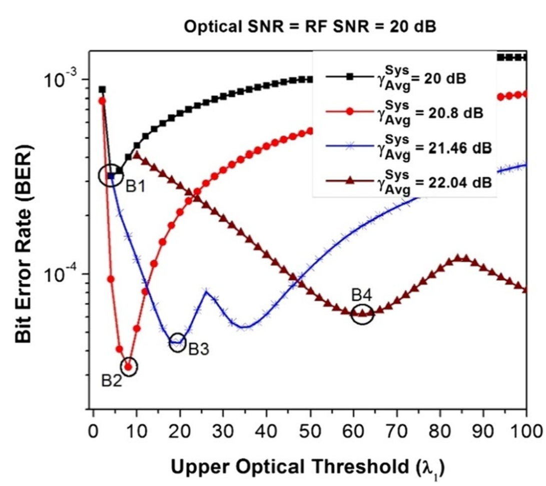

Fig. 2 BER with respect to λ_1

from Fig. 2 that the points B_1, B_2, B_3 and B_4 are local minima for different values of average system SNR. Out of these B_2 is the global minima. The values of λ_1 and $\gamma_{\text{Avg}}^{\text{Sys}}$ corresponding to point B_2 are the optimal values, any other choice of threshold gives a larger value of BER for the same $\gamma_{\text{Avg}}^{\text{Sys}}$.

The optimization problem is solved numerically as follows:

4.2 Steps for numerical solution

Step 1: Select a value for outage probability $\left(P_{\text{Out}}^{\text{Sys}}\right)$ say K_1. We select a practical range of outage probabilities 10^{-3} and 10^{-4}.
Step 2: Select a value for average system SNR $\left(\gamma_{\text{Avg}}^{\text{Sys}}\right)$ say K_2, lying between the minimum and maximum values, as defined by (8).
Step 3: Adopt variable separation method and apply on (7)–(9). Find a closed form expression for BER as a function of λ_1, K_1, and K_2, $\left[\text{i.e., } \text{BER}_{\text{Avg}}^{\text{Sys}}\left(\lambda_1, K_1, K_2\right)\right]$.
Step 4: Compute the minimum of BER with respect to λ_1 for selected value of $\gamma_{\text{Avg}}^{\text{Sys}}$ (selected in step 3). This gives a local minima for $\gamma_{\text{Avg}}^{\text{Sys}} = K_2$.
Step 5: Select another value for $\gamma_{\text{Avg}}^{\text{Sys}}$ say K_2', and find the minimum value of BER with respect to λ_1 for same value of outage probability (K_1). Repeat this step for various values of $\gamma_{\text{Avg}}^{\text{Sys}}$.
Step 6: Find the minimum value of BER among the all minimum values of BERs obtained in steps 4 and 5. This gives a global minima.
Step 7: Compute λ_2, and ϕ_1 using λ_1 and γ_{Avg}^{Sys}, corresponding to MIN $\text{BER}_{\text{Avg}}^{\text{Sys}}$. Finally, optimized values for optical as well as RF thresholds $\left(\lambda_1, \lambda_2, \text{ and } \phi_1\right)$ and average system SNR $\left(\gamma_{\text{Avg}}^{\text{Sys}}\right)$ are obtained which provides minimized value of BER for a fixed outage probability.
Step 8: Repeat all steps 1–7 for different values of outage probability.

Algorithm to compute the optimized values of optical and RF thresholds is as follows:

Start
  Assign: $\bar{\gamma}_o = A$,and $\bar{\gamma}_{rf} = B$;
  Assign: $P_{Out}^{Sys} = \text{K}_1$;
    Loop1: for a range of $\gamma_{Avg}^{Sys} = K_2 : K_2 + \Delta K : K_2^n$
      Loop2: for a range of $\lambda_1 = 0 : L$
        Search: $MIN\langle BER_{Avg}^{Sys}(\lambda_1, \text{K}_1, \text{K}_2)\rangle, wrt\ \lambda_1$
      End **Loop2**
    End **Loop1**
  Search: Global $MIN\langle BER_{Avg}^{Sys}(\lambda_1, \text{K}_1, \text{K}_2)\rangle$, wrt λ_1 and γ_{Avg}^{Sys}
  Compute: λ_2, and ϕ_1 using λ_1 and γ_{Avg}^{Sys}, correspond to $MIN\langle BER_{Avg}^{Sys}\rangle$
  Repeat: for a different set of $\bar{\gamma}_o$ and $\bar{\gamma}_{rf}$
  Repeat: for a different value of P_{Out}^{Sys}
End

5 Numerical results

This section discusses the results of threshold optimization computed through numerical methods and proposed algorithm. We consider a practical range of outage probability i.e., 10^{-3} and 10^{-4} for the optimization process.

5.1 Optimized optical and RF thresholds

Tables 1 and 2 show the variation of optimized values of upper optical threshold (λ_1), lower optical threshold (λ_2) and RF threshold (ϕ_1) for outage probabilities 10^{-3} and 10^{-4}. The optimization is performed for $\bar{\gamma}_{rf} = \bar{\gamma}_o$, and for fixed values of $\bar{\gamma}_{rf}$. Each computed point of thresholds corresponds to the minimum value of BER for a given set of average optical and RF SNR. It may be inferred that the minimum BER can be achieved for a given average optical and RF SNRs by selecting the appropriate threshold levels, which supports the concept of threshold optimization in the hybrid FSO/RF system.

The BER corresponding to optimized and non-optimized thresholds is given in Table 3. For $\bar{\gamma}_{rf} = \bar{\gamma}_o = 20$ dB, and outage probability of 10^{-3}, the optimized thresholds are computed, and they are used to calculate BER. It may be observed from Table 3 that the use of the optimized thresholds yields the minimum BER for a given optical and RF SNRs. Use of thresholds other than the optimized values

Table 1 Optimized thresholds for outage probability = 10^{-3}, all values are in dB

SNR	$\bar{\gamma}_{rf} = \bar{\gamma}_o$			$\bar{\gamma}_{rf}$ = 10 dB			$\bar{\gamma}_{rf}$ = 25 dB		
$\bar{\gamma}_o$	λ_1	λ_2	ϕ_1	λ_1	λ_2	ϕ_1	λ_1	λ_2	ϕ_1
5	2.4304	−37.7171	−3.309	3.9794	−27.2727	−3.719	7.9588	−5.4134	0.8534
10	5.1188	−31.3561	0.965	5.1188	−28.5292	−0.5184	8.6034	−5.6101	3.1743
15	7.2016	−19.7764	2.5507	6.0206	−27.869	1.7726	9.0309	−3.4351	4.4943
20	8.7506	−8.6423	4.4617	6.7669	−25.0561	2.9626	9.7772	−1.118	5.7745
25	10.3141	1.0771	7.1344	7.4036	−23.2166	4.7487	10.5115	1.0837	7.1312
30	12.8443	8.4863	10.9651	7.7815	−18.2392	4.7619	11.2222	3.3645	8.4667

Table 2 Optimized thresholds for outage probability = 10^{-4}, all values are in dB

SNR	$\bar{\gamma}_{rf} = \bar{\gamma}_o$			$\bar{\gamma}_{rf}$ = 10 dB			$\bar{\gamma}_{rf}$ = 25 dB		
$\bar{\gamma}_o$	λ_1	λ_2	ϕ_1	λ_1	λ_2	ϕ_1	λ_1	λ_2	ϕ_1
5	2.4304	−59.5204	−2.3444	3.9794	−41.6387	−6.6234	7.7815	−16.3046	−4.1595
10	5.1188	−49.4554	−0.0574	5.1188	−49.4554	−0.0574	8.1291	−18.9	−0.4661
15	6.9897	−37.187	1.1888	6.2839	−50.2694	3.0686	8.6034	−20.0443	2.5729
20	8.293	−27.7608	3.943	6.5321	−45.7984	3.3628	9.1645	−17.2362	3.6639
25	9.4201	−14.7274	4.9073	7.4036	−37.9557	1.8042	9.4201	−14.7274	4.9073
30	10.607	−2.8581	6.4881	7.5967	−32.7393	1.6876	9.7772	−11.7672	5.9282

Table 3 BER comparison for optimized and non-optimized threshold values for $\bar{\gamma}_{rf} = \bar{\gamma}_o$ = 20 dB, and outage probability of 10^{-3}

λ_1	λ_2	ϕ_1	BER	Remark
8.451	−10.5631	5.4208	3.23×10^{-5}	Non-optimized
8.7506	−8.6423	4.4617	3.13×10^{-5}	Optimized
9.0309	−7.1204	3.7064	3.23×10^{-5}	Non-optimized

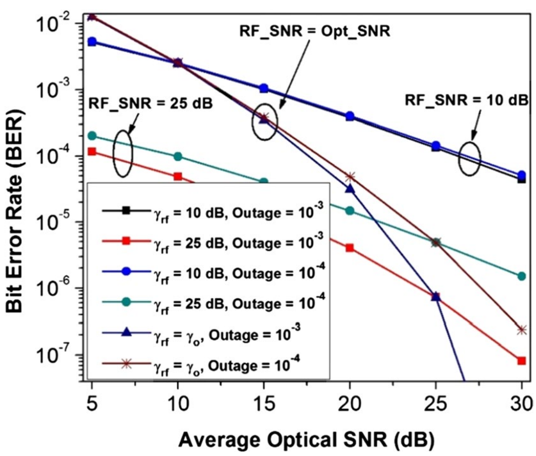

Fig. 3 BER performance of hybrid system versus average optical SNR

yields a larger BER. This validates the statement that optimized thresholds yield minimum BER.

5.2 Minimum BER with optimized thresholds

Figure 3 shows the plot of BER of the hybrid FSO/RF system w.r.t average optical SNR. Here, two cases have been considered (1) when $\bar{\gamma}_{rf}$ is kept fixed at some values (i.e., 10 dB and 25 dB) (2) when $\bar{\gamma}_{rf}$ is varied with $\bar{\gamma}_o$ and $\bar{\gamma}_{rf} = \bar{\gamma}_o$. The BER has been computed from (10). An obvious observation from this plot is that the BER performance of the hybrid FSO/RF system improves for higher values of $\bar{\gamma}_{rf}$ (i.e., 25 dB) in comparison with lower value of $\bar{\gamma}_{rf}$ (i.e., 10 dB), whereas the best performance has been achieved for large value of $\bar{\gamma}_o$ when $\bar{\gamma}_{rf} = \bar{\gamma}_o$.

It may also be observed that for low values of RF SNR (i.e., 10 dB), there is a very slight difference between BER performances for different values of outage probability (i.e., 10^{-3} and 10^{-4}). However, for higher values of RF SNR (i.e., 25 dB), the BER performance degrades for lower outage probability. The above results can be explained as follows: at low RF SNR, the probability of system operating in only RF region is low, which eliminates the impact of RF SNR on BER performance. On the other hand, for higher RF SNR, the probability of operating in only RF region is higher, thus RF SNR shows an impact on BER performance for different values of outage probability.

In general, the outage probability is governed by thresholds. For a given optical SNR and a lower outage probability, the corresponding optical/RF thresholds are low, thus the system operates for lower SNR values leading to a high BER. When the outage probability is high, the thresholds are higher as compared to the previous case. The system now operates only for higher SNR values, hence BER improves.

5.3 Average system SNR

As discussed in Sect. 3, the actual power utilized by the hybrid FSO/RF system needs to be investigated. In Fig. 4, variation in average system SNR of the hybrid FSO/RF system, as given by (8) is presented as a function of the average optical SNR. We see that, in general, average system SNR of the hybrid FSO/RF system is greater than the average optical SNR. At very high average optical SNR, the average system SNR is equal to the average optical SNR since the probability of the hybrid FSO/RF system operating in optical mode is very high, hence, the contribution of RF SNR is negligible. Thus, the average system SNR increases linearly with the increase in optical SNR for higher values of optical SNR. For lower values of optical SNR, the hybrid FSO/RF system operation shifts to the hybrid as well as RF region. This results in usage of RF power in addition to optical power, thus, the actual power utilized by the hybrid FSO/RF system is greater than the average optical power.

It can be observed from Fig. 4 that when outage is 10^{-3}, for a higher average RF SNR (i.e., 25 dB) and lower values of average optical SNR (i.e., below 25 dB), the average

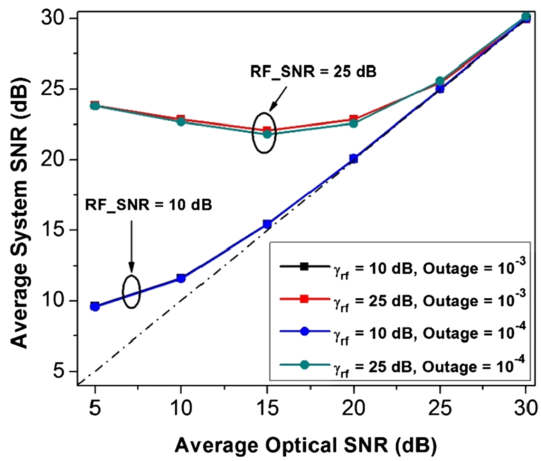

Fig. 4 Variation in average system SNR versus variation in average optical SNR

system SNR is greater than the average optical SNR. Beyond 25 dB of average optical SNR, the average system SNR is equal to the average optical SNR. A similar result is obtained when outage is 10^{-4}. This may be attributed to the fact that while operating in the hybrid or RF mode, greater average power is consumed because average RF SNR is larger. This effect is not prominent in low RF SNR values. Hence, for lower average RF SNR (i.e., 10 dB), the average system SNR has a lesser deviation from the average optical SNR.

Further, it can also be seen that for higher average RF SNR (i.e., 25 dB), the average system SNR is almost constant over a wide range of variation in average optical SNR values. Therefore, two different values of average optical SNR give rise to the same average system SNR. This observation maybe explained as follows:

At low average optical SNR, the hybrid FSO/RF system will operate in either hybrid or RF region. In this case, since the RF SNR is high, probability of the hybrid FSO/RF system operating in RF mode is high. This results in high average system SNR. As the average optical SNR increases, the system moves in hybrid mode, in this situation, the average system SNR depends on both optical as well as RF SNR. As the optical SNR is further increased, the system shifts to optical region and average system SNR is governed by the average optical SNR. Hence, two different optical SNRs will give same average system SNR.

It is worthwhile to investigate the impact of this phenomenon on the BER performance of the system, and will be discussed subsequently.

5.4 Minimum BER versus average system SNR

The power contribution of optical and RF SNRs in the average system SNR is shown in Table 4 and 5 and Figs. 5, 6, 7 and 8 depict the BER versus average system SNR curves.

It can be observed from Fig. 5 that the system gives rise to two different BER performances (point A and B) for the same average system SNR. This observation also corresponds to the fact that two different optical SNRs give rise to the same average system SNR as described above, with reference to Fig. 4. We would like to partition the entire BER curve of Fig. 5 in two segments, to explain this effect as shown in Table 4.

(1) In segment 1, the performance degrades rapidly with increase in average system SNR, this corresponds to lower values of average optical SNR where system primarily operates in RF mode, and thus the RF source mainly contributes to the system power. Further, the thresholds λ_1 and λ_2 are lower for lower values of average optical SNR in order to maintain the required outage probability. Thus, in this situation, the SNR in the hybrid mode of operation (which is $\gamma_o + \gamma_{rf}$) is lower, resulting in higher BER. This region

Table 4 Average system SNR for outage probability = 10^{-3} and RF SNR ($\bar{\gamma}_{rf}$) = 25 dB

$\bar{\gamma}_o$ (dB)	Average system SNR (dB)	BER	Remark
5	23.85	1.15×10^{-04}	Segment 1
10	22.85	4.84×10^{-05}	
12	22.32	3.21×10^{-05}	
15	22.05	1.61×10^{-05}	Critical point
18	22.3	7.25×10^{-06}	Segment 2
20	22.85	4.03×10^{-06}	
22	23.82	2.12×10^{-06}	
25	25.45	7.22×10^{-07}	
30	29.95	8.12×10^{-08}	

Table 5 Average system SNR for outage probability = 10^{-3} and RF SNR ($\bar{\gamma}_{rf}$) = 20 dB

$\bar{\gamma}_o$ (dB)	Average system SNR (dB)	BER	Remark
5	18.7	5.16×10^{-04}	Segment 1
7	18.42	3.90×10^{-04}	
10	18	2.42×10^{-04}	
12	17.98	1.69×10^{-04}	Critical point
15	18.22	9.40×10^{-05}	Segment 2
20	20.8	3.13×10^{-05}	
25	25.08	8.88×10^{-06}	
30	29.94	2.07×10^{-06}	

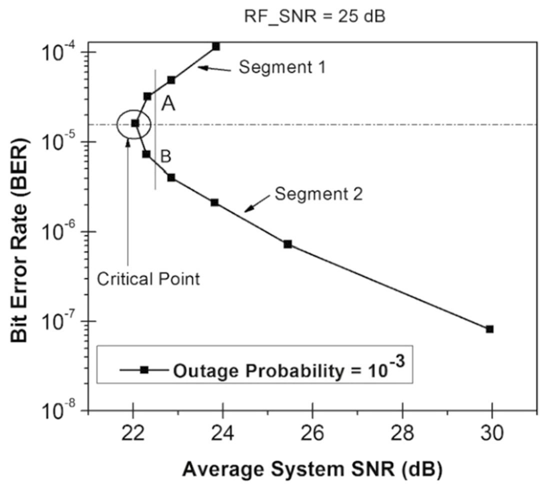

Fig. 5 Error performance versus average system SNR for a fixed average RF SNR ($\bar{\gamma}_{rf}$) = 25 dB

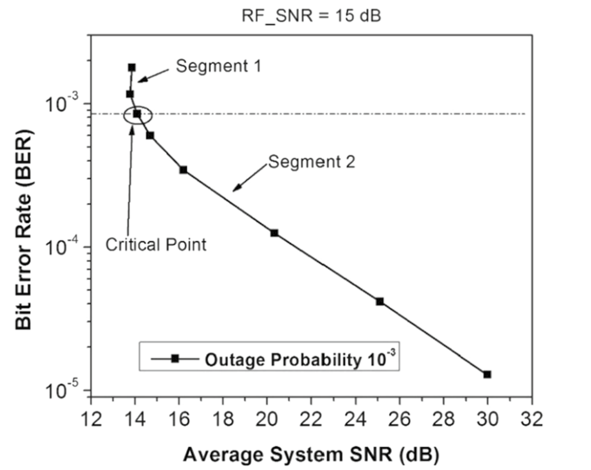

Fig. 7 Error performance w.r.t average system SNR for a fixed average RF SNR ($\bar{\gamma}_{rf}$) = 15 dB

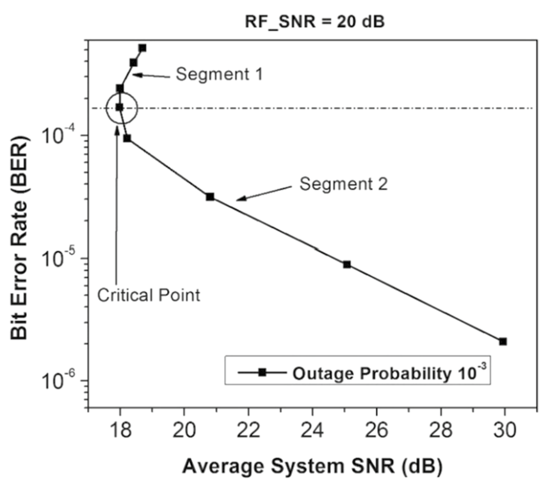

Fig. 6 Error performance w.r.t average system SNR for a fixed average RF SNR ($\bar{\gamma}_{rf}$) = 20 dB

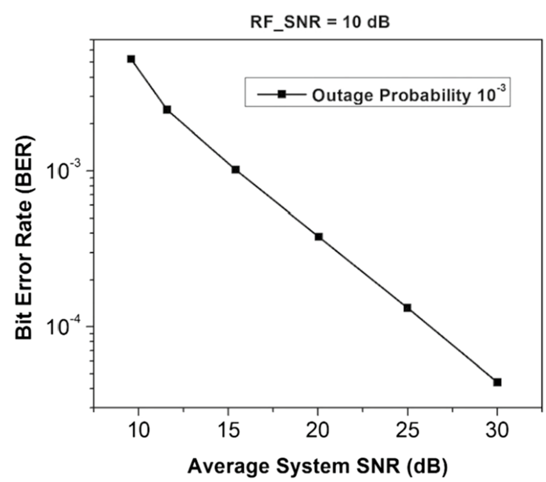

Fig. 8 Error performance w.r.t average system SNR for a fixed average RF SNR ($\bar{\gamma}_{rf}$) = 10 dB

corresponds to optical SNR's in the range of 5–12 dB as marked in Table 4.

(2) The second segment corresponds to higher values of average optical SNR. This region corresponds to optical SNR's in the range of 18–30 dB as marked in Table 4. It can be observed that the performance of the hybrid system improves with increase in average system SNR, this corresponds to high average optical SNR region of Fig. 4, in which system operation is more prominent in the hybrid and optical region.

(3) Based on the above observations, it is recommended to operate in the average optical SNR region which corresponds to the segment 2 of Fig. 5 for better BER performance. This, in turn, motivates us to define a minimum/critical value of average optical SNR for this switching scheme, which corresponds to the minimum allowable value of average optical SNR for a given value of average RF SNR.

From Table 4, we see that $\bar{\gamma}_o = 10$ dB, $\bar{\gamma}_{rf} = 25$ dB and $\bar{\gamma}_o = 20$ dB, $\bar{\gamma}_{rf} = 25$ dB give rise to same value (22.85 dB) of average system SNR. The significant variation in the BER performance at same average system SNR is clearly visible in Fig. 5 and Table 4. In this case, the BER performance of the hybrid FSO/RF system is better when average optical SNR is equal to the average RF SNR, and the system performance improves with an increase in $\bar{\gamma}_o$. Hence, the hybrid FSO/RF system performance has a better performance when $\bar{\gamma}_o \geq \bar{\gamma}_{rf}$. These combinations of $\bar{\gamma}_o, \bar{\gamma}_{RF}$ correspond to segment 2 of Fig. 5.

A similar observation has been made from Table 5 and Fig. 6. The BER value corresponding to segment 2 lie in the

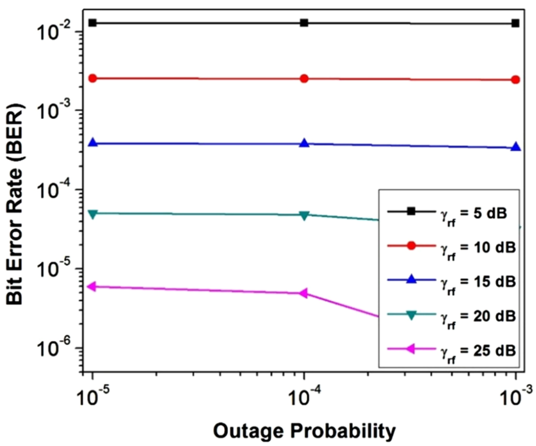

Fig. 9 BER versus outage probability

acceptable range, i.e., the hybrid FSO/RF system should be preferably operated in this segment 2. Here, in segment 2, the average optical SNR is greater than or equal to average RF SNR.

A similar discussion is valid for an average RF SNR of 15 dB and 10 dB. The results are shown in Figs. 7 and 8. The result obtained in Fig. 8 is similar to the result achieved in Fig. 3 for 10 dB RF SNR. This is because, at RF SNR of 10 dB, the average system SNR is approximately equal to average optical SNR as can be seen from Fig. 4. Hence, at very low average RF SNR (~ 10 dB or less), the error performance with respect to average optical SNR is almost identical to the error performance obtained with respect to average system SNR.

5.5 BER versus outage probability

Figure 9 shows the plots of BER with respect to the variation in outage probability when optical SNRs are 5 dB, 10 dB, 15 dB, 20 dB and 25 dB. The system parameters have been chosen as per above discussion, such that the average optical SNR is above the critical point. It is observed that there is a very slight degradation in BER (only at higher values of $\bar{\gamma}_{rf}$) as outage probability of the chosen system decreases from 10^{-3} to 10^{-5}. It is concluded from Fig. 9 that the hybrid system performance can be improved in terms of outage probability without paying any cost on BER by optimized selection of thresholds.

6 Conclusions

In this paper, we have tried to address the important issue of proper/optimized selection of optical and RF SNR thresholds for the modified switching scheme of the hybrid FSO/RF system. The nonlinear constraint optimization problem has been formulated to minimize the BER performance under the constraints on outage probability and transmitted power. The solution of the optimization problem shows that the optimum thresholds are dependent on average optical and RF SNR. Thus, thresholds cannot be chosen arbitrarily.

The actual power consumption comprises contribution from optical as well as RF power. The average system SNR has been defined on the basis of actual power transmission. The actual consumed power is seen to be always greater than or equal to average optical power, and hence, performance should be evaluated with respect to average system SNR rather than average optical/RF SNR.

Numerical results show that various combinations of $\bar{\gamma}_o, \bar{\gamma}_{rf}$ give rise to the same average system SNR, but there is a significant difference in the corresponding BER performances. It is seen that for a given outage probability and average RF SNR, when $\bar{\gamma}_o \geq$ critical optical SNR, there is a remarkable improvement in the BER performance when compared with the scenario of $\bar{\gamma}_o <$ critical optical SNR, for the same average system SNR. This motivates us to recommend that the average optical SNR should always be greater than the critical optical SNR. Finally, the optimal choices of optical and RF thresholds allow some improvement in outage probability with no penalty in terms of BER, by appropriate selection of optical and RF thresholds.

Acknowledgements We would like to thanks the Department of ECE, NSIT, New Delhi, INDIA for providing us necessary tools and facilities for the research.

References

1. Ghassemlooy, G., Popoola, W., Rajbhandari, S.: Optical Wireless Communications: System and Channel Modelling with MATLAB, 1st edn. CRC Press, Boca Raton (2017)
2. Khalighi, M.A., Uysal, M.: Survey on free space optical communication: a communication theory perspective. IEEE Commun. Surv. Tuto. **16**(4), 2231–2258 (2014)
3. Abadi, M.M., Ghassemlooy, Z., Zvanovec, S., Bhatnagar, M.R., Wu, Y.: Hard switching in hybrid FSO/RF link: investigating data rate and link availability. In: IEEE ICC (Workshop 2017), pp. 463–468 (2017)
4. Tapse, H., Borah, D.: Hybrid optical/RF channels characterization and performance study using low-density parity check codes. IEEE Trans. Commun. **57**(11), 3288–3297 (2009)
5. Rakia, T., Yang, H.C., Alouini, M.S., Gebali, F.: Outage analysis of practical FSO/RF hybrid system with adaptive combining. IEEE Commun. Lett. **19**(8), 1366–1369 (2015)
6. Shrivastava, S.K., Sengar, S., Singh, S.P.: A new switching scheme for hybrid FSO/RF communication in the presence of strong atmospheric turbulence. Photon. Netw. Commun. **37**(1), 53–62 (2019)
7. Mai, V.V., Pham, A.T.: Adaptive multi-rate designs for hybrid FSO/RF systems over fading channels. In: Globecom 2014 Workshop on Optical Wireless Communication, pp. 469–474 (2014)
8. Rakia, T., Yang, H.C., Gebali, F., Alouini, M.S.: Power adaptation based on truncated channel inversion for hybrid FSO/RF

transmission with adaptive combining. IEEE Photon. J. **7**(4), 1–13 (2015)
9. Touati, A., Abdaoui, A., Touati, F., Uysal, M., Bouallegue, A.: On the effects of combined atmospheric fading and misalignment on the hybrid FSO/RF transmission. J. Opt. Commun. Netw. **8**(10), 715–725 (2016)
10. Nath, S., Shrivastava, S.K., Sengar, S., Singh, S.P.: Novel architectures for efficient RF usage in hybrid FSO/RF system. In: IEEE International Conference on Advanced Networks and Telecommunications Systems (ANTS), pp. 1–6 (2018).
11. Bag, B., Das, A., Ansari, I.S., Prokes, A., Bose, C., Chandra, A.: Performance analysis of hybrid FSO systems using FSO/RF-FSO link adaptation. IEEE Photon. J. **10**(3), 1–17 (2018)
12. Andrews, L.C., Phillips, R.L.: Laser Beam Propagation Through Random Media, 2nd edn. SPIE Press, Bellingham (2005)
13. Kaushal, H., Jain, V.K., Kar, S.: Free Space Optical Communication, Optical Networks Series. Springer, New Delhi (2017)
14. Djordjevic, G.T., Petkovic, M.I., Spasic, M., Antic, D.S.: Outage capacity of FSO link with pointing errors and link blockage. Opt. Express **24**(1), 219–230 (2016)
15. Popoola, W.O., Ghassemlooy, Z.: BPSK subcarrier intensity modulated free-space optical communications in atmospheric turbulence. J. Lightw. Technol. **27**(8), 967–973 (2009)

Shreesh Kumar Shrivastava holds master's degree in Telecommunication Technology from RGPV University, Bhopal, India, in 2007. He has 8 years teaching experience at UG level. He is currently pursuing a Ph.D. degree in Electronics and Communication Engineering at the University of Delhi, New Delhi. His research interests include free space optics, wireless communication, and multicarrier CDMA.

Sujata Sengar received her B.Sc. Eng. (Hons.) and M.Sc.Eng. (Hons.) degree in Electronics and Communication Engineering from Aligarh Muslim University, Aligarh, India in 1988 and 1990. She holds a Doctoral Degree in Wavelet based multicarrier transmission system from the Indian Institute of Technology, Delhi (IITD), India, in 2014. From 1990 to 1991 she worked as Scientist Fellow at C.E.E.R.I., Pilani. From 1992 to 1994 she worked at Lecturer in Department of Electronics and Communication Engineering, T.I.T. S. Bhiwani. From 1994 to 1997 she worked at Lecturer in Department of Electronics and Communication Engineering, C.R. State College of Engineering, Murthal. From 1997 to 1999 she worked as a Lecturer in the department of Electronics Communication and Computer, at REC Kurukshetra and in 1999 she joined as faculty at Netaji Subhas Institute of Technology, Delhi. Presently she is working as Professor in the Division of Electronics and Communication Engineering, Netaji Subhas University of Technology (formerly Netaji Subhas Institute of Technology), Delhi, India. Her area of interest is speech processing, signal processing, wireless communication.

Shree Prakash Singh received his B.E. degree in Electronics and Communication Engineering from MMM Engineering College, Gorakhpur, India, in 1990 and M.Tech. degree from REC, Kurukshetra, India, in 1994. He holds a Doctoral Degree in Optical Networks from the Indian Institute of Technology, Delhi (IITD), India, in 2007. During 1994–2000, he worked as Lecturer in the Department of Electronics and Computer Engineering at REC Kurukshetra. In October 2000, he joined as Lecturer in the Department of Electronics, Netaji Subhas Institute of Technology (formally known as Delhi Institute of Technology New Delhi). Presently he is Professor at the Division of Electronics and Communication Engineering, Netaji Subhas University of Technology (formerly Netaji Subhas Institute of Technology), New Delhi. His current area of research is Optical Networks, Optical wireless communication.

Soyinka Nath is pursuing her Ph.D. degree in Optical Communications at Delhi University. She received her B.Tech. degree in Electronics and Communications and Master's degree in Digital Communication from IP University. Her research interests include satellite communication, free space optical communication, cognitive radio and optical codes.