Philos Stud (2022) 179:1323–1339
https://doi.org/10.1007/s11098-021-01709-x

Being in a position to know

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Accepted: 29 June 2021 / Published online: 30 August 2021

Abstract The concept of being in a position to know is an increasingly popular member of the epistemologist's toolkit. Some have used it as a basis for an account of propositional justification. Others, following Timothy Williamson, have used it as a vehicle for articulating interesting luminosity and anti-luminosity theses. It is tempting to think that while knowledge itself does not obey any closure principles, being in a position to know does. For example, if one knows both p and 'If p then q', but one dies or gets distracted before being able to perform a modus ponens on these items of knowledge and for that reason one does not know q, one is still plausibly in a position to know q. It is also tempting to suppose that, while one does not know all logical truths, one is nevertheless in a position to know every logical truth. Putting these temptations together, we get the view that being in a position to know has a normal modal logic. A recent literature has begun to investigate whether it is a good idea to give in to these twin temptations—in particular the first one. That literature assumes very naturally that one is in a position to know everything one knows and that one is not in a position to know things that one cannot know. It has succeeded in showing that, given the modest closure condition that knowledge is closed under conjunction elimination (or 'distributes over conjunction'), being a position to know cannot satisfy the so-called K axiom (closure of being in a position to know under modus ponens) of normal modal logics. In this paper, we explore the question of the normality of the logic of being in a position to know in a more far-reaching and systematic way. Assuming that being in a position to know entails the possibility of knowing and that knowing entails being in a position to know, we can demonstrate

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radical failures of normality without assuming any closure principles at all for knowledge. (However, as we will indicate, we get further problems if we assume that knowledge is closed under conjunction introduction.) Moreover, the failure of normality cannot be laid at the door of the K axiom for knowledge, since the standard principle NEC of necessitation also fails for being in a position to know. After laying out and explaining our results, we briefly survey the coherent options that remain.

Keywords Being in a position to know · Epistemic justification · Epistemology · Knowledge · Epistemic logic · Modal logic · Presupposition · Factivity

1 Introduction

The concept of being in a position to know is an increasingly popular member of the epistemologist's toolkit. Some have used it as a basis for an account of propositional justification.[1] Others, following Timothy Williamson,[2] have used it as a vehicle for articulating interesting luminosity and anti-luminosity theses. It is tempting to think that, while knowledge itself does not obey any closure principles, being in a position to know does. It is easy enough to see why closure principles for knowledge might be denied: death or distraction may prevent one from knowing the logical consequences of what one knows. Nevertheless, if one knows—and therefore is in a position to know—both p and $p \rightarrow q$, but one dies or gets distracted before being able to perform a *modus ponens* on these items of knowledge and for that reason one never gets to know q, one was still plausibly *in a position* to know q.[3,4] It is also

[1] See especially Rosenkranz (2016b, 2018).

[2] See Williamson (2000: §4.2) and (e.g.) Smithies (2019: Ch. 7). As far as we can tell, the expression 'in a position to know' came to be widely used in general epistemology due to the influence of Williamson's discussion of luminosity, while earlier it (along with variations on 'can know' and 'is able to know') was sometimes used as glosses on the epistemic logician's 'K' operator (see note 5).

[3] See Williamson (2000: 282).

[4] An anonymous referee pointed out (in effect) that such examples might well be used to argue that being in a position to know also isn't closed under any inference rules: It's natural to describe a sudden death scenario with a sentence along the lines of: 'Since he died before he *could* perform the inference, he was not in a position to know its conclusion.' Fair enough, but such examples don't strike us as very decisive. The trouble is that there are many senses of 'in a position to know', and to do its job the proposed counterexamples would have to work not just for *some* sense (content) 'in a position to know' expresses in some context—that's easy—but for *all* senses 'in a position to know' is capable of expressing that are plausibly the ones that philosophers who think 'in a position to know' has a normal modal logic have in mind. Certainly there is at least *a* sense of 'in a position to know' and a corresponding sense of 'could have known' in which its true that someone who is just about to perform a *modus ponens* but is struck dead before completing the inference was in a position to know and could have known its conclusion, and any philosopher who is dead set on having a notion of 'in a position to know' that obeys a normal modal logic can insist that it is *that* sense (or one of those senses) that he or she has in mind. Cf. Lewis (1976: 150): 'Whenever the context leaves it open which facts are relevant, it is possible to equivocate about whether I can speak Finnish'—and likewise about whether the unfortunate subject was in a position to know or could have known the conclusion of the *modus ponens*. And if the philosopher proposing the counterexample fixes the context in a way that leaves no room for equivocation, then the philosopher who

tempting to suppose that, while one does not know all logical truths, one is nevertheless in a position to know every logical truth.[5] Putting these temptations together, we get the view that being in a position to know has a normal modal logic. As evidence of these temptations, we note that the use of normal epistemic logics is sometimes justified by glossing their epistemic operators using ‘in a position to know that’ and synonyms or near synonyms such as ‘can know that’ and ‘able to know that’.[6] A recent literature has begun to investigate whether it is a good idea to give in to these temptations.[7] That literature assumes very naturally that one is in a position to know everything one knows and that one is not in a position to know things that one cannot know. It has succeeded in showing that, given the modest closure condition that knowledge is closed under conjunction elimination (or ‘distributes over conjunction’), being a position to know cannot satisfy the so-called K axiom (closure of being in a position to know under *modus ponens*) of normal modal logics. In this paper, we explore the question of the normality of the logic of being in a position to know in a more far-reaching and systematic way. Assuming that being in a position to know entails the possibility of knowing and that knowing entails being in a position to know, we can demonstrate radical failures of normality for being in a position to know without assuming any closure principles for knowledge. This means that there is no easy way to rescue a normal modal logic for being in a position to know by denying that knowledge obeys any closure principles. (However, as we will indicate, we get further problems if we assume that knowledge is closed under conjunction elimination.) Moreover, the failure of normality cannot be laid at the door of the K axiom for knowledge, since the standard principle NEC of necessitation (or modal generalization) also fails for being in a position to know. After laying out and explaining our results, we briefly survey the coherent options that remain and give some reasons for preferring our favorite option, which is that

Footnote 4 continued
claims to have a normal modal logic-obeying notion of ‘in a position to know’ can fix the context another way and say: ‘We’re talking at cross purposes. You are right that, *given his untimely death*, the subject was not in a position to know the conclusion, but *given his general cognitive capacities* the subject *was* in a position to know the conclusion, and when I say ‘in a position to know’, I mean *in a position to know given the subject’s general cognitive capacities*’’.

[5] By a ‘logical truth’ we mean simply any theorem of the logic characterized by the axioms and rules under consideration; the notion is syntactic, not semantic. For semantic notions of ‘logical truth’, the idea that one is in a position to know every logical truth is no more plausible than the idea that set of ‘logical truths’, in the relevant semantic sense, is axiomatizable (in some cases it is, in others it isn’t). Part of the appeal of the idea that one is in a position to know all logical truths, we take it, comes from the idea that one could in principle prove any of them in a finite number of steps.

[6] For example, Berto and Hawke (2018: 6) use ‘in a position to know’ to gloss the epistemic operators when describing what they call ‘[t]he standard approach to (multi-agent) epistemic logic’. See also Hilpinen’s (1970) use of ‘in a position to know that’ and Williamson’s (1990: 5–10) use of ‘the subject is able to activate knowledge that’. Admittedly some characterizations of being in a position to know are less suggestive of a normal modal logic: notably Williamson’s (2000: 95) remark that what one is in a position to know is ‘open to one’s view, unhidden, even if one does not yet see it’ could easily be taken to suggest that a subject lacking in logical acumen is not able to know some logical consequence of what he knows due to its being hidden from his view. In what follows we hope to offer more decisive objections to the normality assumption. (Thanks to an anonymous referee for discussion here.).

[7] See Heylen (2016) and Rosenkrantz (2016a, 2016b, 2018: 317–318).

'one is in a position to know' is approximately synonymous with 'one can know', and thus it's no surprise that it doesn't behave like a necessity operator.

2 The logic

Our investigation of the logic of being in a position to know will be conducted using a language of propositionally quantified modal logic. The language has an infinite stock of atomic sentences, an infinite stock of propositional variables p, q, r, …, the standard truth-functional connectives, a universal quantifier $\forall p$ for each propositional variable p, the propositional operators K^P ('one is in a position to know that'), K ('one knows that'), $\Box$ ('necessarily'), @ ('actually'), and the usual formation rules and metalinguistic abbreviations (thus $\Diamond\varphi$ is $\neg\Box\neg\varphi$ and $\exists p\varphi$ is $\neg\forall p\neg\varphi$). We will assume the logic characterized by the following rules and axioms, where the final two are standard axioms for the logic of actuality.[8]

(Taut)	All tautologies
(MP)	Modus ponens
(UG)	If $\vdash \varphi \to \psi$, then $\vdash \varphi \to \forall p\psi$, where p is not free in φ
(UI)	$\forall p\varphi \to \varphi[^{\psi}/_{p}]$, where ψ is free for p in φ
(T_K)	$K\varphi \to \varphi$
($K^P/\Diamond$)	$K^P\varphi \to \Diamond K\varphi$
(K/K^P)	$K\varphi \to K^P\varphi$
($T_\Box$)	$\Box\varphi \to \varphi$
($K_\Box$)	$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$
($NEC_\Box$)	If φ is provable using only the axioms and rules already specified, then $\vdash \Box\varphi$.[9]
($T_@$)	$@\varphi \to \varphi$
($RIG_@$)	$\varphi \to \Box@\varphi$

We will call this logic 'L', and will also write '$\vdash \varphi$' for '$\varphi \in$ L'.[10]

Here are the further candidate axioms and rules that will be discussed below.

[8] See Crossley and Humberstone (1977). In higher-order logic, the existence of a unique property of propositions obeying these axioms can be derived from the axiom of choice: see Goodsell and Yli-Vakkuri (2020).

[9] We don't want to be able to necessitate theorems of the logic of actuality, since some of those truths are contingent. Hence the proviso.

[10] Here is a further axiom that may strike some readers as plausible, which says that being in a position to know is factive:

($T_K{}^P$) $K^P\varphi \to \varphi$.

($T_K{}^P$), however, plays no role in our results. We also lean towards the view that ($T_K{}^P$) has false instances (at least insofar as 'in a position to know' being used in an ordinary and not as a term of art), and that its apparent factivity is the result of something like a presupposition: see Sect. 4.

($K_K{}^P$)	$K^P(\varphi \rightarrow \psi) \rightarrow (K^P\varphi \rightarrow K^P\psi)$
($NEC_K{}^P$)	If $\vdash \varphi$, then $\vdash K^P\varphi$
($DIST_K$)	$K(\varphi \wedge \psi) \rightarrow (K\varphi \wedge K\psi)$

We will call the logic that results from adding any axioms or rules $X_1, \ldots, X_n$ to L '$L + X_1 + \cdots + X_n$', and we will write '$X_1, \ldots, X_n \vdash \varphi$' when $L + X_1 + \cdots + X_n$ includes φ. We will say that the logic of a propositional operator P is *normal in* $L + X_1 + \cdots + X_n$ when $L + X_1 + \cdots + X_n$ includes.

(K_P) $\quad P(\varphi \rightarrow \psi) \rightarrow (P\varphi \rightarrow P\psi)$

and is closed under the rule

(NEC_P) $\quad$ If $\vdash \varphi$, then $\vdash P\varphi$

Thus, for example, the logic of K^P in $L + (K_K{}^P) + (NEC_K{}^P)$ is normal, and $L + (K_K{}^P) + (NEC_K{}^P) \vdash K^P(K\varphi \rightarrow \varphi)$, since $\vdash K\varphi \rightarrow \varphi$.

3 Our main results

In this section we will present our main results informally. The formal proofs are included in the "Appendix".

The sentence

α: $\quad \forall p(p \leftrightarrow @p)$

will play a starring role in our discussion. α has two interesting features. First, α is a logical truth. Second, the truth α expresses is extremely modally fragile: if things had been different in any way, it would have been false. After all, α says that everything is as it actually is, and if things had been different in any way, things would not have been as they actually are. Owing to this fragility, one only gets one shot, so to speak, at knowing α.[11,12] If one doesn't know α, then it is impossible for one to know α, since if one doesn't know α, then, if one had known α, things would have been different than they actually are and α would have been false. Thus, if one doesn't know α, then one could not have known α. Ditto for the conjunction of α

[11] When we say that one 'knows φ', where φ is a sentence, we mean that one knows the proposition expressed by φ—that is, we mean what is formalized by $K\varphi$. The sentence α, of course, would have expressed a truth as long as it had its actual character (in the sense of Kaplan 1989), but it would not have expressed the same truth as it actually does.

[12] One should read our 'only gets one shot' in a modal rather than temporal way.

with anything else: if one doesn't know $\alpha \wedge \varphi$, then one cannot know $\alpha \wedge \varphi$. Formally:

(i) $\vdash \alpha$

(ii) $\vdash \neg K\alpha \rightarrow \neg\Diamond K\alpha$

(iii) $\vdash \neg K(\alpha \wedge \varphi) \rightarrow \neg\Diamond K(\alpha \wedge \varphi)$

Let us say that one *conjunctively knows* φ iff one knows the conjunction of φ and some proposition ($\exists pK(p \wedge \varphi)$, abbreviated as $K^{\wedge}\varphi$[13]). Our first result is:

(1) $(\mathrm{K}_K{}^P) + (\mathrm{NEC}_K{}^P) \vdash K^P\varphi \leftrightarrow K^{\wedge}\varphi$

That is: If the logic of being in a position to know is normal, then one is in a position to know something if and only if one conjunctively knows it.

Informal argument: Suppose that the logic of being in a position to know is normal, so that one is in a position to know all logical truths (that is, suppose $(\mathrm{NEC}_K{}^P)$), and being in a position to know the premises of a *modus ponens* entails being in a position to know its conclusion (that is, suppose $(\mathrm{K}_K{}^P)$). Let φ be anything one is in a position to know. Since α is a logical truth, so is $\varphi \rightarrow (\alpha \wedge \varphi)$. By $(\mathrm{NEC}_K{}^P)$, then, it follows that one is in a position to know $\varphi \rightarrow (\alpha \wedge \varphi)$, and by $(\mathrm{K}_K{}^P)$, that one is in a position to know $\alpha \wedge \varphi$. Since one is in a position to know something only if it is possible for one to know it (i.e., by $(K^P/\Diamond)$), it is possible for one to know $\alpha \wedge \varphi$. But, as we just saw in the previous paragraph (by (iii)), it is possible for one to know $\alpha \wedge \varphi$ only if one actually knows $\alpha \wedge \varphi$, and so only if one actually knows the conjunction of φ with something. It follows that if one is in a position to know φ, then one conjunctively knows φ. What we have just seen is that, if the logic of being in a position to know is normal, then if one is in a position to know something, one conjunctively knows it. We can also use normality to establish that one is in a position to know anything one conjunctively knows. If one knows $\alpha \wedge \varphi$, one is in a position to know $\alpha \wedge \varphi$. Since $(\alpha \wedge \varphi) \rightarrow \varphi$ is a logical truth, one is in a position to know it (by the $(\mathrm{NEC}_K{}^P)$ part of normality). Then one is in a position to know φ (by the $(\mathrm{K}_K{}^P)$ part of normality). Assuming normality for being in a position to know we have now established both directions of the biconditional: one is in a position to know something if and only if one conjunctively knows it. This is result (1).

An immediate consequence of (1) is (2):

(2) The logic of $K^{\wedge}$ in $\mathrm{L} + (\mathrm{K}_K{}^P) + (\mathrm{NEC}_K{}^P)$ is normal

[13] In order to smoothen the presentation, we will pretend that $K^{\wedge}$ is an operator in the language. $K^{\wedge}$ is an operator in the standard loose sense in which a formula like $T(x)$ in one free variable is a predicate. If we had λ-expressions in the language we could avoid this loose talk by proving things about the operator $\lambda p.\exists qK(q \wedge p)$, but that would require adding the axioms of the λ-calculus to the logic and applying them in the derivations in the "Appendix"—a significant increase in complexity with little payoff.

That is: if the logic of being in a position to know is normal, then the logic of conjunctive knowledge is normal.

What happens if the logic of being in a position to know is normal and knowledge distributes over conjunction, in the sense that one who knows a conjunction knows each conjunct? The result is arguably even more disturbing than (1). By (1) it already follows that one is in a position to know something if and only if one conjunctively knows it. If knowledge furthermore distributes over conjunction, then one knows everything one conjunctively knows, and it follows that one is in a position to know something if and only if one knows it. This is our third main result:

(3) $(\mathrm{K}_K{}^P) + (\mathrm{NEC}_K{}^P) + (\mathrm{DIST}_K) \vdash K^P\varphi \leftrightarrow K\varphi$

Our fourth main result states an obvious corollary:

(4) The logic of K in $\mathrm{L} + (\mathrm{K}_K{}^P) + (\mathrm{NEC}_K{}^P) + (\mathrm{DIST}_K)$ is normal.

That is: if the logic of being in a position to know is normal and knowledge distributes over conjunction, then the logic of knowledge is normal.

Let us next see what we can show about the individual components of a normal modal logic for being in a position to know, $(\mathrm{NEC}_K{}^P)$ and $(\mathrm{K}_K{}^P)$, beginning with the former.

Suppose, as $(\mathrm{NEC}_K{}^P)$ states, that one is in a position to know every logical truth. Let λ be an arbitrary logical truth. It follows that $\alpha \wedge \lambda$ is a logical truth, and so that one is in a position to know $\alpha \wedge \lambda$, and so that it is possible for one to know $\alpha \wedge \lambda$. But, once again, it is only possible for one to know $\alpha \wedge \lambda$ if one actually knows $\alpha \wedge \lambda$, and so actually conjunctively knows λ. It follows that, if one is in a position to know every logical truth, then one conjunctively knows every logical truth. This is our fifth main result:

(5) $\mathrm{L} + (\mathrm{NEC}_K{}^P)$ is closed under $(\mathrm{NEC}_K{}^\wedge)$

Our sixth main result states the obvious corollary:

(6) $\mathrm{L} + (\mathrm{NEC}_K{}^P) + (\mathrm{DIST}_K)$ is closed under (NEC_K)

That is: if one is in a position to know every logical truth and knowledge distributes over conjunction, then one knows every logical truth.

Let us finally turn to the hypothesis that being in a position to know is closed under *modus ponens* (that is, $(\mathrm{K}_K{}^P)$). That is, if one is in a position to know the premises of a *modus ponens*, then one is in a position to know its conclusion. Our seventh main result is:

$$\text{(7)} \qquad (\mathrm{K}_K{}^P) \vdash K^P(\varphi \to \psi) \to (K^P\varphi \to \Diamond K\psi)$$

This amounts to the observation that this entails that, if one is in a position to know the premises of a *modus ponens*, then—because being in a position to know entails the possibility of knowing—it is possible for one to know its conclusion.

The example of α serves as a useful reminder of how problematic (7) is. Replacing ψ with $\alpha \wedge \varphi$ in (7), we get:

$$\text{(!??)} \qquad K^P(\varphi \to (\alpha \wedge \varphi)) \to (K^P\varphi \to \Diamond K(\alpha \wedge \varphi))$$

There are two main ways to generate counterexamples to (!??). First, insofar as we are willing to countenance any unknown logical truths at all, we should accept that there are cases in which one is in a position to know $\varphi \to (\alpha \wedge \varphi)$ as well as φ but one does not know either $\varphi \to (\alpha \wedge \varphi)$ or $\alpha \wedge \varphi$. The details can be filled in in a variety of plausible ways. Perhaps φ is some humdrum truth (such as 'We had lunch at Scott's Seafood Restaurant in Mayfair on December 7th, 2019') that one knows, and therefore is in a position to know, but, although one is in a position to know $\varphi \to (\alpha \wedge \varphi)$, one has never considered the issue, and for that reason one knows neither $\varphi \to (\alpha \wedge \varphi)$ nor $\alpha \wedge \varphi$. Second, and perhaps even more decisively, consider someone who knows, and so is in a position to know, the logical truth $\varphi \to (\alpha \wedge \varphi)$ but is merely in a position to know φ, knowing neither φ nor $\alpha \wedge \varphi$. Both kinds of case are counterexamples to (!??): by (!??), if one is in a position to know both φ and $\varphi \to (\alpha \wedge \varphi)$, it is possible for one to know $\alpha \wedge \varphi$, which in turn entails that one does know $\alpha \wedge \varphi$.[14]

4 Alternatives

We have seen that the principle that knowledge entails being in a position to know and that being in a position to know entails possibly knowing, which are axioms of our logic:

$$(K/K^P) \qquad K\varphi \to K^P\varphi$$
$$(K^P/\Diamond) \qquad K^P\varphi \to \Diamond K\varphi$$

produce disastrous results when combined with either component of a normal logic for being in a position to know along with a minimal logic of necessity and actuality. We see two main lines of retreat, one of which we find clearly preferable to the other.

But first we will briefly address three general methodological concerns that a number of people have expressed to us in conversation.

The first concern is that our results are uninteresting because, after all, not even the necessity operator can be true of all logical truths (since the actuality operator

[14] The formal result that underwrites these remarks is this:

$$(\mathrm{K}_K{}^P) \vdash K^P(\varphi \to (\alpha \wedge \varphi)) \to (K^P\varphi \to K(\alpha \wedge \varphi)).$$

generates some contingent ones). There is of course an symmetry here: neither 'being in a position to know' nor 'it is necessary that' will be applicable to all logical truths. But there is also an important asymmetry: In the language with @, while one won't be able to apply 'it is necessary that' to all logical truths (though of course one can still apply that operator to all truth-functional tautologies in the expanded language), there will be no counterexamples to closure for 'It is necessary that'. By contrast, in the expanded language, 'one is in a position to know that' is not only inapplicable to certain logical truths but also fails to obey K.

This is all relevant to a second concern someone might raise, namely that our conception of logical truth is too expansive. One might hope to escape the conclusions of this paper by saying 'I prefer a narrow conception of logical truth. I agree with all your principles concerning actuality, but I deny that those principles have the status of logical truths.' It would take us too far afield to properly engage with modes of conceptualizing logical truth that might motivate this restrictive vision.[15] But in any case they do not provide an escape route. Let us be concessionary and adopt the interlocutor's position. There won't be any clear counterexamples to a rule of necessitation for 'being in a position to know' since the operative conception of logical truth is sufficiently narrow as to avoid the counterexamples of the paper. Still, the counterexamples to closure will be unimpugned, since they relied on the truth, not the logical truth, of the principles concerning actuality.

The third concern is that we do not offer a 'semantics', specifically a 'possible-worlds semantics', for our logic. Some contemporary philosophers are so enamored of the model-theoretic apparatus introduced by Kripke for investigating modal logics[16] that they will only accept our proofs—and in particular our proof of α[17]—after we have given them a possible-worlds model theory on which they are valid. The demand is misguided. Beyond establishing consistency (which is not in doubt in this case) model theory—whether done in the possible-worlds style or any other—cannot be used for justifying axioms or rules of inference. It's rather the other way around: the right model theory is whatever the logic being studied is sound and complete on.[18]

The first line of retreat involves rejecting axiom ($K^P/\Diamond$), according to which being a position to know entails the possibility of knowing, and accepting that being

[15] For the record, we realize that they are not 'logical truths' according to the Tarskian permutation-invariance criterion of logicality, but we deny that this has any bearing on the epistemological issues this paper is concerned with: see Goodsell and Yli-Vakkuri (2020) for discussion.

[16] See Kripke (1959) and (1963).

[17] Lines 13–14 in the proof of (3) in the "Appendix". We maintain that the axioms of actuality derive their ultimate justification from the axiom of choice in higher-order logic. See Goodsell and Yli-Vakkuri (2020) for relevant results and Yli-Vakkuri and Hawthorne (forthcoming) for discussion of implications for epistemology.

[18] See Dorr et al. (2021: Ch. 1, Sec. 6) for some discussion, and Goodsell and Yli-Vakkuri (2020) for a more thorough discussion of appropriate uses of possible-worlds model theory.

in a position know has a normal modal logic.[19] Here is one natural way to develop this thought: The logic of being in a position to know is normal, and one is in a position to know a proposition if and only if one knows it under an idealization to perfect rationality. It need not be metaphysically possible for that idealization to hold, and, under a metaphysically impossible idealization, one may know some propositions without knowing all propositions.[20] This way of thinking may have

[19] There are precedents for giving up (K^P/◇) in the work of David Chalmers and Declan Smithies. We have in mind Chalmers' proposal in *Constructing the World* regarding 'ideal a priori warrant': in some cases 'there exists an (ideal a priori) warrant for believing *p* even though the warrant cannot be used to know *p*.' Chalmers' example, too, involves the actuality operator. However, he has in mind a sentence that expresses a proposition necessarily equivalent to the proposition that snow is white in any world (holding its character fixed) in which snow is white:

> When *p* is the proposition expressed by the semantically fragile sentence *S* discussed at the end of the third excursus ('Snow is white iff actually snow is white'), one can argue that there exists a proof of *p* even though it is impossible to use it to prove *p*. In particular, there exists an abstract proof of *S* using the logic of 'actually'. *S* expresses *p* in the actual world, so this abstract proof of *S* is also an abstract proof of *p*. But if one were to *use* the proof to prove *S*, *S* would express *p'* rather than *p*, so one would not prove *p* (Chalmers 2012: 93).

Smithies' idea is that 'being in an epistemic position to know' is can be a 'finkish' disposition to know under certain circumstances—a disposition that would disappear if those circumstances obtained:

> As a rough heuristic, you're in an epistemic position to know that *p* just in case you would know that *p* if your doxastic response to your epistemic position were sufficiently rational. More precisely, you would know that *p* if you were to properly base a doxastically justified belief that *p* on your propositional justification to believe that *p*. This is only a heuristic because there are finkish cases in which you cannot respond rationally to your epistemic position without thereby changing it (Smithies 2019: 349–350).

The paradigm cases of finkish dispositions to *A* in circumstances *C* discussed in the literature, however, are all ones in which it is metaphysically possible for the bearer of the disposition to *A* in *C*: see Lewis (1997) and Yli-Vakkuri (2010).

Note that Smithies' picture seems to be one on which being in a position to know requires being in fact propositionally justified. This fits poorly with the kind of externalism we favor, according to which one might be disposed to form a safe belief in φ and thus be in a position to know but currently have no evidence for φ and, thus, arguably, no propositional justification.

[20] Another way to develop the thought is to go for something along the following lines: Being in a position to know that φ does not entail possibly knowing φ but rather possibly knowing some proposition suitably similar to φ. But this strategy seems even less promising. It is very much out of step with how the concept of being in a position to know is used in the literature, and we have no good idea of how to develop this thought in a systematic and satisfying way.

Yet another strategy borrows an idea from the literature on Fitch's paradox: being in a position to know φ entails possibly knowing $@\varphi$ (see Edgington 1985 and Schlesinger 1985: 103–6). Our concerns about this idea very much mirror Williamson's concerns about the Edgington/Schlesinger proposal (see Williamson 2000: 292–5). One concern is that, on a fine-grained conception of propositions it is extremely difficult to know $@\varphi$ in a counterfactual situation, because, while @ is a convenient guise for singling out the actual world in the actual world, there is no convenient guise for singling out the actual world in counterfactual situations. Of course, on a coarse-grained conception of propositions (according to which necessarily equivalent propositions are identical) this problem doesn't arise, but then $@\varphi$, if true, will be identical to every necessary truth—there being only one—and the new principle would be a terrible surrogate for the old one, since it will say that being in a position to know a fact entails knowing the necessary truth. There is a lot more to say about how coarse-grained conceptions of propositions interacts with the logic of @ and knowledge: see Yli-Vakkuri and Hawthorne (forthcoming) for more.

A third way to develop the thought is to say that being in a position to know φ entails the possibility of knowing a certain proposition under some guise one actually associates with the *sentence* φ. On one some precedent in natural and social science. An idealization to frictionless surfaces may have an explanatory point even if frictionless surfaces are metaphysically impossible. And perhaps an idealization to market economies that are free of certain 'noise' has a point even if such economies are metaphysically impossible. Similarly, for example, one who does not know a logical truth containing α may know α under an idealization to perfect rationality even when it is metaphysically impossible for one to know α. Drop the assumption that being in a position to know entails the possibility of knowing, and weaken our logic by dropping the corresponding axiom ($K^P/\Diamond$), and you will no longer face the logical problems we have presented for far.

The reader should not underestimate the difficulties here, however. The problem with this proposal is that it doesn't allow one to know that one doesn't know a particular logical truth. As long as the logic of being in a position to know is normal—indeed, as long as ($NEC_K{}^P$) holds, with or without ($K_K{}^P$) or ($K^P/\Diamond$)—we have the following result.

(12) If $\vdash \varphi$, then $\vdash K\neg K\varphi \rightarrow (K^P\varphi \wedge K^P\neg K\varphi)$[21]

By (12), whenever φ is a logical truth one knows oneself to not know, one is in a position to know φ and one is in a position to know that one doesn't know φ. This is inconsistent with the conception of being in a position to know we are entertaining, since, according to that conception, the truth of.

$$K^P\varphi \wedge K^P\neg K\varphi$$

amounts to the truth of the *inconsistent*

$$K\varphi \wedge K\neg K\varphi$$

under an idealization to perfect rationality. That idealization may be metaphysically impossible, but it is not (or so its advocates should hope) inconsistent.

Here is a second observation. As long as the logic of being in a position to know is normal, we can prove, even without ($K^P/\Diamond$):

(13) If $\vdash \varphi$, then $\vdash K\neg K\varphi \rightarrow K^P(\varphi \wedge \neg K\varphi)$

By (13), being in a position to know that one does not know a logical truth φ implies being in a position to know the conjunction: φ and one does not know φ.

Footnote 20 continued
version of this proposal the guise could be just that sentence itself, on another it is the Kaplanian character of φ, on a third it is the primary intension of φ, on a fourth it is the language of thought sentence one correlates with φ.

[21] Suppose $K\neg K\varphi$ and $\vdash \varphi$. By (K/K^P), $K^P\neg K\varphi$. Since $\vdash \varphi$, by ($NEC_K{}^P{}_K$), $K^PK\varphi$.

The idealization picture suggests a further principle:

(NEC$_{K}{}^{P}{}_{K}$) If $\vdash \varphi$, then $\vdash K^{P}K\varphi$

The idea is that one not only knows but knows that one knows each logical truth under an idealization to perfect rationality. But if we have (NEC$_{K}{}^{P}{}_{K}$), we certainly cannot have a normal logic for being in a position to know. If we had both, we could prove, again without ($K^{P}/\Diamond$):

(14) If $\vdash \varphi$, then $\vdash K\neg K\varphi \rightarrow K^{P}\bot$

That is, we would get the result that one is in a position to know that one does not know a logical truth only if one is in a position to know a contradiction.

We thus do not think that the idealization picture provides a happy path to normality. Of course there may yet be other grounds for denying $K^{P}\varphi \rightarrow \Diamond K\varphi$, and perhaps those other paths would provide a better basis for normality for K^{P}. We leave it as a challenge to others to find a well motivated package of this sort.

The second line of retreat, which we endorse, involves giving up both components of normality for being in a position to know: (NEC$_{K}{}^{P}$) and (K$_{K}{}^{P}$). This approach is easy to motivate once one notices that 'in a position to know' is a composition of two operators, 'in a position to' and 'know'.

We begin with the former. We propose that '*x* in a position to *F*' is approximately synonymous with '*x* can *F*' or '*x* is able to *F*'. The standard of synonymy we have in mind here is roughly sameness of Kaplanian character. It's not that these expressions have the same semantic content in a context-independent way, but rather that they are capable of expressing approximately the same range of semantic contents in different contexts—plausibly, including varieties of deontic possibility. It's hard to imagine contexts in which

You are not in a position to say what she just said

and

You can't say what she just said

fail to express approximately the same proposition—for example, the proposition that, in view of *a*'s legal obligations, *a* can't say what *b* just said, where *a* is the person being addressed and *b* is the person referred to by 'she'.[22] Thus, 'one in a

[22] See Lewis (1976) and Kratzer (1977) on the flexibility of 'can' and its relation to 'in view of … can' and related constructions like 'given … can'.

position to know that' has approximately the same context-independent meaning (character) as.

$\lambda p.\Diamond Kp$,

with $\Diamond$ interpreted as expressing the modality determined by the context of speech or writing. When the modality is deontic, axiom (K/K^P) ($K\varphi \rightarrow K^P\varphi$) will not hold, but L is a good enough logic of 'in a position to know', since the senses of 'in a position' that are at issue in epistemology when 'in a position' combines with 'know' are never deontic, as far as we can tell.

Note what follows: even in non-deontic senses of 'in a position to', being in a position to know is not factive. And indeed it is easy to find examples of its non-factivity. For example, each of us is in a position to thump this table. And each of us is also in a position to thump this table while knowing that he is thumping it. But then, since neither of us in fact thumps this table, each of is in a position to know something (namely, that he is thumping this table) that is actually false.

Why is the view that 'in a position to know' is factive so tempting? Note first that 'can know' displays the same appearance of factivity. One does not normally say—at least without further elaboration—either that one 'can know', 'could know' (etc.) or that one 'cannot know', 'could not have known' (etc.) φ unless one takes φ to be true. Consider:

John could not have known that we were out of cocktail olives.

One does not even normally assert a subjunctive conditional with 'might/could have known φ' either in the consequent or the antecedent unless one takes φ to be true:

If you had read the emails, you might have known that the meeting was rescheduled.

If I could have known that the meeting was rescheduled, my lateness would not have been excusable.

We are not looking at a special feature of 'in a position to know' but at a special feature of factive verbs, such as 'realize', 'learn', and 'acknowledge'. The result of combining any of these with either a possibility modal or 'in a position to' is an operator that appears to be factive and moreover 'projects' this appearance of factivity out of a variety of constructions when embedded. It's natural enough to describe this kind of 'projected' content as a 'presupposition'. In fact, the term 'factive' was originally introduced for the kind of presupposition triggered by 'know' and other factive verbs, and that this class of verbs is the *paradigm* of the presupposition trigger in the literature.[23]

[23] In a survey article, David Beaver and Bart Geurts begin their (non-alphabetic) list of 'lexical classes widely agreed to be presupposition triggers' with:

- *factives* (Kiparsky and Kiparsky, 1971).
  Berlusconi knows that he is signing the end of Berlusconism.
  → Berlusconi is signing the end of Berlusconism (Beaver and Geurtz, 2011).

Here they are citing the article that initiated the study presupposition in linguistics, Paul Kiparsky and Carol Kiparsky's 'Fact', which in turn begins:

Now we see the solution to the logical mystery: *Of course* being in a position to know does not have a normal modal logic. Its logic is exactly that of $\lambda p.\Diamond Kp$ ('one can know'), with $\Diamond$ restricted by some condition. For a variety of restricting conditions and a variety of propositions p, one can know p and can know $\neg p$, but one can never know $p \wedge \neg p$, as would be required by a normal modal logic.

Philosophers are of course free to use 'in a position to know' as a term of art. Indeed it is quite clear that it is integral to the use of 'in a position to know' in epistemology that it is supposed to be factive. If our conjecture about the use of 'in a position to know' in English is correct, then this means that it is integral to that practice to be using 'in a position to know' in a way that is a little deviant *vis-à-vis* ordinary English. There need be no great shame in that. However, if you do insist on a using 'in a position to know' in a way that secures factivity, you will have some uncomfortable choices to make. For it is also the habit of epistemologists to reason as if 'in a position to know' has a normal modal logic, that being in a position to know is entailed by knowing, and that it entails the possibility of knowing. So, by the results of this paper, something has to give. If one reasons using 'in a position to know' without being clear about the logic of 'being in a position to know' and what one means by it, there is a danger that one will be left with an expression that is too vague to be interesting. And without such clarity, there is also the danger that epistemologists will tend to fall back on their understanding of the ordinary English 'in a position to know' which, if we are right, is unsuitable as a guide to any factive use of that expression.

Footnote 23 continued

> The object of this paper is to explore the interrelationship of the English complement system. Our thesis is that the choice of complement type is in large measure predictable from a number of basic semantic factors. Among these we single out for special attention *presupposition* by the speaker that the complement of the sentence expresses a true proposition (Kiparsky and Kiparsky, 1971: 345, emphasis in the original).

Kiparsky and Kiparsky introduce the phenomenon using 'Two syntactic paradigms', two lists of 'predicates which take sentences as their subjects', labelled '*Factive*' and '*Non-factive*' (*ibid.*). Lauri Karttunen's (1971) classic paper, in which he argues, *contra* Kiparsky and Kiparsky, that the choice of sentential complement-taking verb does not alone determine whether the speaker presupposes the truth of the complement, begins:

> There is a class of verbs that are commonly called 'factive' verbs. …
>
> There is a general agreement that factive verbs involve presuppositions, though it seems that nobody quite understands what we mean by the term 'presupposition' (Karttunen, 1971: 55).

And 'the hallmark of presuppositions, as well as the most thoroughly studied presuppositional phenomenon, is *projection*' (Beaver and Geurtz, 2011)—projection of the kind we saw examples of in the main text.

Appendix: Proofs of the main results

We begin with

(3) $(\mathrm{K}_K{}^P) + (\mathrm{NEC}_K{}^P) + (\mathrm{DIST}_K) \vdash K^P\varphi \leftrightarrow K\varphi$.

Proof Below is an abbreviated derivation of $K^P\varphi \leftrightarrow K\varphi$ in L + $(\mathrm{K}_K{}^P)$ + $(\mathrm{NEC}_K{}^P)$ + (DIST_K), where '**K**' (boldface) indicates provability using classical logic and $(\mathrm{NEC}_\Box)$ from $(\mathrm{K}_\Box)$, and

$\alpha := \forall p(p \leftrightarrow @p)$
$\alpha^* := \neg\mathrm{K}(\alpha \wedge \varphi)$

1. $\alpha^* \rightarrow \Box\ @\alpha^*$	$(\mathrm{RIG}_@)$
2. $\alpha^* \rightarrow \Box\ (\forall p(p \leftrightarrow @p) \rightarrow (\alpha^* \leftrightarrow @\alpha^*))$	(UI), (NEC$\Box$)
3. $\alpha^* \rightarrow \Box((\alpha^* \leftrightarrow @\alpha^*) \leftrightarrow \alpha^*)$	1, **K**
4. $\alpha^* \rightarrow \Box(\forall p(p \leftrightarrow @p) \rightarrow \alpha^*)$	2, 3, **K**
5. $\alpha^* \rightarrow \Box(\alpha \rightarrow \alpha^*)$	4 abbreviated
6. $\neg K(\alpha \wedge \varphi) \rightarrow \Box(\alpha \rightarrow \neg K(\alpha \wedge \varphi))$,	5, with α^* unabbreviated
7. $\neg K(\alpha \wedge \varphi) \rightarrow \Box((\alpha \wedge \varphi) \rightarrow \neg K(\alpha \wedge \varphi))$,	6, K
8. $\Box(K(\alpha \wedge \varphi) \rightarrow (\alpha \wedge \varphi))$	(T_K), $(\mathrm{NEC}_\Box)$
9. $\neg K(\alpha \wedge \varphi) \rightarrow \Box\neg K(\alpha \wedge \varphi)$	7, 8, K
10. $\neg K(\alpha \wedge \varphi) \rightarrow \neg\Diamond K(\alpha \wedge \varphi)$	9, K
11. $\Diamond K(\alpha \wedge \varphi) \rightarrow K(\alpha \wedge \varphi)$	10
12. $\Diamond K(\alpha \wedge \varphi) \rightarrow K\varphi$	11, (DIST_K)
13. $p \leftrightarrow @p$	$(\mathrm{T}_\Box)$, $(\mathrm{T}_@)$ $(\mathrm{RIG}_@)$
14. $\forall p(p \leftrightarrow @p)$	13, (UG)
15. $K^P(\varphi \rightarrow (\alpha \wedge \varphi))$	14, $(\mathrm{NEC}_K{}^P)$
16. $K^P\varphi \rightarrow K^P(\alpha \wedge \varphi)$	15, $(K_K{}^P)$
17. $K^P(\alpha \wedge \varphi) \rightarrow \Diamond K(\alpha \wedge \varphi)$	$(K^P/\Diamond)$
18. $K^P\varphi \rightarrow \Diamond K(\alpha \wedge \varphi)$	16, 17
19. $K^P\varphi \rightarrow K\varphi$	12, 18
20. $K\varphi \rightarrow K^P\varphi$	(K/K^P)
21. $K^P\varphi \leftrightarrow K\varphi$	19, 20

(1) $(\mathrm{K}_K{}^P) + (\mathrm{NEC}_K{}^P) \vdash K^P\varphi \leftrightarrow K^\wedge\varphi$.

Proof To show this, replace line 12 in the above with $K(\alpha \wedge \varphi) \rightarrow \exists pK(p \wedge \varphi)$, where p is not free in φ (which is equivalent by contraposition and quantifier duality to an instance of (UI)), and proceed:

11. $\Diamond K(\alpha \wedge \varphi) \rightarrow K(\alpha \wedge \varphi)$	
12. $K(\alpha \wedge \varphi) \rightarrow \exists pK(p \wedge \varphi)$	UI
13. $p \leftrightarrow @p$	$(\mathrm{T}_\Box)$, $(\mathrm{T}_@)$ $(\mathrm{RIG}_@)$
14. $\forall p(p \leftrightarrow @p)$	13, (UG)
15. α	14 abbreviated

16. $\varphi \rightarrow (\alpha \wedge \varphi))$	15
17. $K^P(\varphi \rightarrow (\alpha \wedge \varphi)))$	16, $(\mathrm{NEC}_K{}^P)$
18. $K^P\varphi \rightarrow K^P(\varphi \wedge \alpha)$	$(\mathrm{K}_K{}^P)$
19. $K^P(\varphi \wedge \alpha) \rightarrow \Diamond K(\varphi \wedge \alpha)$	$(K^P/\Diamond)$
20. $K^P\varphi \rightarrow \exists pK(p \wedge \varphi)$	11, 12, 18, 19
21. $K(p \wedge \varphi) \rightarrow K^P(p \wedge \varphi)$	(K/K^P)
22. $K^P((p \wedge \varphi) \rightarrow \varphi)$	$(\mathrm{NEC}_K{}^P)$
23. $K^P((p \wedge \varphi) \rightarrow \varphi) \rightarrow (K^P(p \wedge \varphi) \rightarrow K^P\varphi)$	$(\mathrm{K}_K{}^P)$
24. $K^P(p \wedge \varphi) \rightarrow K^P\varphi$	22, 23
25. $K(p \wedge \varphi) \rightarrow K^P\varphi$	21, 24
26. $\neg K^P\varphi \rightarrow \neg K(p \wedge \varphi)$	25
27. $\neg K^P\varphi \rightarrow \forall p\neg K(p \wedge \varphi)$	26, (UG)
28. $\neg\forall p\neg K(p \wedge \varphi) \rightarrow K^P\varphi$	27
29. $\exists pK(p \wedge \varphi) \rightarrow K^P\varphi$	28, definition of $\exists p$
30.$\exists pK(p \wedge \varphi) \leftrightarrow K^P\varphi$	29, 20

(2) The logic of $K^\wedge$ in L + $(\mathrm{K}_K{}^P)$ + $(\mathrm{NEC}_K{}^P)$ is normal.

Proof Immediate from (1).

(4) The logic of K in L + $(\mathrm{K}_K{}^P)$ + $(\mathrm{NEC}_K{}^P)$ + (DIST_K) is normal.

Proof Immediate from (3).

(5) L + $(\mathrm{NEC}_K{}^P)$ is closed under $(\mathrm{NEC}_K{}^\wedge)$.

Proof To show this, let $(\mathrm{NEC}_K{}^P) \vdash \varphi$ and continue from line 11 in the proof of (3) as follows.

11. $\Diamond K(\alpha \wedge \varphi) \rightarrow K(\alpha \wedge \varphi)$	
12. $p \leftrightarrow @p$	$(\mathrm{T}_\Box)$, $(\mathrm{T}_@)$ $(\mathrm{RIG}_@)$
13. $\forall p(p \leftrightarrow @p)$	12, (UG)
14. φ	By hypothesis
15. $\varphi \wedge \alpha$	13, 14
16. $K^P(\varphi \wedge \alpha)$	15, $(\mathrm{NEC}_K{}^P)$
17. $K^P(\varphi \wedge \alpha) \rightarrow \Diamond K(\varphi \wedge \alpha)$	$(K^P/\Diamond)$
18. $K(\alpha \wedge \varphi)$	11, 16, 17
19. $K(\alpha \wedge \varphi) \rightarrow \exists pK(p \wedge \varphi)$	UI and quantifier duality
20. $\exists pK(p \wedge \varphi)$	18, 19

(6) L + $(\mathrm{NEC}_K{}^P)$ + (DIST_K) is closed under (NEC_K).

Proof Immediate from the proof of (5).

(7) $(\mathrm{K}_K{}^P) \vdash K^P(\varphi \rightarrow \psi) \rightarrow (K^P\varphi \rightarrow \Diamond K\psi)$.

Proof Immediate from axiom $(K^P/\Diamond)$.

Acknowledgements We would like to thank David Chalmers, Cian Dorr, Kevin Dorst, Julien Dutant, Peter Fritz, Jeremy Goodman, Alex Kaiserman, Amir Kiani, Clayton Littlejohn, Beau Madison Mount, Chunxiao Qu, Alex Roberts, Sven Rosenkrantz, Declan Smithies, Ching Hei Yau, and audiences at the Dianoia Institute of Philosophy at Australian Catholic University, the 2020 Workshop on Philosophical Applications of Modal Logic at the University of Glasgow, and the Oxford Epistemology Reading Group for helpful comments and discussions.

Data Availability This manuscript has associated data in a data repository.

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