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Discovering new abundant optical solutions for the resonant nonlinear Schrödinger equation using an analytical technique

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Abstract

In our exploration of optical physics, the intricate resonant nonlinear Schrödinger (NLS) equation featuring dual-power law nonlinearity is investigated which is an equation of paramount importance in the field of optics. This equation serves as a key to unlocking the intricacies of optical phenomena, including solitons, nonlinear effects, and wave interactions. Various optical solutions covering a broad variety of mathematical expressions, from trigonometric and hyperbolic functions to rational ones, are revealed by applying the technique of the powerful $(\dot{G}/G,1/G)$-expansion analytical approach which is the main goal of this study. The utmost precision and reliability of our findings are rigorously confirmed via the robust Mathematica software. Furthermore, the dynamic visual representations including 2D, 3D, and contour charts are presented to vividly depict various optical patterns such as single periodic, multi-periodic, singular soliton, and semi-bell-shaped phenomena. These solutions are of the utmost significance in the fields of nonlinear fiber optics and telecommunications, contributing to our comprehension of the fundamental physical concepts underlying the equation. The adaptability and application of our new and standardized technique is demonstrated by applying it to a wide range of mathematical and physical challenges.

Keywords $(\dot{G}/G,1/G)$-expansion scheme · Resonant NLS equation · Optical solutions

1 Introduction

Advancements in nonlinear optics have given rise to a range of NLS equations with broader applicability. NLS equations naturally describe the propagation of pulses in optical fibers, making them a fundamental tool in optics research. The universal applicability of NLS equations extends their usefulness to a broad range of nonlinear physical systems in various domains, encompassing electrical engineering, mathematics, and optical physics (such as light transmission through optical fibers, chaos, and photonics) (Baskonus et al. 2021; Kumar et al. 2020). Among the myriad NLS equations, the resonant NLS equation has gained renown for its apt description of nonlinear optical

Extended author information available on the last page of the article phenomena, particularly in the realms of optical fibers and telecommunications. In the realm of optical fibers, the resonant NLS equation serves as a mathematical description for the journey of optical pulses through these systems, accommodating the influences of nonlinearity, dispersion, and various other factors. This equation takes the following form (Tozar et al. 2021; Zayed and Alurrfi 2016):

$$i\Upsilon_t + p\Upsilon_{xx} + \Upsilon\left(q|\Upsilon|^2 + s|\Upsilon|^4\right) + w\left(\frac{|\Upsilon|_{xx}}{|\Upsilon|}\right)\Upsilon = 0, \quad i = \sqrt{-1} \tag{1}$$

In this equation p, (q, s, w), and $\Upsilon = \Upsilon(x, t)$, respectively symbolize the coefficient of dispersion, coefficients of nonlinearity, and the normalized complex amplitude of the pulse confined within the optical fiber.

Tozar et al. (2021) used the functional variable method to find a variety of soliton solutions with different structures for this model. In Zayed and Alurrfi (2016), the auxiliary equation method proposed by Sirendaoreji and Kudryashov is extended to construct new types of Jacobi elliptic function solutions for Eq. (1).

The resonant NLS equation, serving as a versatile model in the realm of optical physics, holds immense significance for understanding a wide spectrum of optical phenomena. It unravels the mysteries of wave transmission, the formation of optical solitons, and the intricate nonlinear behaviors observed within optical fibers and related media. Esteemed by scholars, this equation delves into the dynamic behavior of optical waves in diverse scenarios. It paves the way for innovative optical solitons, exploration of intricate optical wave interactions, and scrutiny of light pulse propagation in nonlinear optical materials. Researchers employ a blend of mathematical methodologies and numerical simulations to decode the secrets concealed within this equation, thus gaining profound insights into the intricate behavior of optical waves within complex optical systems.

Numerous scholars have developed a wide array of methodologies to extract precise solutions for NLS equations and other nonlinear evolution equations, employing techniques such as the Riccati equation method (Yomba 2005; Elsayed and Alurrfi 2015), Lie symmetries (Jafari et al. 2015; Hosseini et al. 2023a), the $(\dot{G}/G)$-expansion technic (Mohanty et al. 2023; Naher and Abdullah 2012), the extended Jacobi elliptic function method (Hosseini et al. 2023b; Wen and Lü 2009), the (G'/G^2)-expansion method (Rehman et al. 2022a), He's semi-inverse scheme (Mirzazadeh 2015), the new modified simple equation method (Irshad et al. 2017), the functional variable technic (Babajanov and Abdikarimov 2022; Bekir and San 2012), Homogeneous balance method (Wang et al. 1996; Fan and Zhang 1998), the fractional approach (Tandel et al. 2022), the new auxiliary equation method (Islam et al. 2023; Zhang 2013), the sine–Gordon expansion scheme (Kumar et al. 2022), the first integral method (Taghizadeh and Mirzazadeh 2011), the tanh–function method (Parkes and Duffy 1996; Fan 2000), the generalized (G'/G)-expansion technique (Kaur 2014), the tanh–coth method (Kumar and Pankaj 2015; Mamun et al. 2022), the generalized Kudryshov method (Habib et al. 2019; Islam et al. 2015), the $\exp(-\varphi(\xi))$-expansion method (Roshid et al. 2014), the unified method (Fokas and Lenells 2012; Abdel-Gawad and Osman 2015), the first extended rational sinh-Gordon method (Rehman and Ahmad 2023), the Galilean transformation and its bifurcation analysis (Hosseini et al. 2023c), the modified rational sine–cosine and sinh-cosh methods (Rehman et al. 2023), and other different techniques (Akram et al. 2023; Islam et al. 2024; Ahmad et al. 2023; Rehman et al. 2022b; Boakye et al. 2024; Ma 2023; El-Sherif et al. 2005; Ma et al. 2010; Fetoh et al. 2019; Abdel-Gawad and Osman 2013; Ganie et al. 2024).

The double $(\dot{G}/G,1/G)$-expansion method is a highly valuable analytical technique used to solve nonlinear partial differential equations featuring variable coefficients. It relies on representing solutions as power series with coefficients figured out by two variables (G and 1/G), and it derives these coefficients by substituting the series into the equation and matching coefficients of like terms. This method finds extensive application in uncovering solitary and optical wave solutions within a range of NLS equations. A number of investigators (Iqbal et al. 2023; Chowdhury et al. 2023) have utilized this technique to reveal solutions for NLS equations. As of now, there has been no exploration of optical solutions for the resonant NLS equation using the double $(\dot{G}/G,1/G)$-expansion method. The aim of this research is to acquire optical soliton solutions for this nonlinear equation through the application of this method. The paper follows the following structure: (i) Sect. 2 elucidates the methodology employed. (ii) In Sect. 3, we apply the aforementioned method to the Resonant NLS equation and obtain the requisite solutions. (iii) Sect. 4 is dedicated to the exploration of dynamic representations, visually illustrating the captivating behaviors of various solitons using 2D, 3D, and contour graphs. (iv) Sect. 5 offers concluding remarks.

2 Methodology

Within this discussion, we provide a comprehensive overview of the essential steps required for the application of the $(\dot{G}/G,1/G)$-expansion method, a technique introduced by some researchers (Iqbal et al. 2023; Chowdhury et al. 2023), in the analysis of NLS equations. Indeed, facilitating this analytical process necessitates the introduction of an auxiliary linear ordinary differential equation. This auxiliary equation is meticulously formulated to complement the methodology, enabling a systematic exploration of nonlinear phenomena and their solutions, which is constructed as follows:

$$\ddot{G}(\varsigma) + \lambda G(\varsigma) = \beta \tag{2}$$

In the above equation, the sign '.' stands for the differentiation as regards ς in addition the variables agree like this:

$$\Phi = \dot{G}(\varsigma)/G(\varsigma) \text{ and } \Psi = 1/G(\varsigma) \tag{3}$$

Equation (3) satisfied the following relationships:

$$\dot{\Phi} = -\Phi^2 + \beta\Psi - \lambda \quad \text{and} \quad \dot{\Psi} = -\Phi\Psi \tag{4}$$

It is worth noting that Φ and Ψ are the functions of ς.

The Eq. (2) mentioned above yields different results subject to the value of λ, which can be classified into three distinct scenarios:

Case I. If $\lambda > 0$ (positive values).

In this case, Eq. (2) yields the general solution (GS) as follows:

$$G(\varsigma) = C_1 \sin\left(\varsigma\sqrt{\lambda}\right) + C_2 \cos\left(\varsigma\sqrt{\lambda}\right) + \beta/\lambda \tag{5}$$

$$\text{which produces}: \Psi^2 = \left(\frac{\Phi^2 - 2\beta\Psi + \lambda}{\alpha_1\lambda^2 - \beta^2}\right)\lambda \tag{6}$$

where $\alpha_1 = C_1^2 + C_2^2$ stands for the arbitrary constants.
Case II. If $\lambda < 0$ (negative values)
In this consequence, Eq. (2) produces the GS in the following manner:

$$G(\varsigma) = C_1 \sinh\left(\varsigma\sqrt{-\lambda}\right) + C_2 \cosh\left(\varsigma\sqrt{-\lambda}\right) + \beta/\lambda \tag{7}$$

resulting in:

$$\Psi^2 = -\lambda\left(\frac{\Phi^2 - 2\beta\Psi + \lambda}{\alpha_2\lambda^2 + \beta^2}\right), where \alpha_2 = C_1^2 - C_2^2 \tag{8}$$

Case III. If $\lambda = 0$
Within this context, Eq. (2) produces the GS in the following manner:

$$G(\varsigma) = \frac{\beta\varsigma^2}{2} + C_1\varsigma + C_2 \tag{9}$$

$$\text{whichdelivers}: \Psi^2 = \left(\frac{\Phi^2 - 2\beta\Psi}{{C_1}^2 - 2\beta C_2}\right) \tag{10}$$

Now, let's assume the general form of an NLS equation that encompasses three independent variables (*x, y,* and *t*) can be represented as:

$$\mathcal{H}\left(\Upsilon, \Upsilon_x, \Upsilon_{xx}, \Upsilon_y, \Upsilon_{yy}, \Upsilon_{xy}, \Upsilon_t, \Upsilon_{tt}, \Upsilon_{xt}, \ldots\ldots\ldots\right) = 0 \tag{11}$$

Within this context, $\mathcal{H}$ represents a polynomial function that is contingent upon the variables encapsulated in Υ and $\Upsilon_x = \frac{\partial\Upsilon}{\partial x}$, $\Upsilon_y = \frac{\partial\Upsilon}{\partial y}$, $\Upsilon_t = \frac{\partial\Upsilon}{\partial t}$, $\Upsilon_{xx} = \frac{\partial^2\Upsilon}{\partial x^2}$, $\Upsilon_{yy} = \frac{\partial^2\Upsilon}{\partial y^2}$, $\Upsilon_{tt} = \frac{\partial^2\Upsilon}{\partial t^2}$, $\Upsilon_{xt} = \frac{\partial\Upsilon}{\partial x\partial t}$, $\Upsilon_{xy} = \frac{\partial\Upsilon}{\partial x\partial y}$ and so on.
In order to transform Eq. (11), we introduce a new variable ς, which is governed by the following relations:

$$\Upsilon(x,t) = e^{i\chi(x,t)}v(\varsigma), \chi(x,t) = \tau - kx + lt, \varsigma = (x - at) \tag{12}$$

where k is the wave number, τ represents the constant and l is the wave frequency.
Equation (11), now converted into an ordinary differential equation which can be written as follows:

$$J(v, \dot{v}, \ddot{v}, \dddot{v}, \ldots\ldots\ldots) = 0 \tag{13}$$

Here, J represents the new polynomial that contains v with its ordinary derivatives.
Consider the following equation, which represents the GS of Eq. (13) utilizing the previously discussed method:

$$v(\varsigma) = a_0 + \sum_{i=1}^{M} a_i\Phi^i(\varsigma) + \sum_{i=1}^{M} b_i\Phi^{i-1}(\varsigma)\Psi(\varsigma) \tag{14}$$

In the above equation a_0, a_i and $b_i(i = 1, 2, 3, \ldots\ldots\ldots, M)$ are the arbitrary constant fulfilling the requirement $a_M^2 + b_M^2 \neq 0$ and the parameter M is defined as a positive homogeneous balance number in this context. To figure out these arbitrary constants, we employ the previously mentioned method, following these phases:

Phase 1: To find the balance number M, the homogeneous balance method is employed. This method entails balancing the highest-order derivative and the nonlinear terms with the highest degree.

Phase 2: By inserting the value of M into Eq. (14) and subsequently substituting this modified equation into Eq. (13), while utilizing Eqs. (4) and (6) (illustrated through Case I as an example), the left-hand side of Eq. (13) is transformed into a polynomial that incorporates Φ and Ψ. In this polynomial, the degree associated with Ψ does not exceed 1, whereas the degree of Φ ranges from 0 to any integer. Setting the coefficients of terms with matching powers within the polynomial to 0 leads to the establishment of a system of algebraic equations involving a_i, b_i, λ (when $\lambda > 0$), β and others.

Phase 3: Utilizing Mathematica software to solve the algebraic equations obtained in Step II, we determine the values of a_i, b_i, λ (where $\lambda > 0$), and β. Subsequently, these values are substituted into the transformed Eq. (14), allowing us to derive the optical solutions represented by the trigonometric functions as described in Eq. (5). This comprehensive procedure ultimately yields the optical solution for Eq. (1), following the coordinate change outlined in Eq. (12).

Phase 4: Following a process similar to Steps II and III, we obtain the optical solutions for Eq. (13), particularly Eq. (11), which are represented as a combination of hyperbolic functions and rational functions.

3 Method's application

In this part, we implement the aforementioned method outlined in Sect. 2 to acquire the optical solution of the resonant NLS equation.

Upon applying the transformation described in Eq. (12), Eq. (1) can be expressed as an ordinary differential equation after separating the real and imaginary components in the following manner:

Real part:

$$(p + w)\ddot{v} - \left(l + pk^2\right)v + qv^3 + sv^5 = 0 \tag{15}$$

Imaginary part:

$$a = -2kp \tag{16}$$

Operating the homogeneous balance technique, we find out the balance number M to be $\frac{1}{2}$. However, since it is not an integer number, we introduce an additional transformation as follows:

$$\mathcal{F}^{\frac{1}{2}} = v \tag{17}$$

Now, utilizing this transformation in Eq. (15), we obtain:

$$(p + w)\left(2\mathcal{F}\ddot{\mathcal{F}} - \dot{\mathcal{F}}^2\right) - 4\left\{\left(l + pk^2\right)\mathcal{F}^2 - q\mathcal{F}^3 - s\mathcal{F}^4\right\} = 0 \tag{18}$$

Once more, we apply the homogeneous balance law to Eq. (18), resulting in: $M = 1$ and the solution of Eq. (18) can be written as follows:

$$\mathcal{F}(\varsigma) = a_0 + a_1\Phi(\varsigma) + b_1\Psi(\varsigma) \tag{19}$$

In this equation, the constants a_0, a_1, and b_1 are coefficients that need to be figured out. We will now apply the three cases, as discussed in the methodology section.

Case I. $\lambda > 0$ (For trigonometric solutions).

To obtain the required solution, Eq. (19) is substituted into Eq. (18) and Eqs. (2) and (3) are applied, the left side of Eq. (18) is switched to a polynomial that encompasses Φ and Ψ. Setting each coefficient of this polynomial to zero leads to a system of algebraic equations involving the coefficients a_0, a_1, and b_1. The solution to these algebraic systems supplies the values for the arbitrary constants as results:

$$a_0 = 0, a_1 = 0, q = \frac{4s\lambda\beta b_1}{3\sqrt{-\beta^2 + \alpha_1\lambda^2}}, w = \frac{3p\beta^2 - 3p\alpha_1\lambda^2 - 4s\lambda b_1^2}{3\left(-\beta^2 + \alpha_1\lambda^2\right)}$$
$$and l = \frac{3pk^2\beta^2 - 3pk^2\alpha_1\lambda^2 + s\lambda^2 b_1^2}{3\left(-\beta^2 + \alpha_1\lambda^2\right)} \tag{20}$$

Now, employing these computed values in Eq. (19), we obtain the following:

$$\mathcal{F}(\varsigma) = b_1 \frac{1}{C_1 \sin\left(\varsigma\sqrt{\lambda}\right) + C_2 \cos\left(\varsigma\sqrt{\lambda}\right) + \beta/\lambda} \tag{21}$$

where $b_1 \neq 0$.

By reverting Eq. (21) to its initial form with the assistance of Eq. (12) and Eq. (17), we obtain the following:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{3pk^2\beta^2 - 3pk^2\alpha_1\lambda^2 + s\lambda^2 b_1^2}{3\left(-\beta^2+\alpha_1\lambda^2\right)}\right)t\right\}} \left(b_1 \frac{1}{C_1 \sin\left((x-at)\sqrt{\lambda}\right) + C_2 \cos\left((x-at)\sqrt{\lambda}\right) + \beta/\lambda}\right)^{\frac{1}{2}} \tag{22}$$

where $b_1 \neq 0$.

If we set both β and C_2 to zero while ensuring C_1 is non-zero simplifies Eq. (22) to the solitary wave solution as:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{-3pk^2C_1^2 + sb_1^2}{3C_1^2}\right)t\right\}} \left(\frac{b_1}{C_1}\operatorname{cosec}\left((x-at)\sqrt{\lambda}\right)\right)^{\frac{1}{2}} \tag{23}$$

where $b_1 \neq 0$.

Furthermore, if we set both β and C_1 to zero while ensuring that C_2 is non-zero, Eq. (22) simplifies to return the solitary wave solution as follows:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{-3pk^2C_2^2 + sb_1^2}{3C_2^2}\right)t\right\}} \left(\frac{b_1}{C_2}\sec\left((x-at)\sqrt{\lambda}\right)\right)^{\frac{1}{2}} \tag{24}$$

where $b_1 \neq 0$.

Case II. $\lambda < 0$ (For hyperbolic solutions)

In this scenario, we follow a similar procedure that is described in case I to derive the necessary solution. We start the process by inserting Eq. (19) into Eq. (18) and implementing Eqs. (2) and (3). This series of operations results in the left side of Eq. (18) being converted into a polynomial that incorporates Φ and Ψ. Upon equating each coefficient of this polynomial to zero, we derive a system of algebraic equations involving the coefficients a_0, a_1,and b_1. The solution to these algebraic systems provides the values for the arbitrary constants as follows:

Set 1:

$$a_0 = -\frac{\lambda b_1}{\sqrt{\beta^2 + \alpha_2 \lambda^2}}, a_1 = \pm \frac{\sqrt{-\lambda} b_1}{\sqrt{\beta^2 + \alpha_2 \lambda^2}}, q = \frac{8 s \lambda b_1}{3\sqrt{\beta^2 + \alpha_2 \lambda^2}}, w = \frac{-3p\beta^2 - 3p\alpha_2 \lambda^2 + 16 s \lambda b_1^2}{3\left(\beta^2 + \alpha_2 \lambda^2\right)} \text{ and}$$
$$l = \frac{-3pk^2\beta^2 - 3pk^2\alpha_2\lambda^2 - 4s\lambda^2 b_1^2}{3\left(\beta^2 + \alpha_2\lambda^2\right)} \tag{25}$$

By incorporating these values into Eq. (19), we arrive at the solution in the following manner:

$$\mathcal{F}(\varsigma) = -\frac{\lambda b_1}{\sqrt{\beta^2 + \alpha_2\lambda^2}} \pm \frac{\sqrt{-\lambda} b_1}{\sqrt{\beta^2 + \alpha_2\lambda^2}} \sqrt{-\lambda} \frac{\left(C_1 \cosh\left(\varsigma\sqrt{-\lambda}\right) + C_2 \sinh\left(\varsigma\sqrt{-\lambda}\right)\right)}{C_1 \sinh\left(\varsigma\sqrt{-\lambda}\right) + C_2 \cosh\left(\varsigma\sqrt{-\lambda}\right) + \beta/\lambda}$$
$$+ b_1 \frac{1}{C_1 \sinh\left(\varsigma\sqrt{-\lambda}\right) + C_2 \cosh\left(\varsigma\sqrt{-\lambda}\right) + \beta/\lambda} \tag{26}$$

where $b_1 \neq 0$.

By substituting the transformation variables outlined in Eq. (12) into this equation, it is transformed into the following GS form:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{-3pk^2\beta^2 - 3pk^2\alpha_2\lambda^2 - 4s\lambda^2 b_1^2}{3\left(\beta^2 + \alpha_2\lambda^2\right)}\right)t\right\}}$$
$$\times\left(-\frac{\lambda b_1}{\sqrt{\beta^2 + \alpha_2\lambda^2}} \pm \frac{\sqrt{-\lambda} b_1}{\sqrt{\beta^2 + \alpha_2\lambda^2}} \sqrt{-\lambda} \frac{C_1 \cosh\left((x - at)\sqrt{-\lambda}\right) + C_2 \sinh\left((x - at)\sqrt{-\lambda}\right)}{C_1 \sinh\left((x - at)\sqrt{-\lambda}\right) + C_2 \cosh\left((x - at)\sqrt{-\lambda}\right) + \beta/\lambda}\right.$$
$$\left. + b_1 \frac{1}{C_1 \sinh\left((x - at)\sqrt{-\lambda}\right) + C_2 \cosh\left((x - at)\sqrt{-\lambda}\right) + \beta/\lambda}\right)^{\frac{1}{2}} \tag{27}$$

Where $b_1 \neq 0$.

When both β and C_2 are set to zero, with the condition that C_1 is non-zero, Eq. (27) simplifies to supply the solitary wave solution as follows:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{-3pk^2C_1^2 - 4sb_1^2}{3C_1^2}\right)t\right\}} \left(-\frac{b_1}{C_1} \pm \frac{b_1}{C_1} \coth\left((x - at)\sqrt{-\lambda}\right) + \frac{b_1}{C_1} \operatorname{cosech}\left((x - at)\sqrt{-\lambda}\right)\right)^{\frac{1}{2}} \tag{28}$$

Where $b_1 \neq 0$.

Set 2:

$$a_0 = \frac{\lambda b_1}{\sqrt{\beta^2 + \alpha_2\lambda^2}}, a_1 = \pm\frac{\sqrt{-\lambda}b_1}{\sqrt{\beta^2 + \alpha_2\lambda^2}}, q = -\frac{8s\lambda b_1}{3\sqrt{\beta^2 + \alpha_2\lambda^2}}, w = \frac{-3p\beta^2 - 3p\alpha_2\lambda^2 + 16s\lambda b_1^2}{3\left(\beta^2 + \alpha_2\lambda^2\right)} \text{ and}$$
$$l = \frac{-3pk^2\beta^2 - 3pk^2\alpha_2\lambda^2 - 4s\lambda^2 b_1^2}{3\left(\beta^2 + \alpha_2\lambda^2\right)} \tag{29}$$

Incorporating these determined values into Eq. (19), we arrive at the solution in the following manner:

$$\mathcal{F}(\varsigma) = \frac{\lambda b_1}{\sqrt{\beta^2 + \alpha_2\lambda^2}} \pm \frac{\sqrt{-\lambda}b_1}{\sqrt{\beta^2 + \alpha_2\lambda^2}}\sqrt{-\lambda}\frac{\left(C_1\cosh\left(\varsigma\sqrt{-\lambda}\right) + C_2\sinh\left(\varsigma\sqrt{-\lambda}\right)\right)}{C_1\sinh\left(\varsigma\sqrt{-\lambda}\right) + C_2\cosh\left(\varsigma\sqrt{-\lambda}\right) + \beta/\lambda}$$
$$+ b_1\frac{1}{C_1\sinh\left(\varsigma\sqrt{-\lambda}\right) + C_2\cosh\left(\varsigma\sqrt{-\lambda}\right) + \beta/\lambda} \tag{30}$$

where $b_1 \neq 0$.

Upon substituting the transformation variables as defined in Eq. (12) into this equation, it transforms into the following generalized solution form:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{-3pk^2\beta^2 - 3pk^2\alpha_2\lambda^2 - 4s\lambda^2 b_1^2}{3\left(\beta^2 + \alpha_2\lambda^2\right)}\right)t\right\}}$$
$$\times\left(\frac{\lambda b_1}{\sqrt{\beta^2 + \alpha_2\lambda^2}} \pm \frac{\sqrt{-\lambda}b_1}{\sqrt{\beta^2 + \alpha_2\lambda^2}}\sqrt{-\lambda}\frac{C_1\cosh\left((x-at)\sqrt{-\lambda}\right) + C_2\sinh\left((x-at)\sqrt{-\lambda}\right)}{C_1\sinh\left((x-at)\sqrt{-\lambda}\right) + C_2\cosh\left((x-at)\sqrt{-\lambda}\right) + \beta/\lambda}\right.$$
$$\left. + b_1\frac{1}{C_1\sinh\left((x-at)\sqrt{-\lambda}\right) + C_2\cosh\left((x-at)\sqrt{-\lambda}\right) + \beta/\lambda}\right)^{\frac{1}{2}} \tag{31}$$

Where $b_1 \neq 0$.

If both β and C_2 are set to zero, with the condition that C_1 is non-zero, Eq. (31) simplifies to supply the solitary wave solution as follows:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{-3pk^2C_1^2 - 4sb_1^2}{3C_1^2}\right)t\right\}}\left(\frac{b_1}{C_1} \pm \frac{b_1}{C_1}\coth\left((x-at)\sqrt{-\lambda}\right) + \frac{b_1}{C_1}\text{cosech}\left((x-at)\sqrt{-\lambda}\right)\right)^{\frac{1}{2}} \tag{32}$$

where $b_1 \neq 0$.

Set 3:

$$a_0 = 0, a_1 = 0, q = \frac{4s\lambda\beta b_1}{3\sqrt{\beta^2 + \alpha_2\lambda^2}}, w = \frac{-3p\beta^2 - 3p\alpha_2\lambda^2 + 4s\lambda b_1^2}{3\left(\beta^2 + \alpha_2\lambda^2\right)} \text{ and}$$
$$l = \frac{-3pk^2\beta^2 - 3pk^2\alpha_2\lambda^2 - s\lambda^2 b_1^2}{3\left(\beta^2 + \alpha_2\lambda^2\right)} \tag{33}$$

Utilizing these computed values in Eq. (19), the solution can be expressed as follows:

$$\mathcal{F}(\varsigma) = b_1 \frac{1}{C_1 \sinh\left(\varsigma\sqrt{-\lambda}\right) + C_2 \cosh\left(\varsigma\sqrt{-\lambda}\right) + {}^{\beta}/_{\lambda}} \tag{34}$$

where $b_1 \neq 0$.

Reverting Eq. (34) to its original form with the assistance of Eq. (12) and Eq. (17), we arrive at the following:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{-3pk^2\beta^2 - 3pk^2\alpha_2\lambda^2 - s\lambda^2 b_1^2}{3\left(\beta^2 + \alpha_2\lambda^2\right)}\right)t\right\}} \left(b_1 \frac{1}{C_1 \sinh\left((x-at)\sqrt{-\lambda}\right) + C_2 \cosh\left((x-at)\sqrt{-\lambda}\right) + {}^{\beta}/_{\lambda}}\right)^{\frac{1}{2}} \tag{35}$$

where $b_1 \neq 0$.

When β and C_2 to zero while ensuring C_1 is non-zero shortens Eq. (35) to the solitary wave solution as follows:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{-3pk^2C_1^2 - sb_1^2}{3C_1^2}\right)t\right\}} \left(\frac{b_1}{C_1} \operatorname{cosech}\left((x-at)\sqrt{-\lambda}\right)\right)^{\frac{1}{2}} \tag{36}$$

where $b_1 \neq 0$.

Furthermore, when we set both β and C_1 to zero while ensuring that C_2 is non-zero, Eq. (35) simplifies to return the solitary wave solution as follows:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{-3pk^2C_2^2 + sb_1^2}{3C_2^2}\right)t\right\}} \left(\frac{b_1}{C_2} \operatorname{sech}\left((x-at)\sqrt{\lambda}\right)\right)^{\frac{1}{2}} \tag{37}$$

where $b_1 \neq 0$.

Case III. $\lambda = 0$ (For rational solutions)

In this instance, we follow a procedure like the one expressed in cases I and II to obtain the required solution. The process begins with the substitution of Eq. (19) into Eq. (18) and the application of Eqs. (2) and (3). This manipulation converts the left side of Eq. (18) into a polynomial containing Φ and Ψ. By equating each coefficient of this polynomial to zero, we set up a system of algebraic equations that involve the constants a_0, a_1 and b_1. The solution to these algebraic systems furnishes the values for the arbitrary constants as follows:

Set 1:

$$\begin{aligned} a_0 &= \frac{2\beta b_1}{C_1^2 - 2\beta C_2}, a_1 = 0, q = \frac{4s\beta b_1}{-C_1^2 + 2\beta C_2}, w = \frac{-3pC_1^2 + 6p\beta C_2 - 4sb_1^2}{3\left(C_1^2 - 2\beta C_2\right)} \textit{ and} \\ l &= \frac{-pk^2C_1^4 + 4pk^2\beta C_1^2 C_2 - 4pk^2\beta^2 C_2^2 - 4s\beta^2 b_1^2}{\left(C_1^2 - 2\beta C_2\right)^2} \end{aligned} \tag{38}$$

Incorporating these values into Eq. (19), we obtain the solution as follows:

$$\mathcal{F}(\varsigma) = \frac{2\beta b_1}{C_1^2 - 2\beta C_2} + b_1 \frac{1}{\frac{\beta}{2}\varsigma^2 + C_1\varsigma + C_2} \tag{39}$$

where $b_1 \neq 0$.

Substituting the transformation variables defined in Eq. (12) into the above equation yields the following generalized solution form:

$$\Upsilon(x,t) = e^{i\left\{\tau - kx + \left(\frac{-pk^2C_1^4 + 4pk^2\beta C_1^2 C_2 - 4pk^2\beta^2 C_2^2 - 4s\beta^2 b_1^2}{\left(C_1^2 - 2\beta C_2\right)^2}\right)t\right\}} \left(\frac{2\beta b_1}{C_1^2 - 2\beta C_2} + b_1 \frac{1}{\frac{\beta}{2}(x-at)^2 + C_1(x-at) + C_2}\right)^{\frac{1}{2}} \tag{40}$$

where $b_1 \neq 0$.

If both β and C_2 are set to zero, with the condition that C_1 is non-zero, Eq. (40) simplifies to supply the solitary wave solution as follows:

$$\Upsilon(x,t) = e^{i\{\tau - kx - pk^2 t\}} \left(\frac{b_1}{C_1}\frac{1}{(x-at)}\right)^{\frac{1}{2}} \tag{41}$$

where $b_1 \neq 0$.

Set 2:

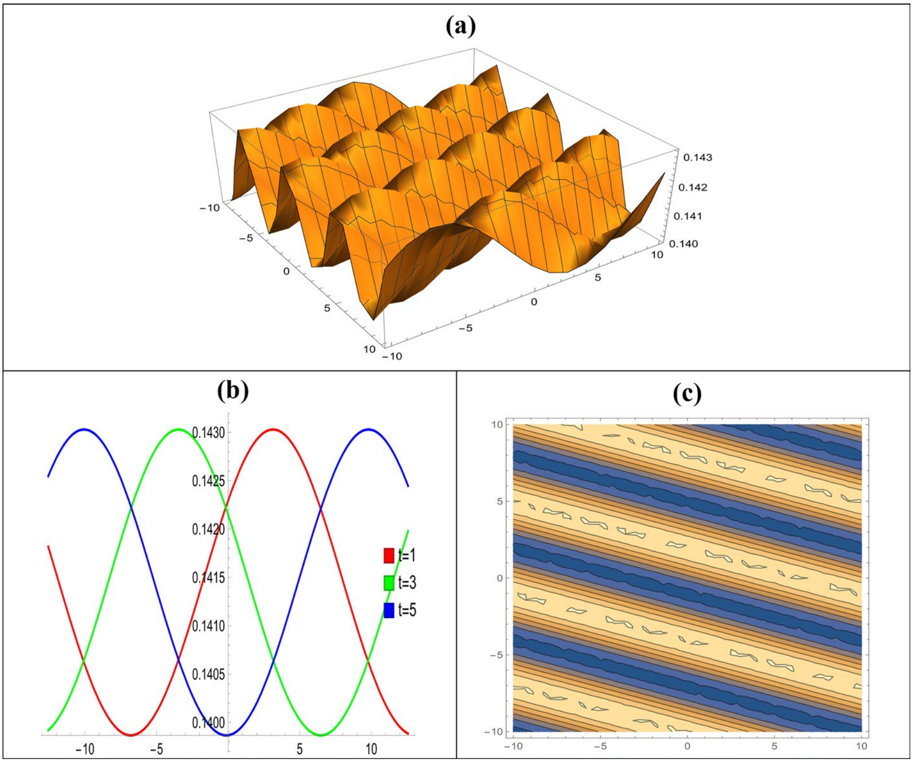

Fig. 1 Graphical representations of the solutions $|\Upsilon(x,t)|$ of Eq. (22) for $b_1 = -.2, \lambda = .1, a = -3.3, C_1 = -.2, C_2 = 0.1$, and $\beta = 1$**:** (a) A 3D representation (b) A 2D representation and (c) Contour representation

$$a_0 = 0, a_1 = 0, q = \frac{4s\beta b_1}{4(C_1^2 - 2\beta C_2)}, w = \frac{-3pC_1^2 + 6p\beta C_2 - 4sb_1^2}{3\left(C_1^2 - 2\beta C_2\right)} \text{ and } l = -pk^2 \tag{42}$$

Incorporating these values into Eq. (19), we obtain the solution as follows:

$$\mathcal{F}(\varsigma) = \frac{b_1}{\frac{\beta}{2}\varsigma^2 + C_1\varsigma + C_2} \tag{43}$$

where $b_1 \neq 0$.

When the transformation variables defined in Eq. (12) are substituted into the equation mentioned above, it results in the following generalized solution form:

$$\Upsilon(x,t) = e^{i\{\tau - kx - pk^2 t\}}\left(\frac{b_1}{\frac{\beta}{2}(x-at)^2 + C_1(x-at) + C_2}\right)^{\frac{1}{2}} \tag{44}$$

where $b_1 \neq 0$.

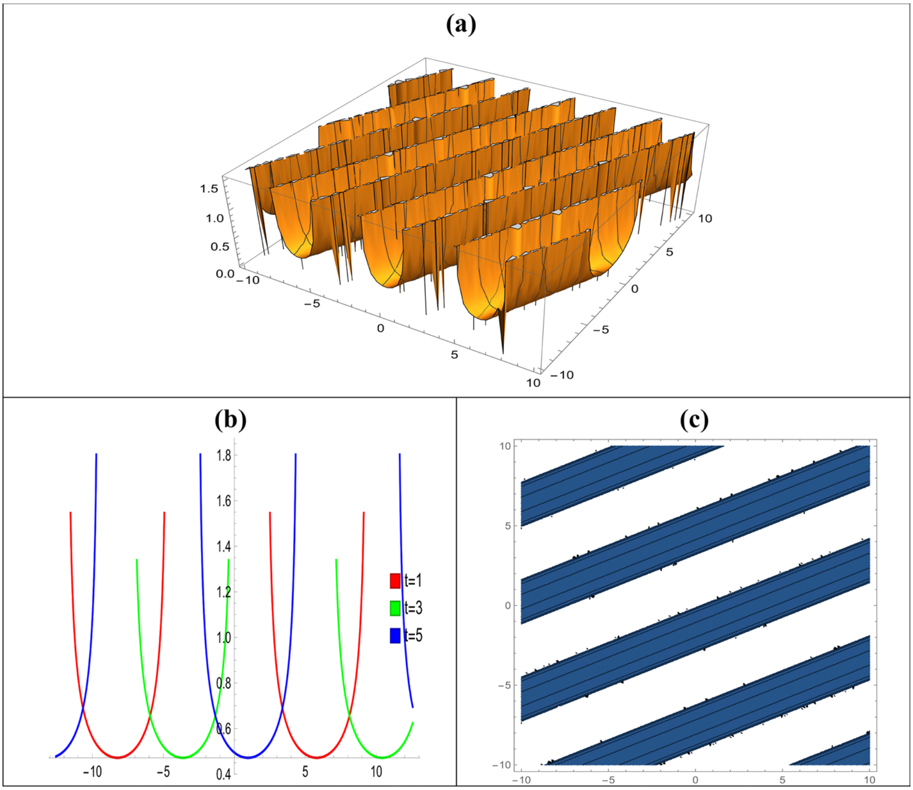

Fig. 2 Graphical representations of the solutions $|\Upsilon(x,t)|$ of Eq. (23) for $b_1 = .2, \lambda = .2, a = 2.3$, and $C_1 = .9$ **:** (a) A 3D representation (b) A 2D representation and (c) Contour representation

4 Graphs and their associated physical interpretations

In this section, we employed Mathematica, a modern mathematical computational tool, to reveal unique graphical patterns displayed by the resonant NLS equation. Our presentation featured a diverse set of visual representations, including 3D renderings, 2D graphical displays, and contour plots. These graphics spanned a broad spectrum of parameter values for each relevant variable. The goal was to provide a comprehensive insight into the graphical behavior of the resonant NLS equation, elucidating its intricacies across a diverse parameter space.

In the interest of clarity and brevity, we have chosen to visually depict a limited selection of five solution sets from our comprehensive results. To keep simplicity, we have standardized the *x*-axis (− 10 to 10) for all the graphs. The constants specific to each graph are provided in the corresponding figure captions. In 2-D graphs, we have consolidated multiple solutions within a single figure by varying the parameter *t*.

Figure 1 is obtained from Eq. (22), effectively illustrates the multi-periodic soliton solution. In (a), we provide a 3D visualization of these solutions, (b) offers a 2D display with time variations, and (c) features a contour representation of the solution. This graphical depiction allows us to discern the recurring and bell-shaped patterns that define these

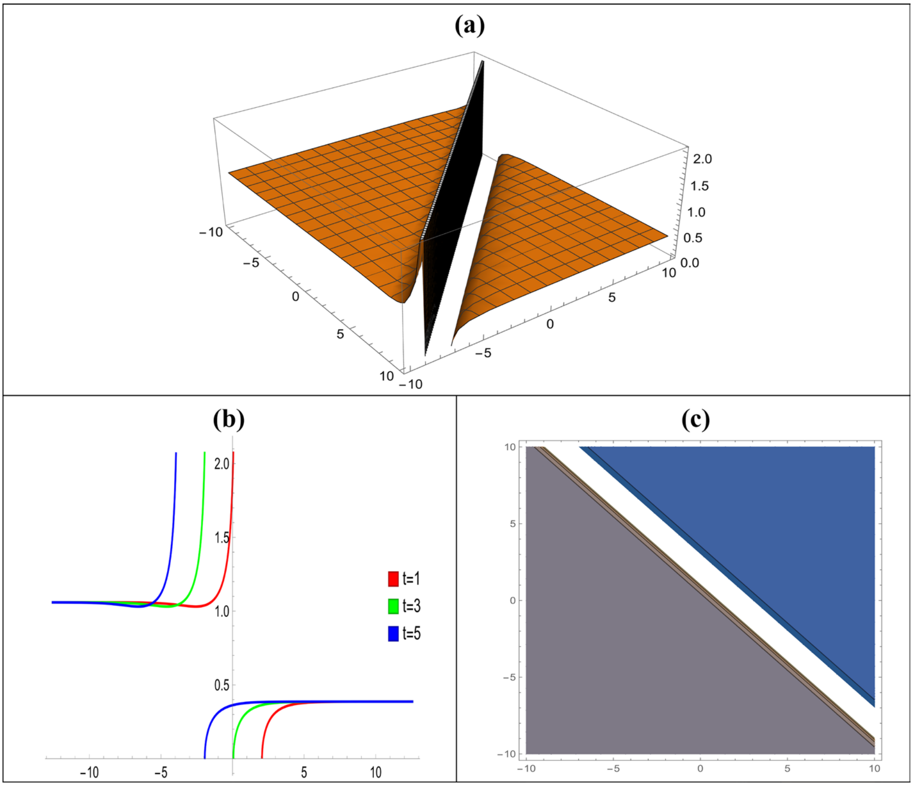

Fig. 3 Graphical representations of the solutions of Eq. (31) for and**:** (a) A 3D representation (b) A 2D representation and (c) Contour representation

solutions, offering a visual understanding of the unique characteristics of the Resonant NLS equation.

It is evident that the behavior of the derived solutions, including the amplitude and width of the solitary wave, was unaffected by the nonlinear parameters.

Figure 2, is found from Eq. (23), effectively illustrates a specific solution derived from Eq. (22), prominently displaying its singular multi-periodic characteristics. In (a), we offer a 3D visualization of these solutions; (b) provides a 2D display showcasing time variations, and (c) includes a contour representation of the solution. This graphical representation enables us to clearly identify the periodic behavior inherent in the solutions within the context of the Resonant NLS equation.

Figure 3, derived from Eq. (31), prominently highlights singular soliton features characterized by a semi-bell-shaped behavior. In (a), a 3D representation of these solutions is provided, while (b) offers a 2D view with time variations, and (c) showcases a contour representation of the solution. This graphical representation affords a clear visualization of the distinctive behavior inherent in the solutions of the Resonant NLS equation.

It is evident that the graph's structure in Figs. 2 and 3 is made up of singular-periodic solutions. We claim that a singular wave solution is essential for studying many physical phenomena. For instance, a singular wave is formed when a sudden force is applied, such an earthquake that might generate a disastrous tsunami wave. Moreover, a sudden temperature shock might also result in a thermal tsunami for a porous medium.

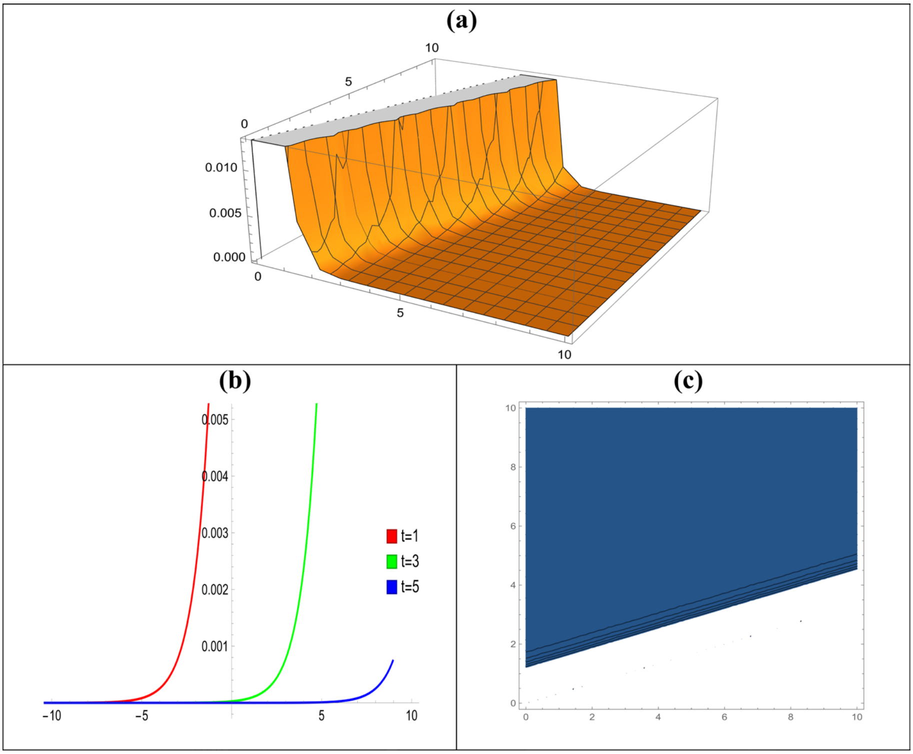

Fig. 4 Graphical representations of the solutions of Eq. (36) for and**:** (a) A 3D representation (b) A 2D representation and (c) Contour representation

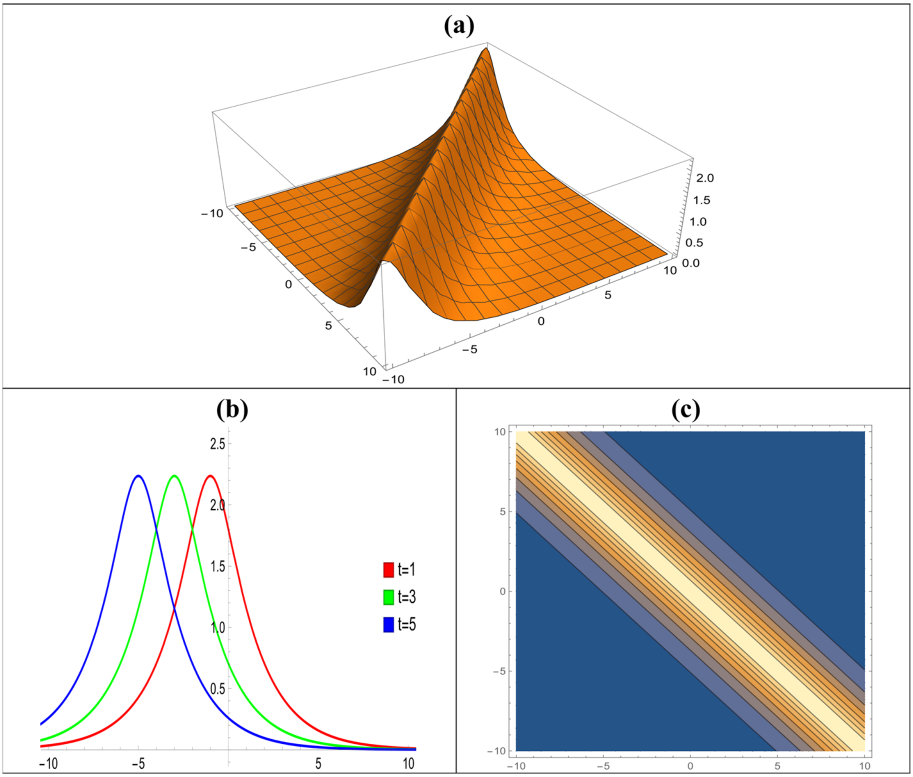

Fig. 5 Graphical representations of the solutions of Eq. (37) for and**:** (a) A 3D representation (b) A 2D representation and (c) Contour representation

Figure 4, obtained from Eq. (36), illustrates a specific solution derived from Eq. (35), emphasizing a bright soliton solution with a half-bell-shaped structure within the singular soliton. In (a), a 3D representation of these solutions is displayed, while (b) presents a 2D view with time variations, and (c) showcases a contour representation of the solution. This graphical depiction offers a clear visualization of the unique bright soliton solution with a half-bell-shaped structure inherent in the solutions of the Resonant NLS equation.

Figure 5, derived from Eq. (37), which is a particular solution of Eq. (35), prominently showcases bright soliton solutions characterized by a specific periodicity and bell-shaped profiles. In (a), a 3D representation of these solutions is presented, while (b) offers a 2D view displaying varying time patterns, and (c) showcases a contour representation of the solution. This graphical depiction provides a clear visualization of the distinct bright soliton solutions with bell-shaped profiles inherent in the solutions of the Resonant NLS equation.

It is evident that the behavior of the derived solutions, including the amplitude and width of the solitary wave, was unaffected by the nonlinear parameters.

By contrasting the results we obtained in this paper with the well-known results from Tozar et al. (2021) and Zayed and Alurrfi (2016), we can conclude that while the other solutions in the paper are new and unpublished, our results (36) and (37) are equivalent to the solutions $W_{13,14}$and$W_{15,16}$ in Tozar et al. (2021), respectively. While our results (23), (24), and (37) are equivalent to the solutions (3.44), (3.47), and (3.54) obtained in Zayed and Alurrfi (2016), respectively.

5 Conclusion

The double $(\dot{G}/G,1/G)$-expansion method is investigated to get numerous precise optical solutions for the resonant NLS equation. This equation is essential to understanding the dynamics of optical soliton in optical fiber theory. By constructing several nonlinear wave structures inside this equation, new traveling pulse responses are obtained. Flexible forms are created that include rational, hyperbolic, and trigonometric functions. A range of behaviors are examined, including brilliant solitons, single solitons, singular and multiperiodic patterns, and semi-bell-shaped structures. These optical solutions, characterized by varying parameter values, hold substantial promise for advancing the field of optical physics, impacting both light and electron optics. The versatility of the double $(\dot{G}/G,1/G)$-expansion method that employed in our study empowered us to explore a diverse spectrum of optical solutions. Significantly, the optical soliton solutions derived through this method underscore its efficacy, reliability, and simplicity when contrasted with alternative techniques. In the near future, this model will be discussed by other different techniques when its coefficients are not constants.

Author contributions MNH and MMM: Conceptualization, Methodology, Software, Validation, Resources, Writing-original draft. AHG: Data curation, Writing-original draft. MSO and WXM: Supervision, Project administration, Funding acquisition, Writing-review editing, Formal analysis.

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Declarations

Competing interests The authors declare no conflict of interest.

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References

Abdel-Gawad, H.I., Osman, M.S.: On the variational approach for analyzing the stability of solutions of evolution equations. Kyungpook Math. J. **53**(4), 661–680 (2013)

Abdel-Gawad, H.I., Osman, M.: On shallow water waves in a medium with time-dependent dispersion and nonlinearity coefficients. J. Adv. Res. **6**, 593–599 (2015)

Ahmad, J., Akram, S., Rehman, S.U., Turki, N.B., Shah, N.A.: Description of soliton and lump solutions to M-truncated stochastic Biswas-Arshed model in optical communication. Result Phys. **51**, 106719 (2023)

Akram, S., Ahmad, J., Rehman, S.U.: Stability analysis and dynamical behavior of solitons in nonlinear optics modelled by Lakshmanan–Porsezian–Daniel equation. Opt. Quant. Electron. **55**, 685 (2023)

Babajanov, B., Abdikarimov, F.: The application of the functional variable method for solving the loaded non-linear evaluation equations. Front. Appl. Math Stat. **8**, 1–9 (2022)

Baskonus, H.M., Osman, M.S., Rehman, H., Ramzan, M., Tahir, M., Ashraf, S.: On pulse propagation of soliton wave solutions related to the perturbed Chen–Lee–Liu equation in an optical fiber. Opt. Quantum Electron. **53**(10), 556 (2021)

Bekir, A., San, S.: The functional variable method to some complex nonlinear evolution equations. J. Mod. Math. Front. **1**(3), 5–9 (2012)

Boakye, G., Hosseini, K., Hinçal, E., Sirisubtawee, S., Osman, M.S.: Some models of solitary wave propagation in optical fibers involving Kerr and parabolic laws. Opt. Quant. Electron. **56**(3), 345 (2024)

Chowdhury, M.A., Miah, M.M., Iqbal, M.A., Alshehri, H.M., Baleanu, D., Osman, M.S.: Advanced exact solutions to the nano-ionic currents equation through MTs and the soliton equation containing the RLC transmission line. Eur. Phys. J. plus **138**, 502 (2023)

Elsayed, E.M.E., Alurrfi, K.A.E.: The generalized projective Riccati equations method and its applications for solving two nonlinear PDEs describing microtubules. Int. J. Phys. Sci. **10**, 391–402 (2015)

El-Sherif, A.A., Shoukry, M.M.: Copper (II) complexes of imino-bis (methyl phosphonic acid) with some bio-relevant ligands. Equilibrium studies and hydrolysis of glycine methyl ester through complex formation. J. Coord. Chem. **58**(16), 1401–1415 (2005)

Fan, E.: Extended tanh-function method and its applications to nonlinear equations. Phys. Lett. A **277**, 212–218 (2000)

Fan, E., Zhang, H.: A note on the homogeneous balance method. Phys. Lett. A **246**, 403–406 (1998)

Fetoh, A., Asla, K.A., El-Sherif, A.A., El-Didamony, H., El-Reash, G.M.A.: Synthesis, structural characterization, thermogravimetric, molecular modelling and biological studies of Co (II) and Ni (II) Schiff bases complexes. J. Mol. Struct. **1178**, 524–537 (2019)

Fokas, A.S., Lenells, J.: The unified method: I Nonlinearizable problems on the half-line. J. Phys. A Math. Theor. **45**(19), 195201 (2012)

Ganie, A.H., Sadek, L.H., Tharwat, M.M., Iqbal, M.A., Miah, M.M., Rasid, M.M., Elazab, N.S., Osman, M.M.: New investigation of the analytical behaviors for some nonlinear PDEs in mathematical physics and modern engineering. Part. Differ. Eqn. Appl. Math. **9**, 100608 (2024)

Habib, M.A., Ali, H.M.S., Miah, M.M., Akbar, M.A.: The generalized Kudryashov method for new closed form traveling wave solutions to some NLEEs. AIMS Math. **4**, 896–909 (2019)

Hosseini, K., Alizadeh, F., Hinçal, E., Baleanu, D., Akgül, A., Hassan, A.M.: Lie symmetries, bifurcation analysis, and Jacobi elliptic function solutions to the nonlinear Kodama equation. Result Phys. **54**, 107129 (2023a)

Hosseini, K., Sadri, K., Hinçal, E., Sirisubtawee, S., Mirzazadeh, M.: A generalized nonlinear Schrödinger involving the weak nonlocality: its Jacobi elliptic function solutions and modulational instability. Optik **288**, 171176 (2023b)

Hosseini, K., Hinçal, E., Ilie, M.: Bifurcation analysis, chaotic behaviors, sensitivity analysis, and soliton solutions of a generalized Schrödinger equation. Nonlinear Dyn. **111**(18), 17455–17462 (2023c)

Iqbal, M.A., Baleanu, D., Miah, M.M., Ali, H.M.S., Alshehri, H.M., Osman, M.S.: New soliton solutions of the mZK equation and the Gerdjikov-Ivanov equation by employing the double (G′/G, 1/G)-expansion method. Result Phys. **47**, 106391 (2023)

Irshad, A., Mohyud-din, S.T., Ahmed, N., Khan, U.: A new modification in simple equation method and its applications on nonlinear equations of physical nature. Result Phys. **7**, 4232–4240 (2017)

Islam, S., Khan, K., Arnous, A.H.: Generalized Kudryashov method for solving some (3+ 1)-dimensional nonlinear evolution equations. New Trends Math. Sci. **57**, 46–57 (2015)

Islam, S.M.R., Arafat, S.M.Y., Wang, H.: Abundant closed-form wave solutions to the simplified modified Camassa–Holm equation. J. Ocean Eng. Sci. **8**, 238–245 (2023)

Islam, M.N., Al-Amin, M., Akbar, A., Wazwaz, A.M., Osman, M.S.: Assorted optical soliton solutions of the nonlinear fractional model in optical fibers possessing beta derivative. Physica Scr. **99**(1), 015227 (2024)

Jafari, H., Kadkhoda, N., Baleanu, D.: Fractional Lie group method of the time-fractional Boussinesq equation. Nonlinear Dyn. **81**, 1569–1574 (2015)

Kaur, L.: Generalized (G’/G)-expansion method for generalized fifth order KdV equation with time-dependent coefficients. Math. Sci. Lett. **3**, 255–261 (2014)

Kumar, A., Pankaj, R.D.: Tanh–coth scheme for traveling wave solutions for Nonlinear Wave Interaction model. J. Egypt. Math. Soc. **23**, 282–285 (2015)

Kumar, D., Park, C., Tamanna, N., Paul, G.C., Osman, M.S.: Dynamics of two-mode Sawada-Kotera equation: mathematical and graphical analysis of its dual-wave solutions. Result Phys. **19**, 103581 (2020)

Kumar, D., Paul, G.C., Seadawy, A.R., Darvishi, M.T.: A variety of novel closed-form soliton solutions to the family of Boussinesq-like equations with different types. J. Ocean Eng. Sci. **7**, 543–554 (2022)

Ma, W.X.: AKNS type reduced integrable bi-Hamiltonian hierarchies with four potentials. Appl. Math. Lett. **145**, 108775 (2023)
Ma, W.X., Huang, T., Zhang, Y.: A multiple exp-function method for nonlinear differential equations and its application. Phys. Scr. **82**(6), 065003 (2010)
Mamun, A.A., Ananna, S.N., An, T., Asaduzzaman, M., Miah, M.M.: Solitary wave structures of a family of 3D fractional WBBM equation via the tanh–coth approach. Partial Differ. Equ. Appl. Math. **5**, 100237 (2022)
Mirzazadeh, M.: Topological and non-topological soliton solutions of Hamiltonian amplitude equation by He's semi-inverse method and ansatz approach. J. Egypt. Math. Soc. **23**, 292–296 (2015)
Mohanty, S.K., Kravchenko, O.V., Deka, M.K., Dev, A.N., Churikov, D.V.: The exact solutions of the 2+ 1–dimensional Kadomtsev-Petviashvili equation with variable coefficients by extended generalized G′ G-expansion method. J. King Saud Univ. - Sci. **35**, 102358 (2023)
Naher, H., Abdullah, F.A.: The basic (G'/G)-expansion method for the fourth order Boussinesq equation. Appl. Math. **03**, 1144–1152 (2012)
Parkes, E.J., Duffy, B.R.: An automated tanh-function method for finding solitary wave solutions to nonlinear evolution equations. Comput. Phys. Commun. **98**, 288–300 (1996)
Rehman, S.U., Ahmad, J.: Stability analysis and novel optical pulses to Kundu–Mukherjee–Naskar model in birefringent fibers. Int. J. Mod. Phys. B (2023). https://doi.org/10.1142/S0217979224501923
Rehman, S.U., Bilal, M., Ahmad, J.: Highly dispersive optical and other soliton solutions to fiber Bragg gratings with the application of different mechanisms. Int. J. Mod. Phys. B **36**(28), 2250193 (2022a)
Rehman, S.U., Bilal, M., Ahmad, J.: The study of solitary wave solutions to the time conformable Schrödinger system by a powerful computational technique. Opt. Quant. Electron. **54**, 228 (2022b)
Rehman, S.U., Ahmad, J., Muhammad, T.: Dynamics of novel exact soliton solutions to Stochastic Chiral Nonlinear Schrödinger Equation. Alex. Eng. J. **79**, 568–580 (2023)
Roshid, H.O., Kabir, M.R., Bhowmik, R.C., Datta, B.K.: Investigation of Solitary wave solutions for Vakhnenko–Parkes equation via exp-function and Exp (− ϕ (ξ))-expansion method. Springerplus **3**, 1–10 (2014)
Taghizadeh, N., Mirzazadeh, M.: The first integral method to some complex nonlinear partial differential equations. J. Comput. Appl. Math. **235**, 4871–4877 (2011)
Tandel, P., Patel, H., Patel, T.: Tsunami wave propagation model: A fractional approach. J. Ocean Eng. Sci. **7**, 509–520 (2022)
Tozar, A., Tasbozan, O., Kurt, A.: Optical soliton solutions for the (1+ 1)-dimensional resonant nonlinear Schröndinger's equation arising in optical fibers. Opt. Quantum Electron. **53**(6), 316 (2021)
Wang, M., Zhou, Y., Li, Z.: Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics. Phys. Lett. A **216**, 67–75 (1996)
Wen, X., Lü, D.: Extended Jacobi elliptic function expansion method and its application to nonlinear evolution equation. Chaos Solitons Fractals **41**, 1454–1458 (2009)
Yomba, E.: General projective Riccati equations method and exact solutions for a class of nonlinear partial differential equations. Chin. J. Phys. **43**, 991–1003 (2005)
Zayed, E.M.E., Alurrfi, K.A.E.: Extended auxiliary equation method and its applications for finding the exact solutions for a class of nonlinear Schrödinger-type equations. Appl. Math. Comput. **289**, 111–131 (2016)
Zhang, Z.Y.: Exact traveling wave solutions of the perturbed Klein-Gordon equation with quadratic nonlinearity in (1+ 1)-dimension, Part I: Without local inductance and dissipation effect. Turkish J. Phys. **37**, 259–267 (2013)

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