



Enhanced laser wakefield acceleration by a circularly polarized laser pulse in obliquely magnetized under-dense plasma

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Received: 4 July 2023 / Accepted: 27 August 2023 / Published online: 7 October 2023
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Abstract

Laser wakefield acceleration is one of the prominent methods to obtain high energy charge particles. This method can be used to accelerate lighter charged particles like electrons to relativistic energy level. Energy enhancement of electrons depends on the properties of laser pulse as well as plasma. In this study, we have used a circular polarized laser pulse propagating (along z - axis) through under-dense plasma. An external oblique transverse static magnetic field is applied an arbitrary angle (at angle θ with Y -axis in x - y plane). Effect of laser pulse strength (a), strength of external magnetic field (B_0), and its arbitrary angle (θ) on generated longitudinal wakefield, wake potential, change in relativistic factor and energy enhancement are calculated theoretically and curves are drawn. Optimized oblique angle for maximum wakefield and potential is calculated for selected parameters. This study is useful for obtaining an energy efficient acceleration technique for electron.

Keywords Laser wakefield acceleration · Circular polarized laser pulse · Oblique magnetic field · Energy gain · Energy efficiency

1 Introduction

High energy charged particles are of great importance and significance due to several applications. Internal structure of atoms can be studied with the help of X- ray diffraction experiment. Production of X-rays is possible when fast-moving electrons collide with heavy ions or atoms. Similarly controlled nuclear fusion is possible with the help of fast-moving charged particles. During past few decades, the phenomenon of acceleration of charged particles using laser beam is dominant over other conventional methods. Laser beat wave accelerator [LBWA], Plasma wakefield acceleration [PWFA], Laser wakefield acceleration [LWFA] are some of the major techniques to produce high potential gradient.

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In Laser wake-field acceleration, a laser beam produces a wakefield in plasma, which accelerates the electrons in longitudinal direction. This phenomenon of LWFA was first introduced theoretically by Tajima and Dawson (1979) in 1979. This phenomenon was later studied and discussed by Amiranoff et al. (1998). In their study on electrons, they estimated a maximum energy gain of 1.6 MeV and maximum longitudinal electric field of 1.5 GV/m. Shukla (1993) have studied the excitation of plasma wave due to the propagation of circular polarized pulse in axial magneto plasma. In his theoretical study, He concluded that plasma wakefield generated by right hand circularly polarized EM pulse can be enhanced by using external magnetic field. Singh (2004) have studied direct laser acceleration of electrons by circular polarized laser pulse propagating through magnetized plasma. They concluded that electrons with less initial energy can gain more energy due to long interaction time. Gupta and Ryu (2005) have used circular polarized light in the presence of oblique magnetic field in vacuum to accelerate electrons in relativistic regime. They concluded that at oblique angle of 30° , maximum electron gain can be generated. Shokri and Mirzaie (2005) have studied the pulse shape effect of circular polarized laser pulse on electron acceleration in the presence of axial magnetic field. They compared electron acceleration produced by Gaussian and piece wise circular polarized light. Fortin et al. (2010) have used radially polarized light in electron acceleration. They showed that proper optimization of beam waist and pulse duration can produce maximum acceleration. Salamin (2010) have studied the acceleration of electrons, α -particles and oxygen bare nuclei at very small diffraction angle using radially polarized laser light. Jha et al. (2012) have investigated the generation of longitudinal wakefield by a circular polarized laser pulse in the presence of axial magneto plasma. They concluded that reverse magnetic field increases the maximum energy gain of test electron. Payeur et al. (2012) have shown that fast moving electron beam can be produced by using radially polarized laser pulse. Their method can be implemented on positrons like charged particles. Mohammed et al. (2017) have studied the concept of electron acceleration by assuming the ellipsoid bubble regime. They also obtained the equation of motion of electrons and energy gain and shown that a proper external magnetic field can enhance the loss of energy of accelerated electrons. Abedi-Varaki (2018) has used circularly polarized electromagnetic wave for electron acceleration in the presence of wiggler magnetic field. In his study, he showed that proper use of laser intensity can increase electron energy. Ghotra and Kant (2018) have studied the effect of laser polarization and wiggler magnetic field on acceleration of electron. They also compared the results for linear and circular polarized laser and concluded that circular polarized light is more efficient for electron acceleration in the presence of wiggler magnetic field. Zhang et al. (2018) have demonstrated the LWFA effect by using two laser pulses using different polarization states. Time delay between laser beams plays a crucial role in LWFA effect. Singh (2018) have compared the energy gain of electrons in vacuum under the influence of linear and circular polarized light and concluded that in magnetized plasma, circular polarized laser pulse can accelerate electrons more effectively. In a simulation study, Wen et al. (2019) have used radially polarized laser pulse in parabolic plasma micro-channel to obtain electron beams of MeV energy at very low emittance and divergence. Kumar et al. (2021a) have used circularly polarized Gaussian laser pulse to accelerate electrons in plasma bubble regime under combined effect of direct laser action and laser wakefield acceleration. They concluded that circular polarized light could accelerate the electrons more effectively under the combined effect of both acceleration schemes as compared to individual acceleration scheme. Middha et al. (2022) have compared electron acceleration in vacuum using linear and Quadratic chirped laser pulses and concluded that a beating of quadratic chirped pulses requires less chirp parameter for energy enhancement

of electrons. Sharma and Kumar (2022) have studied the effect of frequency chirp on electron acceleration in LWFA and concluded that positive chirped pulse can generate much effective wakefield for electron acceleration. Abedi-Varaki and Kant (2022) have studied the effect of Gaussian and Super Gaussian laser pulses on electron energy enhancement in magneto plasma. Recently, Kim et al. (2023) have used self-steeping laser pulse to accelerate electrons. Other non-linear phenomenon in laser plasma interaction, like self-focussing (Thakur and Kant 2018), second harmonic generation (Kant et al. 2019), THz generation (Kumar et al. 2021b), third harmonic generation (Sharma et al. 2019) etc. are affected to a great extent by external magnetic field. Xia et al. (2022) have studied the electron acceleration in inhomogeneous underdense plasma. They concluded that inhomogeneity of plasma reduces the electron acceleration slightly. Zhu et al. (2021) have successfully achieved electron bunches of very low energy spread (3%) and very low divergence (4 mrad). Abedi-Varaki (2018) have studied electron acceleration due to interaction of circular polarized laser pulse with wiggler magnetized plasma by direct laser acceleration mechanism. They also described the path of accelerated electron and the impact of magnetic field on its path.

Kad and Singh (2022) have studied the electron acceleration using Laguerre-Gaussian laser pulse. They have used spatio-temporal coupled effect to accelerate electrons. Recently, Sharma et al. (2023) have used asymmetric chirped laser pulses to enhance wakefield excitation.

In the current paper, a circular polarized laser pulse is propagated through under-dense plasma in presence of transverse static oblique magnetic field. Maxwell's equation, equation of continuity and equation of motion are solved to obtain longitudinal wakefield, wake potential, energy gain and change in relativistic factor. Effect of magnetic field strength, oblique angle and laser strength parameter are investigated and plotted on curves and the results are compared. Section 2 deduces the longitudinal wakefield, wake potential, energy gain, and change in relativistic factor analytically. Section 3 investigates and discusses the wakefield and energy augmentation produced by laser pulses with various characteristics. Conclusions are provided in Sect. 4. The references are at the final section of the paper.

2 Analytical study of LWFA

Consider a circular polarized laser pulse propagating along z-direction through under-dense plasma in the presence of obliquely incident transverse magnetic field $\vec{B}_{ext} = B_0(-\hat{i}\sin\theta + \hat{j}\cos\theta)$ where, B_0 is the strength of external oblique magnetic field and θ is the angle of external magnetic field with y-axis.

Electric field \vec{E} of such laser pulse is given by Gupta and Ryu (2005),

$$\vec{E} = E_0(r, z, t) \{ \cos(kz - \omega t)\hat{i} - \sin(kz - \omega t)\hat{j} \} = \hat{i}E_x + \hat{j}E_y \quad (1)$$

From the definition of Lorentz force,

$$\frac{d(\vec{v})}{dt} + (\vec{v} \cdot \nabla)(\vec{v}) = -\frac{e}{m} \left\{ \vec{E} + \frac{\vec{v}}{c} \times (\vec{B} + \vec{B}_{ext}) \right\} \quad (2)$$

\vec{v} is electron velocity and \vec{B} is laser magnetic field. By solving Eq. (2) using \vec{B}_{ext} and \vec{E} , we get second order solution under quasistatic approximation as

$$\frac{\partial \overline{V^{(2)}}}{\partial t} = -\frac{e}{m} \overline{E^{(2)}} - \frac{c^2}{2} \{ \overline{\nabla a^2} \} - \frac{eB_0}{mc} (c \sin(kz - \omega t) \cos \theta \hat{k} + c \cos(kz - \omega t) \sin \theta \hat{k}) \quad (3)$$

Here $a = \frac{eE_0}{mc\omega}$ represents normalized laser strength parameters that are less than one.

Let us consider the envelop of laser pulse as Gaussian. Laser pulse strength in terms of new defined parameter ξ , (where $\xi = z - ct$), spot size (r_0) and pulse length (L) of Gaussian laser pulse can be written as.

$$a = a_0 e^{-\frac{(x^2+y^2)}{r_0^2}} \sin \frac{\pi \xi}{L} \text{ where } 0 \leq \xi \leq L \text{ and under paraxial approximation, } \frac{(x^2+y^2)}{r_0^2} \ll 1$$

So,

$$a = a_0 \sin \frac{\pi \xi}{L} \quad (4)$$

Here, a_0 is amplitude of laser strength parameter.

Solving for longitudinal direction using Eqs. (3) and (4)

$$\frac{\partial V_z^{(2)}}{\partial \xi} = \frac{e}{mc} E_z^{(2)} + \frac{c}{2} \frac{\partial}{\partial \xi} \{ a^2 \} + \frac{eB_0}{m} (a \sin(k\xi + \theta)) \quad (5)$$

Second order equation for generated fields are given by Maxwell’s equation.

$$\overline{\nabla} \times \overline{B^{(2)}} = \frac{4\pi}{c} \overline{J^{(2)}} + \frac{1}{c} \frac{\partial \overline{E^{(2)}}}{\partial t}$$

Taking z- component $\frac{\partial E_z^{(2)}}{\partial \xi} = \frac{4\pi}{c} J_z^{(2)}$

Solving this equation using $J_z^{(2)} = -neV_z^{(2)}$, we get general differential equation for the wake potential as

$$\frac{\partial^2 E_z^{(2)}}{\partial \xi^2} + k_p^2 E_z^{(2)} = -\frac{k_p^2 mc^2}{2e} \frac{\partial}{\partial \xi} \left\{ a_0^2 \sin^2 \left(\frac{\pi \xi}{L} \right) + \frac{eB_0 a_0}{mc} \left[\frac{\sin \left(\frac{\pi \xi}{L} - k\xi - \theta \right)}{\frac{\pi}{L} - k} - \frac{\sin \left(\frac{\pi \xi}{L} + k\xi + \theta \right)}{\frac{\pi}{L} + k} \right] \right\} \quad (6)$$

By solving it for $a_0^2 \ll 1$, the generated longitudinal wakefield behind the laser pulse ($\xi < 0$) is (Sprangle et al. 1988)

$$\begin{aligned} E_z^{(2)} = & \frac{k_p^2 mc^2}{2e} \left[\frac{2\pi^2 a_0^2}{k_p} \left\{ \frac{(\sin(k_p(L - \xi) + \sin(k_p \xi))}{k_p^2 L^2 - 4\pi^2} \right\} \right. \\ & + \frac{LeB_0 a_0}{2mc} \left[\frac{1}{\frac{\pi}{L} - k} \left\{ \frac{\cos\{kL + \theta + (L - \xi)k_p\} + \cos\{\theta - \xi k_p\}}{kL - \pi + Lk_p} \right. \right. \\ & \left. \left. - \frac{\cos\{kL + \theta + (\xi - L)k_p\} + \cos\{\theta + \xi k_p\}}{-kL + \pi + Lk_p} \right\} \right. \\ & \left. + \frac{1}{\frac{\pi}{L} + k} \left\{ \frac{\cos\{kL + \theta + (\xi - L)k_p\} + \cos\{\theta + \xi k_p\}}{kL + \pi - Lk_p} \right. \right. \\ & \left. \left. + \frac{\cos\{kL + \theta + (L - \xi)k_p\} + \cos\{\theta - \xi k_p\}}{kL + \pi + Lk_p} \right\} \right] \quad (7) \end{aligned}$$

In this equation, the first term represents the wakefield generated in plasma by pulse only. Remaining terms are responsible for effects produced by laser pulse and oblique magnetic field as these terms contain variables like B_0, a_0 and θ etc.

Generated wake potential can be calculated as $\Phi = -\int E_z dz = -\int E_z d\xi$. By solving and simplifying it, we get

$$\begin{aligned} \Phi = & -\frac{k_p^2 mc^2}{2e} \left[\frac{2\pi^2 a_0^2}{k_p^2} \left\{ \frac{\cos(k_p(L - \xi)) - \cos(k_p \xi)}{k_p^2 L^2 - 4\pi^2} \right\} \right. \\ & - \frac{LeB_0 a_0}{2mck_p} \left[\frac{L}{\pi - kL} \left\{ \frac{\sin\{kL + \theta + (L - \xi)k_p\} + \sin\{\theta - \xi k_p\}}{(kL - \pi + Lk_p)} \right. \right. \\ & \left. \left. + \frac{\sin\{kL + \theta + (\xi - L)k_p\} + \sin\{\theta + \xi k_p\}}{-kL + \pi + Lk_p} \right\} \right. \\ & - \frac{L}{\pi + kL} \left\{ \frac{\sin\{kL + \theta + (\xi - L)k_p\} + \sin\{\theta + \xi k_p\}}{kL + \pi - Lk_p} \right. \\ & \left. \left. - \frac{\sin\{kL + \theta + (L - \xi)k_p\} + \sin\{\theta - \xi k_p\}}{kL + \pi + Lk_p} \right\} \right] \end{aligned} \tag{8}$$

Change in relativistic factor is given by $\Delta\gamma = \frac{-e}{mc^2 \left\{1 - \frac{1}{\beta}\right\}} \int E_w d\xi$ here, $\beta = c/v_g$

$$\begin{aligned} \Delta\gamma = & -\frac{1}{\left\{1 - \frac{1}{\beta}\right\}} \left[\frac{\pi^2 a_0^2}{k_p^2} \left\{ \frac{\cos k_p(L - \xi) - \cos k_p \xi}{k_p^2 L^2 - 4\pi^2} \right\} \right. \\ & + \frac{L_n^2 eB_0 a_0}{4mck_p} \left[\frac{1}{\pi - kL} \left\{ \frac{-\sin\{kL + \theta + k_p(L - \xi)\} - \sin\{\theta - k_p \xi\}}{kL - \pi + Lk_p} \right. \right. \\ & \left. \left. - \frac{\sin\{kL + \theta + k_p(\xi - L)\} + \sin\{\theta + k_p \xi\}}{-kL + \pi + Lk_p} \right\} \right. \\ & \left. + \frac{1}{\pi + kL} \left\{ \frac{\sin\{kL + \theta + k_p(\xi - L)\} + \sin\{\theta + k_p \xi\}}{kL + \pi - Lk_p} - \frac{\sin\{kL + \theta + k_p(\xi - L)\} + \sin\{\theta + k_p \xi\}}{kL + \pi + Lk_p} \right\} \right] \\ & - \pi^2 a_0^2 \left\{ \frac{(\cos(k_p L) - 1)}{k_p^2 L^2 - 4\pi^2} \right\} - \frac{L_n^2 eB_0 a_0}{4mck_p} \left[\frac{1}{\pi - kL} \left\{ \frac{-\sin\{kL + \theta + k_p L\} - \sin\{\theta\}}{(kL - \pi + Lk_p)} - \frac{\sin\{kL + \theta - k_p L\} + \sin\{\theta\}}{-kL + \pi + Lk_p} \right\} \right. \\ & \left. \left. - \frac{1}{\pi + kL} \left\{ \frac{\sin\{kL + \theta - k_p L\} + \sin\{\theta\}}{kL + \pi - k_p L} - \frac{-\sin\{kL + \theta + kL + \pi - k_p L\} - \sin\{\theta\}}{kL + \pi + k_p L} \right\} \right] \end{aligned} \tag{9}$$

Thus, the electron energy gain is achieved, by employing $\Delta W = mc^2 \Delta\gamma$.

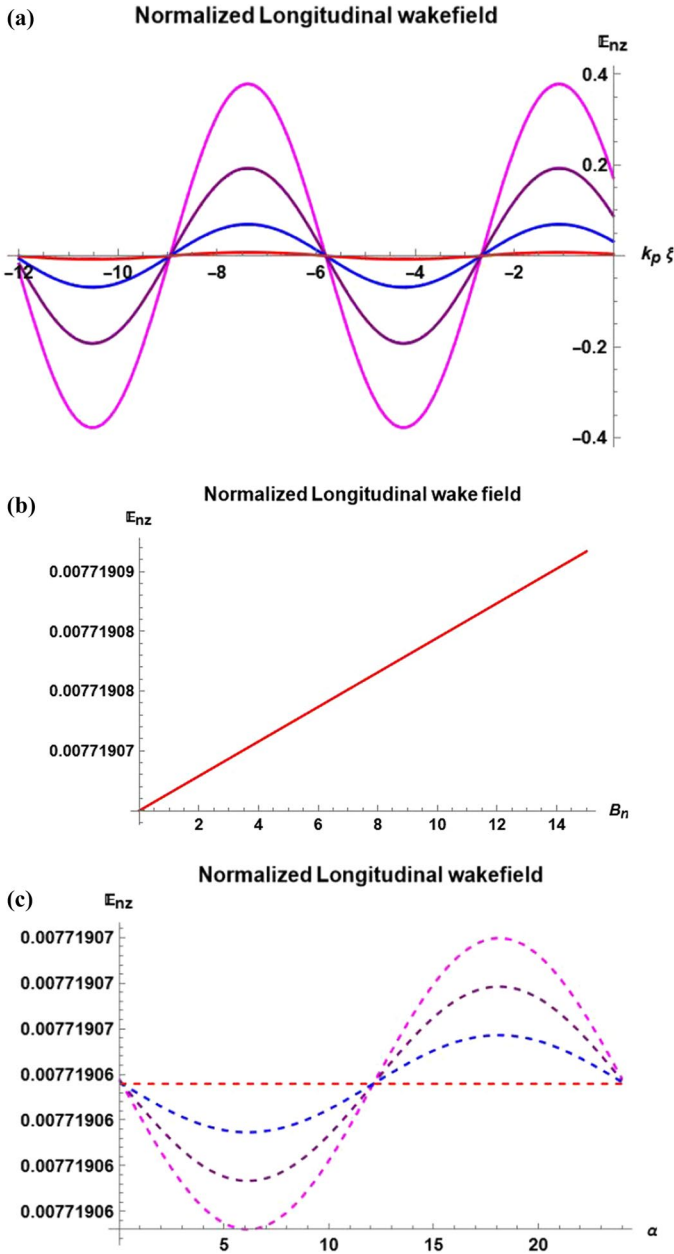


Fig. 1 **a** Variation of normalized wakefield with normalized distance for $a_0 = 0.1$ (Red), 0.3 (Blue), 0.5 (Purple), 0.7 (magenta). $B_n = 0.15$, $\theta = 3\pi/2$. Other parameters are same as stated above. **b** Variation of normalized maximum wakefield with magnetic field for $a_0 = 0.1$, $\theta = 3\pi/2$. Other parameters are same as stated above. **c** Variation of normalized Max. wakefield with α , where oblique angle $\theta = \alpha\pi/12$, for $a_0 = 0.1$, $B_n = 0$ (red), 0.05 (blue), 0.1 (purple), 0.15 (magenta). Other parameters are the same as stated above

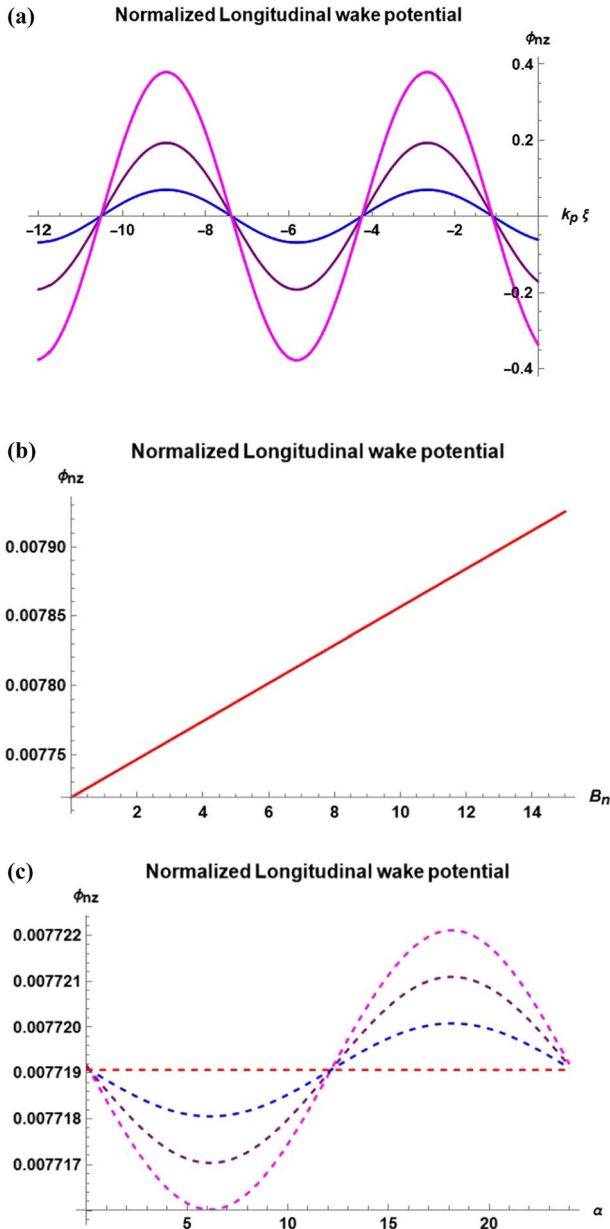


Fig. 2 **a** Variation of normalized wake potential with normalized distance for $a_0 = 0.1$ (Red), 0.3 (Blue), 0.5 (Purple)and 0.7 (magenta). $B_n = 0.15, \theta = 15\pi/12$. Other parameters are same as stated above. **b** Variation of normalized maximum wake potential with magnetic field for $a_0 = 0.1, \theta = 15\pi/12$. Other parameters are same as stated above. **c** Variation of normalized Max. wake potential with oblique angle for $a_0 = 0.1, B_n = 0$ (black), 0.03 (red), 0.06 (blue), 0.09 (green). Other parameters are same as stated above

$$\begin{aligned}
 \Delta W = & -\frac{mc^2}{\left\{1 - \frac{1}{\beta}\right\}} \left[\pi^2 a_0^2 \left\{ \frac{\cos k_p(L - \xi) - \cos k_p \xi}{k_p^2 L^2 - 4\pi^2} \right\} \right. \\
 & + \frac{L_n^2 e B_0 a_0}{4mck_p} \left[\frac{1}{\pi - kL} \left\{ \frac{-\sin\{kL + \theta + k_p(L - \xi)\} - \sin\{\theta - k_p \xi\}}{kL - \pi + Lk_p} \right. \right. \\
 & \left. \left. - \frac{\sin\{kL + \theta + k_p(\xi - L)\} + \sin\{\theta + k_p \xi\}}{-kL + \pi + Lk_p} \right\} \right] \\
 & + \frac{1}{\pi + kL} \left\{ \frac{\sin\{kL + \theta + k_p(\xi - L)\} + \sin\{\theta + k_p \xi\}}{kL + \pi - Lk_p} - \frac{\sin\{kL + \theta + k_p(\xi - L)\} + \sin\{\theta + k_p \xi\}}{kL + \pi + Lk_p} \right\} \Bigg] \\
 & - \pi^2 a_0^2 \left\{ \frac{(\cos(k_p L) - 1)}{k_p^2 L^2 - 4\pi^2} \right\} - \frac{L_n^2 e B_0 a_0}{4mck_p} \left[\frac{1}{\pi - kL} \left\{ \frac{-\sin\{kL + \theta + k_p L\} - \sin\{\theta\}}{(kL - \pi + Lk_p)} - \frac{\sin\{kL + \theta - k_p L\} + \sin\{\theta\}}{-kL + \pi + Lk_p} \right\} \right. \\
 & \left. \left. - \frac{1}{\pi + kL} \left\{ \frac{\sin\{kL + \theta - k_p L\} + \sin\{\theta\}}{kL + \pi - k_p L} - \frac{-\sin\{kL + \theta + kL + \pi - k_p L\} - \sin\{\theta\}}{kL + \pi + k_p L} \right\} \right] \Bigg] \tag{10}
 \end{aligned}$$

Equation (7), (8), (9) and (10) give generated wakefield, wake potential, change in relativistic factor and energy gain by electron. The first term in each of these equations reflects the influence of laser pulse intensity solely, whereas the following terms contain additional variables such as laser propagation constant, magnetic field, and oblique angle, which are responsible for creating the effect of polarised pulse and magnetic field on LWFA. The argument of angles in these equations contain variables like oblique angle, pulse length, laser propagation constant and plasma frequency. Oblique angle corresponding to maximum energy gain varies with other parameters.

3 Result and discussion

In present analysis, we have considered a circularly polarized laser pulse of wavelength 0.8 μm propagating through plasma. Corresponding angular frequency of laser pulse (ω_L) is 2.36×10^{15} rad/sec. Density of plasma (n) is $4.1 \times 10^{23} m^{-3}$ corresponding plasma frequency (ω_p) is 3.6×10^{13} rad/sec and plasma wavelength is 52.21 μm. Plasma frequency is less than frequency of laser pulse ($\omega_p < \omega_L$), which is the required condition of under-dense plasma.

Normalizing the quantities as $\left(\frac{eE_z^{(2)}}{mc\omega_p}\right) \rightarrow E_{nz}$ normalized generated wakefield, $k_p L \rightarrow L_n$ normalized pulse length, $\frac{k}{k_p} \rightarrow k_n$ normalized propagation constant, $k_p \xi \rightarrow \xi_n$ normalized distance, $\frac{eB_0}{mk_p c} \rightarrow B_n$ normalized external magnetic field, $\frac{e\Phi}{mc^2} \rightarrow \Phi_n$ normalized wake potential.

For numerical calculations laser pulse strength parameter is chosen as 0.1, 0.3, 0.5 and 0.7 (<1). Laser intensity corresponding to such pulses are $1.07 \times 10^{20} W/m^2$, $9.66 \times 10^{20} W/m^2$, $2.68 \times 10^{21} W/m^2$ and $5.26 \times 10^{21} W/m^2$ respectively. Normalized pulse length is taken as 4.

Normalized external static magnetic field is taken as 0, 0.05, 0.1 and 0.15 which are corresponding to magnetic field of 0 T, 10.27 T, 20.53 T and 30.80 T, respectively. Random values of oblique angle are taken as $\theta = \alpha\pi/12$, where α is taken from 0 to 24 to obtain values of θ from 0° to 360° in degree.

(i) Variation in generated longitudinal wakefield intensity

In Fig. 1a, normalized longitudinal wakefield (Eq. 6) is plotted with normalized distance for different laser pulse strength ($a_0 = 0.1, 0.3, 0.5$ and 0.7). It is clear from Fig. 1a that the normalized wakefield increases sharply (varies as a_0^2) with increase

in laser strength parameter. It is because a high-intensity laser pulse may create a significant ponderomotive force on electrons, resulting in a significant plasma wave, which effectively generates wakefield. The position at which maximum longitudinal wakefield is obtained, does not depend on laser strength.

Maximum normalized wakefield is calculated for different values of normalized magnetic field and represented in Fig. 1b. The normalized maximum wakefield increases slightly with an increase in normalized magnetic field strength.

Normalized wakefield is calculated for different magnetic field strength and oblique angle $\theta = \alpha\pi/12$ and where α is taken from 0 to 24 to obtain values of θ from 0° to 360° . Figure 1c shows the variation of normalized wakefield with oblique angle. Peak of the wakefield curve is obtained at $\theta = 270^\circ$. So, to obtain maximum wakefield with the above selected parameters, magnetic field should be at $\theta = 270^\circ$ from y axis and perpendicular to the direction of propagation of laser pulse. It can be seen through Eq. (7), that generated normalized wakefield depends on oblique angle, pulse length, laser propagation constant and plasma frequency. Oblique angle corresponding to maximum laser wakefield varies with other parameters.

(ii) Variation in generated wake potential

In Fig. 2a, normalized longitudinal wake potential (Eq. 8) is plotted with normalized distance for different laser pulse strength ($a_0 = 0.1, 0.3, 0.5$ and 0.7). It is clear from that the normalized wake potential increases sharply with increase in laser strength parameter. The position at which maximum longitudinal wake potential is obtained does not depend on laser strength.

Maximum normalized wake potential is calculated for different values of normalized magnetic field and represented in Fig. 2b. The normalized maximum wake potential increases slightly with an increase in normalized magnetic field strength. For small B_n , wake potential is almost constant. With the increase in B_n , variation in wake potential increases, which leads to enhanced wakefield acceleration.

Maximum normalized wake potential is calculated for different magnetic fields and plotted in Fig. 2c. Peak of the wakefield curve is obtained at $\theta = 270^\circ$ from y axis. It can be seen through Eq. (8), that generated normalized wake potential depends on oblique angle, pulse length, laser propagation constant and plasma frequency. Oblique angle corresponding to maximum laser wake potential varies with other parameters.

(iii) Variation in relativistic factor ($\Delta\gamma$) and energy gain by electron

In Fig. 3a, change in relativistic factor (Eq. 9) is plotted with normalized distance for different laser pulse strength ($a_0 = 0.1, 0.3, 0.5$ and 0.7). $\Delta\gamma$ increases sharply with the increase in laser strength parameter. The position at which maximum longitudinal wake potential is obtained does not depend on laser strength. For $a_0 = 0.1$, maximum change in relativistic factor is 0.06 GeV. It increases rapidly with a_0 and becomes 0.52 GeV for $a_0 = 0.3$, 1.43 GeV for $a_0 = 0.5$. Maximum electron gain with selected feasible parameters is 2.82 GeV for $a_0 = 0.5$. So, Laser pulse intensity plays the key role in electron energy gain in LWFA.

Figure 3b depicts a very gradual linear growth in $(\Delta\gamma)$ with increasing external magnetic field intensity.

The effect of pulse length on change in relativistic factor is shown in Fig. 3c. For the chosen set of other variables, the maximum change in relativistic factor is obtained at normalized length 4 and it does not depend on laser intensity. Normalized pulse length

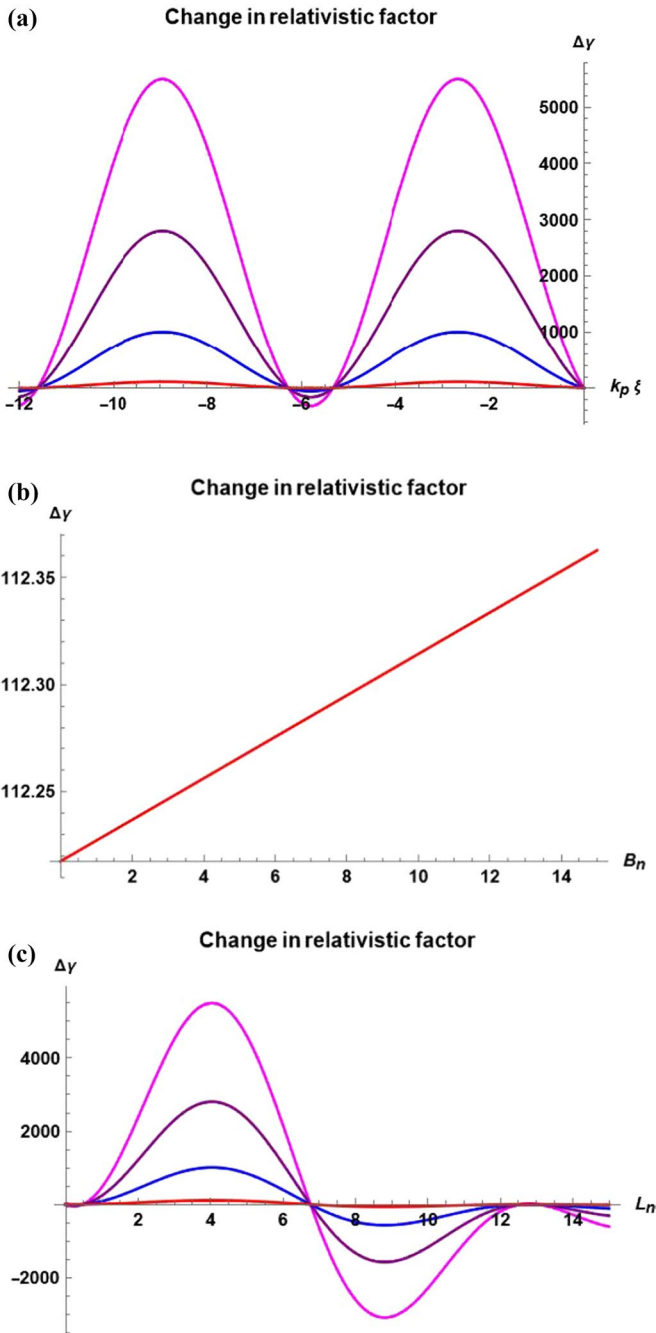


Fig. 3 **a** Variation of change in relativistic factor with normalized distance for $a_0 = 0.1$ (Red), 0.3 (Blue), 0.5 (Purple) and 0.7 (magenta), $\theta = 15\pi/12$, $B_n = 0.15$ Other parameters are same as stated above. **b** Variation of change in relativistic factor with magnetic field for $a_0 = 0.1$, $\theta = 15\pi/12$. Other parameters are same as stated above. **c** Variation of change in relativistic factor with pulse length for $a_0 = 0.1$ (Red), 0.3 (Blue), 0.5 (Purple) and 0.7 (magenta), $B_0 = 0.15$, $\theta = 15\pi/12$. Other parameters are same as stated above

corresponds to the 0.64 times plasma wavelength. It concludes that for effective electron acceleration, pulse length should be of the order of plasma wavelength.

A similar study was conducted by Kumar et al. (2021a) in relativistic regime to verify that a circular polarized light can develop much more effective electron acceleration due to combined effect of direct laser action and LWFA than a linearly polarized pulse and a maximum 3 GeV energy gain was observed. Jha et al. (2012) have used axial static magnetic field with circular polarized laser pulse to study longitudinal LWFA. In our study, for specific parameters ($a_0 = 0.7$, $B_n = 0.15$), a maximum energy gain of 2.82 GeV can be achieved using transvers oblique static magnetic field with circular polarized laser pulse.

4 Conclusion

In this study, we have used a circular polarized laser pulse propagating (along z- axis) through under-dense plasma. An external oblique transverse static magnetic field is applied an arbitrary angle (at angle θ with Y-axis in x-y plane). Effect of laser pulse strength (a_0), strength of external magnetic field (B_0), and its arbitrary angle (θ) on generated longitudinal wakefield, wake potential and change in relativistic factor are calculated theoretically and curves are drawn. In our results, optimized oblique angle of transverse magnetic field for efficient longitudinal wake field is obtained as 270° . Generated wakefield, wake potential and electron energy gain depend on oblique angle, pulse length, laser propagation constant and plasma frequency. Oblique angle corresponding to maximum laser wake field effect varies with other parameters. Generated wakefield, wake potential and change in relativistic factor increases sharply with increase in laser pulse strength and increases slightly with increase in magnetic field strength. A maximum energy gain of 2.82 GeV is achieved using specific parameters ($a_0 = 0.7$, $B_n = 0.15$). Effect of pulse length is also investigated for energy efficient electron acceleration. Our current analysis will help researchers pick magnetic field and laser pulses to develop effective and energy efficient LWFA.

Author contribution VS: derivation, methodology, analytical modeling, and graph plotting; SK: numerical analysis; NK: numerical analysis and result discussion; VT: supervision, reviewing, and editing.

Funding Not applicable.

Data availability The data that support the findings of this study are available from the corresponding authors upon reasonable request.

Declarations

Conflict of interest The authors declare no competing interest.

Ethical approval Not applicable

Consent to participate Not applicable.

Consent for publication Not applicable

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