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Resemblance theories of properties

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Abstract The paper aims to develop a resemblance theory of properties that technically improves on past versions. The theory is based on a comparative resemblance predicate. In combination with other resources, it solves the various technical problems besetting resemblance nominalism. The paper's second main aim is to indicate that previously proposed resemblance theories that solve the technical problems, including the comparative theory, are nominalistically unacceptable and have controversial philosophical commitments.

Keywords Resemblance nominalism · Properties · Comparative resemblance predicate · Coextension problem · Imperfect community problem · Companionship problem · Mere intersections problem

1 Introduction

Resemblance nominalism faces a host of technical problems in its bid for extensional adequacy.[1]

[1] Some of these problems were raised in Goodman (1966) against Carnap's phenomenological version of resemblance nominalism; see also Carnap (1928, §§70 & 72). Writing in the mid twentieth-century, Price claimed that most metaphysicians of properties at the time were resemblance nominalists (1953, p. 13). Today the situation is reversed; Goodman's discussion may well have been responsible for the decline in resemblance nominalism's popularity.

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The coextension problem

It might be that anything that instantiates property *F* instantiates property *G* and that anything that instantiates *G* instantiates *F*. A strong version: *F* and *G* are necessarily coextensive.

The problem of imperfect community

Some particulars might pairwise resemble without sharing a common property. For example: *a* instantiates *F* and *G*, *b* instantiates *G* and *H*, and *c* instantiates *F* and *H*.

The companionship problem

The class of entities instantiating *F* might be a proper subclass of the class of entities instantiating *F**. In that case *F** is said to be a companion to *F*. In iterated companionship, a companion *F** to *F* is itself accompanied by a property *F***; a further iteration arises if *F*** is in turn accompanied by some property *F****, and so on. Note that if conjunctive properties exist so do companions: *having mass m* and *having charge c* are companions to *having mass m and having charge c*. Companionship is also a consequence of taking determinables as well as determinates to be genuine properties, e.g. *having mass* is a companion to *having mass m*.

Why are these problems for resemblance nominalism? A property *F* corresponds to a class *C*, which is then called a *property class*, when each of the members of *C*, and nothing else, has property *F*. Property classes are a type of *maximal resemblance class*. A maximal resemblance class is a resemblance class in that any two of its members resemble; and it is a maximal such class, in that anything not in the class does not resemble at least one of its members. Resemblance nominalists must try to account for properties, and in particular property classes, on the basis of maximal resemblance classes. The coextension problem is therefore a problem for recovering properties from property classes, and the imperfect community and companionship problems are problems for recovering property classes from maximal resemblance classes.[2]

Finally, suppose a resemblance nominalist has the resources to express the fact that a class is of resemblance degree *n*—the resemblance nominalist counterpart of the fact that some particulars share *n* properties. In trying to define a property class as a maximal resemblance class of degree *n*, the resemblance nominalist faces one further problem.[3]

The mere intersections problem

Suppose that conjunctive properties sometimes do not exist, i.e. for some *F* and *G* there is no conjunctive property $F \wedge G$. In that case particulars instantiating *F* and *G* are a maximal resemblance class of degree 2 that does not correspond to a property class.

[2] For the sake of simplicity we focus on (unary) properties rather than relations.

[3] The terminology is from Rodriguez-Pereyra (2002).

Resemblance nominalist solutions to all four problems must have two aspects: a technical aspect, that is to say, an argument to the effect that all the facts a theory of properties must account for are accounted for; and a philosophical aspect, that is, an argument to the effect that the theory's resources and commitments are acceptable. The paper accordingly has two parts: a technical one (§§2–5) and a philosophical one (§6). In §2 we consider three resemblance theories of properties and their technical solutions (if any) to the problems. The first is a generic version of traditional resemblance nominalism; the second is due to Lewis (1983); and the third to Rodriguez-Pereyra (2002). In §3 we present a new theory based on a comparative resemblance relation that is technically superior to these three resemblance theories. In §§4–5 we show how the comparative theory technically improves on the other resemblance theories, taking in the problems' infinite cases in §5. In §6 we argue that the primitives of Lewis' theory, Rodriguez-Pereyra's theory and the comparative theory are nominalistically unacceptable, and that they also have controversial commitments.

The paper therefore has two main aims. The first is the technical development of a new and technically superior resemblance theory of properties, the comparative theory. The paper's second main aim is to argue that previously proposed resemblance theories of properties—including the comparative theory—that solve the technical problems are nominalistically unacceptable and have controversial philosophical commitments. Our discussion rigorously sets out the trade-off resemblance theories face between extensional adequacy and an attractive primitive theory of properties. It suggests the broader moral that any extensionally adequate resemblance theory of properties is not a form of nominalism.[4]

2 Three resemblance theories[5]

2.1 The traditional R^2-theory

Traditional versions of resemblance nominalism are based on the two-place resemblance predicate R^2 (x, y), interpreted as 'x resembles y'. Property realists would understand this predicate as 'x and y share at least one property'. Maximal resemblance classes may be defined in terms of R^2:

$$C \text{ is a maximal resemblance class} \equiv_{\text{def}} \forall x \in C \forall y \in C\big(R^2(x,y)\big) \\ \wedge \forall x \notin C \exists y \in C(\neg R^2(x,y))$$

Some variants of resemblance nominalism (e.g. versions based on paradigms) modify this definition in ways that do not significantly affect our discussion. The first three problems are well-known precisely because they are problems for

[4] We won't consider other objections to resemblance nominalism, for example the regress problem pressed in Russell (1912).

[5] For the sake of legibility this paper is sloppy about use and mention, e.g. we use the same letter to denote a predicate and the relation it expresses, and we often drop quotation marks or corner quotes where they are formally required.

constructing properties from property classes and property classes from maximal resemblance classes. These three problems are thus problems for traditional R^2-based resemblance nominalism. Invoking possibilia solves the problem of coextension, as contingently coextensive properties may be distinguished. But it does not solve the problem of necessary coextension. The imperfect community and companionship problems are evidently not solved by the R^2-theory.[6] This motivates the search for a technically stronger resemblance theory.

2.2 Lewis

In a brief passage (1983, pp. 14–15), Lewis proposes basing resemblance nominalism on a contrastive and variably polyadic resemblance predicate.[7] The intended interpretation of this predicate, which we may label $x_1, x_2,\ldots R^L y_1, y_2,\ldots$ —the 'L' stands for 'Lewis'—is:

$x_1, x_2, \ldots$resemble one another and do not likewise resemble any of $y_1, y_2, \ldots$

In property realist terms: $x_1, x_2,\ldots$ share a property which is not shared by any of y_1, $y_2,\ldots$. Lewis remarks that the adicity of the variable strings may be infinite, even uncountably infinite. The collective resemblance predicate $R^n(x_1, x_2,\ldots,x_n)$, interpreted as '$x_1, x_2,\ldots,x_n$ resemble', is just R^L applied to n-many x_i and to no y_i, and so is expressible in Lewis' theory. In particular, we recover traditional resemblance nominalism's predicate R^2 when $n = 2$.

Lewis defines property classes by (1983, p.15, fn. 9):

$$X \text{ is a property class}$$
$$\equiv_{\text{def}} \exists y_1, \exists y_2, \ldots, \forall z(z, x_1, x_2, \ldots R^L y_1, y_2, \ldots \equiv z = x_1 \vee z = x_2 \vee \ldots)$$

where x_1, $x_2,\ldots$ are the particulars that are X's members. The idea is that X is a property class iff anything that resembles all the X-members is already one of them. The theory technically solves the imperfect community, companionship and mere intersections problems. Particulars that form an imperfect community do not stand in the relation R^L to anything else, whereas particulars that form a perfect community do. If F_0 is accompanied by F_1 then the F_0-particulars stand in the relation R^L to a particular that instantiates F_1 but not F_0, so the F_0-resemblance class is distinguishable from the F_1-resemblance class and is not 'swallowed' by it. And if the classes corresponding to properties F and G overlap (without subsumption), then $F \wedge G$ exists iff the particulars instantiating both F and G stand in the relation R^L to two particulars instantiating F but not G and G but not F respectively.

Finally, the problem of coextension can be technically solved by appealing to possibilia, since the property classes corresponding to merely contingently coextensive properties are different. This seems to be a necessary and sufficient condition for the R^L-theory to technically solve the coextension problem (see §6(b)

[6] The mere intersections problem apparently does not arise, since the R^2-theory seems incapable of defining the notion of being a resemblance class of degree n.

[7] Lewis is considering the best form that resemblance nominalism might take. He is not advocating the theory himself.

for a general argument to that effect). However, as we shall see in §4, the R^L-theory cannot solve the problem of necessary coextension.

2.3 Rodriguez-Pereyra

Rodriguez-Pereyra (2002) develops a resemblance theory that aims to solve all four problems. It attempts to solve the coextension problem by adopting Lewisian realism and taking resemblance relations as obtaining among possible as well as actual particulars; it attempts to solve the imperfect community problem's finite cases by defining the infinitely many collective resemblance predicates R^n $(x_1, x_2,\ldots,x_n)$, for n finite, using set-theoretic apparatus; and it attempts to solve the companionship problem's finite cases by taking as primitive the infinitely many (two-place) degree resemblance predicates R_n (x_1, x_2), for n finite. From the property realist perspective the predicate 'R_n (x_1, x_2)' should be interpreted as 'x_1 and x_2 share exactly n properties'. (Henceforth 'R_n' will abbreviate the two-place n-degree resemblance predicate $R_n(x_1, x_2)$ and 'R^n' will abbreviate the n-place collective resemblance predicate $R^n(x_1,\ldots,x_n)$.) It cannot solve the infinite cases of the imperfect community problem and the companionship problem (more on this in §5).

Rodriguez-Pereyra defines R^k using set theory as follows: $R^k(a_1, a_2,\ldots a_k)$ obtains when R^2 holds between any two pair sets whose members are themselves pair sets, whose members' members are also pair sets, etc., and the union of whose urelements is the class made up of $a_1, a_2,\ldots a_k$. In other words, $R^k(a_1, a_2,\ldots a_k)$ iff R^2 holds between any two nth-rank pair sets whose transitive closure contains $a_1,\ldots,a_k$ as its only individuals.[8] This can be achieved by going sufficiently high in the hierarchy of pair sets based on $a_1, a_2,\ldots a_k$. Thus two nth-rank pair-sets resemble one another iff all the particulars out of which they are ultimately made up—the individuals in their transitive closure—share a common property. A metaphysician who reifies properties takes the right-hand side of this equivalence as primitive; Rodriguez-Pereyra in contrast takes its left-hand side as primitive.

The informal motivation for the biconditional is that particulars $a_1, a_2,\ldots a_k$ resemble each other (in property realist terms: share a property) just when sets containing them as urelements resemble each other. An abundant property example: Galba, Otho, Vitellius and Vespasian resemble each other because they were all Roman emperors in AD 69, hence the sets {Galba, Otho} and {Vitellius, Vespasian} also resemble each other. Conversely, sets with urelements that do not resemble each other do not share any properties, at least no properties other than abstract ones such as *being a set*. Rodriguez-Pereyra does not count the latter as genuine properties, because his theory is a theory of sparse properties, that is, a theory of natural or

[8] The set theory in question has all the individual particulars as its urelements (i.e. entities in the domain of quantification that are not sets). Ranks are as standardly defined in set theory: the set $\{a, b\}$ (where a and b are urelements) has rank 1, the set $\{\{a, b\}\}$ has rank 2, and so on. A pair set is a set with two members. An nth-rank pair set is a pair set of rank n such that all its elements are pair sets, elements of its elements are pair sets, and so on for n steps until urelements are reached. The transitive closure of a set X is defined as the smallest set containing X and closed under the union operation. Thus a set's urelements are the individuals (non-sets) in its transitive closure. All the set theory used in this paper may be found in Goldrei (1996).

fundamental properties investigated by basic science. So his theory is not about *being a set* or *being a Roman emperor* (which he takes to be abundant properties), but about sparse properties such as *having mass m* and *having charge c*.

The definition allows his theory to technically solve finite cases of the imperfect community problem by allowing it to express R^m for any finite m. The companionship problem's finite cases are solved by taking the degree resemblance predicates R_n as primitive. If F_0 is accompanied by F_1 (and perhaps some other properties) then the F_0-particulars form a resemblance class of degree $n + 1$, whereas the F_1-particulars form a resemblance class of degree n. A resemblance class of degree n is formally defined in terms of R_n as follows:

$$C \text{ is a maximal resemblance class of degree } n$$
$$\equiv_{\text{def}} \forall x \in C \forall y \in C \exists k \geq n(R_k(x, y)) \wedge \exists x \in C \exists y \in C(R_n(x, y)) \wedge$$
$$\forall x \notin C \exists y \in C \exists k < n(R_k(x, y))$$

The problem of coextension could be similarly solved (*modulo* the other problems), since a property class corresponding to two properties has resemblance degree 2. However, Rodriguez-Pereyra does not opt for this solution. He prefers instead to invoke possibilia and to argue that there are no cases of necessary coextension.

The mere intersections problem is solved in the following way. Define an ultimate class to be a maximal resemblance class that is not a proper subclass of *any* maximal resemblance class (and not just any resemblance class of the same degree as itself). We then define the following function from classes to numbers[9]:

$R * \text{diff}(C) \equiv_{\text{def}}$ 0 **if** C is not a maximal resemblance class;
the resemblance degree of C **if** C is an ultimate class;
the resemblance degree of C minus the sum of the R∗ diff−values of all of C's superclasses **if** C is a non−ultimate maximal resemblance class.

Property classes then correspond to maximal resemblance classes with an R*diff-value of 1. Intuitively, the R*diff-value of a class is its net resemblance contribution. A mere intersection of degree n brings no net resemblance contribution over and above the contributions to its resemblance degree made by the property classes of which it is the mere intersection, hence its R*diff-value is 0. A property class in contrast always makes a net contribution of 1, which allows the theory to solve the mere intersections problem.[10]

[9] X is a superclass of Y iff Y is a subclass of X.

[10] Observe that if conjunctive properties never exist then Rodriguez-Pereyra's definition is in fact logically equivalent (for finite cases) to the following simpler definition: a maximal resemblance class is a property class iff it is not the intersection of any proper maximal resemblance superclasses. This simpler definition also extends to cases of infinite resemblance, whereas the R*diff-definition does not, since a class's net resemblance contribution is not in general equal to the difference between the sum of some infinite cardinals from some infinite cardinal (however one extends the notion of subtraction to the transfinite). We come back to this point in §5.

3 Comparative resemblance

So far we have presented three resemblance theories and considered whether, and if so how, they solve the technical problems introduced in §1. As we have observed, each of them fails to solve at least one of the technical problems. This section presents a new resemblance theory of properties, based on a comparative resemblance relation, which solves all four.[11]

The comparative theory is based on two ideas. The first is to allow resemblance to hold between sets of arbitrary size and not just *n*th-rank pair sets as in Rodriguez-Pereyra's theory. The second is to take resemblance as a comparative relation. In this section we develop the theory on the basis of the comparative predicate $R^+(x_1, x_2)$, which for the time being we understand as 'x_1 resembles itself more than x_2 resembles itself'. From the property realist point of view, $R^+(x_1, x_2)$ should be interpreted as 'x_1 instantiates more properties than x_2 instantiates'. The predicate R^+ allows us to make a claim such as that a particle with mass m_1 and charge c_1 and no other (sparse) properties resembles itself more than a particle with just mass m_1 resembles itself. If x_1 or x_2 are classes, $R^+(x_1, x_2)$ holds just when, from the property realist perspective, the urelements of x_1 jointly instantiate more properties than the urelements of x_2 do. (If x is not a class we may take its one and only urelement to be itself.) In §6 we shall see that the predicate R^+ is interdefinable with the perhaps more natural primitive $\mathscr{R}^+(x_1, x_2, x_3, x_4)$, which we may for the time being understand as 'x_1 resembles x_2 more than x_3 resembles x_4'. In property realist terms $\mathscr{R}^+(x_1, x_2, x_3, x_4)$ states that the number of properties shared by x_1 and x_2 is greater than the number of properties shared by x_3 and x_4. Both R^+ and $\mathscr{R}^+$ have surprising features. To give just one example, two distinct particulars that share several properties can stand in the relation $\mathscr{R}^+$ to a single particular and itself if the latter instantiates fewer properties than are shared by the first two. To appreciate the theory's technical development, we suppress these and other qualms about R^+ and $\mathscr{R}^+$ until §6, where we give vent to them. Until then, for the sake of technical simplicity we develop the theory in terms of R^+ rather than $\mathscr{R}^+$.

We showed earlier how Lewis' theory attempts to define a property class. Lewis' primitive R^L can in fact be defined in terms of R^+ by:

$$x_1, x_2, \ldots R^L y_1, y_2, \ldots \equiv_{\text{def}} (\forall y \in Y)(XR^+(X \cup \{y\}))$$

[11] Of the resemblance nominalisms founded on predicates other than R^2, I know of none technically superior to the one presented here. For example, in a section of a paper discussing Carnap's *Aufbau* (1975, pp. 68–73), Eberle proposed founding resemblance nominalism on a three-place predicate, call it R^E, whose intended interpretation is 'x_1 exactly resembles x_2 in a certain respect but not x_3'. In property terms: 'there is some property P that x_1 and x_2 instantiate but x_3 does not'. Eberle's proposal does not solve the imperfect community problem. Consider the following four-particular communities. The first community consists of particulars a_1, a_2, a_3, a_4, where a_1 instantiates F_2, F_3 and F_4, a_2 instantiates F_1, F_3 and F_4, a_3 instantiates F_1, F_2 and F_4, and a_4 instantiates F_1, F_2 and F_3. The second community consists of particulars a_1, a_2, a_3, a_4, where a_1 instantiates G, F_2, F_3 and F_4, a_2 instantiates G, F_1, F_3 and F_4, a_3 instantiates G, F_1, F_2 and F_4, and a_4 instantiates G, F_1, F_2 and F_3. The second community is thus the first community with some extra G-instantiations tacked on. The first community is imperfect (the a_i do not share a property) and the second is perfect (the a_i share a property: G). But in both cases any three particulars stand in the relation R^E.

where X's members are x_1, x_2,…,and Y's members are y_1, y_2,…. For if the left-hand side obtains then the members of X collectively share a property that the members of $X \cup \{y\}$ do not collectively share for any y in Y. And if the right-hand side obtains then the members of X collectively share a property not shared by any of the members of Y. It follows that the comparative theory can technically solve the problems of imperfect community, companionship, and mere intersections, since the R^L-theory can. The problem of coextension can be technically solved in similar fashion by appeal to possibilia.

Unlike Lewis' theory, the comparative theory also affords a technical solution to the problem of necessary coextension. We say that C is property class of degree n iff C is a property class corresponding to n properties. The ability to express this notion affords a technical solution to the necessary coextension problem, since a class corresponding to two or more properties may then be distinguished from a class corresponding to just one property. We define:

$$XR^{=}Y \equiv_{\text{def}} \neg XR^{+}Y \wedge \neg YR^{+}X$$
$$XR^{+=}Y \equiv_{\text{def}} XR^{+}Y \vee XR^{=}Y$$

From the property realist perspective, $XR^{+=}Y$ iff the urelements of X share no fewer properties than the urelements of Y. Next, we define $\mathscr{P}$ to be the class of all possibilia.[12] We then recursively define the infinitely many predicates $\mathrm{P}(X, n)$, interpreted as 'X is a class of degree n' as follows, where the quantifiers range over all possibilia classes (note that X may be a class of degree n even if it is not a property class, e.g. it may be a subset of a property class):

$$\mathrm{P}(X,0) \equiv_{\text{def}} \neg XR^{+}\mathscr{P}$$
$$\mathrm{P}(X,n+1) \equiv_{\text{def}} \exists Y(\mathrm{P}(Y,n) \wedge XR^{+}Y) \wedge \forall Z(\exists Y(\mathrm{P}(Y,n) \wedge ZR^{+}Y) \rightarrow ZR^{+=}X)$$

The idea behind the basis clause is that the members of X do not share a property iff they do not share more properties than are shared by all the possibilia. The modal assumption underlying this clause is that there is no property shared by all possibilia. The idea behind the recursion clause is that the members of a class of degree $n+1$ share more properties than the members of a class of degree n, but no more than any other class whose members share more properties than the members of a class of degree n. In other words, the assumption is that there are no property gaps across modal space, i.e. for any n, some particulars across modal space instantiate exactly n properties. A property class C then corresponds to n properties iff $\mathrm{P}(C, n)$. This is how the comparative theory solves the problem of necessary coextension.

Some observations are in order. First, note that if the quantifiers are restricted to classes of actual particulars, the definition will be correct only if the actual world happens to have two particulars in it that share no properties and the actual world

[12] We call this and other collections 'classes' to preserve neutrality on whether they are sets or proper classes. Since standard set theories with urelements take the class of urelements to be a set, it would not be controversial to assume that P, and indeed all the other classes mentioned in this paper, are sets.

has no property degree gaps. If our world is of this kind, that is a contingent fact about it (see §6(b)). Second, the use of property talk in this and similar contexts to elucidate the official definitions is supposed to be motivational and not part of the official theory itself: 'property slang' is acceptable only once the translation procedures into primitive resemblance terminology in terms of R^+ (and perhaps ultimately in terms of $\mathscr{R}^+$) are in place. In §6(a), we will argue that this primitive terminology is not in fact truly nominalist. Third, recursive definitions of this kind are a standard set-theoretic method, available in pretty much any set theory (including ZFC). The final observation is that given P(X, n) we can define various resemblance notions, for example:

x_1 and x_2 resemble exactly to degree n('$R_n(x_1, x_2)$' above) $\equiv_{\text{def}}$ $\text{P}(\{x_1, x_2\}, n)$

$x_1, x_2, \ldots, x_m$ resemble('$R^m(x_1, x_2, \ldots, x_m)$' above) $\equiv_{\text{def}}$

$\exists n \geq 1\ \text{P}\left(\left\{x_1, x_2, \ldots, x_m\right\}, n\right)$

4 Advantages: the finite cases

The comparative theory is technically superior to the R^2-theory, as should be evident. We now compare it to the other two resemblance theories encountered in §2.

4.1 Lewis

Lewis' theory cannot solve the problem of necessary coextension, since it has no way to distinguish necessarily coextensive resemblance classes on the basis of R^L. Let scenario 1 be a scenario in which a transworld class of possibilia C is the class corresponding to property F, and let scenario 2 be a scenario in which C corresponds to properties F and G. (A scenario is a consistently describable situation; it may or may not be metaphysically possible.) Scenarios 1 and 2 are R^L-indistinguishable, yet as we have seen they are R^+-distinguishable. This implies that R^+ cannot be defined in terms of R^L even in the presence of set theory and quantification over possibilia, since scenarios with necessarily coextensive properties can be distinguished by R^+ but not by R^L. Given that R^L can be defined in terms of R^+, it follows that the comparative theory is stronger than Lewis'.

It remains to be seen whether this is a genuine problem for Lewis' theory, that is, whether scenarios with necessarily coextensive properties correspond to genuine possibilities. This is not the place to tackle this question.[13] We note only that

[13] One reason for believing in necessarily coextensive properties, that they appear in mathematics, does not sit well with the comparative theory. If mathematical properties such as *being trilateral* and *being triangular* exist then any two classes presumably share properties such as *being a set*, contrary to an important motivating idea behind the comparative theory, that classes share the properties shared by their urelements. And if the property theory is intended to be a theory of sparse properties, that is another reason for thinking that mathematical examples don't qualify, since mathematical properties don't seem to be sparse. But this last claim is controversial. Sober (1982) argues that *being trilateral* and *being triangular* have different causal roles (a device could be designed to detect one but not the other) and are therefore distinct. There are other arguments for necessarily coextensive properties, for example that there could be a property determinate with just one determinable.

inability to solve this problem is a mark against a resemblance theory for any metaphysician who takes necessarily coextensive properties to be an epistemic possibility. It is certainly a problem for metaphysicians such as Rodriguez-Pereyra who believe that a theory of properties must account for conceptual possibilities: "If Resemblance Nominalism cannot do this [solve the companionship problem] then it is wrong, for accompanied properties are at least a conceptual possibility." (2002, p. 151). (Here he is speaking about accompanied properties but his point is more general.) Assuming that the existence of necessarily coextensive properties is not conceptually ruled out, the necessary extension problem is a problem for any such metaphysician.

The second problem with Lewis' theory is that it is triply infinitary: its basic predicate is infinitary, its background logic allows infinitely many disjunctions and conjunctions, and it also allows quantification over infinitely many variables. These are of course essential aspects of the theory, on pain of not being able to express the fact that an infinite class of particulars share a property. But it seems that the only way we can understand such predicates is derivatively, as a set-theoretic construction from finitary predicates. To fix ideas, consider the predicate x_1, $x_2, \ldots R^L y_1, y_2, \ldots$, where the x_i are $\aleph_1$-many and the y_i are $\aleph_2$-many. This predicate is of relatively small infinitary size; but it is still uncountably large, doubly so. We cannot utter it or write it without ellipsis. We can only grasp it indirectly, by deploying some combination of finite concepts of resemblance and set-theoretic understanding. Lewis' theory therefore seems a poor candidate for the basic resemblance nominalist theory.

One could reformulate Lewis' theory to overcome this problem. Let R^{L^*} be the relation that obtains between classes X and Y iff the relation R^L obtains between the members of X and the members of Y. Then develop the theory in terms of R^{L^*}. The R^{L^*}-theory overcomes the second problem, but not the problem of necessary coextension.

4.2 Rodriguez-Pereyra

A point of difference between the comparative theory and Rodriguez-Pereyra's is that for him resemblance relations hold only between certain kinds of classes—pair sets—whereas in the comparative theory they hold between any sets whatsoever. But Rodriguez-Pereyra (2002) contains no cogent argument for the claim that the relata of the resemblance relation must be *n*th-rank pair sets if they are sets at all, and thus no reason for privileging pair sets over other kinds of sets.[14] What does the work in his theory is the fundamental equivalence that individuals have a property in common iff two sets of a certain kind whose transitive closure contains precisely these individuals resemble one another. There is nothing in this idea that requires the sets in question to be pair sets.

Rodriguez-Pereyra also insists that a primitive resemblance relation cannot be both binary and *N*-ary for *N* greater than 2 (2002, pp. 80–81). For the nature of

[14] Rodriguez-Pereyra (2002, pp. 175–176) contains an argument that presupposes his argument against a collective notion of resemblance, discussed in the next paragraph.

resemblance is such that if N particulars resemble then any pair of them also resemble one another (and indeed any M of them for $M \leq N$). That N particulars are in the N-ary resemblance relation does not, however, logically entail that two of them resemble. So the facts that if $a_1,\ldots a_N$ collectively resemble then a_1 and a_2 pairwise resemble, and a_1 and a_3 pairwise resemble,…, all have to be *stipulated* in such a theory. Rodriguez-Pereyra objects that such a stipulation is no explanation for the implication. Contrast his resemblance theory, which does explain it, he says:

> The explanation of this is, I maintain, that resemblance links *pairs* of particulars; so that what makes the members of any class resemble is that they resemble *pairwise*, which entails that the members of all its subclasses also resemble. (2002, p. 81).

For this reason he restricts the resemblance relation to at most two terms.

One could object to the argument that since we are dealing with interpreted predicates what matters is conceptual rather than logical entailment. Patently, the collective resemblance of $a_1,\ldots,a_N$ does conceptually entail a_1 and a_2's pairwise resemblance. A further, *ad hominem* response would be to point out that since Rodriguez-Pereyra takes the resemblance predicates R_n as primitive, his theory has similar resemblance links, namely $\forall x_1 \forall x_2[R_n(x_1, x_2) \rightarrow \neg R_m(x_1, x_2)]$ for $n \neq m$ (recall that $R_n(x_1, x_2)$ is interpreted as 'x_1 and x_2 share exactly n properties'). In any case, valid or not, the objection does not touch the comparative theory. It is at best a reason against taking more than one collective resemblance degree as primitive. It therefore doesn't apply to the comparative theory, which is founded on the single primitive R^+ (or $\mathscr{R}^+$).

An advantage of the comparative theory is a gain in ideological economy. (We count this as a 'technical' advantage, but it could also be considered a more 'philosophical' advantage—a bookkeeping issue.) Rodriguez-Pereyra's theory takes the infinitely many predicates $R_n(x, y)$ as primitive. He insists that he does not wish to reduce these infinitely many predicates to a single three-place predicate $R(x, y, n)$, because this would make resemblance into a three-place rather than a two-place or one-place relation (2002, p. 80).[15] Thus adopting the comparative theory in favour of Rodriguez-Pereyra's greatly reduces the theory's primitives, from infinitely many to one. Furthermore, a theory with infinitely many primitives cannot be grasped by us unless it is understood on the basis of a finitary theory, for example by understanding the infinitely many predicates $R_n(x, y)$ as instances of the single predicate $R(x, y, n)$, or, as in §3, $P(\{x_1, x_2\}, n)$. Rodriguez-Pereyra's theory thus cannot be the basic resemblance theory.

The comparative theory is also technically superior to Rodriguez-Pereyra's in that it can technically solve the technical problems' infinite cases whereas his cannot. This is the topic of the next section, the paper's most technical one. Readers eager to move on to the philosophical assessment of the theories can head straight for §6.

[15] Given that he has to quantify over the subscript in 'R_k' in giving a definition of a maximal resemblance class of degree n as we saw at the end of §2, it is not even clear that this is a consistent position.

5 The problems' infinite cases

5.1 The infinitary hypothesis…

In §4 we encountered a scenario in which particulars instantiate infinitely many properties. Consider another scenario. Imagine a world with infinitely many particulars a_1,a_2,…, each instantiating exactly one property: a_1 instantiates F_1, a_2 instantiates F_2, and so on. Suppose further that any two particulars sharing a property attract (as may be verified within this world by investigating the particulars' interactions). Thus any two F_1-particulars attract, any two F_2-particulars attract, and so on. Now suppose that there is another particular in this world, *b*, that instantiates each of the properties F_1, F_2, F_3,… (and no other). *b* thus attracts each of a_0, a_1, a_2,…, in virtue of sharing a property with each of them. *b*'s instantiation of the infinitely many properties F_1, F_2, F_3,…, could in principle be confirmed by inhabitants of the world in question by considering *b*'s interactions with the particulars a_0, a_1, a_2,….[16]

Isn't the scenario just described a coherent one? Doesn't it seem to capture a metaphysical possibility? If so, there are worlds in which particulars instantiate infinitely many properties. Moreover, some of these worlds have inhabitants that could come to know that their world is of this kind, as in the described scenario.

Call the possibility of a scenario of this type the 'infinitary hypothesis' and its impossibility the 'finitary hypothesis'. Before exploring how to deal with the four technical problems if the infinitary hypothesis is true, we rebut two arguments Rodriguez-Pereyra advances against it in the context of defending his theory, which he recognises cannot deal with infinite cases. (Given that he thinks a theory of properties must account for conceptual possibilities as well as metaphysical ones, he is committed to the infinitary hypothesis not even being a conceptual possibility.) This rebuttal does not add up to a conclusive case for the infinitary hypothesis. But in light of the hypothesis' initial plausibility, it provides motivation for investigating how resemblance theories might account for it.

Rodriguez-Pereyra's first argument against the infinitary hypothesis is contained in this short passage:

> …sparse properties are those that basic science tries to make an inventory of. Science also tries to discover the basic laws of nature, which are general facts about particulars having different sparse properties. But if particulars could have infinitely many sparse properties then science would be a project in principle impossible to complete. (2002, p. 173)

This argument is not persuasive. First, perhaps science *is* a project in principle impossible to complete. We have no guarantee that our world is amenable to complete scientific description. To suppose so is to confuse an ambition of the scientific project for its guaranteed outcome. Second, perhaps science could find some finite way of describing an infinity of different determinable properties and

[16] This confirmation could be inductive, or by exhaustion of instances (e.g. the world might have infinitely many inhabitants, or it might have finitely many, some of whom are capable of supertasks).

maintain that, say, some particulars have a determinate instance from each of them. For all that Rodriguez-Pereyra has said, scientists might justifiably postulate infinitely many sparse determinables. Third, the point at issue is not whether the science of our world is completable. Rodriguez-Pereyra's version of resemblance nominalism is no different from others: it aims to give a metaphysics of properties for all possible worlds.[17] Hence what must be argued is that there are no particulars instantiating infinitely many properties in *any* possible world, not just our actual world. The completeness of the science of our world is a red herring, since it is compatible with there being other possible worlds in which particulars instantiate infinitely many properties, even if no actual particular does.

From the point of view of his own theory, Rodriguez-Pereyra's argument also proves too much. Consider a world similar to that contemplated at the start of this section, in which infinitely many particulars a_1, a_2,…, each instantiate exactly one determinate from each of infinitely many distinct determinables. According to his argument, the science of such a world is incompletable; hence no such world can exist. But Rodriguez-Pereyra himself does not want to rule out worlds in which particulars instantiate finitely many properties; he only wants to rule out worlds in which particulars instantiate infinitely many.

Rodriguez-Pereyra's second argument against the infinitary hypothesis is:

> …suppose *x*, *y*, and *z* have each [the same number of] infinitely many sparse properties, *x* and *z* share all their sparse properties, and all sparse properties of *y* are properties of *x* but not vice versa. It is clear that *x* and *y* resemble each other to a lesser degree than *x* and *z* resemble each other but since *x* and y share the same number of properties, infinitely many, which *x* and *z* share, *x* and y resemble each other to the same degree that *x* and *z* do! The way to escape this paradoxical result is, I think, to reject the idea that particulars can have infinitely many sparse properties. (2002, p. 174).

This argument shows that a cardinality criterion of resemblance and the infinitary hypothesis (as elaborated in the example) together lead to a counter-intuitive conclusion. That particulars could instantiate infinitely many properties is, as just explained, plausible. The cardinality criterion of resemblance on the other hand is implausible. A particular that has infinitely many properties in common with another particular resembles it less than it does a third particular with which it shares these and infinitely more properties of the same infinity κ, despite the fact that $\kappa = \kappa + \kappa$ for infinite κ,[18] so that the two (cardinal) numbers of shared properties are identical. Rodriguez-Pereyra has therefore faulted the wrong principle. The objectionable conclusion follows from the premise that there could be particulars instantiating infinitely many properties and the cardinality criterion of resemblance; but the latter is less plausible than the former.

[17] As he puts it: "Resemblance Nominalism, as a theory about what makes particulars have the properties they have, is not based on any contingent feature of the world" (2002, p. 98). See also §6(b).

[18] '+' here denotes cardinal addition.

We conclude that the infinitary hypothesis' plausibility is not thrown into doubt by Rodriguez-Pereyra's arguments. Let us explore what follows if particulars can instantiate infinitely many properties.

5.2 …and how to account for it

Suppose that the κ particulars a_i ($1 \leq i < \kappa$) each instantiate all but one of the κ properties F_i ($1 \leq i < \kappa$) (and no others), where κ is infinite. For concreteness, say particular a_i instantiates all the κ-many F-properties with the exception of F_i. Then any λ of the a_i where $\lambda < \kappa$ collectively resemble one another, since they all share a property (in fact, they share κ-many properties)[19]; but the a_i don't share a property—it cannot be any F_j, since a_j omits it—and thus they don't collectively resemble one another.[20] If such scenarios are genuine possibilities, resemblance theories must be capable of expressing the collective resemblance predicate R^κ for any cardinal κ. (If there is a limit on how great property instantiations can be and still describe genuine possibilities, a resemblance theory is only required to express the corresponding predicate up to that limit.)

Rodriguez-Pereyra's theory expresses R^m for any finite m; but since it cannot express R^κ for infinite κ, the infinite cases of imperfect community remain insoluble. To illustrate this, consider the simplest version of the earlier example, in which the countably infinite particulars a_0, a_1,…, each possess all but one of countably many properties F_0, F_1,….—say a_n lacks F_n. The particulars a_0, a_1,…, share no property—it cannot be any F_n, since a_n lacks it—yet they all resemble each other. But any finite selection of these infinitely many particulars is a perfect community: any m of the particulars a_0, a_1,…, share all but m of the infinitely many F-properties. So the n^{th}-rank pair sets made up from these particulars all resemble one another. The account therefore lacks the means to distinguish perfect from imperfect infinite communities. Similarly, Rodriguez-Pereyra's theory is incapable of resolving the companionship problem's infinite cases, because its stock of predicates includes only R_n for n finite. Yet if the members of the F-class share infinitely many properties and G is an immediate companion to F, the F-class and G-class have the same infinite degree of resemblance.

In contrast, both the comparative theory and Lewis' theory can deal with infinite imperfect community and companionship cases because they can define the notion of a property class. For example, suppose that some property F_0 has companion F_1, which in turn has companion F_2, and so on ad infinitum. Then $\{x\colon F_0x\}R^+\{x\colon F_1x\}$, so the F_0-class can be distinguished from the F_1-class. Likewise, the members of F_0 stand in the relation R^L to the members of F_1. Infinite cases of this kind might be called *purely ordinal* because the F_0-particulars have the same infinite degree of

19 λ-many of the a_i omit λ-many of the κ properties. Since $\lambda < \kappa$ and κ is infinite, the smallest cardinal x such that $x + \lambda = \kappa$ is κ itself.

20 In this case, all but one—the improper one—of the κ subclasses of the class made up of the a_i also collectively resemble.

resemblance as the F_1-particulars, etc., and thus, unlike the finite cases, are not technically soluble merely by using the degree resemblance predicates R_κ for κ, finite or infinite. One way in which such cases would arise is if infinitely conjunctive properties can exist (recall that one source of finite companionship cases is the existence of conjunctive properties). A property such as $F_1 \wedge F_2 \wedge F_3 \wedge F_4 \wedge \ldots \wedge F_n \wedge \ldots$, if it exists, is accompanied by the infinitely many properties $F_2 \wedge F_3 \wedge F_4 \wedge F_5 \wedge \ldots \wedge F_n \wedge \ldots$, $F_3 \wedge F_4 \wedge F_5 \wedge F_6 \wedge \ldots \wedge F_n \wedge \ldots$, etc.

Turning to the mere intersections problem, the comparative theory technically solves infinite versions of this problem in the same manner as above. We note in passing that the R*diff method cannot solve infinite cases of mere intersections, that is, cases in which the particulars with properties F and G resemble each other to infinite degree (e.g. cases in which F and G are both accompanied by infinitely many properties). For if the resemblance degree of the intersection of the F-class and the G-class is infinite, then it will be the same infinite degree whether or not $F \wedge G$ exists, since $\kappa + 1 = \kappa$ for infinite cardinals κ. However we extend the notion of subtraction into the transfinite, the R*diff method will not distinguish between the cases in which in the intersection of the the F-class and the G-class is a property class and cases in which it isn't.[21]

We turn finally to how the comparative theory deals with infinite cases of the necessary coextension problem. The obvious way to supplement §3's recursion clause is:

$$\mathrm{P}(X,\lambda) \equiv_{\mathrm{def}} \forall\kappa < \lambda \exists Y(\mathrm{P}(Y,\kappa) \wedge XR^{+}Y) \\ \wedge \forall Z(\forall\kappa < \lambda\exists Y(\mathrm{P}(Y,\kappa) \wedge ZR^{+}Y) \rightarrow ZR^{+=}X)$$

However, this will not do. Consider the simplest infinite case, in which $\lambda = \aleph_0$, and suppose that the members of some class C share $\aleph_0$-many properties, say F_0, F_1, $F_2, \ldots$, so that P(C, $\aleph_0$). There might be a subclass of C, say D, whose members share F_1, $F_2, \ldots$. For example, suppose that F_0 is the first in an infinite companionship series F_0, F_1, $F_2, \ldots$, and consider the particulars instantiating F_1 (and thus F_2, $F_3, \ldots$,) but not F_0. In this scenario P(C, $\aleph_0$) and P(D, $\aleph_0$) but not $DR^{+=}C$; rather, $CR^{+}D$.

An improved definition must hold on to the failed attempt's first conjunct, that the members of a resemblance class of degree λ resemble more than the members of a resemblance class of lesser degree. But its minimality condition has to take into account the fact that there could be many other classes with that property that resemble themselves less than other resemblance classes of degree λ. Indeed there could be an infinite sequence of classes X_1, X_2, $X_3, \ldots$, such that $X_1R^{+}X_2R^{+}X_3\ldots$, all of the members of the sequence having resemblance degree λ. The crucial observation is that any such sequence has at most λ members, because X_1 has resemblance degree λ. Intuitively, if a list of the properties shared by the members of a class has length λ and one deletes the properties from the list (one by one or

[21] If some property classes have infinite degree of resemblance and no conjunctive properties exist then the alternative definition to the R*diff method—that a maximal resemblance class is a property class iff it is not the intersection of any proper maximal resemblance superclasses—will work.

more than one at a time), then it cannot take any more than λ deletions to remove all the properties from the list. This points to the following, this time correct, definition:

$$\mathrm{P}(X,\lambda) \equiv_{\mathrm{def}} \forall\kappa<\lambda\exists Y(\mathrm{P}(Y,\kappa) \wedge XR^{+}Y)$$
$$\wedge\ \forall s(\text{if } s \text{ is a sequence of classes } X_1, X_2, X_3, \ldots, \text{with}$$
$$\text{the property that for each } X_i \text{ in the sequence} \forall\kappa<\lambda\exists Y(\mathrm{P}(Y,\kappa) \wedge X_iR^{+}Y),$$
$$\text{and } XR^{+}X_1R^{+}X_2R^{+}X_3, \ldots, \text{then } s \text{ is of length no greater than } \lambda)$$

Thus the comparative theory can solve infinite cases of necessary coextension, should there be any.

6 Against resemblance nominalism

The technical parts of the paper introduced a new resemblance theory of properties and argued that it is technically superior to previous ones. The present section argues for three main claims. First, the three non-traditional resemblance theories are nominalistically unacceptable (§6(a)). Second, an extensionally adequate resemblance nominalism must invoke possibilia. Prima facie, this commits it to a controversial Lewisian hyper-realism about modality (§6(b)). Third, any extensionally adequate resemblance nominalism is committed to primitive resemblance relations between collections (§6(c)). Consequently, the prospects for a resemblance theory that is both technically and philosophically adequate are dim.

We have so far presented the comparative theory mainly in terms of R^{+}. However, as mentioned, a more natural choice of primitive is $\mathscr{R}^{+}(x_1, x_2, x_3, x_4)$, which in property realist terms is interpreted as 'x_1 and x_2 share more properties than x_3 and x_4'. More generally, if the x_i are classes, the property realist interprets $\mathscr{R}^{+}(x_1, x_2, x_3, x_4)$ as stating that x_1 and x_2's urelements share more properties than x_3 and x_4's urelements. As before, any non-class has by stipulation itself as its only urelement. The two predicates are interdefinable:

$$R^{+}(x_1,x_2) \equiv_{\mathrm{def}} \mathscr{R}^{+}(x_1,x_1,x_2,x_2)$$
$$\mathscr{R}^{+}(x_1,x_2,x_3,x_4) \equiv_{\mathrm{def}} R^{+}(\{x_1,x_2\},\{x_3,x_4\})$$

Perhaps an even more natural choice of comparative resemblance primitive would be the variably polyadic predicate 'x_1, x_2,…, x_n $R^{V+}y_1$, y_2,…, y_m', interpreted as 'x_1, x_2,…, x_n resemble each other more than y_1, y_2,…, y_m do' for n and m finite, though a cloud hangs over this strategy given the concerns about the legitimacy of variably polyadic predicates.[22] This demonstrates that there is a wide range of theories based on comparative resemblance predicates that technically solve the problems as in the previous sections, *modulo* the initial definitions. There is an intramural debate to be had about which of these approaches is the best. However, if our criticisms of the $\mathscr{R}^{+}$-theory are sound, they apply to all comparative theories.

[22] If variably polyadic predicates are not acceptable, that would be another reason to reject Lewis's theory based on the primitive R^{L}.

6.1 Nominalistic acceptability

The dominant motivation behind resemblance nominalism is the desire for a one-category ontology of particulars in contrast to the universalist's two-category one. However, resemblance nominalisms of the past have been plagued by the need to posit resemblances in virtue of some respect or other. This will not do because respects are nothing but reified properties in disguise. As Rodriguez-Pereyra remarks: "[W]hat are these respects if not the properties the Resemblance Nominalist proposes to account for in terms of resemblance?" (2002, p. 158). The first chapter of Price's *Thinking and Experience*, today considered the objection's *locus classicus*, itself calls this one of the 'classical objections' to resemblance nominalism (1953, p. 20). An extreme version of this problem is incurred by any theory that posits, for each property F, a primitive resemblance relation $R_F xy$, read 'x resembles y in the F-respect'. This move reintroduces primitive properties not so much by the back door as the front one. Any resemblance theory that purports to be a form of nominalism must show how it meets this important objection.

Alas, Rodriguez-Pereyra's theory falls prey to the objection, because each of the primitive predicates $R_n(x, y)$ is nothing other than the relation 'x and y have n properties in common'. Consider for example the claim that $R_n(x, x)$. There is no way to understand it as saying anything other than that x has exactly n properties. His theory reifies, indeed enumerates, respects of resemblance in a problematic way from the outset.

Lewis' primitive R^+ is also guilty of implicitly reifying properties. For $x_1, x_2, \ldots R^L y_1, y_2, \ldots$, iff $x_1, x_2, \ldots$, resemble one another but do not resemble each of $y_1, y_2, \ldots$, *in the same respect*. Consider for example the scenario in which a and b have properties F and G, c has F only and d has G only. Both abR^Lc and abR^Ld obtain because a and b share G which is lacked by c, and they also share F which is lacked by d. But it is not the case that abR^Lcd, since there is no property shared by a and b lacked by both c and d. Thus the predicate $x_1, x_2, \ldots R^L y_1, y_2$ with just two right-hand variables cannot be understood as

$$x_1, x_2, \ldots R^L y_1 \wedge x_1, x_2, \ldots R^L y_2.$$

It must instead be understood as

$$x_1, x_2, \ldots \left(R^L\right)_F y_1 \wedge x_1, x_2, \ldots \left(R^L\right)_F y_2$$

for the *same* F, where $(R^L)_F$ specifies the resemblance property F possessed by the left-hand variables and lacked by the right-hand one. R^L therefore also reifies properties.

As for the comparative theory, its primitive $\mathscr{R}^+$ is understood so that, from the property realist perspective, what matters is the number of properties shared rather than the ratio of properties shared to properties possessed (similarly for R^+). It therefore seems to enumerate properties at the outset. We now strengthen the prima facie case for this conclusion by presenting and responding to perhaps the strongest defence of the comparative theory's claim to nominalism.

The defence begins by noting that predicates have a positive, comparative and superlative form derived from their corresponding adjectives' positive, comparative and superlative forms. For example, the positive form of the predicate derived from the adjective 'big' is 'x is big', its comparative form is 'x is bigger than y' and its superlative form is 'x is the biggest'. When the original predicate is two-place then the comparative becomes four-place; for example, the comparative of 'x likes y' is 'x likes y more than z likes w'. The predicate $\mathscr{R}^+$ is similarly the comparative of R^2, since 'x_1 resembles x_2 more than x_3 resembles x_4' is the comparative of 'x_1 resembles x_2'. Now there might be various reasons to take a positive rather than comparative predicate as one's primitive when developing some theory or other. But the fact that the comparative but not the positive form presupposes *X*s, where the *X*s have nothing to do with the conversion of the predicate from positive to comparative, as in the case before us where *X*s are properties, cannot be such a reason. It would be akin to claiming that 'x is bigger than y' is committed to a particular account of size whereas 'x is big' is not. If this argument is sound, $\mathscr{R}^+$ is on a par with R^2 and is therefore nominalistically acceptable, assuming the latter is.

This defence of $\mathscr{R}^+$'s nominalistic acceptability is in fact unsound. Let's agree for the sake of argument that a predicate's comparative form is nominalistically acceptable iff its positive form is. But the predicate $\mathscr{R}^+(x_1, x_2, x_3\ x_4)$, whose property realist understanding is that x_1 and x_2 share more properties than x_3 and x_4 do, is *not* the comparative of 'x_1 and x_2 resemble'. In other words, $\mathscr{R}^+.(x_1, x_2, x_3\ x_4)$ is not correctly interpreted as 'x_1 resembles x_2 more than x_3 resembles x_4'. It has a different interpretation.

There are at least two ways of seeing this. First, everything resembles itself to the maximum degree. (Whether we are thinking in terms of abundant or sparse properties.) This is arguably a conceptual truth about resemblance. It is certainly entrenched in our thinking about resemblance and formal treatments of it.[23] But $\mathscr{R}^+$ does not satisfy this principle. For example, if *a* has a single property *F*, *b* has three properties *G*, *H* and *I*, and *c* has three properties *G*, *H* and *J*, then $\mathscr{R}^+bcaa$. In general, many pairs of qualitatively different particulars resemble each other more than some particulars resemble themselves. Thus $\mathscr{R}^+(x_1, x_2, x_3\ x_4)$ cannot be understood as a resemblance relation. It can only be understood as stating that the number of properties shared by x_1 and x_2 is greater than those shared by x_3 and x_4.

This objection cannot be dismissed as an 'ordinary language argument'. In its polemical sense the expression applies to arguments that crucially depend on a language's more or less idiosyncratic features. However, the objection is not of that sort, since it is robust with respect to the substitution of 'similarity' or 'likeness' or similar terms for 'resemblance'. And it is also robust with respect to related terms in other languages, for example the French 'ressemblance' or the German 'Ähnlichkeit'. The point is about the nature of resemblance, not about an idiosyncratic feature of the English word 'resemblance'.

The second point against the defence is that $\mathscr{R}^+$ neglects similarities between properties (notably determinates of a single determinable) though resemblance

[23] E.g. the four-term comparative resemblance relation Williamson (1988) sees as underpinning the notion of degrees of similarity has this property.

does not. Suppose for example that a and b share their F-determinate, but that their G, H, and J determinates are towards the opposite ends of the scale; and that c and d have determinates of these four determinables that are extremely close on the scale (if they are quantities such as *having mass m*, perhaps they only differ at the millionth decimal place say), but do not exactly share any. Clearly, c resembles d more than a resembles b; yet $\mathscr{R}^+abcd$. This demonstrates once more that $\mathscr{R}^+(x_1, x_2, x_3\ x_4)$ cannot be understood as 'x_1 resembles x_2 more than x_3 resembles x_4', and that the $\mathscr{R}^+$-theory is not a resemblance theory. It is covertly a property theory, just like Rodriguez-Pereyra's and Lewis'.[24] Notice that these remarks do not merely parry the objection. They show that the comparative theory is not a form of nominalism.

6.2 Modality

Consider a world w_1 containing just two particulars a and b, in which a instantiates F and b instantiates G (and in which there are no other sparse property facts), and a world w_2 in which c instantiates F and G and d instantiates H and I (and in which there are no other sparse property facts). How is the resemblance nominalist to distinguish w_1 from w_2? Let c_1 and c_2 be constants respectively denoting a and b in w_1 and c and d in w_2. The R^2-facts in w_1 and w_2 are expressible as: $R^2c_1c_1$, $\neg R^2c_1c_2$, $\neg R^2c_2c_1$, $R^2c_2c_2$. The R^n-facts are more generally the same in both w_1 and w_2 for $n \geq 2$. The comparative resemblance facts are also the same in both worlds (in terms of R^+: $\neg R^+c_1c_1$, $\neg R^+c_1c_2$, $\neg R^+c_2c_1$, $\neg R^+c_2c_2$). A resemblance nominalist is faced with four options: (i) reject these scenarios' possibility; (ii) take some resemblance predicates R_n as primitive, or at any rate take as primitive some predicate that specifies how many properties a particular instantiates; (iii) claim that a resemblance theory of properties need only be contingently true, and hope that our world is such that the resemblance analysis of property class is contingently true; (iv) invoke possibilia to distinguish w_1 from w_2.

The first option is unpromising. It does grave violence to our modal beliefs to take scenarios of the kind described in the given example and throughout this paper as impossible. At any rate, the resemblance nominalist must have powerful independent arguments up her sleeve to shrink our usual conception(s) of the extent of modal space so radically. The second option is ruled out because it reifies properties at the outset, as explained in §6(a). As for the third option, the orthodox conception of metaphysical theories about the nature of properties is that, if true, they are necessarily true. Every metaphysician of properties—not just resemblance nominalists—must give a non-contingent account of what it is for a particular to instantiate a property. The account of what it is for some particular a to be F (or for a and b to be F, etc.) should not depend on any specific features of this world; it should generalise. Thus even if our world happens to be such that the various

[24] Of course for all that has been said, $\mathscr{R}^+$ (or R^+) could be definable from a predicate that is nominalistically acceptable; but there is no reason to think so.

technical problems do not arise for it, resemblance nominalists still owe us a necessary account that does not exploit this contingency. For example, a resemblance nominalist cannot identify property classes with maximal resemblance classes even if in our world property classes happen to correspond one-to-one to maximal resemblance classes. She must offer a more general necessary account, equivalent to the maximal resemblance class account for the special case of the actual world when the latter's contingent features are plugged in. It is hard to see how this orthodox conception of the task of a metaphysics of properties could be challenged if the metaphysics of properties is to remain the broadly philosophical project it has always been. Furthermore, since we are far from knowing all the (sparse or abundant) property facts about our world, we currently have an extra reason for providing as general an account as possible, to cover all the epistemic possibilities.

This leaves the fourth option: resemblance nominalism must quantify over possibilia if it is to be extensionally adequate. For example, there are possibilia denoted by c_3 and c_4 satisfying $R^2c_1c_3$, $R^2c_1c_4$ and $\neg R^+c_3c_4$ where the constants denote the entities in w_2, but this is not the case if the constants denote the particulars in w_1 since c but not a instantiates more than one property.

How is the appeal to possibilia to be cashed out? Lewis' (1986) modal hyper-realism, with its straightforward apparatus of possibilia and transworld resemblance classes, is one option. If you cannot stomach modal hyper-realism, however, you are left with the challenge of accounting for talk of possibilia. This is not the place to explore whether the challenge can be met, but there are initial grounds for thinking it will be difficult for resemblance nominalists to do so. For example, the ersatzist about modality cannot appeal to properties in his account, nor to states of affairs or propositions if these are understood as compounded out of properties. A fictionalist about modality will end up being a fictionalist about properties, an uncomfortable slide. No surprise, then, that one of the reasons Lewis offers on behalf of his modal realism is that it allows a literal rendering of transworld comparative similarity statements (1986, p. 13). It remains unclear whether there is a satisfactory alternative to Lewis' in the restricted context of a nominalist theory of properties.

In sum, on the orthodox conception of a metaphysics of properties, resemblance nominalists have to give a necessary account of what it is for a particular to instantiate a property. To take degree resemblance predicates as primitive is to give up on resemblance nominalism at the outset. Consequently, the only apparent way to distinguish property-wise distinct but resemblance-wise identical possibilities, assuming that they are indeed possibilities, is to quantify over possibilia. The challenge for resemblance nominalists is to show that Lewisian realism is not indispensable for a resemblance theory that quantifies over possibilia, since few metaphysicians are willing to swallow this view—Lewis and Rodriguez-Pereyra being notable exceptions.

6.3 Set theory

Resemblance nominalists' acceptance of set theory is not in itself problematic.[25] What *is* problematic is commitment to primitive resemblance relations between 'collections'—be they pluralities or sets or mereological sums or some other form of collective grouping. Consider the infinitely many collective resemblance predicates R^0, R^1, R^2,…., needed to solve the imperfect community problem. A theory with infinitely many primitives is parasitic on a more basic finitary theory, on pain of not being graspable by finite subjects. This problem afflicted Lewis' theory: its primitive predicate R^L must be grounded in other predicates (e.g. R^{L^*}), as we saw. The comparative theory expresses the fact that the particulars $a_1,\ldots, a_n$ resemble one another in terms of resemblances between sets with $a_1,\ldots, a_n$ as members, as does Rodriguez-Pereyra's theory. As the collective resemblance of any finite sample of particulars fails to guarantee the collective resemblance of a larger group from which the sample is drawn, it seems that ability to express the fact that collections of particulars resemble is a necessary component of any extensionally adequate resemblance nominalism.

This is no more than a plausibility argument for the claim that resemblance nominalism must appeal to resemblance relations between classes of particulars, or to resemblance relations between mereological sums of particulars, or to some cooked up mereological-cum-plural ersatz for set theory.[26] If sound, it generates the following problem. The collections in question do not seem to have the required properties if resemblance facts about them are to act as surrogates for collective resemblance facts about their members. For example, the comparative theory relies on the assumption that the set of particles with mass m_1 and charge c_1 shares more properties with the set of particles with mass m_1 and charge c_2 than set of particles with mass m_2 and charge c_3 shares with the set of particles with mass m_3 and charge c_4. But arguably these sets are abstract and therefore don't have sparse properties and hence don't share any sparse properties with one another. And even if they are not abstract, it is difficult to accept that the sets themselves—as opposed to their

[25] Rejection of sets and rejection of universals (or more generally primitive properties) spring from different sources. For instance, a broadly empiricist acceptance of spatiotemporal (or immanent or *in re*) universals goes naturally with a rejection of mathematical objects. Conversely, accepting abstract mathematical objects but not universals is a coherent philosophical stance, as exemplified by Quine and others. The difference between the two kinds of metaphysical stance is apt to be blurred by the modern tradition of giving the rejection of sets (and abstract objects more generally) the traditional name for the rejection of universals, viz. 'nominalism'. But these two types of nominalism, as we are now accustomed to calling them, are different. Moreover, it should be clear that the philosophical work done by sparse properties is different from that of classes, a point made forcefully in Lewis (1983). Not only is resemblance nominalism logically compatible with acceptance of standard set theory, then, depending on the source of one's nominalism it may also be naturally allied with it.

[26] It seems that the theory must also rely on set theory in another, less problematic way. When criticising Rodriguez-Pereyra's resemblance nominalism we saw that resemblance nominalism cannot take the infinitely many degree predicates R_0, R_1, R_2,…, as primitive. Since it must be capable of expressing them in order to technically solve the companionship problem, it must define them. But any such definition apparently requires that a transfinite recursion be effected on an argument place of some predicate or other. If so, the ability to effect such a recursion is a necessary ingredient of any extensionally adequate resemblance nominalism.

members—have sparse properties. Alternatively, suppose that the resemblance theory is intended as a theory of abundant properties. Sets have different abundant properties from their members, so the number of properties shared by sets is not a good guide to how many properties their members share. For example, $\{a, b\}$ and $\{c, d\}$ resemble simply in virtue of being sets, whether or not a and b and c and d share any properties.

In sum, resemblance theories must invoke primitive collective resemblance relations between collections of entities. Either these relations only relate (non-set) particulars, in which case the theory cannot be the basic resemblance theory, which must be finitary. Or they can relate set-like collections, in which case resemblance relations among sets do not reflect resemblance relations among their members.

7 Conclusion

The $\mathscr{R}^+$-based comparative resemblance theory is technically superior to traditional R^2-based resemblance nominalism, Lewis' R^L-based resemblance nominalism, and Rodriguez-Pereyra's resemblance nominalism based on the degree resemblance predicates R_0, R_1, R_2,…. But its primitive $\mathscr{R}^+$ is nominalistically unacceptable, and it is most likely committed to Lewisian modal hyper-realism and to primitive resemblance relations among sets. We suggested that these objections generalise: they seem to apply to any extensionally adequate resemblance theory. If that is right, resemblance theories of properties can be either extensionally adequate or nominalist; but they cannot be both.

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