



# New optical quantum effects of ferromagnetic electroosmotic phase

Talat Körpınar<sup>1</sup> · Zeliha Körpınar<sup>2</sup>

Received: 30 July 2023 / Accepted: 28 August 2023 / Published online: 28 September 2023  
© The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2023

## Abstract

In this paper, we study electroosmotic microfluidic  $\tau_{\nabla, \phi(\alpha)}$ ,  $\tau_{\nabla, \phi(t)}$ ,  $\tau_{\nabla, \phi(s)}$  magnetic torques density with spherical fields. Also, we obtain optical geometric microfluidic  $\tau_{\nabla, \phi(\alpha)}$ ,  $\tau_{\nabla, \phi(t)}$ ,  $\tau_{\nabla, \phi(s)}$  magnetic torques phase microscale in wave phenomena. Then, we illustrate optical microfluidic ferromagnetic electroosmotic  $\tau_{\nabla, \phi(\alpha)}$ ,  $\tau_{\nabla, \phi(t)}$ ,  $\tau_{\nabla, \phi(s)}$  magnetic torques density. Thus, we present microfluidic ferromagnetic  $\tau_{\nabla, \phi(\alpha)}$ ,  $\tau_{\nabla, \phi(t)}$ ,  $\tau_{\nabla, \phi(s)}$  magnetic torques phase microscale in wave phenomena.

**Keywords** Optical quantum model · Electroosmotic microfluidic · Magnetic torque density · Magnetic phase

## 1 Introduction

The modeling of hybrid optical microfluid is an important dynamic analysis area in optical microfluidics and perspective of constructing optical models. Unique in flux devices with different views have been advanced by some directions geometric optics. Optical microfluidic is constructed by optophotonics, optoelectronics, environmental science, remote sensing, metamaterials, biomedicine and quantum geometric optics (Ashkin et al. 1986; Ashkin 1970; Dholakia and Zemánek 2010; Burns et al. 1989; Wang et al. 2005; Körpınar 2020; Körpınar et al. 2020, 2021a, b; Körpınar and Körpınar 2021a, b).

Optical modeling of spherical cooling, binding, trapping, sorting, optical forces, transporting has designed intensive physical applications. Optical modeling is important tool in optics, atomic physics, chemistry, biological science. Also, by a rapid progression of optical nanotechnology, geometric features have constructed diverse progress with microwave and nanometer phase (Chaumet and Nieto-Vesperinas 2001; Almaas and Brevik 2013; Moffitt et al. 2008; Chang et al. 2009; Fazal and Block 2011; Reiserer et al. 2013; Körpınar

---

✉ Talat Körpınar  
talatkorpınar@gmail.com

Zeliha Körpınar  
zelihakorpınar@gmail.com

<sup>1</sup> Department of Mathematics, Muş Alparslan University, 49250 Muş, Turkey

<sup>2</sup> Department of Administration, Muş Alparslan University, 49250 Muş, Turkey

et al. 2021a, b; Körpınar and Körpınar 2021a, b; Neirameh 2021; de Azevedo et al. 1982; Chou and Qu 2001; Marı Beffa et al. 2002; Calini and Ivey 2005).

Optical electroosmotic microfluidic of quasi electromagnetic and spherical geometric design for flux modeling is numerically important optical illustrations. Optical microfluidic flux is important significance in mathematical solitons, optical applications, wave geomety with electromagnetic energy (Körpınar et al. 2019a, b, c; Balakrishnan et al. 1990; Bliokh 2009; Marı Beffa 2009; Marı Beffa and Olver 2010; Calini et al. 2009; Wo and Qu 2007; Li et al. 2010; Körpınar and Körpınar 2021; Arbind et al. 2019; Körpınar 2020; Körpınar et al. 2020, 2021a, b; Körpınar and Körpınar 2021; Gürbüz 2022a, b, 2021).

The establishment of our paper is as follows. First, we study electroosmotic microfluidic  $\tau_{\nabla_i\phi(\alpha)}$ ,  $\tau_{\nabla_i\phi(t)}$ ,  $\tau_{\nabla_i\phi(s)}$  magnetic torques density with spherical fields. Also, we obtain optical geometric microfluidic  $\tau_{\nabla_i\phi(\alpha)}$ ,  $\tau_{\nabla_i\phi(t)}$ ,  $\tau_{\nabla_i\phi(s)}$  magnetic torques phase microscale. Then, we illustrate optical Microfluidic ferromagnetic electroosmotic  $\tau_{\nabla_i\phi(\alpha)}$ ,  $\tau_{\nabla_i\phi(t)}$ ,  $\tau_{\nabla_i\phi(s)}$  magnetic torques density. Thus, we present microfluidic ferromagnetic  $\tau_{\nabla_i\phi(\alpha)}$ ,  $\tau_{\nabla_i\phi(t)}$ ,  $\tau_{\nabla_i\phi(s)}$  magnetic torques phase microscale.

## 2 Spherical magnetic torque phase microscale

### 2.1 Microfluidic ferromagnetic $\tau_{\nabla_i\phi(\alpha)}$ magnetic torque phase

$\tau_{\nabla_i\phi(\alpha)}$  magnetic torque of  $\nabla_i\phi(\alpha)$  is constructed by

$$\tau_{\nabla_i\phi(\alpha)} = \Lambda \left( \beta_1 + \left( \Upsilon \beta_1 + \frac{\partial \beta_2}{\partial \sigma} \right) \omega \right) \mathbf{t}.$$

Thus, fluid of  $\tau_{\nabla_i\phi(\alpha)}$  magnetic torque is

$$\begin{aligned} \nabla_i \tau_{\nabla_i\phi(\alpha)} &= -\Lambda \beta_1 \left( \beta_1 + \left( \Upsilon \beta_1 + \frac{\partial \beta_2}{\partial \sigma} \right) \omega \right) \alpha \\ &+ \Lambda \frac{\partial}{\partial t} \left( \beta_1 + \left( \Upsilon \beta_1 + \frac{\partial \beta_2}{\partial \sigma} \right) \omega \right) \mathbf{t} + \Lambda \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \beta_1 + \left( \Upsilon \beta_1 + \frac{\partial \beta_2}{\partial \sigma} \right) \omega \right) \mathbf{s}. \end{aligned}$$

✱ *Electroosmotic microfluidic  $\tau_{\nabla_i\phi(\alpha)}$  magnetic torque density is*

$$\begin{aligned} \mathcal{V}_{\tau_{\phi(\alpha)}} &= \Lambda \beta_1^2 \left( \beta_1 + \left( \Upsilon \beta_1 + \frac{\partial \beta_2}{\partial \sigma} \right) \omega \right) \\ &+ \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right)^2 \Lambda \left( \beta_1 + \left( \Upsilon \beta_1 + \frac{\partial \beta_2}{\partial \sigma} \right) \omega \right). \end{aligned}$$

✱ *Microfluidic  $\tau_{\nabla_i\phi(\alpha)}$  magnetic torque phase microscale is given*

$$\begin{aligned} \mathcal{I}_{\tau_{\phi(\alpha)}} &= \tau_\rho^\alpha \int \int_{\mathcal{W}} \left( \beta_1^2 \Lambda \left( \beta_1 + \left( \Upsilon \beta_1 + \frac{\partial \beta_2}{\partial \sigma} \right) \omega \right) \right. \\ &\left. + \Lambda \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right)^2 \left( \beta_1 + \left( \Upsilon \beta_1 + \frac{\partial \beta_2}{\partial \sigma} \right) \omega \right) \right) d\mathcal{W}, \end{aligned}$$

where  $\tau_0^\alpha$  is magnetic torque potential for  $\nabla_i\phi(\alpha)$ .

Also, we easily get

$$\phi(\alpha) \times \nabla_{\sigma}^2 \phi(\alpha) = \frac{\partial \Upsilon}{\partial \sigma} \alpha.$$

※ Microfluidic ferromagnetic electroosmotic  $\tau_{\nabla, \phi(\alpha)}$  magnetic torque density is

$$\mathcal{V}_{\tau_{\phi(\alpha)}}^{\mathcal{F}} = -\Lambda \beta_1 \frac{\partial \Upsilon}{\partial \sigma} \left( \beta_1 + \left( \Upsilon \beta_1 + \frac{\partial \beta_2}{\partial \sigma} \right) \omega \right).$$

※ Microfluidic ferromagnetic  $\tau_{\nabla, \phi(\alpha)}$  magnetic torque phase microscale is

$$\mathcal{T}_{\tau_{\phi(\alpha)}}^{\mathcal{F}} = \tau_{\rho}^{\alpha} \int \int_{\mathcal{W}} \left( -\beta_1 \frac{\partial \Upsilon}{\partial \sigma} \Lambda \left( \beta_1 + \left( \Upsilon \beta_1 + \frac{\partial \beta_2}{\partial \sigma} \right) \omega \right) \right) d\mathcal{W},$$

where  $\tau_{\rho}^{\alpha}$  is magnetic torque potential for  $\nabla_t \phi(\alpha)$ .

With the flexibility of microfluidic ferromagnetic  $\tau_{\nabla, \phi(\alpha)}$  magnetic torque phase microscale, a variety of spherical effects can be illustrated for electroosmotic microfluidic  $\tau_{\nabla, \phi(\alpha)}$  magnetic torque metasurfaces, with diverse potential optical applications in Fig. 1.

### 2.2 Microfluidic ferromagnetic $\tau_{\nabla, \phi(t)}$ magnetic torque phase

$\tau_{\nabla, \phi(t)}$  magnetic torque of  $\nabla_t \phi(t)$  is presented by

$$\tau_{\nabla, \phi(t)} = -\Lambda \left( \beta_1 + \omega \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \right) \alpha + \Lambda \frac{\partial \omega}{\partial t} \omega t + \omega \left( \beta_1 + \omega \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \right) \Lambda s.$$

Flow of  $\tau_{\nabla, \phi(t)}$  magnetic torque is

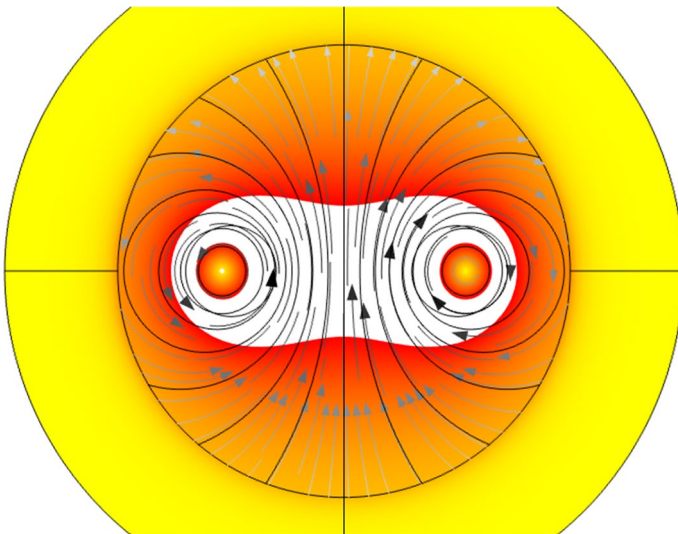


Fig. 1 Flexibility of microfluidic ferromagnetic  $\tau_{\nabla, \phi(\alpha)}$

$$\begin{aligned} \nabla_t \tau_{\nabla_t \phi(\mathbf{t})} = & -\Lambda \left( \frac{\partial}{\partial t} \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) + \beta_1 \left( \frac{\partial \omega}{\partial t} \right) \right. \\ & + \left( \omega \left( \beta_1 + \omega \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \right) \right) \beta_2 \boldsymbol{\alpha} + \Lambda \left( \frac{\partial}{\partial t} \left( \frac{\partial \omega}{\partial t} \right) - \beta_1 \left( \beta_1 \right. \right. \\ & + \left. \left. \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) - \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \omega \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} \right. \right. \right. \right. \\ & + \left. \left. \left. \Upsilon \beta_1 \right) \omega \right) \right) \right) \mathbf{t} + \Lambda \left( \frac{\partial}{\partial t} \left( \omega \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \right) - \beta_2 \left( \beta_1 \right. \right. \\ & \left. \left. + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) + \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \omega \frac{\partial \omega}{\partial t} \right) \right) \mathbf{s}. \end{aligned}$$

✱ Electroosmotic microfluidic  $\tau_{\nabla_t \phi(\mathbf{t})}$  magnetic torque density is

$$\begin{aligned} \mathcal{V}_{\tau_{\phi(\mathbf{t})}} = & \Lambda \omega \beta_2 \left( \frac{\partial}{\partial t} \left( \omega \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) + \beta_1 \right) + \beta_1 \left( \omega \frac{\partial \omega}{\partial t} \right) \right. \\ & + \left( \omega \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \right) \beta_2 - \Lambda \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} \right. \right. \\ & + \left. \left. \Upsilon \beta_1 \right) \omega \right) \left( \frac{\partial}{\partial t} \left( \omega \frac{\partial \omega}{\partial t} \right) - \left( \beta_1 + \omega \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \right) \beta_1 - \left( \Upsilon \beta_1 \right. \right. \\ & + \left. \left. \frac{\partial \beta_2}{\partial \sigma} \right) \left( \omega \left( \beta_1 + \omega \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \right) \right) \right) + \Lambda \left( -\beta_2 \right. \\ & + \left. \frac{\partial \omega}{\partial t} \right) \left( \frac{\partial}{\partial t} \left( \left( \beta_1 + \omega \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \right) \omega \right) - \beta_2 \left( \beta_1 \right. \right. \\ & \left. \left. + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) + \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \omega \frac{\partial \omega}{\partial t} \right) \right). \end{aligned}$$

✱ Microfluidic  $\tau_{\nabla_t \phi(\mathbf{t})}$  magnetic torque phase microscale is given

$$\begin{aligned} \mathcal{T}_{\tau_{\phi(\mathbf{t})}} = & \tau_\rho^t \int \int_{\mathcal{W}} \left( \left( -\beta_2 + \frac{\partial \omega}{\partial t} \right) \Lambda \left( \frac{\partial}{\partial t} \left( \omega \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \right) \right. \right. \\ & - \beta_2 \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) + \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \omega \frac{\partial \omega}{\partial t} \right) + \beta_2 \omega \Lambda \left( \frac{\partial}{\partial t} \left( \beta_1 \right. \right. \\ & + \left. \left. \omega \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \right) + \beta_1 \left( \omega \frac{\partial \omega}{\partial t} \right) + \left( \omega \left( \beta_1 + \omega \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \right) \right) \beta_2 \right) \\ & - \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \Lambda \left( \frac{\partial}{\partial t} \left( \frac{\partial \omega}{\partial t} \right) - \beta_1 \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \right) \\ & \left. - \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \omega \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \right) \right) d\mathcal{W}, \end{aligned}$$

where  $\tau_\rho^t$  is magnetic torque potential for  $\nabla_t \phi(\mathbf{t})$ .

Thus, we have

$$\begin{aligned} \phi(\mathbf{t}) \times \nabla_{\sigma}^2 \phi(\mathbf{t}) &= \omega \left( \omega \frac{\partial \Upsilon}{\partial \sigma} + 2 \frac{\partial \omega}{\partial \sigma} \Upsilon \right) \boldsymbol{\alpha} + \left( \frac{\partial^2 \omega}{\partial \sigma^2} \right. \\ &\quad \left. + (\omega - \Upsilon)(\omega \Upsilon + 1) \right) \mathbf{t} - \left( \frac{\partial \Upsilon}{\partial \sigma} \omega + 2 \Upsilon \frac{\partial \omega}{\partial \sigma} \right) \mathbf{s}. \end{aligned}$$

✳ *Ferromagnetic electroosmotic  $\tau_{\nabla_t \phi(\mathbf{t})}$  magnetic torque density is*

$$\begin{aligned} \mathcal{V}_{\tau_{\phi(\mathbf{t})}}^{\mathcal{F}} &= -\Lambda \left( \frac{\partial}{\partial t} \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) + \beta_1 \left( \omega \frac{\partial \omega}{\partial t} \right) + \left( \omega \left( \beta_1 \right. \right. \right. \\ &\quad \left. \left. + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \right) \beta_2 \omega \left( \omega \frac{\partial \Upsilon}{\partial \sigma} + 2 \Upsilon \frac{\partial \omega}{\partial \sigma} \right) + \Lambda \left( \frac{\partial}{\partial t} \left( \omega \frac{\partial \omega}{\partial t} \right) \right. \\ &\quad \left. - \beta_1 \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) - \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \omega \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} \right. \right. \right. \right. \right. \\ &\quad \left. \left. + \Upsilon \beta_1 \right) \omega \right) \right) \left( \frac{\partial^2 \omega}{\partial \sigma^2} + (\omega - \Upsilon)(\omega \Upsilon + 1) \right) - \Lambda \left( \frac{\partial}{\partial t} \left( \omega \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \right) \right) \\ &\quad \left. - \beta_2 \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) + \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \omega \frac{\partial \omega}{\partial t} \right) \right) \left( \frac{\partial \Upsilon}{\partial \sigma} \omega + 2 \Upsilon \frac{\partial \omega}{\partial \sigma} \right). \end{aligned}$$

✳ *Microfluidic ferromagnetic  $\tau_{\nabla_t \phi(\mathbf{t})}$  magnetic torque phase microscale is*

$$\begin{aligned} \mathcal{T}_{\tau_{\phi(\mathbf{t})}}^{\mathcal{F}} &= \tau_{\rho}^t \int \int_{\mathcal{W}} \left( \left( \frac{\partial}{\partial t} \left( \omega \frac{\partial \omega}{\partial t} \right) - \beta_1 \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) - \left( \beta_1 \Upsilon \right. \right. \right. \\ &\quad \left. \left. + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \omega \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \right) \right) \left( \frac{\partial^2 \omega}{\partial \sigma^2} + (\omega - \Upsilon)(\omega \Upsilon + 1) \right) \Lambda - \left( \frac{\partial}{\partial t} \left( \beta_1 \right. \right. \\ &\quad \left. \left. + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) + \beta_1 \left( \omega \frac{\partial \omega}{\partial t} \right) + \left( \omega \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \right) \beta_2 \right) \omega \left( \omega \frac{\partial \Upsilon}{\partial \sigma} \right. \\ &\quad \left. + 2 \Upsilon \frac{\partial \omega}{\partial \sigma} \right) \Lambda - \left( \frac{\partial}{\partial t} \left( \omega \left( \beta_1 + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) \right) - \beta_2 \left( \beta_1 \right. \right. \\ &\quad \left. \left. + \left( \frac{\partial \beta_2}{\partial \sigma} + \Upsilon \beta_1 \right) \omega \right) + \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \omega \frac{\partial \omega}{\partial t} \right) \right) \left( \frac{\partial \Upsilon}{\partial \sigma} \omega + 2 \Upsilon \frac{\partial \omega}{\partial \sigma} \right) \Lambda \right) d\mathcal{W}, \end{aligned}$$

where  $\tau_{\rho}^t$  is magnetic torque potential for  $\nabla_t \phi(\mathbf{t})$ .

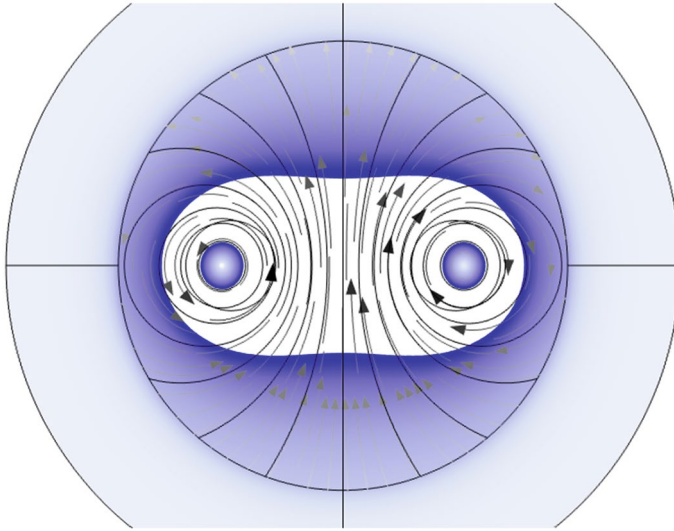
With the flexibility of microfluidic ferromagnetic  $\tau_{\nabla_t \phi(\mathbf{t})}$  magnetic torque phase microscale, a variety of spherical effects can be illustrated for electroosmotic microfluidic  $\tau_{\nabla_t \phi(\mathbf{t})}$  magnetic torque metasurfaces, with diverse potential optical applications in Fig. 2.

### 2.3 Microfluidic ferromagnetic $\tau_{\nabla_t \phi(\mathbf{s})}$ magnetic torque phase

$\tau_{\nabla_t \phi(\mathbf{s})}$  magnetic torque microscale is

$${}^B \mathcal{M}_{\phi(\mathbf{s})} = \tau_{\rho}^s \int \int_{\mathcal{W}} \nabla_t \phi(\mathbf{s}) \cdot \nabla_t \tau_{\phi(\mathbf{s})} d\mathcal{W}.$$

$\tau_{\nabla_t \phi(\mathbf{t})}$  magnetic torque of  $\nabla_t \phi(\mathbf{t})$  is presented by



**Fig. 2** Flexibility of microfluidic antiferromagnetic  $\tau_{\nabla, \phi(t)}$

$$\tau_{\nabla, \phi(s)} = -\Lambda \frac{\partial \omega}{\partial t} \alpha - \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 + \beta_1 \omega \right) \Lambda t + \frac{\partial \omega}{\partial t} \omega \Lambda s.$$

Thus, fluid of  $\tau_{\nabla, \phi(s)}$  magnetic torque is

$$\begin{aligned} \nabla_t \tau_{\nabla, \phi(s)} = & \left( \beta_1 \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 + \beta_1 \omega \right) - \frac{\partial}{\partial t} \frac{\partial \omega}{\partial t} - \beta_2 \left( \frac{\partial \omega}{\partial t} \omega \right) \right) \Lambda \alpha \\ & - \left( \frac{\partial}{\partial t} \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 + \beta_1 \omega \right) + \frac{\partial \omega}{\partial t} \beta_1 + \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \frac{\partial \omega}{\partial t} \omega \right) \right) \Lambda t \\ & + \left( \frac{\partial}{\partial t} \left( \frac{\partial \omega}{\partial t} \omega \right) - \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 + \beta_1 \omega \right) \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) - \frac{\partial \omega}{\partial t} \beta_2 \right) \Lambda s. \end{aligned}$$

✳ *Electroosmotic microfluidic  $\tau_{\nabla, \phi(s)}$  magnetic torque density is*

$$\begin{aligned} \mathcal{V}_{\tau_{\phi(s)}} = & \frac{\partial \omega}{\partial t} \Lambda \left( \frac{\partial}{\partial t} \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 + \beta_1 \omega \right) + \frac{\partial \omega}{\partial t} \beta_1 \right. \\ & + \left. \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \frac{\partial \omega}{\partial t} \omega \right) \right) + \Lambda \left( \beta_1 \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 + \beta_1 \omega \right) \right. \\ & - \left. \frac{\partial}{\partial t} \frac{\partial \omega}{\partial t} - \beta_2 \left( \frac{\partial \omega}{\partial t} \omega \right) \right) \beta_1 \omega - \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega \left( \frac{\partial}{\partial t} \left( \frac{\partial \omega}{\partial t} \omega \right) \right) \\ & - \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 + \beta_1 \omega \right) \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) - \frac{\partial \omega}{\partial t} \beta_2 \right) \Lambda. \end{aligned}$$

✳ *Microfluidic  $\tau_{\nabla, \phi(s)}$  magnetic torque phase microscale is given*

$$\begin{aligned} \mathcal{T}_{\tau_{\phi(s)}} = & \tau_{\rho}^s \int \int_{\mathcal{W}} \left( - \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega \left( \frac{\partial}{\partial t} \left( \frac{\partial \omega}{\partial t} \omega \right) - \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 \right. \right. \right. \\ & \left. \left. \left. + \beta_1 \omega \right) \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) - \frac{\partial \omega}{\partial t} \beta_2 \right) \Lambda + \left( \beta_1 \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 \right. \right. \right. \\ & \left. \left. \left. + \beta_1 \omega \right) - \frac{\partial}{\partial t} \frac{\partial \omega}{\partial t} - \beta_2 \left( \frac{\partial \omega}{\partial t} \omega \right) \right) \beta_1 \omega \Lambda + \Lambda \frac{\partial \omega}{\partial t} \left( \frac{\partial}{\partial t} \left( \left( \frac{\partial \beta_2}{\partial \sigma} \right. \right. \right. \right. \\ & \left. \left. \left. + \beta_1 \Upsilon \right) \omega^2 + \beta_1 \omega \right) + \frac{\partial \omega}{\partial t} \beta_1 + \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) \left( \frac{\partial \omega}{\partial t} \omega \right) \right) d\mathcal{W}, \end{aligned}$$

where  $\tau_0^s$  is magnetic torque potential for  $\nabla_t \phi(s)$ .

Also, we easily get

$$\phi(s) \times \nabla_{\sigma}^2 \phi(s) = \omega \left( \omega \frac{\partial \Upsilon}{\partial \sigma} + 2 \frac{\partial \omega}{\partial \sigma} \Upsilon \right) \alpha + 2 \frac{\partial \omega}{\partial \sigma} \omega s.$$

✱ *Microfluidic ferromagnetic electroosmotic  $\tau_{\nabla_t \phi(s)}$  magnetic torque density is*

$$\begin{aligned} \mathcal{V}_{\tau_{\phi(s)}}^{\mathcal{F}} = & \left( \beta_1 \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 + \beta_1 \omega \right) - \frac{\partial}{\partial t} \frac{\partial \omega}{\partial t} \right. \\ & \left. - \beta_2 \left( \frac{\partial \omega}{\partial t} \omega \right) \right) \omega \left( \omega \frac{\partial \Upsilon}{\partial \sigma} + 2 \frac{\partial \omega}{\partial \sigma} \Upsilon \right) \Lambda + \left( \frac{\partial}{\partial t} \left( \frac{\partial \omega}{\partial t} \omega \right) - \left( \left( \frac{\partial \beta_2}{\partial \sigma} \right. \right. \right. \\ & \left. \left. \left. + \beta_1 \Upsilon \right) \omega^2 + \beta_1 \omega \right) \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) - \frac{\partial \omega}{\partial t} \beta_2 \right) 2 \frac{\partial \omega}{\partial \sigma} \omega \Lambda. \end{aligned}$$

✱ *Microfluidic ferromagnetic  $\tau_{\nabla_t \phi(s)}$  magnetic torque phase microscale is*

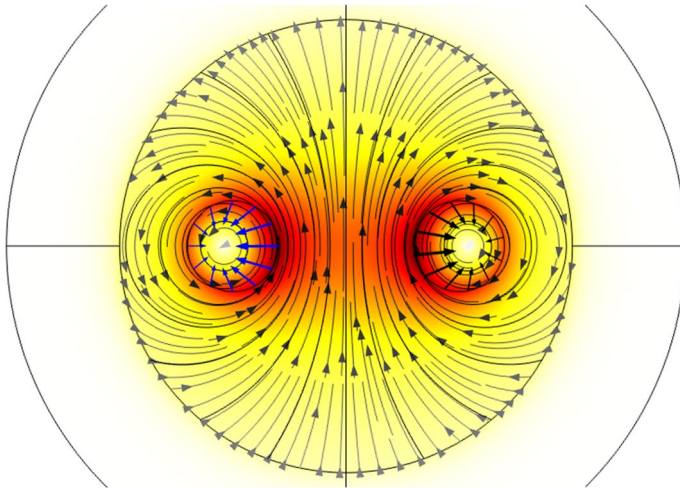
$$\begin{aligned} \mathcal{T}_{\tau_{\phi(s)}}^{\mathcal{F}} = & \tau_{\rho}^s \int \int_{\mathcal{W}} \left( 2\Lambda \frac{\partial \omega}{\partial \sigma} \omega \left( \frac{\partial}{\partial t} \left( \frac{\partial \omega}{\partial t} \omega \right) - \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 \right. \right. \right. \\ & \left. \left. \left. + \beta_1 \omega \right) \left( \beta_1 \Upsilon + \frac{\partial \beta_2}{\partial \sigma} \right) - \frac{\partial \omega}{\partial t} \beta_2 \right) + \Lambda \left( \beta_1 \left( \left( \frac{\partial \beta_2}{\partial \sigma} + \beta_1 \Upsilon \right) \omega^2 \right. \right. \right. \\ & \left. \left. \left. + \beta_1 \omega \right) - \frac{\partial}{\partial t} \frac{\partial \omega}{\partial t} - \beta_2 \left( \frac{\partial \omega}{\partial t} \omega \right) \right) \omega \left( \omega \frac{\partial \Upsilon}{\partial \sigma} + 2 \frac{\partial \omega}{\partial \sigma} \Upsilon \right) \right) d\mathcal{W}, \end{aligned}$$

where  $\tau_{\rho}^s$  is magnetic torque potential for  $\nabla_t \phi(s)$ .

With the flexibility of microfluidic ferromagnetic  $\tau_{\nabla_t \phi(s)}$  magnetic torque phase microscale, a variety of spherical effects can be illustrated for electroosmotic microfluidic  $\tau_{\nabla_t \phi(s)}$  magnetic torque metasurfaces, with diverse potential optical applications in Fig. 3.

### 3 Conclusions

The concept for geometric microfluidic is constructed by magnetic nano fluids, electromagnetic nanoparticles with optical heat transfer fluid and some optical applications (Ashkin et al. 1986; Ashkin 1970; Dholakia and Zemánek 2010; Burns et al. 1989; Chaumet and Nieto-Vesperinas 2001; Almaas and Brevik 2013; Körpınar and Körpınar 2021; Körpınar et al. 2021a, b; Yépez-Martínez et al. 2022; Rehman et al. 2022; Bhambere and Durge 2022; Zafar et al. 2023; Singh et al. 2023; Raza et al. 2023; Kang et al. 2022; Viscarra



**Fig. 3** Flexibility of microfluidic ferromagnetic  $\tau_{\nabla_i\phi(s)}$

and Urzagasti 2022; Körpınar et al. 2023a; Körpınar and Körpınar 2023a, b, c; Körpınar et al. 2023b; Körpınar and Körpınar 2023d, e; Körpınar et al. 2023; Körpınar and Körpınar 2023f; Körpınar et al. 2023; Körpınar and Körpınar 2023).

In this paper, we obtain optical geometric microfluidic  $\tau_{\nabla_i\phi(\alpha)}$ ,  $\tau_{\nabla_i\phi(t)}$ ,  $\tau_{\nabla_i\phi(s)}$  magnetic torques phase microscale. Then, we illustrate optical microfluidic ferromagnetic electroosmotic  $\tau_{\nabla_i\phi(\alpha)}$ ,  $\tau_{\nabla_i\phi(t)}$ ,  $\tau_{\nabla_i\phi(s)}$  magnetic torques density. Thus, we present microfluidic ferromagnetic  $\tau_{\nabla_i\phi(\alpha)}$ ,  $\tau_{\nabla_i\phi(t)}$ ,  $\tau_{\nabla_i\phi(s)}$  magnetic torques phase microscale.

**Author contributions** All authors of this research paper have directly participated in the planning, execution, or analysis of this study; All authors of this paper have read and approved the final version submitted.

**Funding** No funding was received for the study.

**Data availability** No data was used for the research described in the article.

## Declarations

**Conflict of interest** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Ethical approval** The contents of this manuscript have not been copyrighted or published previously; The contents of this manuscript are not now under consideration for publication elsewhere.

## References

- Almaas, E., Brevik, I.: Possible sorting mechanism for microparticles in an evanescent field. *Phys. Rev. A* **87**, 063826 (2013)
- Almaas, E., Brevik, I.: Possible sorting mechanism for microparticles in an evanescent field. *Phys. Rev. A* **87**, 063826 (2013)



- Arbind, A., Reddy, J.N., Srinivasa, A.R.: A nonlinear 1-D finite element analysis of rods/tubes made of incompressible neo-Hookean materials using higher-order theory. *Int. J. Solids Struct.* **166**, 1–21 (2019)
- Ashkin, A.: Acceleration and trapping of particles by radiation pressure. *Phys. Rev. Lett.* **24**, 156–159 (1970)
- Ashkin, A.: Acceleration and trapping of particles by radiation pressure. *Phys. Rev. Lett.* **24**, 156–159 (1970)
- Ashkin, A., Dziedzic, J.M., Bjorkholm, J.E., Chu, S.: Observation of a single-beam gradient force optical trap for dielectric particles. *Opt. Lett.* **11**, 288–290 (1986)
- Ashkin, A., Dziedzic, J.M., Bjorkholm, J.E., Chu, S.: Observation of a single-beam gradient force optical trap for dielectric particles. *Opt. Lett.* **11**, 288–290 (1986)
- Balakrishnan, R., Bishop, A.R., Dandoloff, R.: Geometric phase in the classical continuous antiferromagnetic Heisenberg spin chain. *Phys. Rev. Lett.* **64**(18), 2107 (1990)
- Bhambere, C.M., Durge, N.G.: Synthesis, growth and characterization of L-Leucine magnesium Nitrate Hexahydrate crystal. *J. Nonlinear Opt. Phys. Mater.* **31**(02), 2250007 (2022)
- Bliokh, K.Y.: Geometrodynamics of polarized light: Berry phase and spin Hall effect in a gradient-index medium. *J. Opt. A Pure Appl. Opt.* **11**(9), 094009 (2009)
- Burns, M.M., Fournier, J.-M., Golovchenko, J.A.: Optical binding. *Phys. Rev. Lett.* **63**, 1233–1236 (1989)
- Burns, M.M., Fournier, J.-M., Golovchenko, J.A.: Optical binding. *Phys. Rev. Lett.* **63**, 1233–1236 (1989)
- Calini, A., Ivey, T.: Finite-gap solutions of the vortex filament equation genus one solutions and symmetric solutions. *J. Nonlinear Sci.* **15**, 321–361 (2005)
- Calini, A., Ivey, T., Marí Beffa, G.: Remarks on KdV-type flows on star-shaped curves. *Phys. D* **238**, 788–797 (2009)
- Chang, D.E., Thompson, J.D., Park, H., Vuletić, V., Zibrov, A.S., et al.: Trapping and manipulation of isolated atoms using nanoscale plasmonic structures. *Phys. Rev. Lett.* **103**, 123004 (2009)
- Chaumet, P.C., Nieto-Vesperinas, M.: Optical binding of particles with or without the presence of a flat dielectric surface. *Phys. Rev. B* **64**, 035422 (2001)
- Chaumet, P.C., Nieto-Vesperinas, M.: Optical binding of particles with or without the presence of a flat dielectric surface. *Phys. Rev. B* **64**, 035422 (2001)
- Chou, K.S., Qu, C.Z.: The KdV equation and motion of plane curves. *J. Phys. Soc. Jpn.* **70**, 1912–1916 (2001)
- de Azevedo, L.G., de Moura, M.A., Cordeiro, C., Zeks, B.: Solitary waves in a 1D isotropic Heisenberg ferromagnet. *J. Phys. C Solid State Phys.* **15**(36), 7391–7396 (1982)
- Dholakia, K., Zemánek, P.: Colloquium: gripped by light: optical binding. *Rev. Mod. Phys.* **82**, 1767–1791 (2010)
- Dholakia, K., Zemánek, P.: Colloquium: gripped by light: optical binding. *Rev. Mod. Phys.* **82**, 1767–1791 (2010)
- Fazal, F.M., Block, S.M.: Optical tweezers study life under tension. *Nat. Photon.* **5**, 318–321 (2011)
- Gürbüz, N.E.: The evolution of the electric field with Frenet frame in Lorentzian Lie groups. *Optik* **247**, 167989 (2021)
- Gürbüz, N.E.: The change of electric field and relaxed elastic line via anholonomic coordinates with Darboux frame in R13. *Optik* **270**, 170023 (2022)
- Gürbüz, N.E.: The evolution of electric field in pseudo-Galilean 3-space G13. *Optik* **269**, 169818 (2022)
- Kang, J., Chen, S., Yang, W.: Measuring far-field beam divergence angle of supercontinuum fiber sources. *J. Nonlinear Opt. Phys. Mater.* **31**, 2350003 (2022)
- Körpınar, T., Körpınar, Z.: Antiferromagnetic viscosity model for electromotive microscale with second type nonlinear heat frame. *Int. J. Geom. Methods Modern Phys.* 2350163 (2023)
- Körpınar, T., Körpınar, Z.: New optical geometric recursional electromagnetic ferromagnetic microscale. *Int. J. Modern Phys. B* 2450092 (2023)
- Körpınar, Z., Körpınar, T.: New optical recursional spherical ferromagnetic flux for optical sonic microscale. *J. Nonlinear Opt. Phys. Mater.* 2350051 (2023)
- Körpınar, T., Körpınar, Z.: Optical visco microfluidic optimistic hybrid optical electromotive microscale. *Int. J. Modern Phys. B* 2450159 (2023)
- Körpınar, T.: Optical directional binormal magnetic flows with geometric phase: Heisenberg ferromagnetic model. *Optik* **219**, 165134 (2020)
- Körpınar, T.: Optical directional binormal magnetic flows with geometric phase: Heisenberg ferromagnetic model. *Optik* **219**, 165134 (2020)
- Körpınar, T., Körpınar, Z.: Timelike spherical magnetic  $S_N$  flux flows with Heisenberg sphericalferromagnetic spin with some solutions. *Optik* **242**, 166745 (2021)

- Körpınar, T., Körpınar, Z.: Spherical electric and magnetic phase with Heisenberg spherical ferromagnetic spin by some fractional solutions. *Optik* **242**, 167164 (2021)
- Körpınar, Z., Körpınar, T.: New approach for optical  $S$ -spherical electromagnetic phase by Landau Lifshitz approach. *Optik* **247**, 167906 (2021)
- Körpınar, Z., Körpınar, T.: Optical tangent hybrid electromotives for tangent hybrid magnetic particle. *Optik* **247**, 167823 (2021)
- Körpınar, T., Körpınar, Z.: Optical spherical  $S_s$ -electric and magnetic phase with fractional  $q$ -HATM approach. *Optik* **243**, 167274 (2021)
- Körpınar, T., Körpınar, Z.: Timelike spherical magnetic  $S_N$  flux flows with Heisenberg spherical ferromagnetic spin with some solutions. *Optik* **242**, 166745 (2021)
- Körpınar, T., Körpınar, Z.: New version of optical spherical electric and magnetic flow phase with some fractional solutions in  $S_{\text{Heis}}^2$ . *Optik* **243**, 167378 (2021)
- Körpınar, T., Körpınar, Z.: Antiferromagnetic Schrödinger electromotive microscale in Minkowski space. *Opt. Quantum Electron.* **55**(8), 681 (2023)
- Körpınar, T., Körpınar, Z.: Antiferromagnetic complex electromotive microscale with first type Schrödinger frame. *Opt. Quantum Electron.* **55**(6), 505 (2023)
- Körpınar, T., Körpınar, Z.: Optical phase of recursion hybrid visco ferromagnetic electromagnetic microscale. *Phys. Lett. A* **462**, 128651 (2023)
- Körpınar, T., Demirkol, R.C., Körpınar, Z.: Soliton propagation of electromagnetic field vectors of polarized light ray traveling along with coiled optical fiber on the unit 2-sphere  $S^2$ . *Rev. Mex. Fis.* **65**(6), 626–633 (2019)
- Körpınar, T., Demirkol, R.C., Körpınar, Z.: Soliton propagation of electromagnetic field vectors of polarized light ray traveling in a coiled optical fiber in Minkowski space with Bishop equations. *Eur. Phys. J. D* **73**(9), 203 (2019)
- Körpınar, T., Demirkol, R.C., Körpınar, Z.: Soliton propagation of electromagnetic field vectors of polarized light ray traveling in a coiled optical fiber in the ordinary space. *Int. J. Geom. Methods Modern Phys.* **16**(8), 1950117 (2019)
- Körpınar, T., Körpınar, Z., Demirkol, R.C.: Binormal schrodinger system of wave propagation field of light radiate in the normal direction with  $q$ -HATM approach. *Optik* **235**, 166444 (2020)
- Körpınar, T., Körpınar, Z., Yeneroğlu, M.: Optical energy of spherical velocity with optical magnetic density in Heisenberg sphere space  $S_{\text{Heis}}^2$ . *Optik* **247**, 167937 (2021)
- Körpınar, T., Sazak, A., Körpınar, Z.: Optical effects of some motion equations on quasi-frame with compatible Hasimoto map. *Optik* **247**, 167914 (2021)
- Körpınar, T., Demirkol, R.C., Körpınar, Z.: Polarization of propagated light with optical solitons along the fiber in de-sitter space. *Optik* **226**, 165872 (2021)
- Körpınar, T., Demirkol, R.C., Körpınar, Z.: Approximate solutions for the inextensible Heisenberg antiferromagnetic flow and solitonic magnetic flux surfaces in the normal direction in Minkowski space. *Optik* **238**, 166403 (2021)
- Körpınar, T., Demirkol, R.C., Körpınar, Z.: Magnetic helicity and electromagnetic vortex filament flows under the influence of Lorentz force in MHD. *Optik* **242**, 167302 (2021)
- Körpınar, T., Demirkol, R.C., Körpınar, Z.: New analytical solutions for the inextensible Heisenberg ferromagnetic flow and solitonic magnetic flux surfaces in the binormal direction. *Phys. Scr.* **96**(8), 085219 (2021)
- Körpınar, T., Demirkol, R.C., Körpınar, Z.: Polarization of propagated light with optical solitons along the fiber in de-sitter space. *Optik* **226**, 165872 (2021)
- Körpınar, T., Demirkol, R.C., Körpınar, Z.: Approximate solutions for the inextensible Heisenberg antiferromagnetic flow and solitonic magnetic flux surfaces in the normal direction in Minkowski space. *Optik* **238**, 166403 (2021)
- Körpınar, T., Ünlütürk, Y., Körpınar, Z.: A novel approach to the motion equations of null Cartan curves via the compatible Hasimoto map. *Optik* **290**, 171220 (2023)
- Körpınar, T., Demirkol, R.C., Körpınar, Z.: On the new conformable optical ferromagnetic and antiferromagnetic magnetically driven waves. *Opt. Quantum Electron.* **55**(6), 496 (2023)
- Körpınar, Z., Inc, M., Körpınar, T.: Ferromagnetic recursion for geometric phase timelike SN-magnetic fibers. *Opt. Quantum Electron.* **55**(4), 382 (2023)
- Körpınar, T., Körpınar, Z., Asil, V.: Optical electromotive microscale with first type Schrödinger frame. *Optik* **276**, 170629 (2023)
- Li, Y.Y., Qu, C.Z., Shu, S.C.: Integrable motions of curves in projective geometries. *J. Geom. Phys.* **60**, 972–985 (2010)
- Marí Beffa, G.: Hamiltonian evolution of curves in classical affine geometries. *Phys. D* **238**, 100–115 (2009)

- Marí Beffa, G., Olver, P.J.: Poisson structure for geometric curve flows in semi-simple homogeneous spaces. *Regul. Chaotic Dyn.* **15**, 532–550 (2010)
- Marí Beffa, G., Sanders, J.A., Wang, J.P.: Integrable systems in three-dimensional Riemannian geometry. *J. Nonlinear Sci.* **12**, 143–167 (2002)
- Moffitt, J.R., Chemla, Y.R., Smith, S.B., Bustamante, C.: Recent advances in optical tweezers. *Annu. Rev. Biochem.* **77**, 205–228 (2008)
- Neirameh, A.: Solitary wave solutions to the multidimensional Landau-Lifshitz equation. *Adv. Math. Phys.* **2021**, 5538516 (2021)
- Raza, N., Arshed, S., Butt, A.R., Inc, M., Yao, S.W.: Investigation of new solitons in nematic liquid crystals with Kerr and non-Kerr law nonlinearities. *J. Nonlinear Opt. Phys. Mater.* **32**(2), 2350020 (2023)
- Rehman, S.U., Bilal, M., Inc, M., Younas, U., Rezazadeh, H., Younis, M., Mirhosseini-Alizamini, S.M.: Investigation of pure-cubic optical solitons in nonlinear optics. *Opt. Quantum Electron.* **54**(7), 1–21 (2022)
- Reiserer, A., Nölleke, C., Ritter, S., Rempe, G.: Ground-state cooling of a single atom at the center of an optical cavity. *Phys. Rev. Lett.* **110**, 223003 (2013)
- Singh, D., Sharma, B.S., Singh, M.: Quantum corrections on threshold and growth rate of modulational amplification in semiconductor magneto-plasmas. *J. Nonlinear Opt. Phys. Mater.* **32**(01), 2350009 (2023)
- Viscarra, M.A., Urzagasti, D.: Dark soliton solutions of the cubic-quintic complex Ginzburg-Landau equation with high-order terms and potential barriers in normal-dispersion fiber lasers. *J. Nonlinear Opt. Phys. Mater.* **31**(01), 2250003 (2022)
- Wang, M.M., Tu, E., Raymond, D.E., Yang, J.M., Zhang, H., et al.: Microfluidic sorting of mammalian cells by optical force switching. *Nat. Biotechnol.* **23**, 83–87 (2005)
- Wo, W.F., Qu, C.Z.: Integrable motions of curves in  $S^1$ . *J. Geom. Phys.* **57**, 1733–1755 (2007)
- Yépez-Martínez, H., Rezazadeh, H., Gómez-Aguilar, J.F., Inc, M.: A new local fractional derivative applied to the analytical solutions for the nonlinear Schrödinger equation with third-order dispersion. *J. Nonlinear Opt. Phys. Mater.* **31**(03), 2250011 (2022)
- Zafar, A., Shakeel, M., Ali, A., Rezazadeh, H., Bekir, A.: Analytical study of complex Ginzburg-Landau equation arising in nonlinear optics. *J. Nonlinear Opt. Phys. Mater.* **32**(01), 2350010 (2023)

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.