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A paradox for some theories of welfare

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Abstract Sometimes people desire that their lives go badly, take pleasure in their lives going badly, or believe that their lives are going badly. As a result, some popular theories of welfare are paradoxical. I show that no attempt to defend those theories from the paradox fully succeeds.

Keywords Value · Welfare · Desire · Hedonism · Paradox

Sometimes people desire that their lives go badly. Such desires have sometimes been thought to be counterexamples to desire satisfactionist theories of welfare—contrary to desire satisfactionism, getting what you want is not always good for you.¹ Recently it has been pointed out that such desires might not be mere counterexamples; they seem to create a paradox for desire satisfactionism. Given the existence of these desires, and given the truth of desire satisfactionism, sometimes a person's life goes well if and only if it does not go well.² The only extended discussion of this paradox appears in Heathwood (2005). The paradox deserves more attention, in light of the following facts: (1) Desire-satisfactionism is currently one of the most popular theories of welfare. (2) The scope of the paradox has not been appreciated; similar paradoxes arise for other theories of welfare, as well as for axiological theories about the values of things such as consequences and possible worlds. (This paper, however, focuses on theories of welfare.) (3) Welfare theory is a vital part of many subfields of ethics and political philosophy, so the paradox has wide-ranging implications. In what follows I explain the paradox for desire-satisfactionism, I show that a similar

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¹ See Kraut (1994, pp. 40–41), Adams (1999, pp. 89–90), and Carson (2000, pp. 88–92) for discussions of this problem.

 $^{^{2}}$ Feldman (2004, p. 17) seems to be the first to point out the paradox in print; also see Heathwood (2005, pp. 502–503).

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paradox arises for other welfare theories as well, and I discuss some attempts to defend those theories from paradox. I claim that no defense fully succeeds.

The paradox formulated³

Desire-satisfactionism

According to a simple version of desire-satisfactionism (DS), when someone satisfies one of his desires, his life thereby goes better for him, and when someone's desire is frustrated, his life thereby goes worse for him. How good or bad it is that a desire is satisfied or frustrated depends on the intensity of the desire. To get a paradox for DS, we need a case where someone desires his life to go badly, and where his life is close to the threshold between good and bad. Suppose DS is true, and suppose Epimenides has just two desires. His first desire, D_a , is a desire of intensity +5 for an apple. He does not get the apple, so his life includes a desire frustration of value -5. His second desire, $D_{\rm b}$, is a desire of intensity +10 that his life goes badly for him. Is $D_{\rm b}$ satisfied? If it is, then Epimenides' life contains a desire-satisfaction of value +10, in which case his life has an overall value of +5 (it goes well for him), in which case $D_{\rm b}$ is not satisfied after all. If $D_{\rm b}$ is not satisfied, then his life contains a desire frustration of value -10, in which case his life has an overall value of -15 (it goes badly for him), in which case $D_{\rm b}$ is satisfied. Thus if DS is true, $D_{\rm b}$ is satisfied if and only if it is not satisfied, and Epimenides' life goes well if and only if it does not go well.⁴ DS is paradoxical.

This is a problem only if the life I have described for Epimenides is genuinely possible. Granted, it would be difficult for someone to have only two desires. But the life is described that way only to make the math easy; if you want a more complicated life, add more desire-satisfactions and frustrations, so long as they balance out. If Epimenides' life is not possible as described, it must have something to do specifically with $D_{\rm b}$. That desire does not seem *inherently* problematic; in most cases, it does not create a paradox even if DS is true. I discuss some arguments for the claim that $D_{\rm b}$ is problematic in Section II below.

TAIAH

Paradox also infects at least one version of hedonism: Fred Feldman's "Truth-Adjusted Intrinsic Attitudinal Hedonism." The idea behind TAIAH is that the value of a pleasure depends on the truth-value of its propositional object. One might be led to such a view by consideration of well-known examples such as Nozick's experience machine and Nagel's deceived businessman (Nozick, 1974, pp. 42–45; Nagel, 1979, p. 4). According to TAIAH, it is intrinsically better to be pleased that you climbed the mountain when you actually did climb it than when you merely had simulated

³ I sometimes refer to "the paradox" as if there is a single paradox faced by a number of different theories. Perhaps it is better to say that, for each theory, there is a distinct paradox. Nevertheless, the paradoxes seem quite similar, as all involve a person having some attitude towards the proposition that his or her life goes badly. Perhaps it is better to think of "the paradox" as being rather a family of paradoxes. Along the same lines, I sometimes refer to the "welfare paradox." This is for convenience, and not meant to imply that all theories of welfare face a paradox.

⁴ See Heathwood (2005, p. 502) for a nice statement of the paradox.

mountain-climbing experiences, and better to be pleased that others like you when they actually do like you than when they hate you.

According to TAIAH, the value of a pleasure is a function of its intensity and the truth-value of its object. Just to make things precise, suppose that pleasures taken in truths ("true pleasures") are twice as valuable as pleasures of similar intensity taken in falsehoods ("false pleasures"), so that a degree 10 pleasure has intrinsic value of +10 when it is taken in a false proposition, but +20 when it is taken in a true proposition. Suppose that Epimenides takes pleasure to degree 10 in the fact that his life is, on the whole, a bad one. Call this pleasure P. Suppose that there is only one other hedonic or doloric episode in Epimenides' life: an experience of pain with intrinsic value of -15. Is P a true pleasure or a false pleasure? If it is a true pleasure, then its intrinsic value is +20, which means his life has intrinsic value of +5, which means P is not a true pleasure after all. If P is a false pleasure, then P has intrinsic value of +10, which means his life has intrinsic value of -5, which means P was in fact a true pleasure. If TAIAH is true, then P is a true pleasure if and only if it is a false pleasure, and Epimenides' life is good if and only if it is bad. TAIAH is paradoxical.

Other theories

This sort of paradox arises for other theories of welfare too. For example, there are theories of welfare according to which *achievement*, or the completion of projects, affects a person's welfare (Keller, 2004; Hurka, 2001, pp. 12–13).⁵ To get a paradox, we just imagine a person whose achievement or project involves having his life go badly for him. Sometimes, that project or goal will be completed or achieved if and only if it is not completed or achieved. Achievement and project views are very similar to desire-satisfaction views, so I will henceforth assume that what can be said about DS can also be said for achievement views. There are also views according to which *true belief* or *knowledge* affects a person's welfare—*ceteris paribus*, the more propositions one knows, or the more true beliefs one has, the better off one is (Moore, 1993, Ch. VI; Ross, 1988, pp. 138–140; Hurka, 2001, pp. 12–13).⁶ Call this view TB. Nobody thinks TB is a *complete* theory of welfare, but as long as it is part of the story, paradox ensues. To get a paradox for TB, we just imagine a person who believes that his life is going badly. Sometimes, his belief is true if and only if it is false, and his life is going badly if and only if it is not.

The paradoxical theories have a common feature. According to each theory, how well things go for a person is determined by (i) the person's attitudes towards states of affairs or propositions (desiring them, believing them, taking pleasure in them), and (ii) whether those states of affairs are *true*. Call all such theories "correspondence theories." Not every theory of welfare is a correspondence theory. For example, according to simple versions of hedonism, the value of a pleasure does not depend on the truth-value of its object; thus, there is no way to formulate an analogous paradox for simple hedonism. On the other hand, since it is not a correspondence theory, simple hedonism is thought to fall victim to counterexamples involving experience machines and deceived businessmen. So even if there really is a problematic paradox for correspondence theories, it could still be claimed that some

⁵ Keller places no restrictions on which achievements make one's life go better, limiting his options in dealing with the paradox.

⁶ Moore's view is about intrinsic value *simpliciter*, not personal welfare.

correspondence theory is true, provided that only correspondence theories can give a satisfactory account of the values of lives in experience machine examples.

I turn now to discussing ways defenders of correspondence theories might avoid paradox. I will focus on just three correspondence theories: DS, TAIAH, and TB.

Attempts to avoid the paradox

Since the paradox has not been discussed extensively, formulating responses to it involves a certain amount of speculation. All the responses discussed here, besides Heathwood's, are attempts to precisify ideas that seem to recur in informal discussions of the paradox.

Before discussing attempts to avoid the paradox, I note in passing that there are those who think the appropriate response to some paradoxes is to embrace true contradictions—in this case, to admit that there are some lives that are, in the same respect, both good and not-good. I have nothing interesting to say about this response. Australian-rules logicians need not read on.

Worthiness

The paradox might be avoided by placing restrictions on the desire-satisfactions or pleasures that make one's life go better. One might place a rationality or morality constraint on desires or pleasures, so that only *rational* or *moral* pleasures or desire satisfactions make one's life go better.⁷ Feldman suggests that the value of a pleasure should be adjusted not only for the *truth* of its object, but also for its *pleasure-worthiness* (Feldman, 2004, p. 121). This view ("Desert-Adjusted Intrinsic Attitudinal Hedonism") might entail that the state of affairs consisting of one's life going badly is undeserving of pleasure, and pleasures taken in that state of affairs might turn out to be worthless. Many desire-satisfactionists believe something similar about desires.⁸

This response has no plausibility as a defense of TB. The analogous move for TB would be to distinguish belief-worthy from non-belief-worthy propositions, and to say that the belief that one's life is going badly is not belief-worthy. But it would seem that what makes a proposition belief-worthy is, at least in part, that it is true. If so, then there will be another paradox for TB: a paradox of belief-worthy if and only if it is false, and therefore not belief-worthy.

The worthiness response is more promising as a defense of DS or TAIAH. But it requires the desire-satisfactionist or hedonist to give an account of what distinguishes desire- and pleasure-worthy states from non-desire- and pleasure-worthy states—no small feat. And that account must be shown to entail that the objects of paradox-inducing desires and pleasures are not worthy of desire or pleasure. It is not obvious that a plausible account of desire-worthiness would entail that one's life

⁷ Thanks to David Sobel for this suggestion; see Adams (1999, pp. 84–93) and Carson (2000, ch. 3) for further discussion.

⁸ One might wonder why Feldman seems to be more sanguine about the prospects for an account of pleasure-worthiness than he is about the prospects for an account of desire-worthiness (compare Feldman, 2004, p. 17; with Feldman, 2004, pp. 121–22).

going badly is unworthy of desire. Suppose Jeff has performed some very bad actions; suppose he also has a strong (but intermittent) sense of justice. Upon reflection, Jeff might well desire that things go badly for himself. Were he to do so, we might judge him more favorably than if he continued to desire a satisfied life; we might think he demonstrated an admirable sense of morality. We might also say Jeff is rational to have that desire—not that it is in his interest to have it, but that, in the broadest sense, he *ought* to have it.⁹ The same may be said about taking pleasure in one's life going badly.

At this point one might appeal to a difference between "intrinsic" and "extrinsic" desires. To desire something intrinsically is to desire it for its own sake; to desire something extrinsically is to desire it for the sake of something else. Perhaps it is only the satisfaction or frustration of *intrinsic* desires that matters to welfare. Jeff's desire to have a bad life seems to be an extrinsic desire, since he desires a bad life only for the sake of justice, not for itself; thus, its satisfaction or frustration does not make Jeff's life better or worse.

This response merely relocates the problem. If Jeff's desire to have his life go badly is merely extrinsic since it is based on his desire for justice, then it will be his desire for justice that creates the paradox. Suppose Jeff's life goes badly. If, in virtue of his life going badly, his desire for justice is satisfied, then his life will contain an extra satisfaction, and therefore (given the appropriate stipulations, and contrary to our hypothesis) his life does not go badly after all. Paradox.

The hierarchical solution

One might say that it is only "first-order" desire-satisfactions, pleasures or beliefs that make one's life go better.¹⁰ The desire-satisfactionist might say that, since what makes it true that one's life goes badly is facts about the satisfaction of one's desires, the desire for one's life to go badly is a "second-order" desire—a desire about one's desires. Similarly, the pleasure one takes in living a bad life (not getting enough pleasure to outweigh one's pleasures; the belief that one's life is going badly is a second-order pleasure, since it is a pleasure taken in facts about one's pleasures; the belief that one's life is going badly is a second-order belief.

But this is too quick; there is a complication about how the contents of propositional attitudes are individuated. If DS is true, but Epimenides does not believe that it is true, and desires his life to go badly, does Epimenides desire that his desires be frustrated? If not, the hierarchical solution is a non-starter, because the desire to have one's life go badly is not a second-order desire after all.¹¹ We can ignore this complication, however, because the hierarchical solution fails for independent reasons.

⁹ See Carson (2000, pp. 90–91) for discussion of this issue.

¹⁰ Thanks to Jussi Suikkanen, Richard Chappell, Scott Wilson and Campbell Brown for discussion of this idea.

¹¹ There is a related problem about how the hierarchical solution would work for TB. Nobody thinks TB is a complete theory of welfare, so it is not obvious that a belief that one's life is going badly is a second-order belief. Perhaps, ignoring the previous complication about individuating contents of propositional attitudes, the belief that one's life is going badly could be identified with the belief that the positive value of one's true beliefs, pleasures, and achievements is outweighed by the negative value of one's false beliefs, pains and failures; perhaps such a belief counts as a second-order belief, since it is partly about one's beliefs.

Consider a smoker who desires to smoke, but also desires to be rid of the desire to smoke; when that second-order desire is satisfied, the desire-satisfactionist should say that her life goes better. If she takes pleasure in the fact that she no longer gets pleasure from smoking, the hedonist should say that her life goes better in virtue of that second-order pleasure. Or consider someone who, after reading a lot of Epicurus, rids herself of most of her first-order desires; subsequently she regrets this, and desires to get some of those old desires back. If that second-order desire is satisfied, the desire-satisfactionist should say that her life goes better. If she is pleased to be able to take pleasure in the satisfaction of those desires again, the hedonist should say that her life goes better in virtue of that second-order pleasure. We need some principled reason to say that such pleasures and desire-satisfactions are not valuable. It would be ad hoc for the defender of DS or TAIAH to deny their value simply to avoid paradox.

Defenders of TB ought to reject this move as well. It seems to curtail, arbitrarily, the intrinsic value of self-knowledge. Having an accurate self-conception, including true beliefs about the accuracy of one's own beliefs, seems like just the sort of thing a TB defender ought to think makes one's life intrinsically better.

The anti-globality solution

Bertrand Russell once said, in introducing his theory of types, that "whatever involves *all* of a collection must not be one of the collection" (Russell, 1971, p. 63). One might apply this idea to the paradox of welfare, and say that the paradox shows either (i) that there can be no truly "global" desires (desires about one's *entire* life); or (ii) that satisfaction of such desires is not intrinsically good; or, following Russell's suggestion most closely, (iii) that global desires are not themselves part of one's life.

But this will not help either. Consider the following ridiculously simplified example. Suppose I live for 2 days, Tuesday and Wednesday. Each day I have just one desire. On Tuesday, I have the following desire:

(D1) The desire that my life not go well on Wednesday. On Wednesday, I have the following desire:

(D2) The desire that my life go well on Tuesday.

Neither D1 nor D2 is a global desire. Neither D1 nor D2 is paradoxical on its own; nor are they jointly paradoxical, unless DS is true. Suppose that DS is true. Is D1 satisfied? If it is, then two things are true. (i) My life goes well on Tuesday, since I have just one desire on that day, and it is satisfied. (ii) My life goes does not go well on Wednesday, which means the one desire I have that day, D2, is not satisfied. But if D2 is not satisfied, then my life goes badly on Tuesday, in which case my Tuesday desire, D1, must not be satisfied after all. So if my life goes well on Tuesday, then it does not. Suppose D1 is not satisfied. (ii) My life goes well on Wednesday, in which case D2 is satisfied. But that means my life does go well on Tuesday after all. So if my life goes well on Tuesday after all. So if my life does not go well on Tuesday if and only if it does not. Paradox. We could construct similar examples, *mutatis mutandis*, for TAIAH and TB.

The liar-parasitic response

One might well be thinking something like the following: "This is a paradox of self-reference; in that respect, it is like the liar, which is a problem for everybody. Why not just take the best solution to the liar and apply it, suitably modified, to the paradoxes for correspondence theories? Let the logicians sort this thing out."¹²

To be sure, the welfare and liar paradoxes have a similar self-referential flavor. Hence, it is unsurprising that there are similarities between the hierarchical and antiglobality solutions to the welfare paradox for and similar well-known solutions to the liar paradox. But as we've seen, those solutions to the welfare paradox are not plausible.

Furthermore, it cannot simply be assumed that a solution to the liar can be adapted to apply to the welfare paradox. Consider the view that liar sentences are self-contradictory. According to this view, an utterance of the liar sentence, 'This sentence is false,' implicitly asserts that it is true; so utterances of the liar sentence attribute both truth and falsity to themselves; so they are not paradoxical, but self-contradictory and therefore simply false (Prior, 1961; Kirkham, 1995, pp. 294–295). Whatever the merits of this solution to the liar paradox, it cannot be adapted to solve the welfare paradox unless it can be shown that the desire to have one's life go badly somehow implicitly involves a desire to have one's life go well. This seems extremely implausible.

In general, the liar-parasitic strategy cannot be fully successful. In the case of the liar, there is obviously something defective or abnormal about the liar sentence; the challenge is to say just how it is defective. But the corresponding proposition at stake here, the proposition that one's life is going badly, does not seem defective in any way; nor does the belief in it, nor pleasure taken in it, nor the desire that it be true (but see the next section). We get a paradox only given the assumption that DS, TAIAH, or TB is the correct theory of welfare. Thus, we have good reason to wonder whether the paradox for correspondence theories is really so similar to the liar paradox after all.

Heathwood's response

Finally, there is the response given by Chris Heathwood. Heathwood is defending DS, but we will examine whether his solution generalizes to other correspondence theories. Heathwood attempts to get everyone else in the same boat as the desire-satisfactionist:

But not just desire-satisfaction theorists are mired in paradox. Analogous paradoxes get off the ground without assuming a desire theory of welfare. Imagine a person who desires, to intensity 10, that his net balance of desire satisfaction over frustration at some time be negative. Suppose he gets 6 units of desire frustration at that time. It would seem his balance is -6. But if it is, then his intensity 10 desire is satisfied, and so his net balance is +4. But, then his intensity 10 desire is not satisfied. In short, this desire is satisfied if and only if it is not satisfied... however the more basic paradoxes of desire are solved so will the paradoxes for desire-satisfaction theories be solved (Heathwood, 2005, pp. 502–503).

¹² Something along these lines was suggested to me by JC Beall and an anonymous referee.

The idea is that the paradox of desire arises independently of any theory of welfare; even if DS is not true, it is sometimes paradoxical for a person to desire that his desires be frustrated. So DS faces no *special* problem here.

Heathwood's strategy might also be employed by those who think true belief is intrinsically good. Paradox can arise merely from the existence of a person who believes that most of his beliefs are false; at least sometimes, that belief is true if and only if it is false. Again, this is independent of any theory of welfare.

The strategy does not generalize completely. The paradox for TAIAH arises only because we assign differential *values* to pleasures depending on the truth-values of their objects. There does not seem to be a value-independent paradox of pleasure analogous to the paradox of desire. There seems to be nothing paradoxical about a person being pleased that he is getting more pain than pleasure. (Sometimes such pleasures are *false* pleasures merely in virtue of their own existence; this is a bit odd, but not paradoxical.) So TAIAH cannot be defended in the same way Heathwood defends DS.¹³

As for DS, despite Heathwood's efforts, desire-satisfactionists cannot get themselves into the same boat as the rest of us. Suppose that in order to avoid the paradox of desire, we must say that the desire to have one's desires be, on the whole, mostly frustrated, is at least sometimes impossible to have (Heathwood, 2005, p. 503).¹⁴ If DS were true, the impossibility of that desire would entail the impossibility of desiring one's life to go badly on the whole. But *that* desire does not seem paradoxical. So there's a cost to DS here—there is at least one desire that is paradoxical if, but *only* if, DS is true. TB faces the same cost—it does not seem paradoxical to believe one's life is going badly, even if one's life is very close to the threshold between good and bad.

Perhaps the best the DS or TB defender can do is bite this bullet, and say that the desire to be badly off or the belief that one is badly off really is paradoxical in some circumstances; hence, the Epimenides story in I.a does not describe a possible life. They might argue that the Epimenides story really does seem impossible to one who has accepted DS or TB as the true theory of welfare (perhaps mentioning something about one philosopher's modus tollens being another's modus ponens). Whether this is a reasonable response depends on whether DS or TB is so much more attractive than a paradox-immune theory that it is best, all things considered, to bite this bullet. One might argue as follows: it is vitally important that a theory of welfare entail that a life on the experience machine is not a terrifically good life; only correspondence theories (TB, DS, TAIAH) can yield this result; it is therefore worthwhile on the whole to endorse a correspondence theory, even if it requires us, for example, to say that it is impossible for someone living a mediocre life to desire that his life go badly. Evaluation of this sort of response would take us beyond the scope of this paper, since it would require an assessment of all the arguments for and against each theory of welfare. For now, it is enough to note that, despite what Heathwood says, the paradox exacts a price from correspondence theories.

¹³ The TAIAH defender could use Heathwood's response to get around a slightly different paradox, involving a person who takes pleasure in the fact that most of his pleasures are false pleasures.

¹⁴ Richard Chappell suggests this strategy for avoiding the paradox; see http://pixnaps.blogspot.com/2005/02/this-desire-is-thwarted.html and http://pixnaps.blogspot.com/2005/02/is-immoral-value-possible.html and

Taking stock

We have seen five responses to the paradox on behalf of correspondence theories. The hierarchical and anti-globality responses are entirely unsuccessful. The worthiness response is entirely unsuccessful as a defense of TB. As a defense of DS or TAIAH, its plausibility depends on an account of the desire- or pleasure-worthiness of propositions that entails that one's life going badly cannot be worthy of desire or pleasure. There are good reasons to be pessimistic about this. The liar-parasitic response seems unsatisfactory due to the fact that there is no reason to think that solutions to the liar can be adapted to the welfare paradox, and the fact that there seems to be nothing defective about the proposition that one's life is going badly, nor about the relevant desires, pleasures, and beliefs. Finally, Heathwood's response does not work for TAIAH. It also seems unsatisfactory as a defense of DS or TB, since the desire that one's life go badly, or belief that it is going badly, does not seem paradoxical independently of the assumption that DS or TB is the correct theory of welfare.

I conclude that more work needs to be done on behalf of correspondence theories to show either that they can avoid the welfare paradox, or that the costs associated with the best solutions to the paradox are worth paying given the independent plausibility of correspondence theories. I hope welfare theorists take up the challenge; it might turn out that this paradox will be as provocative a topic in the theory of welfare as the liar has been in the theory of truth.

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