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Development of a new power spectrum of refractive-index fluctuations for ocean turbulence

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Abstract

We develop a new power spectrum of the refractive-index fluctuations turbulent oceanic based on the marine atmospheric spectrum developed in the literature. This power spectrum is global, it allows to describe the refractive index variations in turbulent oceanic, turbulent biological tissue and turbulent marine atmosphere.

Keywords New spectrum · Refractive-index fluctuations · Turbulent oceanic

1 Introduction

The determination of the propagation properties of laser beams through random media, such as the atmosphere, the ocean and the biological tissue, represents the subject of several research active fields, such as free space optical communication, underwater optics communications, remote sensing, imaging and targeting systems (Baykal 2016; Khannous and Belafhal 2018; Doronin et al. 2019; Luo et al 2018; Chib et al. 2020, 2023a, b). Furthermore, the ocean environment has different effects on the propagation of optical waves through it compared with the atmosphere above land or ocean. This medium is characterized by temperature and salinity fluctuations which lead to spatial variations of its refractive index. Therefore, the introduction of the spatial of the power spectrum of oceanic refractive-index fluctuations, which describes the spatial changes of the index of refraction, is important for researchers to obtain the accurate and reliable theoretical model of optical communication in the oceanic turbulence. In the literature, some models have been investigated and developed to describe the power spectrum of oceanic turbulence (Nikishov and Nikishov 2000; Yao et al. 2017; Li et al. 2019). Recently, Li et al. (2019) have developed a new spectrum of the refractive-index fluctuations for the unstable stratification ocean based on the linear combination of the temperature spectrum, salinity spectrum and coupling spectrum that all include the outer scale. This spectrum is used to describe the behavior of turbulence in a turbulent oceanic environment.

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On other hand, Chib et al. (2023b) are proposed a global spectrum model characterizing the refractive index variations in turbulent biological tissue based on the spectrum model of Khannous and Belafhal (2018). Also, since the atmosphere, the ocean and the biological tissue mediums are the turbulent media. So, we must have a single model that describes its three mediums and which depends on the characteristic parameters of each medium. In this manuscript, we propose a new spectrum model which describes the fluctuations of the index of refraction in the ocean medium. An analytical expression of this spectrum has been developed based on the general form which was introduced by Khannous and Belafhal (2018). The rest parts of the paper are organized as follows: In Sect. 2, we present the theoretical model of the power spectrum of the refractive-index fluctuations turbulent oceanic. Finally, conclusion of the present work is presented in Sect. 3.

2 Theoretical model

Oceanic turbulence is represented by the spectrum of oceanic refractive-index variations, which are primarily brought on by changes in temperature and salinity. Therefore, we aim to find relations between variations in temperature and salinity and changes in oceanic refractive index. According to the study (Nikishov and Nikishov 2000), in the linear approximation, we can structure the refractive-index fluctuating n as a linear function of the temperature and salinity fluctuations

$$n = -AT + BS, \tag{1}$$

where $A = 2.6 \times 10^{-4}\text{liter}/\,\text{deg}$, $B = 1.75 \times 10^{-4}\text{liter}/\,\text{gram}$, T indicates the temperature fluctuations of the refraction index and S is the salinity fluctuations of the refraction index.

The three correlation functions of temperature, salinity, and coupling fluctuations may be used to determine the structure function of the refractive-index fluctuations for locally homogeneous and isotropic turbulence (Nikishov and Nikishov 2000; Yao et al. 2017). Using the relationship between the spatial power spectrum and the scalar spectrum (Lu et al. 2006), as well as the spectral expansion of the structural function of the refractive-index variations, the Nikishov's spatial power spectrum of oceanic refractive-index fluctuations is written as (Nikishov and Nikishov 2000; Yao et al. 2017)

$$\Phi_n^c(\kappa) = A^2\Phi_T^c(\kappa) + B^2\Phi_S^c(\kappa) - 2AB\Phi_{TS}^c(\kappa), \tag{2}$$

where κ represents the spatial wave number, $\Phi_T^c(\kappa)$ is the scalar spectrum of temperature fluctuations, $\Phi_S^c(\kappa)$ is the scalar spectrum of salinity fluctuations and $\Phi_{TS}^c(\kappa)$ is the scalar spectrum of coupling fluctuations.

In the following, we will propose the three separate spatial power spectra of oceanic turbulence based on the approximations of the oceanic spectra (Yao et al. 2017) of the temperature, salinity, and coupling fluctuations

$$\Phi_i^c(\kappa) = \varphi\left(\kappa_0, \kappa\right)\chi_i G_i(\kappa\eta) F_i(\kappa\eta),\ \ 0 < \kappa < \infty,\ \ with\ (i = T,\ S,\ TS), \tag{3}$$

where $\kappa_0 = \frac{2\pi}{L_0}$, L_0 is the outer scale of the turbulence, χ_T is the rate of the dissipation of mean-squared temperature, χ_S is the rate of the dissipation of mean-squared salinity, χ_{TS} is the rate of the dissipation of mean-squared coupling and they are related by the equation (Nikishov and Nikishov 2000; Lu et al. 2006) $\chi_n = A^2\chi_T + B^2\chi_S - 2AB\chi_{TS}$,

$G_i(\kappa\eta) = \exp\left[-(\kappa\eta)^2/R_i^2\right]$, $\eta \equiv \left(\nu^3/\varepsilon\right)^{1/4}$ is the inner scale of turbulence with ν is the kinematic viscosity and ε represents the rate of dissipation of turbulent kinetic energy of fluid, $R_i = \sqrt{3}\left[W_i - 1/3 + 1/(9W_i)\right]^{3/2}/Q^{3/2}$, with Q is the non-dimensional constant, $W_i = \left\{\left[\mathrm{Pr}_i^2/\left(6\beta Q^{-2}\right)^2 - \mathrm{Pr}_i/\left(81\beta Q^{-2}\right)\right]^{1/2} - \left[1/27 - \mathrm{Pr}_i/\left(6\beta Q^{-2}\right)\right]\right\}^{1/3}$, $\mathrm{Pr}_{TS} = 2\,\mathrm{Pr}_T\,\mathrm{Pr}_s/\left(\mathrm{Pr}_T + \mathrm{Pr}_s\right)$, Pr_T and Pr_S are the Prandtl numbers of the temperature and salinity, respectively, and β is the Obukhov-Corrsin constant. $\varphi\left(\kappa_0, \kappa\right) = \frac{A_n\kappa_0^3\Gamma(D/2)}{\pi^{3/2}2^{(5-D)/2}\left(1+\kappa_0^2\kappa^2\right)^{D/2}}$ is the power spectral density that characteristics the light scattering with D being the fractal dimension and determines the shape of the distribution.

Substituting Eq. (3) into Eq. (2), the spectrum of the oceanic refractive-index fluctuations can be expressed as

$$\Phi_n^c(\kappa) = \varphi\left(\kappa_0, \kappa\right)\left[A^2\chi_T G_T(\kappa\eta)F_T(\kappa\eta) + B^2\chi_S G_S(\kappa\eta)F_S(\kappa\eta) - 2AB\chi_{TS}G_{TS}(\kappa\eta)F_{TS}(\kappa\eta)\right]. \tag{4}$$

If we consider the same correction of the corresponding spectra, one obtains

$$F_T(\kappa\eta) = F_S(\kappa\eta) = F_{TS}(\kappa\eta) = F(\kappa\eta), \tag{5}$$

where

$$F(\kappa\eta) = 1 + b\frac{\kappa\eta}{\kappa_L}\exp\left[-\alpha\frac{(\kappa\eta)^2}{\kappa_L^2}\right] + c\left(\frac{\kappa\eta}{\kappa_L}\right)^{7/6}\exp\left[-\beta\frac{(\kappa\eta)^2}{\kappa_L^2}\right] \tag{6}$$

Consequently, Eq. (2) becomes

$$\Phi_n^c(\kappa) = \varphi\left(\kappa_0, \kappa\right)F(\kappa\eta)\left[A^2\chi_T G_T(\kappa\eta) + B^2\chi_S G_S(\kappa\eta) - 2AB\chi_{TS}G_{TS}(\kappa\eta)\right] \tag{7}$$

We will choose that $\chi_s = \frac{A^2}{B^2\varpi^2\theta}\chi_T$ and $\chi_{TS} = \frac{A(1+\theta)}{2\varpi B\theta}\chi_T$ with θ being the eddy diffusivity ration and ϖ defines the contributions of the temperature and salinity distributions to the distribution of the refractive-index (Lu et al. 2006). Note that the eddy diffusivity ration θ, in the unstable stratification, is expressed as (Elamassie et al. 2017)

$$\theta = \frac{|\varpi|}{R_F} = \begin{cases} 1/\left(1 - \sqrt{(|\varpi| - 1)/|\varpi|}\right) & |\varpi| \geq 1 \\ 1.85|\varpi| - 0.85 & 0.5 \leq |\varpi| \leq 1 \\ 0.15|\varpi| & |\varpi| \leq 0.5 \end{cases}, \tag{8}$$

where R_F is the eddy flux ratio. By replacing χ_S and χ_{TS} by their expressions in Eq. (7), this last equation becomes

$$\begin{aligned}\Phi_n^c(\kappa) &= \varphi\left(\kappa_0, \kappa\right)F(\kappa\eta)A^2\chi_T\left[G_T(\kappa\eta) + \frac{1}{\varpi^2\theta}G_S(\kappa\eta) - \frac{(1+\theta)}{\varpi\theta}G_{TS}(\kappa\eta)\right] \\ &= A^2\Phi_T^c(\kappa) + B^2\Phi_S^c(\kappa) - 2AB\Phi_{TS}^c(\kappa),\end{aligned} \tag{9}$$

where

$$\Phi_T^c(\kappa) = \varphi\left(\kappa_0, \kappa\right) F(\kappa\eta) \chi_T \exp\left[-\frac{(\kappa\eta)^2}{R_T^2}\right], \tag{10.a}$$

$$\Phi_S^c(\kappa) = \varphi\left(\kappa_0, \kappa\right) F(\kappa\eta) \chi_T \frac{A^2}{B^2 \varpi^2 \theta} \exp\left[-\frac{(\kappa\eta)^2}{R_S^2}\right], \tag{10.b}$$

and

$$\Phi_{TS}^c(\kappa) = \frac{A}{2B} \frac{(1+\theta)}{\varpi\theta} \varphi\left(\kappa_0, \kappa\right) F(\kappa\eta) \chi_T \exp\left[-\frac{(\kappa\eta)^2}{R_{TS}^2}\right] \tag{10.c}$$

By the use of the following expression

$$\frac{A^2}{B^2} = \varpi^2 \theta \frac{\chi_s}{\chi_T}, \tag{11}$$

$$\frac{\chi_T A^2 / B^2}{\varpi^2 \theta} = \chi_S, \tag{12}$$

and

$$\chi_T \frac{A}{2B} \frac{(1+\theta)}{\varpi\theta} = \chi_{TS}, \tag{13}$$

we can write

$$\Phi_S^c(\kappa) = \varphi\left(\kappa_0, \kappa\right) F(\kappa\eta) \chi_S \exp\left[-\frac{(\kappa\eta)^2}{R_S^2}\right], \tag{14}$$

and

$$\Phi_{TS}^c(\kappa) = \varphi\left(\kappa_0, \kappa\right) F(\kappa\eta) \chi_{TS} \exp\left[-\frac{(\kappa\eta)^2}{R_{TS}^2}\right] \tag{15}$$

Finally, our oceanic spectrum of the refractive-index in the unstable stratification case ($\theta \neq 1$) is given by

$$\begin{aligned}\Phi_n^c(\kappa) &= \varphi\left(\kappa_0, \kappa\right) F(\kappa\eta) A^2 \chi_T \\ &\times \left\{ \exp\left[-\frac{(\kappa\eta)^2}{R_T^2}\right] + \frac{1}{\varpi^2\theta} \exp\left[-\frac{(\kappa\eta)^2}{R_S^2}\right] - \frac{(1+\theta)}{\varpi\theta} \exp\left[-\frac{(\kappa\eta)^2}{R_{TS}^2}\right] \right\}, \; 0 < \kappa < \infty.\end{aligned} \tag{16}$$

Equation (16) is our main result which is used to describe the refractive index variations in turbulent oceanic. b, c, α and β are constants to be determined. κ_l is a parameter inversely proportional to the inner scale of turbulence.

3 Conclusion

In summary, the oceanic optical turbulence is primarily caused by fluctuations in temperature and salinity concentration and might grow significantly in areas where cold and warm waters mechanically interact. Turbulence in the ocean boundary layer during rains is one example of this. In this paper, we propose a new spatial power spectrum of oceanic refractive-index fluctuation expressed by Eq. (16). In this analyze, the structure function of the refractive-index fluctuations for spatially homogeneous and isotropic turbulence is identified by using the three correlation functions of temperature, salinity, and coupling fluctuations.

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Declarations

Conflict of interest The authors have no financial or proprietary interests in any material discussed in this article.

Ethical approval This article does not contain any studies involving animals or human participants performed by any of the authors. We declare that this manuscript is original, and is not currently considered for publication elsewhere. We further confirm that the order of authors listed in the manuscript has been approved by all of us.

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