



# Otto Selz's phenomenology of natural space

Klaus Robering<sup>1</sup> 

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## Abstract

In the 1930s Otto Selz developed a novel approach to the psychology of perception which he called “synthetic psychology of wholes”. This “synthetic psychology” is based on a phenomenological description of the structural relationships between elementary items (tones, colors, smells, etc.) building up integral wholes. The present article deals with Selz’s account of spatial cognition within this general framework. Selz *Zeitschrift für Psychologie*, 114, 351–362 (1930a) argues that his approach to spatial cognition delivers answers to the long-discussed question of the epistemological status of the laws of geometry. More specifically he tries to derive (a subset of) the Euclidean axioms from the structural laws valid for phenomenal space. After a brief description of the discussion of the status of geometry in the 1920s/1930 (section 2), the present article explains Selz’s understanding of “phenomenology” (section 3). Section 4 then deals with Selz’s attempt to derive the Euclidean laws from the structural phenomenological laws of space. Selz’s attempted derivation suffers from some formal shortcomings, which however can be repaired. The question arises, though, whether the necessary improvements do not rely upon more intricate geometric intuitions and thus render Selz’s attempt to base geometry upon the phenomenology of spatial cognition circular.

**Keywords** Spatial cognition · Gestalt psychology · Phenomenal vs. physical space · Foundations of geometry

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✉ Klaus Robering  
robering@sdu.dk

<sup>1</sup> Institute of Design and Communication, University of Southern Denmark, Kolding, Denmark

## 1 Introduction

In a series of nine articles written between 1929 and 1943, the psychologist Otto Selz<sup>1</sup> (1929; 1930a, b; 1934; 1936; 1941a; b; 1949) developed a novel account of the structure of the phenomenal world, i.e., the world as it appears to us in perception. Selz calls this new general theoretical framework “synthetic psychology of wholes” (“synthetische Ganzheitspsychologie” or “synthetische Psychologie der Ganzen,” cf. Selz (1941a, 176) for the first term and (1949, 91) for the second). This synthetic psychology of wholes is opposed to both associationism, which – building upon the ideas of the British Empiricists – had dominated psychology in the nineteenth century, and gestaltism as it has been developed in various schools in Austria and Germany since the beginning of the twentieth century.<sup>2</sup> In spite of their contradictory views concerning the relationship between phenomenal wholes and their constituents, associationism and gestaltism share, according to Selz (1941a, 173), a common theoretical principle, namely: that the characteristic properties of gestalts must be explained dynamically by means of forces. Whereas associationists, however, assume that the force of association composes unanalyzable psychic elements into more comprehensive wholes, gestaltists conversely assume such wholes to be primary and explain their articulation into parts as a selforganization of the perceptual field by its inner forces. Such explanations, if adequate, explain the emergence of wholes; they are alternative answers to the question *why* a whole of a certain kind develops. However, they cannot explain the specific way *how* this whole is structured; they cannot disclose its “constitutional or structural laws” (“Aufbau- oder Strukturgesetze,” Selz 1941a, 175). Both the associationists and the gestaltists aim at a causal-genetic explanation of wholes; finding the structural laws of these wholes, however, is the task of a phenomenological analysis. Such an analysis will reveal how structured wholes are composed out of simpler items and will thus open the way back – “for long considered impossible in psychology” (Selz 1941a, 176) – from analysis to synthesis.

Within Selz’s project of a synthetic psychology of wholes the analysis of phenomenal space and time are of special importance since more complicated wholes like patterns of colors or tones build upon the order of places in space and/or that of moments in time. Selz’s theory of phenomenal space and time thus plays a foundational role for his entire project. Phenomenal or, as he (1930a, 357) says, “natural” space is the system of possible positions (“Raumlagen”) or loci (“Örter”) at which we locate our experiences, section 4.3 below. The basic principles of natural space, its constitutional structural laws are found by a

<sup>1</sup> Selz (1881–1943) originally studied law and later philosophy and psychology in Munich under Theodor Lipps and in Bonn under Oswald Külpe. Being of Jewish descent, he was deported to the Auschwitz concentration camp where he was killed in 1943. A collection of his basic writings on the psychology of perception and thinking (Selz 1991) has been edited by Alexandre Métraux and Theo Herrmann, who also give a brief account of his life and work in the introduction to their edition. An extensive historiographic study of Selz’s psychology has been provided by Seebohm (1970) in his PhD dissertation. Today Selz is mainly remembered for his impact upon Popper’s philosophy of science and epistemology (ter Hark 2003). Furthermore, he is seen as a forerunner of the “cognitive revolution” (van Strien and Faas 2004) because of his contribution to “Denkpsychologie”.

<sup>2</sup> In the first section of his article from 1934 Selz explains the differences between his psychology of wholes and the gestalt psychology of the Berlin school (represented by Wolfgang Köhler) and that of the Leipzig school (represented by Felix Krueger) by means of their different analyses of the circle shape.

phenomenological analysis of our spatial experience. They have the character of non-trivial analytic statements, and are thus a priori valid. According to Selz, the long-discussed philosophical problems concerning the nature of space, the epistemological status of Euclid's axioms, and the question of their validity for physical space find solutions within his theory of natural space. The discussion of these problems as it took place at the time of his writing is briefly described in the following section 2. Section 3, then, explains Selz's special conception of phenomenology and his notion of a constitutional or structural law, which plays a crucial role in his synthetic psychology of wholes. According to Selz, Euclid's axioms are derivable from the structural laws of phenomenal space and thus inherit the status of necessary a priori principles from them. His attempt to derive (some of) these axioms from the structural laws governing natural space is discussed in section 4.

## 2 The problem of space and geometry

The discovery of non-Euclidean geometries in the nineteenth century triggered an intensive discussion of the epistemological status of the geometric axioms. In view of alternative geometries, the question naturally arose which of the mathematically conceivable possibilities actually applies to spatial reality and whether the axioms of the "correct" geometry are a priori valid principles or just empirical facts or, perhaps, even merely useful conventions accepted for the sake of the measurement of distances, areas, and volumes. At the beginning of the twentieth century this discussion was enforced by two events: namely, the appearance of Hilbert's seminal *Foundations of Geometry* (1899) and the Einsteinian revolution. Einstein adopts the modern conception of the axiomatic method as it is already implicit in Hilbert's book and has been worked out by Hilbert in later works.<sup>3</sup> When applying this method, we have to distinguish between the formal theory, considered as a purely syntactic structure on the one hand and the semantic interpretation of this formal theory on the other. The axioms of a formal theory are only schemes which become statements with a definite truth value first when the terms occurring in them are provided with interpretations relating them to a specific subject domain. A complete set of interpretations for all terms which renders all axioms of the formal theory true is a model of that theory. The question whether physical space is a model of Euclidean geometry, considered as a formal system, asks for such physical interpretations of its basic terms which render its formal axioms true assertions. We may, for example, determine that we understand by a point a tiny mark on a rigid body and by a line a ray of light. If we in this way supplement the formal theory of Euclidean geometry by interpretations of its basic terms, the question whether we thus get a model of Euclidean geometry turns into a question of physics.<sup>4</sup> Experience decides upon this question and the thus interpreted axioms of Euclidean geometry are a posteriori empirical statements about reality.

<sup>3</sup> Cf., the first pages of Einstein's address to the Prussian Academy of Science from 1921 where he deals with "axiomatics". His views as explained there are obviously inspired by Hilbert's work.

<sup>4</sup> In his talk about "Geometry and experience" (cited in the previous footnote) Einstein (1921, 6) explicitly explains: "[...] we may consider it [namely: the thus interpreted geometry] as just the most ancient branch of physics."

The same views on the status of geometry are put forward by Hilbert (1922, 78–91) in a talk having the same title *Geometry and Experience* as Einstein’s academy address cited in fns. 3 and 4.<sup>5</sup> Hilbert explains that setting up a formal theory starts with the construction of a “conceptual framework” (“Begriffsfachwerk,” Hilbert 1922, 83) and proceeds with the formulation of suitable axioms which fix the relations between the “concepts” of the framework – understood, as explained above, as purely formal schemes. A formal theory is applied to a certain domain by assigning “real things” (“Dinge der Wirklichkeit,” Hilbert 1922, 84) to these schematic terms. We thus apply a theory by pointing out a model of it built from items of reality. The construction of the conceptual framework and the formulation of the axioms will be guided by the aims which one pursues with the axiomatic theory. By a formal system of Euclidean geometry we may for instance aim at an account of human spatial intuition. In that case the task of setting up the axioms – i.e., the “fundamental principles” of Euclidean geometry – is “tantamount to the logical analysis of our intuition of space” (Hilbert 1899, 1).<sup>6</sup>

However, there is no inherent connection between a formal axiomatic theory and a specific interpretation. The theory’s framework of formal concepts may be applicable to quite different aspects of reality. Hilbert (1922, 81–86) illustrates this by means of an axiomatic theory of linear order. This theory may be conceived of as describing the order of points on a line and the relationships between inheritable features of the common fruit fly (*Drosophila melanogaster*). Hence it has both a geometric and a biological model. The decision whether a proposed interpretation really is a model of a formal theory requires factual knowledge about the realm to which the interpretation refers, e.g., genetic knowledge about the fruit fly. A formal theory originally designed to describe our intuition of space may be re-interpreted by providing its basic terms with physical meanings. As already said above, we may, for instance, understand by a point a tiny mark on a rigid body and provide the remaining terms with similar interpretations. The question whether such a re-interpretation of the theory is a model does then no more depend upon our intuition of space, rather it is an empirical question of physics. In his 1922 lectures Hilbert considers the physical interpretation of the geometric terms as the main and proper one. Thus he repeats Einstein’s *dictum* cited in fn. 4: “Geometry is nothing else than a branch, the oldest one, of physics [...]” (Hilbert 1922, 89). The appearance of intuitive evidence and apriority which many of the geometric axioms have is explained by Hilbert by the fact that we are accustomed to them since childhood. We thus do not, for instance, need a physical institute in order to find out that three points determine a plane. That experience, however, really is necessary becomes obvious in more complicated cases such as that of the principle that the sum of the angles of a triangle equals two right ones. Whether this really holds true for physical triangles can only be settled by experiments and one thus really needs

<sup>5</sup> This talk belongs to a series of lectures, which Hilbert held at the University of Göttingen in the winter term 1922/23 and which have been recorded and worked out by his then assistant Wilhelm Ackerman.

<sup>6</sup> Selz (1934, 378, fn. 1), too, cites this sentence; however, he is not aware that, if taken in isolation, it is not an appropriate statement of Hilbert’s view on geometry; cf. the following paragraph of the main text.

instruments and institutes in this case (“braucht man also sehr wohl Apparate und Institute,” Hilbert 1922, 89).<sup>7</sup>

Selz is well aware of the discussion on the epistemological status of the geometric laws going on in philosophy, mathematics, and physics since the second half of the nineteenth century.<sup>8</sup> One goal of his project is to re-establish the rigorous evidence of the Euclidean axioms that has been lost in the course of the development of modern axiomatics (Selz 1929, 352; 1930c, 362). For Hilbert such evidence – as far as it exists at all – is due to the fact that we already in early childhood adapt to simple and obvious physical traits of our environment. Such a kind of evidence, however, relies upon experience and is open to revision; therefore it does not provide our geometric knowledge with an epistemologically distinguished status. Selz (1929, 353), on the other hand, considers the evidence of the Euclidean axioms to be due to the structural laws of “natural space,” i.e., space as disclosed to us in perception.<sup>9</sup> From these laws the Euclidean axioms can be achieved by logical deduction. To disclose the laws of natural space is the task of a phenomenological analysis. By their deduction from the structural laws governing natural space the Euclidean axioms inherit their evidence. In this way the Euclidean laws regain the evidence which they have lost in the modern discussion about the epistemological status of geometry. As becomes clear, however, from a consideration undertaken by Selz (1949, 114) in another context, this inherited evidence is not considered by him to be the highest form of “‘insight’ or rational comprehensibility” since it only relies upon inferences and is therefore indirect. The structural phenomenological principles from which the Euclidean axioms can be inferred are however open to *direct* insight (“unmittelbar einsichtig”), their truth is “phenomenologically exhibitable” (“phänomenologisch aufweisbar”), and they concern the essence (“Wesen”) of the objects with which they deal (1949, 114).

### 3 Selz and phenomenology

As has been stated above, Selz considers it a task of phenomenology to determine the structural laws underlying spatial cognition. However, Selz himself neither explains what he precisely means by *phenomenology* nor does he expound what a phenomenological analysis consists in. In the present section first Selz's relation to Husserl's phenomenology is discussed (section 3.1); then it is argued (in section 3.2) that

<sup>7</sup> The principle at issue is equivalent to the axiom of parallels. Hilbert (1922, 89) suggests that Gauss, who had measured the sum of the angles of a large triangle determined by the peaks of three mountains of the Harz, did so in order to empirically check the validity of that axiom. As is shown by Breitenberger (1984), however, this interpretation of Gauss's geodesic research is untenable. Nevertheless, it seems that Gauss held views on the epistemological status of geometry similar to those of Hilbert. In a letter to the astronomer Olbers he wrote: “Perhaps in another life we reach at other insights into the nature of space, which are inaccessible for us now. Until then one would have to rank geometry not together with arithmetic, which is purely a priori, but for instance together with mechanics” (Gauss 1900, 177).

<sup>8</sup> As the inventory of Selz's literary estate indicates he took sixteen pages of excerpts from Carnap's PhD thesis from 1922, which deals with this debate (Selz 2013, card D II 1). The intensiveness of the discussion is testified by the bibliography of Carnap's dissertation, which comprises no less than 275 titles, most of them from the two first decades of the 20th, the rest from the second half of the nineteenth century.

<sup>9</sup> Though the idea of natural space is provided to us by perception, it is not “space as perceived,” cf. p. 9 below.

Selz's views on phenomenology coincide with those of Carl Stumpf rather than with those of Husserl. This is confirmed in section 3.3 dealing with the question in what sense space can be an appropriate topic of phenomenology (as understood by Stumpf and Selz) at all. There it is explained that Selz's treatment of natural space in many, though not all, respects has been inspired by Stumpf.

### 3.1 Selz and Husserl

Given the time of Selz's writing, a first plausible interpretative hypothesis is that he refers by the term *phenomenology* to the specific method and the doctrines of Husserl. This hypothesis is supported by the fact that Selz gave a very sympathetic presentation of Husserl's doctrine in his talk "Husserl's phenomenology and its relation to questions of psychology" held in 1912 as a trial lecture in the course of his habilitation procedure at the University of Bonn (Selz 1912). However, though he recognizes the relevance and importance of Husserl's phenomenology for psychology, Selz (1912, 84) nevertheless identifies a basic difference of interest between the phenomenologist and the psychologist: whereas the ultimate goal of the former is knowledge of "ideal essences," descriptive psychology "only aims at the determination of the characteristics of classes of real experiences."<sup>10</sup> Selz considers the structural principles of psychology to relate to the essences of the objects with which they deal, cf. the statement from his 1949 article cited at the end of the previous section (p. 4). Hence the difference between phenomenology and empirical psychology cannot concern the notion of essence; that notion plays a role in both disciplines. Thus, when Selz contrasts "ideal essences" with "real experiences," the decisive opposition must be that between *ideal* and *real*. Empirical psychology is interested in real rather than in ideal objects.

Furthermore, empirical psychology is concerned with psychic objects only. It "has rightly banned from its domain all investigations concerning the phenomenological verification [Nachweis] of real or ideal nonpsychic kinds of objects" (Selz 1912, 84). Since Selz held his lecture in 1912, the main bibliographic sources of Husserlian phenomenology available for him have been the first edition of the *Logical Investigations* (1901), the bibliographic report on German contributions to logic from 1903/04, and Husserl's *Logos*-article from 1911; cf. Selz's (1912, 73f) own indication of his sources. It is thus by no means surprising that he conceived of Husserl's investigations of ideal non-psychic objects (such as, e.g., proposition and concepts) "as the main goal of the phenomenological analysis of consciousness, which only by an analysis of this kind becomes an appropriate foundation for epistemology and logic" (Selz 1912, 84). Later work of Husserl – such as, e.g., his investigations about "thing and space"<sup>11</sup> and his "phenomenological psychology" – is closer to and highly relevant for Selz's psychological endeavors. However, this work has been presented in lectures

<sup>10</sup> It is clear from the context that the term *descriptive psychology* is not used here in Brentano's sense but rather refers just to psychology as an empirical discipline aiming at adequate descriptions and explanations of psychic phenomena.

<sup>11</sup> Cf. Giorello and Sinigaglia (2007) for a succinct presentation of Husserl's views on the perception and constitution of space as developed in his lectures on "Ding und Raum".

(1907 and 1925, respectively) which only have been published posthumously in Husserl (1973) and 1962 and thus have not been available to Selz.<sup>12</sup>

Though the ideas on perception and space which Husserl developed after the *Logical Investigations* could not be known to Selz in the 1930s and 1940s, he could have found some hints towards them in the publications of Husserl's disciples. As far as I can see, however, Selz's nine articles at issue here contain only one single reference of this kind, namely a critical remark on Wilhelm Schapp's PhD-thesis (written under Husserl; cf. Schapp 1910) from 1910 on the phenomenology of perception (Selz 1934, 376). Most remarkable is Selz's neglect of Oskar Becker's "Habilitationsschrift" from 1923 which is devoted to the phenomenological justification of geometry and its physical applications.<sup>13</sup> Becker's analyses as well as Husserl's own doctrines developed in the lectures on "thing and space" and "phenomenological psychology" depend upon methodological innovations implemented by Husserl after the *Logical Investigations*. Selz, however, in his nine articles from the 1930s/40s does not refer to Husserl's later works. Presumably Selz's neglect of them is the main reason that historiographic studies consider his psychological research to be only loosely connected to Husserl's phenomenology. Seebohm (1970, 161), devoting a whole chapter of his biography to "Otto Selz als 'Phänomenologe'", just states that it is obvious that Selz does not use the term *phenomenology* in Husserl's sense despite of his great appreciation of that philosopher. Spiegelberg (1972, 63) calls Selz's debt to Husserl to be "even weaker" than that of the psychologist Narziß Ach (as Selz a member of the Würzburg School of Psychology<sup>14</sup>), whose debt already was "hardly [...] substantial".<sup>15</sup> The question arises then what Selz's own conception of phenomenology is? An answer is provided in the following subsection.

### 3.2 Selz and Stumpf

In the winter term 1901 Selz, who else studied in Munich, participated in the psychological seminars of Carl Stumpf at the University of Berlin (Seebohm 1970, 10f).<sup>16</sup> Since,

<sup>12</sup> There is no indication either that Selz has been accustomed with Husserl's considerations in the posthumously article published 1939 by Eugen Fink in the *Revue internationale de philosophie* (1939), which considers the problem of the origin of geometry from a perspective quite different from that taken by Selz.

<sup>13</sup> As is obvious from a letter (from 9 April 1922) to Hermann Weyl, Husserl considered Becker's analysis of physical geometry as definitive. According to Husserl (1994, 293f), Becker had shown that Einstein's theory when complemented with Weyl's infinitesimal geometry is "the only possible and ultimately understandable one" and he asks the rhetorical question: "What will Einstein say to this when it is proven that nature postulates a relativistic structure because of a priori reasons of phenomenology rather than because of positivistic principles?"

<sup>14</sup> Selz received his "Habilitation" in 1912 under Oswald Külpe, who had moved from Würzburg to Bonn in 1909.

<sup>15</sup> As regards Husserl's knowledge of Selz's work it may be remarked here that he seems to have been acquainted at least with Selz's contribution to the psychology of thinking. In July 1922 Karl Bühler, when leaving the Technical University of Dresden for a professorship in Vienna, asked Husserl to support Selz as his successor in Dresden (Husserl 1994, 45). He refers Husserl to Selz's "new great book," i.e., to the monograph from 1922. Already two days later Husserl (1994, 247) sent a letter to Selz asking him for a copy of the book and received it September 7th; cf. Schuhmann (1977, 183).

<sup>16</sup> Stumpf has been the academic teacher of "nearly all of the founders or leading co-workers of Gestalt theory" (Ash 1998, 34). As will be shown in the present section, he also had a decisive impact upon Selz's synthetic psychology of wholes, which Selz considered to provide an answer to the question *how* gestalts are built up by simpler items, a question left open, according to Selz, by the gestalt psychologists; cf. section 1 above.

furthermore, Selz (1949, 120) calls his Berlin teacher a “grand master [Altmeister] of phenomenological analysis,” the interpretational hypothesis that Selz’s conception of phenomenology is due to that of Stumpf does not seem to be completely implausible. Stumpf gave a first, brief account of his views of phenomenology in a treatise published in 1906 in the proceedings of the Prussian Academy of Science.<sup>17</sup> In this treatise, which deals with the classification of the sciences, Stumpf (1906, 26–32) locates phenomenology as a “neutral discipline” (“neutrale Wissenschaft”) between the natural sciences (“Naturwissenschaften”) on the one hand and the humanities (“Geisteswissenschaften”) on the other.<sup>18</sup> Both the natural sciences and the humanities start from appearances (“Erscheinungen”). The sciences derive hypothetical external objects from them and study the laws governing the temporal, spatial, and causal relationships obtaining between these objects. The humanities are concerned with the psychic functions or acts relating to the appearances such as notifying them and their interrelationships, combining them, forming concepts from them, judging, comprehending, feeling, and willing (Stumpf 1906, 20). Though the disciplines of both groups thus start with appearances, the ultimate aims of their interests are other entities. In contrast to them, phenomenology is concerned with appearances as such.

By an appearance Stumpf (1906, 30, 46f) does not mean a subjective experience of a percipient but rather the objective entity which is experienced in such a psychic occurrence. Thus, e.g., colors and tones themselves rather than the subjective experiences of seeing colors or hearing tones are appearances. The laws established by phenomenology are not causal laws but immanent structural laws (“immanente Strukturgesetze,” Stumpf 1906, 28, 61–64). Such laws do not describe the causal succession of events but describe how parts are organized within a whole, thus, for example, how the chromatic hues are organized within the color solid or how the musical tones build up the tonal space. Non-causal dependencies are a special type of such structural relationships, a type which is of special importance within the area of the psychic (Stumpf 1906, 61).<sup>19</sup> The structural laws of phenomenology admit formulations matching the standards of mathematical preciseness (Stumpf 1906, 28).

In his treatise from 1906 Stumpf does not explicitly discuss the epistemological status of the phenomenological structural laws though he says that both observation and experimentation may be necessary in order to discover them (Stumpf 1906, 31f). As he later explains in his posthumously published monograph, this does by no means imply that they are contingent empirical truths. Rather, the structural laws state necessary facts about the elementary appearances, facts which cannot be otherwise (Stumpf 1939/40,

<sup>17</sup> More detailed and comprehensive explanations were given by Stumpf in an extensive monograph on epistemology published posthumously in two volumes in 1939 and 1940; cf. Stumpf (1939/40). However, Stumpf’s monograph was probably unknown to Selz, who since 1939 lived as an emigrant under difficult conditions in Amsterdam.

<sup>18</sup> Besides phenomenology Stumpf assumes two other neutral foundational disciplines, namely “eidology” (“Eidologie”) – concerned with such structures (“Gebilde”) as concepts and states of affairs – and “general relation theory” (“allgemeine Verhältnislehre”) – dealing with relational concepts such as similarity, identity, part (Stumpf 1906, 32ff, 37ff).

<sup>19</sup> Such dependencies play an important role in Stumpf’s theory of space and space perception. Thus, for instance, spatial extension and color mutually depend on each other: neither can we represent extensionless color nor colorless extension. Stumpf (1873, ch. I, § 5) works out his account of such dependencies in his theory of psychological parts, which is the starting point for Husserl’s (1901, II/1, ch. III, §§ 2–4) celebrated investigation concerning parts and wholes.



169). Appearances are provided to us either by our outer senses (“Sinneserscheinungen,” Stumpf 1939/40, 180) or by the activities of the soul (“die erlebten eigenen Seelentätigkeiten”). The thus given appearances are the material for concept formation, which as a mental function differs from the bare having of appearances. However, this function needs some input: all our concepts, so Stumpf (1939/40, 180) explains, are ultimately based upon appearances delivered either by the senses or by the activity of the soul. For Stumpf, this empiricism with respect to concepts does however not imply an empiricism for propositions. Propositions concerning the inner constitution of appearances do not depend upon experience though observation and experiment may be necessary to find out their truth values. Each appearance is originally a unitary whole (“etwas Einheitliches”); in most cases, however, an appearance is not something simple (“absolut Einfaches”) but a complex whole (Stumpf 1939/40, 174). We come to know the parts of such a whole, the relations between its parts, and the relations between such relations by bringing the appearance to clearest consciousness (“zum deutlichsten Bewußtsein zu bringen;” Stumpf 1939/40, 174) and to fathom it as deeply and intensively as possible (“so tief und intensiv als möglich einzudringen,” Stumpf 1939/40, 174). This phenomenological analysis relies upon a kind of perception (“eine Art von Wahrnehmen,” Stumpf 1939/40, 175), which Stumpf compares with reading. A phenomenological analysis of this kind may reveal necessary connections between the parts of the appearance under analysis. Thus, for example, we recognize that an area seen always has some color or that a tone heard always has a certain strength. Since such connections are discovered by an analysis, the propositions describing them – such as “The visually extended is colored” or “Tones have volume” – exemplify “the type of the analytical judgement” (Stumpf 1939/40, 176). They are analytical in the same sense as the proposition that an ill man is a man (Stumpf 1939/40, 174). But while this is obvious in the trivial case of the latter proposition, a phenomenological analysis is necessary to see it in the first two cases. Observation and experiment may be helpful in a phenomenological analysis by uncovering hidden parts or by disclosing non-obvious relationships between the parts of a whole. But by this they do not impair the analytic and a priori character of the structural law resulting from the analysis. These two methods are just auxiliary means, the only proper method for establishing structural constitutional laws is still the analysis of the content delivered by the senses.<sup>20</sup>

Selz (1929, 340) takes over the notion of a structural law from Stumpf. He explains that structural laws are a second, special group of psychological laws besides the class of the empirical ones, cf. p. 12 below. They are not found by induction, rather their validity is due to the structure of our perceptions, which is revealed by psychological analysis. Like Stumpf, Selz, too, emphasizes the quasi-mathematical precision of the structural laws. Such similarities support the interpretational hypothesis that Selz's conception of phenomenology coincides with that of his Berlin teacher. However, this still leaves open the question how their common understanding of phenomenology relates to that of Husserl. In a footnote Stumpf (1906, 63, fn. 61) traces his conception

<sup>20</sup> Cf. Stumpf (1939, 173): “The analysis of the content of the senses in itself has still the last word to say.”

of phenomenology as a discipline aiming at structural laws back to ideas put forward by Dilthey<sup>21</sup> and Brentano. In his later epistemological monograph Stumpf (1939/40, 185) furthermore includes Husserl's early phenomenology in the same tradition.<sup>22</sup> But according to Stumpf the main concern of the scholars working in the Brentano tradition are "thought experiences" ("Denkerlebnisse," cf. the quotation in Fn. 22).<sup>23</sup> But then their endeavors belong in Stumpf's classification of the sciences to the humanities rather than to phenomenology as understood by him as a discipline concerned with appearances rather than with thought. Wider objections against Husserl's phenomenology have been put forward by Stumpf in a section of his epistemological monograph bearing the title "Critique of Husserlian Phenomenology" (Stumpf 1939/40, 188–200). According to Stumpf, the methods of eidetic intuition ("Wesensschau") and bracketing (epoché, "Einklammerung") introduced by Husserl (1913) in his *Ideen* keep away from phenomenology everything factual ("Tatsächliche") – and thus especially sense perception.<sup>24</sup> As already said above (cf. fn. 17), Selz, was probably unaware of Stumpf's criticism of Husserl put forward in his monograph from 1939/40. However, the considerations at the end of his trial lecture from 1912, where he confronts Husserl's interest in "ideal essences" with the descriptive psychologist's interest in "real experiences" converge with Stumpf's concerns.

### 3.3 Space as a topic of phenomenology

Selz (1930a, 351) starts his article on the foundations of geometry by the apodictic statement that the location ("Ort") of a visual perception is – just "like color" – a sense phenomenon.<sup>25</sup> Such a parallelizing comparison of color and place is directly opposed to Kant's account of space. Kant (1902ff, vol. III, 50) = (1998, 155f) uses the term *Erscheinung* – the German equivalent of the foreign word *Phänomen*<sup>26</sup> – in order to denote the "undetermined object" of an "empiric intuition," i.e., of an intuition which is related to its object by sensation. With respect to appearance, Kant distinguishes between its matter, which "corresponds to sensation" and its form, "which allows the

<sup>21</sup> Stumpf refers to Dilthey's Academy lecture from 1894, the very lecture which gave rise to the Dilthey-Ebbinghaus-controversy on the status of psychology and its proper methodological procedure; cf., e.g., Galliker (2010). Stumpf remarks however that he would like to have separated Dilthey's notion of a "teleological connection of life"

("teleologischen Lebenszusammenhanges") from that of a structural law. On Husserl's view upon the controversy cf. Husserl (1968, § 1).

<sup>22</sup> "What E. Husserl originally understood by 'pure phenomenology' was nothing else than Brentano's descriptive or phenomenological psychology, especially the analysis of thought experiences [Denkerlebnisse]." In the printed text the first component *Denk-* of the composite noun *Denkerlebnisse* is highlighted by letter-spacing.

<sup>23</sup> One might object here that Stumpf ignores both Brentano's (1907) work on the psychology of sensation ("Sinnespsychologie") and Husserl's concerns with perception in the *Logical Investigations*; cf., e.g., Mulligan (1995).

<sup>24</sup> A similar criticism is put forward by Husserl's modern interpreter Mulligan (1995, 168), who explains that "Husserl lost interest in describing the things and processes in the real world." Therefore Mulligan, in his article on Husserl's work on perception, restricts himself to the early works, which "are relatively free of the mysteries of Husserl's transcendental and idealist turns."

<sup>25</sup> "Like the color of a visual perception, its location, too, is a sensory phenomenon."

<sup>26</sup> Thus, the entry "Phänomen" in Schmid's (1798, 420) dictionary of Kantian terminology consists in nothing more than a reference to the article "Erscheinung".

manifold of appearance to be intuited as ordered in certain relations". Space is "pure form" devoid of any sensation whereas color *is* sensation.<sup>27</sup> Hence, from Kant's point of view, the very first sentence of Selz's article involves a blunt confusion of form and matter. Selz (1929, 338) himself points out his opposition to Kant on this issue: "Kant, on the contrary, makes a very clear distinction here: color, smell and flavor are 'drawn' [«tirées»], that is to say, abstracted, from external phenomena; space and time, as subjective forms of our intuition, are formative conditions prior to any particular sensory experience."

The term *phenomenology* is used by Kant in two different meanings both of which, however, are related to his notion of appearance. The last chapter of his *Metaphysical Foundations of Natural Science* (1786) is called "phenomenology" and deals with movement "as appearance of the outer senses;" Kant (1902ff, IV, 477) = (2011, 12). Phenomenology in this special sense is of no concern in the present context. Sixteen years before, however, Kant had used the term *phenomenology* in quite another sense in a letter to Johann Heinrich Lambert (Kant 1902ff, X, 96–99).<sup>28</sup> There he argues that metaphysics, which exclusively deals with pure reason, has to be preceded by a philosophical discipline which investigates the validity and scope of the "principles of sensitivity" and he suggests the name "phaenomenologia generalis" for this discipline (Kant 1902ff, X, 98).<sup>29</sup> This conception of phenomenology resembles that of Stumpf and Selz dealt with in the previous subsection. Yet Kant never presented such a theory under the name *phenomenology* and instead used this term, as just mentioned, for another purpose. However, in the second edition of his *Critique of Pure Reason* Kant (1902ff, III, 50) = (1998, 156) explicitly explains: "I call a science of all principles of a priori sensibility the transcendental aesthetic." Thus both Selz's disciple Julius Bahle and his biographer Seebohm recognize a strong relationship between Kant's transcendental aesthetics and Selz's project as developed in the nine articles mentioned in the introduction above. In a letter (from February 2nd, 1937), Bahle asks Selz whether his theory may be understood as a "psychologisation of the Kantian transcendental consciousness" (Seebohm 1970, appendix B, 36). Seebohm (1970, 161) explains that Selz, in order to realize his project, "had to put forward a 'phenomenology of the transcendental aesthetics'."

Seebohm's excerpts from the correspondence between Selz and Bahle unfortunately do not contain an answer to Bahle's question. But given Selz's parallelization of space and time with the qualities of sensations, it is clear that Selz's phenomenology of space and time cannot be a "psychologisation" of Kant's transcendental aesthetics. Selz

<sup>27</sup> Kant is quite explicit about this: he mentions color as something which "belongs to sensation" and points to the spatial features of extension and form as something which remains "if we separate from the representation that which the understanding thinks about it [...] as well as that which belongs to sensation" Kant (1902ff, III, 50) = (1998, 156).

<sup>28</sup> In 1764 Lambert had published a comprehensive treatise *New Organon* by which he hoped more completely to achieve that what Aristotle and Bacon intended in their "Organa". Lambert's treatise comprises four parts the last of which is called "Phenomenology or The Doctrine of Appearance" ("Phänomenologie oder Lehre von dem Schein") and contains the "theory of appearance and its influence upon the correctness and incorrectness of human cognition" (Lambert 1764, 645).

<sup>29</sup> Two years later, in 1772, in a letter to Marcus Herz, the term "Die Phänomologie [sic] überhaupt" re-occurs as the heading of the first part in a sketch for a planned comprehensive work on *Die Grenzen der Sinnlichkeit und Vernunft* (Kant 1902ff, X, 129). Thus "phenomenology" was intended to take that role in the planned work which later in the *Critique of Pure Reason* is taken over by the transcendental aesthetics.

(1929, 338) considers it to be an achievement of post-Kantian psychology that “it, contrary to the views of Kant, brings back to the impressions of sensible experience and to their elaborations the spatial and temporal determinations that we attribute to objects as well as the qualitative determinations of things, color, smell, or flavor.” A major role in this theoretical development is played by the investigations of the origins of spatial representations undertaken by Selz’s Berlin teacher Stumpf (1873). The first chapter of Stumpf’s book includes a severe critique of “Kant’s theory of subjective forms”. With regard to a representation (“Vorstellung”), so Stumpf (1873, 25) explains, we can distinguish three aspects: (1) its content, (2) the act, i.e., the activity (or state) of representing and (3) the outer (physical and physiological) and inner (psychological) conditions of its emergence. Stumpf (1873, 27) argues that space must be “content” like the sensual qualities since it obviously does not belong to the two last kinds. Thus there is complete agreement on this point between Selz and his teacher Stumpf.<sup>30</sup>

The spatial appearances that we experience in touch and vision are a topic to be investigated in phenomenology as Selz and Stumpf understand it. But what exactly is the phenomenal “natural space” which they fill? It obviously differs from physical space (Selz 1930a, 362). Selz’s (1930a, 531) brief discussion of Hering’s (1879) theory of the “spatial sense” (“Raumsinn”) could be interpreted as suggesting that natural space coincides with visual space. But this cannot be the case either. Visual space is the space of the “Sehdinge,” the things as seen. Hering (1879, 345) = (1942, 2) points to the downing sun as an example. As a “Sehding” it is a flat, circular disc which consists of yellow-red, thus – as Hering says – of a visual sensation (“Gesichtsempfindung”).<sup>31</sup> This sensation is neither in the perceiver’s eye nor in her/his brain but rather “there where the sun appears to us,” namely at a place in visual space (“Sehraum”). Since not all of the principles which according to Selz (1930a) are valid for natural space hold true for visual space, the two spaces have to be distinguished. Natural space is, as Selz (1930a, 357) says a “homogeneous, unbounded and infinite continuum” whereas Selz (1941a, 182) declares visual space to be finite. Furthermore, the Euclidean axiom of parallels holds true for natural space Selz (1930a, 361) whereas this was debated among psychologists for visual space both at the time of Selz’s writing (cf., e.g., Allesch 1931) and even two or three decades before (Boring 1942, 294–296).<sup>32</sup> Selz (1941a, 182) explicitly declares that natural space differs from both physical space and visual space. Natural space is characterized by him as being the “space of our sensible intuition” (“Raum unserer sinnlichen Anschauung”).

This characterization provokes the question, of course, whether the admittance of an intuitive space besides visual and physical space is not just a retreat to Kant’s doctrine of “subjective forms,” which has been rejected by both Stumpf and Selz when they declared space to be an appearance “just like color”. However, one should remember here Stumpf’s characterization of the structural laws: though they are found by means

<sup>30</sup> In his *Erkenntnislehre* Stumpf (1939/40, 157) formulates the issue in a sentence resembling the first sentence of Selz’s (1930a) article: “Spatial extension is as well an attribute of visual sensation as color quality and lightness,” cf. fn. Fn. 25 above.

<sup>31</sup> In the English translation the German term “Sehding” is rendered as “visual object” and “Gesichtsempfindung” is translated simply by “perception”; cf. Hering (1942, 2).

<sup>32</sup> That the “geometry of visibles” is non-Euclidean had been suggested already in 1764 by Thomas Reid, thus 60 years before the possibility of non-Euclidean geometries was recognized by the mathematicians Lobachevsky and Bolyai in the 1820s; cf., e.g., Daniels (1972).

of observation and experiment, analysis has the last word; cf. fn. 20. Such an analysis may reveal that a structural relationship else well attested in observation does not occur because of accidental restrictions. Thus, for instance, we find that we can iteratively divide a finite line into parts of decreasing size by placing a pencil mark between the endpoints. That the process cannot be continued indefinitely we ascribe – abstracting away from such factors as the size of the pencil tip and the precision of our hand movements – to the limited resolution ability of our eyes thus considering the operation of cutting a line into two halves to be “in itself” repeatable without restriction. There is nothing in the operation itself which restricts it to be executable only a finite number of times. This suggests to us the idea of a space which is densely filled by points in the sense that there always is a third point between two given ones although points fuse in visual space when their physical distance remains under a certain *limen*. Selz (1941a, 182) calls this space in which the structural laws are not restricted by accidental factors the “phenomenal space” and characterizes it as “the space of our world of appearances,” the “system of the spatial positions [Raumlagen] or loci [Örter]”. Hence it must be the space which he called the “natural space” in his earlier article (1930a, 357).

## 4 The Euclidean structure of natural space

For Selz the acceptance of phenomenological structural laws governing natural space renders superfluous the distinction between the matter and the form of appearances drawn by Kant in order to explain the apriority of the laws of geometry; cf. section 4.1. Structural laws characterize the arrangement of phenomenal items in various types of orders in accordance with an increase or decrease of one of their characteristic traits; cf. section 4.2. The laws of natural space describe specifically the spherical order of loci from the viewpoint of an observer located at its center. With respect to this central position, each locus determines a direction of space; the order of loci thus induces a corresponding order of directions; cf. section 4.3. Using the notion of direction, Selz defines lines and planes and tries to derive the Euclidean laws valid for them from his definitions and his structural laws of natural space; cf. section 4.4.

### 4.1 The epistemological status of geometric Laws

According to Selz, Kant mistakenly conceives of space as a form of intuition in order to solve two interrelated problems: namely to explain (1) the unity of experience and (2) the a priori status of the laws of Euclidean geometry. Our experiences build up a structured system of lawfully connected items rather than a “Gewühl von Erscheinungen” (“swarm of appearances”); Kant (1902ff, IV, 84) = (1998, 234).<sup>33</sup> Were our experiences such a “muddle,” we could at best obtain from them empirical laws by means of induction. Whereas such laws admit of exceptions, the geometric laws are necessary and absolutely exact (Selz 1929, 339). By declaring the axioms and theorems of geometry to be laws of

<sup>33</sup> The English rendering of the term *Gewühl* by *swarm* is not quite adequate since there are orderly swarms (e.g., of birds) while a “Gewühl” always lacks any structure. The term occurs only in the first edition of the *Critique*. A few pages later Kant (1902, IV, 89) = (1989, 239) speaks of the “unruly heaps” (“regellose Haufen”) which our representations would build if they “reproduced one another [...] just as they fell together.” This passage again has not been taken over to the *Critique*'s second edition.

the form of spatial intuition, Kant simultaneously solves both of his problems. First, space and time provide a framework of lawful order relations by which the muddle of appearances receives a coherent structure.<sup>34</sup> Secondly, the laws of geometry can be known a priori and recognized as necessary since the framework which they define is prior to its accidental content provided by experience.

This interpretation of Kant seems to be inspired by that of the Southwestern School of Neo-Kantianism. In his nine articles at issue here, Selz mentions Kant's talk about the muddle of appearances twice: first in his French article from 1929 («un fouillis de qualités», 1929, 340) and later in the article on the concept of number (Selz 1941b). The citation of the latter article is not quite precise; Selz quotes Kant as talking about the “Gewühl von Empfindungen” rather than about the “Gewühl von Erscheinungen”. In Kantian terminology an “Erscheinung” is the object of an “empirische Anschauung,” i.e., an intuition which is related to its object by “sensation” (“Empfindung”); Kant (1902ff, III, 50) = (1998, 155). Exactly the same replacement of *Erscheinung* by *Empfindung* as in Selz's article can be found in Lask (1912, 58) and in Bauch (1923, 259). The hypothesis thus suggests itself that Selz used Lask or Bauch as a source rather than Kant's original text. There is an important difference, though, between Lask and Bauch on the one hand and Selz on the other. Whereas the former – as well as Kant himself in the passage at issue, which comes from the transcendental analytics – are concerned with the role of the categories for the constitution of a coherent and structured experience, Selz deals with the role of space and time which, according to him, are given in appearances in the same way as, e.g., the colors.<sup>35</sup> Whereas both Kant (1902ff, III, 221 = 1998, 312) and his neo-kantian followers Lask and Bauch are concerned with “the synthetic unity of the manifold in concepts,” thus with reason, Selz deals with “the form of things in space,” thus with intuition.

Selz (1929, 339) explains Kant's acceptance of space and time as forms of intuition by the fact that the only psychological “laws” known at his times were merely empirically valid rules like Hume's “laws of association”.<sup>36</sup> Since Kant thus could not know of the notion of a phenomenological structural law of constitution,<sup>37</sup> he instead drew the distinction between the matter and the form of appearances in order to

<sup>34</sup> Selz thus seems to ascribe to Kant what Falkenstein (1995, 78f) calls the “heap thesis” according to which “sense dumps its deliverances on us all in a heap”. For a complete interpretation of Kant, this thesis has furthermore to be supplemented either by the hypothesis that space is an inborn mechanism for ordering the items of this heap (Falkenstein's “form as mechanism”; 1995, 77–81) or an orderly pattern of “ready-made, empty containers” to be filled with them (Falkenstein's “form as representation”). Falkenstein argues in his book that such an interpretation, while it fits for Kant's dissertation (from 1770), misrepresents Kant's position in the *Critique*. According to him (1995, 93), Kant, in his later works, considered the ordered “spatiotemporal sensory manifold” not to be the product of “any cognitive processing” but rather “to be *originally received* as the immediate effect of impression of the senses” [emphasis in the original]. Such a position would be much closer to that of Stumpf and Selz.

<sup>35</sup> As has been explained in the previous section (cf. p. 9 above), Selz here follows his Berlin teacher Stumpf. On the difference between Stumpf's view on sensation and perception and those of the neo-Kantians cf. Dewalque (2014).

<sup>36</sup> Selz adds that psychology continued in that state “until the present time” (“jusqu'à l'époque actuelle”). For Selz, it is a recent discovery of phenomenology due to the work of Hering and Stumpf that there are also necessary structural laws of constitution governing perception.

<sup>37</sup> One may object here that Kant (1902ff, III, 116f) = (1998, 291) assumes that sensations (of the same kind) are ordered by their “degree” (“Grad”), which is an “intensive magnitude” (“intensive Größe”). Such orderings are, if not identical, so very similar to Selz's increaselements of degree of strength; cf. section 4.2 below. However, Kant does not state structural laws for intensive magnitudes.

secure the a priori status of the necessary laws of geometry. Yet this distinction is untenable for any psychologist who assumes, as Selz following his teacher Stumpf does, that the (allegedly) “formal” characteristics of appearances are given in precisely the same way as their “material” ones; cf. section 3.3 above. As soon as the notion of a structural law of constitution is available, Kant’s problematic distinction can be abandoned. The Euclidean laws follow logically from the more fundamental laws of constitution of natural space and thus inherit from the latter the status of necessary a priori truths. Though structural laws of constitution deal with items (like colors, tones, smells, and loci) given in experience, they are both necessary and absolutely exact. They possess that kind of “rigor which we admire of the mathematical laws” (Selz 1929, 340) and thus make up a second class of psychological laws besides the merely empirical ones. Laws of that latter kind are obtained by induction and thus only have a certain degree of probability; in contrast to them the necessary structural laws are reached by phenomenological analysis.

Like the fundamental structural laws of natural space from which they are derived, the Euclidean laws of geometry, too, have a specific subject matter, namely natural space. This conception of geometry is quite different from that of Hilbert dealt with in section 2 above. Whereas for Hilbert the axiomatic systems of geometry are formal frameworks which can be filled by quite different contents (i.e., have different models),<sup>38</sup> which even need not necessarily be connected with “space” (visual, intuitive, physical) at all, geometry is the science of natural space for Selz. From Hilbert’s point of view, natural space would be only one possible model of his axiom system for Euclidean geometry. Whereas Hilbert’s geometry is a formal system admitting for different interpretations, Selz’s geometry of natural space is an interpreted theory with a definite topic.

## 4.2 Series and their structural laws

Selz’s (1930a, 355ff) derivation of (some of) the Euclidean axioms is based on a phenomenological systematics of point-like loci (“Örter”). Loci may differ with respect to two attributes: quality or tone (“Ortston”) at the one hand and degree of strength (“Stärkegrad”) on the other. With respect to their tone we distinguish loci which are, e.g., LEFT or RIGHT, TOP or BOTTOM, or FRONT or BACK. Of two loci which both are LEFT one will be more to the LEFT than the other; this is a difference of strength. By accepting point-like loci as the final constituents of his natural space, Selz takes a position contrary to that of his Berlin teacher Stumpf, whom he else follows – as we have seen (cf. sections 3.2 and 3.3) – in many other respects. Stumpf (1873, 81) rejects the view that space is composed out of points since, as he explains, no finite extension can be composed out of nothing but zeroes. To this Selz (1930b, 530) objects that “it is no more inconsistent to conceive of a local continuum [Ortskontinuum] as being constituted by continuously varying loci which in themselves lack extension than it is to think of [...] a pitch continuum being built up from extensionless pitches”. Just

<sup>38</sup> However, it may happen – and does so in the case of Hilbert’s own system of Euclidean geometry because of his “axiom of completeness” (Hilbert 1899, 30, Axiom V.32) – that all models are isomorphic to each other. Such a theory is called categorical. Identifying isomorphic models, one may say that a categorical theory has a uniquely defined topic.

as we, for example, hear a swelling tone filling a temporal interval, so do we get the impression of a local tone filling a spatial continuum when we, starting from some point of fixation, let our gaze wander upward. In such a case our gaze meets a series of loci with the same tone but increasing strength.<sup>39</sup> Both experiences, the auditory and the visual one, are “phenomena of increase” (“Steigerungspänomene,” Selz 1930b, 530). Ironically, the notion of *increase* here applied by Selz is borrowed from Stumpf (1883, I, 96, 109–111; 1906, 27), who within his theory of relations (“Verhältnislehre”) considers “increase” (“Steigerung”) to be an undefinable basic relation (“Grundverhältnis”).

Building upon Stumpf’s account of “increases,” Selz modifies and supplements it in various ways. Perceptual phenomena like colors and loci are orderly arranged in “increasement series” (“Steigerungsreihen”).<sup>40</sup> It belongs to the tasks of phenomenology to discover the laws governing such series. There are two types of them: “increases of degree” (“Gradsteigerungen”) and “increases of cardinality” (“Mengensteigerungen”).<sup>41</sup> Increasement series belonging to the latter type provide the basis of arithmetic (Selz 1941b), which thus also has a root in the phenomenal world. An auditory tone increasing in volume and a local tone increasing in strength (becoming, e.g., more and more TOP) exemplify increases of degree of strength (Selz 1934, 327). Besides them there is another type of increases: increases of degrees of purity (“Gradsteigerung nach Reinheitsgraden”). Thus, for example, the colors of the grayscale range from pure WHITE to pure BLACK with mixtures of these two pure hues in between. Traversing the scale from WHITE to BLACK, one meets hues which become less pure as shades of WHITE and purer as shades of BLACK. The temporal qualities BEFORE and AFTER behave quite otherwise. They do not merge but exclude each other (Selz 1930a, 352). However, each of them determines a series of increase and these two series can be combined into a single one which starts from a “common point of indifference” (“gemeinsame Indifferenzstelle”) in opposite directions. Such a bidirectional series is called an “antagonistic series of increase” (“antagonistische Steigerungsreihe”) by Selz. Of the two quality tones involved in such an antagonistic series Selz says that they “simultaneously drop out” (“gleichzeitig ausfallen”) at the point of indifference.

A series of increase can display various structural properties such as, e.g., continuity, homogeneity, unboundedness, and infinity (Selz 1930a, 352–354). A continuum is defined by Selz (1930a, 353) as a gapless and dense series. A gapless series is complete by containing all the phenomena which lie between two given items of the series. The series of all the pitches between a certain root and the corresponding perfect fifth is gapless whereas a scale has gaps (Selz 1930a, 352). A series is dense if it contains with two items always an item in between them. There are gapless series which are not dense, e.g., the series 2, 3, ..., 8 of cardinalities. However, each gapless series of increasing degrees of a phenomenal quality is dense according to Selz (1930a, 353).

<sup>39</sup> The notion of a “phenomenal continuum” is analyzed in more depth by Selz in his article posthumously published in 1949.

<sup>40</sup> This term also occurs in Husserl’s *Logical Investigations* (1901, II/2, 720, 735f, 808) = (1970, II/2, 598, 614f, 700). His translator Findlay renders it by three English phrases, namely: “ascending series,” “graded series,” and “ascending scale”.

<sup>41</sup> It is rather difficult to provide an adequate English translation for Selz’s German term. “Increases of number” would be misleading since Selz considers the notion of number to be derived from that of “Mengensteigerung”. Nor would “Increases of amount” fit since it would include increases in the quantity of continuous masses while Selz is thinking of the phenomenal growth of collections of discrete “units” (Selz 1941b). Hence I have borrowed the technical term *cardinality* from set theory here.



Selz (*ibid.*) calls a series homogeneous “if the same types of increase are possible everywhere within its domain of graded qualities”. This is hard to interpret; perhaps Selz means that each part of a homogeneous series is isomorphic – i.e., structurally similar – to each other part.<sup>42</sup> A series is unbounded if each of its items has a successor and it is open in both direction (“beiderseits offen”; Selz 1930a, 354) if, besides being unbounded, each of its items also has a predecessor. In cyclical series<sup>43</sup> each item has a successor, but such series may nevertheless consist only of a finite number of items since moving along such a series may take one back to one’s start point.<sup>44</sup>

Selz’s account of the various types of series and their structural properties reminds of the theory of relations as developed in formal logic and set theory in the three decades preceding Selz’s writings. Selz was well accustomed with these developments; thus he (1930a, 353, fn.1; 1934, 77, fn. 1; 1941b, 181, fn.1, 1949, 95, fn. 4) cites, e.g., relevant works by Bertrand Russell – as Stumpf (1906, 3) does, too, – and Abraham Fraenkel. Furthermore, the card D I 3 of the inventory of Selz’s literary estate (Selz 2013) refers to six pages of handwritten excerpts from Edward Huntington’s monograph on *The Continuum and other Types of Serial Order*; cf. Fn. 42. Hence it may be assumed that, besides Stumpf’s “allgemeine Verhältnislehre,” mathematical order theory has been an inspiration for Selz, too. However, Selz’s doctrine of series of increase is not intended to be a contribution to that theory and he definitely does not write as a mathematical logician. Rather he is interested in the “psychological sources of concepts of apparently non-sensory [nicht-sinnlicher] descent” (Selz 1934, 377). Thus, for instance, relations as conceived by (standard)<sup>45</sup> mathematical logic have an inherent direction. For Selz this directedness of relations is founded in our experience of phenomena of increase.

### 4.3 Natural space

Cyclical order, though it plays a prominent part in Selz’s theory of space, was only briefly mentioned in the previous section. However, Ewald Hering’s (1920) account of the cyclical order of the chromatic colors has been a major source of inspiration for Selz. For Hering, a phenomenological analysis of colors has to precede any physiological investigation of color perception. The aim of such an analysis is to establish a “natural color system” (Hering 1920, ch. 2), i.e., a systematics of colors which is “based solely on the properties of the colors themselves” (Hering 1920, 23) = (1964, 24).<sup>46</sup> Selz’s “natural

<sup>42</sup> Huntington’s (1917) little book on the continuum was known to Selz. From that he might have known that any two continua (i.e., orders of Cantor’s type  $\theta$ ) are isomorphic (“ordinally similar”; Huntington 1917, 49) to each other. This could have motivated Selz’s notion of homogeneity.

<sup>43</sup> Strictly speaking, the expression *cyclical series* – as well as Selz’s *zyklische Reihe* – involve a *contradictio in adjecto* since the notion of a series normally is understood to comprise linearity.

<sup>44</sup> Selz (1930a, 35) says that they are finite, thereby overlooking the possibility that such series may be dense and thus infinite.

<sup>45</sup> This is not the case for the notion of relation as applied in situation theory: “There is no reason to suppose the argument places of an arbitrary relation are intrinsically ordered” (Barwise 1981, 180).

<sup>46</sup> Hering’s methodological procedure markedly differs from that of his opponent Helmholtz, who, in the second volume of his *Treatise of physiological Optics*, prepares his treatment of color in §§ 19 and 20 by two paragraphs considering, respectively, the “Stimulation of the Organ of Vision” (§ 17) and the “Stimulation by Light” (§ 18), cf. Helmholtz (1856: II, 3–41) = (1924: II, 1–46). There is a whole series of further differences between Helmholtz and Hering on the issue of color as well as on other matters of visual perception, cf. Turner (1994).

space” is intended to be a systematics of loci which achieves for the experience of space what Hering has accomplished for color perception. Hering arranges the chromatic hues – such as, e.g., RED, BLUE, GREEN, and YELLOW – in a cyclical order, a “closed series” (Hering 1924, 42) = (1964, 41), cf. Fig. 1a. The four hues just mentioned are “primary colors”. All the other chromatic hues are mixtures of two of the primary ones. Pure GREEN (“g” in Fig. 1a), for instance, is followed by “greenish” hues with an increasing admixture of BLUE (“b”) until the pure shade of this hue is reached. Similarly, pure BLUE is succeeded by a series of hues becoming continuously more reddish until pure RED (“r”) is reached. Then the shades of RED become more and more yellowish. Finally, after having arrived pure YELLOW (“y”), the hues approach pure GREEN which thus completes the cycle.

Such cyclic arrangements of colors are by no means uncommon in color theory. What makes Hering’s cycle peculiar is his doctrine of “opponent colors” (“Gegenfarben”); Hering (1920, 49) = (1964, 50). The primary colors are grouped into two pairs of opponent colors: GREEN is opposed to RED and BLUE to YELLOW. Whereas there cannot be any mixture of opposing hues (thus, e.g., no “greenish red” or “reddish green”), non-opposing hues combine to new hues such as, for instance, TEAL, which is “greenish-blue”. This means that we can “see” – i.e., visually detect – the primary colors GREEN and BLUE in TEAL, not that this color can be produced by blending blue and green pigments or that the radiation characteristic for that hue can be mixed from blue and green light; Hering (1920, 46) = (1964, 50). There are thus four kinds of mixed hues corresponding to the four 90°-sectors of the entire cycle of hues (assuming a co-ordinate system as that indicated by the dotted lines in Fig. 1a). A sector is bounded by the two pure primary hues which are the components of the “intermediate hues” in between them; Hering (1920, 43) = (1964, 42). Intermediate hues of the same sector differ only with respect to the proportion of their two primary components. This is indicated by the half-moon-shaped areas in Fig. 1a; cf. the colored Plate I following p. 48 (= p. 42) in Hering (1920) = (1964). Applying Selz’s terminology explained in the previous subsection, we may describe each of the four sectors as a series of increase of purity. Seen from the perspective of BLUE, for instance, the purity of that hue continuously increases when we approach it starting from the zero level at GREEN by traversing the upper right sector of the circle of hues.

Selz’s (1930a, 351ff) systematics of local tones differs from Hering’s color system just by the presence of a third pair of opponent or, as Selz prefers to say, “antagonistic,” tones. There are three pairs of antagonistic basic local tones: RIGHT – LEFT, TOP – BOTTOM, and FRONT – BACK. We may thus visualize the resulting system of local tones by a sphere; cf. Fig. 1b.<sup>47</sup> In this figure, the sphere is presented as seen from the perspective of the reader who is looking “forward” into the sphere so that her/his glance enters it at the BACK and leaves it through the FRONT. The local tones LEFT and RIGHT are thus located to the left and right of the reader. Corresponding to the mixtures of primary hues, we have “intermediate local tones” (“Zwischenortstöne”) such as, e.g., FRONT-RIGHT-TOP, marked as FRT in Fig. 1b. Each local tone, so Selz (1930a, 356) explains, determines a phenomenon of increase, which he identifies with

<sup>47</sup> Selz himself does not mention this obvious possibility. However, he (1930a, 355) observes that “[t]he perception of curved surfaces, e.g., the surfaces of spheres, requires a three-dimensional system of local qualities.”

the “directions” (“Richtungen”) of natural space. A direction determined by a certain local tone is thus the complete series of loci of that tone ordered by increasing strength. According to Selz's typology of phenomenal orders, a direction is thus a special series of increase of degree of strength. Since there is a one-one correspondence between local tones and directions, concepts defined for the latter may be transferred to the former so that we, for instance, can talk about opposite directions: directions are opposite to each other if the corresponding local tones are so (Selz 1930a, 356). Furthermore, we can use the names of local tones also in order to refer to directions.

When using his system of directions in order to describe the geometry of natural space in the second part of his article, Selz (1930a, 356ff) changes terminology by talking now about “points” (“Punkte”) rather than “loci” (“Örter”). Directions are used by him (1930a, 356) to determine the relative position of points: a point  $B$  may lie in a certain direction  $d$  from point  $A$ . Selz does not seem to be aware of a major problem involved in this notion of relative position. Let us call the local tones of  $A$  and  $B$  “ $tA$ ” and “ $tB$ ,” respectively. These tones fix the directions of  $A$  and  $B$  relative to the center of the sphere of local tones which represents the position of the observer.<sup>48</sup> Thus localizing  $A$  and  $B$  in directions  $tA$  and  $tB$  is a case of what is called “egocentric” positioning in spatial cognition research (Klatzky 1998). In contrast to this, localizing  $B$ , as being situated in direction  $d$  from point  $A$  is an “allocentric” description of  $B$ 's position:  $A$ , instead of the observer's position, is used as the point of reference. To make sense out of his allocentric notion of position, Selz should have specified a method to calculate the direction  $d$  from  $A$  to  $B$  from the egocentric positions  $tA$  and  $tB$  of these two points. Given the vector-like character of Selz's directions, such a method is well conceivable; however, Selz does not specify one.

#### 4.4 Selz's derivation of Euclidean geometry from the phenomenology of loci

Making use of his relation of allocentric localization (as seen from point  $A$ , point  $B$  lies in direction  $d$ ), Selz (1930a, 355–362) tries to derive a fragment of Euclidean geometry from his phenomenology of natural space.<sup>49</sup> This fragment comprises roughly the theory of incidence and order (axiom groups I and II of Hilbert 1899) as well as the axiom of parallels (the single axiom of Hilbert's group IV). One may thus describe it as a system of ordered affine geometry.<sup>50</sup> Selz (1930a 358–361) defines that a collection  $a$  of points is a line if there is a direction  $d$  such that, for any arbitrarily chosen point  $A$  of  $a$ ,  $a$  comprises exactly  $A$  itself and those points which lie either in direction  $d$  or in the

<sup>48</sup> Note that this center must lack a local tone since it is the point of indifference of any antagonistic series of increase composed of two directions opposite to each other. Selz explicitly says that the two characteristic tones of an antagonistic series simultaneously “drop out” at the point indifference. There is thus no local tone HERE and natural space has a punctual hole at its center.

<sup>49</sup> It should be noted here that Selz, being not trained as a mathematician, probably tried to get expert advice for his geometrical endeavors. One card (1930a, A I 5 3) of his literary estate mentions “6 pages and 2 sheets” of handwritten notes from a conversation with Arthur Rosenthal, then professor at the University of Heidelberg and an expert in the axiomatics of geometry.

<sup>50</sup> Affine geometry may be characterized as that part of Euclidean geometry which can be developed without the use of the notion of congruence.

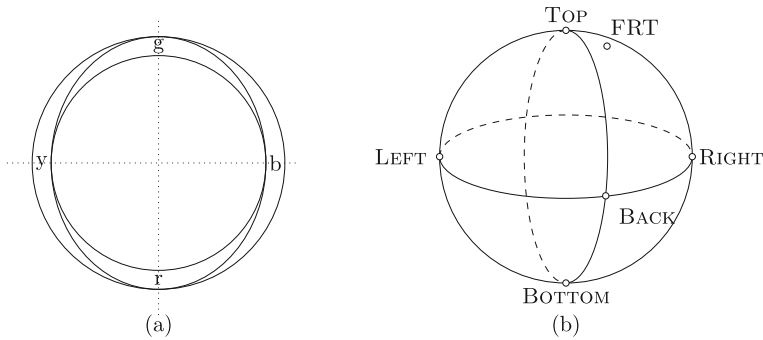


Fig. 1 Hering's circle of chromatic hues

opposite direction  $\bar{d}$  from  $A$ ; cf. Fig. 2a. Each line is thus characterized by a pair of opposite directions. Parallelism is coincidence of direction (Selz 1930a, 358). Planes are defined by Selz by means of the great circles of the sphere of local tones.<sup>51</sup> Selz (1930a, 357) explains that a plane is a collection  $\alpha$  of points for which there is some cyclical order  $\Gamma$  of directions corresponding to the items on a great circle of the sphere of local tones such that, for any arbitrary chosen point  $A$  of  $\alpha$ ,  $\alpha$  comprises exactly  $A$  itself and all those point which lie in some direction from  $\Gamma$  to  $A$ ; cf. Fig. 2. Figure 2b.

Selz's strategy now is to prove from his structural laws of natural space and from his definitions of line and plane that Hilbert's axioms hold true for the thus defined notions. His endeavors are open to criticism both for principled reasons and for reasons relating to the formal mathematical details. As regards objections of the first kind, one may wonder how the derivations envisaged by Selz can be possible at all. Common modern axiomatizations of geometry, when talking about, e.g., incidence, parallelism, or congruence, apply the notion of relation as it is understood in formal logic even when natural language rather than the symbolism of formal logic is used. Above it has been mentioned in passing that Selz conceives of the logician's "apparently non-sensory" conception of relations as having roots in phenomena of increase. But then to formalize the structural laws governing series of increase by means of relations as understood in formal logic seems to involve a circle: one states these laws by means of a conceptual device dependent upon them. Selz (1930a, 333), furthermore, criticizes the psychology of perception of his time for its inability to provide an adequate account of phenomena of increase and he ascribes this inability to its "sticking to a concept of relation residing in logic". This seems to suggest that he considers it to be principally impossible to provide an adequate account of such phenomena within the framework of the logic of relations. But then the questions arise (a) what precisely the logical framework is which underlies Selz's own account of phenomena of increase and (b) how it is possible to derive from statements formulated within that phenomenological more basic framework statements conforming to the standard framework of the logic of relations.

<sup>51</sup> Intuitively, a great circle of a sphere is a circle on it which has maximal circumference. Thus, on the sphere of local tones, the equator leading through FRONT, LEFT, BACK, and RIGHT back to FRONT is a great circle as is, e.g., the path leading from FRONT via TOP, BACK, and BOTTOM back to FRONT.

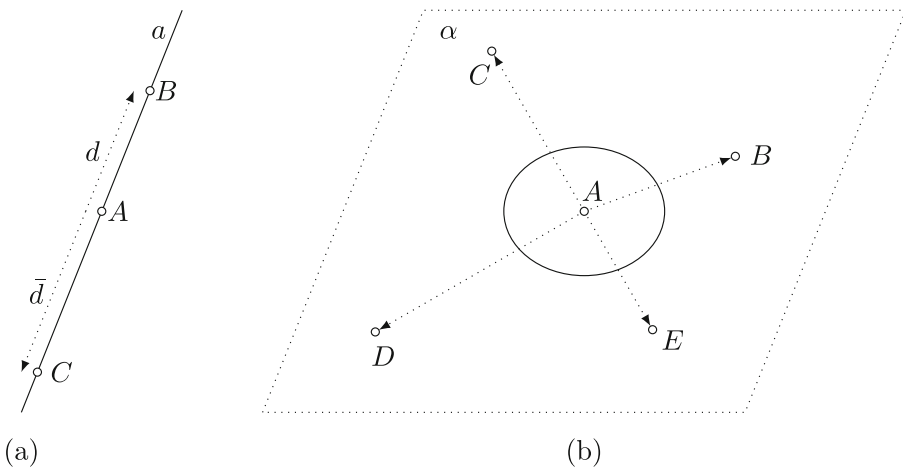


Fig. 2 Selz's definitions of lines and planes

Besides these foundational problems there is the question of the formal correctness of Selz's attempted derivation of affine geometry from his structural laws of natural space. His proofs (Selz 1930a, 358–361) are probably intended to be only samples exemplifying that his project can actually be carried out; in any case, they do not constitute full-fledged formal derivations satisfying all the demands of the mathematician. Though there are gaps in Selz's proofs, it is, on the other hand, not too difficult to see how to fill them to the satisfaction of the mathematician. There are, however, (at least) two problems with such "improvements". The first of them concerns the question whether all improvements required would match Selz's intentions. A critical point at issue here, for instance, is his notion a continuum (Selz 1930a, 353). Phenomenal continua are without gaps ("lückenlos"). But though it seems that Selz means by this more than just "denseness" (i.e., that there is always a third item between two given ones), it remains unclear whether being without gaps (taken in his sense) is the same as being Dedekind complete.<sup>52</sup> The second problem arising from improvements of Selz's demonstrations is whether they introduce principles into his system which lack a phenomenological foundation. Many of the gaps in Selz's proof are due to the fact that his description of the sphere of local tones does not deliver all the facts which are needed as intermediate steps in his demonstrations. An obvious solution here would be to add these missing facts by drawing them from some axiom system of spherical geometry, which describes the arrangement of the great circles and the order of the

<sup>52</sup> A linear order is Dedekind complete if it cannot be split up into two halves such that both the first one lacks a last and the second one a first element. Thus, e.g., each rational number either is smaller or greater than the square root of 2, which itself is "irrational". This square root thus divides the rational numbers into two disjoint subsets one of which has no greatest and the other no smallest element. The smaller-greater-order of the rational numbers is thus not Dedekind complete though it is dense. The question whether the concept of Dedekind completeness makes sense for phenomenal continua is discussed by Brentano (1976, 3–59) in a treatise "Vom Kontinuierlichen" from 1914. It should be noted that Dedekind himself by no means considered it necessary that space ("der Raum") is continuous (in his sense), cf. Dedekind (1872: 11) = (1996: 772) He points out that it would still have many of its usual properties even if it were not continuous. If one identifies space with the collection of all points which can be constructed by straightedge and compass, as Kant – according to Friedman (1996) – does, then that space (as seen from the viewpoint of modern mathematics) contains "holes" and would thus not be continuous; cf. Friedman (1985, 464).

points on them. These relationships exactly match those obtaining in the realm of local tones; cf. Fig. 1b. Though such an amendment would fill some of the gaps in Selz's proofs, it raises the question whether the thus added axioms would really rely on a phenomenological insight into the order of spatial directions or rather upon our geometric intuition of the order of points on a sphere.

## 5 Conclusion

It may be conceded to Selz that a subsystem of Euclidean geometry can be derived from his structural laws describing natural space when (a) these laws are formulated within a conceptual framework consistent with that commonly used for the statement of geometrical axioms and (b) his description of the system of directions in natural space is supplemented in a suitable way; cf. section 4.4.

For Selz the derivation of the Euclidean laws from the structural laws of natural space settles the question of the epistemological status of the Euclidean axioms; cf. section 2. He agrees with both Kant and Stumpf that the Euclidean laws are a priori. However, as Stumpf does, so Selz, too, rejects Kant's doctrine of space as a form of intuition. In Selz's view, Kant set forth this doctrine because he only knew of such empirical laws of psychology as the rules of association and was not familiar with the notion of a phenomenological structural law which only has been developed much later; cf. section 3.3. The a priori character of Euclid's laws is due to their derivability from the structural laws governing "natural space," i.e., the phenomenological system of loci. As analytic principles (cf. p. 7 above) these structural laws are a priori and transmit this property to the Euclidean laws deduced from them. Unlike Stumpf, Selz thus assumes point like loci ("Örter") to be the basic constituents of natural space. That they, though lacking extension, nevertheless can build up an extended continuum becomes understandable as soon as such a continuum is conceived of as a phenomenon of increase; cf. section 4.2. Since natural space is the space of our experience, there is no reason to assume that its structural laws also apply to the space of physics. This is an empirical question for Selz (1985, 362), who thus agrees with Hilbert and Einstein on this issue.

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