Philos Stud (2010) 151:1–18
DOI 10.1007/s11098-009-9420-8

Indiscernibility and bundles in a structure

Sun Demirli

Published online: 4 June 2009

Abstract The bundle theory is a theory about the internal constitution of individuals. It asserts that individuals are entirely composed of universals. Typically, bundle theorists augment their theory with a *constitutional approach to individuation* entailing the thesis 'identity of constituents is a sufficient ground for numerical identity' (CIT). But then the bundle theory runs afoul of Black's duplication case—a world containing two indiscernible spheres. Here I propose and defend a new version of the bundle theory that denies 'CIT', and which instead conjoins it with a *structural diversity thesis*, according to which being separated by distance is a sufficient ground for numerical diversity. This version accommodates Black's world as well as the *three-spheres world*—a world containing three indiscernible spheres, arranged as the vertices of an equilateral triangle. In this paper, I also criticize Rodriguez-Pereyra's alternative attempt to defend the bundle theory against Black's case and the case of the *three-spheres world*.

Keywords Bundle theory · Identity of indiscernibles · Individuation · Universals · Compresence · Structuralism · Diversity

1. There are two separate issues that philosophers often run together when they discuss the nature of individual substances.[1] First, there is the question of internal composition. Along with this, there is also the question of intra-world identity and

[1] Hector-Neri Castaneda makes a similar distinction (1975, pp. 131–133). He distinguishes what he calls the "ontological issue" about the internal constitution of an individual substance from what he calls the "epistemological issue" about how individuals are individuated. In Castaneda's view, the individuation problem is clearly epistemological in character and is not discussed at length in that work. In this paper, I take both of these issues to be ontological in character.

S. Demirli (✉)
Department of Philosophy, Bogazici University, Bebek, Istanbul, Turkey
e-mail: sdemirli@gmail.com

diversity; this second question concerns how we individuate various individuals and how we count the population of a given world.

In answering the internal constitution question, we may begin an inquiry about the various categories that go into the composition of individual substances and hope that at the end of this inquiry we will come up with a list of ingredients that constitute various individuals. Just as a certain recipe in a cook book provides us with a list of ingredients and instructions for mixing these ingredients together, we may maintain that the list or the recipe of individual substances—God's recipe book, so to say—will tell us what items from various categories are used, and how these items are combined.

The bundle theory is, first and foremost, a theory about the internal constitution of individual substances. It asserts that the constituents of individual substances are exhausted by the (pure) properties we associate with them. And, typically, its defenders construe properties as immanent universals; so, they hold that where distinct individuals have the same property, there is a single item—an immanent universal—that enters into the composition of each. And thus, as this theory is traditionally formulated, *each individual is a bundle of universals*.

In explaining how a number of universals are wrapped together to form a bundle, bundle theorists agree that there is a contingent sui generis relation—compresence—tying various individuals into bundles, and that once universals are brought together, the end product—i.e., bundle theoretic composition—turns out to be an entity that is categorically different from the universals going into its composition.

2. A typical introduction of the question concerning the identity and diversity of individual substances is this. Consider a case of intra-world duplication as it is illustrated in Max Black's example (Black 1952). Suppose a world containing two qualitatively identical spheres in a relational space. Both of these spheres have the same color, size, mass and so on, and they are both 5 m apart from each other. Philosophers, introducing this problem as such, ask what explains the numerical diversity between these two spheres. What accounts for our saying that there are two spheres rather than one in Black's world?

This problem, illustrated in terms of distinct indiscernible spheres, is a special case of the more general problem about the intra-world identity or diversity of any two individuals. Whether we are concerned with Black's world or with more familiar worlds, we need to give a principle for individuating various individuals, and maintain that such a principle will provide us with the right method of counting the population of a world.

One such principle is offered by the 'constitutional approach to individuation' (Loux 2006, p. 215). According to this approach, what accounts for the identity and diversity must be (in the Aristotelian lingo) immanent in the substance. For the defenders of this approach, the list of constituents, which serves as an answer to the internal constitution question, is also used in individuating substances. That is: the constituents of individual substances are taken to be the only grounds according to which we account for their identity and diversity. The constitutional approach to individuation entails what is called here the 'constitutional identity thesis':

(CIT) Identity of constituents is a sufficient ground for numerical identity.

The bundle theory, proposed as a theory concerning the constitution of individuals, is independent of the constitutional approach to individuation, and does not require the truth of ‘CIT’. Nevertheless, its traditional defenders conjoin the bundle theory with the constitutional approach, and require that identity of constituents be a ground for numerical identity. However, then, as is commonly noted, the bundle theory cannot accommodate the possibility of distinct indiscernible individuals. Consider again Black’s world. Since both of Black’s spheres have the same color, size and so on and they are 5 m distant from each other, by the bundle theory’s lights, they have exactly the same constituents. Yet if the bundle theory is augmented by the constitutional approach that entails the ‘constitutional identity thesis’, it cannot allow these two spheres to be distinct; so the possibility of Black’s world seems to undermine the bundle theory construed as such.

3. In defending the bundle theory against Black-type counterexamples, John Hawthorne agrees with the traditional understanding of this theory and does not deny the ‘constitutional identity thesis’. Thus he maintains that distinct indiscernible individuals cannot be allowed (1995, pp. 191–196). Nevertheless, he claims, Black’s case provides no reason for rejecting the bundle theory. According to Hawthorne, Black’s case concerns a world that we ordinarily describe as containing two spheres, each 5 m from the other. But, he argues, since individuals are bundles of universals, individuals also, just like universals that compose them, must be the sorts of entities that are capable of multiple locations. Therefore, rather than describing Black’s case as containing two indiscernible spheres, Hawthorne suggests, bundle theorists have to reinterpret this world as containing *one single sphere located in two different places*—as one sphere-bundle 5 m away from itself. In the light of this gloss, Hawthorne concludes, Black’s case falls short of undermining the bundle theory.

Jan Cover and Hawthorne discuss what they call the ‘triplication case’ as a likely objection against Hawthorne’s view (1998, p. 124). Black’s duplication case was about a world that we ordinarily describe as containing two spheres, each 5 m from each other. According to these authors, by the bundle theorist’s light, Black’s world must rather be described as a world where one single sphere-bundle is located in two different places, each 5 m apart from the other. Their triplication case is a variant on Black’s duplication case and concerns a world that we ordinarily describe as containing three indiscernible spheres, each 5 m distant from each other—thus making up an equilateral triangle. So, by the bundle theory’s light, Hawthorne and Cover concludes, this world must be reinterpreted as a *tri-located sphere world* where one single sphere-bundle is located in three different places, each 5 m distant from each other.

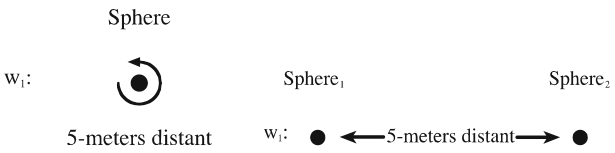
But distance relations are often taken to be dyadic. So there can be no fact that obtains in the *tri-located sphere world* but not in Black’s world; in both worlds the same fact obtains—a single sphere-bundle is 5 m distant from itself. Thus, to the extent that distance relations are understood as dyadic, these two worlds cannot be distinguished. Yet they need to be distinguished, since the *tri-located sphere world* describes a genuine possibility, which is as obviously possible as Black’s case.

Hawthorne and Cover reply to this possible objection by claiming that distance relations must be construed as irreducibly polyadic. In their view, the difference between these two worlds can be captured by invoking triadic distance relations holding in the world where the sphere-bundle is tri-located, but not in the world where the sphere-bundle is bi-located. Hawthorne and Cover claim that, "unless there is some obligation to restrict oneself to dyadic facts or to insist that all triadic facts supervene on dyadic facts, … the [tri-located sphere world] is no objection to the bundle theory …" (ibid.).

In a recent paper, Gonzalo Rodriguez-Pereyra argues against this solution appealing to polyadic distance relations (and thus against Hawthorne's defense of the bundle theory) by denying the *irreducibility* of such relations (2004, p. 75). On his view, to say that x, y and z are mutually distant under a triadic distance relation is merely to say that x, y and z are pair-wise distant under a dyadic distance relation: there cannot be irreducibly polyadic distance relations. He writes: "The triadic relation in the world where the bundle is tri-located is a relation that obtains between x, y and z if and only if x and y are [5 m] apart, y and z are [5 m] apart and x and z are [5 m] apart" (ibid.). But if so, he argues, this triadic distance relation also obtains in Black's world, where the sphere-bundle is bi-located, because there it is also the case that x and y are 5 m distant, y and z are 5 m distant and x and z are 5 m distant. So, he concludes, triadic relations can be of no help in distinguishing *the tri-located sphere world* from Black's world (ibid.).

Rodriguez-Pereyra's case against Hawthorne's defense turns on the intelligibility of irreducibly polyadic distance relations. I believe that Rodriguez-Pereyra is too hasty in denying the irreducibility of relations as such. As I will argue in due course, irreducibly polyadic distance relations can be used to amend Rodriguez-Pereyra's version of the bundle theory.

However, Hawthorne's defense is still ineffective. Independently of the question of whether triadic distance relations are reducible to dyadic distance relations, Hawthorne's view is subject to two major criticisms. First of all, by suggesting that there may be individuals at spatial distance from themselves, Hawthorne's bundle theory goes against our ordinary method of counting, which takes being separated by distance to be a sufficient condition for numerical diversity. More significantly, as Hawthorne himself contends, although his bundle theory may reinterpret Black's world as containing *one single sphere* (as depicted in w_1), it rules out worlds with *two distinct indiscernible spheres* (as depicted in w_2) as impossible (Hawthorne and Sider 2002, pp. 53–76; see also Rodriguez-Pereyra 2004, p. 74). Therefore, it fails to address the very problem of individuation that has been presented as a challenge to the bundle theory.



4. Rodriguez-Pereyra develops a version of the bundle theory that allows numerically distinct indiscernible individuals (2004, pp. 72–81). He maintains that there may be distinct individuals, composed of exactly the same constituents, and denies the traditional conception of the bundle theory that requires the

'constitutional identity thesis' (2004, p. 76). He begins his account by drawing a distinction between a *bundle of universals* and its *instance* (2004, p. 78). On his view, the instance is entirely composed of the universals that compose the bundle, but instance and bundle are distinct entities: whereas bundles are the sort of entities that are capable of multiple-location, their instances cannot be in more than one location at once. Having introduced this distinction, he maintains that an individual substance must be identified not with a bundle but with an *instance* of this bundle (ibid.).

Rodriguez-Pereyra's proposal looks promising. On his view, an individual is entirely composed of universals; but by drawing a distinction between bundles and their instances, he maintains that there can be more than one instance of a certain bundle; so, on his view, there may be more than one individual with exactly the same constituents—each of which is a different instance of this bundle.

Rodriguez-Pereyra argues that the bundle theory, developed as such, can easily get around the problem of intra-world duplication cases (2004, pp. 78–79). He maintains that in Black's world there is one single bundle—a sphere-bundle—located in two different places. However, in his view this sphere-bundle has two instances, each of which is singly located. Since individuals are identified with sphere-instances, he argues, his version of the bundle theory can explain the existence of the plurality in Black's world.

Yet the critical question that needs to be answered is what accounts for the diversity of sphere-instances. Given that, let us say, Castor and Pollux are two *distinct* sphere-instances of the sphere-bundle, it follows that Castor is numerically distinct from Pollux. However, it does not follow from the assertion that Castor and Pollux are both *instances* of the sphere-bundle, that Castor and Pollux are distinct. What could be the grounds for saying that there are two distinct sphere-instances in Black's world?

Surely, Rodriguez-Pereyra may refuse to account for the diversity of sphere-instances. He may say: "Instances come to us as fully individuated, and we do not need to account for their diversity." The only explanation that these two sphere-instances are distinct is the fact that they have distinct primitive identities.

Appealing to primitive individuation may in itself be undesirable. But that is not the only objection. In a recent paper, Michael Della Rocca shows that any theory invoking primitive individuation must allow an empty possibility: the *piling* case (2005). My point will be similar. Embracing the view that sphere-instances are individuated primitively will lead us to the empty possibility that there may be more than one single sphere-instance piled up in exactly the same place at the same time. To see this, consider Black's world again. Suppose that one claims there are two distinct sphere-instances in this world, each of which is distinguished from the other in terms of the brute facts of diversity. Yet once one recognizes the brute facts of diversity in Black's case, she must also allow, let us say, the 'twenty-sphere world' in which there are twenty sphere-instances, every ten of which are piled up in exactly the same place at the same time. That is, if she contends for the existence of primitive diversity in Black's world, there will be no way of avoiding the case of the 'twenty-sphere world' where each sphere-instance is distinguished from the others by virtue of the fact that it has a distinct identity from each of the others.

Obviously, the 'twenty-sphere world' where ten distinct sphere-instances are located in exactly the same place at the same time is an empty possibility. It will be absurd to say that one must be open to the possibility that what we ordinarily describe as a single individual substance is in fact ten distinct substances.

5. In fact, Rodriguez-Pereyra has a way of accounting for the diversity of sphere-instances without appealing to their primitive identities. He writes: "The sphere-instances [in Black's world] are not identical to each other, since they are in different places" (2004, p. 78). So he claims that sphere-instances can be distinguished in virtue of the fact that they are in different spatial locations.

But what are spatial locations? Now any theory that presupposes spatial locations must choose between two competing views of space: absolutism and relationism. Rodriguez-Pereyra maintains that both of these views are compatible with his version of the bundle theory (2004, pp. 79–80).

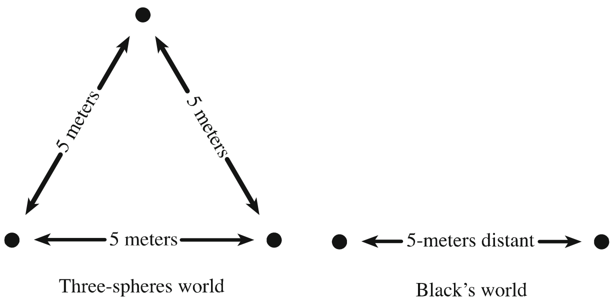
Consider first the absolutism. In the case of absolutism, locations will be understood as sets or sums of spatial points. According to Rodriguez-Pereyra, spatial points are "*instances* of the universal pointhood" (2004, p. 80). So, on his view, the sphere-instances in Black's world will be distinguished in virtue of the fact that they occupy different *instances* of pointhood. But this will not do. In this case, there will be an explanation for the diversity of sphere-instances, but there will be no explanation for the diversity of *instances* of pointhood. How can Rodriguez-Pereyra account for the diversity of point-instances without presupposing that they are primitively individuated? Therefore, one must conclude, although *instances* of pointhood can be used to secure the distinctness of sphere-instances, they will themselves be subject to the very problem that their introduction is meant to resolve.

Then, it seems, Rodriguez-Pereyra must embrace relationism. Relationism states that spatial locations cannot be taken as real in themselves but must be regarded as constructions out of spatial relations that hold among the various items in one's ontology. Is relationism compatible with his version of the bundle theory?

Rodriguez-Pereyra thinks that it is. He says: "The location of a thing stems from its relation to other things. So the location of an instance is given by its spatial relations to other instances" (ibid.). This proposal seems to work just fine in Black-type duplication cases. In Black's world, sphere-instances are 5 m from each other, and their being 5 m distant makes them in different places, and this latter will in turn guarantee that they are numerically distinct from one another.

However, when confronted with what I here call 'three-spheres world', Rodriguez-Pereyra's account does not stand up and he is forced to appeal to the view that instances are primitively individuated. The *three-spheres world* is a variant on Hawthorne and Cover's *tri-located sphere world*. As will be recalled, the *tri-located sphere world* describes a world where *one single sphere* is located in three different places, every two of which are 5 m distant from each other (thus this single sphere forms an equilateral triangle); the *three-spheres world*, on the other hand, concerns a world containing *three distinct indiscernible spheres*, each of which is 5 m distant from the others (thus these three indiscernible spheres form a similar equilateral triangle).

How can Rodriguez-Pereyra account for the distinction between this *three-spheres world* and Black's world? As seen earlier, when criticizing Hawthorne's defense of the bundle theory and arguing that in Hawthorne's account the *tri-located sphere world* cannot be distinguished from Black's world, Rodriguez-Pereyra claims that there can be no irreducibly triadic distance relations. However, to the extent that triadic distance relations are understood as reducible to dyadic distance relations, he himself must fail to distinguish the *three-spheres world* from Black's world.



Let us say that, in the *three-spheres world*, sphere-instances are x, y and z. Suppose we say: x is 5 m from y, y is 5 m from z, and x is 5 m from z. Each of these dyadic facts entails that there are two distinct spheres-instances. But this is also true of Black's world, and unless we combine these facts with the supposition that x, y and z are numerically diverse, we cannot conclude that there are three distinct instances.

Rodriguez-Pereyra in fact considers the *three-spheres world* (2004, pp. 79–80). In distinguishing this world from Black's world, he makes use of the idea that sphere-instances are primitively individuated. He claims that the difference between these worlds can be captured by the fact that in the former world there are three sphere-instances while in the latter there are only two instances. So, in order to secure the distinctness between these two worlds, he says: "Something exists in one world that does not exist in the other" (2004, p. 80). But he can say this only by presupposing the bare numerical diversity of sphere-instances.

It may be that Rodriguez-Pereyra's account can be salvaged if, contrary to what he argues, there are irreducibly polyadic distance-relations. In that case, the *three-spheres world* can be distinguished from Black's world by invoking such distance-relations holding in the former world, but not in the latter world. In the rest of this paper, I shall propose an alternative account of the bundle theory and will defend it with the help of triadic distance-relations as such. As will be argued, this alternative account has an additional comparative advantage over Rodriguez-Pereyra's account.

6. Now I agree with Rodriguez-Pereyra that the traditional conception of the bundle theory requiring the 'constitutional identity thesis' is refuted by Black-type counterexamples. Like Rodriguez-Pereyra, I believe that Hawthorne's defense of this traditional view is not effective. However, my reason for rejecting Hawthorne's defense is different from Rodriguez-Pereyra's. As will be recalled, Rodriguez-Pereyra's case against Hawthorne turns on the intelligibility of irreducibly polyadic distance relations. On his view, since triadic distance relations can be reduced to dyadic distance relations, Hawthorne must fail to distinguish the *tri-located sphere*

world from Black's world. But, contrary to Rodriguez-Pereyra, I believe that triadic distance relations cannot be reduced to dyadic ones. Therefore, I maintain, Rodriguez-Pereyra's reason for rejecting Hawthorne's defense is not valid. But that doesn't mean that I do not find fault with Hawthorne's way of defending the traditional conception of the bundle theory. As I mentioned in Sect. 3, my criticism of Hawthorne's view is twofold: (i) by reinterpreting Black's world as containing one single sphere, Hawthorne rules out worlds with *two distinct indiscernible spheres* as impossible; and (ii) by suggesting that there may be individuals which are at spatial distance from themselves, Hawthornes's view goes against our ordinary method of counting, which takes being separated by distance to be a sufficient condition for numerical diversity. In the light of these two criticisms, I concluded that Hawthorne's defense of the traditional conception of the bundle theory requiring the truth of the 'constitutional identity thesis' must be rejected.

The new version of the bundle theory that I propose here, just like Rodriguez-Pereyra's view, denies the constitutional approach to individuation and the 'constitutional identity thesis' entailed by this approach. Thus, contrary to the traditional conception of the bundle theory, this new version allows the possibility of numerically distinct individual substances, composed of exactly the same constituents. But unlike Rodriguez-Pereyra's account, in this version of the bundle theory, there will be no distinction to be drawn between bundles and their instances, and individuals will be identified with bundles.

My overall project is to conjoin the bundle theory with a *structural approach to individuation*, according to which bundles are positions in structures and are distinguished by the distance relations they bear to the other positions in the structures to which they belong. This approach entails both a sufficiency for numerical diversity and a sufficiency thesis for numerical identity. But within the confines of this paper I will propose and defend only the sufficiency thesis for numerical diversity.[2] This thesis is:

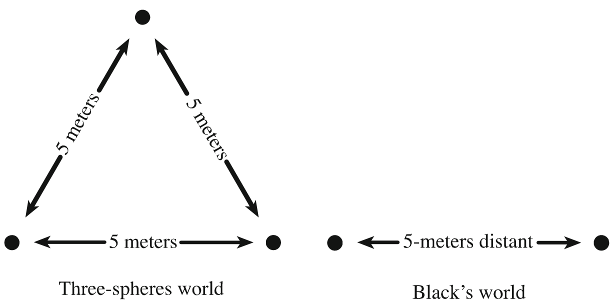
(SDT) If bundles *x* and *y* are separated by distance, then they are numerically diverse.

Note that the bundle theory augmented by 'SDT' can easily explain the existence of plurality in Black's world. In Black's world, there are two bundles, and these bundles are 5 m distant from each other. According to this version, this will be expressed by saying that there are *x* and *y* bundles, and that the *x* bundle is 5 m distant from the *y* bundle. Since being separated by distance is a sufficient condition

[2] As it stands, the sufficiency thesis for numerical identity, viz., that if *x* and *y* are at no distance from one another then they are numerically identical, is open to some criticisms. One criticism is that the possibility of coinciding objects must be rule out. Following Locke, a metaphysician may wish to allow the diversity of a piece of bronze and the statue made out of it. But the bundle theory coupled with this sufficiency thesis for numerical identity cannot allow this possibility. Of course, this Lockean metaphysics may well be false; therefore there cannot be coinciding objects. But this should be decided independently of whether one subscribes to the bundle theory or not. An anonymous referee of this journal suggests a more significant criticism. In Special Theory of Relativity, the notion of our usual spatial distance is replaced by the notion of a relativistic spatio-temporal 'interval'. Now the spatio-temporal interval of any two 'bundles' lying on a given light ray is zero. But then the bundle theory coupled with the sufficiency thesis for numerical identity cannot diversify between these two.

for the diversity between the *x* and *y* bundles, we will also thus establish that there are two distinct sphere-bundles in Black's world.

Let us evaluate now the present version of the bundle theory *vis-à-vis* the case of the *three-spheres world* (which, as I argued, forces Rodriguez-Pereyra to revoke the idea of primitive individuation). By endorsing the 'SDT', it successfully explains the plurality in Black's world. In this world, there are the *x* and *y* bundles, and the *x* bundle is at a certain distance from the *y* bundle. And the dyadic fact that they are separated by distance from each other accounts for the diversity of these two bundles. Nevertheless, it may seem, the bundle theory understood as such cannot account for the *three-spheres world*, a world containing three indiscernible spheres, each 5 m from the other two, so that these three spheres make up an equilateral triangle. Along with the *three-spheres world*, one may talk about the *four-spheres world*, a world where there are four indiscernible spheres, each of which is 5 m from the other three in 3D Euclidean space, so that these four spheres make up an equilateral pyramid. On the surface of a 3D sphere considered as a 2D hyper-space, there can be four points an equal distance apart, which suggests an analogous symmetry in a 3D hyper-space where the case of a *five-spheres world* arises. Without loss of generality, I will concentrate here on the *three-spheres world*.[3] Now although the *three-spheres world* describes a distinct possibility, in so far as distance relations are taken to be binary, the bundle theory that takes distance relations between bundles to be a sufficient ground for their diversity will not be able to distinguish this world from Black's world.



Let us say that, in the three-spheres world, the sphere-bundles are *x*, *y* and *z*, and that the *x* bundle is 5 m distant from the *y* bundle, the *y* bundle is 5 m from the *z* bundle, and the *x* bundle is 5 m from the *z* bundle. Each of these dyadic facts entails that there are two distinct bundles. But this is also true of Black's world, and unless we combine these facts with the supposition that these three bundles are all numerically diverse, we cannot conclude that there are three distinct sphere-bundles. So, in order to establish that there are three distinct sphere-bundles, we need to presuppose the very thing we wish to establish. Thus to the extent that distance relations are binary, we cannot distinguish the *three-spheres world* from Black's world.

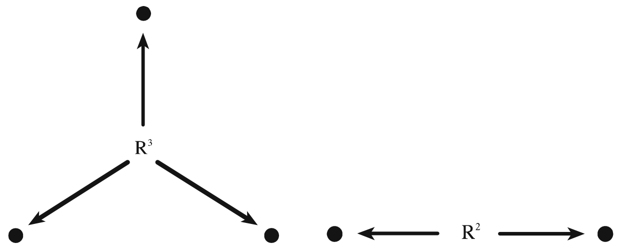
As previously suggested, this problem will be resolved if one adopts the idea proposed by Hawthorne and Cover, namely that distance relations must be

[3] I am indebted to an anonymous referee of this journal for this point.

understood as *irreducibly* polyadic. Suppose that there are such irreducible polyadic relations. Then the 'structural diversity thesis' may be amended in the following way:

(RSDT) If bundles $x_1 \ldots x_n$ are mutually distant under the distance relation R^n, then $x_1 \ldots x_n$ are all mutually diverse (where n is a finite number).[4]

Accordingly, a triadic distance relation R^3 that x, y and z enter mutually (in such a way that they are arranged as the vertices of an equilateral triangle) will be defined, and it will be maintained that if any three things x, y and z are mutually distant under R^3 then they are all numerically diverse.



Three-spheres world Black's world

The triadic distance relation R^3 obtains in the *three-spheres world*. So since x, y and z are all mutually distant under the triadic relation R^3, then there must be three distinct sphere-bundles in this world. However, this triadic relation R^3 does not obtain in Black's world since in this world no three things are arranged as the vertices of an equilateral triangle. So by invoking a triadic distance relation like R^3 bundle theorists will capture the distinction between the *three-spheres world* and Black's world.

One may make the following objection against this proposal. It is true: to say that x, y and z are mutually distant under R^3 cannot merely be to say that any two of x, y and z are pair-wise distant. So this triadic relation, arranging x, y and z as the vertices of an equilateral triangle, cannot be reduced just to dyadic distance relations. However, this does not mean that it is not reducible to some or other dyadic relations. For example, as mentioned earlier, the triadic distance relation in question can be reduced to dyadic distance relations, where these three things entering into relations as such are all numerically diverse. That means: although it is not reducible solely to dyadic distance relations, it is reducible to dyadic distance relations and to non-identity. So to say that x, y and z are mutually distant under R^3 can only mean that x, y and z *are pair-wise distant* and *are also pair-wise diverse* from each other. But if there is such an obvious reduction of the relation in question, then this should not be ignored.

[4] I am grateful to an anonymous referee of this journal for helping me see that the bundle theory's irreducibly polyadic distance relations must be of finite adicity. Consider again the notion of a relativistic spatio-temporal interval. On the path of a given light ray there are infinitely many 'bundles', for any pair of which spatio-temporal interval is zero. This will force us to postulate infinitary distance relations and to understand them as primitives. Not only that but even relations as such cannot distinguish two possible cases: (i) a case where, relative to some choice of coordinates, on a given light ray with the t coordinate, there are integers-many 'bundles'; and (ii) a case where, relative to the same choice of coordinates, on the same light ray with the t coordinate, there are even-integers-many 'bundles'.

In replying to this objection, it is important to note that the claim that the triadic distance relation R^3 is reducible involves the idea of primitive individuation. That is: if the relation that arranges three indiscernible spheres as the vertices of an equilateral triangle is reducible, this reduction, as argued above, presupposes primitive non-identities between these three spheres. So the claim that the triadic distance relation R^3 is reducible can only be defended on the grounds that there are primitive non-identities. But, as previously shown, appealing to brute facts of diversity leads to the empty possibility that there can be more than one individual piled up in exactly the same place at the same time. So the present version of the bundle theory that has a claim to account for multiplicity must be well advised not to allow the reducibility of the triadic distance relation in question, which comes at the expense of invoking primitive non-identities.

7. The bundle theory, augmented with 'RSDT', can successfully account for the plurality in both Black's world and the *three-spheres world*. But there is a serious objection against grounding the diversity of sphere-bundles by invoking the distance relations that they bear to one another. Here is the objection. It is the nature of universals to be capable of repetition; they are the kind of items that can be repeated in many different places. So a sufficiency principle like 'RSDT' cannot apply to them; because one and the same universal may well be distant from itself. However, in the present view, bundles are entirely composed of universals, and thus do not have any constituent other than universals. This suggests that bundles too, just like universals that compose them, must be capable of repetition and that they too must be able to be distant from themselves. So unless there is a reason for claiming that bundles should not be thought of like universals in this respect, being separated by distance cannot be used as a sufficient condition for the diversity of bundles.[5]

Now this objection would be valid if bundles were mereological sums of universals. But the bundle theory does not need to take bundles to be mereological compositions. In fact, as mentioned earlier, its recent defenders agree that there is a sui generis relation—compresence—tying various universals together and that, once a number of universals are bundled together under a 'compresence' relation, the end result (i.e. the bundle) must be construed as a sui generis composition (Hawthorne and Sider 2002, pp. 56–57). Let me explain first what is meant by 'compresence' in terms of four theses.

Firstly, compresence is not 'colocation'. Traditionally, compresence is explained in terms of 'colocation'. According to this analysis, to say that universals F and G are compresent is just to say that F and G are at the same location. But this cannot be true. As David Lewis notes, colocation is transitive, but compresence that is supposed to tie various universals together cannot be transitive (Lewis 1983, p. 345; see also Armstrong 1989, pp. 70–73). Suppose F is compresent with G and G is compresent with H. Contrary to transitivity, it is not necessary that F is compresent with H. To see this, consider the following example. Being spherical is compresent with redness, because there is a red sphere. Being cubical also is compresent with redness, because there is a red cube. But being spherical cannot be compresent with being cubical, because there is nothing that is both spherical and cubical.

5 See also Russell (1911).

Secondly, compresence is an *irreducibly* plural and multigrade relation. In presenting his version of the bundle theory, Bertrand Russell construes compresence as a binary relation holding between two universals (1948, p. 312). And by appealing to this relation, he explains the sense in which individual substances are bundles of universals. Accordingly, he suggests that a certain individual is a bundle of universals (a 'complex of compresence' in his choice of words) whenever each universal is compresent with each other member of the bundle. So, on the Russellian view, a given individual is a bundle of, let us say, F, G and H just in case F, G and H are all pair-wise compresent.

Nevertheless, contrary to what Russell maintains, the fact that universals F, G and H are all pair-wise compresent cannot guarantee that a certain individual substance contains them all. Here is why. Consider a world containing three individuals: the first individual is F and G, the second is G and H and the third is F and H; but no fourth individual has all of F, G and H. But, according to Russell's account, since F, G and H are all pair-wise compresent, there must also be an FGH bundle—a fourth individual substance—along with the FG, GH and FH bundles. But this conflicts with our presupposition that this world does not contain a fourth individual with all three properties (Armstrong 1978, pp. 98–99, 1989, pp. 70–71; see also Goodman 1977, pp. 146–147).

In response to this difficulty, bundle theorists such as Jan Cover and John Hawthorne maintain that compresence is an irreducibly plural relation (1998, pp. 213–214). On these authors' view, the fact that there is an FGH bundle should not be analyzed in terms of F, G and H's being pair-wise compresent; bundle theorists need to introduce an irreducibly polyadic compresence relation wrapping F, G and H together and that facts involving this polyadic relation such as 'F, G and H are compresent' do not supervene on facts involving dyadic relations such as 'F and G are compresent', 'G and H are compresent' and so on. But still there is a further point that needs to be clarified. It may be that there are various *n*-ary compresence relations—a dyadic compresence for individuals with two properties, a triadic one for individuals with three properties and so on. Alternatively, there may be one single multigrade compresence relation. Although Cover and Hawthorne (1998) leave this question undecided, I maintain that compresence should rather be construed as a multigrade relation. Note that the 'overlapping' relation is very similar to 'compresence' in this respect. It is unintuitive to insist that there are various distinct 'overlapping' relations—a 'two-place overlapping' when two things overlap, a 'three-place overlapping' when three things overlap and so on. It would be more proper to claim that there is one single multigrade 'overlapping' relation, holding between any numbers of items. A fortiori, I believe that it is also more reasonable to claim that there is one single multigrade compresence, with the help of which bundle theorists may describe various bundles by stating that *F_1 … F_n are compresent*, for any finite *n* (see also Hawthorne and Sider 2002, pp. 53–54).[6] So, in the case where a certain individual has only exactly single property—let us say, an F

[6] Following Hawthorne and Sider (2002) I construe the multigrade compresence as a relation of finite adicity. This seems to be a safe bet as long as the number of natural properties (corresponding to universals) is limited in number.

bundle—this will be expressed in bundle-theoretic language as ‘F is compresent’; when an individual has exactly two properties—let us say, an FG bundle—this will be expressed as ‘F and G are compresent’; when an individual has exactly three properties—let us say, an FGH bundle—this will be expressed as ‘F, G and H are compresent’; and so on.

Thirdly, compresence cannot all by itself secure the numerical diversity between the bundles that it composes. It may seem that the compresence relation, as it is developed here, comes very close to what medieval philosophers understood by thisnesses.[7] Sketched in very rough terms, thisnesses are thought of as ‘unifying principles’—as items bringing various properties of substances together and thus explaining their unity. But explaining how a number of items composes a seamless whole—a unity—is not the only role thisnesses play. They are also thought of as ‘principles of individuation’—as items guaranteeing numerical diversity between individual substances. These two roles attributed to thisnesses are independent from one another. Explaining how various properties compose a certain unity does not need to guarantee that this unity is numerically distinct from the rest; and conversely, grounding the diversity of a thing does not need to explain how various items come together to compose that thing.[8]

Note that if there were a single thisness unifying various properties into more than one whole then it would be hard to understand how it could play the second role—being the guarantor of the numerical diversity of the wholes that it composes. So there needs to be a multiplicity of thisnesses, each having its distinct identity and each bestowing this identity into the whole that it enters as a ‘unifying principle’. And it is only in virtue of having such a primitive identity that a certain thisness can secure the diversity of the whole that it enters as a ‘unifying principle’ from other wholes that are unified by various other thisnesses.

Certainly, compresence plays a role that is very similar to that of a thisness conceived as a ‘unifying principle’. Just like a certain thisness unifying properties of a substance into a whole, the bundle theorists’ compresence brings various universals together and gives an account of how these universals compose a bundle. But, according to the bundle theory presented here, there is no multiplicity of compresences, each of which is distinguished from other compresences in virtue of its primitive identity. As mentioned above, there is one single compresence and this single compresence is the item tying universals together to form various bundles. Therefore, one must conclude that the bundle theorists’ compresence relation, unlike thisnesses, cannot secure the numerical diversity between the bundles that it composes.

To illustrate this point in a different way, consider the following. Suppose you and I are talking about FGH bundles. As a bundle theorist, you report your FGH bundle by stating, “F, G and H are compresent.” And I report my FGH bundle in the

[7] I am especially grateful here to an anonymous referee of this journal for helping me to clarify this point. As he notes, construing the bundle theorists’ compresence relation in a way that is similar to thisnesses certainly goes against the spirit of the bundle theory.

[8] It may be helpful to note that the two roles attributed to thisnesses are foreshadowed by a distinction, drawn earlier in this paper, between two questions concerning the nature of individual substances: (i) the question concerning internal structure of substances and (ii) the question concerning their numerical diversity.

same way: “F, G and H are compresent”. Now what is told here in bundle-theoretic language, viz., *F, G and H are compresent* and *F, G and H are compresent*, does not entail that there is one single bundle—because your FGH bundle may well be distinct from my FGH bundle. Nor does it entail that they are numerically distinct however—because our bundles may well be one and the same and there may be in fact a single FGH bundle. Therefore, in order to conclude that there are two distinct bundles, something more needs to be claimed—something to the effect that our FGH bundles are separated by distance. And only this will entail that they are two numerically distinct bundles.

Fourthly, compresence is a relation; but unlike the universals that it ties together, it should not be thought of as a relational universal that can be included in the bundle theory’s ground floor ontology (Ehring 2001, pp. 163–168). Suppose that compresence, bundling various universals together, is itself another universal. Then a new tie will be needed to tie this relational universal with the universals that it is supposed to tie. This new tie will be understood as a second-order compresence—another universal. Then a third-order compresence will be needed to tie the second-order compresence, the first-order compresence and the universals together. We are thus left with an infinite regress.

The regress mentioned here is analogous to Bradley’s regress facing substance-attribute theories (see Armstrong 1997, p. 114; see also Baxter 2001, p. 449). Roughly put, Bradley’s regress is the following. Consider the ‘instantiation’ relation that is supposed to tie a particular *p* with its property F, construed as a universal. If this instantiation were a two-place universal that can be added into the substance-attribute theory’s ontology, then a second-order instantiation relation would be required to tie the first-order instantiation, the particular *p* and the universal F together. And this would go on ad infinitum.

In response to Bradley’s regress, the right thing to say is that the ‘instantiation’ relation, bringing a particular such as *p* with its property F together, should not be included in the ontology of the substance-attribute theory—it should rather be taken as a part of this theory’s ideology.[9] Similarly, the regress facing the bundle theory can be avoided when it is denied that the ‘compresence’ relation is a relational universal. Therefore ‘compresence’ must also be understood as a primitive—a part of bundle theorists’ ideology (see also Hawthorne and Sider 2002, p. 54).

It may be objected that the bundle theory’s primitive ‘compresence’ is rather mysterious. However, compresence is in fact no more mysterious than the substance-attribute theory’s primitive ‘instantiation’.[10] Compresence of universals, then, is what bundle theorists use instead of the ‘instantiation’ relation that is supposed to hold between particulars and universals. Where substance-attribute theorists would say that a certain particular *p* instantiates F, G and H, bundle theorists say ‘F, G and H are compresent’.

[9] Vallicella (2000) produces a similar response to the Bradleyan regress. Rather than saying that ‘instantiation’ relation is part of the ideology of the substance-attribute theory, he says that ‘instantiation’, tying a particular with its properties, is an operation that the mind imposes on the world.

[10] This point is defended in Cover and Hawthorne (1998, p. 219); see also van Cleeve (1985, p. 104).

Those who follow David Armstrong's ontology of states of affairs might like to regard bundle-theoretic facts such as 'F, G and H are compresent' as states of affairs. In his *A World of States of Affairs*, Armstrong gives his own gloss on the substance-attribute theory, and proposes to take, for example, *p's instantiating F* to be a state of affairs (1997). For Armstrong, *p's instantiating F* is *not* a mereological sum of the particular constituent *p* and the universal constituent F, but rather is a sui generis composition of these two constituents. In a similar way, bundle theorists may take bundle theoretic facts such as *F, G and H's being compresent* to be states of affairs. Likewise, this state of affairs, rather than being a mereological sum of the universals F, G and H, will be understood as a sui generis composition of these three universals.

Now if bundles, expressed in terms of bundle-theoretic facts like *F, G and H's being compresent*, are not mereological sums of universals, but rather sui generis compositions, then we are not obliged to maintain that bundles, just like universals, can be repeated in different places, and this allows us to account for their diversity in terms of distance relations.

8. Note that, according to Rodriguez-Pereyra's version of the bundle theory, bundles of universals can be in more than one location at once. However, he does not explain why *bundle-instances* should not be thought of in the same way as bundles in this respect. On his view, "the instance is entirely constituted by the universals of the bundle" (2004, p. 78). If so, the bundle instance, just like the bundle, must also be capable of multiple locations. Then the diversity of, let us say, the sphere-instances in Black's world cannot be grounded in their being 5 m distant from each other. Therefore, Rodriguez-Pereyra will be forced once again to appeal to primitive individuation in explaining the diversity of bundle instances. So, given the fact that the idea of irreducibly polyadic distance relations is intelligible, the bundle theory that I defend here can successfully explain the diversity of bundles by appealing to the distance relation that they bear to one another, but Rodriguez-Pereyra's account fails to explain the diversity of bundle-instances in a similar way.

It may be objected that the advantage of the present version of the bundle theory over Rodriguez-Pereyra's bundle theory is unwarranted. First, let us go over this comparative advantage. I maintain that the present version is in a better position to answer the worry that being separated by distance may not be sufficient for the numerical diversity of bundles of universals, since it is not sufficient for the numerical diversity of the constituent universals of such bundles. For, on my preferred version, since bundles are not mereological sums (but rather sui generis compositions) of the constituent universals, they are categorically different from universals, and hence this point can be used in explaining why, whereas universals may be in more than one location at once, the same is not true of bundles themselves. But when we look at Rodriguez-Pereyra's account, we see that, on his view, bundles of universals, just like the constituent universals, can be at different places at once, and thus being separated by distance cannot be a sufficient ground for the diversity of bundles. And since Rodriguez-Pereyra further states that *instances* of bundles are entirely constituted by the universals of bundles, distance relations must also fail to be sufficient for the diversity of bundle-instances. There cannot be an asymmetry between bundles and their instances in this respect.

So Rodriguez-Pereyra owes us an explanation as to why distance relations can be a sufficient ground for the diversity of instances of bundles.

Rodriguez-Pereyra may try to explain this asymmetry by denying that bundles and their instances are composed of the same kind of items. According to this reply, while the bundle is constituted out of universals, the instance is constituted by the *instances* of these universals. So bundle of universals, like their constituents, can be at different locations at once; but the same cannot be true of their instances, because, unlike the bundles that they are instances of, the bundle-instances are composed of the *instances* of universals, but not of universals themselves. Therefore, since, unlike universals, *instances* of universals cannot be at different places at the same time, distance relations can be a sufficient ground for the diversity of bundle-instances.[11]

But I believe that this reply will not do for Rodriguez-Pereyra. By denying that bundles of universals and their instances are composed of the same type of items, Rodriguez-Pereyra may explain why, whereas bundles of universals can be in more than one location at once, the same cannot be true of their instances. But if this is his solution, then we may wonder in which sense, he thinks, bundle-instances are instances of bundles, because bundle-instances cannot be truly said to be *instances* of bundles if bundles and their instances are composed of different type of items. Furthermore, if Rodriguez-Pereyra chooses to say that bundle-instances are composed not by universals but by universal-instances, then this means that he is abandoning the bundle of universals view and is replacing it with the bundle of tropes view.

Alternatively, it may be argued that, in my preferred version of the bundle theory, I am making heavy use of 'compresence' in order to show that while universals may be in more than one place at once the same is not true of bundles of universals. And it may be said that Rodriguez-Pereyra may try to explain the asymmetry between bundles and their instances by making a corresponding point about bundle-instances.[12] Accordingly, Rodriguez-Pereyra's views can be modified in the following way. Bundles of universals are mereological sums of universals, and thus bundles, just like universals that go into their compositions, can be repeated in more than one location at once. But although bundles and their instances are composed of the same universals, the instance (unlike the bundle) is a sui generis composition, brought together under the primitive 'compresence'. So the bundle-instance will be categorically different from its constituent universals. And that will be enough of a difference to show that while a bundle of universal can be repeated in many places, its instance cannot.

I maintain that this second attempt to restore the asymmetry will not do either. According to this attempt, a bundle of universals is a mereological composition, but its instance is a sui generis composition of the same universals, tied together under the primitive 'compresence'. But if the bundle and its instance are different kinds of compositions, then it will not be acceptable to say that the instance is an *instance* of the bundle.

[11] I am indebted to Douglas Ehring for this point.

[12] See footnote 11.

9. In a recent article, Jonathan Lowe argues that spatial relations cannot solve the problem of individuation because, by their very nature, spatial relations presuppose the diversity of the things that enter into such relations (2003, p. 84). Many other authors have defended a similar position by saying things like: 'relations presupposes numerical diversity of their relata and so cannot account for it' (Russell 1911, pp. 1–12; Allaire 1965, pp. 16–21). Fraser MacBride expresses this idea more openly in the following way. Suppose that *x* is separated by distance from *y*. But the fact that *x* is numerically diverse from *y* must be ontologically prior to the obtaining of this distance relation. He writes: "[*x* and *y*] must be numerically diverse 'before' the [distance] relation can even obtain; if they are not individuated independently of the obtaining of [this] relation then there are simply no items available for the relation in question to obtain between" (MacBride 2006, pp. 66–68). So the obtaining of distance relations presupposes the whole framework of already-individuated substances.

Bertrand Russell and Edwin Allaire are the only writers who present arguments for the conclusion that distance relations presuppose numerical diversity and so cannot account for it (Russell 1911, pp. 113–124; Allaire 1965, p. 19). Their arguments are roughly put as follows. Consider again Black's world with two indiscernible spheres; and assume for reductio that distance relations that they bear to each other do not presuppose their diversity. But if the obtaining of distance relation did not presuppose numerical diversity, then that would leave the possibility that one and the same thing is at a certain distance from itself. If so, then why not say that there is one single sphere that is at a distance from itself? To the extent that one denies that distance relations do not presuppose numerical diversity, this cannot be ruled out in any way. However, saying that there is one single thing at a certain distance from itself is unacceptable. Therefore, the assumption that distance relations do not presuppose numerical diversity must be rejected.

I grant that Russell and Allaire are right in their claim that nothing can be separated by distance from itself. However, the reason for this claim is not necessarily that distance relations presuppose numerical diversity. Their opponents may deny that something can be at a distance from itself, simply on the grounds that being separated by distance is a sufficient ground for numerical diversity.[13] Therefore, the contention that if distance relations did not presuppose numerical diversity then we could as well say that in Black's world there is one single thing that is at a distance from itself, amounts to a rejection of the opponents' view that distance relations constitute diversity.

It follows that Russell and Allaire can only argue for the result that distance relations presuppose numerical diversity by way of arguing that relations as such cannot provide a sufficient ground for diversity (and not the other way around).

But why should one accept that distance relations could not ground diversity? When we look at Russell's argument carefully, we see that his main target is the bundle theory. He does not argue that *numerical diversity cannot be explained in terms of distance relations*, but rather argues that *diversity of bundles cannot be explained in this way*. In showing this, he relies upon the premise that bundles, like the universals

[13] In his "Do Relations Individuate?", Meiland (1966, pp. 65–66) makes a similar point.

that compose them, are themselves capable of repetition. If one and the same bundle is capable of being distant from itself, he argues, then being separated by distance cannot constitute the diversity of the bundles (Russell 1911, p. 118).

However, once again I need to emphasize that bundle theorists are not required to contend that bundles have universal-like natures. According to the bundle theory defended here, bundles, being sui generis compositions, are categorically different from the universals going into their compositions. So there is no necessity to accept the view that bundles themselves must be capable of repetition. Therefore, contrary to what Russell argues, it should be granted that the diversity of bundles might well be grounded in the distance relations that they bear to each other.

Acknowledgments I am grateful to Stephen Voss, Barry Stocker, Fiona Tomkinson, Lucas Thorpe, Tonguc Rador, an anonymous reviewer of this journal, and especially John Heil and Berna Kilinc for helpful comments and suggestions on earlier drafts of this paper.

References

Allaire, E. (1965). Another look at bare particulars. *Philosophical Studies, 16*, 16–21. doi:10.1007/BF00398838.

Armstrong, D. (1978). *Universals and scientific realism volume I: Nominalism and realism*. Cambridge: Cambridge University Press.

Armstrong, D. (1989). *Universals: An opinionated introduction*. Boulder: Westview Press.

Armstrong, D. (1997). *A world of states of affairs*. Cambridge: Cambridge University Press.

Baxter, D. (2001). Instantiation as partial identity. *Australasian Journal of Philosophy, 79*, 449–464.

Black, M. (1952). The identity of indiscernibles. *Mind, 61*, 153–164. doi:10.1093/mind/LXI.242.153.

Castaneda, H.-N. (1975). Individuation and non-identity: A new look. *American Philosophical Quarterly, 12*, 131–140.

Cover, J., & O'Leary-Hawthorne, J. (1998). A world of universals. *Philosophical Studies, 91*, 205–219.

Della Rocca, M. (2005). Two Spheres, twenty spheres, and the identity of indiscernibles. *Pacific Philosophical Quarterly, 86*, 480–492.

Ehring, D. (2001). Temporal parts and the bundle theory. *Philosophical Studies, 104*, 163–168. doi:10.1023/A:1010327401920.

Goodman, N. (1977). *The structure of appearance*. Dordrecht: D. Reidel Publishing Company.

Hawthorne, J., & Sider, T. (2002). Locations. *Philosophical Topics, 30*, 53–76.

Lewis, D. (1983). New work for a theory of universals. *Australasian Journal of Philosophy, 61*, 343–377. doi:10.1080/00048408312341131.

Loux, M. (2006). Aristotle's constituent ontology. In D. Zimmerman (Ed.), *Oxford studies in metaphysics* (Vol. 2). New York: Oxford University Press.

Lowe, J. (2003). Individuation. In M. Loux & D. Zimmerman (Eds.), *The Oxford handbook of metaphysics*. New York: Oxford University Press.

MacBride, F. (2006). What constitutes the numerical diversity of mathematical objects? *Analysis, 66*, 63–69. doi:10.1111/j.1467-8284.2006.00590.x.

Meiland, J. (1966). Do relations individuate. *Philosophical Studies, 17*, 65–69. doi:10.1007/BF00398597.

O'Leary-Hawthorne, J. (1995). The bundle theory of substance and the identity of indiscernibles. *Analysis, 55*, 191–196. doi:10.2307/3328579.

Rodriguez-Pereyra, G. (2004). The bundle theory is compatible with distinct but indiscernible spheres. *Analysis, 64*, 72–81. doi:10.1111/j.0003-2638.2004.00463.x.

Russell, B. (1911). On the relations of universals and particulars. *Proceedings of the Aristotelian Society, 12*, 1–24.

Russell, B. (1948). *Human knowledge, it scopes and limits*. New York: Simon and Schuster.

Vallicella, W. (2000). Three conceptions of states of affairs. *Nous (Detroit, Mich.), 34*, 237–259. doi:10.1111/0029-4624.00209.

Van Cleeve, J. (1985). Three versions of the bundle theory. *Philosophical Studies, 47*, 95–104.