# **PHASE BOUNDARIES AND INTERFACIAL ENERGY IN QUASIBINARY BORIDE AND METAL CERAMIC EUTECTIC SYSTEMS**

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*The a priori pseudopotential method is employed to propose a model by which phase boundaries form in quasibinary eutectic systems. Interfacial energy for boride and cermet quasibinary eutectic composites, LaB<sub>6</sub>–TiB<sub>2</sub> and LaB<sub>6</sub>–ZrB<sub>2</sub>, is calculated for different temperatures.* 

*Keywords: eutectic, phase boundary, interfacial energy, a priori pseudopotential.* 

## **INTRODUCTION**

Interactions between atoms at the interface of eutectic phases are studied in the papers [1, 2]. The energy states of the atoms in boundary layers and inside eutectic phases significantly differ. Computational experiments confirm that the eutectic is not a mechanical mixture of the phases but a single system of interacting phases.

Regular interface of the eutectic phases is associated with the minimum energy between components, i.e., interfacial energy. The minimum interfacial energy is reached when a combination of symmetry, orientation, and lattice parameters corresponds to the greatest number of atomic matches [3]. The interface means a transition zone separating two phases or components that are insoluble in each other [4].

The eutectic solidified in normal conditions has an interface of about  $1 \text{ m}^2 \times 1 \text{ cm}^3$  [4]. The extremely large interface resulting from solidification is indicative of eutectic compositions and predetermines their specific properties.

To determine the interfacial characteristics, various theories can be applied: macroscopic (for example, thermodynamics gives average interfacial characteristics); atomistic, allowing conclusions on the interfacial structure and development of geometric and energy models; electron, describing interactions at the boundary between eutectics in terms of quantum mechanics [4, 5].

There is currently no comprehensive description of all interfacial phenomena in eutectic composites. In this regard, it is difficult to provide their comprehensive overview. Moreover, most studies address intergranular boundaries or outer surfaces of the samples [4, 5]. Being so important for eutectic materials, the phase boundaries are just starting to receive researchers' attention. As pointed out in [5], there are no experimental data for heterogeneous interface. For this reason, it is difficult to describe interaction between atoms through the interface, especially in case of unlike phases. The interface between two phases can be coherent or locally coherent. The local coherent regions are presented as 'cells' in [6, 7]. The notion of 'supercell' is introduced for complete description of

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the actual interface. The supercells consist of crystalline lattices of two phases and are separated by a layer of vacuum. Nevertheless, the notion of 'virtual cell' is more productive. The virtual cell is located along the interface between the components, allowing for contact of crystalline lattices forming a quasicoherent boundary. The firstprinciples calculations of energy in the electron–ion system of the components and the entire composite [8, 9] serves as a basis for developing a model to describe the formation of interfaces in eutectic systems.

### **EXPERIMENTAL PROCEDURE, CALCULATION RESULTS, AND DISCUSSION**

In the  $LaB_6–MeB_2$  system, the phases are connected at the interface through boron atoms belonging to two components. There are 0.75 LaB<sub>6</sub> molecule and 0.25 TiB<sub>2</sub>B molecule at the interface in the eutectic LaB<sub>6</sub>–TiB<sub>2</sub> composite. The number of boron atoms is common for the two components at the interface and amounts to five. The common atoms make the boundary strong enough. The number of common atoms depends on lattice parameters of the components. For TiB<sub>2</sub>, ZrB<sub>2</sub>, and HfB<sub>2</sub>, these parameters are very close and contact energies are almost the same, which is not the case for  $VB_2$  and  $CrB_2$ . Because of the small lattice parameter (*c* is the hexagonal lattice parameter of transition metal diborides), the degree of possible contact between the two components increases with higher likelihood that LaB<sub>6</sub> (002), (004), and MeB<sub>2</sub> (002) atomic planes match, leading to greater contact energy. These statements have been supported by calculations.

Since the interface is characterized by the energy of interaction between molecules (atoms) of the two phases, there is excess internal energy in the composite at the interface compared to the energies of the components. The energy of the components and system was calculated applying the pseudopotential method [8, 9]. For this purpose, we used the scheme [10], according to which the total energy of components A and B that are not in interaction becomes

$$
U_1 = (C^2 U_{AA} + CU_A) + ((1 - C)^2 U_{BB} + CU_B). \tag{1}
$$

The  $U_A$  and  $U_B$  values include the kinetic energy of free electron gas, exchange-correlation effect, and electron energy in the first-order perturbation theory by pseudopotential for respective molecules [8];  $U_{AA}$  and  $U_{BB}$  are the energy of interaction between A*–*A and B*–*B; *C* is the concentration of component A.

The energy of the  $(A-B)$  system allowing for interaction between components A and B  $(U_{AB})$  can be written as

$$
U_2 = C^2 U_{AA} + C U_A + (1 - C)^2 U_{BB} + C U_B + 2C(1 - C)U_{AB}.
$$
\n(2)

The excess energy determining the surface factor is eventually equal to

$$
\Delta U = U_1 - U_2 = -2C(1 - C) U_{AB}.
$$
\n(3)

The energy of interaction between the two components  $U_{AB}$  (calculated with the a priori pseudopotential method) can be represented as a sum of interatomic pair potentials  $\Phi_{AB}(\vec{R}_{i,j})$  [8, 9]:

$$
U_{AB} = \frac{1}{N} \sum_{i,j} \Phi_{AB} (\vec{R}_{i,j})
$$
\n(4)

where  $\vec{R}_{i,j} = \vec{R}_i(A) - \vec{R}_j(B)$  is the distance between atoms *i* and *j* belonging to molecules A and B.

Since the energy of interaction between two molecules does not depend on the position of other molecules but is only a function of  $k_F$  (Fermi momentum) or  $Z/\Omega$  [11], we used approximation for the potential of interaction between the molecules similarly to that for the atoms to calculate the energy of interaction between molecules of different types located at distance *R* from each other [10]:

$$
\Phi_{AB}(R) \approx \left[ 18\pi Z_A Z_B \frac{V_A (2k_F) V_B (2k_F)}{k_F^2} \right] \frac{\cos(2k_F R)}{(2k_F R)^3},\tag{5}
$$

where  $Z_A$ ,  $Z_B$  and  $V_A$ ,  $V_B$  are the number of free electrons and pseudopotentials (calculated at  $q = 2k_F$ ) for A and B. The Fermi momentum was determined through the volume and charge per molecule. In this case, the molecule



*Fig. 1*. Unit cell of a virtual crystal: *MNPQ* is the interface of two phases

consists of two parts at the interface: molecule A with concentration  $C$  and molecule B with concentration  $(1 - C)$ . The volume of this molecule is

$$
\Omega = C\Omega_{\rm A} + (1 - C)\Omega_{\rm B},\tag{6}
$$

and the charge is

$$
Z = CZ_A + (1 - C)Z_B,
$$
\n<sup>(7)</sup>

where  $\Omega_A$  and  $\Omega_B$  are the volume of components A and B;  $Z_A$  and  $Z_B$  are the number of external elements of components A and B, respectively. The distance *R* between molecules was determined from the minimum of function (5)  $(k_F \approx 1.09)$ .

To determine the contact surface of two components, we used a 'virtual cell' with volume (6) and charge (7). Figure 1 represents a cube (unit cell of a virtual crystal) and contact surface between the two components. The average interfacial area in a virtual cell can be found if we present  $\Omega = A^3$  (virtual cell parameter,  $A = MQ$ ) and then the contact surface area becomes

$$
S = S_{MNPQ} = A^2. \tag{8}
$$

The interfacial energy of two components per unit contact area is

$$
\gamma = -U_{\rm AB}/S,\tag{9}
$$

or

$$
\gamma = -U_{AB}(R) / \Omega^{2/3}.
$$
\n(10)

TABLE 1. Volume ( $\Omega$ ), Interfacial Area (S), Molecule Interaction Energy ( $U_{AB}$ ), and Interfacial Energy ( $\gamma$ )

in Eutectic Systems	
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*Note:* a.u.—atomic units (here and in Tables 2 and 3).



*Fig. 2.* Interface of  $LaB<sub>6</sub>$  and  $MeB<sub>2</sub>$  atomic planes filled with boron atoms

The results (Table 1) show that the surface contact energy is small in the  $LaB<sub>6</sub>–MeB<sub>2</sub>$  system compared to the  $B_4C$ –Ti $B_2$ ,  $B_4C$ –SiC, and Ti $B_2$ –SiC systems. The energy of contact between the two components is so weak in the  $LaB_6-MeB_2$  system that it can be neglected and the composite can be regarded as a mechanical mixture, which is not the case for the TiB<sub>2</sub>–SiC, B<sub>4</sub>C–SiC, and B<sub>4</sub>C–TiB<sub>2</sub> systems. In the TiB<sub>2</sub>–SiC system, components contact through the TiC clusters, locally appearing at the contact surface. The contact energy is higher in this case, as confirmed by calculations.

Two phases or two components (if they are insoluble) in the quasibinary eutectic systems have different crystalline lattices and different sizes of atoms and molecules. It is rather difficult to account for these effects to determine the interfacial energy for two components in quantum mechanics calculations. To solve this issue, likely contact between crystalline structures of the components should be considered first in determining the contact surface. In the LaB<sub>6</sub>–MeB<sub>2</sub> system, two components are connected through boron atoms. Base MeB<sub>2</sub> (002) and  $LaB<sub>6</sub>$  (004) surfaces are in the same plane (Fig. 2). The boundary is quasicoherent with this contact of the two

System	<i>S</i> , a.u.	$\gamma$ , J/m <sup>2</sup>
$LaB6-TiB2$	38.03	4.928
$LaB_6-ZrB_2$	37.78	5.123
$LaB6-HfB2$	39.15	4.799

TABLE 2. Calculated Surface Areas  $(S)$  and Interfacial Energies  $(\gamma)$  in Eutectic Systems

TABLE 3. Average Volumes and Contact Surface Energies at Different Temperatures in Virtual Crystal Approximation

T, K	$LaB6-TiB2$		$LaB6-ZrB2$	
	$\Omega$ , a.u.	$\gamma$ , J/m <sup>2</sup>	$\Omega$ , a.u.	$\gamma$ , J/m <sup>2</sup>
$\theta$	406.107	3.454	400.980	3.582
300	408.941	3.407	403.770	3.535
500	410.830	3.371	405.613	3.497
750	413.191	3.331	407.891	3.457
1000	415.670	3.290	410.222	3.416
1500	420.273	3.214	414.832	3.336
2000	424.955	3.139	419.440	3.271
2500	429.716	3.056	424.051	3.109
2750	435.011	3.010	428.360	3.106

components. Figure 2 shows the contact of boundaries with a heavy line. The distance between the boron atoms for two structures,  $LaB_6$  and MeB<sub>2</sub> (Me=Ti, Zr, Hf), is almost the same.

Within one LaB<sub>6</sub> unit cell, the length of contact line is approximately 2*d*, where  $d = a\sqrt{2}/4$  is the distance between boron atoms on (004) plane in LaB<sub>6</sub> and *a* is lattice parameter. The contact area,  $S = 2dc$ , corresponds to the minimum contact of two borides in the eutectic system  $(c$  is the hexagonal MeB<sub>2</sub> lattice parameter). The unit cell volume was determined with Eq.  $(6)$  and the energy of interaction between the components  $U_{AB}$  with Eqs.  $(4)$ and (5). The results are presented in Table 2.

The two approaches differ in how the contact surface is determined. The contact surface between two components is average in the 'virtual' crystal model (Fig. 1). The contact energy is  $2.44$  J/m<sup>2</sup> at the maximum contact surface (area being formed with face diagonals). For more accurate determination of the contact surface, the ratio of cell parameters needs to be taken into account. If we consider that contacts with common boron atoms can form in the systems even when the base MeB<sub>2</sub> surfaces do not match the  $\text{LaB}_6(001)$  planes, the 'virtual' cell option can be perceived as the average energy of possible contacts in the systems.

Applying the quasiharmonic approximation model proposed in [12], we calculated the interfacial energy for the systems over a range from zero to eutectic temperature. The interaction energy of the components and the volume per average molecule were obtained for each temperature. The interfacial energy at different temperatures was calculated with the respective equations. The results for the  $LaB_6$ -Ti $B_2$  and  $LaB_6$ -ZrB<sub>2</sub> systems are summarized in Table 3.

## **CONCLUSIONS**

The computational experiment for boride and metal ceramic quasibinary eutectic composites allows the following conclusion: eutectics are not a mechanical mixture of components but represent a system of interacting crystalline components connected with common atoms or atomic clusters at the interface. When composite temperature increases, the interfacial energy decreases but the system retains mechanical properties up to melting points (eutectic temperatures). The interfacial area depends on the ratio of crystalline lattice parameters of the components.

This interfacial structure will be observed at any composition in eutectic systems. The interfacial energy chosen from extremum condition of the thermodynamic function minimizes the system energy only at eutectic composition.

The proposed method can be used to calculate the interfacial energy for any composites provided that the components are insoluble in each other.

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