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Dynamical perspective of bifurcation analysis and soliton solutions to (1+1)-dimensional nonlinear perturbed Schrödinger model

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Abstract
This work simulates the (1+1)-dimensional nonlinear perturbed Schrödinger model (NLPSM). Hydrodynamics, elastic media, nonlinear optical fiber communication, and plasma physics are just a few of this model's mathematical physics and engineering applications. The study aims to accomplish two main objectives. First, it seeks to find unique soliton solutions such as solitary, dark, periodic, and plane wave solutions that haven't been found in the literature before using the modified Sardar sub-equation approach (MSSEA). Second, a novel approach to analysis called bifurcation analysis is used to investigate the dynamic behavior of the model. Physical compatibility findings are supported by density, 3-D, and 2-D illustrations made with parametric variables. The analysis shows that the approach used to quickly acquire complete and typical answers was successful. This approach works well for solving challenging problems in physics, engineering, mathematics and fiber optic phenomena.

Keywords (1+1)-dimensional nonlinear perturbed Schrödinger model · Modified Sardar sub-equation approach · Soliton solutions · Phase portrait analysis

1 Introduction

The analysis of nonlinear events that arise in a range of models across many domains has made nonlinear partial differential equations (NLPDEs) an indispensable tool. In a wide range of scientific domains, such as dynamics, physics, geochemistry, fluid mechanics, geophysics, plasma physics, optical fibers, and many more, the NLPDEs are crucial in characterizing the physical behavior of actual objects and dynamical processes. In today's cutting-edge scientific period, nonlinear phenomena are one of the most exciting subjects for analysts, because of their significant role in the knowledge of the actual aspects of

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the systems, finding the exact or analytical solutions has been a research area of interest Rehman et al. (2021, 2022). It has become increasingly vital for academics to find exact answers to challenging algebraic computations using efficient computing methods. Wave theory in mathematical physics has a special place for determining the precise answers Xu and Pruess (2001). These NLPDEs solutions provide improved support for the physical structures. To acquire the precise solution for nonlinear physical models, several robust and efficient techniques Rehman et al. (2023); Zulfiqar et al. (2022), were established, and these techniques are the Hirota bilinear technique Li et al. (2023), the F-expansion technique Sumantha and Suresha (2023), the sinh-Gordon function technique Wang (2023), the Darboux transformation technique Almusawa et al. (2021), new ϕ^6-model expansion approach Shahzad et al. (2023), the modified extended tanh-function technique Ghanbari and Kuo (2019), Boulaaras et al. (2023), the sin-cosine technique Fahad et al. (2023) and several others. More recently, several approaches have also been considered for various lump solutions Ghanbari (2019), Rehman et al. (2023). The modified Sardar sub-equation approach Ghanbari and Gómez-Aguilar (2019); Rehman et al. (2022) is a relatively new mathematical method, that has been applied to a range of NLPDEs. Below is a summary of some of the most significant works that have used this approach: Using the modified Sardar sub-equation technique, wave solutions for the doubly dispersive equation are found by Rashida et al. Rasool et al. (2023), exploration of novel solitons in photonic media with more complex dispersive and nonlinear properties. This work investigates the innovative soliton solution and dynamical phase portrait analysis of (1+1)-dimensional NLPSM using the modified Sardar sub-equation approach and bifurcation analysis Rehman et al. (2022, 2023), respectively. The authors in Maghsoudi-Khouzani and Kurt (2024), discuss the numerical techniques. The authors in Kurt and Bektas (2023); Yalçnkaya et al. (2022); Özkan and Ali (2022); Durur et al. (2020); Tasbozan et al. (2019), study the analytical and numerical techniques and employed these techniques to NLPDEs to acquire multiple types of soliton solutions. Extensive research on the NLPSM has yielded important theoretical and experimental insights into the behavior of nonlinear waves in a variety of physical systems. Among other applications, the NLPSM has been used to study the behavior of plasma waves in fusion reactors and the dynamics of optical pulses in fiber optic communications. Investigations have also been conducted on soliton in the NLPSM, which has important applications in optical fiber communications and other fields. In a nonlocal NLPSM, the dynamics of dark solitons were investigated by Ozisik (2022); Turbiner (2016). They found that it would be possible to use the dark solitons to create long-lasting oscillations in the wave field, which might be useful for information processing and storage.

The main objective of the article is to find the novel soliton solutions of the given below (1+1)-dimensional nonlinear perturbed Schrödinger model Tariq et al. (2022), by using a modified Sardar sub-equation approach.

$$\psi_4 \frac{\partial^2 \mathcal{B}}{\partial x\, \partial x} + i\frac{\partial \mathcal{B}}{\partial t} - i\left(\psi_3 \frac{\partial \mathcal{B}}{\partial x} - \psi_2\left(\mathcal{B}\frac{\partial |\mathcal{B}|^2}{\partial x}\right) - \psi_1 \mathcal{B}\frac{\partial |\mathcal{B}|^2}{\partial x}\right) + \psi \mathcal{B}|\mathcal{B}|^2 = 0, \tag{1}$$

where i represents the imaginary unit, $\mathcal{B}$ represents the amplitude envelope, (x) is spatial terms and t is temporal term, ψ, ψ_1, ψ_2, ψ_3 and ψ_4 are real constant. In many physical systems, such as optics, plasma physics, Bose-Einstein condensates, and water waves, the dynamics of nonlinear waves are represented by the (1+1)-dimensional NLPSM. The (1+1)-dimensional NLPSM is used to describe nonlinear wave propagation among other scientific phenomena. A few particular uses are as follows. The model is applied to the study of light behavior in planar waveguides and nonlinear optical fibers, where nonlinear effects are important. It behaves nonlinearly in the domain of ultracold atomic gases, represented by NLPSM, including perturbed versions such as the NLPSM. The model aids in the comprehension of soliton dynamics, in which solitons are self-reinforcing lone waves that continue to propagate at the same speed and form. It can be used to examine how wave functions behave under perturbations in a single spatial dimension in quantum systems. Based on the observation that many physical systems exhibit nonlinear behavior, which results in complex and frequently unanticipated events emerging from interactions among system components, Eq. (1) is included. This paper, supported by earlier studies, seeks to provide common, practical, and widely compatible solutions for the model, advancing a better comprehension of the fundamental ideas. The paper's following sections are arranged as follows: Presented is the mathematical analysis in Sect. 2. In Sect. 3, the model described in Eq. (1) is applied, and figures and solutions are produced using the MSSEA approach. Section 4 provides a full explanation of the bifurcation analysis, and Sec. 5 includes the results and comments. A summary of the study's conclusions and next steps is provided in Sect. 6.

2 General description of modified Sardar sub-equation approach

This approach has been successfully used for the solution of NLPDEs in mathematics and science several times. Let's assume the general form of NLPDEs:

$$\mathcal{K}(\mathcal{B},\ \mathcal{B}_x,\ \mathcal{B}_t,\ \mathcal{B}_{xx},\ \mathcal{B}_{xt}, ...) = 0. \tag{2}$$

Step-1. Apply the wave transformation

$$\mathcal{B} = \mathcal{R}(\xi)e^{i\phi}, \qquad \xi = x - \gamma t \qquad \phi = \mu_2 t - \mu_1 x, \tag{3}$$

into Eq. (2), the nonlinear ordinary differential equations (NLODEs) is obtained as,

$$\mathcal{K}(\mathcal{R},\ \mathcal{R}',\ \mathcal{R}'', ...) = 0. \tag{4}$$

Step 2. The given assertion, using the given technique, clarifies the general solution for Eq. (4).

$$\mathcal{R}(\xi) = \ell_0 + \sum_{i=1}^{J} \ell_i U^i(\xi), \qquad \ell_i \neq\ 0, \tag{5}$$

where $\mathcal{R} = \mathcal{R}(\xi)$, and ℓ_i is constant which assures

$$U(\xi)' = \sqrt{\delta_0 + \delta_2 U(\xi)^4 + \delta_1 U(\xi)^2}, \tag{6}$$

where $\delta_0 \neq 1$, δ_1 and $\delta_2 \neq 0$ are integers. Calculating the constants ℓ_0 and ℓ_1 and additionally, it is invertible for ℓ_i to be zero. Determined the value of J using the balance principle. Following are the Clusters to Eq. (6) satisfying the Eq. (6) with q is integration constant. **Cluster-1:** If $\delta_0 = 0,\ \delta_1 > 0$ and $\delta_2\ \neq 0$, we get

$$U_1(\xi) = \sqrt{-\frac{\delta_1}{\delta_2}}\text{sech}\left(\sqrt{\delta_1}(\xi + q)\right), \tag{7}$$

$$U_2(\xi) = \sqrt{\frac{\delta_1}{\delta_2}}\operatorname{csch}\left(\sqrt{\delta_1}(\xi + q)\right). \tag{8}$$

Cluster-2:

For constants k_1 and k_2, If $\delta_0 = 0,\ \delta_1 > 0$ and $\delta_2 = +4k_1k_2$, we get

$$U_3(\xi) = \frac{4k_1\sqrt{\delta_1}}{\left(4k_1^2 - \delta_2\right)\sinh\left(\sqrt{\delta_1}(\xi + q)\right) + \left(4k_1^2 - \delta_2\right)\cosh\left(\sqrt{\delta_1}(\xi + q)\right)}. \tag{9}$$

Cluster-3:

For constants E_1 and E_2, If $\delta_0 = \frac{\delta_1^2}{4\delta_2}, \delta_1 < 0$ and $\delta_2 > 0$, we get

$$U_4(\xi) = \sqrt{-\frac{\delta_1}{2\delta_2}}\tanh\left(\sqrt{-\frac{\delta_1}{2}}(\xi + q)\right), \tag{10}$$

$$U_5(\xi) = \sqrt{-\frac{\delta_1}{2\delta_2}}\coth\left(\sqrt{-\frac{\delta_1}{2}}(\xi + q)\right), \tag{11}$$

$$U_6(\xi) = \sqrt{-\frac{\delta_1}{2\delta_2}}\left(\tanh\left(\sqrt{-\frac{\delta_1}{2}}(\xi + q)\right) + i\operatorname{sech}\left(\sqrt{-2\delta_1}(\xi + q)\right)\right), \tag{12}$$

$$U_7(\xi) = \sqrt{-\frac{\delta_1}{8\delta_2}}\left(\tanh\left(\sqrt{-\frac{\delta_1}{8}}(\xi + q)\right) + \coth\left(\sqrt{-\frac{\delta_1}{8}}(\xi + q)\right)\right), \tag{13}$$

$$U_8(\xi) = \frac{\sqrt{-\frac{\delta_1}{2\delta_2}}\left(\sqrt{E_1^2 + E_2^2} - E_1\cosh\left(\sqrt{-2\delta_1}(\xi + q)\right)\right)}{E_1\sinh\left(\sqrt{-2\delta_1}(\xi + q)\right) + E_2}, \tag{14}$$

$$U_9(\xi) = \frac{\sqrt{-\frac{\delta_1}{2\delta_2}}\cosh\left(\sqrt{-2\delta_1}(\xi + q)\right)}{\sinh\left(\sqrt{-2\delta_1}(\xi + q)\right) + i}. \tag{15}$$

Cluster-4:

If $\delta_0 = 0,\ \delta_1 < 0$ and $\delta_2 \neq 0$, we get

$$U_{10}(\xi) = \sqrt{-\frac{\delta_1}{\delta_2}}\sec\left(\sqrt{-\delta_1}(\xi + q)\right), \tag{16}$$

$$U_{11}(\xi) = \sqrt{-\frac{\delta_1}{\delta_2}} \csc\left(\sqrt{-\delta_1}(\xi + q)\right). \tag{17}$$

Cluster-5:

If $\delta_0 = \frac{\delta_1^2}{4\delta_2}$, $\delta_1 > 0$ and $\delta_2 > 0$ and $E_1^2 - E_2^2 > 0$, we get

$$U_{12}(\xi) = \sqrt{\frac{\delta_1}{2\delta_2}} \tan\left(\sqrt{\frac{\delta_1}{2}}(\xi + q)\right), \tag{18}$$

$$U_{13}(\xi) = -\sqrt{\frac{\delta_1}{2\delta_2}} \cot\left(\sqrt{\frac{\delta_1}{2}}(\xi + q)\right), \tag{19}$$

$$U_{14}(\xi) = -\sqrt{\frac{\delta_1}{2\delta_2}}\left(\tan\left(\sqrt{2\delta_1}(\xi + q)\right) - \sec\left(\sqrt{2\delta_1}(\xi + q)\right)\right), \tag{20}$$

$$U_{15}(\xi) = \sqrt{\frac{\delta_1}{8\delta_2}}\left(\tan\left(\sqrt{\frac{\delta_1}{8}}(\xi + q)\right) - \cot\left(\sqrt{\frac{\delta_1}{8}}(\xi + q)\right)\right), \tag{21}$$

$$U_{16}(\xi) = \frac{\sqrt{\frac{\delta_1}{2\delta_2}}\left(\sqrt{E_1^2 - E_2^2} - E_1 \cos\left(\sqrt{2\delta_1}(\xi + q)\right)\right)}{E_2 + E_1 \sin\left(\sqrt{2\delta_1}(\xi + q)\right)}, \tag{22}$$

$$U_{17}(\xi) = \frac{\sqrt{\frac{\delta_1}{2\delta_2}} \cos\left(\sqrt{2\delta_1}(\xi + q)\right)}{\sin\left(\sqrt{2\delta_1}(\xi + q)\right) - 1}. \tag{23}$$

Cluster-6:

If $\delta_0 = 0$, $\delta_1 > 0$, we get

$$U_{18}(\xi) = \frac{4\delta_1 e^{\sqrt{\delta_1}(\xi+q)}}{e^{2\sqrt{\delta_1}(\xi+q)} - 4\delta_1\delta_2}, \tag{24}$$

$$U_{19}(\xi) = \frac{4\delta_1 e^{\sqrt{\delta_1}(\xi+q)}}{1 - 4\delta_1\delta_2 e^{2\sqrt{\delta_1}(\xi+q)}}. \tag{25}$$

Cluster-7:

If $\delta_0 = 0$, $\delta_1 = 0$ and $\delta_2 > 0$, we get

$$U_{20}(\xi) = \frac{1}{\sqrt{\delta_2}(\xi + q)}, \tag{26}$$

$$U_{21}(\xi) = \frac{i}{\sqrt{\delta_2}(\xi + q)}. \tag{27}$$

Step 3. A polynomial written in terms of the power of $U(\xi)$ is obtained by combining Eq. (5) with Eq. (1), using Eq. (6), and the second-order derivatives needed for Eq. (4).

Step 4. Using the same powers, collect all of the $U(\xi)$ coefficients, then set them all to zero. After this procedure, the algebraic system for $\ell_0,\ \ell_n \quad (n = 1, 2, 3, ...)$ is obtained.

Step 5. Lastly, use Wolfram Mathematica to solve the algebraic equation systems and get the values of the parameters. The solutions for Eq. (1) are obtained by substituting these values into Eq. (4). Accurate solutions to NLPDEs, such as the (1+1)-dimensional NLPSM, may be obtained with efficiency using the MSSEA.

3 Implementation

For an exact solution, take into consideration using Eq. (1). The corresponding NLODEs is obtained by replacing an wave transformation Eq. (3) with Eq. (1).

$$\mathcal{R}^3(\xi)\big(\psi - \mu_1\psi_2\big) + \psi_4\mathcal{R}''(\xi) - \mathcal{R}(\xi)\left(\mu_1^2\psi_4 + \mu_2\psi_1 + \mu_1\right) = 0, \tag{28}$$

where $\mathcal{R}(\xi)$ is complex valued function, $\psi,\ \psi_1,\ \psi_2,\ \psi_3,\ \psi_3,\ \psi_4,\ \mu_1$ are all constants and μ_2 is wave speed. Using Eq. (5) and the homogeneous balance principle, we determine that $J = 1$ by finding a balance between the terms $\mathcal{R}^3(\xi)$ and $\mathcal{R}''(\xi)$.

$$\mathcal{R}(\xi) = \ell_1 U(\xi) + \ell_0. \tag{29}$$

Calculate the derivative Eq. (9). After doubling over and taking into account Eq. (6), we obtain

$$\mathcal{R}''(\xi) = \ell_1 U(\xi)\big(\delta_1 + 2\delta_2 U(\xi)^2\big). \tag{30}$$

A polynomial to the power of $U(\xi)$ may be obtained by inserting Eqs. (9) and (10) into Eq. (7). After gathering all the coefficients with identical powers of the $U(\xi)$, set all of the coefficients to zero. The resulting algebraic equation system is as follows for $\ell_0,\ \ell\delta_1,\ \ell\delta_1$ and μ_1.

$$\begin{aligned} -\mu_1\psi_2\ell_0^3 - \mu_1\psi_1\ell_0 - \mu_1^2\psi_4\ell_0 - \mu_2\ell_0 + \psi\ell_0^3 &= 0 \\ +\big(\delta_1\psi_4\ell_1 - 3\mu_1\psi_2\ell_1\ell_0^2 - \mu_1\psi_1\ell_1 - \mu_1^2\psi_4\ell_1 - \mu_2\ell_1 + 3\psi\ell_1\ell_0^2\big) &= 0 \\ +\big(3\psi\ell_0\ell_1^2 - 3\mu_1\psi_2\ell_0\ell_1^2\big) &= 0 \\ +\big(2\delta_2\psi_4\ell_1 - \mu_1\psi_2\ell_1^3 + \psi\ell_1^3\big) &= 0. \end{aligned} \tag{31}$$

Following the algebraic system of equations solution, we determined that family 1 is

$$\left\{\delta_0 \to 0, \quad \delta_1 \to \frac{\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}}{\sqrt{\psi_2^2\left(\mu_2-\delta_1\psi_4\right)+\psi_4\psi^2+\psi_1\psi_2\psi}}, \right. \tag{32}$$
$$\left. \mu_1 \to \frac{\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}-\psi_1}{2\psi_4}\right\}.$$

Use the above family-1 and find the following novel soliton solutions of Eq. (1).

$$\mathcal{B}_{1,1}(x,\ t) = \frac{\sqrt{\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}\operatorname{sech}\left(\sqrt{\delta_1}(q-\gamma t+x)\right)}{\sqrt{\psi_2^2\left(\mu_2-\delta_1\psi_4\right)+\psi_4\psi^2+\psi_1\psi_2\psi}}$$
$$\times \exp\left(i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right), \tag{33}$$

$$\mathcal{B}_{1,2}(x,\ t) = \frac{\sqrt{\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}\operatorname{csch}\left(\sqrt{\delta_1}(q-\gamma t+x)\right)}{\sqrt{\psi_2^2\left(\mu_2-\delta_1\psi_4\right)+\psi_4\psi^2+\psi_1\psi_2\psi}}$$
$$\times \exp\left(i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right), \tag{34}$$

$$\mathcal{B}_{1,3}(x,\ t) = \frac{4\sqrt{\delta_1}k_1\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}}{\sqrt{\psi_2^2\left(\mu_2-\delta_1\psi_4\right)+\psi_4\psi^2+\psi_1\psi_2\psi}\left(\left(4k_1^2-\delta_2\right)\sinh\left(\sqrt{\delta_1}(q-\gamma t+x)\right)\right)}$$
$$\times \frac{\exp\left(i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)}{+\left(4k_1^2-\delta_2\right)\cosh\left(\sqrt{\delta_1}(q-\gamma t+x)\right)}, \tag{35}$$

$$\mathcal{B}_{1,4}(x,\ t)=\frac{\sqrt{-\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}\tanh\left(\frac{\sqrt{-\delta_1}(q-\gamma t+x)}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{\psi_2^2\left(\mu_2-\delta_1\psi_4\right)+\psi_4\psi^2+\psi_1\psi_2\psi}}\times\exp\left(i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right),\tag{36}$$

$$\mathcal{B}_{1,5}(x,\ t)=\frac{\sqrt{-\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}\coth\left(\frac{\sqrt{-\delta_1}(q-\gamma t+x)}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{\psi_2^2\left(\mu_2-\delta_1\psi_4\right)+\psi_4\psi^2+\psi_1\psi_2\psi}}\times\exp\left(i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right),\tag{37}$$

$$\mathcal{B}_{1,6}(x,\ t)=\frac{\sqrt{-\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}\left(\left(\tanh\left(\sqrt{2}\sqrt{-\delta_1}(q-\gamma t+x)\right)\right)\right.}{\sqrt{2}\sqrt{\psi_2^2\left(\mu_2-\delta_1\psi_4\right)+\psi_4\psi^2+\psi_1\psi_2\psi}}\left.+i\operatorname{sech}\left(\sqrt{2}\sqrt{-\delta_1}(q-\gamma t+x)\right)\right)\times\exp\left(i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right),\tag{38}$$

$$\mathcal{B}_{1,7}(x,\ t)=\frac{\exp\left(i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\left(\tanh\left(\frac{\sqrt{-\delta_1}(q-\gamma t+x)}{2\sqrt{2}}\right)+i\coth\left(\frac{\sqrt{-\delta_1}(q-\gamma t+x)}{2\sqrt{2}}\right)\right)}{2\sqrt{2}\sqrt{\psi_2^2\left(\mu_2-\delta_1\psi_4\right)+\psi_4\psi^2+\psi_1\psi_2\psi}}\times\sqrt{-\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)},\tag{39}$$

$$\mathcal{B}_{1,8}(x,\ t)=\frac{\exp\left(i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\left(\sqrt{E_1^2+E_2^2}-E_1\cosh\left(\sqrt{2}\sqrt{-\delta_1}(q-\gamma t+x)\right)\right)}{\sqrt{2}\sqrt{\psi_2^2\left(\mu_2-\delta_1\psi_4\right)+\psi_4\psi^2+\psi_1\psi_2\psi}\left(E_1\sinh\left(\sqrt{2}\sqrt{-\delta_1}(q-\gamma t+x)\right)+E_2\right)}\times\sqrt{-\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4\left(\delta_1\psi_4-\mu_2\right)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)},\tag{40}$$

$$\mathcal{B}_{1,9}(x,\ t) = \frac{\exp\left(i\left(\mu_2 t - \frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\cosh\left(\sqrt{2}\sqrt{-\delta_1}(q-\gamma t+x)\right)}{\sqrt{2}\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\psi^2+\psi_1\psi_2\psi}\left(\sinh\left(\sqrt{2}\sqrt{-\delta_1}(q-\gamma t+x)\right)+i\right)} \times \sqrt{-\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}, \tag{41}$$

$$\mathcal{B}_{1,10}(x,\ t) = \frac{\exp\left(i\left(\mu_2 t - \frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\sec\left(\sqrt{-\delta_1}(q-\gamma t+x)\right)}{\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\psi^2+\psi_1\psi_2\psi}} \times \sqrt{-\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}, \tag{42}$$

$$\mathcal{B}_{1,11}(x,\ t) = \frac{\exp\left(i\left(\mu_2 t - \frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\csc\left(\sqrt{-\delta_1}(q-\gamma t+x)\right)}{\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\psi^2+\psi_1\psi_2\psi}} \times \sqrt{-\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}, \tag{43}$$

$$\mathcal{B}_{1,12}(x,\ t) = \frac{\exp\left(i\left(\mu_2 t - \frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\tan\left(\frac{\sqrt{\delta_1}(q-\gamma t+x)}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\psi^2+\psi_1\psi_2\psi}} \times \sqrt{\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}, \tag{44}$$

$$\mathcal{B}_{1,13}(x,\ t) = \frac{\exp\left(i\left(\mu_2 t - \frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\cot\left(\frac{\sqrt{\delta_1}(q-\gamma t+x)}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\psi^2+\psi_1\psi_2\psi}} \times \sqrt{\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\psi\psi_4\right)}, \tag{45}$$

$$\mathcal{B}_{1,14}(x,\ t) = -\frac{\exp\left(i\left(\mu_2 t - \frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\left(\tan\left(\sqrt{2}\sqrt{\delta_1}(q-\gamma t+x)\right)\right)}{\sqrt{2}\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\Psi^2+\psi_1\psi_2\Psi}} - \sec\left(\sqrt{2}\sqrt{\delta_1}(q-\gamma t+x)\right) \times\sqrt{\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\Psi\psi_4\right)}, \tag{46}$$

$$\mathcal{B}_{1,15}(x,\ t) = \frac{\exp\left(i\left(\mu_2 t - \frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\left(\tan\left(\frac{\sqrt{\delta_1}(q-\gamma t+x)}{2\sqrt{2}}\right)\right)}{2\sqrt{2}\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\Psi^2+\psi_1\psi_2\Psi}} - \cot\left(\frac{\sqrt{\delta_1}(q-\gamma t+x)}{2\sqrt{2}}\right) \times\sqrt{\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\Psi\psi_4\right)}, \tag{47}$$

$$\mathcal{B}_{1,16}(x,\ t) = \frac{\exp\left(i\left(\mu_2 t - \frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\left(\sqrt{E_1^2-E_2^2}-E_1\cos\left(\sqrt{2}\sqrt{\delta_1}(q-\gamma t+x)\right)\right)}{\sqrt{2}\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\Psi^2+\psi_1\psi_2\Psi}\left(E_1\sin\left(\sqrt{2}\sqrt{\delta_1}(q-\gamma t+x)\right)+E_2\right)} \times\sqrt{\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\Psi\psi_4\right)}, \tag{48}$$

$$\mathcal{B}_{1,17}(x,\ t) = \frac{\exp\left(i\left(\mu_2 t - \frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)\cot\left(\sqrt{2}\sqrt{\delta_1}(q-\gamma t+x)\right)}{\sqrt{2}\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\Psi^2+\psi_1\psi_2\Psi}} \times\sqrt{\frac{\delta_1}{\delta_2}}\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\Psi\psi_4\right)}, \tag{49}$$

$$\mathcal{B}_{1,18}(x,\ t)=\frac{\exp\left(\sqrt{\delta_1}(q-\gamma t+x)+i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)}{\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\Psi^2+\psi_1\psi_2\Psi}\left(e^{2\sqrt{\delta_1}(q-\gamma t+x)}-4\delta_1\delta_2\right)} \times 4\delta_1\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\Psi\psi_4\right)}, \tag{50}$$

$$\mathcal{B}_{1,19}(x,\ t)=\frac{\exp\left(\sqrt{\delta_1}(q-\gamma t+x)+i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right)}{\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\Psi^2+\psi_1\psi_2\Psi}\left(1-4\delta_1\delta_2 e^{2\sqrt{\delta_1}(q-\gamma t+x)}\right)} \times 4\delta_1\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\Psi\psi_4\right)}, \tag{51}$$

$$\mathcal{B}_{1,20}(x,\ t)=\frac{\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\Psi\psi_4\right)}}{\sqrt{\delta_2}\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\Psi^2+\psi_1\psi_2\Psi}(q-\gamma t+x)} \times\exp\left(i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right), \tag{52}$$

$$\mathcal{B}_{1,21}(x,\ t)=\frac{i\sqrt{-\delta_2\psi_4\left(\psi_2\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}+\psi_1\right)+2\Psi\psi_4\right)}}{\sqrt{-\delta_2}\sqrt{\psi_2^2(\mu_2-\delta_1\psi_4)+\psi_4\Psi^2+\psi_1\psi_2\Psi}(q-\gamma t+x)} \times\exp\left(i\left(\mu_2 t-\frac{x\left(\sqrt{4\psi_4(\delta_1\psi_4-\mu_2)+\psi_1^2}-\psi_1\right)}{2\psi_4}\right)\right). \tag{53}$$

4 Bifurcation analysis

We shall now analyze the phase portraits of bifurcation analysis using Eq. (28). Equation (28) yields the first-order differential equations, from which the dynamical system is obtained as below:

$$\begin{cases} \frac{d\mathcal{R}}{d\xi} = W = C_1, \\ \frac{dW}{d\xi} = H_1\mathcal{R} - H_2\mathcal{R}^3 = C_2, \end{cases} \tag{54}$$

where $H_1 = \frac{\psi_4\mu_1^2+\mu_1\psi_1+\mu_2}{\psi_4}$ and $H_2 = \frac{\psi-\psi_2\mu_1}{\psi_4}$.

Case (i): $H_1 < 0$ **and** $H_2 < 0$. By using $\psi_4 = 2, \psi_1 = -0.7$, $\psi_2 = 0.7$ and $\mu_1 = 0.7$, the equilibrium points $q_1 = (0,0)$, $q_2 = (\sqrt{2}, 0)$, $q_3 = (-\sqrt{2}, 0)$ have been retrieved from system (54) shown in Fig. 7. It is observed, by the phase portrait, that H_2 shows center points, whereas H_1 is a cuspidal point.

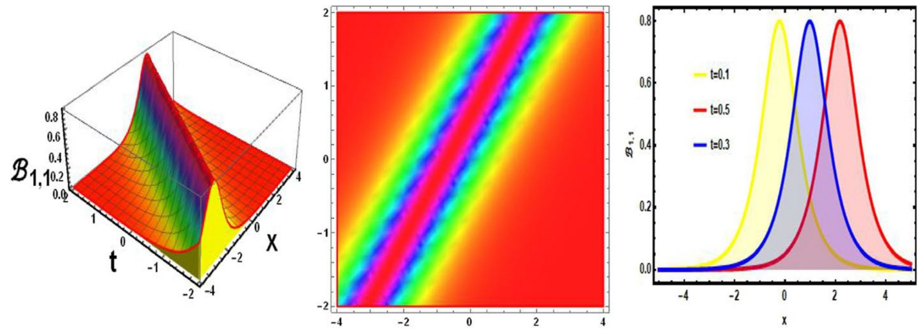
Case (ii): $H_1 > 0$ **and** $H_2 < 0$. By using $\psi_4 = 1.4$, $\psi_1 = 0.7, \psi_2 = 0.7$ and $\mu_1 = 0.7$, there exists only one real equilibrium point $H_1 = (0,0)$, which has been obtained from system (54) and shown in Fig. 7. It is observed that H_1 is a cuspidal point.

Case (iii): $H_1 > 0$ **and** $H_2 > 0$. By using $\psi_4 = 1.2$, $\psi_2 = -0.7$, $\psi_1 = -0.7$ and $\mu_1 = 0.7$, there exists only one real equilibrium point $H_1 = (0,0)$, which has been obtained from system (54) and shown in Fig. 7. It is observed that H_1 is a saddle point.

Case (iv): $H_1 < 0$ **and** $H_2 > 0$. By using $\psi_4 = 1.2$, $\psi_2 = -0.7$, $\psi_1 = -0.7$ and $\mu_1 = 0.7$, there exists only one complex equilibrium point $H_1 = (0,0)$, which has been obtained from system (54) and shown in Fig. 7. It is observed that H_1 is a center point.

5 Results and discussions

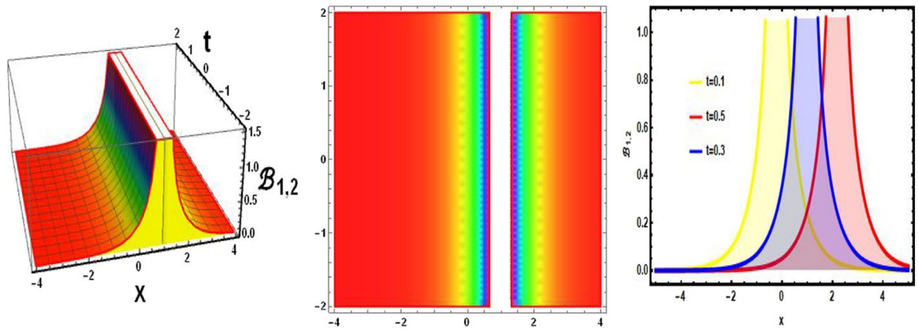
The thorough comparison of the obtained results with the previously appraised result in this section demonstrates the originality of this work. The governing model's solutions were obtained by utilizing GREFT. This study was modified by our research to examine other techniques. This part's unique contribution to the research is demonstrated by a detailed contrast of the assessed findings with the previously calculated results. The authors in Tariq et al. (2022), study the (1+1)-dimensional NLPSM, by using different analytical techniques to find exact solutions. Our inquiry synthesized this research to apply more effective techniques. The modified Sardar sub-equation approach and bifurcation analysis to a wide range of fields in mathematics, physics, and engineering. It may provide solutions with physical interpretations and analytic properties. This makes it a helpful tool for understanding the behavior of complex nonlinear systems as well as for drawing new theoretical conclusions and experimental predictions. Comparing our achievements to their findings reveals the uniqueness of our calculated results, which have never been obtained in published literature previously. The evaluation's findings are fresh and original, and they have the potential to greatly further the field of applied mathematics research. We just wanted to let you know that the obtained solutions may be of tremendous use for future studies of higher-order NLPDEs. This research analyses solutions of the rational type, periodic solutions, dark solutions, and singular solitons. In mathematics, a non-singular solution is one in which there are no singularities anywhere in its domain. This is important because mathematical equations may break down due to singularities. Contour plots represent a 3-dimensional surface by plotting constant z slices, known as contours, on a 2-dimensional plane. These plots display relationships between two independent variables (X and Y) and an outcome (Z), with each contour line representing points of equal value to the outcome variable Z. In contrast, density plots show the distribution of data in a 2-dimensional space, often representing



(a) 3D (b) Density plot (c) 2D

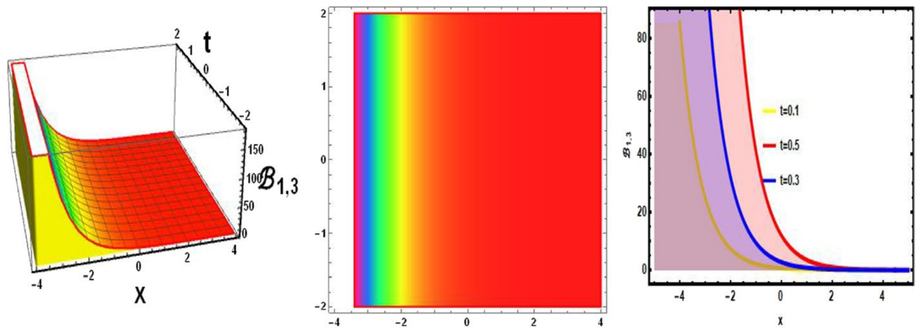
Fig. 1 Graphical illustration of bright soliton solution of Eq. (33), by using appropriate parameter values are $\delta_1 = 0.7,\ \delta_2 = 0.8,\ \delta_0 = 0,\ \psi_1 = 0.1,\ \psi = 0.9,\ \psi_2 = 0.3,\ \psi_3 = 0.6,\ \psi_4 = 0.5,\ q = 0.1,\ \mu_1 = 0.54$ and $\mu_2 = 0.65$

the probability density function of the data. While contour plots focus on displaying contours of equal values, density plots emphasize the distribution of data points and their relative concentrations. Contour plots are particularly useful when drawing in three dimensions is inconvenient, providing a clear visualization of relationships between variables in a 2-dimensional format. In mathematical physics, NLPSM is used to model and comprehend complicated processes like soliton dynamics. It is essential for simulating the behavior of stable, localized wave packets known as optical solitons. For fiber-optic communication systems to preserve signal integrity over extended distances, these solitons are crucial. NLPSM plays a role in improving our comprehension of the dynamics of nonlinear wave propagation phenomena in a variety of physical systems, including nonlinear optical fibers and Bose-Einstein condensates. These phenomena include solitary waves and soliton solutions. Applications of the model can be found in quantum physics, namely in the investigation of the behavior of quantum systems under nonlinear interactions and disturbances. On the other hand, density plots are valuable for understanding the density of data points across different regions of the plot. Non-singular solutions are favored since they are precisely specified and allow for accurate computations and forecasts. Non-singular solutions are often associated with wave equations because, in physics, differential equations with non-singular solutions may be used to explain waves. Exact solutions to nonlinear equations can be efficiently found using the modified Sardar-sub equation approach. It offers answers to many different kinds of nonlinear equations, including ones with different nonlinearities. The modified Sardar-sub equation strategy performs competitively when compared to other recent methods, such the Sardar sub-equation approach. The modified Sardar-sub equation method is effective, although it may need intricate mathematical derivations. The Schrödinger equation's non-singular solution can be utilized to simulate the behavior of particles or wave packets in quantum physics. The (1+1)-dimensional NLPSM solutions found by the MSSEA approach may find use in optical fiber communication and plasma physics, among other areas. The paper also draws attention to several problems and unanswered issues in the area, such as the impact of higher-order nonlinear factors, stability analysis, and generalization to different equations. The dynamics of several reported solutions are shown in Figs by selecting the proper parametric values Figs. 1, 2, 3, 4, 5 and 6 as 3-D, 2-D and density plots.



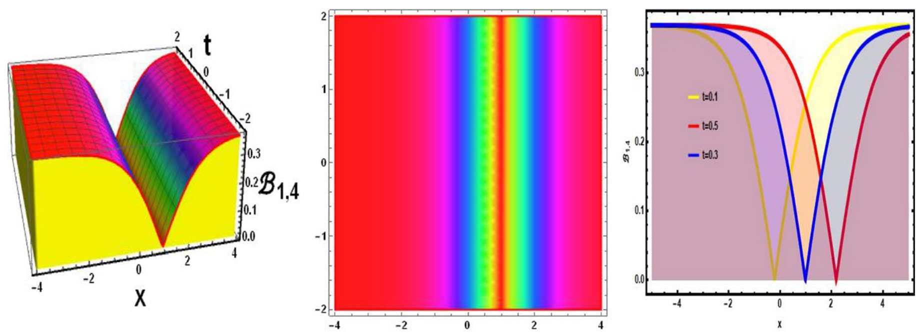
(a) 3D (b) Density plot (c) 2D

Fig. 2 Graphical illustration of singular soliton solution of Eq. (34), by using appropriate parameter values are $\delta_1 = 0.7,\ \delta_2 = 1,\ \delta_0 = 0,\ \psi_1 = 0.1,\ \psi = 0.9,\ \psi_2 = 0.3,\ \psi_3 = 0.6,\ \psi_4 = 0.5,\ q = 0.1,\ \mu_1 = 0.54$ and $\mu_2 = 0.65$



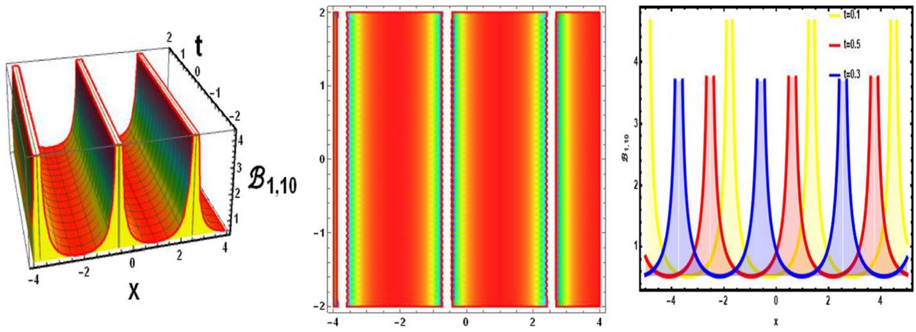
(a) Density plot (b) Density plot (c) 2D

Fig. 3 Graphical illustration of hyperbolic soliton solution of Eq. (35), by using appropriate parameter values are $\delta_1 = 0.7,\ \delta_2 = 1,\ \delta_0 = 0,\ \psi_1 = 0.1,\ \psi = 0.9,\ \psi_2 = 0.3,\ \psi_3 = 0.6,\ \psi_4 = 0.5,\ q = 0.1,\ \mu_1 = 0.54$ and $\mu_2 = 0.65$



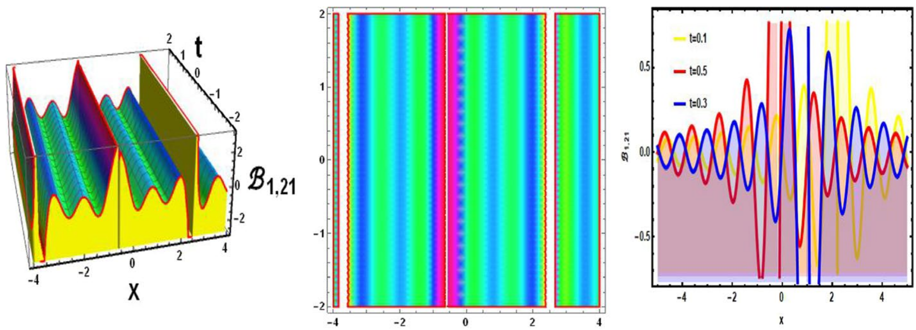
(a) 3D (b) Density plot (c) 2D

Fig. 4 Graphical illustration of dark soliton solution of Eq. (36), by using appropriate parameter values are $\delta_1 = -0.7,\ \delta_2 = 0.8,\ \delta_0 = 0.3,\ \psi_1 = 0.1,\ \psi = 0.9,\ \psi_2 = 0.3,\ \psi_3 = 0.6,\ \psi_4 = 0.5,\ q = 0.1,\ \mu_1 = 0.54$ and $\mu_2 = 0.65$



(a) 3D (b) Density plot (c) 2D

Fig. 5 Graphical illustration of periodic solution of Eq. (42), by using appropriate parameter values are $\delta_1 = 0.7,\ \delta_2 = 0.8,\ \delta_0 = 0,\ \psi_1 = 0.1,\ \psi = 0.9,\ \psi_2 = 0.3,\ \psi_3 = 0.6,\ \psi_4 = 0.5,\ q = 0.1,\ \mu_1 = 0.54$ and $\mu_2 = 0.65$

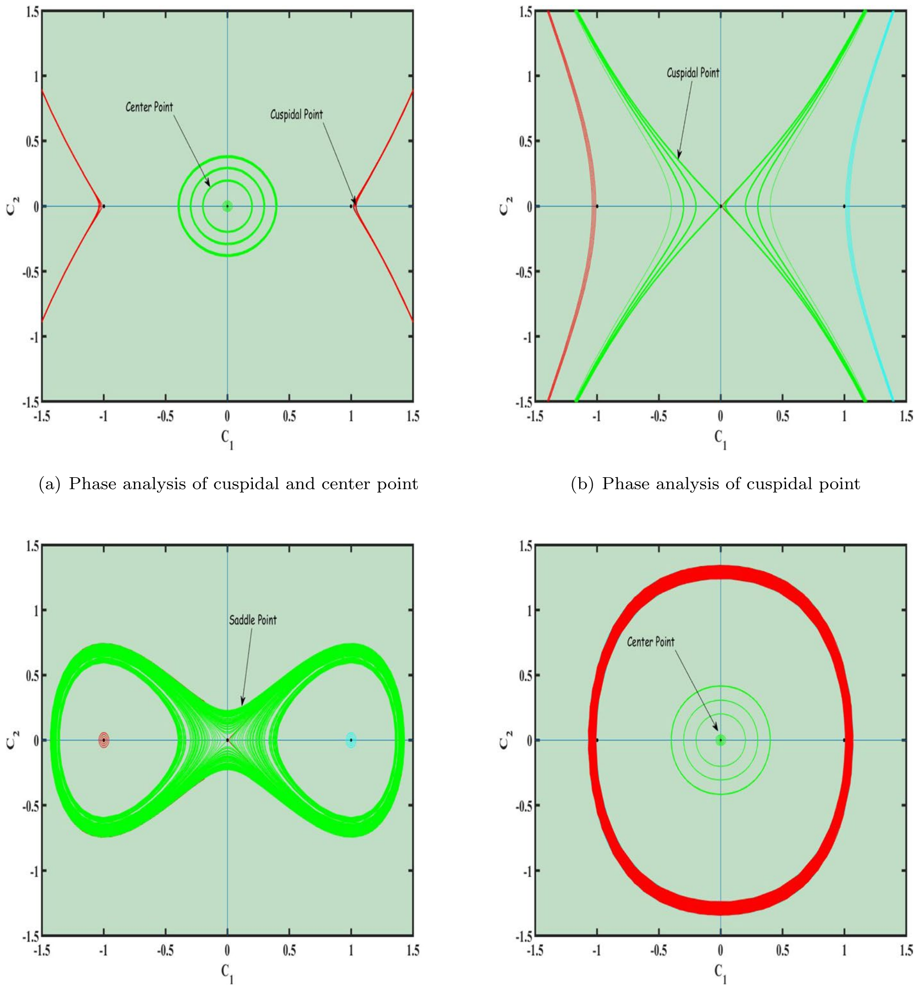


(a) 3D (b) Density plot (c) 2D

Fig. 6 Graphical illustration of rational soliton solution of Eq. (53), by using appropriate parameter values are $\delta_1 = 0,\ \delta_2 = 0.8,\ \delta_0 = 0,\ \psi_1 = 0.1,\ \psi = 0.9,\ \psi_2 = 0.3, \delta_1 = 0,$

6 Conclusions

This study discusses optical physical phenomena and emphasizes the value and usefulness of the (1+1)-dimensional NLPSM. Even though the model's characteristics and behavior have already been thoroughly examined in the literature, our approach offers fresh perspectives. Effective evolution of soliton solutions is achievable. Bifurcation analysis is carried out by the transformation of the governing model into a dynamical system. Numerous diverse industries may benefit from the novel ideas presented in this study. We verified our findings by using Mathematica software to analyze and visually represent several wave patterns with various system properties. Our solutions offer distinctiveness in contrast to traditional methods. The results of this work should spark more conversations in the nonlinear physical sciences. The approach is computationally efficient in finding exact wave solutions. Our suggested approaches might be extended in the future to handle other nonlinear models. The distinct and fascinating solutions found may aid in the understanding of mathematical models of various domains. In the future, these findings will be improved and



(a) Phase analysis of cuspidal and center point
(b) Phase analysis of cuspidal point
(c) Phase analysis of saddle point
(d) Phase analysis of center point

Fig. 7 Graphical illustration of phase portrait analysis by using appropriate parameter values

refined by further study. The soliton's mean free velocity will be obtained by using soliton perturbation theory, which incorporates perturbation terms and takes into account stochastic perturbation components. The (1+1)-dimensional NLPSM will also be integrated using a variety of integration methods.

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Data availability Since no datasets were created or examined during the current investigation, information sharing is not as relevant to this topic

Declarations

Conflict of interest The authors have no relevant financial or non-financial interests to disclose

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