Check for
updates

A novel synchronization between two different chaotic systems (Convert Lorenz chaotic system to Chua chaotic system)

Dina Ahmed Kafi[1] · Salam K. Mousa[2] · Raied K. Jamal[1]

© The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2022

Abstract

In this work, a new synchronization was achieved between two different chaotic systems in behavior (Chua-Lorenz) by coupling them together with a coupling factor (k) and it was noted that Lorenz's behavior completely transformed Chua's behavior by changing the value of this factor. It is observed that when the values of the coupling factor are zero, the two systems are asynchronous, and when its value is 100, there is partial synchronization, while its value is 100000, so we get perfect synchronization. Where it is possible to use this synchronization in the field of secure communications.

Keywords Chaos · Chen system · Time series · Attractor · Synchronization

1 Introduction

There are many successful attempts that have been made by researchers working in the field of Chaos world, including Chen, Ikeda, Chua and others(Chua 1980; Lorenz 1963; Rossler 1976; Cascais et al. 1983; Chua et al. 1993; Nakagawa and Saito 1996; Chen and Ueta 1999). Researcher Chua invented his simple electric circuit in 1983(Chua 1980). These numerous attempts became theoretically and practically largely supportive of the idea of chaos that began with Researcher Lorenz(Lorenz 1963). Chua proved, through his theory, that chaos is an existing physical phenomenon and not just an illusion resulting from computer approximation errors. Chua has demonstrated through his work that chaotic behavior accompanies all areas of our daily lives (Ibrahim et al. 2016; Ibrahim and Jamal 2016). He designed his simple electronic circuit that displays chaotic behavior (vibrating irregular behavior) in which the wave does not repeat itself over time. The Chua system is considered a standard model for studying chaos because of its ease of construction. There are many previous studies that were presented to solve many problem(Jamal et al. 2021;

✉ Salam K. Mousa
salam.khalaf@uoanbar.edu.iq

1   College of Science, Physics Department, University of Baghdad, Baghdad, Iraq

2   College of Education for Pure Sciences, Physics Department, University of Anbar, Anbar, Falluja, Iraq

Abdulaali et al. 2021; Hamadi et al. 2022; Mousa and Jamal 2021), including the issue of secure communications (Jamal and Kafi 2016, 2019a, b; Kafi et al. 2016; Ibrahim et al. 2018), but the most important of these studies conducted now is an attempt to transform the behavior of a chaotic system into another, through linking and synchronizing them.

2 Modling

In this work, the behavior of two different systems, namely Lorenz and Chua, was studied and they were coupling together with a coupling factor k and the effect of this coupling on the behavior of the Lorenz system was studied.

The Lorenz system is defined by three ordinary differential equations, as follows(Chua 1980):

$$
\begin{aligned}
\mathrm{d/dt}\,(x_1) &= \sigma\,(y_1 - x_1) \\
\mathrm{d/dt}\,(y_1) &= (x_1)\,(r - z_1) - y_1 \\
\mathrm{d/dt}\,(z_1) &= (x_1)\,(y_1) - \beta\,(z_1)
\end{aligned}
\tag{1}
$$

where the Lorenz parameters system [r, σ, and β] are real positive number that equal [28, 15, 2.1] respectively and the system exhibt chaotic behavior for these value. The initial conditions of the Lorenz system [$xi1$, $yi1$, $zi1$] were chosen to be [0, 0.1, 0], respectively.

For the Chua system, its electronic circuit can be analyzed using Kirchhoff's laws. The dynamics of an electronic circuit can be modeled with three nonlinear ordinary differential equations in the variables x, y, and z, as follows (Lorenz 1963).

$$
\begin{aligned}
\mathrm{d/dt}\,(x_2) &= a\,(y_2 - x_2 - \mathrm{g}) \\
\mathrm{d/dt}\,(y_2) &= x_2 - y_2 + z_2 \\
\mathrm{d/dt}\,(z_2) &= -b\,(y_2) + \gamma\,(z_2) \\
\mathrm{g} &= \mathrm{c}\,(x_2) + (0.5)\big((d - c)\big(\mathrm{abs}(x_2 + 1) - \mathrm{abs}(x_2 - 1)\big)\big)
\end{aligned}
\tag{2}
$$

where Chua parameters system are [a, b, c, d, and γ] equal [15, 25.58, -0.714286, -1.14287, and 0.001] respectively and initial conditions [x_{i2}, y_{i2}, z_{i2}] are [0, 0.1, 0] respectively.

To achieve complete synchronization between the Lorenz system and the Chua system, the coupling terms are added to all the equations of the Lorenz system, as shown below:

$$
\begin{aligned}
\mathrm{d/dt}\,(x_1) &= \sigma\,(y_1 - x_1) + \mathrm{k}\,(x_2 - x_1) \\
\mathrm{d/dt}\,(y_1) &= (x_1)\,(r - z_1) - y_1 + \mathrm{k}\,(y_2 - y_1) \\
\mathrm{d/dt}\,(z_1) &= (x_1)\,(y_1) - \beta\,(z_1) + \mathrm{k}\,(z_2 - z_1)
\end{aligned}
\tag{3}
$$

where k(x_2-x_1), k(y_2-y_1), and k(z_2-z_1) are coupling terms and k is called coupling factor and is real numbers.

3 Results and discussion

To analyze the results obtained from the mathematical model, the Matlab program was used, where the differential equations of all systems were solved using fourth-degree Runga–Kutta integration, as follows:

Figure 1 represent the time series of Lorenz system at initial conditions equal [0, 0.1, 0], while Fig. 2 represent the strange attractor was shown. The shape of the Lorenz attractor, when plotted graphically, it be seen to resemble a butterfly.

Figure 3 represent the time series of Chua system at initial conditions equal [0, 0.1, 0], while Fig. 4 represent the strange attractor (double scroll) was shown. The double-scroll
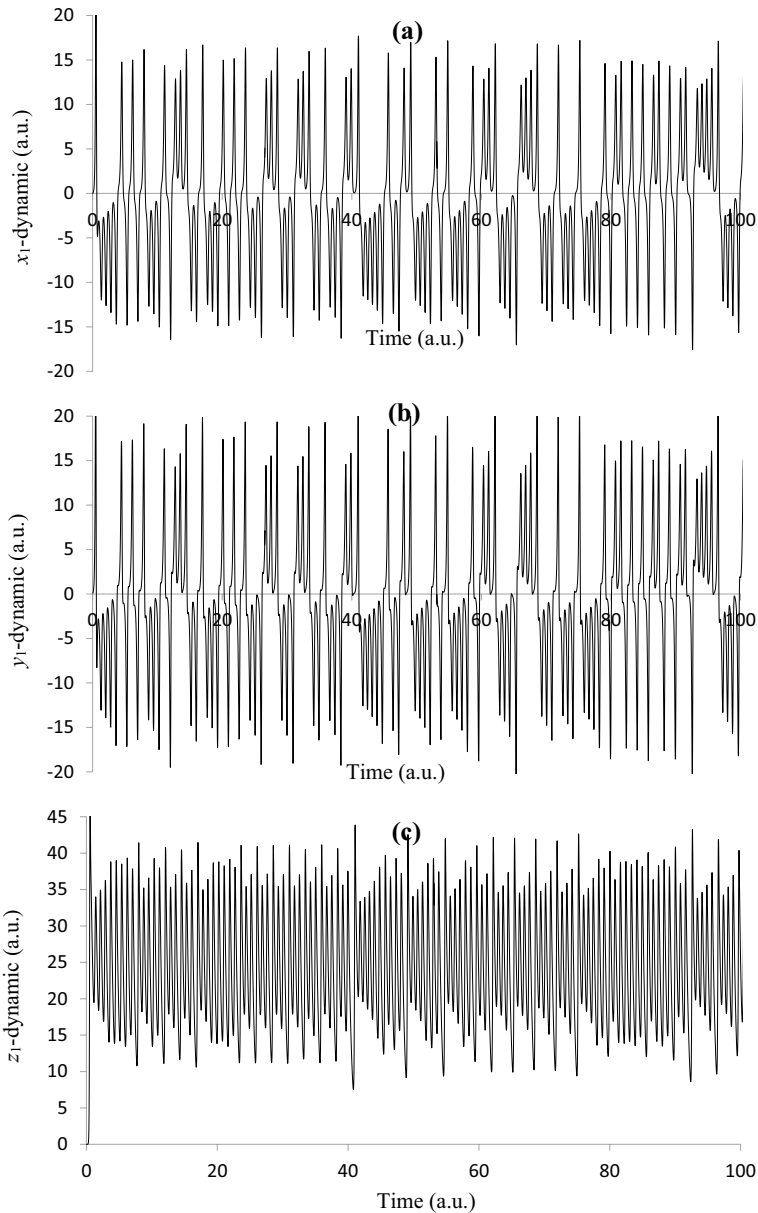

Fig. 1 Time series of Lorenz system at initial conditions $x_i = 0$, $y_i = 0.1$, and $z_i = 0$, (a) x_1-dynamic, (b) y_1-dynamic, z_1-dynamic
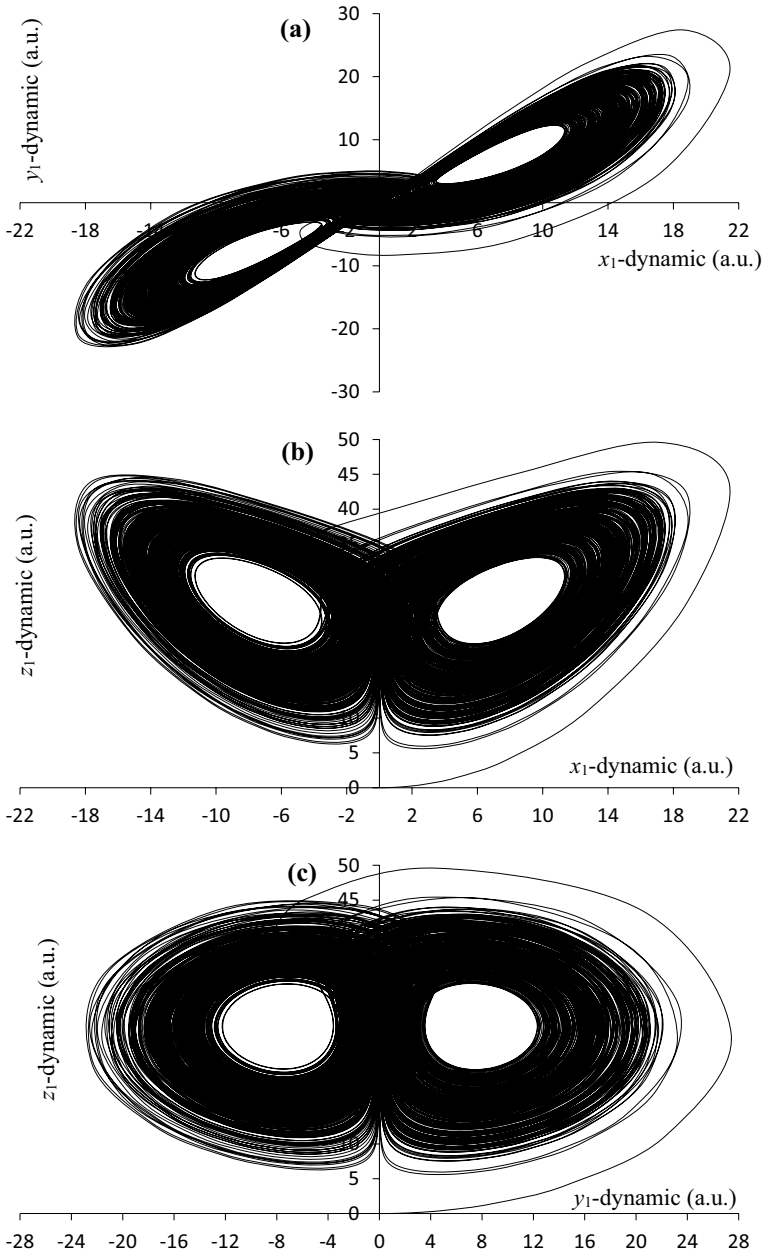
Fig. 2 Attractor of Lorenz system at initial conditions $x_i = 0$, $y_i = 0.1$, and $z_i = 0$, (a) y_1 vs. x_1, (b) z_1 vs. x_1, (c) z_1 vs. y_1

system is often described by a system of three nonlinear ordinary differential equations and a 3-segment piecewise-linear equation (Chua's equations). This makes the system easily simulated numerically and easily manifested physically due to Chua's circuits' simple design. The double scroll attractor contains an infinite number of fractal-like layers.
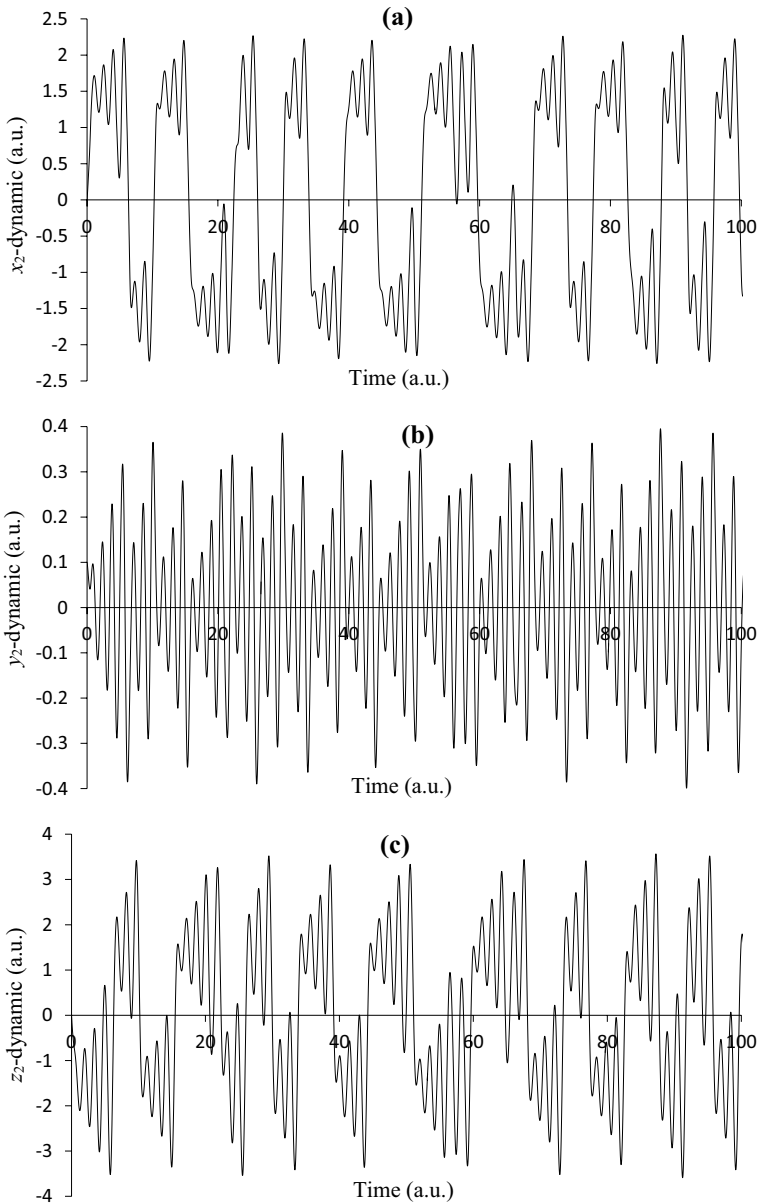
Fig. 3 Time series of Chua chaotic system at initial conditions $x_i = 0$, $y_i = 0.1$, and $z_i = 0$, (a) x_2-dynamic, (b) y_2-dynamic, z_2-dynamic

To investigation the coupling between two chaotic systems, Fig. 5 represents the correlation between to chaotic system at coupling factor $k = 0$, where (a) represent the relation between x_2-dynamic versus x_1-dynamic, (b) represent the relation between y_2-dynamic versus y_1-dynamic, and (c) represent the relation between z_2-dynamic
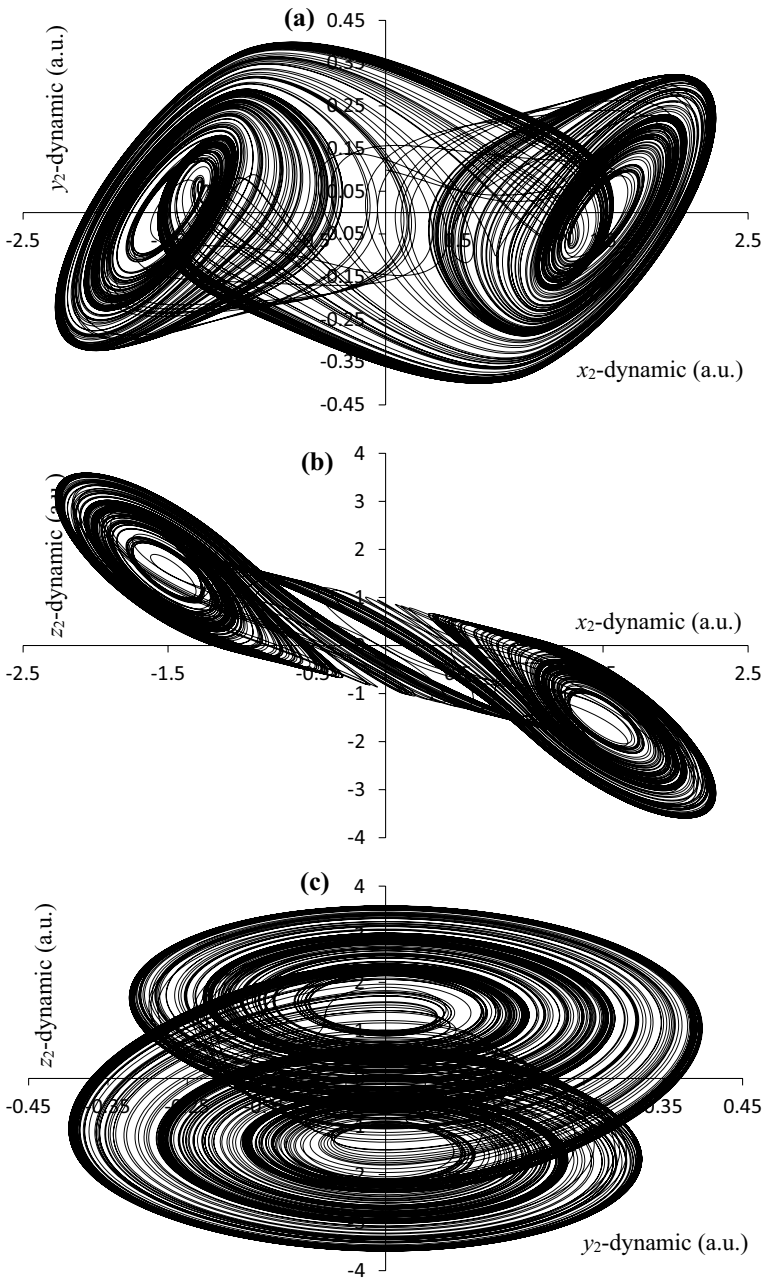
Fig. 4 Attractor of Chua chaotic system at initial conditions $x_i = 0$, $y_i = 0.1$, and $z_i = 0$, (a) y_2 vs. x_2, (b) z_2 vs. x_2, (c) z_2 vs. y_2
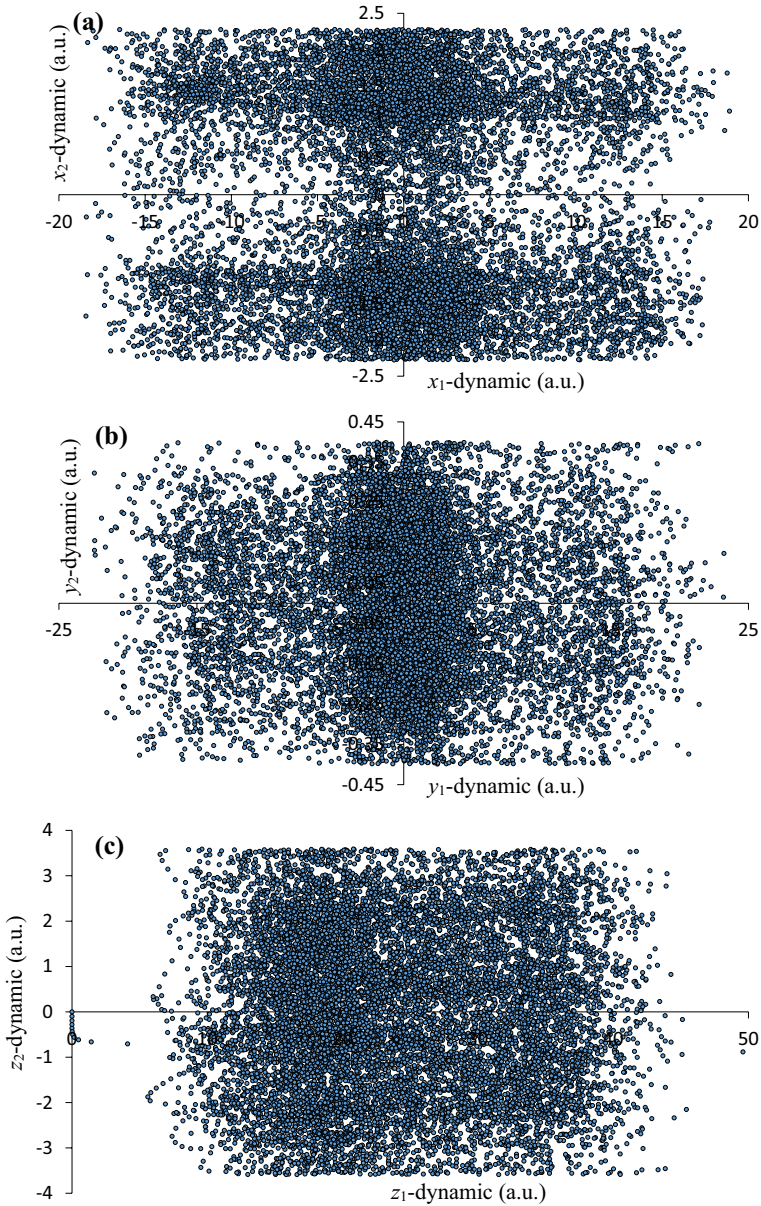
Fig. 5 Correlation between to chaotic system at coupling factor k = 0, (a) x_2 vs. x_1, (b) y_2 vs y_1, (c) z_2 vs z_1

versus z_1-dynamic. The correlation figure represents the distribution of diffuse points in the axis space and collects around the original point.

When increasing the coupling factor k to be 100, Fig. 6 represent the time series of two different chaotic systems, Lorenz system is blue line and Chua system is red line, (a) x-dynamic, (b) y-dynamic, (c) z-dynamic. While Fig. 7 represent the attractor of two
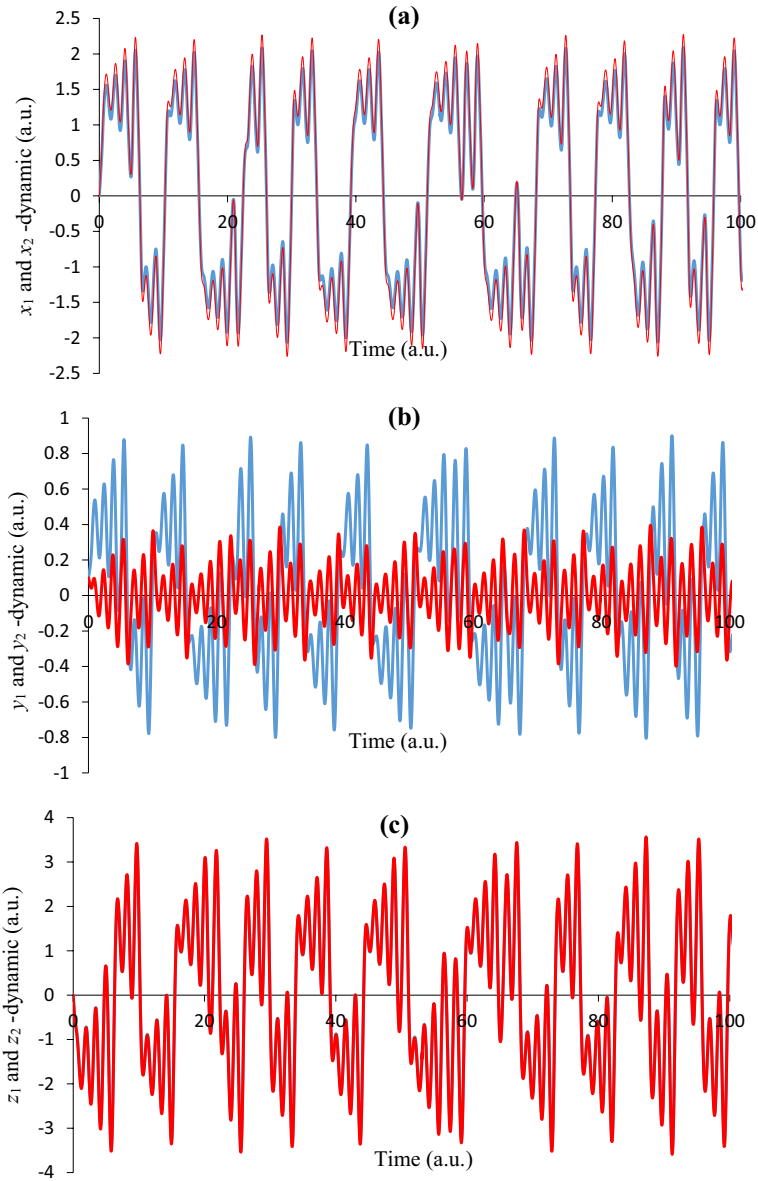
Fig. 6 Time series of two different chaotic systems at k = 100, Lorenz system is blue line and Chua system is red line, **a** x-dynamic, **b** y-dynamic, **c** z-dynamic

different chaotic systems where the Lorenz system is blue line and red line of Chua system (a) y-dynamic versus x-dynamic, (b) z-dynamic versus x-dynamic, (c) z-dynamic versus y-dymaic. Where we notice that the attractor is identical in (x–z) plane for both systems, with the difference of the attractor in (x–y) plane and (y–z) plane. This means that the system at this value of the coupling factor will force the Lorenz system to follow the behavior of the Chua system for dynamics x and z. This is clearly evident through the correlation
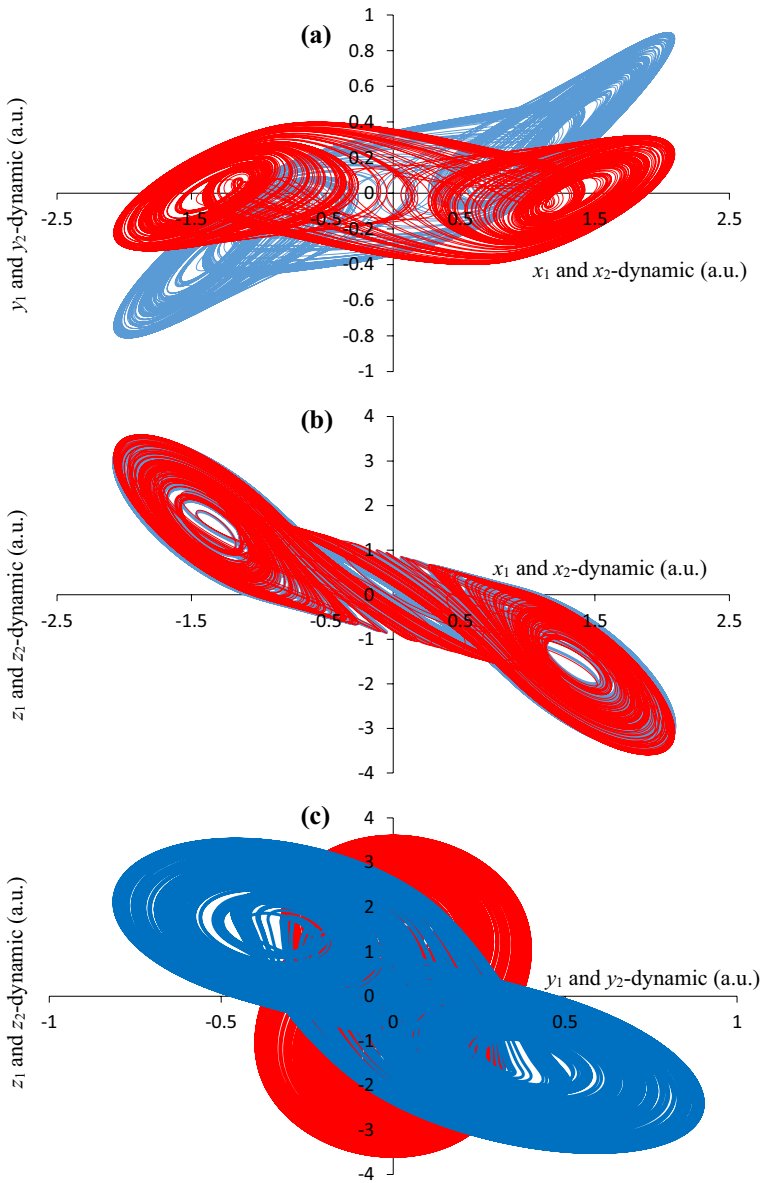
Fig. 7 Attractor of two different chaotic systems at k = 100, Lorenz system is blue line and Chua system is red line, **a** y vs. x, **b** z vs. x, **c** z vs. y

relationship, as in Fig. 8, where a straight line was obtained that makes an angle of 45^0 with the x-axis of the dynamic x and z, and this confirms the state of synchronization in these two dynamics, while the dynamic y remains in the asynchronous state of two system.

To achieve full synchronization between the two chaotic systems, the value of the coupling factor is changed to 100,000, as shown in Fig. 9, which represents the time series of the two systems, where notice a complete transformation of the Lorenz system
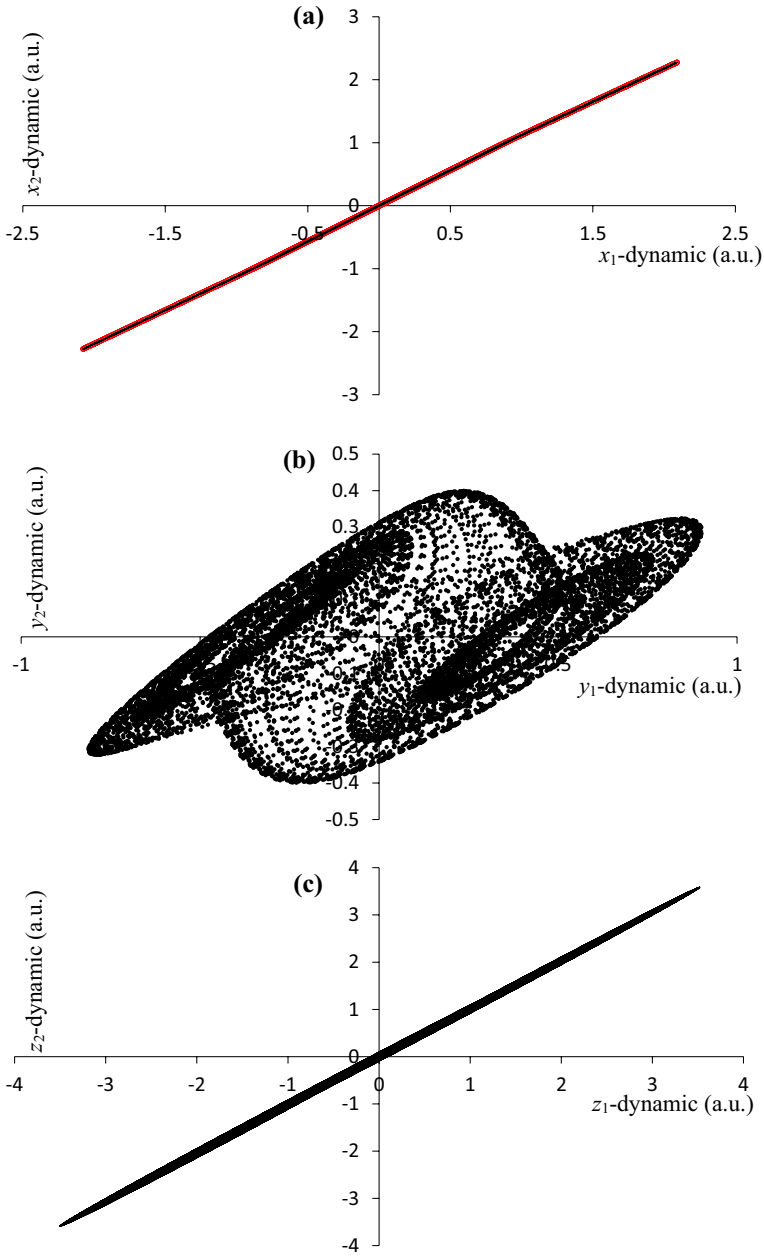
Fig. 8 Correlation between to chaotic system at coupling factor k = 100, **a** x_2 vs. x_1, **b** y_2 vs y_1, **c** z_2 vs z_1

into the Chaotic Chua system with the directions of the three dynamics x, y and z. The fact that the two chaotic systems are identical, or rather, the complete transformation of the behavior of the Lorenz system into the behavior of the Chua system, is the attractor and the correlsion, as in Figs. 10 and 11. As Fig. 10 shows the congruence of the
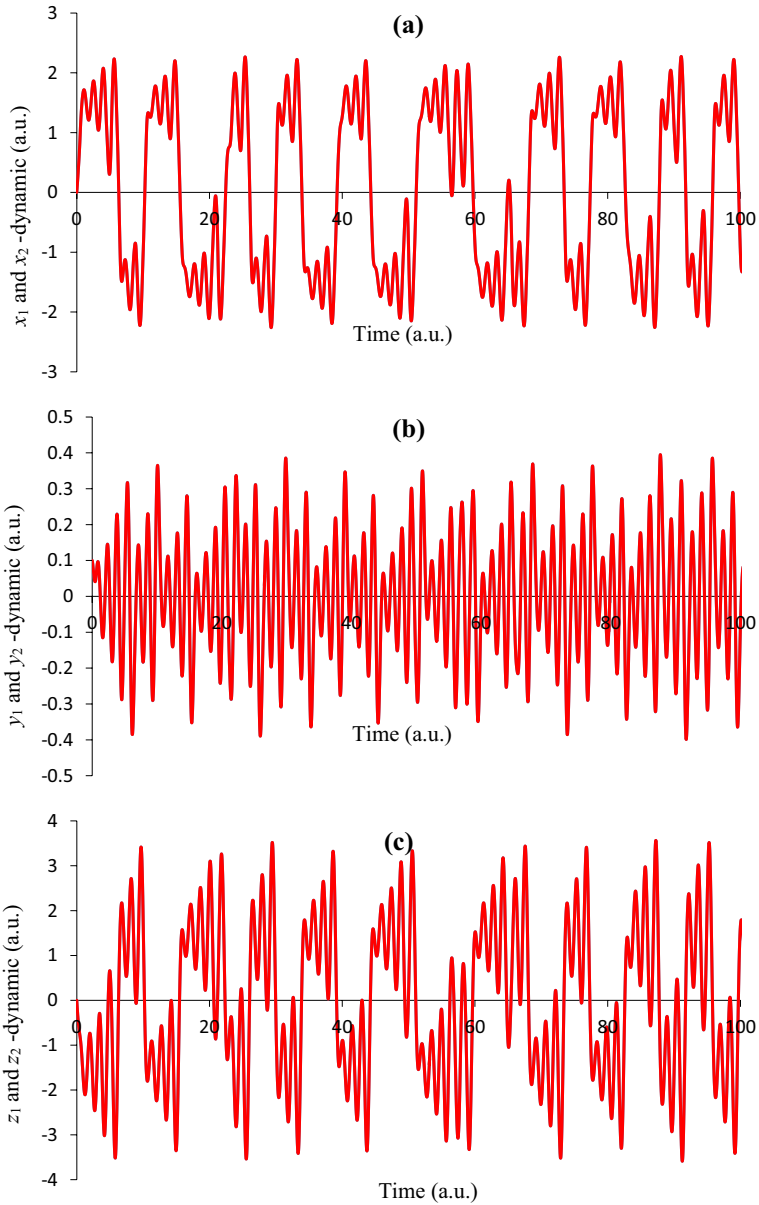
Fig. 9 Time series of two different chaotic systems at k = 100,000, Lorenz system is blue line and Chua system is red line, **a** x-dynamic, **b** y-dynamic, **c** z-dynamic
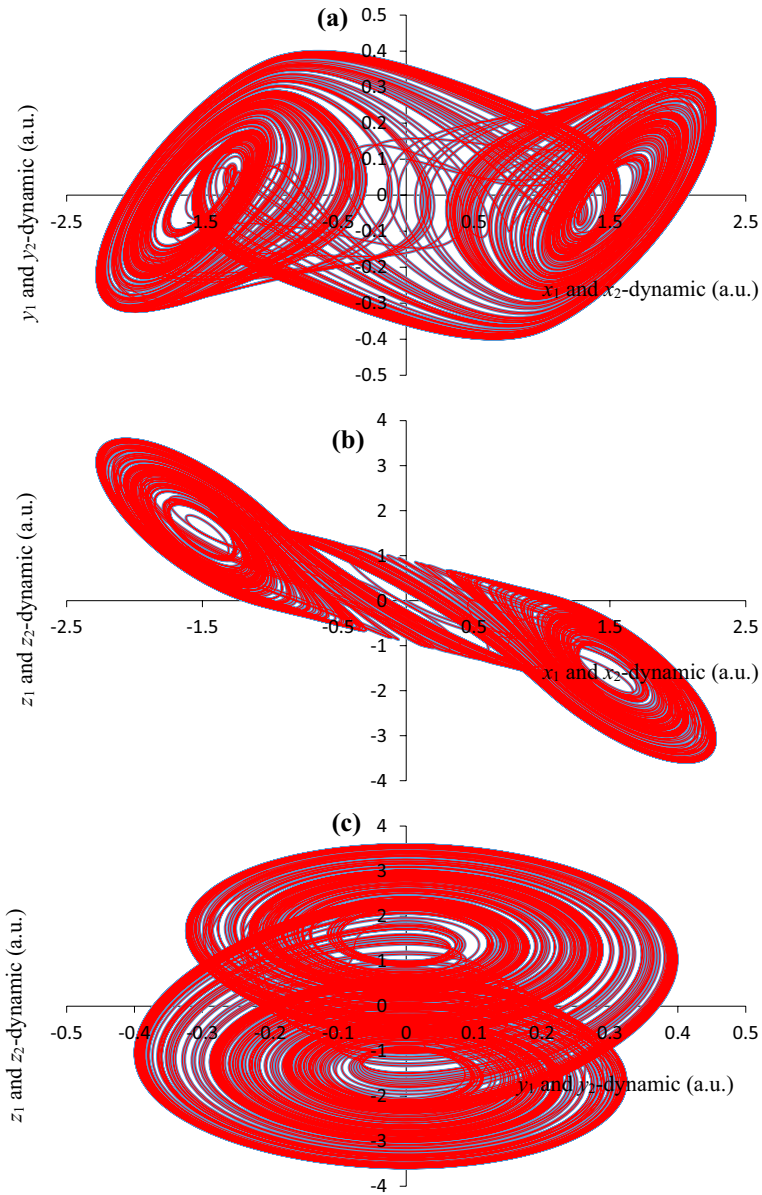
Fig. 10 Attractor of two different chaotic systems at k = 100,000, Lorenz system is blue line and Chua system is red line, **a** y vs. x, **b** z vs. x, **c** z vs. y

(a)

*x*₂-dynamic (a.u.) vs *x*₁-dynamic (a.u.)

(b)

*y*₂-dynamic (a.u.) vs *y*₁-dynamic (a.u.)

(c)

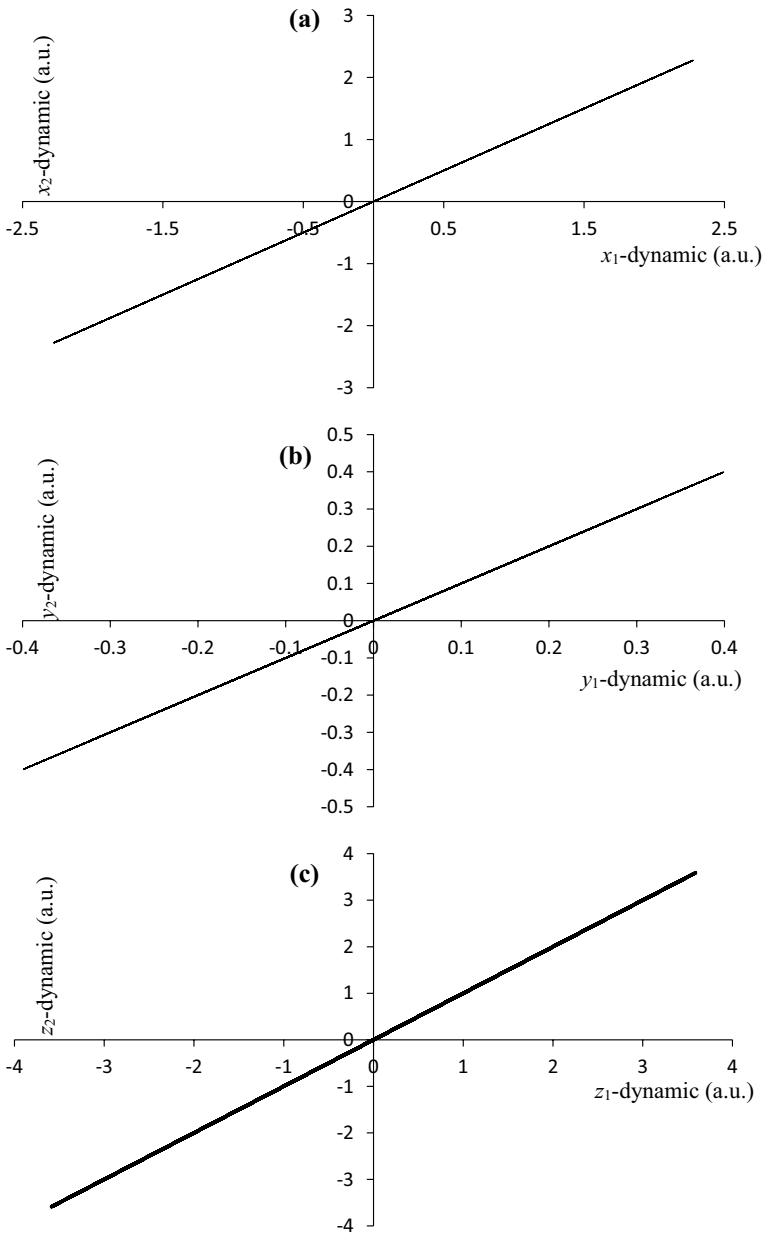*z*₂-dynamic (a.u.) vs *z*₁-dynamic (a.u.)

Fig. 11 Correlation between to chaotic system at coupling factor $k = 100,000$, **a** x_2 vs. x_1, **b** y_2 vs y_1, **c** z_2 vs z_1

strange attractor for all dynamics, as well as the appearance of the correlation in straight lines and at an angle of 45^0 for all dynamics.

4 Conclusion

Through the results, we can conclude that it is possible to change the behavior of any chaotic system to a different behavior when it is coupling to another system that is completely different from it, through the coupling factor.

Declarations

Competing interests The authors have not disclosed any competing interests.

References

Abdulaali, R.S., Jamal, R.K., Mousa, S.K.: Generating a new chaotic system using two chaotic Rossler-Chua coupling systems. Opt. Quant. Electron. **53**, 667 (2021)

Cascais, J., Dilao, R., Da Noronha, A., Costa.: Chaos and reverse bifurcation in a rcl circuit. Phys. Lett. A **93**(5), 213–216 (1983)

Chen, Guanrong, Ueta, Tetsushi, Yet another chaotic attractor. International Journal of Bifurcation and Chaos. **9**(7),1465–1466 (1999)

Chua, L.O.: "Dynamic nonlinear networks: state of the art," IEEE Trans. Circuits Sysr. **CAS-27**, 1059–1087 (1980)

Chua, L.O., Wu, C.W., Huang, A., Zhong, G.-Q.: A universal circuit for studying and generating chaos. i. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications **40**(10), 745–761 (1993)

Hamadi, I.A., Jamal, R.K., Mousa, S.K.: Image encryption based on computer generated hologram and Rossler chaotic system. Opt. Quant. Electron. **54**, 33 (2022)

Kejeen M., Ibrahim, Raied K. Jamal, Kais A. Al-Naimee. Complex Dynamics in incoherent source with ac-coupled optoelectronic Feedback. Iraqi Journal of Science, 2016, Special Issue, Part B, pp: 328–340.

Ibrahim, K.M., Jamal, R.K., Ali, F.H.: Chaotic behaviour of the Rossler model and its analysis by using bifurcations of limit cycles and chaotic attractors. IOP Conf. Series: Journal of Physics: Conf. Series 1003, 012099 (2018)

Ibrahim, K.M., Jamal, R.K.: Full Synchronization of 2×2 Optocouplers Network Using LEDs. Aust. J. Basic Appl. Sci. **10**(16), 8–13 (2016)

Jamal, R.K., Kafi, D.A.: Secure communications by chaotic carrier signal using Lorenz model. Iraq. J. Phys. **14**(30), 51–63 (2016)

Jamal, R.K., Kafi, D.A.: Secure Communication Coupled Laser Based on Chaotic Rössler Circuits. Nonlinear Optics, Quantum Optics **51**, 79–91 (2019b)

Jamal, R.K., Ali, F.H., Mutlak, F.A.-H.: Studying the Chaotic Dynamics Using Rossler-Chua Systems Combined with A Semiconductor Laser. Iraqi J Sci **62**(7), 2213–2221 (2021)

Jamal, R.K., Kafi, D.A.: Secure Communication Coupled semiconductor Laser Based on Rössler Chaotic Circuits. IOP Conf. Series: Materials Science and Engineering **571**, 012119 (2019a)

Kafi, D.A., Jamal, R.K., Al- Naimee, K.A.: Lorenz model and chaos masking /addition technique. Iraq. J. Phys. **14**(31), 51–60 (2016)

Lorenz, E.N.: Deterministic nonperiodic flow. J. Atmos. Sci. **20**(2), 130–141 (1963)

Mousa, S.K., Jamal, R.K.: Realization of a novel chaotic system using coupling dual chaotic system. Opt. Quant. Electron. **53**, 188 (2021)

Nakagawa, S., Saito, T.: An rc ota hysteresis chaos generator. In 1996 IEEE International Symposium on Circuits and Systems. Circuits and Systems Connecting the World. ISCAS 96. **3**, 245–248. IEEE (1996)

Rossler, O.E.: An equation for continuous chaos. Phys. Lett. A. **57**(5), 397–398 (1976)

Publisher's Note   Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.