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A propositional semantics for substitutional quantification

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Abstract The standard truth-conditional semantics for substitutional quantification, due to Saul Kripke, does not specify what proposition is expressed by sentences containing the particular substitutional quantifier. In this paper, I propose an alternative semantics for substitutional quantification that does. The key to this semantics is identifying an appropriate propositional function to serve as the content of a bound occurrence of a formula containing a free substitutional variable. I apply this semantics to traditional philosophical reasons for interest in substitutional quantification, namely, theories of truth and ontological commitment.

Keywords Substitutional quantification · Semantics · Truth · Ontological commitment · Kripke

1 Introduction

What proposition, if any, is expressed by a sentence, like (1), containing a substitutional quantifier?

$$(\Sigma x)\ x \text{ is a dog} \tag{1}$$

The purpose of this paper is to answer this question.

There are several reasons to think that an answer to this question matters. One is that the substitutional quantifier is standard equipment in the philosophical logician's toolbox—useful for theories of truth and for getting clear about ontological commitment—yet there has been little discussion of the *modal* properties of substitutionally quantified sentences like (1). To investigate these modal properties, we must inquire into the truth or falsity of (1) at various different

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possible worlds. But to say that (1) is true (false) at a possible world *w* is to say that the proposition expressed by (1) in @ (the actual world) would be true (false) were *w* the actual world (Soames 2011, p. 126). Thus any investigation into the modal properties of (1) presupposes that (1) expresses a proposition in the actual world.[1]

A second reason to think that an answer to this question matters is that merely knowing under what conditions (1) is true does not suffice for knowing what (1) means. To know what (1) means requires knowing what (1) says, or what proposition it expresses. But the standard semantic treatment of substitutional quantification, as presented for example in Kripke (1976), specifies only under what conditions (1) is true. Thus the standard semantic treatment of substitutional quantification does not tell us what sentences like (1) mean.[2]

One might resist these first two reasons on the grounds that the particular substitutional quantifier 'Σ' is a formal device that has only as much meaning as we stipulate it to have. There is, however, a third reason to think that an answer to our original question matters. Sentences like (2) are strong prima facie evidence for the existence of substitutional quantification in natural language:

Whenever Sara says that so and so, you should believe that so and so. (2)

The phrase 'so and so' in (2) appears to be bound by the adverb 'whenever', but it is not an objectual variable. The phrase 'so and so' occurs in (2) after the complementizer 'that', where grammatically we require a complete sentence. Thus the natural interpretation of 'so and so' as it occurs in (2) is that it is a substitutional variable.[3] If this is the correct interpretation of 'so and so' as it occurs in (2), then without a propositional semantics for substitutional quantification, we are missing an important piece of a complete semantic theory for natural language. This reason avoids the worry about 'Σ' being a formal device.

In this paper, I will focus primarily on the first two of these issues—the modal status of sentences like (1) and what sentences like (1) mean. Thus for the majority of this paper, I focus on the semantics for the particular substitutional quantifier 'Σ', and compare it to the existential objectual quantifier '$\exists$'. The key to the semantics I propose is identifying the appropriate propositional function to serve as the content of an occurrence of a formula containing a free substitutional variable. The

[1] Soames's point is directly analogous to a well-known observation about rigid designation. To say that a rigid designator designates the same object in every possible world in which that object exists (and nothing else in any world where the object does not exist) is to say that the rigid designator, *as we actually use it* does this (Kaplan 1989, pp. 493–494). It would be a mistake to conclude that the proper name 'Tally' fails to rigidly designate my border collie on the grounds that there are possible worlds in which I use the name to refer to some other dog.

[2] Peter Van Inwagen (1981) presses a version of this argument, focusing on the connection between knowing what proposition is expressed by (1) and *understanding* (1). The semantics proposed in the present paper directly addresses Van Inwagen's argument.

[3] Christopher Hill (1999, pp. 101–102) gives this argument in favor of substitutional quantification in natural language. Joseph Camp (1975) discusses a different kind of example: 'there are things I have always wanted that don't exist'. Many of the points I raise at the end of the paper about (2) can be made about Camp's example. For a different view of examples like Camp's, see Priest (2005).

semantics presented here suggests that the difference between substitutional and objectual quantification is primarily a difference between the interpretations of the variables bound by the quantifier, rather than a fundamental semantic difference between the quantifiers themselves. This final point about variables, I suggest, is relevant to some of the issues raised by (2).

The outline of this paper is as follows. In the first section, I briefly sketch the standard truth-conditions for substitutional quantification. In the second section, I raise an objection to the standard truth-conditions: either they get the modal profile of sentences like (1) wrong, or they leave it a mystery what proposition is expressed by sentences containing a substitutional quantifier. In the third section, I consider two alternative semantic rules for substitutional quantification, and argue in favor of one of them. One of the arguments turns on the possibility of interpreting ‘so and so’ in (2) as a substitutional variable. In the forth section, I apply the lessons of the propositional semantics defended in section three to two traditional philosophical reasons for interest in substitutional quantification: theories of truth and ontological commitment.

2 Substitutional quantification

The quantifier ‘Σ’ in (1) is called ‘substitutional’ because the variable that it binds, rather than ranging over objects in a domain of discourse, is semantically associated with a set expressions standardly called a ‘substitution class’ (following Kripke, I will call the members of the substitution class ‘terms’). These terms are not assigned to substitutional variables in the way that objects are assigned to variables in objectual quantification. Instead, the substitutional variable is merely a placeholder for the terms in the substitution class. On the standard treatment of substitutional quantification, (1) is true if and only if there is some term t in the substitution class such that ⌜t is a dog⌝ is true (where ⌜t is a dog⌝ is the result of writing t, then a space, then ‘is a dog’).

One need not restrict one’s substitution classes to names. Let ϕ(x) be any formula containing free occurrences of the substitutional variable ‘x’, and for any term t in a specified substitution class, let ϕ(x/t) be the expression that results from substituting t for ‘x’ wherever the latter occurs freely in ϕ(x). Then the standard semantic treatment of substutitional quantification (due to Kripke 1976) is Sub:

$$\ulcorner \Sigma x\ \phi(x) \urcorner \text{ is true if and only if } \exists t(\phi(x/t) \text{ is true}) \qquad \text{(Sub)}$$

Here, the role of substitution is explicit in our definition of ‘ϕ(x/t)’. The substitution class could equally well be a set of binary truth-functional connectives, or the singleton set containing just the left parenthesis. I assume for this paper that we have built in suitable restrictions on the substitution class so that the result of replacing the free substitutional variable in ‘x is a dog’ with arbitrary terms in the substitution class is a sentence (Kripke 1976, p. 329). Then according to Sub, the truth conditions for (1) are given by (3):

$$(\Sigma x)\, \text{x is a dog} \tag{1}$$

$$(\exists t)\ulcorner \text{t is a dog}\urcorner \text{ is true.} \tag{3}$$

These are just the truth conditions for (1) stated in the previous paragraph.

3 The modal problem

The semantic rule Sub above states conditions under which sentences containing the particular substitutional quantifier are true. According to Sub, (1) and (3) are equivalent. Yet they are not necessarily equivalent. Let the substitution class be the singleton set {'Tally'}. Let *w* be a world in which Tally (my border collie) is the only dog that exists, but in *w*, Tally has no name.[4] Then (3) is false at *w*, because in *w* there is no term t (no expression in the substitution class) such that the sentence ⌜t is a dog⌝ is true. Yet (1) is true at *w*, because (1) is entailed by (4) ((1) is true at every world at which (4) is true), which is true at *w*:

$$\text{Tally is a dog.} \tag{4}$$

Thus Sub gets the modal profile of (1) wrong.

3.1 Objections and responses

I have heard several different objections to the modal problem. Below, I present the four most common or illuminating objections, and respond to each. Some of these objections raise important points that clarify or further illuminate the basic issues raised by the modal problem. Some of them rest on confusions that it is important to dispel.

Objection 1 One might object to the above argument as follows: how can (4) be true at *w*, while it is not true at *w* that there is a term t such that the sentence ⌜t is a dog⌝ is true? If (4) is true at *w*, then the sentence 'Tally is a dog' is true at *w*. But if the sentence 'Tally is a dog' is true at *w*, then there is a term t (namely, 'Tally') such that ⌜t is a dog⌝ is true at *w*. If this is right, then (1) and (3) do have the same truth value in *w* (they are both true), and the argument against Sub fails.

Response This objection overlooks a basic point about the evaluation of sentences like (4) at, or relative to, worlds like *w*. When we evaluate (4) relative to *w*, we consider the proposition actually expressed by (4), and assess whether this proposition would be true were *w* the actual world (Soames 2011, p. 126). (4) actually expresses the singular proposition that predicates doghood of Tally. Were *w* the actual world, this proposition would be true. It is in this sense that we say that (4)—the sentence—is true at *w*. Yet the proposition actually expressed by (3) would

[4] It is not strictly required for the example that Tally be the only dog that exists. All that is strictly required is (i) that Tally exists, and (ii) that no existing dog be named 'Tally'. But I find that the case in which Tally is the only dog that exists and has no name makes it easier to focus on the relevant intuition.

not be true were *w* the actual world. The proposition actually expressed by (3) is metalinguistic: it says that there is a term t such that ⌜t is a dog⌝ is true. But in *w*, there is no such term, because the substitution class is {'Tally'} and no dog in *w* is named 'Tally'. It is in this sense that (3) is false at *w*.

There is another way to see this point. The sentence 'Tally = Tally' is (actually) logically true. So by an intuitive rule of necessitation, we can derive a further logical truth: 'Necessarily, Tally = Tally'.[5] Yet there are possible worlds at which the name 'Tally' either does not exist or does not refer to anything (such as *w* from the modal problem above). If such a world were actual, the sentence 'Tally = Tally' would either not exist or be meaningless, but the proposition that Tally = Tally (the proposition actually expressed by 'Tally = Tally') would be true. Thus to preserve the intuitive rule of necessitation, we must evaluate the proposition actually expressed by 'Tally = Tally' at alternative worlds.

Objection 2 Another objection to the modal problem is that we shouldn't rely on any intuitions about the modal profile of (1). Different reasons have been given to justify this claim, but the underlying motivation is the same: since 'Σ' is an expression of a formal language, it is up to us to *stipulate* the modal behavior of 'Σ'.

Sometimes this objection appears as a protest that someone doesn't have intuitions about the modal profile of (1) becase he or she doesn't understand (1).[6] But here again, the underlying reason for the protest seems to be that the substitutional quantifer is a formal device whose meaning is simply underdetermined by the semantic theories that have been offered for it so far.[7]

Response This objection misconstrues the argument above. My claims about the modal profile of (1) were not motivated by a direct appeal to intuitions about the meaning of 'Σ'. Rather, they were motivated by two further claims: (i) that (4) is true at the world *w* described above, and (ii) that (given a substitution class that contains the name 'Tally') (4) entails (1). The second claim is in turn motivated by the desire to take substitutional quantification seriously *as a species of quantification*. Insofar as we take 'Σ' to be a quantifier, we expect it to be governed by basic inference rules that govern quantifiers, including basic introduction rules. Our semantics should validate such rules.

Given such an introduction rule (and, again, given an appropriate substitution class), '(Σx) x = Tally' is an immediate consequence of 'Tally = Tally'. Since the latter is logically true, the former should be as well. Yet the same considerations as we raised in the response to objection 1 show that given the rule Sub, 'Necessarily,

[5] I call this an 'intuitive' rule of necessitation to distinguish it from the rule Necessitation stated for formal languages of modal logic.

[6] I have not determined whether all those who lodge this version of the objection have read Van Inwagen, but it is just how I would imagine he would protest. See note 2.

[7] Perhaps the most extreme variation on this objection is Lycan's (1979) claim that the particular substitutional quantifier is 'semantically mute'. According to Lycan, we don't understand sentences like (1) because there is simply nothing that they *say*, or no proposition that they express. The semantics I present in Sect. 4 of this paper directly answers Lycan's claim.

(Σx) x = Tally' is false. Thus the modal problem also raises a challenge for what I called above 'an intuitive rule of necessitation'.[8]

Thus to reject the modal problem it is not enough to protest that one does not fully understand substitutional quantification. One must explain how to maintain that substitutional quantification is a species of quantification while allowing that (4) does not entail (1).

Objection 3 There is, however, a sophisticated version of the second objection that merits further attention. This objection challenges the claim that (4) entails (1). Specifically, the challenge is to rule out the possibility that the relation between (4) and (1) is something other than entailment without a direct appeal to intuitions about the meaning of the substitutional quantifier.

As an example of an alternative to the entailment relation, it may be helpful to draw a comparison to the operator 'actually'. For any contingently true sentence S, the inference from ⌜Actually S⌝ to S is justified *a priori*, but ⌜Actually S⌝ does not *entail* S. Since S is true (i.e., true at the actual world), ⌜Actually S⌝ is necessarily true, but since S is only contingently true, there will be worlds at which ⌜Actually S⌝ is true while S is false. Similarly, we might understand the inference from (4) to (1) as justified *a priori*, without it being the case that (4) entails (1). We might then still claim that substitutional quantification was a species of quantification, governed by appropriate introduction rules, while challenging the reasons given for the modal problem.[9]

Response My response to this objection is to concede it, to a point. There might be different ways of understanding substitutional quantification. We might understand it along the lines described in the objection. But we might also understand it along the lines proposed in the original statement of the modal problem. (Or we might *try* to understand it this way.) It remains to be seen whether it is possible to construct a semantics that avoids the modal problem. With different semantic proposals under our belt, we may then debate their relative merits.

Aside on Substitutional Quantification and Time With this much of the modal problem clarified, we can see that an analogous problem arises for the temporal profile of (1) on temporalist views of propositions—views according to which the truth value of at least some propositions can vary from one time to another (Brogaard 2012). Suppose that in the actual world, there was a canine apocalypse shortly after Tally was born in the wild, and Tally alone survived. Several days after this canine apocalypse, I found, named, and adopted Tally. Thus we are supposing that there was a time t_0 in the actual world (between apocalypse and adoption) at which Tally was the only dog that existed, but at which time she had no name.[10] An argument directly analogous to the argument for the modal problem shows that on a temporalist view, the propositions now expressed by (1) and (3) would differ in truth value at t_0.

[8] I thank an anonymous referee for this observation.

[9] Thanks to Brian Cutter for discussion.

[10] Again, it is not strictly required that Tally be the only dog that existed. See note 4.

This argument does not apply to eternalist views of propositions, because on such views the truth value of a proposition does not vary from one time to another. But it is just this feature of temporalism that is required for the argument. A referee has suggested, however, that if we consider an utterance of ‘that is a dog’ at t_0 we could generate the same kind of problem, even on an eternalist view of propositions. The thought is this: consider the context c_0 of an utterance of ‘that is a dog’ at t_0, where the speaker is pointing at Tally (so that t_0 is the time of c_0). Relative to c_0, (1) is a consequence of ‘that is a dog’, via a basic introduction rule for ‘Σ’.[11] Thus (1) is true relative to c_0, since ‘that is a dog’ is true relative to c_0. But (3) is false relative to c_0, because there is no term t (no member of the substitution class) such that $\ulcorner$t is a dog$\urcorner$ is true relative to c_0 (since at t_0 Tally has not yet been named). Thus we have a version of the temporal problem even with eternalist propositions.

There is, however, a gap in this argument. (1) is not a consequence of ‘that is a dog’ relative to c_0, given that the substitution class has been specified as {‘Tally’}. If we allowed the substitution class instead to be {‘Tally’, ‘that’}, then (1) would be a consequence of ‘that is a dog’ relative to c_0, but in that case (1) and (3) would not differ in truth value, because (3) would also be true relative to c_0.[12] Thus as far as I can see, the problem arises only for temporalist views of propositions. Given the restricted appeal (currently) of such views, I will continue to call the problem articulated above ‘the modal problem’. Temporalists about propositions may call it ‘the modal/temporal problem’ as they wish.[13]

Objection 4 A final objection to the modal problem is that the rule Sub should not be generalized in the way I have suggested. A modal semantics should specify under what conditions a sentence ϕ is true at a possible world. Abbreviating ‘is true at a possible world w’ as ‘$\models_w$’, there are two ways we might extend Sub to a modal semantic rule:

$$\models_w \ulcorner \Sigma \mathrm{x}\ \phi(\mathrm{x}) \urcorner \text{ if and only if } \models_w \exists \mathrm{t}(\phi(\mathrm{x/t}) \text{ is true}) \tag{Sub$_1$}$$

$$\models_w (\ulcorner \Sigma \mathrm{x})\ \phi(\mathrm{x}) \urcorner \text{ if and only if } \exists \mathrm{t} \models_w \phi(\mathrm{x/t}) \tag{Sub$_2$}$$

The modal problem above turns on the assumption that Sub_1 is the appropriate modal semantic rule, but if Sub_2 is the correct rule, then the modal problem does not arise.[14]

Response This objection is correct as far as it goes. According to Sub_2, (1) is true at w. Furthermore, Sub_2 makes sense of our reasons for taking (1) to be true at w: that

[11] For discussion of the notion of logical consequence relative to a context, see Georgi (2014).

[12] The discussion above exposes a tension in our understanding of Sub. Strictly, this argument requires a modified version of Sub along the following lines:

$$\ulcorner \Sigma \mathrm{x}\ \phi(\mathrm{x}) \urcorner \text{ is true in } c \text{ if and only if } \exists \mathrm{t}(\phi(\mathrm{x/t}) \text{ is true in } c) \tag{Sub$_c$}$$

Yet with this modified version in place, it is unclear what role the truth of (3) relative to a context plays in the evaluation of (1). A related issue is addressed in Objection 4 below.

[13] Thanks to an anonymous referee for helping me to see the issues raised in this aside.

[14] Thanks to Michael Kremer for discussion.

(4) entails (1), and that (4) is true at w. This is good evidence that Sub_2 is the correct modal semantic rule. But this point raises more forcefully than before the basic question of this paper: what proposition is expressed by (1)? The rule Sub_1 can be interpreted as a straightforward answer to this question: (1) expresses the metalinguistic proposition expressed by (3).[15] But this proposal, as we have seen, is subject to the modal problem, and is rejected by (at least some) advocates for substitutional quantification.[16] How are we to extract from Sub_2 an alternative proposal? The semantics in the next section does exactly this.[17]

3.2 Summary

The above objections and responses highlight an important point about any attempt to state a propositional semantics for substitutional quantification. The reason that the proposition expressed by (3) is false at w is that the proposition actually expressed by (3) is metalinguistic—it is about sentences, and involves objectual quantification over expressions in the substitution class. If, as I have argued, (1) is true at w, this means that the proposition actually expressed by (1) cannot be metalinguistic. If the proposition expressed by (1) were metalinguistic, it would be a mystery how (4) could entail (1), since (4) says nothing at all about any sentence or expression.[18]

4 A propositional semantics for substitutional quantification

The previous discussion highlights three desiderata for a propositional semantics for substitutional quantification: (i) the proposition expressed by (1) should not be metalinguistic; (ii) the truth conditions for (1) in the actual world should be equivalent to (3); and (iii) (1) should be true in the world w described in the previous section. The semantics below satisfies all three of these desiderata. In addition, it clarifies the significance of the rule Sub_2.

4.1 The rule PS

I propose the following semantic rule for substitutional quantification (where v_S is any substitutional variable, f is an assignment of objects to objectual variables, and c is a context):

[15] Note that the most plausible way of understanding the reasoning in the first objection is that it fallaciously moves from a true claim based on Sub_2 (that if (4) is true at w, then there is some term t such that ⌜t is a dog⌝ is true at w) to a false claim apparently based on Sub_1 (that (3) is true at w).

[16] That substitutional quantification is not merely metalinguistic objectual quantification has been emphasized in the literature on substitutional quantification since at least Dunn and Belnap (1968, pp. 184–185). (Though Kripke at least hints that he would accept such an interpretation of substitutional quantification (1976, p. 356).)

[17] According to the rule Sub_2, a substitutionally quantified sentence like (1) is necessarily equivalent to the disjunction of all of its substitution instances. See note 21 for more discussion of this observation.

[18] Several of the issues raised in the objections and responses are discussed by Soames in the Appendix of Chapter 3 of *Understanding Truth* (1999).

PS The proposition expressed by $\ulcorner(\Sigma v_S)\ \phi\urcorner$ relative to f, c, and substitution class S is $\langle$SOME, $g_S\rangle$, where SOME is the property of being a function that maps at least one thing to a true proposition, and g_S is the function that maps each term t of S to the proposition expressed (relative to f and c) by $\phi(v_S/\ \mathrm{t}\ )$ (and is undefined otherwise).[19]

According to PS, the proposition expressed by (1) (relative to an assignment f, context c, and substitution class $S = \{$'Tally'$\}$) is (5), where g_S^* is the function that maps the term 'Tally' to the proposition expressed by (4) relative to f and c:

$$\left\langle \text{SOME}, g_S^* \right\rangle \tag{5}$$

The function g_S^* is the propositional content of the bound *occurrence* (relative to f, c, and S) of the open formula 'x is a dog' in (1).

Our semantics satisfies desideratum (i). (5) is not a metalinguistic proposition, in the following sense: let us say that a proposition is metalinguistic if and only if either it contains an expression as a constituent, or it contains a metalinguistic propositional function. A propositional function g is metalinguistic if and only if for any object o, $g(o)$ is metalinguistic. This fairly captures the required sense of 'metalinguistic proposition'.[20] (5) is the proposition $\left\langle \text{SOME}, g_S^* \right\rangle$, where for any term t, g_S^*(t) is the proposition expressed by $\ulcorner$t is a dog$\urcorner$. 'Tally' is the only term in S, and g_S^*('Tally') (the proposition expressed by (4)) does not contain any expression, nor does it contain any metalinguistic proposition. Thus the propositional function g_S^* is not metalinguistic. Since the only constituents of (5) are g_S^* and SOME, (5) is not metalinguistic.

4.2 Truth and the modal problem revisited

The truth of propositions like (5) is given by the following rule:

Truth A proposition $\langle$SOME, $g\rangle$ is true if and only if g has the property SOME (i.e., g maps at least one thing to a true proposition).

According to Truth, (5) is true if and only if there is at least one term t such that g_S^*(t) is a true proposition. Given our substitution class $S = \{$'Tally'$\}$, this means that (5)

[19] Two notes about the rule PS: (i) I adopt a neo-Russellian picture of structured propositions, but that does not mean that I identify propositions with ordered n-tuples. Rather, n-tuples stand in for propositions in the statement of the rule. For a selection of current views of structured propositions, see King et al. (2014). (ii) A consequence of PS is that (1) expresses different propositions relative to different substitution classes. But this seems to me to be the right result (or at least not obviously the wrong result), similar to the different proposition expressed by 'there is no beer' relative to different domain restrictions in different contexts.

[20] Example: the proposition expressed by '$\forall y \exists z \forall r(r =$ 'apple'$)$' is $\langle$EVERY, $h\rangle$, where for any o, $h(o)$ is the proposition $\langle$SOME, $h'\rangle$, where for any o, $h'(o)$ is the proposition $\langle$EVERY, $h''\rangle$, where for any o, $h''(o)$ is the proposition that $o =$ 'apple'. The propositional function h'' is metalinguistic because for any o, $h''(o)$ contains the expression 'apple'. The propositional function h' is metalinguistic because for any o, $h'(o)$ contains the metalinguistic propositional function h''. The propositional function h is metalinguistic because for any o, $h(o)$ contains the metalinguistic propositional function h'. The proposition $\langle$EVERY, $h\rangle$ is metalinguistic because it contains the metalinguistic propositional function h.

is true, because g^*_S('Tally') is the proposition expressed by (4), and the proposition expressed by (4) is true. Our propositional semantics yields the same truth conditions as Sub, at least for the actual world. Thus our semantics satisfies desideratum (ii).

Finally, our semantics also satisfies desideratum (iii): (1) is true at the world w described in the previous section. Recall that w is a world in which Tally (my border collie) is the only dog that exists, but in w, Tally has no name. Recall also that (1)—the sentence—is true at w if and only if the proposition that (1) actually expresses (namely (5)) would be true were w the actual world. But (5) would be true were w the actual world, because g^*_S('Tally') is the proposition expressed by (4), and the proposition expressed by (4) would be true were w the actual world.

The key to this result is that the truth of (5) relative to w does not require the existence of any terms of the substitution class in w. This is what generated the modal problem for Sub. Thus the present semantics solves the modal problem for substitutional quantification, while specifying what proposition is expressed by sentences containing the particular substitutional quantifier 'Σ'.[21]

4.3 Substitutional and objectual variables

The treatment of substitutional quantification in Sect. 4.1 is directly analogous to the standard Russellian semantics for the existential objectual quantifier (where v_R is any objectual variable, and f and c are as above):

PO The proposition expressed by $\ulcorner(\exists v_R)\ \phi\urcorner$ relative to c and f is $\langle$SOME, $g\rangle$, where SOME is the property of being a function that maps at least one thing to a true proposition, and g is the function that maps each object o to the proposition expressed by ϕ relative to c and $f^o_{v_R}$.

According to PO, the proposition expressed by (6) relative to an assignment f and context c is (7), where g' is the function that maps any object o to the proposition expressed by 'x is a dog' relative to $f^o_{x'}$ and c:

$$(\exists x)\ x \text{ is a dog} \tag{6}$$

$$\langle \text{SOME}, g' \rangle \tag{7}$$

The function g' in (7) is the propositional content of the bound occurrence of the open formula 'x is a dog' in (6) (Salmon 2006). The key to PO is the use of objectual propositional functions like g'that map objects to propositions. Similarly, the key to PS is the use of substitutional propositional functions like g^*_S that map terms to propositions.

[21] An alternative proposal, suggested by an anonymous referee, is to identify the proposition expressed by (1) with the (perhaps infinite, given an infinite substitution class) disjunction of the propositions in the range of g^*_S. This proposal also satisfies desiderata (i)–(iii). I see at least two disadvantages to this proposal: first, it is questionable whether, on this account, we could understand a sentence like (1), if understanding a sentence requires grasping the proposition that it expresses. We would need some account of what it is to grasp an infinite proposition. Second, on this view the form of the proposition expressed by S is nothing like the form of the sentence that expresses it.

It is important, however, to recognize the different roles played by substitutional variables in PS and by objectual variables in PO. Our semantics is consistent with Kripke's in maintaining this difference:

> Note that free [objectual] variables play a genuine semantical role, but the free [substitutional] variables do not: formulae with free [substitutional] variables are assigned no interpretation. (Kripke 1976, p. 355)

According to PS, *occurrences* of formulae with free substitutional variables play a genuine semantical role, but the formulae themselves do not. Nowhere in the semantics is a formula containing a free substitutional variable given an interpretation (relative to an assignment or anything else). Yet PO explicitly requires that formulae containing free objectual variables express propositions relative to assignments of objects to variables. Thus PS is not in this way a radical departure from the standard semantic treatment of substitutional quantification.

One of the notable features of PS is that the semantic content of the substitutional quantifier 'Σ' is the same as the semantic content of the objectual quantifier '$\exists$': the property of being a function that maps at least one thing to a true proposition. Strictly, according to PS (and PO), the quantifiers are synonymous. On this view, the difference between substitutional and objectual quantification is a difference in the variables bound by the quantifier, not a difference between two distinct quantifiers.[22]

4.4 An alternative to PS

An alternative to this view would locate the difference between substitutional and objectual quantification in a semantic difference in the quantifiers. Consider, for example, the following rule:

PR The proposition expressed by $\ulcorner(\Sigma v_S)\ \phi\urcorner$ relative to f, c and substitution class S is $\langle \text{SOME}_S, g\rangle$, where SOME_S is the property of being a function such that there is some expression e in S and some object o such that (i) e actually refers to o (relative to f and c) and (ii) the function maps o to a true proposition, and g is the function that maps each object o to the proposition expressed by ϕ relative to c and $f^{o}_{v_S}$.

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[22] It is this point, I suggest, more than any other, that Van Inwagen (see note 2) fails to understand about substitutional quantification:

> I shall assume at the outset that my readers agree with me on one fundamental point: it is not the case that there is something called 'the existential—or particular—quantifier' of which philosophers of logic have offered two 'interpretations' or 'readings' , *viz*. the objectual or referential interpretation and the substitutional interpretation. It would be better to say that there are two quantifiers, two distinct variable-binding operators: the objectual or referential and the substitutional. (1981, p. 281)

I do not agree. On the view defended in this paper, neither option is correct. There is just one particular quantifier, with one interpretation, but two distinct kinds of variables to be bound.

According to this rule, there is no difference between substitutional and objectual *variables*; both are assigned objects by assignment functions. The difference between substitutional and objectual quantification is a difference in the properties expressed by the quantifiers.[23]

PR also satisfies our three desiderata. According to PR, (1) expresses the proposition $\langle \mathrm{SOME}_S, g'' \rangle$, where for any o, $g''(o)$ is the proposition expressed by ‘x is a dog’ relative to an assignment of o to the substitutional variable ‘x’. This proposition is not metalinguistic (desideratum (i)), it is true in the actual world (desideratum (ii)), and it is true at the world w described in Sect. 4 (desideratum (iii)).[24] (For the last one, note that the rule PR requires that ‘Tally’ refers to Tally in the actual world, not in w.)

There are, however, several reasons to prefer PS to PR. One reason is that PR applies only to those cases of substitutional quantification where the substitution class consists of referential expressions. But substitutional quantification is more general than this. If our substitution class S includes the left parenthesis, then (8) is true:

$$(\Sigma \mathrm{x})\ \mathrm{x}\exists y)\ y \text{ is a dog} \tag{8}$$

Yet the rule PR predicts that (8) is false. According to PR, (8) expresses (9) relative to a context c, assignment f, and our substitution class S:

$$\langle \mathrm{SOME}_S, h' \rangle \tag{9}$$

(where h' is the content of the occurrence of ‘$\mathrm{x}\exists y)\ y$ is a dog’ in (8)). Yet there is no expression e in S and no object o such that (i) e refers to o and (ii) ‘$\mathrm{x}\exists y)\ y$ is a dog’ expresses a true proposition relative to an assignment of o to ‘x’. Thus h' does not have the property SOME_S.

PS, on the other hand, gets the right result for (8). If S is as in the previous paragraph, then the propositional function h^*_S that maps any term t from S to the proposition expressed by $\ulcorner \mathrm{t}\exists y)\ y \text{ is a dog} \urcorner$ maps at least one term (the left parenthesis) to a true proposition. Thus h^*_S has the property SOME.

A second reason to prefer PS to PR is based on the example (2) from the introduction:

Whenever Sara says that so and so, you should believe that so and so. (2)

I argued in the introduction that (2) provides prima facie evidence for the existence of substitutional quantification in natural language, because the phrase ‘so and so’ in (2) appears to be bound by the quantifier ‘whenever’, but does not appear to be an objectual variable. If this is correct, then the occurrence of ‘whenever’ in (2) must

[23] Thanks to David Braun for suggesting to me the rule PR.

[24] One might worry that the proposition $\langle \mathrm{SOME}_S, g'' \rangle$ shows that our earlier definition of ‘metalinguistic proposition’ is in fact inadequate. For while the proposition satisfies that definition, it may seem intuitively metalinguistic insofar as SOME_S includes some kind of objectual quantification over expressions. But one might also think that substitutional quantification is metalinguistic in this sense.

be interpreted as a substitutional quantifier. Elsewhere, however, ‘whenever’ is clearly functioning as an objectual quantifier. The objectual interpretation of ‘whenever’ is apparent in (10):

$$\text{Whenever I go outside, it rains.} \tag{10}$$

Thus if ‘so and so’ in (2) is a substitutional variable, and substitutional quantification requires a semantically distinct quantifier, then we are committed to the claim that ‘whenever’ is ambiguous between an objectual and a substitutional interpretation. It seems to me, however, that there is no such ambiguity in ‘whenever’, and hence that we should reject the claim that substitutional quantification requires a semantically distinct quantifier.

This last argument is contingent on the interpretation of ‘so-and-so’, as it appears in (2), as a substitutional variable. I have not, in this paper, addressed the plausibility of this interpretation. But the semantics for substitutional quantification proposed in Sect. 4.1 may help to allay some worries about the plausibility of this interpretation, on precisely the grounds that we can now see how to allow for substitutional quantification in English without requiring that English have special substitutional quantifiers. All we would require is that English have special substitutional variables. The above interpretation of (2) proposes that ‘so and so’ is just such a variable.

5 Substitutional quantification, truth, and ontology

My primary concern in this paper has been to present a semantics for substitutional quantification that identifies what proposition is expressed by sentences like (1). This concern may seem far removed from traditional philosophical reasons for interest in substitutional quantification. Two of the biggest reasons for interest in substitutional quantification have traditionally been (i) the possibility of using substitutional quantification in deflationary theories of truth, and (ii) the promise of an ‘ontologically neutral’ species of quantification. Yet the semantics for substitutional quantification I have proposed here has reprecussions for each of these two traditional reasons for interest in the topic. In this section, I briefly discuss these reprecussions, starting with truth.[25]

5.1 Substitutional quantification and deflationism

Deflationism about truth is less of a specific theory and more of a set of loosely linked theses about the philosophical significance of truth. Two key deflationist theses are (i) that the concept of truth is loaded with neither metaphysical (correspondence) nor epistemic (coherence) significance, and (ii) that the (non-paradoxical) instances of schema T capture what is essential to truth:

[25] Note that in order to maintain contact with these traditional debates, I will frame the discussion in terms of two quantifiers ‘$\exists$’ and ‘Σ’. This is despite the consequence above that there is really only one particular quantifier.

T The proposition that P is true if and only if P

The most straightforward way to achieve these two theses is to identify one's theory of truth with the set of every non-paradoxical instance of T. This is, in effect, Paul Horwich's minimal theory of truth (Horwich 1998).

Yet Horwich's minimal theory is inadequate as a theory of truth. While it does (trivially) entail every non-paradoxical instance of T, it does not entail important generalizations about truth that an adequate theory should entail. An example of such a generalization is G:

G If the proposition p is true, and the conditional proposition whose antecedent is p and whose consequent is the proposition q is also true, then q is true.

It is impossible to derive G from the non-paradoxical instances of T alone. Yet G seems to be a basic fact about truth, one that an adequate theory of truth should entail.[26]

One way around this problem is to employ substitutional quantification, as in (11):

$$(\forall x)\ x \text{ is true if and only if } (\Sigma \text{s})(\text{s and } x = \text{ the proposition that s}) \tag{11}$$

(11) entails every non-paradoxical instance of T, but it also entails important generalizations about truth.[27] Thus (11) avoids the problem raised above for Horwich's minimal theory, while still avoiding any of the substantial philosophical claims about truth that deflationists about truth want to resist.

This solution to the problem of interesting generalizations about truth works only if we can characterize substitutional quantification without any appeal to truth. Mark Platts (1972) has argued on this basis that this solution to the problem of interesting generalizations about truth won't work, because the semantics for substitutional quantification characterizes substitutional quantification in terms of truth:

> Yet consider the interpretation of the substitutional quantifier. The interpretation of the existential quantification '[(Σx) Fx]' is this: there is a name which, when concatenated with the predicate 'F', produces a *true* sentence...The problem is clear: substitutional quantification is defined in terms of truth, and so cannot itself be used to define truth. (p. 15)

Thus according to Platts, (11), as a theory of truth, is circular.

In response, Soames (1999, p. 91) has argued that Platts mistakes the metalinguistic truth conditions of sentences like (1) for the propositions that such sentences express. This is just the kind of mistake that we saw above in our discussion of the modal problem. The rule Sub does not give the meaning of the particular substitutional quantifier. To give the meaning of the particular substitutional quantifier is to specify what contribution it makes to the propositions expressed by sentences in which it appears. Sub, as we have seen, does not do this.

26 Both Gupta (1993) and Soames (1999) level this criticism against Horwich's minimal theory.

27 For a derivation of one important generalization about truth from a close cousin of (11) (see below), see Hill (1999).

Thus to argue on the basis of Sub that the particular substitutional quantifier is defined in terms of truth is to misunderstand the role of Sub in the semantics for substitutional quantification.

Soames's response to Platts is correct as far as it goes, but the appeal to propositions does not save (11) as a theory of truth. Soames offers no account of what proposition is expressed by sentences containing substitutional quantifiers, but the account offered in the previous section suggests that Platts's objection is well-founded. The semantic rule PS does appeal to truth. The property SOME is the property of mapping at least one thing to a *true* proposition. Thus understanding substitutional quantification requires a prior grasp of the concept of truth, and any attempt to give an analysis of truth in terms of substitutional quantification is circular.[28]

A very different response to this objection is to attempt to characterize substitutional quantification via introduction and elimination rules, rather than via any kind of semantic rule. Chistopher Hill (1999) has proposed such rules for a version of substitutional quantification for what Hill calls *thoughts*. Thus Hill defends an alternative to (11) as a theory of truth:

$$(\forall x)\ x \text{ is true if and only if } (\Sigma \mathrm{s})(\mathrm{s} \text{ and } x = \text{ the thought that s})$$

Hill distinguishes thoughts from Russellian propositions (structured propositions of the kind required by PS). The appeal to thoughts complicates Hill's view in ways that take us too far afield in this paper, but my main objection to Hill's view is that his introduction and elimination rules appeal to what he calls "open thoughts"—thoughts that contain a free substitutional variable:

$$\frac{\underline{\text{Existential Elimination}}}{\dfrac{(\Sigma \mathrm{p})(\ldots \mathrm{p} \ldots) \qquad \text{If } (\ldots \mathrm{q} \ldots), \text{ then T}}{\mathrm{T}}}$$

In this rule, '(...q...)' is an open thought, and 'q' is substitutional variable that is not free in any of T, '(Σp)(...p...)', or any further premise on which T depends (Hill 1999, p. 99). As applied to Hill's thoughts, my objection to this rule is that I have no idea what thoughts containing a free substitutional variable would be like, nor whether I have ever had any.

A similar problem undermines the attempt to apply a rule like the one proposed by Hill for sentences (instead of thoughts). It is unclear to me what it means to perform or endorse inferences containing free substitutional variables. The problem is that sentences containing free substitutional variables are semantically inert—a substitutional variable is merely a placeholder for the members of the associated substitution class. We do not make or endorse inferences using semantically inert

[28] It may be tempting to respond to this objection to (11) as follows: the content of 'Σ' in PS is the property of being a function that maps at least one thing to a true proposition, but there are other neo-Russellian semantic treatments of quantification according to which the content of a quantifier is the property of being a non-empty set (e.g. Soames (1987, p. 73)). Understanding this property does not require grasp of the concept of truth. But this proposal has the unfortunate consequence that (1) is about sets, contrary to our linguistic intuitions.

sentences, so some further account is required to explain what it means to endorse such an inference.

5.2 Substitutional quantification and ontology

Substitutional quantification is supposed to be 'ontologically neutral' (Marcus 1972, p. 245). Because the truth of a substitutionally quantified sentence (according to Sub) does not depend on the satisfaction of any open sentence by any object, substitutional quantification does not carry the ontological commitments that Quine has so often emphasized for objectual quantification (Quine 1969, 1980). Yet on the semantics proposed in this paper, the truth of (1) (relative to the substitution class {'Tally'}) depends on the truth of the proposition expressed by (4), and this proposition requires for its truth that Tally exists. It thus appears that substitutional quantification is not as ontologically neutral as it has been taken to be.

So much the worse, I say, for the supposed ontological neutrality of substitutional quantification. But it is important not to overstate the result. To clarify, suppose that E is either '∃' or 'Σ', and call the widest scope occurrence of E in $\ulcorner(\mathrm{Ev})\phi\urcorner$ the *primary occurrence* of E in $\ulcorner(\mathrm{Ev})\phi\urcorner$. We can now define 'existential import' for a primary occurrence of a quantifier E:

Definition The primary occurrence of E in $\ulcorner(\mathrm{Ev})\phi\urcorner$ has *existential import* if and only if ($\ulcorner(\mathrm{Ev})\ \phi\urcorner$ is true relative to a context c, assignment f, and substitution class S only if there is some object o and some objectual variable v_R that does not occur freely in ϕ, such that $\phi(\mathrm{v}/v_R)$ is true relative to c, $f^{o}_{v_R}$, and S).

Trivially, every primary occurrence of '∃' has existential import. This is an immediate consequence of the definition above and the semantics for '∃'.

Examples like (1), however, show that some primary occurrences of 'Σ' also have existential import.[29] The occurrence of 'Σ' in (1) is a primary occurrence, and (1) is true (relative to the substitution class {'Tally'}) only if (4) is true, which is itself true only if 'x is a dog' is true relative to an assignment of Tally to 'x'.

But it is false that every primary occurrence of 'Σ' has existential import—witness the occurrence of 'Σ' in (8). Unlike a primary occurrence of '∃', the existential import of the primary occurrence of 'Σ' in $\ulcorner(\Sigma\mathrm{x})\phi\urcorner$ is not an immediate consequence of the semantics for 'Σ', but instead depends on the truth conditions of

[29] Alex Orenstein (1984) has also called the ontological neutrality of substitutional quantification into question, on grounds very much like those raised here. One must approach his argument with care, however, as Orenstein is not always careful to distinguish between quantifiers, occurrences of quantifiers, and sentences in which quantifiers appear. In the following passage, he attributes what he calls 'referential force' to a whole sentence, but elsewhere he attributes referential force to quantifiers (or uses of quantifiers):

> The referential force and ontological import of '[(Σx) x is a Siamese cat]' would not be due to [Sub] but to the referential aspect of the truth condition for the instances it depends upon. (p. 147)

It seems to me that Orenstein is here concerned about the (primary) *occurrence* of 'Σ' in '(Σx) x is a Siamese cat'. Thus I take Orenstein to be arguing for the same claim as I have stated above.

the substitution instances in virtue of which ⌜$(\Sigma x)\phi$⌝ is true (Orenstein 1984, p. 147).[30]

6 Conclusion

In this paper, I have proposed a propositional semantics for substitutional quantification: the rule PS. PS clarifies how substitutional quantification is not merely metalinguistic quantification over expressions, how substitutional quantification behaves differently from standard objectual quantification, and yet how substitutional quantification is clearly a species of quantification. The possibility of such a treatment of substitutional quantification seems to me to be of interest in its own right for the insight it offers into the nature of quantification, but PS is interesting also for the possibility of its application to natural languages like English, and for the light it sheds on various traditional philosophical reasons for interest in substitutional quantification.

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References

Brogaard, B. (2012). *Transient truths: An essay on the metaphysics of propositions*. Oxford: Oxford University Press.

Camp, J. L. (1975). Truth and substitution quantifiers. *Noûs*, *9*(2), 165–185.

Dunn, J. M., & Belnap, N. D. (1968). The substitution interpretation of the quantifiers. *Noûs*, *2*(2), 177–185.

Georgi, G. (2014). Logic for languages containing referentially promiscuous expressions. Manuscript.

Gupta, A. (1993). Minimalism. *Philosophical Perspectives*, *7*, 359–369.

Hill, C. S. (1999). Truth in the realm of thoughts. *Philosophical Studies*, *96*(1), 87–121.

Horwich, P. (1998). *Truth* (2nd ed.). New York: Oxford University Press.

Kaplan, D. (1989). Demonstratives. In J. Almog, J. Perry, H. K. Wettstein, & D. Kaplan (Eds.), *Themes from Kaplan*. New York: Oxford University Press.

King, J. C., Soames, S., & Speaks, J. (2014). *New thinking about propositions*. Oxford: Oxford University Press.

Kripke, S. (1976). Is there a problem about substitutional quantification? In G. Evans & J. McDowell (Eds.), *Truth and meaning: Essays in semantics*. Oxford: Clarendon Press.

Lycan, W. G. (1979). Semantic competence and funny functors. *The Monist*, *62*(2), 209–222.

Marcus, R. B. (1972). Quantification and ontology. *Noûs*, *6*(3), 240–250.

Orenstein, A. (1984). Referential and nonreferential substitutional quantifiers. *Synthese*, *60*(2), 145–157.

MdB, Platts. (1972). *Ways of meaning*. London: Routledge and Kegan Paul.

Priest, G. (2005). *Towards non-being*. Oxford: Clarendon Press.

Quine, W. V. O. (1969). *Ontological relativity and other essays*. New York: Columbia University Press.

Quine, W. V. O. (1980). *From a logical point of view*. Cambridge, MA: Harvard University Press.

Salmon, N. (2006). A theory of bondage. *Philosophical Review*, *115*(4), 415–448.

Soames, S. (1987). Direct reference, propositional attitudes, and semantic content. *Philosophical Topics*, *15*, 47–87.

[30] See also Kripke (1976, p. 333).

Soames, S. (1999). *Understanding truth*. Oxford: Oxford University Press.
Soames, S. (2011). True at. *Analysis*, *71*(1), 124–133.
Van Inwagen, P. (1981). Why I don't understand substitutional quantification. *Philosophical Studies*, *39*, 281–285.