



Investigation of internal dynamics of soliton with the help of traveling wave soliton solution of Hamilton amplitude equation

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Received: 9 March 2022 / Accepted: 18 June 2022 / Published online: 18 July 2022

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Abstract

The φ^6 -model expansion technique is used in this work to obtain dark, bright, periodic, singular, dark-bright, combined singular soliton and rational solutions to the Hamiltonian amplitude equation. This equation is used to analyze the stability of the exact solutions as well as to determine modulated wave train instabilities. The obtained results are mostly applicable in the study of nonlinear waves in plasma in the setting of a non-magnetized fluid-type plasma, the dispersion of the Langmuir (electrostatic) wave when it generates some low-frequency acoustic waves, such as the ion-acoustic wave, and other fields. The behavior of the traveling wave is analyzed using the frequency values, which is one of the internal dynamics of the dark soliton. This discussion is supported by graphs and the effects on wave behavior of different densities are simulated.

Keywords φ^6 -model expansion method · Hamilton amplitude equation · Traveling wave solution · Wave frequency · Internal dynamics of soliton

1 Introduction

Nonlinear evolution equations (NLEEs) are studied in many areas of applied mathematics and theoretical physics, including engineering and biological sciences. These equations are crucial in explaining important scientific phenomena. There are several parameters and variables connected in a real-life phenomenon under the imperious rule of that phenomenon. When we present the relationships between the parameters and variables in mathematical terms, we generally derive a mathematical model of the problem, which may be an equation, a differential equation, a system of integral equations, or something else (Jalil and Heidari 2019).

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The study of exact solutions of NLEEs, as scientific methods of the concepts, will help one to clarify these phenomena. Many successful methods for obtaining exact solutions of NLEEs, such as the finite forward difference method (Asif 2018), sine-Gordon expansion method (Yokus Asif 2018), the Adomian's decomposition method (Doğan and Yokus 2002), the $\left(\frac{1}{G'}\right)$ -expansion method (Yokuş Asif et al 2021), the $\left(\frac{G'}{G}\right)$ -expansion method (Yokus Asif and Münevver Tuz 2017), generalized exponential rational function method (Hulya 2021), the modified sub-equation method (Duran Serbay et al. 2021; Duran and Karabulut 2022), the 4th order residual power series method (Tariq et al. 2021), the q -homotopy analysis transform method (Gao Wei et al. 2020), the auto-Bäcklund transformation method (Kaya et al. 2020), the Modified Extended Tanh method (Yuming 2021), the φ^6 -model expansion method (Behzad and Baleanu 2019; Zhou 2015; Zayed Elsayed ME and Abdul-Ghani Al-Nowehy 2017; Zayed Elsayed ME et al. 2018), the modified simple equation method (Arnous 2017), the extended auxiliary equation method (Sun 2003), improved tanh method (Yokuş et al. 2022), Hirota bilinear operator method (Liu et al. 2021), the Jacobi elliptic function expansion method (Tarla et al. 2022), multiple Exp-function method (Wan et al. 2020), long-wave method (Ali and Yilmazer 2022), symbolic computational method (Ali et al. 2021), etc have been introduced in the last few decades.

Solitons are solutions to Nonlinear partial differential equations that describe a single moving wave. Optical solitons, the result of a perfect balance between dispersion (or diffraction) and nonlinearity in a nonlinear medium, are frequently used in telecommunications and electromagnetics. Optical fibers are solutions to the Nonlinear Schrodinger equations. Soliton solutions contain particle-like structures, such as magnetic monopoles, as well as extended structures, such as domain walls and cosmic strings, which have implications for the cosmology of the early universe. Periodic solutions, such as $\cos(x+t)$, are periodic traveling wave solutions (Jalil and Heidari 2019; Zhou 2015). Dark soliton refers to solitary waves with lower intensity than the background, bright soliton refers to solitary waves with higher peak intensity than the background, and singular soliton refers to a solitary wave with discontinuous derivatives; examples of such solitary waves include compactons, which have finite (compact) support, and peakons, whose peaks have a discontinuous derivative, dark solitons are moduled by the \tanh functions whilst bright solitons are moduled by the $\sec h$. Wadati et al Wadati et al. (1992) published the new Hamiltonian amplitude equation in 1992. According to the scientists, this equation is obviously not integrable because it lacks the Painleve property, but it is a Hamiltonian equivalent of the Kuramoto-Sivashinsky equation, which appears in dissipative systems (Wadati et al. 1992).

Many scholars have attempted a variety of approaches to find exact solutions to the Hamiltonian amplitude equation, such as modified expansion method (Duran and Kaya 2021), the extended Jacobi elliptic expansion method (Zafar et al. 2020), the general projective Riccati equation method (Yomba 2005), the sinh-Gordon expansion method (Krishnan and Yan 2006), the extended F -expansion method (Feng et al. 2007), the first integral method (Taghizadeh and Mirzazadeh 2011), the $\left(\frac{G'}{G}\right)$ -expansion method (Taghizadeh and Najand 2012), $\left(\frac{1}{G'}\right)$ -expansion method (Duran and Yokuş 2021), Lie symmetry method (Kumar et al. 2012), the simplest equation method (Eslami and Mirzazadeh 2013), He's semi-inverse variational principle method and Ansatz method (Mirzazadeh 2015), modified simplest equation method (Mirzazadeh 2014), the mapping method (Peng 2003), the $\left(\frac{G'}{G}, \frac{1}{G'}\right)$ -expansion method (Duran 2021), the Lie classical and the $\left(\frac{G'}{G}\right)$ -expansion method (Kumar Sachin et al. 2012). The new Hamiltonian amplitude model is studied in

this research using the newly developed φ^6 -model expansion method, which results in the restoration of dark, bright, singular, rational, and periodic solitary wave solutions.

The following is the outline of this paper. In the following section we give the analysis of the Hamiltonian amplitude equation. In Sect. 3, the description of the φ^6 -model expansion method is given. In Sect. 4, the φ^6 -model expansion method is applied to get new soliton solutions such as bright, dark, singular, periodic and rational solutions for the Hamiltonian amplitude equation. The physical structure of the traveling wave solution is graphically displayed in the related 3D, 2d and contour graphs, and also the physical dynamics of the soliton solutions are explored in Sect. 5, while the conclusions will be drawn in Sect. 6.

2 Mathematical analysis of the model

The Hamiltonian amplitude equation is discussed in this section (Wadati et al. 1992; Mirzazadeh 2015; Peng 2003).

$$ip_t + p_{xx} + 2\sigma|p|^2p = 0, \quad \sigma = \pm 1 \quad (1)$$

$$ip_x + p_{tt} + 2\sigma|p|^2p = 0, \quad \sigma = \pm 1 \quad (2)$$

$$iq_x + q_{tt} + 2\sigma|q|^2q - \epsilon q_{xt} = 0, \quad \sigma = \mp 1, \quad \epsilon \ll 1. \quad (3)$$

The Eqs. (1), (2), (3) are known to be Hamiltonian equations. Equation (1) is the famous Schrödinger equation in the literature. This equation has an important place in applied sciences and has come to the fore in quantum mechanics in nonlinear optics, deep sea waves and Langmuir waves in plasma. Equation (2) is the reversed version of the variable in the Schrödinger equation. Renamed to emphasize the roles of the variables, this equation is designed as a nonlinear modulation model for stable plane waves in an unstable medium. This equation is known as the unstable nonlinear Schrödinger equation. Equation (3), which appears as the Hamiltonian amplitude equation, is an improved version of the Eq. (2), some studies show that Eq. (3), which we consider in this study, has the same generality as Eq. (1) (Feng et al. 2007).

The study of nonlinear waves in plasmas in the setting of a non-magnetized fluid-type plasma is the focus of this research. The dispersion of the Langmuir (electrostatic) wave when it generates some low-frequency acoustic waves, such as the ion-acoustic wave, is of interest to us. The nonlinearity of the Langmuir wave's ponderomotive influence on the local charge density allows for this process (Latifi and Leon 1991). Langmuir waves cannot propagate in a nonideal plasma, according to an extension of the Landau theory. In theoretical and numerical studies of nonideal plasmas, however, both Langmuir and ion-acoustic waves have been discovered. Experimental observations published in Morozov and Norman (2005) backed up these findings. Despite this, the significance of these findings has never been properly recognised. As a result, Langmuir wave features such as dispersion and damping rate have received little attention.

Equation (3) is an equation that governs such instabilities of modulated wave trains, with the additional term $-\epsilon q_{xt}$ eliminating the unpredictable nonlinear Schrodinger equation's ill-posedness. It is a Hamiltonian analog of the Kuramoto-Sivashinsky equation, which appears to be non-integrable in dissipative systems.

The nonlinear Schrodinger's equation explains many nonlinear physical processes in applied sciences, including solitons in nonlinear optical fibers, solitons in the mean-field theory of Bose-Einstein condensates, and rogue waves in oceanography.

To solve Eq. (3), we use the wave transformation shown below.

$$q(x, t) = P(\zeta)e^{i(-kx+wt+\theta)}, \quad (4)$$

where $P(\zeta)$ is the shape of the pulse and

$$\zeta = \mu(x - vt), \quad (5)$$

here v donates the soliton's velocity. In the phase factor, k represents the soliton frequency, the phase constant θ , and ω the soliton wave number. Substituting Eq. (4) into Eq. (3) results in the following couple of equations after breaking into real and imaginary parts (Wadati et al. 1992; Mirzazadeh 2015; Peng 2003):

$$1 - 2v\omega - \epsilon kv - \epsilon\omega = 0, \quad (6)$$

and

$$\mu^2(v^2 + \epsilon v)P'' - (\epsilon k\omega + \omega^2 - k)P + 2\sigma P^3 = 0, \quad (7)$$

using Eq. (6), we get

$$v = \frac{1 - \epsilon\omega}{\epsilon k + 2\omega}. \quad (8)$$

In the following part, Eq. (7) will be examined.

3 Description of the method

According to Zayed et al. Zayed Elsayed ME et al. (2018) the following are the key steps of a recent φ^6 -model expansion method:

Step-1: Consider the following nonlinear evolution equation (NLEE) for $q = q(x, t)$.

$$W(q, q_x, q_t, q_{xx}, q_{xt}, q_{tt}, \dots) = 0, \quad (9)$$

there W is a polynomial of $q(x, t)$ and its highest order partial derivatives, including its nonlinear terms.

Step-2: Making use of the wave transformation

$$q(x, t) = q(\zeta), \quad \zeta = \mu(x - vt), \quad (10)$$

where v represents wave velocity and Eq. (10) can be converted into the nonlinear ordinary differential equation shown below

$$\Omega(q, q', -vq', q'', \dots) = 0, \quad (11)$$

where the derivatives with respect to ζ are represented by prime.

Step-3: Suppose that the formal solution to Eq. (11) exists:

$$q(\zeta) = \sum_{i=0}^{2N} \alpha_i U^i(\zeta), \tag{12}$$

where $\alpha_i (i = 0, 1, 2, \dots, N)$ are to be determined constants, N can be obtained using the balancing rule, and $U(\zeta)$ satisfies the auxiliary NLODE;

$$\begin{aligned} U'^2(\zeta) &= h_0 + h_2 U^2(\zeta) + h_4 U^4(\zeta) + h_6 U^6(\zeta), \\ U''(\zeta) &= h_2 U(\zeta) + 2h_4 U^3(\zeta) + 3h_6 U^5(\zeta), \end{aligned} \tag{13}$$

where $h_i (i = 0, 2, 4, 6)$ are real constants that will be discovered later.

Step-4: It is well known that the answer to Eq. (13) is as follows;

$$U(\zeta) = \frac{P(\zeta)}{\sqrt{fP^2(\zeta) + g}}, \tag{14}$$

provided that $0 < fP^2(\zeta) + g$ and $P(\zeta)$ is the Jacobi elliptic equation solution

$$P'^2(\zeta) = l_0 + l_2 P^2(\zeta) + l_4 P^4(\zeta), \tag{15}$$

where $l_i (i = 0, 2, 4)$ are unknown constants to be determined, whilst f and g are given by

$$\begin{aligned} f &= \frac{h_4(l_2 - h_2)}{(l_2 - h_2)^2 + 3l_0l_4 - 2l_2(l_2 - h_2)}, \\ g &= \frac{3l_0h_4}{(l_2 - h_2)^2 + 3l_0l_4 - 2l_2(l_2 - h_2)}, \end{aligned} \tag{16}$$

under the restriction condition

$$h_4^2(l_2 - h_2)[9l_0l_4 - (l_2 - h_2)(2l_2 + h_2)] + 3h_6[-l_2^2 + h_2^2 + 3l_0l_4]^2 = 0. \tag{17}$$

Step-5: According to Zayed Elsayed ME et al. (2018), it is well known that the Jacobi elliptic solutions of Eq. (15) can be calculated when $0 < m < 1$. We can have the exact solutions of Eq. (9) by substituting Eqs. (14) and (15) into Eq. (12).

Function	$m \rightarrow 1$	$m \rightarrow 0$	Function	$m \rightarrow 1$	$k \rightarrow 0$
$sn(\zeta, m)$	$\tanh(\zeta)$	$\sin(\zeta)$	$ds(\zeta, m)$	$\operatorname{csch}(\zeta)$	$\operatorname{csc}(\zeta)$
$cn(\zeta, m)$	$\operatorname{sech}(\zeta)$	$\cos(\zeta)$	$sc(\zeta, m)$	$\sinh(\zeta)$	$\tan(\zeta)$
$dn(\zeta, m)$	$\operatorname{sech}(\zeta)$	1	$sd(\zeta, m)$	$\sinh(\zeta)$	$\sin(\zeta)$
$ns(\zeta, m)$	$\operatorname{coth}(\zeta)$	$\operatorname{csc}(\zeta)$	$nc(\zeta, m)$	$\operatorname{cosh}(\zeta)$	$\operatorname{sec}(\zeta)$
$cs(\zeta, m)$	$\operatorname{csch}(\zeta)$	$\operatorname{cot}(\zeta)$	$cd(\zeta, m)$	1	$\cos(\zeta)$

4 Application of the φ^6 -model expansion method

From Eq. (7), we get $N = 1$ by balancing P'' with P^3 , we obtain the following by substituting $N = 1$ in Eq. (12).

$$P(\zeta) = \alpha_0 + \alpha_1 U(\zeta) + \alpha_2 U^2(\zeta), \tag{18}$$

where α_0, α_1 and α_2 are constants to be determined.

We obtain the following algebraic equations by substituting Eq. (18) along with Eqs. (13) and (14) into Eq. (7) and setting the coefficients of all powers of $U^i(\zeta)$, $i = 0, 1, \dots, 6$ to be equal to zero

$$\begin{aligned}
 U^0(\zeta); & \quad k\alpha_0 - \omega^2\alpha_0 - k\omega\epsilon\alpha_0 + 2\sigma\alpha_0^3 + 2v^2\mu^2h_0\alpha_2 + 2v\epsilon\mu^2h_0\alpha_2 = 0, \\
 U^1(\zeta); & \quad \alpha_1(k - \omega^2 - k\omega\epsilon + v(v + \epsilon)\mu^2h_2 + 6\sigma\alpha_0^2) = 0, \\
 U^2(\zeta); & \quad \alpha_2(k - \omega^2 - k\omega\epsilon + 4v(v + \epsilon)\mu^2h_2) + 6\sigma\alpha_0(\alpha_1^2 + \alpha_0\alpha_2) = 0, \\
 U^3(\zeta); & \quad 2\alpha_1(v(v + \epsilon)\mu^2h_4 + \sigma(\alpha_1^2 + 6\alpha_0\alpha_2)) = 0, \\
 U^4(\zeta); & \quad 6\alpha_2(v(v + \epsilon)\mu^2h_4 + \sigma(\alpha_1^2 + \alpha_0\alpha_2)) = 0, \\
 U^5(\zeta); & \quad 3\alpha_1(v(v + \epsilon)\mu^2h_6 + 2\sigma\alpha_2^2) = 0, \\
 U^6(\zeta); & \quad 8v(v + \epsilon)\mu^2h_6\alpha_2 + 2\sigma\alpha_2^3 = 0.
 \end{aligned}$$

We get the following result after solving the resulting system:

$$\begin{aligned}
 \alpha_0 = 0, \quad \alpha_1 &= \frac{\sqrt{-vh_4}\sqrt{v + \epsilon\mu}}{\sqrt{\sigma}}, \quad \alpha_2 = 0, \\
 h_2 &= \frac{-k + \omega^2 + k\omega\epsilon}{v(v + \epsilon)\mu^2}, \quad h_4 = h_4, \quad h_6 = 0.
 \end{aligned} \tag{19}$$

In view of Eqs. (14), (18) and (19) along with the Jacobi elliptic functions in the above table, we obtain the following exact solutions of Eq. (3).

1. If $l_0 = 1, l_2 = -(1 + m^2), l_4 = m^2, 0 < m < 1$, then $P(\zeta) = sn(\zeta, m)$ or $P(\zeta) = cd(\zeta, m)$ and we have

$$q_1(x, t) = \frac{\sqrt{-vh_4}\sqrt{v + \epsilon\mu}}{\sqrt{\sigma}} \left[\frac{sn(\zeta, m)}{\sqrt{f(sn(\zeta, m))^2 + g}} \right] e^{i(-kx + wt + \theta)}, \tag{20}$$

or

$$q_{1,1}(x, t) = \frac{\sqrt{-vh_4}\sqrt{v + \epsilon\mu}}{\sqrt{\sigma}} \left[\frac{cd(\zeta, m)}{\sqrt{f(cd(\zeta, m))^2 + g}} \right] e^{i(-kx + wt + \theta)}, \tag{21}$$

such that $0 < c, \zeta = \mu(x - vt)$ and f and g are given by

$$\begin{aligned}
 f &= \frac{(1 + m^2 + h_2)h_4}{1 - m^2 + m^4 - h_2^2}, \\
 g &= \frac{-3h_4}{1 - m^2 + m^4 - h_2^2},
 \end{aligned}$$

under the restriction condition

$$-h_4^2(-1 - m^2 - h_2)(-1 + 2m^2 - h_2)(-2 + m^2 + h_2) = 0.$$

If $m \rightarrow 1$, then the dark soliton is obtained

$$q_{1,2}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \tanh(\zeta)}{\sqrt{\frac{h_4(3-(2+h_2)\tanh^2(\zeta))}{-1+h_2^2}}} e^{i(-kx+wt+\theta)}, \tag{22}$$

such that

$$-h_4^2(2+h_2)[-1+h_2]^2 = 0.$$

If $m \rightarrow 0$, then the periodic solution is obtained

$$q_{1,3}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \sin(\zeta)}{\sqrt{\frac{h_4(-3+(2+h_2)\sin^2(\zeta))}{-1+h_2^2}}} e^{i(-kx+wt+\theta)}, \tag{23}$$

such that

$$h_4^2(-1+h_2)[(-2-h_2)(-1+h_2)] = 0.$$

2. If $l_0 = 1 - m^2, l_2 = 2m^2 - 1, l_4 = -m^2, 0 < m < 1$, then $P(\zeta) = cn(\zeta, m)$ therefore

$$q_2(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{cn(\zeta, m)}{\sqrt{f(cn(\zeta, m))^2 + g}} \right]}{e^{i(-kx+wt+\theta)}}, \tag{24}$$

where f and g are determined by

$$f = -\frac{(-1+2m^2-h_2)h_4}{1-m^2+m^4-h_2^2},$$

$$g = \frac{3(-1+m^2)h_4}{1-m^2+m^4-h_2^2},$$

under the constraint condition

$$h_4^2(-1+2m^2-h_2)[(-2+m^2+h_2)(1+m^2+h_2)] = 0.$$

If $m \rightarrow 1$, then the bright soliton is retrieved

$$q_{2,1}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\mu}}{\sqrt{\sigma}} \operatorname{sech}(\zeta)}{\sqrt{\frac{-(-3+(1+h_2)\operatorname{sech}^2(\zeta))h_4}{-1+h_2^2}}} e^{i(-kx+wt+\theta)}, \tag{25}$$

provided that

$$h_4^2(-1-h_2)[h_2^2-h_2-2] = 0.$$

If $m \rightarrow 0$, then the periodic solution is obtained

$$q_{2,2}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \sin(\zeta)}{\sqrt{\frac{-(-3+(1+h_2)\sin^2(\zeta))h_4}{-1+h_2^2}}} e^{i(-kx+wt+\theta)}, \tag{26}$$

such that

$$h_4^2(-1 - h_2)[(-2 + h_2)(1 + h_2)] = 0.$$

3. If $l_0 = m^2 - 1, l_2 = 2 - m^2, l_4 = -1, 0 < m < 1$, then $P(\zeta) = dn(\zeta, m)$ which gives

$$q_3(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{dn(\zeta, m)}{\sqrt{f(dn(\zeta, m))^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{27}$$

where f and g are determined by

$$f = \frac{(-2 + m^2 + h_2)h_4}{1 - m^2 + m^4 - h_2^2},$$

$$g = \frac{-3(-1 + m^2)h_4}{1 - m^2 + m^4 - h_2^2},$$

under the restriction condition

$$h_4^2(2 - m^2 - h_2)[-(-1 + 2m^2 + h_2)(1 + m^2 + h_2)] = 0.$$

If $m \rightarrow 1$, then the bright soliton is obtained

$$q_{3,1}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \operatorname{sech}(\zeta)}{\sqrt{\frac{-h_4\operatorname{sech}^2(\zeta)}{1+h_2}}} e^{i(-kx+wt+\theta)}, \tag{28}$$

provided that

$$h_4^2(1 - h_2)[-2 + h_2 + h_2^2] = 0.$$

If $m \rightarrow 0$, then the rational solution is obtained

$$q_{3,2}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}}}{\sqrt{\frac{-h_4}{1+h_2}}} e^{i(-kx+wt+\theta)}, \tag{29}$$

such that

$$h_4^2(2 - h_2)[(1 + h_2)^2] = 0.$$

4. If $l_0 = m^2$, $l_2 = -(1 + m^2)$, $l_4 = 1$, $0 < m < 1$, then $P(\zeta) = ns(\zeta, m)$ or $P(\zeta) = dc(\zeta, m)$ then

$$q_{4,1}(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{ns(\zeta, m)}{\sqrt{f(ns(\zeta, m))^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{30}$$

or

$$q(x, t)_{4,2} = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{dc(\zeta, m)}{\sqrt{f(dc(\zeta, m))^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{31}$$

where f and g are given by

$$f = \frac{(1 + m^2 + h_2)h_4}{1 - m^2 + m^4 - h_2^2},$$

$$g = \frac{-3m^2h_4}{1 - m^2 + m^4 - h_2^2},$$

under the constraint condition

$$h_4^2(-1 - m^2 - h_2) [-(-1 + 2m^2 - h_2)(-2 + m^2 + h_2)] = 0.$$

If $m \rightarrow 1$, then the dark-singular solution is obtained

$$q_{4,3}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \coth(\zeta)}{\sqrt{\frac{-(-1+h_2+(2+h_2)\operatorname{csch}^2(\zeta))h_4}{-1+h_2^2}}} e^{i(-kx+wt+\theta)}, \tag{32}$$

such that

$$h_4^2(-2 - h_2) [(-1 + h_2)^2] = 0.$$

If $m \rightarrow 0$, then the periodic solution is obtained

$$q_{4,4}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \csc(\zeta)}{\sqrt{\frac{h_4(3-(2+h_2)\operatorname{csc}^2(\zeta))}{-1+h_2^2}}} e^{i(-kx+wt+\theta)}, \tag{33}$$

such that

$$h_4^2(-2 - h_2) [(-1 + h_2)^2] = 0.$$

5. If $l_0 = -m^2$, $l_2 = 2m^2 - 1$, $l_4 = 1 - m^2$, $0 < m < 1$, then $P(\zeta) = nc(\zeta, m)$ and we have

$$q_5(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{nc(\zeta, m)}{\sqrt{f(nc(\zeta, m))^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{34}$$

where f and g are given by

$$f = \frac{-(-1 + 2m^2 - h_2)h_4}{1 - m^2 + m^4 - h_2^2},$$

$$g = \frac{3m^2h_4}{1 - m^2 + m^4 - h_2^2},$$

under the constraint condition

$$h_4^2(-1 + 2m^2 - h_2)[(-2 + m^2 + h_2)(1 + m^2 + h_2)] = 0.$$

If $m \rightarrow 1$, then the solitary wave solution is obtained

$$q_{5,1}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \cosh(\zeta)}{\sqrt{\frac{h_4(-3+(1-h_2)\cosh^2(\zeta))}{-1+h_2^2}}} e^{i(-kx+wt+\theta)}, \tag{35}$$

such that

$$h_4^2(1 - h_2)[-2 + h_2 + h_2^2] = 0.$$

If $m \rightarrow 0$, then the periodic solution is obtained

$$q_{5,2}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \sec(\zeta)}{\sqrt{\frac{-h_4 \sec^2(\zeta)}{-1+h_2}}} e^{i(-kx+wt+\theta)}, \tag{36}$$

such that

$$h_4^2(-1 - h_2)[(-2 + h_2)(1 + h_2)] = 0.$$

6. If $l_0 = -1, l_2 = 2 - m^2, l_4 = -(1 - m^2), 0 < m < 1$, then $P(\zeta) = nd(\zeta, m)$ and we have

$$q_6(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{nd(\zeta, m)}{\sqrt{f(nd(\zeta, m))^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{37}$$

where f and g are given by

$$f = \frac{(-2 + m^2 + h_2)h_4}{1 - m^2 + m^4 - h_2^2},$$

$$g = \frac{3h_4}{1 - m^2 + m^4 - h_2^2},$$

under the constraint condition

$$h_4^2(2 - m^2 - h_2)[-(-1 + 2m^2 - h_2)(1 + m^2 + h_2)] = 0.$$

7. If $l_0 = 1, l_2 = 2 - m^2, l_4 = 1 - m^2, 0 < m < 1$, then $P(\zeta) = sc(\zeta, m)$ and we have

$$q_7(x, t) = \frac{\sqrt{-vh_4}\sqrt{v + \epsilon\mu}}{\sqrt{\sigma}} \left[\frac{sc(\zeta, m)}{\sqrt{f(sc(\zeta, m))^2 + g}} \right] e^{i(-kx + wt + \theta)}, \tag{38}$$

where f and g are given by

$$f = \frac{(-2 + m^2 + h_2)h_4}{1 - m^2 + m^4 - h_2^2},$$

$$g = \frac{-3h_4}{1 - m^2 + m^4 - h_2^2},$$

under the constraint condition

$$h_4^2(2 - m^2 - h_2)[-(-1 + 2m^2 - h_2)(1 + m^2 + h_2)] = 0.$$

If $m \rightarrow 1$, then the solitary wave solution is obtained

$$q_{7,1}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v + \epsilon\mu}}{\sqrt{\sigma}} \sinh(\zeta)}{\sqrt{\frac{(3 + (1 - h_2) \sinh^2(\zeta))h_4}{-1 + h_2^2}}} e^{i(-kx + wt + \theta)}, \tag{39}$$

such that

$$h_4^2(1 - h_2)[-2 + h_2 + h_2^2] = 0.$$

If $m \rightarrow 0$, then the periodic wave solution is obtained

$$q_{7,2}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v + \epsilon\mu}}{\sqrt{\sigma}} \tan(\zeta)}{\sqrt{\frac{(3 - (-2 + h_2) \tan^2(\zeta))h_4}{-1 + h_2^2}}} e^{i(-kx + wt + \theta)}, \tag{40}$$

such that

$$h_4^2(2 - h_2)[(1 + h_2)^2] = 0.$$

8. If $l_0 = 1, l_2 = 2m^2 - 1, l_4 = -m^2(1 - m^2), 0 < m < 1$, then $P(\zeta) = sd(\zeta, m)$ and we have

$$q_8(x, t) = \frac{\sqrt{-vh_4}\sqrt{v + \epsilon\mu}}{\sqrt{\sigma}} \left[\frac{sd(\zeta, m)}{\sqrt{f(sd(\zeta, m))^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{41}$$

where f and g are given by

$$f = \frac{(-1 + 2m^2 - h_2)h_4}{1 - m^2 + m^4 - h_2^2},$$

$$g = \frac{-3h_4}{1 - m^2 + m^4 - h_2^2},$$

under the constraint condition

$$h_4^2(-1 + 2m^2 - h_2)[(-2 + m^2 + h_2)(1 + m^2 + h_2)] = 0.$$

9. If $l_0 = 1 - m^2, l_2 = 2 - m^2, l_4 = 1, 0 < m < 1$, then $P(\zeta) = cs(\zeta, m)$ and we have

$$q_9(x, t) = \frac{\sqrt{-vh_4}\sqrt{v + \epsilon\mu}}{\sqrt{\sigma}} \left[\frac{cs(\zeta, m)}{\sqrt{f(cs(\zeta, m))^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{42}$$

where f and g are given by

$$f = \frac{(-2 + m^2 + h_2)h_4}{1 - m^2 + m^4 - h_2^2},$$

$$g = \frac{3(-1 + m^2)h_4}{1 - m^2 + m^4 - h_2^2},$$

under the constraint condition

$$h_4^2(2 - m^2 - h_2)[-(-1 + 2m^2 - h_2)(1 + m^2 + h_2)] = 0.$$

If $m \rightarrow 1$, then the singular solitary wave solution is obtained

$$q_{9,1}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v + \epsilon\mu}}{\sqrt{\sigma}} csch(\zeta)}{\sqrt{\frac{-csch^2(\zeta)h_4}{1+h_2}}} e^{i(-kx+wt+\theta)}, \tag{43}$$

such that

$$h_4^2(1 - h_2)[-2 + h_2 + h_2^2] = 0.$$

If $m \rightarrow 0$, then the periodic wave solution is obtained

$$q_{9,2}(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu} \cot(\zeta)}{\sqrt{\sigma}} e^{i(-kx+wt+\theta)}, \tag{44}$$

$$\sqrt{\frac{(3+(2-h_2)\cot^2(\zeta))h_4}{-1+h_2^2}}$$

such that

$$h_4^2(2-h_2)\left[(1+h_2)^2\right] = 0.$$

10. If $l_0 = -m^2(1-m^2)$, $l_2 = 2m^2-1$, $l_4 = 1$, $0 < m < 1$, then $P(\zeta) = ds(\zeta, m)$ and we have

$$q_{10}(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{ds(\zeta, m)}{\sqrt{f(ds(\zeta, m))^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{45}$$

where f and g are given by

$$f = \frac{-(-1+2m^2-h_2)h_4}{1-m^2+m^4-h_2^2},$$

$$g = \frac{-3m^2(-1+m^2)h_4}{1-m^2+m^4-h_2^2},$$

under the constraint condition

$$h_4^2(-1+2m^2-h_2)\left[(-2+m^2+h_2)(1+m^2+h_2)\right] = 0.$$

11. If $l_0 = \frac{1-m^2}{1\pm sn(\zeta, m)^4}$, $l_2 = \frac{1+m^2}{2}$, $l_4 = \frac{1-m^2}{4}$, $0 < m < 1$, then $P(\zeta) = nc(\zeta, m) \pm sc(\zeta, m)$ or $P(\zeta) = \frac{cn(\zeta, m)^4}{1\pm sn(\zeta, m)}$ and we have

$$q_{11,1}(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{nc(\zeta, m) \pm sc(\zeta, m)}{\sqrt{f(nc(\zeta, m) \pm sc(\zeta, m))^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{46}$$

or

$$q_{11,2}(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{\frac{cn(\zeta, m)}{1\pm sn(\zeta, m)}}{\sqrt{f\left(\frac{cn(\zeta, m)}{1\pm sn(\zeta, m)}\right)^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{47}$$

where f and g are given by

$$f = \frac{-8(1 + m^2 - 2h_2)h_4}{1 + 14m^2 + m^4 - 16h_2^2},$$

$$g = \frac{12(-1 + m^2)h_4}{1 + 14m^2 + m^4 - 16h_2^2},$$

under the constraint condition

$$h_4^2 \left(\frac{1}{2} (1 + m^2 - 2h_2) \right) \left[\frac{1}{16} (1 + (-6 + m)m + 4h_2) (1 + m(6 + m) + 4h_2) \right] = 0.$$

If $m \rightarrow 1$, then The combined singular solitary wave solution is obtained

$$q_{11,3}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} (\sinh(\zeta) + \cosh(\zeta))}{\sqrt{\frac{-(\sinh(\zeta)+\cosh(\zeta))^2 h_4}{1+h_2}}} e^{i(-kx+wt+\theta)}, \tag{48}$$

or dark-bright optical soliton

$$q_{11,4}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left(\frac{\operatorname{sech}(\zeta)}{1+\tanh(\zeta)} \right)}{\sqrt{\frac{-\left(\frac{\operatorname{sech}(\zeta)}{1+\tanh(\zeta)}\right)^2 h_4}{1+h_2}}} e^{i(-kx+wt+\theta)}, \tag{49}$$

such that

$$h_4^2 (1 - h_2) [-2 + h_2 + h_2^2] = 0.$$

If $m \rightarrow 0$, then the periodic wave solution is obtained

$$q_{11,5}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} (\sec(\zeta) + \tan(\zeta))}{2\sqrt{\frac{(-5+4h_2+(1+4h_2)\sin(\zeta))h_4}{(16h_2^2-1)(-1+\sin(\zeta))}}} e^{i(-kx+wt+\theta)}, \tag{50}$$

or

$$q_{11,6}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \frac{\cos(\zeta)}{(1+\sin(\zeta))} e^{i(-kx+wt+\theta)}}{2\sqrt{\frac{((2-4h_2)\cos^2(\zeta)+3(1+\sin(\zeta))^2)h_4}{(1+\sin(\zeta))^2(16h_2^2-1)}}}, \tag{51}$$

such that

$$h_4^2 \left(\frac{1}{2} - h_2 \right) \left[\frac{1}{16} (1 + 4h_2)^2 \right] = 0.$$

12. If $l_0 = \frac{-(1-m^2)^2}{4}$, $l_2 = \frac{1+m^2}{2}$, $l_4 = \frac{-1}{4}$, $0 < m < 1$, then $P(\zeta) = mcn(\zeta, m) \pm dn(\zeta, m)$ and we have

$$q_{12}(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{mcn(\zeta, m) \pm dn(\zeta, m)}{\sqrt{f(mcn(\zeta, m) \pm dn(\zeta, m))^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{52}$$

where f and g are given by

$$f = \frac{-8(1+m^2-2h_2)h_4}{1+14m^2+m^4-16h_2^2},$$

$$g = \frac{12(-1+m^2)^2h_4}{1+14m^2+m^4-16h_2^2},$$

under the constraint condition

$$h_4^2 \left(\frac{1}{2}(1+m^2-2h_2) \right) \left[\frac{1}{16}(1+(-6+m)m+4h_2)(1+m(6+m)+4h_2) \right] = 0.$$

13. If $l_0 = \frac{1}{4}, l_2 = \frac{1-2m^2}{2}, l_4 = \frac{1}{4}, 0 < m < 1$, then $P(\zeta) = \frac{sn(\zeta, m)}{1 \pm cn(\zeta, m)}$ and we have

$$q_{13}(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{\frac{sn(\zeta, m)}{1 \pm cn(\zeta, m)}}{\sqrt{f \left(\frac{sn(\zeta, m)}{1 \pm cn(\zeta, m)} \right)^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{53}$$

where f and g are given by

$$f = \frac{8(-1+2m^2+2h_2)h_4}{1-16m^2+16m^4-16h_2^2},$$

$$g = \frac{-12h_4}{1-16m^2+16m^4-16h_2^2},$$

under the constraint condition

$$h_4^2 \left(\frac{1}{2} - m^2 - h_2 \right) \left[\frac{1}{16} + 2m^2 - 2m^4 + \left(\frac{1}{2} - m^2 \right) h_2 + h_2^2 \right] = 0.$$

If $m \rightarrow 1$, then the dark-bright solitary wave solution is obtained

$$q_{13,1}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left(\frac{\tanh(\zeta)}{1+\operatorname{sech}(\zeta)} \right) e^{i(-kx+wt+\theta)}}{2\sqrt{\frac{(3-2(1+2h_2)\left(\frac{\tanh(\zeta)}{1+\operatorname{sech}(\zeta)}\right)^2)h_4}{-1+16h_2^2}}}, \tag{54}$$

such that

$$h_4^2 \left(\frac{-1}{2} - h_2 \right) \left[\frac{1}{16}(1-4h_2)^2 \right] = 0.$$

If $m \rightarrow 0$, then the periodic wave solution is obtained

$$q_{13,2}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left(\frac{\sin(\zeta)}{1+\cos(\zeta)} \right) e^{i(-kx+wt+\theta)}}{2\sqrt{\frac{(3+2(1-2h_2)\left(\frac{\sin(\zeta)}{1+\cos(\zeta)}\right)^2)h_4}{-1+16h_2^2}}}, \tag{55}$$

such that

$$h_4^2 \left(\frac{1}{2} - h_2 \right) \left[\frac{1}{16} (1 + 4h_2)^2 \right] = 0.$$

14. If $l_0 = \frac{1}{4}, l_2 = \frac{1+m^2}{2}, l_4 = \frac{(1-m^2)^2}{4}, 0 < m < 1$, then $P(\zeta) = \frac{sn(\zeta, m)}{cn(\zeta, m)\pm dn(\zeta, m)}$ and we have

$$q_{14}(x, t) = \frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left[\frac{\frac{sn(\zeta, m)}{cn(\zeta, m)\pm dn(\zeta, m)}}{\sqrt{f\left(\frac{sn(\zeta, m)}{cn(\zeta, m)\pm dn(\zeta, m)}\right)^2 + g}} \right] e^{i(-kx+wt+\theta)}, \tag{56}$$

where f and g are given by

$$f = \frac{-8(1+m^2-2h_2)h_4}{1+14m^2+m^4-16h_2^2},$$

$$g = \frac{-12h_4}{1+14m^2+m^4-16h_2^2},$$

under the constraint condition

$$h_4^2 \left(\frac{1}{2} (1 + m^2 - 2h_2) \right) \left[\frac{1}{16} (1 + (-6 + m)m + 4h_2) (1 + m(6 + m) + 4h_2) \right] = 0.$$

If $m \rightarrow 1$, then singular solitary wave solution is obtained

$$q_{14,1}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \sinh(\zeta)}{\sqrt{\frac{(3+(1-h_2)\sinh^2(\zeta))h_4}{-1+h_2^2}}}, e^{i(-kx+wt+\theta)}, \tag{57}$$

such that

$$h_4^2 (1 - h_2) [-2 + h_2 + h_2^2] = 0.$$

If $m \rightarrow 0$, then the periodic wave solution is obtained

$$q_{14,2}(x, t) = \frac{\frac{\sqrt{-vh_4}\sqrt{v+\epsilon\mu}}{\sqrt{\sigma}} \left(\frac{\sin(\zeta)}{1+\cos(\zeta)} \right) e^{i(-kx+wt+\theta)}}{2\sqrt{\frac{(3+2(1-2h_2)\left(\frac{\sin(\zeta)}{1+\cos(\zeta)}\right)^2)h_4}{-1+16h_2^2}}}, \tag{58}$$

such that

$$h_4^2 \left(\frac{1}{2} - h_2 \right) \left[\frac{1}{16} (1 + 4h_2)^2 \right] = 0.$$

5 Result and discussion

The new Hamiltonian amplitude equation solutions always introduce new features of practical applications. As a result, developing new methods for solving equations is important at all times. The φ^6 -model expansion approach is used in this paper to provide the precise solutions to the Hamiltonian amplitude model, it is an equation that regulates modulated wave train instabilities, with the additional term $-cq_{,xt}$ eliminating the unpredictable nonlinear Schrodinger equation's ill-posedness, the obtained solution to this model is in the form of Jacobi's elliptic functions. When $m \rightarrow 1$ the secured solutions approach hyperbolic functions, whilst $m \rightarrow 0$ the solutions degenerate into trigonometric solutions. These solutions include rational, bright, dark, singular, dark-bright, and trigonometric solitary wave solutions.

Many scholars have attempted a variety of approaches to find exact solutions to the Hamiltonian amplitude equation, such as the extended trial equation method, the extended Jacobi elliptic expansion method, the general projective Riccati equation method, the $\left(\frac{G'}{G}\right)$ -expansion method, the functional variable method, Lie symmetry method, the simplest equation method, He's semi-inverse variational principle method and Ansatz method, modified simplest equation method, the mapping method, the $\left(\frac{G'}{G}, \frac{1}{G}\right)$ -expansion method, the Lie classical and the $\left(\frac{G'}{G}\right)$ -expansion method etc. several solutions such as dark, bright, dark-bright and singular soliton solutions were found using the mentioned methods. When we compare our results in this paper to the results in [25-39], we conclude that our results are unique and have not been found elsewhere. Also, we used a computer ready package program to check our answers by plugging them back into the original Eq. (3).

Abundant traveling wave solutions have been generated to the Hamiltonian amplitude equation using a new φ^6 -model expansion technique. This study will play an important role in the study of nonlinear waves in plasmas in the setting of a non-magnetized fluid-type plasma, the dispersion of the Langmuir (electrostatic) wave when it generates some low-frequency acoustic waves, such as the ion-acoustic wave. In this study, the mathematical importance of the travelling wave solution presented in Eq. (21) may create physically different arguments for the magnetic moment and velocity phenomenon of special relativity. Therefore, let us present graphs of the traveling wave solution presented by Eq. (15), representing the standing wave at any instant, in which we will discuss their behavior.

The graphics presented in Fig. 1 are the behavior of dark soliton at any given moment, which plays an important role in the transfer of energy from one place to another. Also, to add a different dimension to the discussion, let's consider the physical meanings of the parameters in the transformation, known as the classical wave transform and presented by the Eqs. (1) and (4). The parameters in the solution of Eq. (22) traveling waves, which contain many constants, mathematically, have physical meanings. It is the internal dynamics of the traveling wave for different values of each parameter. We can say that there will be changes in traveling wave behavior for different values of each. In this discussion, we can observe the changes in the behavior of the dark soliton, which is presented as Eq. (22) for different values of k frequency, which is one of the internal dynamics of the traveling wave.

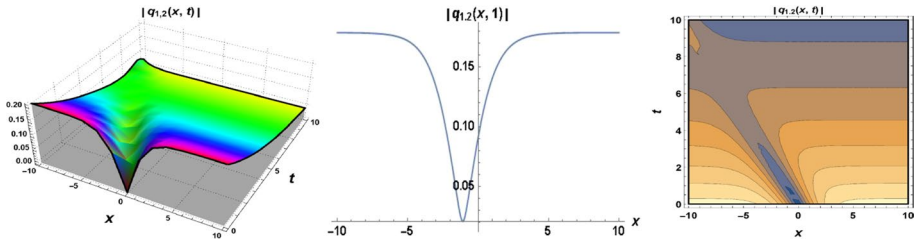


Fig. 1 3D, 2D and contour plots of Eq. (22), using $k = 0.6$, $\epsilon = 0.5$, $\mu = 0.5$, $\sigma = 4$, $r = 1$.

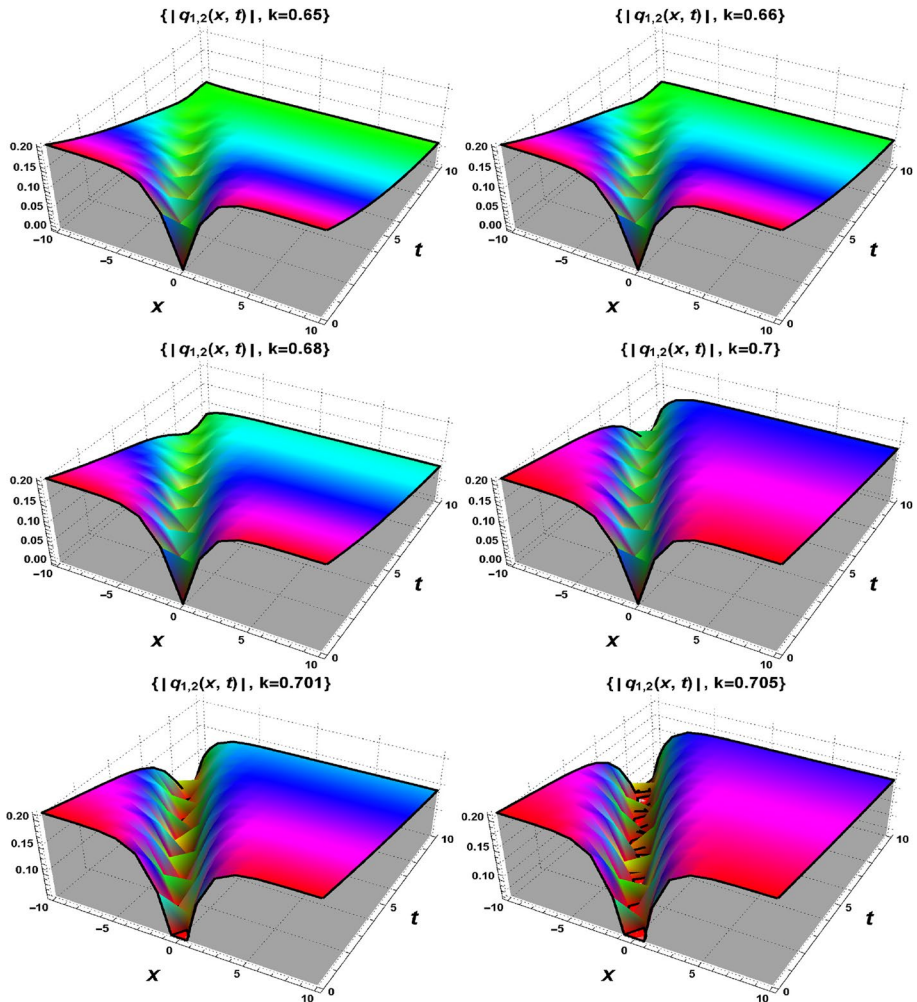


Fig. 2 3D plots of Eq. (22), using $\epsilon = 0.5$, $\mu = 0.5$, $\sigma = 4$, $r = 1$.

As seen in Fig. 2, the number of waves increases as the k frequency value increases. Changes in the amplitude and wavelength of the traveling wave are the answer given by the created solution if distinct internal dynamics other than frequency are not active. In this case, it can be observed in Fig. 2. Also, as the frequency increases, it can be observed that there are distortions at the endpoints of the traveling wave.

In order to observe the changes in the dark soliton more clearly, the simulation is done for different values of the wave frequency. Similarly, similar discussions can be made both for different physical parameters and for different traveling wave solutions.

6 Conclusion

In relation to the findings, it is worth mentioning that the chosen scheme generates a variety of novel solutions that are both interesting and useful for the governing model. The acquired results in this paper are considered to describe some of the physical implications of the Hamiltonian amplitude equation. This study will play an important role in the study of nonlinear waves in plasmas in the setting of a non-magnetized fluid-type plasma, the dispersion of the Langmuir (electrostatic) wave when it generates some low-frequency acoustic waves, such as the ion-acoustic wave. The behavior of dark soliton selected from fourteen different solutions produced for different values of frequency is analyzed. The effect of frequency, which is one of the internal dynamics of the traveling wave, on wave number, wavelength and wave amplitude is discussed. In addition, traveling wave solutions calculated with the mathematical technique used in this study confirm the equation. As a result, it can be said that it is an effective, powerful and usable mathematical instrument for obtaining traveling wave solutions of nonlinear partial differential equations.

Funding The authors have not disclosed any funding.

Declarations

Conflict of interest The authors have not disclosed any competing interests.

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