Philos Stud (2019) 176:259–295
https://doi.org/10.1007/s11098-017-1015-1

Is reality fundamentally qualitative?

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Published online: 19 December 2017

Abstract Individuals play a prominent role in many metaphysical theories. According to an *individualistic metaphysics*, reality is determined (at least in part) by the pattern of properties and relations that hold between individuals. A number of philosophers have recently brought to attention alternative views in which individuals do not play such a prominent role; in this paper I will investigate one of these alternatives.

Keywords Qualitative · Haecceitism · Nihilism · Individuals

Individuals play a prominent role in many metaphysical theories. According to an *individualistic metaphysics*, reality is determined (at least in part) by the pattern of properties and relations that hold between individuals. A number of philosophers have recently brought to attention alternative views in which individuals do not play such a prominent role; in this paper we will investigate one of these alternatives.[1]

The possible motivations for such views are various. Some are very general: worlds that are qualitatively alike—worlds that differ only concerning which individuals play which qualitative roles—are observationally equivalent. At least

[1] See, for example, O'Leary-Hawthorne and Cortens (1995), Dasgupta (2009), Turner (2011), Rayo (2017) and Russell (2016, 2017).

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one line of thought driving the search for individual free metaphysics is so that we can eliminate observationally equivalent worlds.[2]

By contrast, the views I explore here recreate every physical possibility the individualistic metaphysics postulates: if it postulates two worlds whose differences consist only in which particles occupy which qualitative roles, my theory will recreate surrogates of those possibilities except without the particles.

Other motivations come more directly from physics. In some physical theories (such as classical field theories) the postulation of particles is optional: the theory would work just as well if we took them out. Ontological economy suggests, in these cases, that particles should not be considered part of the fundamental furniture of the world—we should avoid postulating redundant structure where possible. It is interesting to note, then, that many physical theories can be elegantly formulated geometrically in terms of the possible trajectories a physical system can take through a class of states. In this formalism, states are not presupposed to have internal structure—they need not consist of individuals standing in relations to each other—they are rather primitive points, and the structure needed to state the theory is given in terms of the relations between the states and not the intrinsic structure of the states. (Indeed, for some physical theories, such as quantum mechanics, this outlook is sometimes more transparent.) The metaphysical view explored here reflects that perspective on physical theories rather straightforwardly.[3]

Even if you do not find these motivations persuasive, there are still reasons to want to work these views out. The question of whether individuals are necessary for the correct description of reality is open, and interesting in its own right. Moreover, it's healthy for a discipline to occasionally revisit its entrenched presuppositions: even if we are not ultimately moved to revise our individual centric world-view, we might get clearer on why individualistic metaphysics are to be preferred in the process of surveying the alternatives. Finally, there are lessons to be learnt. Those who reject individualistic theories because they posit invisible differences should take caution: removing individuals from your theories doesn't always remove the invisible differences, as views of the sort considered here demonstrate, nor does having individuals automatically generate invisible differences.

Although I will abstract from the physical details, the geometric perspective on the state-spaces of physical theories is the inspiration for the sorts of anti-individualistic views I'll be investigating here. According to these views, fundamentally speaking there is just a bunch of different qualitative states the world can be in: these states stand in various relations to one another, but the

[2] At least historically, such constraints have been motivated by crude theistic or verificationist concerns. Most who endorse such a constraint today, think that it is a defeasible constraint stemming from Occam's-razor-style considerations (see, e.g., Pooley 2013).

[3] There is also a distinct motivation from quantum mechanics—which I won't touch on here—that has to do with the interpretation of states in quantum field theory. In a classical theory there are typically four possibilities for a two particle system where each particle can possess one of two properties F and G. In QFT there arise situations where we would only count three: both F, both G, and one of each. The strategies I'm considering in this paper take metaphysics involving individuals and outputs individual-free surrogates. If it turns out that QFT cannot be described in individualistic terms to begin with, our algorithm will have nothing to say about it.

differences between them are not manifested by differences between objects in those worlds.

There are two challenges this sort of position faces. One is to say precisely what it means to reject individualistic metaphysics. In Sects. 3–6 I outline a version of the view that, although there are individuals and truths about them as our everyday judgments demand, they are not fundamental.[4] To make this idea precise we adopt the framework of higher-order logic: on my preferred way of articulating this view all fundamental structure is determined by things whose type does not involve the type of individuals—the sorts of things that can be specified in a language that contains sentence letters, operators, quantifiers binding into sentence and operator position, and similar devices, but which does not contain things like singular terms, predicates or quantifiers binding into the position of a singular term or predicate. (This constraint precludes a common way of articulating anti-individualistic metaphysics in terms of Quine's functorese, discussed in Sect. 8.)

The second challenge is to give a concrete account of how reality can be described without individuals. We will show how to redescribe a simple physical theory—classical mechanics—without talking about particles or space-time points. The crucial insight here is that although the worlds don't have any internal structure, there is modal structure concerning relations *between* worlds that can be used to formulate the theory. The abstract idea generalizes fairly straightforwardly to other physical theories, and in Sect. 12 we unpack the assumptions needed for these generalizations to work.

1 Qualitativeness

At a first gloss, a proposition is qualitative if it is not *about* any particular individual. By contrast, we shall call a proposition that is not qualitative *haecceitistic*. To illustrate consider 1 and 2:

1. Sparky is an electron.
2. There are electrons.

1 is about a particular thing, Sparky, whereas 2 is not about any particular thing.

The distinction we are after is not a linguistic one—it concerns propositions and individuals—but the linguistic analogy is instructive: a qualitative proposition corresponds roughly to the sort of thing you can express without employing singular terms.[5] A natural way to make this anology precise would be to adopt a metaphysics of propositions in which they are structured in a way that mirrors the way that sentences are structured. Thus a proposition is literally composed of properties, relations and individuals, and is qualitative when in doesn't contain any individuals as constituents.

[4] Compare with Russell's statement of 'quantifier generalism' (Russell 2017).

[5] This analogy isn't perfect. 'Someone is German' doesn't contain a singular term, but expresses a proposition that is indirectly about Germany (it says, roughly, that someone is from Germany).

However we shall go a different route in this paper, and develop a theory that is consistent with a fairly coarse-grained theory of propositions.[6] That is, we shall work under the assumption:

Booleanism Boolean equivalent propositions are identical.

Roughly, Boolean equivalence means an equivalence that is provable in the propositional calculus. For example, unlike the structured theory, $p \wedge q$ and $q \wedge p$ are the same proposition because they are Boolean equivalent. Booleanism is validated, for example, on the view that propositions are sets of worlds. But Booleanism does not entail that necessarily equivalent propositions are identical, and is thus compatible with more fine-grained views of propositions as well.

Rather than analysing qualitativeness in terms of propositional constituency—a notion that's not obviously well-defined given Booleanism—we shall take qualitativeness to be a primitive property of propositions. Given qualitativeness as primitive, we can outline natural axioms. It seems clear, for example, that qualitativeness is closed under the Boolean operations: if p and q are not about any particular individuals, and since conjunction does not introduce reference to any individual $p \wedge q$ is also not about any particular individuals.

Even though we take the notion of qualitativeness as primitive, we can nonetheless connect it to other related concepts, thus widening the circle of analysis.

1. ABOUT NOTHING: Intuitively the proposition that Ruth Barcan Marcus is standing is *about* Ruth Barcan Marcus, and not, say, Saul Kripke. A proposition is qualitative if there is no individual that it is about.
2. QUALITATIVE INDISTINGUISHABILITY: Two worlds are qualitatively indistinguishable if the differences between them consist only in which qualitative roles the individuals occupy. A proposition, conceived as a set of worlds, is qualitative iff it is closed under qualitative indistinguishability: if $w \in p$, and w' is qualitatively indistinguishable from w, then $w' \in p$.
3. QUALITATIVE ISOMORPHISM: The notion of qualitative indistinguishability can likewise be understood in terms of isomorphism. Two worlds, w and w', are qualitatively indistinguishable iff there exists a bijection, ρ, between the individuals in w and w' that preserves all the qualitative properties and relations. That is: for each qualitative relation R, $Ra_1 \ldots a_n$ is true at w iff $R\rho(a_1)\ldots\rho(a_n)$ is true at w'.

Note that the last connection cannot serve as a completely reductive definition of a qualitative proposition as it is ultimately circular: it requires us to already possess the notion of a qualitative relation of arbitrary arity, and a qualitative proposition

[6] One important reason to seek a more coarse-grained theory of propositions is that the naïve version of the structured theory is actually inconsistent due to the Russell–Myhill paradox, and is thus unsuitable as a foundation of the metaphysics of propositions (see, for example, the discussion in Dorr 2016, Section 6). There are consistent coarser-grained theories of propositions that keep some elements of the structured theory: for example Dorr's (2016) theory distinguishes propositions that have a different number of occurrences of a component, and in Goodman's (2017) theory propositions can be different in virtue of the individuals that are their constituents, but both theories ignore other aspects of the structure.

just is a 0-ary qualitative relation. Even though these connections are not reductive definitions, they help us get a handle on the notion, and entail that qualitative propositions have certain non-obvious structural features.[7]

I propose the following model for thinking about these connections, drawing from the technology developed in Fine (1977). The intricacies of this model will not be crucial in what follows, but it is helpful to fix ideas. We suppose, for simplicity, that every world has the same domain of individuals, so that the bijections of QUALITATIVE ISOMORPHISM are just permutations of that single domain. Since qualitatively indistinguishable worlds agree about all qualitative matters, and disagree only concerning which individuals occupy which qualitative roles we may talk of *permuting* the roles that individuals occupy at a world, w, to get a qualitatively indistinguishable world. Given a world, w, and a permutation of individuals, π, we write πw, for the unique world qualitatively like w but where the qualitative role each individual occupies has been permuted according to π.[8] For any two qualitatively indistinguishable worlds, w and v, there is always some permutation such that $\pi w = v$—the permutation that maps a to whichever individual occupies, at v, the qualitative role that a occupies at w.

If a proposition is qualitative it cannot say something that is true at one world, but false at a qualitatively indistinguishable world. For otherwise it would be saying something about particular individuals, since the only differences between qualitatively indistinguishable worlds concern which individuals are doing what. Thus if p is qualitative and $w \in p$, $\pi w \in p$ for any permutation π. An alternative way to say this is that π *fixes* p for every permutation π; i.e. that $\pi p = p$ where πp is defined as $\{\pi w \mid w \in p\}$. (The intuitive action of π on propositions is illustrated as follows: if π maps John to Matthew, then π maps the proposition that John is sitting to the proposition that Matthew is sitting. Thus, intuitively, if p involves no individuals, it will not be moved by any permutation.)

On the other hand, suppose that p says something about John, but doesn't say anything about anyone else—suppose it says that John is sitting. Then consider a world w where John is sitting and Matthew is standing, and the permuted world πw, where John and Matthew have swapped qualitative roles. Then John is sitting at w but not at πw, so our chosen p, *that John is sitting*, is not qualitative. We can also capture the idea that p is about John (in particular) and nobody else, by noting that if John is standing at w, then John is standing at πw for any permutation that fixes John: for given that John plays the same qualitative role at w and πw he is either standing at both or at neither. By contrast, if π is a permutation that fixes Matthew, there is no guarantee that if John is standing at w, he is also standing at πw, for π might switch John for someone else who isn't standing. In summary, p is about John

[7] Such as being closed under Boolean operations. We shall discuss further structural features later.

[8] One might deny that there are worlds like this for all permutations of individuals: perhaps I could not have occupied the qualitative role of a boiled egg, so the permutation that switches me for a boiled egg cannot be applied to the actual world. Perhaps it is not metaphysically possible for me to be a hard-boiled egg, but the relevant sense of possibility at stake here might be a broader one (see Bacon 2017). More restricted versions of this idea can be also be developed: perhaps individuals can be partitioned into *kinds*, and we must restrict attention to permutations of individuals that preserves the kind they belong to.

and nobody else because (i) p is fixed by all permutations that fix John and (ii) p is not fixed by arbitrary permutations (it is not qualitative). More generally[9]:

> ABOUTNESS: p is about a finite collection of individuals, $a_1 \ldots a_n$, iff (i) every permutation that fixes $a_1 \ldots a_n$ fixes p, (ii) this does not hold for any proper subset of $a_1 \ldots a_n$.

According to this definition, a proposition is about nothing whatsoever if it is fixed by all permutations. Thus our model gives a precise way of spelling out the notion of aboutness and the notion of qualitative indistinguishability mentioned above, and shows that being about nothing, and being closed under qualitative indistinguishability are equivalent ways of saying that a proposition is qualitative.

(The definitions above can be generalized from propositions to properties and relations. We may think of an n-ary relation as a set of tuples where $(w, a_1, \ldots, a_n) \in R$ iff $Ra_1..a_n$ is true at w. The notion of qualitative indistinguishability can be extended to tuples: $(w, a_1, \ldots, a_n)$ is isomorphic to $(w', b_1, \ldots, b_n)$ iff there is a qualitative isomorphism ρ between w and w' such that $\rho(a_i) = b_i$. A relation is thus qualitative iff it is closed under qualitative indistinguishability. The definition of aboutness for relations is similar.[10])

2 Are all truths qualitative?

A conspicuous way to articulate the anti-individualistic sentiment is as follows:

Qualitativism All truths are qualitative.

Of course, one particularly radical way to be a qualitativist is to reject the existence of individuals altogether:

Nihilism There are no individuals.

For if there aren't any things, there can't be any facts about those things. One might, however, hope for a version of qualitativism that isn't quite so revisionary: a view that would accept all the ordinary qualitative truths that we are accustomed to,

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[9] The generalization of this definition to infinite sets of individuals is non-trivial. If F is a qualitative property, then $Fa_1 \wedge \cdots \wedge Fa_n$ is intuitively about $a_1 \ldots a_n$. But it is also fixed by any permutation that fixes all individuals except $a_1 \ldots a_n$, since permutating the conjuncts of a conjunction leaves it alone, making this conjunction count additionally as being about every individual except for $a_1 \ldots a_n$ (this is a general version of a counterexample mentioned by Fine (1977), although I got the generalization from Harvey Lederman). This is perhaps desirable, since this proposition can be equivalently expressed (in the constant domain setting) as 'everyone except for $a_{n+1}, a_{n+2}, \ldots$ is F'. Be this as it may, on the assumption that there are infinitely many individuals apart from $a_1 \ldots a_n$, this puzzle falls outside the purview of our definition. Fine also raises an issue to do with infinite disjunctions, and discusses a number of different alternative definitions of aboutness; however, he proves that for propositions concerning only finitely many individuals the definitions are all equivalent.

[10] One feature of my definition of aboutness is that if a proposition p is about distinct individuals $\{a_1 \ldots a_n\}$, then there is some qualitative relation R such that $p = Ra_1 \ldots a_n$. This qualitative relation is not in general unique, however there is always a unique *strongest* qualitative relation that p can be decomposed into.

such as 2 (that there are electrons) whilst rejecting the existence of apparent haecceitistic truths like 1 (that Sparky is an electron).

Is such a moderate qualitativism possible? Could the proposition that something is an electron exist and be true without there being any true propositions about particular electrons? I think a straightforward argument is available against this view. Formalising 'it's qualitative whether p' as 'Qp', and 'x is an electron' as 'Ex' we have:

1. $\forall x \neg QEx$ (assumption)
2. $\exists x Ex$ (assumption)
3. $\exists x(Ex \wedge \neg QEx)$ (from 1 and 2 in free logic)
4. $\exists p(p \wedge \neg Qp)$ (from 3 and propositionally quantified logic).

The first premise states the (obvious) fact that for any individual x it is not qualitative whether x is an electron. Given that there are electrons, it follows that there exists an x such that x is an electron and it's not qualitative x is an electron. From this one can infer that there's at least one truth that's not qualitative.[11]

3 Fundamental Qualitativism and Nihilism

It seems, then, that if we are to accept Qualitativism we must be Nihilists. But I think Qualitativism is not necessarily the only question that is of interest to the metaphysician. When doing fundamental metaphysics ordinary objects are often of secondary concern; usually we are more interested with what there is *fundamentally*. There are thus weakenings of the above theses that are worthy of investigation[12]:

Fundamental Qualitativism All fundamental truths are qualitative.

Fundamental Nihilism There are no fundamental individuals.

The latter claim can be simplified, using singular quantifiers, to 'nothing is fundamental'; however this formulation should not be understood to preclude the idea that we can predicate fundamentality to things occupying sentence position, predicate position, and other non-singular categories. Indeed, if we are to make any sense of Fundamental Nihilism, without relinquishing the notion that there are fundamental facts at all, we will need to draw heavily on the idea that the fundamental structure of the world can be completely described by a collection of fundamental propositions, operators, properties (and so on) that does not include any fundamental individuals.

[11] Note that we must employ some sort of free logic in these contexts so that Nihilism has a chance of being true. Classical logic has the theorem that there is at least one thing: $\exists x\, x = x$.

[12] Fundamental Qualitativism is very close to a principle articulated in Russell (2017), that states that all *determinate* truths are qualitative. However it is not true in general that 'fundamental' and 'determinate' are synonymous: in particular, one can know non-fundamental truths whereas, insofar as I can make sense of the notion, indeterminate propositions are always unknown. For these reasons it is worth distinguishing these different theses; my theory of fundamentality is fleshed out below.

As with Qualitativism and Nihilism, it's natural to think that these stand or fall together. Clearly if there are no fundamental individuals then all haecceitistic truths are about non-fundamental individuals, and it is natural to think these cannot therefore be fundamental truths.[13] Conversely, suppose there is at least one fundamental property, F. If Fundamental Nihilism is false there is a fundamental individual, a. Assuming that fundamentality is closed under application, it follows that Fa is also fundamental. Thus we have a non-qualitative fundamental proposition. If this proposition is true then there are non-qualitative fundamental truths, contradicting Fundamental Qualitativism, and if the proposition is false then its negation is true and, arguably, fundamental, also contradicting Fundamental Qualitativism.[14]

4 Type theory

In order to spell out the notion of fundamentality as it applies to these different semantic categories, it will be convenient to help ourselves to the framework of simple type theory. In the syntax of formal languages there are sharp prohibitions on the ways we can combine expressions of different syntactic categories such as singular terms, sentences and predicates: a singular term cannot take the place of a sentence or predicate, a predicate can't take the place of a sentence or term, and so on. Type theory systematizes these sorts of relations as follows. There are two *basic* types, e and t, corresponding to the types of singular terms and sentences respectively. Whenever σ and τ are types, then there is another *functional* type $\sigma \to \tau$, the type of things that take type σ things as arguments and returns something of type τ. Since a unary predicate returns a sentence when provided with a singular term, its type is $e \to t$; since a binary predicate gives a sentence when provided two singular terms in succession its type is $e \to (e \to t)$, and so on. We can also construct things of higher-types as well: $(e \to t) \to t$ corresponds to a quantifier phrase, since it returns a sentence when given a predicate, $(e \to t) \to (e \to t)$ is a predicate modifier, and so on. We adopt here the convention that type brackets associate to the right, so that we can for example, write $(e \to t) \to e \to t$ instead of $(e \to t) \to (e \to t)$.

A model of type theory consists of a collection of domains D_σ for each type, intuitively representing the candidate denotations for expressions of type σ. There is moreover a collection of binary *application* functions, $App^{\sigma,\tau}(\cdot,\cdot)$, which takes an element f of $D_{\sigma\to\tau}$ and a of D_σ and returns an element of D_τ, which is informally the

[13] I am assuming here the principle that if P is fundamental, and is about some individual a, then a is also fundamental.

[14] Both of these arguments rest on subtleties that were not present in the argument from Qualitativism to Nihilism. Assuming Booleanism, it's not always true that propositions of the form Fa are haecceitistic, since $\lambda x(Fx \vee \neg Fx)a = \top$, and $\top$ is surely qualitative. But plausibly contingent properties like being an electron, do not yield qualitative propositions when applied to individuals. Similarly, no principle I have stated so far guarantees that fundamental truths can't be about non-fundamental individuals, although it seems plausible.

result of applying f to a. D_e and D_t can thus be thought of the set of individuals and propositions respectively. A standard choice for $D_{\sigma\rightarrow\tau}$ is the set of all functions from D_σ to D_τ, and $App^{\sigma,\tau}$ is simply ordinary function application. On this interpretation a predicate of type $e \rightarrow t$, for instance, is interpreted by a function mapping each individual to a proposition. For example, the interpretation of 'is an electron' maps Sparky to the proposition that Sparky is an electron.

It must be noted that these models should not be taken too literally. Functions are particular kinds of abstract individuals, and they can be denoted by singular terms: if they have a place in the real type hierarchy, it's in type e. In particular, the intended interpretation of a predicate cannot be a function without violating the type constraints: a predicate must denote something of type $e \rightarrow t$. It's also worth emphasizing how hard it is to express many of these ideas in English: even the idea of a predicate *denoting* something is strictly speaking incorrect, since grammatically *denotes* takes singular terms as arguments, not predicates. We will continue to speak sloppily in what follows, since it is hard to do otherwise in English, but it is a useful exercise for the reader to think about how this sloppy talk can be restated more carefully in the language of type theory.

5 Fundamentality and qualitativeness

Others who have addressed this question have tried to understand fundamentality in terms of grounding (see Dasgupta 2009), or in terms of indeterminacy (Russell 2017). However, since I reject both of these accounts it will be worth our while to spend a little time being explicit about our theory of fundamentality and qualitativeness.

Fundamentality, as I am understanding it, isn't something that can only be possessed by individuals and propositions—it's a feature that can be possessed by things belonging to any domain of our type hierarchy.

Fundamentality Some collection of individuals, propositions, properties, operators, etc. are *fundamental*, others are not.

In order to model this idea, we pick, for each type σ, a subset Fun_σ of the domain D_σ representing the fundamental entities of that type.

Awkward questions can arise on this conception of fundamentality. For example, in order to specify complex fundamental structure we plausibly might want some of the truth functional connectives to be fundamental. However many equally good bases for the truth functional connectives exist, and consequently there are an embarrassing number of hypotheses about what truth-functional structure is fundamental. One might, for example, conjecture that negation and conjunction are fundamental and that disjunction is a non-fundamental defined notion, or one could instead take disjunction and negation as fundamental and conjunction as defined (see Sider 2011, Chapter 10).

In what follows I shall often have reason to theorize in terms of a related notion. Let $D = \bigcup_\sigma D_\sigma$ be a model of type theory as discussed in the previous section.

Suppose that $X \subseteq D$. Write X^* for the set of elements of D that are *definable* from elements of X.

We elaborate on the notion of definability shortly. Crucial to our conception is that definability is not a linguistic notion but a metaphysical one: we shall use the term *metaphysical definability* to emphasize this. Something is *fundamental** iff it is definable from fundamental individuals, propositions, properties etc., thus even if only negation and disjunction are fundamental, all truth functional connectives should count as fundamental*.[15]

Qualitativeness, like Fundamentality, can be possessed by things belonging to any portion of the type hierarchy:

Qualitativeness Some propositions, properties, operators, etc. are *qualitative*, others are not.

We similarly model this by introducing, for each type σ, a set $Qual_\sigma \subseteq D_\sigma$ corresponding to the qualitative entities of that type. In this case the model must be subject to the constraint that $Qual_e = \emptyset$—given what it means to be qualitative, no individual is qualitative.

The final constraint on our model relates qualitativeness and definability. Anything you can define out of qualitative things is also qualitative. The motivation behind this constraint is intuitive: if individuals aren't involved in the ingredients, as it were, they won't be involved in the output.

Closure of Qualitativeness Anything metaphysically definable from qualitative properties, propositions, operators, etc. is also qualitative.

More concisely: $Qual^* = Qual$. This can be illustrated with examples. For instance, we suggested earlier that the negation of a qualitative proposition is qualitative. This follows from our principle given the plausible assumption that negation is qualitative: if p is qualitative then $\neg p$ can be defined in terms of qualitative things. Similarly, our earlier claim that the proposition that there are electrons is qualitative can be justified from the closure principle and the assumption that electronhood and existential quantification are qualitative.

[15] One might attempt to avoid the awkwardness we raised earlier by insisting that fundamentality is closed under definability, thus eliminating the distinction between fundamentality and fundamentality*. An objection to this idea is that it involves redundancy in what is fundamental. Adopting a metaphor from Sider (2011), when God is writing the 'book of the world', it seems like it would be redundant for him to include negation, disjunction and conjunction, when in fact the first two would have sufficed. However, this metaphor is prone to mislead if we imagine that God's language is like English or standard presentations of first-order logic. One could imagine alternative languages in which there's no difference between including negation and conjunction, negation and disjunction and having all three. Consider, for example, Ramsey's notation for the propositional calculus, in which conjunction and disjunction are represented as $\wedge$ and $\vee$ as usual, and negation is represented by turning the formula upsidedown—it is simply not possible to include $\vee$ and negation in this language without including $\wedge$, and vice versa. A full defense of this conception of fundamentality, as closed under definability, would take us too far afield. We will thus continue to draw the distinction between what is fundamental and fundamental*, but leave it open whether they amount to the same thing.

A pressing shortcoming of the foregoing discussion is that it is unclear what ‘definability’ means in the present context. Definability is usually a linguistic notion, and involves concepts from syntax, such as variables or lambdas, that don’t obviously have any corresponding meaning when we’re talking about reality. We want a metaphysical notion of definability. This difference is even more evident under the assumption of Booleanism, where the structure of reality does not mirror the structure of language in any straightforward sense.

Definability is also usually understood relative to a background collection of primitives, even if that set of primitives is just the logical connectives and quantifiers. Here we are in a context where it is optional whether even the logical connectives are part of the fundamental furniture (how else would we raise the question of whether disjunction can be defined from negation and conjunction?)

We solve both of these problems by offering a non-linguistic criterion for definability: roughly, an element a of D_σ is definable from a set $X \subset \bigcup_\tau D_\tau$, if as behaviour is completely fixed by the behaviour of things in X. There is a criterion in logic which captures this idea more formally.[16] If D is a typed collection of domains, a permutation is a typed collection of permutations π_σ of each domain D_σ with the constraint that $\pi_{\sigma\to\tau}(f) = \pi_\tau \circ f \circ \pi_\sigma^{-1}$. A permutation fixes an element of D if it maps that element to itself.

> THE PERMUTATION CRITERION: a is *metaphysically definable* from X if and only if every permutation of D that fixes all the elements of X fixes a.

Other weaker criteria in a similar vein are possible, but they all share the features I’ll appeal to in what follows.[17]

The fine mechanics of this definition will not be important for our purposes, it will suffice to highlight only a few of its consequences.

Firstly, anything definable purely from λs and variables in the simply typed λ-calculus counts as definable from the empty set, and will thus count as vacuously qualitative. So, for example, if X is a predicate variable (of type $e \to t$) and y an individual variable (type e), then there is something of type $(e \to t) \to e \to t$, *property application* defined as $\lambda X \lambda y Xy$, which takes a property and an individual and returns a proposition of type t—the result of applying that property to that individual. The function in $D_{(e\to t)\to e\to t}$ this picks out is mapped to itself by every permutation of D, and thus is definable from $\emptyset$. Another example of a λ definable thing is operator composition, $\lambda X \lambda Y \lambda x XYx$, which takes two operators X and Y and returns the operator you get by composing them.

[16] The criterion that follows is similar to those explored by McGee (1996) and Fine (1977) (see, especially, Theorem 14). However both of these authors assume that truth functional and quantificational structure are basic; my notion is more general.

[17] A natural alternative is that metaphysical definability means definability from the *combinators*: operations in the type hierarchy that can be defined using only λs and variables (although I’ve described them syntactically here, these operations can be given a non-syntactic characterization). However this conception is somewhat limited—for example, it doesn’t allow us to do things like definition by cases. The Permutation Criterion provides a much more comprehensive list of definitional devices by contrast.

Another consequence of this definition is that qualitativeness is closed under application. Any permutation that fixes a function F of type $\sigma \to \tau$ and fixes an argument a of type σ will also fix the result of applying F to a.[18] Thus, if the quantifier *something* is qualitative (an element of $Qual_{(e\to t)\to t}$) and the property of being an electron is qualitative (an element of $Qual_{e\to t}$), then the proposition that something is an electron is qualitative (a member of $Qual_t$).

The truth-functional connectives are not fixed by every permutation, and so are not metaphysically definable from the empty set: it is a substantive metaphysical hypothesis that they are either qualitative or fundamental* on this conception. On the other hand, any permutation that fixes negation and conjunction fixes every other truth-functional connective. It thus follows that if negation and conjunction *are* qualitative, so are the other truth-functional connectives. In fact this follows from the two observations we made above: that λ definable things are qualitative, and that application preserves qualitativeness. Suppose Y is a variable of the same type as conjunction (that is, $t \to t \to t$), X the same type as negation ($t \to t$) and x and y of propositional type t. Then $\lambda X \lambda Y \lambda x \lambda y\, X(Y(Xx)(Xy))$ is qualitative, because it is λ definable, and if $\neg$ and $\wedge$ are qualitative, then the result of applying the former to the latter two is qualitative: $\lambda x \lambda y \neg(\wedge(\neg x)(\neg y))$. This result, given Booleanism and some plausible background logic, is just disjunction, so we have shown that disjunction is qualitative if negation and conjunction are.[19] It follows by a similar argument that the other truth functional connectives are qualitative too. (The same argument may be used to show that all truth-functional connectives are fundamental*, if any truth-functionally complete set of connectives is.)

Lastly, note that a permutation of a model of type theory is a qualitative isomorphism, in the sense of Sect. 1, iff it fixes (i.e. preserves) all the qualitative propositions, properties, and relations.[20] Thus we can restate our Sect. 1 definition ABOUTNESS, saying when a proposition is about a finite collection of individuals, in our present vernacular as follows: p is about $a_1 \ldots a_n$ iff p is metaphysically definable from $a_1 \ldots a_n$ with the qualitative entities, and is not metaphysically definable from any proper subset of $a_1 \ldots a_n$ with the qualitative entities. It follows, as a special case, that the Closure of Qualitativeness falls out of our toy model of qualitativeness, given in Sect. 1, in which an entity is qualitative iff it's closed under qualitative isomorphisms.

Putting all of this together we can formulate a principle that is more general than either Fundamental Qualitativism or Fundamental Nihilism:

[18] Reason: if $\pi_{\sigma\to\tau}(F) = F$ and $\pi_\sigma(a) = a$, then $Fa = \pi_{\sigma\to\tau}(F)\pi_\sigma(a) = \pi_\tau F \pi_\sigma^{-1} \pi_\sigma a = \pi_\tau(Fa)$.

[19] It follows by Booleanism that $\vee(x)(y) = \neg(\wedge(\neg x)(\neg y))$. It follows that $\lambda y \lambda x \vee (x)(y) = \lambda y \lambda x \neg(\wedge(\neg x)(\neg y))$ by the ξ rule, which says that if $\vdash \alpha = \beta$ then $\vdash \lambda x\, \alpha = \lambda x\, \beta$ (this step can also be made using the functionality principle discussed in Sect. 8, but this is stronger than we need). Finally $\vee = \lambda y \lambda x \neg(\wedge(\neg x)(\neg y))$ follows from two applications of the η-rule, which says that $\lambda x\, \alpha x = \alpha$, to the left hand side.

[20] In order to make this precise we assume that the model is one in which D^t, the domain of propositions, just consists of all sets of possible worlds. We will also assume that the Boolean connectives (including the infinitary Boolean connectives) are qualitative and, for simplicity, that the domain of each world is the same, so that qualitative isomorphisms are just permutations. Then it can be shown that the qualitative isomorphisms of Sect. 1 are exactly the permutations defined here that fix the qualitative entities.

All fundamental entities are qualitative: for every type σ, $Fun_\sigma \subseteq Qual_\sigma$.

Note that this entails Fundamental Nihilism, since we know that $Fun_e \subseteq Qual_e = \emptyset$, and Fundamental Qualitativism since $Fun_t \subseteq Qual_t$, and so every fundamental proposition, and thus every fundamental truth, is qualitative.

Note, finally, that since qualitativeness is closed under metaphysical definability, it follows that everything definable from fundamental things must also be qualitative: $Fun^* \subseteq Qual$. This puts some non-trivial constraints on what can be fundamental. To illustrate this constraint, note that if you can define any propositions from fundamental entities, then there can't be any fundamental things of type $t \rightarrow e$, for if $f \in Fun_{t \rightarrow e}$ and $p \in Fun_t$, then we could construct a fundamental*, and thus qualitative, individual $f(p) \in Fun_e$.[21]

6 Supervenience

It's worth noting that it is an entrenched piece of orthodoxy that, if the notion of fundamentality is to do the job it's supposed to, everything should be metaphysically definable from the fundamental. On this picture Fundamental Qualitativism would entail Qualitativism, and thus, given our earlier argument, Nihilism. For assuming Fundamental Qualitativism all fundamental things are qualitative. But if everything is definable from the fundamental, then everything is definable from the qualitative. By the closure of qualitativeness under metaphysical definability it follows that everything is qualitative—in particular, that all true propositions are qualitative. Thus, if we are to explore Fundamental Qualitativism without it collapsing into Nihilism, we must reject the orthodoxy! We should not think that all propositions are definable from fundamental entities.

The entrenched view notwithstanding, I believe this conclusion to be welcome. For arguably most ordinary statements do not express propositions that can be defined from the fundamental. The proposition that there are tables and chairs, for example, is *vague*, and thus cannot be defined from completely fundamental entities. For surely fundamental entities are precise, as are things definable in terms of precise things.[22] Since most of the propositions ordinary people assert are vague, few of these propositions can be defined in terms of the fundamental.

Since one cannot define haecceitistic truths from qualitative ones, one might wonder what the relation between the non-fundamental haecceitistic propositions and the qualitative propositions is. One might attempt to fill this lacuna by appealing to a notion of ground. However such theories typically rest on structured theories of propositions, and are not particularly friendly to Booleanism. For example, theories of ground often maintain that conjunctions are always grounded by their conjuncts, and never vice versa. However given Booleanism, the conjunction $p \wedge (q \vee \neg q)$ is

[21] An example of an expression of type $t \rightarrow e$ is the complementizer *that*. When you apply 'that' to a sentence, you get back something that behaves a bit like a singular term (e.g. 'that snow is white').

[22] That is, we assume that precision is also closed under metaphysical definability. We also assume that there are vague propositions (as opposed to merely vague sentences), as defended in Bacon (2018).

identical to its conjunct p, and so by Leibniz's law, p can only ground $p \wedge (q \vee \neg q)$ if also $p \wedge (q \vee \neg q)$ grounds p.

I shall continue to explore a relatively coarse-grained theory of propositions, and will adopt a modal formulation of the connection:

Supervenience on the Qualitative All propositions supervene on the qualitative propositions.

Here 'supervenience' just means that every truth is metaphysically necessitated by some qualitative truth. This is just one form of supervenience, stating a modal constraint between the domain of propositions and the qualitative propositions. One could also explore supervenience theses at other types: for example, one could explore the idea that all properties (of type $e \rightarrow t$) supervene on qualitative properties; in the higher-order setting there are also similar supervenience theses concerning higher-order properties and relations, and so on.

Evidently the supervenience of all truths on the qualitative propositions follows, given Fundamental Qualitativism, from another attractive supervenience thesis:

Supervenience on the Fundamental Everything supervenes on the fundamental.

As before, there are lots of different ways to formulate this. We will simply focus on the thesis that every truth is necessitated by some truth that can be metaphysically defined from fundamental entities.[23] I will simply refer to these two supervenience ideas as Supervenience.

We will show shortly that, assuming the Boolean connectives are qualitative, every proposition is necessarily equivalent to a qualitative proposition. Thus a suggestive surrogate for grounding would be necessary equivalence: every non-fundamental haecceitistic proposition is necessarily equivalent to a qualitative proposition.

One immediate consequence of Supervenience, along with the plausible hypothesis that not all truths are qualitative, is that this sort of theory is committed to the use of metaphysically impossible worlds. Suppose for the sake of argument that in addition to Booleanism we assume that propositions form a *complete atomic* Boolean algebra, so that each proposition is effectively isomorphic with the set of maximally strong consistent propositions (the atoms) that entail it. Let's call these maximally strong consistent propositions *world propositions*. Now suppose p is a haecceitistic proposition that is necessarily equivalent to some qualitative proposition q. Since p and q are distinct, it follows from general facts about atomic Boolean algebras, that there's some world proposition w that entails p but not q or vice versa. But since p and q are necessarily equivalent, and thus are entailed by exactly the same metaphysically possible world propositions, it follows that w must be a world proposition that is metaphysically impossible. Thus, just like the

[23] One way in which this would be true would be if each truth was simply necessitated by a fundamental truth. However, my formulation is neutral on the question of whether there *are* any fundamental propositions. The idea that there are no fundamental propositions might flow naturally from a 'no redundancy' conception of fundamentality. For example, it would be redundant to include the proposition that Sparky is an electron as fundamental, if Sparky and electronhood are fundamental.

grounding theory, our theory of fundamentality requires a hyperintensional theory of propositions (in the sense that necessarily equivalent propositions needn't be identical) but unlike those theories we keep Booleanism.

We suggested earlier that the Boolean connectives are qualitative. Given this assumption we can shed some light on the structure of the qualitative propositions, $Qual_t$. Here is one important consequence:

Closure of qualitative propositions under Boolean operations If p and q are qualitative, so is $\neg p$, $p \wedge q$ and other Boolean combinations of p and q.

Recall that above we showed that qualitativeness is closed under application. So it follows that if p and $\wedge$ are qualitative that $\wedge(p)$ is qualitative, and moreover if q is also qualitative it follows that $\wedge(p)(q)$ is qualitative—i.e. whenever p and q are qualitative so is their conjunction. Similar conclusions follow for the other Boolean connectives.

Similar arguments show that if infinitary disjunction and conjunction are qualitative, infinite disjunctions and conjunctions of qualitative propositions are also qualitative. Lastly, it follows that if the underlying algebra of propositions is complete and atomic, then the algebra of qualitative propositions is also atomic.[24] Putting this all together, the qualitative propositions form a complete atomic Boolean algebra under the Boolean operations.

It is now routine to show that, given Supervenience, every proposition is necessarily equivalent to a qualitative proposition. Given any proposition p, the weakest qualitative proposition, q, that necessitates p will be necessarily equivalent to p. For if there were a metaphysical possibility where p was true but q false, then the maximally strong consistent qualitative proposition that contains that metaphysical possibility could be disjoined with q and the resulting disjunction would also metaphysically necessitate p, contradicting the assumption that q was the weakest qualitative proposition with this feature.[25]

Let us briefly pause to assess the significance of this result. One might worry that this fact erases any important difference between Fundamental Nihilism/Qualitativism and ordinary Nihilism/Qualitativism. For we have learned, given Supervenience, that even if there are non-qualitative propositions they are always modally equivalent to qualitative propositions. One might thus wonder what work the non-

[24] If the underlying algebra of propositions is complete and atomic, then for any world proposition w (possible or impossible), the conjunction of qualitative propositions w entails is also a consistent qualitative proposition that w entails. Since every world proposition entails a maximally consistent qualitative proposition it follows that the algebra is atomic.

[25] Here is the argument in a little more detail. Suppose w is a metaphysical possibility where p is true and q false. Since the qualitative propositions are complete, the conjunction, r, of all qualitative propositions entailed by w is qualitative, and settles any other qualitative proposition by entailing either it or its negation. If w' were another metaphysical possibility where r is true, then w and w' would agree about all qualitative matters but disagree about some truth (e.g. whether w obtains or not). This contradicts supervenience, so w is the only metaphysical possibility where r is true. Every metaphysical possibility where $q \vee r$ is true is either identical to w or a world where p is true. Thus any metaphysical possibility that makes $q \vee r$ true is a metaphysical possibility that makes p true. So $q \vee r$ is a qualitative proposition that necessitates p, contradicting our assumption that q was the weakest such proposition.

qualitative propositions do for us, since they can always be replaced, at least in modal contexts, by qualitative surrogates. To this charge I must concede, of course, that as far as doing fundamental metaphysics is concerned there is no role for non-qualitative propositions. But this is, after all, exactly the point of Fundamental Nihilism: that while haecceitistic propositions exist, they do not feature in fundamental theorizing about the world. But there are many non-fundamental sciences where hyperintensional differences are important. Semantics is one example, and an advantage of Fundamental Nihilism is that one can give a straightforward semantics for ordinary sentence involving reference to and quantification over individuals. It's also worth noting that lots of contemporary metaphysics is concerned with the relation between the fundamental and non-fundamental, and this is often understood in hyperintensional terms.

We can form an intuitive picture of the relation between the qualitative and haecceitistic as follows. Given the atomicity of the qualitative propositions, it follows that logical space can be carved up into equivalence classes, or *cells*, consisting of maximally strong consistent qualitative propositions. A qualitative proposition is any proposition that is a union of cells, and haecceitistic propositions can sometimes cut across these cells. Moreover, assuming that metaphysical modality goes by a logic of S5, it follows that the modal accessibility relation will also partition logical space into equivalence classes, where x and y are equivalent if according to x, y is possible.[26] Given the supervenience of everything on the qualitative, it follows that the intersection of any modal equivalence class with any cell cannot contain more than one world. For worlds in the same cell agree about all qualitative matters, and if two worlds in the qualitative same cell were modally accessible to one another, that would mean there's two metaphysical possibilities consistent with the same qualitative facts, contradicting Supervenience.[27] See Fig. 1.

7 Why can't we be nihilists about propositions?

We are now in a position to investigate Fundamental Qualitativism and Fundamental Nihilism. As we noted earlier, it's natural to think that these stand or fall together.

[26] Given S5 this relation is reflexive, symmetric and transitive.

[27] An anonymous referee has noted that the above appears to contradict something claimed in the introduction: that our theory recreates metaphysical possibilities corresponding to the swapping of qualitative roles in an individualistic metaphysics (although the recreated possibilities will be qualitatively different). One might be tempted to think this because each world in a cell 'corresponds', as it were, to the different ways of switching the roles of individuals in a individualistic theory. If only one world in a cell is metaphysically possible, then it looks like there can't be two metaphysically possible worlds corresponding to a switching of qualitative roles. However, the above reasoning does not preclude there being two metaphysically possible, but switched, worlds in different cells. Indeed, in our reconstruction of these haecceitistic possibilities, there are primitive qualitative propositions that distinguish them, forcing them to belong to different cells. We shall treat this more thoroughly later; see Fig. 4 and the surrounding discussion.

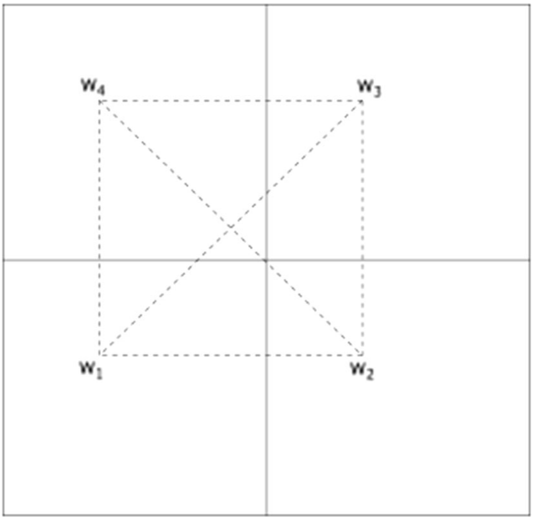

Fig. 1 The qualitative propositions carve out logical space into four disjoint cells, where worlds within a cell agree about all qualitative matters. $w_1, \ldots, w_4$ represent an equivalence class of worlds that are metaphysically possible relative to one another. No more than one member of an equivalence class belongs to any cell

The picture that arises from such a view is that there are no fundamental things of type e. This follows from our general principle that the fundamental is qualitative: in particular, Fun_e is empty since $Fun_e \subseteq Qual_e = \emptyset$. Given the closure constraint on qualitativeness this means that nothing of type e can be defined from fundamental entities either: if one can define an individual from some entities, those entities are not qualitative, and are thus not all fundamental.

In order for such a view to be viable, there must be a rich set of fundamental entities belonging to other types. To illustrate this point, consider our thesis that everything supervenes on the fundamental. A metaphysics on which there are no fundamental entities in any type would not be able to accommodate this supervenience idea. In particular, it follows that there are no fundamental (or even fundamental*) propositions of type t. But since there are many metaphysical possibilities (more than one, at least) there simply aren't enough fundamental (or fundamental*) propositions to make supervenience true.

In order to respect Supervenience, it has to be possible to specify every metaphysical possibility using only fundamental operations. That is to say, for every metaphysical possibility w one has to be able to find a (consistent) fundamental* proposition that necessitates it. (Such a proposition 'uniquely specifies' a possible world w in the sense that w is the only metaphysical possibility consistent with it. But there will in general be other impossible worlds that are also consistent with that proposition—recall that in general at most one world in any cell is metaphysically possible.) If you can't construct enough propositions from the fundamental things, then supervenience is false.

In the context of Nihilism—the view that there are no individuals, fundamental or otherwise—a similar constraint emerges. According to this theory sentences involving first-order existential quantification are strictly speaking *false*, but one needs some sort of theory telling us why they are nonetheless assertable or helpful. This usually goes by way of a *paraphrase*: for every false individualistic sentence, there is a truth in the vicinity that doesn't entail the existence of individuals. In this case one needs a rich set of *propositions*, fundamental or otherwise, in order to for there to be enough truths around to paraphrase each false sentence. Roughly, one needs there to be as many true propositions around as there *would have been* had the individualistic theory been true.[28] And so it is crucial to realize that if this view is to be viable, the domains of non-individual type have to be populated. Although there are many parallels, there are some important differences between the reconstructive projects for Nihilism and for Fundamental Nihilism, so I shall keep both in the discussion in what follows.

8 Functorese

The standard way to populate the domain of things of type t is to postulate fundamental properties and relations (such as, for example, the property of being an electron, of type $e \to t$) and fundamental individuals (such as Sparky), from which one can construct propositions (such as, Sparky is an electron). But there are alternative ways to populate the domain of things of type t that don't involve postulating any fundamental things of type e. One proposal that has received a lot of attention recently is *functorese*.[29] As with an individualistic metaphysics, one takes some properties and relations as fundamental: that is, we populate the domains of unary predicates of type $e \to t$, binary predicates of type $e \to (e \to t)$ and so on. However instead of introducing fundamental individuals we introduce *predicate* functors. Syntactically, a predicate functor is something that takes a predicate, or a sequence of predicates to another predicate of possibly a different arity (one treats sentences as 0-ary predicates). In the present type theory, an example of a predicate functor is something of type $(e \to t) \to t$, that takes unary predicate to a 0-ary predicate. Thus one can see how one could construct a proposition from a fundamental predicate and a fundamental predicate functor without invoking any

[28] It's worth considering why this is so. Couldn't we get by with two propositions, a single falsehood and a single truth which is the target of each false but seemingly true individualistic sentence? I take it that one of the reasons for pursuing a paraphrase strategy is to explain why it's often useful to assert falsehoods. Suppose that there is both a bear and a rabbit behind you: it's more urgent that I warn you that there's a bear behind you, than that I warn you that there's a rabbit behind you. If these warnings had the same true paraphrase we couldn't explain this urgency. These considerations suggest that the nihilist needs there to be as many propositions as there *would have been* had the individualistic theory been true.

[29] See Dasgupta (2009), Turner (2011). The basic idea was first articulated by Quine, and axiomatizations were later provided by Bacon (1985) and Kuhn (1983). The project is also closely related to the program of eliminating variables from theories, originating in Curry's combinatory (Curry 1958). (See also variable free approaches in linguistics) (see Jacobson 1999).

fundamental individuals. Functorese can also be deployed to assist the Nihilist, this time postulating properties and predicate functors to avoid individuals altogether.

Here are some examples. To conjoin two predicates F and G in the simply typed λ-calculus we need singular variables: we first apply both predicates to a singular variable, x, and form a conjunction $Fx \wedge Gx$, and then we λ abstract to get another predicate $\lambda x(Fx \wedge Gx)$. In functorese this would be treated with a primitive predicate functor, of type $(e \to t) \to (e \to t) \to e \to t$, which takes two predicates and directly returns their conjunction. There is also a primitive predicate functor Δ which when given a predicate returns a sentence: roughly if C is the predicate of being a cat, then $\Delta(C)$ expresses what we would ordinarily express by saying 'something is a cat'.[30] (Turner 2011, for example, suggests we pronounce this proposition 'it is catting', in analogy with expressions like 'it is raining' or 'it is cold'. The latter sorts of sentences do not overtly predicate a property to any particular thing, they merely say that certain features are manifest (such as cold) without saying that any particular individuals have those features.)

This language offers useful resources for both the Nihilist and the Fundamental Nihilist. It's possible to prove that no individuals can be defined from predicates and the standard predicate functors.[31] Yet clearly many propositions can be defined from these resources, suggesting that there might be a way to specify each metaphysical possibility using only predicates and predicate functors.

A simple way to see this is possible is to introduce some non-standard predicate functors to play the role of individuals. For any individual—say, Socrates—there is a predicate functor of *Socratizing*, of type $(e \to t) \to t$, which expresses the operation that takes a property as argument and returns (what we would ordinarily describe as) the proposition that property is instantiated by Socrates. (Of course, it cannot really return the proposition that Socrates instantiates the property, since there is no such thing as Socrates on this picture—the thing returned is merely a surrogate for propositions about Socrates.) Now suppose that we have a description of each metaphysical possibility in individualistic terms—using (non-fundamental) individuals and properties. It's possible to find an equivalent description of each possibility by invoking the same properties and relations, but replacing each individual by its associated predicate functor instead. (Thinking about it another way, this view is a version of the *bundle theory*, in which individuals are identified

[30] Since I am presenting a version of functorese that is consistent with the simple theory of types outlined in Sect. 4, there are some minor differences between my presentation and standard presentations. For example, in the usual version of functorese Δ is an operation that takes a predicate of any arity greater than 0, and returns another predicate of one less arity. As such this operation has no type, since the arity of the argument of a functor is uniquely determined by its type. Technically in the present setting we need an infinite collection of predicate functors Δ_n, one for each arity.

[31] For every individual you can construct a permutation that moves it, but fixes all the standard predicate functors. Indeed for any permutation π of D_e, there is a permutation of the entire structure generated by π on the individuals, and the identity permutation on the propositions, which fixes all the predicate functors.

with collections of properties; in this variant they are properties of properties, things that map properties to propositions.[32])

Let us examine the application of functorese in the context of Nihilism and Fundamental Nihilism, in turn. Although functorese has been touted as a valuable tool for the Nihilist, there are some serious obstacles to this approach to reconstructing reality in Nihilistically acceptable terms. Probably the most urgent problem is that the functorese replacement for singular quantification is not obviously any different from the orthodox account of quantification that assumes individuals. That account, which traces back to Frege, similarly treats singular quantification as a kind of predicate functor—an operation of type $(e \to t) \to t)$—and leaves the task of dealing with variables to the λ abstractor.[33] The main difference between functorese and that account of quantification is not in the treatment of the quantifier, but in the handling of jobs typically dealt with by using variables—functorese does it in a variable free way (see Turner 2011). If this is right, then it is not at all obvious that functorese is as ontologically innocent as it appears to be.

The functorese nihilist also falls afoul of an attractive functionality principle, capturing the intuition that properties are completely determined by their application behavior:

Functionality If $Fx = Gx$ for every x then $F = G$.

Here F and G are predicates, and x a singular variable, but the principle can be generalized to any functional type. Functionality is guaranteed in the models we described earlier by the fact that we identified the domain $D_{e \to t}$ as a set of functions from individuals to propositions.

The reason Functionality is troublesome for the functorese Nihilist is that if there are no individuals it follows that there is at most one property (since the antecedent of Functionality is always vacuously true). But if there is at most one property then we can't draw the distinctions necessary to single out all metaphysical possibilities in the way that the functorese nihilist has suggested. If being an electron and being a proton are the same property, for example, then we can't describe in functorese a world where there are electrons but no protons.

Functionality should be distinguished from the much stronger (and much less plausible) extensionality principle, which says that *coextensive* properties are identical—the principle that if $Fx \leftrightarrow Gx$ for every x then $F = G$. Functionality is sometimes opposed on the grounds that there is contingent existence (see Bealer 1989): suppose, for the sake of argument, that for any actual individual the proposition that they are a unicorn is the inconsistent proposition $(\forall x(Ux = \bot))$, but there could have been an individual such that it is not inconsistent that they are a unicorn—then the property of being a unicorn is not identical to the inconsistent property even though they map the same actual individuals to the same propositions.

[32] The view in which individuals are *sets* of properties, as opposed to properties of properties, is not particularly plausible anyway since, on the assumption that an individual has a property if it contains it, it follows that individuals have all their properties essentially.

[33] See for example Carpenter (1997, Chapter 2).

But even in this case there is a weakening of functionality that is also quite attractive (also suggested in Bealer 1989):

Modalized Functionality If it's logically necessary that $Fx = Gx$ for every x then $F = G$

Here a proposition is a 'logical necessity' if it is the same proposition as the Boolean tautology, $\top$.[34] This principle is true in natural possible world, variable domain, models of type theory that don't posit any logically impossible individuals.[35] By a similar argument it follows that if being an electron and being a proton are different properties, then it's at least logically possible that Nihilism is false, for the principle entails that it's logically possible that there's a proton that's not an electron or an electron that's not a proton.

The core insight that these functionality principles capture is the idea that a property is essentially determined by its behavior on individuals (even if, perhaps, some of those individuals only possibly exist). Differences between properties have to be grounded in differences between what those properties do. The Nihilist wants there to be primitive differences between properties like being an electron and being a proton, even though these properties have exactly the same behavior.[36]

It is worth noting that functionality does not only present a problem for the functorese Nihilist, but for other anti-individualistic views that try to construct the world out of properties: an example of this sort is a version of the bundle theory that rejects individuals but attempts to reconstruct them as bundles of properties.

Let us now turn to Fundamental Nihilism. It should be noted that neither of the above objections apply straightforwardly to the Fundamental Nihilist. A Nihilist who uses functorese appears to be quantifying over individuals when they say things like $\Delta(E)$, undermining the central Nihilist thesis. But the Fundamental Nihilist already accepts the existence of individuals and should have no problem with this. She could drop all pretense that Δ is anything other than the existential quantifier, and maintain that the existential quantifier is fundamental while no individual is fundamental or definable from the fundamental. As we saw earlier it is perfectly consistent to maintain that there are fundamental properties, that Δ, the existential quantifier, and the other standard predicate functors are fundamental, and simultaneously maintain that no individuals are fundamental or definable from the fundamental.

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[34] The operator $= \top$ behaves like a very broad necessity operator—see Bacon (2017). In the present context it corresponds to be true in all possible and impossible worlds.

[35] The feature of these sorts of models that is important for validating Modalized Functionality is that properties are uniquely determined by functions from worlds and individuals to truth values.

[36] An anonymous referee has suggested to me a version of functorese nihilism in which predicates are not fundamentally analyzed in terms of things of type $e \rightarrow t$ at all, but rather in terms of things whose types only involve ts. I am broadly sympathetic to this sort of response, and I pursue a similar line of thought in Sect. 9. But it is not clear to me that any such view can really be counted as a version of predicate functorese, since, as I have been using the term, a predicate simply is something of type $e \rightarrow t$. (Of course, if the view described in Sect. 11 can be understood as a version of predicate functorese under a more liberal interpretation of 'predicate', then I have no objection to predicate functors; but this is just a verbal issue.)

More specific problems arise, however, if we are pursuing particular reconstructive strategies involving non-standard predicate functors. For example, even though the non-standard predicate functors associated with individuals aren't individuals, it follows by the Permutation Criterion that the associated individual is definable in terms of it: any permutation that fixes the Sparky predicate functor, for example, also fixes Sparky.[37] (Recall that, unlike the Nihilist, there is such a thing as Sparky for the Fundamental Nihilist.) Thus any account that postulates the Sparky predicate functor as a fundamental predicate functor must also accept Sparky as a fundamentally definable individual.[38] However, for all we've said, it may be possible to reconstruct individualistic metaphysical theories without appealing to individualistic predicate functors. The viability of a functorese version of Fundamental Nihilism thus strikes me as open.

9 The propositional hierarchy

Recall that in our model we made the assumption that the domain $D_{\sigma\to\tau}$ is a set of functions with domain D_σ and codomain D_τ—an assumption which encodes the Functionality principle. Given Nihilism and our modeling assumptions, it follows that the domain D_σ of any type σ containing an e will be boring in a certain sense: it will either be empty, have exactly one element, or be equivalent to a type that only involves ts. Here are three representative examples: any type of the form $\sigma \to e$ with σ non-empty will be empty; any type of the form $e \to \sigma$ will have at most one element (the empty function with empty domain); and any type of the form $(e \to \sigma) \to \sigma$ will be in one-one correspondence with σ. In the last case, the correspondence is given by a simple rule: each element a of D_σ can be identified with the function in $D_{(e\to\sigma)\to\sigma}$ that maps the only element of $D_{e\to\sigma}$ to a, and vice versa.[39]

[37] Here we assume that the $D_{e\to t}$ consists of all functions from D_e to D_t. One can show this by considering the result of applying the Sparky predicate functor to the haecceity of Sparky: the property that maps Sparky to $\top$ and every other element of D_e to $\bot$.

[38] This argument relies on the permutation criterion for definability. A weaker criterion that delivers many of the same results identifies definability with definability using combinators (i.e. things definable using only variables and λ). In the latter case it can be proven that no individuals can be defined from *any* set of predicates and predicate functors. This follows from the Curry-Howard isomorphism, which states that at least one expression of type τ can be defined from things of type $\sigma_1 \ldots \sigma_n$ iff τ is provable from $\sigma_1 \ldots \sigma_n$ in intuitionistic logic, treating the $\to$ symbol as the conditional and e and t as propositional letters. It is easy to see that e cannot be proved from the types of predicates and predicate functors. Consider the following classical model (which is therefore also a model of intuitionistic logic): e is false and t is true. Every n-ary predicate type—t, $e \to t$, $e \to e \to t$, and so on—is easily verified to be true. A predicate functor is of the form $\sigma_1 \to \sigma_2 \to \cdots \sigma_n \to \tau$, where $\sigma_1, \ldots, \sigma_n, \tau$ are all predicates. It follows that the type of any predicate functor is true in this model, since the type of any predicate is true. Since e is false one cannot prove e from the types of predicates and predicate functors.

[39] More generally, every e-involving relational type is equivalent to a non-e-involving relational type. The relational types are defined as follows: t is a relational type, and if σ and τ are relational types then so are $\sigma \to \tau$ and $e \to \tau$. First we define a translation from relational types to relational types over base types t and 1: $t^* \mapsto t$, $(\sigma \to \tau)^* \mapsto \sigma^* \to \tau^*$, $(e \to \tau)^* \mapsto 1$. Then we define a translation from this type signature to the types only involving t as follows: $t' \mapsto t$, $(1 \to \sigma)' \mapsto \sigma'$, $(\sigma \to \tau)' \mapsto \sigma' \to \tau'$. Finally, the result of

This suggests that if we are to recover all the metaphysical possibilities in a Nihilistically friendly way, we should look for devices whose types only involve t. Thus we should *not* be looking at things like predicate functors. Devices we should concentrate on include things taking sentence position (type t), operator position (type $t \to t$), quantifiers into sentence position (type $(t \to t) \to t$), quantifiers into operator position (type $((t \to t) \to t) \to t$), and so on; we shall call the types of these devices hereditarily propositional types.[40]

An analogous project exists for reconciling Fundamental Nihilism with Supervenience. This time we don't merely ban individuals from being fundamental, but any devices of types involving individuals, including properties and predicate functors:

Fundamental Propositionalism The fundamental operations have hereditarily propositional type.

Note that, even if Fundamental Propositionalism is true, it's not true that all things definable from fundamental operations have hereditarily propositional type. For example, predicate identity $\lambda X\, X$ of type $(e \to t) \to e \to t$ is definable from the fundamental (indeed, it's definable from nothing), but does not have hereditarily propositional type.

As we noted above, it's extremely natural for a Nihilist to be a Propositionalist given our functional model theory. By contrast, there's no similarly straightforward argument from Fundamental Nihilism to Fundamental Propositionalism—we saw, for example, that it's consistent even with our strong notion of metaphysical definability that there are fundamental properties and relations, that the first-order existential quantifier and the Boolean operations are fundamental, and that no individuals are fundamental or definable from the fundamental. However, even though we are not forced to be a Propositionalist, it is a natural avenue to explore nonetheless.

10 A reconstruction of classical mechanics

There are many ways to be a Fundamental Propositionalist. For instance, for each possibility that we would ordinarily describe in terms of individuals and their properties, we could postulate a fundamental qualitative proposition saying that things are that way. Such a theory is infinitary, and it's unlikely that any true laws will have a simple or even finitary form. As with all metaphysical theorizing, we must appeal to considerations such as power and simplicity to adjudicate between

Footnote 39 continued
composing these two translations results maps each e-involving type to a type only involving t. A simple induction shows that there is a bijection between D_σ and $D_{\sigma^{*\prime}}$ where we let $D_e = \emptyset$, $D_1 = \{*\}$, D_t may be any set, and we take the full space of functions when we define $D_{\sigma \to \tau}$. (I suspect a more complicated argument would establish a similar result for arbitrary types.)

[40] More formally, t is of hereditarily propositional type, and if σ and τ are of hereditarily propositional type, so is $\sigma \to \tau$.

these possible theories. In particular, if it turned out the above sort of theory is the best the Propositionalist could muster then I think that would be a strong consideration against Propositionalism.

The challenge at hand is to show how to take a concrete individualistic theory and give it a Propositionalist redescription that is of comparable simplicity. A natural place to look for a precisely circumscribed individualistic theory of this sort is physics. In what follows I shall show how to redescribe classical mechanics—a theory usually stated in terms of particles and space-time points—in a way that does away with individuals.[41] I tease out the assumptions needed to make it work for an arbitrary individualistic theory in Sect. 12. The strategy described here takes an individualistic theory and outputs a nihilistic one. However, the application to Fundamental Nihilism is equally straightforward; I sketch how to do that at the end of this section.

According to classical particle mechanics the state of the world at any given time can be completely described by stating the location and momentum of every particle at that time. Moreover every possible assignment of locations and momenta to particles corresponds to some possible state of the world at some time.[42] The heart of classical mechanics consists of a collection of laws telling us how these states evolve over time—it will tell us, for example, what state the world will be in ten minutes given the state it is in now, and things like that. Taken at face value, classical mechanics is committed to an individualistic metaphysics. It freely quantifies over particles and space-time points so it is inconsistent with Nihilism. Moreover, if the theory as stated above is any guide to how things are fundamentally, it suggests that at least particles and space-time points are fundamental, perhaps also momenta and the mathematical objects needed to state the theory.

Now let us suppose that all of the states posited by the theory outlined above exist, but let us suppose that in accordance with Nihilism (and unlike the description above) the states do not consist of individuals with different locations and momenta. On this picture each state represents a qualitative way for the world to be, and there is a different state for each individualistic description of the world. However these different ways for the world to be are not different in virtue of any individual's being different, they are merely *different*—when two states are different there are qualitative propositions that are true at one but not the other, and there is no more to it than that. See Fig. 2.

Now it is natural to wonder how one could go about reconstructing a theory like classical mechanics in this setting. How, for example, would we write down laws that will tell us, given the qualitative state of the universe now, what the qualitative state will be in ten minutes time? Since, on this picture, states don't have any intrinsic structure it is hard to see how to get any traction: we don't have any way to

[41] This sort of redescription is similar to the sort of strategy outlined in broad strokes by Sider (2008) (for a slightly different purpose). Thanks to a referee for pointing this connection out to me.

[42] This therefore includes states where two particles are assigned the same location. Such states are in fact needed in order to account for collisions between point particles, although this raises some subtle issues that we don't need to get into.

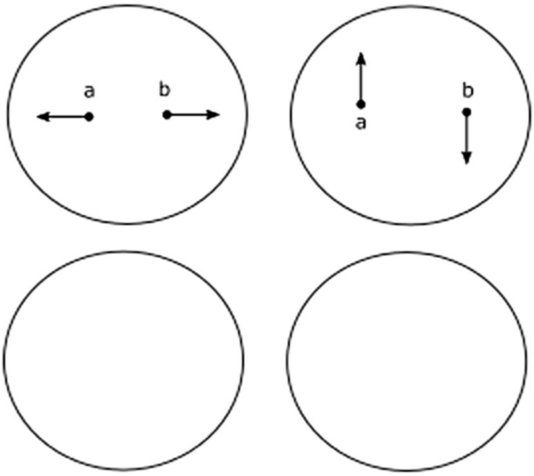

Fig. 2 On top: two classical worlds depicting particles, *a* and *b*, with two different location and momentum properties. Below: the nihilistic surrogates of these worlds

say *how* the states differ, and thus we seem to have no parameters with which to say how the state changes over time. (To make the challenge vivid, consider the two nihilistic worlds depicted in Fig. 2. What would it mean to say that the state depicted by the left world is part of a legal future from the present state, say, and not the right one?)

Luckily there is a reformulation of classical mechanics, *Hamiltonian mechanics*, that can help us see how this can be achieved. The basic idea is that although states don't have intrinsic structure, the set of states as a whole has its own geometric structure.

This is how the Hamiltonian reformulation of classical mechanics works. Recall that every possible state involving *N* particles can be completely specified by assigning each particle a position in a three-dimensional space of locations, and a position in a three-dimensional space of momenta. Thus the entire space of states has 6*N* dimensions, and inherits its geometric structure from the structure of the location and momentum spaces (both of which are three dimensional spaces with the structure of $\mathbb{R}^3$). There is, for example, a well-defined notion of distance between states: two states will be close if the differences between each particle's positions and momenta in the two states is small, and gets bigger as those pairwise comparisons get larger. There is also the notion of a smooth path throughout state-space, and the dynamics of classical mechanics will single out, for any state, a smooth path traveling through that state representing the unique legal history that is compatible with that state.

Each state can be assigned a total measure of energy—something that depends on the momenta of the particles, and the relations they bear to the ambient fields. This forms a scalar field over the state space called the *Hamiltonian*: each state is assigned a real value representing its total energy. Since energy is conserved over time, a dynamically legal path through state-space will trace out a line over a surface of constant energy in state-space. One of the primary innovations of the Hamiltonian formulation of mechanics, is that the direction and speed of these legal paths throughout state-space can be completely determined by the total energy at neighboring states. Roughly if the total energy of states in a neighborhood of a given state is decreasing in some direction at some rate, then the path the system traces out through that state will evolve in a direction perpendicular to that direction, at the same rate.

Ultimately the mathematical details of this formulation do not matter. What matters is that we have a formulation of classical mechanics that is stated entirely in terms of a geometric structure defined over the possible *states* of the system, and not in terms of the intrinsic structure of the states. Moreover, since states can be thought of as maximally strong consistent propositions, relations between and properties of states can be understood in terms of connectives—things which, incidentally, all have hereditarily propositional types. Universal propositional quantification and universal quantification over operators and connectives, all have hereditarily propositional types as well, thus all of this structure can be fixed by a set of primitives that belong only to hereditarily propositional types. For example, the metric structure of state-space can be recovered with the following connectives, called *betweenness* and *congruence* (given also the truth functional connectives and quantifiers into sentence position):

- A ternary connective $Bet: t \rightarrow t \rightarrow t \rightarrow t$, that maps p, q, r to the set of all states if p, q and r are all world propositions, and q lies on a straight line in phase space between p and r, and to the empty set of states otherwise.
- A quaternary connective $Cong: t \rightarrow t \rightarrow t \rightarrow t \rightarrow t$, that maps p, q, r, s to the set of all states if they are all world propositions, and the distance between p and q is the same as the distance between r and s, and to the emptyset of states otherwise.

A similar pair of connectives can be employed to determine the structure of the Hamiltonian. The analogue of *Bet* tells us that the value of the Hamiltonian at world q is between the values at p and r, and the analogue of *Cong* tells us that the gap between the values at p and q is the same as the gap between the values at r and s. Facts about congruence and betweenness are not only sufficient to pin down the structure of the metric and Hamiltonian, but they also provide us with a rich enough language to formulate differential equations and state laws in a relatively simple fashion.[43]

Let us now briefly look at how this machinery could be leveraged to provide a version of classical mechanics suitable for the Fundamental Nihilist. This time we produce, for each world the classical theory posits, a proposition giving a complete description of the world in all fundamental, qualitative matters (see the ‘cells’ of Fig. 1). We want each of these cells to be qualitative: if they contain an (im)possible

[43] See Field (1980). Field shows how to state derivatives using congruence and betweenness relations, which suffice for us to be able to state the two differential equations of the Hamiltonian formulation of classical mechanics.

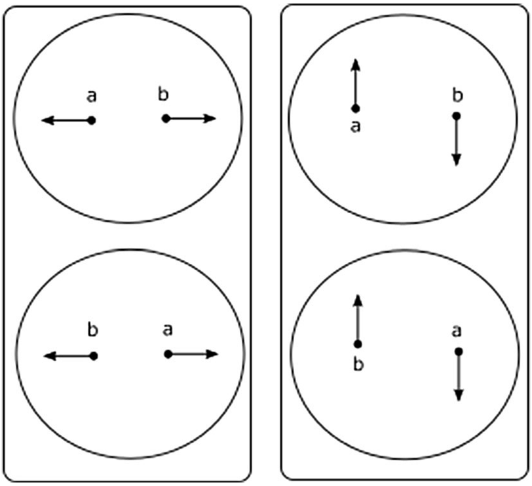

Fig. 3 The qualitative surrogates of the two classical worlds depicted in Fig. 2. Two 'cells' (i.e. maximally strong consistent qualitative propositions) closed under qualitative isomorphisms

world, they contain every (im)possible world qualitatively indistinguishable from it, so the cells will have the sort of structure depicted in Fig. 3.

The theory is then formulated with *Bet* and *Cong* interpreted as expressing relations between cells.

Two things should be noted about this picture. Firstly, note that in this setting there will in general be many qualitatively indiscernable (im)possible worlds (see the left pair and the right pair of worlds in Fig. 3). This is an inevitable feature of Fundamental Qualitativism, for given our definition of a qualitative proposition, each cell must be closed under qualitative isomorphisms. But if cells are not singletons, then the members of each cell consists of multiple qualitatively indiscernable (im)possible worlds. (If the cells are singletons, we are in the degenerate case where every proposition is fundamental, since every proposition will be a disjunction of cells.) But two things are worth emphasizing about this fact. Firstly, while the proposition that corresponds to the singleton of the top left world in Fig. 3 (say) draws a distinction that isn't qualitative, this proposition isn't the sort of thing we can define from fundamental qualitative entities since it cuts across a cell. This is thus consonant with the general concession that there are many *non*-fundamental propositions about particular particles. Secondly, at most one of the top left and bottom left worlds is metaphysically possible, since (as in Fig. 1) no more than one metaphysically possible world can exist in any cell. Thus even though there are qualitatively indistinguishable worlds, the picture is consistent with the supervenience of everything on the qualitative and on the fundamental.

Finally, note that for every world the classical theory posits, our theory reconstructs a qualitative surrogate. In particular, this means that, since the classical theory we started out with posits qualitatively duplicate worlds, our theory will produce surrogates of those possibilities. See Fig. 4. Note that the worlds in the first

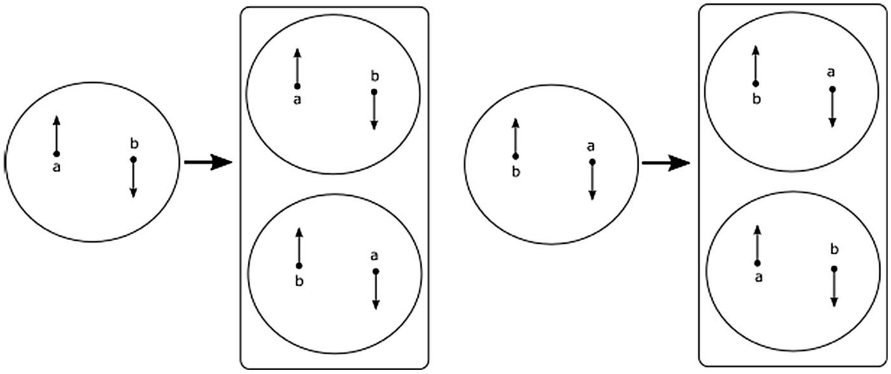

Fig. 4 Two qualitatively identical classical worlds and the two cells corresponding to them

cell of Fig. 4 are *not* qualitative duplicates of the worlds in the second cell. There will be a primitive qualitative proposition true at the two worlds in the left cell which is not true in the right: in this respect, the diagram is misleading since it only depicts differences in momentum and location. The reader may well be thinking (justifiably) that this is a cheat. But let me emphasize this is just the name of the game here. Note, by analogy, that if the Nihilistic version of the theory is to even get off the ground we have to postulate primitive qualitative differences between worlds, because according to that theory *all differences* are qualitative: no proposition is about any individual, since there are no individuals (compare the two empty worlds in Fig. 2: if they're different they're qualitatively different). If one is happy to contemplate views that imply the existence of two or more empty worlds—views such as Nihilism—then one must make ones peace with these sorts of primitive qualitative propositions that can differentiate these worlds.

However, it would also be somewhat unsatisfactory if we couldn't analyse these qualitative propositions further, much as the individualistic metaphysician can decompose a fundamental haecceitistic proposition such as the proposition that Sparky has momentum *m*, into a fundamental individual, Sparky, and a fundamental momentum property. We turn to this issue in the next section.

11 Particles as degrees of freedom

The primitives discussed in the last section are sufficient for stating the laws of classical mechanics. With these concepts we can say which sets of states are legal paths in state-space, and work out what state a given state will evolve into and so forth. However, the sorts of propositions we can state in terms of these primitives are the sorts of propositions that are true at all states, if true at all. This can be seen from the observation that our primitive congruence and betweenness relations only ever output the set of all states or the empty set.

But the challenge we outlined for the Fundamental Nihilist was to identify a set of fundamental primitives from which each metaphysical possibility can be specified: a set of fundamental entities from which every cell of logical space could be defined. In the individualistic theory they can be generated by primitive individuals (the particles) and properties (momenta and locations): it would be nice to have something analogous for the Fundamental Nihilist. Similarly, we suggested that the Nihilist needs a paraphrase for a false individualistic theory, which also suggests a need to find surrogates for particles and for momentum and location properties.

In what follows we appeal to the fact that in classical mechanics momenta and locations are properties of individuals that can be varied independently. This is born out in the fact that the set of all states can be represented by a $6N$ dimensional space, where each dimension represents an independently varying parameter. The first coordinate represents the value of the first particle's x coordinate in space, the second two its y and z coordinates, the next three its momentum coordinates, and then the next set of six coordinates represent the same for the second particle, and so on. In terms of state-space, a particle can be thought of as an equivalence relation, each of whose equivalence classes represents a 6-dimensional subspace. If a is a particle, the equivalence relation on state-space a generates is the following:

***a*-Equivalence** $s \sim_a s'$ if and only if s and s' agree about the positions and momenta of all particles except, possibly, for a

As is well known, an equivalence relation on a set of worlds corresponds to a modal operator with a logic of S5. If a is a particle, we write this operator $[a]P$, and its truth conditions are as follows:

> $[a]P$ is true at state s if and only if P is true at every state that agrees with s concerning the positions and momenta of all particles apart from a.

In more intuitive terms, $[a]P$ says that Ps truth value does not depend on a. You can vary a's location and momenta however you like, P will remain true so long as the other particles are held fixed.

Above I have described the relation $\sim_a$ and the corresponding operator $[a]$ by assuming a metaphysics involving states containing individuals with momenta and location properties. However, even though this is the most natural order of explanation for someone already familiar with the individualistic version of classical mechanics, it does not necessarily reflect the order of metaphysical explanation. For all we've said, it could be that the operator $[a]$, or the relation on states, $\sim_a$, is metaphysically prior to the individual a. Or, indeed, it could be that there are no individuals at all, but there are nonetheless fundamental relations and operators on states behaving structurally just like $\sim_a$ and $[a]$ would if there had been an individual a.

To make that idea plausible it would be nice to have an intrinsic characterization of the operators of the form $[a]$ that does not require us to explicitly refer to an individual. For example, if a and b are two distinct particles, then it is easily verified that the following principles concerning $[a]$ and $[b]$ hold:

Commutativity $[a][b]P \leftrightarrow [b][a]P$

Church–Rosser $\langle a\rangle[b]P \rightarrow [b]\langle a\rangle P$

Here $\langle a \rangle$ is short for the dual of $[a]$, $\neg[a]\neg$, and in the Church–Rosser principle a must be distinct from b. These principles are characteristic of the *product logic*. In addition to the principles of S5 these form a natural starting point for characterising particle operators without reference to particles.

For each particle operator $[a]$ it will often be useful to introduce what we'll call the *obverse* operator $[a]^*$: the operator that sees whether its argument remains true if we vary the properties of every particle *except* for a:

> $[a]^*P$ is true at s if and only if P is true at every state that agrees with s about the momentum and location of a.

Following this rule, $([a][b])^*$ stands for the operator that quantifies over states that agree about the momentum and location of both a and b.[44]

Let us now consider the fundamental properties in the individualistic version of classical mechanics. For the time being we shall forget that locations and momentum can be thought of as relations to space or to numbers; we shall instead think of each position and location property as a primitive monadic fundamental property. For the sake of concreteness, focus on the property of having momentum p. Even though this is a property of particles, there is a natural property of *states* that corresponds to having that momentum: the property a state has if *some* particle has that momentum. For each momentum p let A^p be the proposition that is true at a state iff some particle has momentum p, and similarly for each location x let A^x be the proposition that is true at a state iff some particle has location x:

> A^p is true at s iff some particle in s has momentum p.
> A^x is true at s iff some particle in s has location x.

These propositions 'correspond' to the properties of having momentum p and location x. The proposition that particle a has momentum p, for instance, is just the result of applying the obverse particle operator $[a]^*$ to the momentum proposition A^p:

> FACT: $[a]^*A^p$ is true at a state s if and only if the particle a has momentum p in state s.

The reason this is true is because momentum and location are properties that particles can have or lack independently of whether other particles have or lack them. Thus, if all the states that agree with s about the momentum and location of a are states where something has momentum p, then in particular the state s' that agrees with s about the momentum and location of a, and is such that every particle apart from a has momentum distinct from p, is a state where something has momentum p. So it follows that a must have momentum p in s', and thus in s as well. This sort of reasoning extends to ranges of momenta and to the property of belonging to an extended region: all one needs is a state where every particle apart

[44] Note, crucially, that this is not the same as the composite operator $[a]^*[b]^*$—this is one reason why it's preferable to take the particle operators and the obverse relation as primitive, rather than the obverses of particle operators as primitive.

from a is in the range or region.[45] Indeed it's straightforward to show that any unary relation definable from particular momentum and location properties using (possibly infinitary) Boolean operations can be captured in the above way via a proposition. Note, however, that not every definable binary (or higher arity) relation is capturable by a proposition using the tools we have so far: for example, the relation Rxy that holds when x has momentum p and y location l is not obviously capturable by a proposition, even though for any pair of individuals a and b, we state the proposition that a has momentum p and b has location l (namely $[a]^*A^p \wedge [b]^*A^l$). (The obvious candidate proposition—the proposition that something has momentum p and something has location l—would correspond to a symmetric relation if it corresponded to a relation at all.[46]) We will return to this in Sect. 12.

It's tempting to think of particle operators (or, perhaps, their obverses) as the analogue of particles, and the position and momentum propositions described above as the analogue of position and momentum properties.

Indeed, since we know that each metaphysical possibility can be completely described once we have specified the momentum and location of every particle, the above makes salient an extremely natural fundamental basis for our collection of primitives for classical mechanics: the particle operators and location and momentum propositions, along with the (finitary and infinitary) truth functional connectives. Three features of these primitives are noteworthy. (i) These primitives have hereditarily propositional types, (ii) from these primitives one can define every simple proposition stating the momentum or location of each particle (by Fact, above) (iii) since every state is a conjunction of such propositions, every state is definable from these primitives and (infinitary) conjunction.

This means that we have answered one of the demands we made of the Fundamental Nihilist: to find some basis of fundamental entities sufficient to define every metaphysical possibility. For every state the individualistic theory postulates, we ought to be able to provide a qualitative surrogate of it stated in fundamental terms (ensuring supervenience on the fundamental). And we have seen above that every such state can be picked out by our choice of fundamental primitives: the maximally strong consistent conjunctions of these propositions are in one-one correspondence with the possible states.

Note moreover that the above observations give us the resources to meet one of the demands we made against the Nihilist: to find true paraphrases of the false sentences involving individuals. Suppose we also had as a primitive a property of operators, $\mathcal{P}$, of type $(t \to t) \to t$, expressing the property of being the obverse of a particle operator. Then we can design a paraphrase $(\phi)^+$ for each individualistic sentence ϕ by replacing simple predications of relations and properties in the way

[45] This argument doesn't hold when the range is empty or encompasses every possible value. In these two special cases our Fact holds vacuously.

[46] If we wrote $A^{p,l}$ for this proposition, then $([a][b])^*A^{p,l}$ would correspond to the proposition that *either* a has momentum p and b location l *or* b has momentum p and a location l.

described above, and treating existential quantification via restricted quantification over obverse particle operators.[47]

12 Generalizing

Let us now attempt to generalize the foregoing to an arbitrary individualistic theory. In doing this we shall get clearer on what aspects of classical mechanics make this work.

Our goal is as follows. Given a model of type theory D, we will understand an individualistic metaphysics to be a choice of fundamental entities Fun that is generated by a basis of fundamental individuals $Fun_e \subseteq D_e$ and fundamental properties $Fun_{e\to t} \subseteq D_{e\to t}$ (we consider monadic properties first and generalize later), along with perhaps some logical primitives like the existential quantifier and the Boolean connectives. If the theory meets certain conditions we will then provide a procedure for generating an alternative set of primitives, over a model type theory D' such that $D'_t = D_t$ and $D'_e = \emptyset$, whose types are hereditarily propositional, and which suffice to generate the same propositions as the initial individualistic metaphysics from the Boolean operations. The resulting theory will thus satisfy of everything on the fundamental (and thus supervenience of everything on the qualitative) if the original individualistic metaphysics does.

The feature of classical mechanics that allowed us to do this seemed to be the fact that each state was generated by ascribing to each particle one of a set of mutually exclusive, exhaustive and independent particle states: the momentum and location of each particle. To generalize, then, we consider individualistic metaphysics that posit a set S of mutually exclusive, exhaustive and independent *object states*. That is, a set of monadic properties with the following two features. The first is:[48]

Statehood Nothing can possibly be in more than one state, and necessarily, everything is in at least one state.

In a possible worlds framework, a property can be thought of as a set of world-individual pairs, were a pair (w, a) is in the property if a has that property at w. In this formalism Statehood becomes:

[47] For each variable x_i of type e we choose an operator variable X_i of type $t \to t$, and for each atomic unary relation P we have a propositional letter A_P expressing the proposition corresponding to P:

$$(a)^+ := [a]^*$$
$$(x_i)^+ := X_i$$
$$(Pt)^+ := (t)^+ A_P$$
$$(\neg\phi)^+ := \neg(\phi)^+$$
$$(\phi \wedge \psi)^+ := (\phi)^+ \wedge (\psi)^+$$
$$(\exists x_i \phi)^+ = \exists X_i(\mathcal{P}X_i \wedge \phi).$$

To illustrate the translation informally, the sentence ‘some particle is located in region r’, gets paraphrased as ‘for some particle operator X: X(some particle is located in region r)’.

[48] In the following we assume that there is no contingency concerning what exists. Without this assumption the following conditions would need a slightly more careful formulation—these complications are mostly orthogonal to our concerns here.

S is a *partition* of the set $W \times D$ of all world-individuals pairs.

That is, no two sets in S overlap, and $W \times D$ is the union of S.

The second constraint corresponds to the idea that every possible world is determined by the ascription of object states to individuals, and that the object states are independent.

Combinatorialism For every assignment of states to objects, $f: D \to S$, the set $\bigcap_{a \in D}\{w \mid (w, a) \in f(a)\}$ is a singleton.

If f assigns the object a_1 the state s_1, a_2 the state s_2 and so on, then the set described above just represents the conjunction: 'a_1 has s_1 and a_2 has s_2 and ...'. Combinatorialism encodes two ideas. First, the fact that this intersection is consistent (non-empty) for each assignment of states to objects captures the idea that the states are *independent*: every assignment of states to different objects is possible. Second, the fact that the intersection contains at most one world shows that every world can be uniquely determined by specifying which states each object is in.

We can now introduce object operators and their obverses as we did in the previous section. By Statehood, there is a unique object state that a possesses at a world w. Thus we can say that $[a]^*P$ is true at w iff P is true in every world in which a possesses that state. Similarly, for each object state s there is a proposition A^s that corresponds to it: the set of worlds where some individual has that state. Using Combinatorialism we can show, in a completely analogous, way that $[a]^*A^s$ is true at a world iff a has s at that world. As before, any relation definable from these monadic object state properties using Boolean connectives will also have a corresponding proposition.

It should be noted that the conditions under which there is *some* partition of the set of world-individual pairs into a collection of object states that satisfy Statehood and Combinatorialism is pretty weak. If there is a set S' such that the cardinality of $D^{S'}$ is the same as the cardinality of the set of worlds W then it is possible.[49] The interesting case is presumably when the collection of object state properties are fundamental, or at least definable from the fundamental. For any individualistic theory that is given by a collection of fundamental* object states of this form, there is a parallel non-individualistic theory that postulates fundamental object operators instead of individuals and propositions instead of object states.

As was noted earlier, the choice to formulate the individualistic version of classical mechanics in terms of monadic location and momentum properties was artificial. The restriction to theories generated by monadic properties excludes many individualistic theories, including a straightforward variant of the theory we

[49] The basic intuition is that given the cardinality constraint we can think of each world as a specification of the state of each object where the states are represented by S'. Suppose that $\rho: W \to D^S$ is a bijection. For each $s' \in S'$ we define an object state as follows: $s := \{(w, a) \mid \rho(w)(a) = s'\}$. It is then routine to show that the set of object states generated this way is a partition of $W \times D$ and satisfies Combinatorialism.

considered in which you posit more individuals, *space-time points*, and a fundamental non-symmetric location relation between them.

It is possible to generalize along this dimension as well. Above we considered individualistic theories where each world could be completely described by the distribution of fundamental monadic properties of individuals. According to other individualistic theories, each world can be described by the distribution of monadic *and* dyadic relations between individuals. More generally there are individualistic theories where one needs a fundamental collection of relations of any arity to describe the world. (An example of a physical theory of this sort is quantum mechanics: every state of the system can be described by assigning a state to the tuple consisting of all N particles, but these states can't always be decomposed into a conjunction of monadic states assigned to each individual particle.)

Since the generalization to higher arities will be obvious, let's focus on the dyadic case for concreteness. We can translate between monadic properties and dyadic relations in which the second argument place is redundant,[50] so that when we talk about describing the world in terms of 'individuals, monadic properties and dyadic relations' we can omit the 'monadic properties' without loss of generality.

Instead of talking about object states we shall now talk about *pair* states: a set S of mutually exclusive and exhaustive dyadic relations an (ordered) pair of objects can be in. We spell this out as before:

Dyadic Statehood No pair can possibly stand in more than one of these relations to each other, and necessarily every pair of objects stands in one of these relations to each other.

Modeling relations as ordered triples of a world and two objects this amounts to:

S is a partition of $W \times D \times D$

The assumption of Combinatorialism must also be generalized:

Dyadic Combinatorialism For every assignment of states to pairs, $f: D \times D \to S$, the set $\bigcap_{(a,b)\in D\times D}\{w \mid (w,a,b) \in f(a,b)\}$ is a singleton.

For each pair of individuals the individualistic theorist posits, (a, b), we can introduce a *pair* operator $[ab]$ similar to our object operators. $[ab]P$ is true at a world w iff P holds at every world that agrees with w about the state of (a, b).

Given Dyadic Combinatorialism we can now paraphrase talk of binary relations applying to pairs of individuals—something we couldn't do given Monadic Combinatorialism. For every pair state s, we can introduce a proposition A^s that corresponds to it—the proposition that some pair is in that state—and it follows by

[50] We could also identify a monadic property with a relation whose first argument is redundant (the converse of our choice). A monadic property and the two dyadic relations it generates are all metaphysically interdefinable (given, e.g., the Permutation Criterion), so it actually doesn't matter which we add to our basis.

exactly the same reasoning as before that $[ab]^{*}A^{s}$ is true at a world iff (a, b) has s at that world. The generalization of all the above to higher arities is straightforward.[51]

What if we want to talk about relations of arbitrary arity? If n-adic Combinatorialism is true then we can paraphrase relations of arity n or less using a proposition via the methods described above. It is very much an open question whether it's possible to devise counterparts of relations of arity $m > n$. Two positive things are worth mentioning here. Firstly, one can formulate an infinitary generalisation of Combinatorialism—ω-adic Combinatorialism—in which the states are ω-ary relations that partition the set $W \times D^{\omega}$ and satisfy the obvious analogues of n-adic Combinatorialism. Given ω-adic Combinatorialism it is possible to introduce n-tuple states, and n-tuple operators for *arbitrary* n. And n-ary relation R is equivalent to an ω-ary relation R^{*} in which all but the first n arguments are redundant, and R has a counterpart proposition: that some ω-tuple satisfies R^{*}.

Secondly, although I have no strategy for dealing with arbitrary relations given n-adic Combinatorialism, for finite n, there is a sense in which this doesn't matter: we have counterparts for every proposition definable out of them and individuals. Suppose, for example, that we are assuming Monadic Combinatorialism, and I have the metaphysically definable relation $Rxy := (s_1(x) \wedge s_2(y)) \vee (s_3(x) \wedge s_4(y))$ and I want to apply it to a and b. Although I have not suggested a propositional surrogate for R and operator for the pair (a, b), I can nonetheless express the proposition $Rab = (s_1(a) \wedge s_2(b)) \vee (s_3(a) \wedge s_4(b))$, since I know how to express $s_1(a)$, $s_2(b)$, and so on.

13 Conclusion

We have described a version of Fundamental Nihilism. It is worth asking how it holds up against the motivations for that view.

A nice feature of many physical theories is that they can often be given a Hamiltonian formulation, in which the laws are described in terms of the geometry of the space of possible states of the system rather than formulating them directly in terms the features of the individuals in these states. The sort of view I have described fits very well with physical theories of this sort.

However if one is attracted to an anti-individualistic metaphysics on the basis that it rejects invisible differences—observationally identical worlds that differ concerning which individuals occupy which qualitative roles—then it's not obvious that our suggestion has much to offer. In particular, our anti-individualistic metaphysics recreates every metaphysical possibility the individualistic

[51] It should be noted that theories that involve particles, space-time points and an asymmetric location relation do not straightforwardly satisfy Dyadic Combinatorialism: that would imply that the pair (a, b) and (b, a) could be assigned to a pair state that's contained in the location relation, implying that a is located at b and b is located at a (thanks to Jeremy Goodman for spotting this fact). A space-time theory that *does* satisfy Dyadic Combinatorialism takes the symmetric relation $Sxy :=$ *x is located at y or y is located at x* as primitive, along with $Lx :=$ *x is a location* and $Px :=$ *x is a particle*. In this theory the ordinary asymmetric location relation is a defined relation: $Px \wedge Ly \wedge Sxy$.

metaphysics postulates. One immediate consequence of this is that, since the individualistic metaphysics of classical mechanics postulates qualitatively identical but distinct metaphysical possibilities—for example, pairs of states where two particles have switched their locations and momenta—our reconstruction recovers these possibilities.[52] Of course, in the reconstruction the states corresponding to these qualitatively identical states are not qualitatively identical (since, given Qualitativism or Fundamental Qualitativism, qualitatively identical states are simply identical). But that does not mean they are any more respectable: they are surely also observationally equivalent and just as superfluous to science as the original pair of states the individualistic theory postulated.[53]

It is not at all obvious that individuals are responsible for invisible differences, if the invisible differences still arise in theories formulated without individuals. This raises an important moral: If we are worried about invisible differences we should be more concerned with the content (understood broadly) of the theories that generate them, such as classical mechanics, rather than the choice of whether to formulate those theories in terms of individuals or not.

Acknowledgements Thanks to: Jeremy Goodman, Cian Dorr, and to two anonymous referees for this journal for some very helpful feedback. Thanks also to the audience of Metaphysics on the Mountain 2017; thanks, in particular, to my commenter, Louis deRosset, who provided me with some great feedback and spotted several mistakes.

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[52] This corresponds to a sort of reflection in state-space. If these states were removed there would be gaps in state-space and it's not obvious how to formulate the laws in this setting (but see the theory of symplectic reduction which provides one framework for thinking about this issue; Butterfield 2006).

[53] If they are indeed superfluous to science; see footnote 52.

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