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An exploration of novel soliton solutions for propagation of pulses in an optical fiber

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Abstract
In this article, the propagation of pulses in optical fiber has been studied by considering the nonlinear partial differential equation (NPDE). The proposed model is investigated using two analytical techniques namely the Sine-Gordon expansion (SGE) procedure and the modified auxiliary equation (MAE) method. The trigonometric function, hyperbolic function, and rational function solutions have been extracted from the proposed methods. The employed procedures are compatible in obtaining traveling wave solutions. Moreover, the obtained results are assisted with 3D graphs to demonstrate the physical significance and dynamical behaviors by using different parameter values.

Keywords Traveling wave transformation · Soliton · Sine-Gordon expansion method · Modified auxiliary equation method · Schrödinger equation

1 Introduction

Within the past few years, exact solutions of NPDEs have gained considerable attention. For this reason many distinct procedures have utilized by researchers. We can list some of them as follows. Wazwaz have derived periodic soliton solutions of the Dodd-Bullough-Mikhailov and the Tzitzeica-Dodd-Bullough equations by tanh technique Wazwaz (2005). Akbulut et al. employed the modified simple equation method to the the fifth-order KdV equation Akbulut et al. (2021a) and verified trivial conservation laws and solitary wave solutions for the fifth order Lax equation Akbulut et al. (2021b). Akinyemi et al. have implemented the improved Sardar sub-equation method to the perturbed nonlinear Schrödinger-Hirota equation with spatio-temporal dispersion Akinyemi et al. (2021). Aksoy et al. utilized the exponential rational function method for space–time fractional differential equations Aksoy et al. (2016). Ma et al. have considered the Hirota-Maccari system via the first integral procedure to extract bright, singular, and dark soliton solutions Ma et al. (2021). Sun et al. have utilized the Hirota bilinear

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technique to the (2+1)-dimensional B-Kadomtsev-Petviashvili equation to find M-lump solutions Sun et al. (2022). Durur et al. sub-equation procedure to the KdV6 equation Durur et al. (2020). Hosseini et al. have analyzed soliton solutions of the Hirota-Satsuma-Ito equation via the linear superposition principle Hosseini et al. (2021). Raza et al. have applied the Painleve approach to a nonlinear Kudryashov's equation Raza et al. (2021). Kumar have founded travelling waves, kink waves, rational function, lump-type solitons, multi-solitons, hyperbolic function, and trigonometric solutions by generalised exponential rational function method Kumar (2021). Osman et al. verified some travelling wave solutions of the 2D-chiral nonlinear Schrodinger equation Osman et al. (2020). Inc et al. have founded exact analytic solutions for the (2+1)-dimensional Ito equation by using simplest equation technique Inc et al. (2021). Akbulut et al. obtained the conservation laws of time fractional modified Korteweg–de Vries (mkdv) equation Akbulut and Tascan (2017a), searched some soliton solutions for various equations Akbulut et al. (2022), and applied conservation theorem and modified extended tanh-function procedure to nonlinear coupled Klein–Gordon–Zakharov equation Akbulut and Tascan (2017b). Mirzazadeh et al. employed the improved F-expansion method to find different wave solutions Mirzazadeh et al. (2022). Hosseini et al. obtained conservation laws and kink solitons of the Sharma–Tasso–Olver–Burgers equation Hosseini et al. (2022). In this paper, to explain the propagation pulse in optical fiber, we are considering a new NPDE, in the following form

$$if_t + f_{xx} + \alpha_1 f|f|^{2m-2n} + \beta_1 f|f|^{2m-n} + \gamma_1 f|f|^{2m} + \delta_1 f|f|^{2m+n} + \lambda_1 f|f|^{2m+2n} = 0. \tag{1}$$

In Eq. (1), the complex function $f(x, t)$ representing optical wave, where m and n are usually rational numbers (not necessarily integers) α_1 , β_1 , γ_1, and λ_1 are the parameters. Equation (1) is the generalization of the well known nonlinear Schrödinger equation. At $m = 0$ Eq. (1) get the following form

$$if_t + f_{xx} + \alpha_1 f|f|^{-2n} + \beta_1 f|f|^{-n} + \delta_1 f|f|^{n} + \lambda_1 f|f|^{2n} = 0. \tag{2}$$

The motivation of this paper is to study Eq. (1). For investigating Eq. (1), two partial differential equations have been obtained from Eq. (1) by taking $m = n$ and $m = 2n$. Upon putting $m = n$, Eq. (1) becomes

$$if_t + f_{xx} + \alpha_1 f + \beta_1 |f|^{n} f + \gamma_1 |f|^{2n} f + \delta_1 |f|^{3n} f + \lambda_1 |f|^{4n} f = 0, \tag{3}$$

and when we consider $m = 2n$, we obtain the equation with polynomial nonlinearity in the following expression

$$if_t + f_{xx} + \alpha_1 |f|^{2n} f + \beta_1 |f|^{3n} f + \gamma_1 |f|^{4n} f + \delta_1 |f|^{5n} f + \lambda_1 |f|^{4n} f = 0. \tag{4}$$

The paper consists of the following sections: We presented the mathematical analysis of the considered model in Sect. 2. Then, we gave a description of utilized techniques, respectively the SGE and the MAE techniques in Sect. 3. The extraction of soliton solutions for the proposed models was given in Sect. 4. Section 5 gives graphical illustrations of the obtained results. Finally, the conclusion of the whole research in Sect. 6.

2 The mathematical analysis

In order to find the exact soliton solution of Eqs. (3) and (4), the following traveling wave transformation is considered, as

$$f(x,t) = F(\Upsilon)e^{i(kx-wt)}, \quad \Upsilon = x - ct, \tag{5}$$

where c is the speed and Υ is the amplitude of traveling wave. Using the transformation Eq. (5), the real parts of Eqs. (3) and (4) have been transformed into following ODEs, respectively as,

$$F'' - \left(w - k^2 + \alpha_1\right)F + \beta_1 F^{n+1} + \gamma_1 F^{2n+1} + \delta_1 F^{3n+1} + 2\lambda_1 F^{4n+1} = 0. \tag{6}$$

and

$$F'' - \left(w - k^2\right)F + \alpha_1 F^{2n+1} + \beta_1 F^{3n+1} + \gamma_1 F^{4n+1} + \delta_1 F^{5n+1} + 2\lambda_1 F^{6n+1} = 0. \tag{7}$$

The imaginary parts of Eqs. (3) and (4) gives the speed of traveling wave as

$$c = 2k. \tag{8}$$

In order to obtain the closed form solutions for Eq. (6), apply the following transformation

$$F = V^{\frac{1}{2n}} \tag{9}$$

Eq. (6) takes the following form

$$\frac{1}{2n}\left(\frac{1}{2n} - 1\right)(V')^2 + \frac{1}{2n}VV'' + (w - k^2 + \alpha_1)V^2 + \beta_1 V^{\frac{5}{2}} + \gamma_1 V^3 + \delta_1 V^{\frac{7}{2}} + \lambda_1 V^4 = 0. \tag{10}$$

For obtaining closed form solutions for Eq. (10), the constraint conditions $\beta_1 = \delta_1 = 0$ have been imposed. Equation (10) becomes

$$\frac{1}{2n}\left(\frac{1}{2n} - 1\right)(V')^2 + \frac{1}{2n}VV'' + (w - k^2 + \alpha_1)V^2 + \gamma_1 V^3 + \lambda_1 V^4 = 0. \tag{11}$$

In order to obtain the closed form solutions for Eq. (7), apply the following transformation

$$F = V^{\frac{1}{3n}} \tag{12}$$

Eq. (7) takes the following form

$$\frac{1}{3n}\left(\frac{1}{3n} - 1\right)(V')^2 + \frac{1}{3n}VV'' + (w - k^2)V^2 + \alpha_1 V^{\frac{8}{3}} + \beta_1 V^3 + \gamma_1 V^{\frac{10}{3}} + \delta_1 V^{\frac{11}{3}} + \lambda_1 V^4 = 0. \tag{13}$$

For obtaining closed form solutions for Eq. (13), the constraint conditions $\alpha_1 = \gamma_1 = \delta_1 = 0$ have been imposed. Equation (13) becomes

$$\frac{1}{3n}\left(\frac{1}{3n} - 1\right)(V')^2 + \frac{1}{3n}VV'' + (w - k^2)V^2 + \beta_1 V^3 + \lambda_1 V^4 = 0. \tag{14}$$

3 Description of utilized techniques

In this section, we gave a description of utilized techniques, SGE Alquran and Krishnan (2016)- Inc et al. (2018) and MAE Khater et al. (2019) respectively.

3.1 The Sine-Gordon expansion method

We take into consideration:

$$f_{xx} - f_{tt} = a^2 \sin f, \tag{15}$$

where $f = f(x, t)$ and $a \neq 0$.

Then, we apply the traveling wave transformation Eqs. (5), (15) reduces to

$$F'' = \frac{a^2}{1-c^2} \sin F. \tag{16}$$

Integrating the above equation gives,

$$\left[\left(\frac{F}{2}\right)'\right]^2 = \frac{a^2}{1-c^2} \sin^2\left(\frac{F}{2}\right) + X, \tag{17}$$

Here X is an integration constant. If we set $X = 0$, $\frac{F}{2} = \vartheta(\Upsilon)$ and $h^2 = \frac{a^2}{1-c^2}$, then Eq. (17) becomes

$$\vartheta' = h \sin(\vartheta). \tag{18}$$

If we take $h = 1$ in Eq. (18), then we find

$$\vartheta' = \sin(\vartheta). \tag{19}$$

Afterwards, we solve Eq. (19) to find

$$\sin\vartheta = \sin(\vartheta(\Upsilon)) = \frac{2de^{\Upsilon}}{d^2e^{2\Upsilon}+1}|_{d=1} = \operatorname{sech}(\Upsilon), \tag{20}$$

$$\cos\vartheta = \cos(\vartheta(\Upsilon)) = \frac{d^2e^{2\Upsilon}-1}{d^2e^{2\Upsilon}+1}|_{d=1} = \tanh(\Upsilon). \tag{21}$$

We predict the solution as follows:

$$F(\Upsilon) = \sum_{i=1}^{N} \tanh^{i-1}(\Upsilon)\left[B_i sech(\Upsilon) + A_i \tanh(\Upsilon)\right] + A_0. \tag{22}$$

Using Eqs. (20), (21), (22) becomes

$$F(\vartheta) = \sum_{i=1}^{N} \cos^{i-1}(\vartheta)\left[B_i \sin(\vartheta) + A_i \cos(\vartheta)\right] + A_0. \tag{23}$$

Here N is the balancing number, which will be determined according to the homogenous balance technique. Then, we insert Eq. (23) into ODE to find an equation system for A_0, A_i, B_i. This system is founded if we equate the coefficients of each power of $\sin^p(\vartheta)\cos^q(\vartheta)$ to zero. Solving the resulting system for A_0, A_i, B_i.

3.2 Modified auxiliary equation (MAE) method

According to MAE method the solution of transformed ODE has the following form

$$F(\Upsilon) = p_0 + \sum_{k=1}^{N} \left[q_k (w^G)^k + r_k (w^G)^{-k} \right], \tag{24}$$

p_0, q_k and r_k are constants. $G(\Upsilon)$ satisfies the differential equation

$$G^{'}(\Upsilon) = \frac{\beta + \alpha w^{-G} + \sigma w^G}{\ln w}, \tag{25}$$

β, α, σ are constants along with $w > 0$ and $w \neq 1$. The differential equation (25) has three types of solutions which are given below

Type 1:

When $\beta^2 - 4\alpha\sigma < 0$ and $\sigma \neq 0$ then

$$w^{G(\xi)} = \frac{-\beta + \sqrt{4\alpha\sigma - \beta^2} \tan\left(\frac{\sqrt{-\beta^2 + 4\alpha\sigma}\Upsilon}{2}\right)}{2\sigma}.$$

or

$$w^{G(\xi)} = -\frac{\beta + \sqrt{4\alpha\sigma - \beta^2} \cot\left(\frac{\sqrt{-\beta^2 + 4\alpha\sigma}\Upsilon}{2}\right)}{2\sigma}.$$

Type 2:

When $\beta^2 - 4\alpha\sigma > 0$ and $\sigma \neq 0$ then

$$w^{G(\xi)} = -\frac{\beta + \sqrt{\beta^2 - 4\alpha\sigma} \tanh\left(\frac{\sqrt{\beta^2 - 4\alpha\sigma}\Upsilon}{2}\right)}{2\sigma}.$$

or

$$w^{G(\xi)} = -\frac{\beta + \sqrt{\beta^2 - 4\alpha\sigma} \coth\left(\frac{\sqrt{\beta^2 - 4\alpha\sigma}\Upsilon}{2}\right)}{2\sigma}.$$

Type 3:

When $\beta^2 - 4\alpha\sigma = 0$ and $\sigma \neq 0$ then

$$w^{G(\xi)} = -\frac{2 + \beta\xi}{2\sigma\Upsilon}.$$

4 Construction of solutions using proposed methods

This section gives the extraction of soliton solutions for the proposed models Eqs. (3) and (4) by employing the most efficient analytical procedures, SGE and the MAE techniques.

4.1 Method-I (SGE) method

In this subsection, we will obtain soliton solutions of Eqs. (3) and (4) via the SGE procedure. For this purpose, we balance V'^2 with V^4 in Eq. (11) and find $N = 1$. Therefore, the solution takes the form:

$$V(\Upsilon) = B_1 \operatorname{sech}(\Upsilon) + A_1 \tanh(\Upsilon) + A_0. \tag{26}$$

Then, we use the solution procedure as explained earlier in Sect. 3. The values of unknowns A_0, A_1, B_1 are calculated as:
SET 1:

$$A_0 = \frac{1+n}{4n^2\gamma_1},\ \ A_1 = \frac{1+n}{4n^2\gamma_1},\ \ B_1 = \frac{i(1+n)}{4n^2\gamma_1},\ \ \lambda_1 = -\frac{n^2(1+2n)\gamma_1^2}{(1+n)^2},\ \ w = k^2 - \frac{1}{4n^2} - \alpha_1.$$

SET 2

$$A_0 = \frac{1+n}{n^2\gamma_1},\ \ A_1 = -\frac{1+n}{n^2\gamma_1},\ \ B_1 = 0,\ \ \lambda_1 = -\frac{n^2(1+2n)\gamma_1^2}{4(1+n)^2},\ \ w = k^2 - \frac{1}{n^2} - \alpha_1.$$

SET 3

$$A_0 = 0,\ \ A_1 = 0,\ \ B_1 = \pm\frac{\sqrt{1+2n}}{2n\sqrt{\lambda_1}},\ \ w = k^2 - \frac{1}{4n^2} - \alpha_1.$$

Bright-dark soliton solutions can be founded for **SET 1** as

$$f_1(x,t) = e^{i(kx-wt)}\left[\left(\frac{\imath(1+n)}{4n^2\gamma_1}\right)\operatorname{sech}(\Upsilon) + \left(\frac{1+n}{4n^2\gamma_1}\right)\tanh(\Upsilon) + \frac{1+n}{4n^2\gamma_1}\right]^{\frac{1}{2n}}. \tag{27}$$

Bright soliton solutions can be founded for **SET 2** as follows

$$f_2(x,t) = \pm e^{i(kx-wt)}\left[\left(\frac{\sqrt{1+2n}}{2n\sqrt{\lambda_1}}\right)\operatorname{sech}(\Upsilon)\right]^{\frac{1}{2n}}. \tag{28}$$

Dark soliton solutions are obtained for **SET 3** as follows

$$f_3(x,t) = e^{i(kx-wt)}\left[-\left(\frac{1+n}{n^2\gamma_1}\right)\tanh(\Upsilon) + \frac{1+n}{n^2\gamma_1}\right]^{\frac{1}{2n}}. \tag{29}$$

Moreover, we will also find soliton solutions of Eq. (4) via SGE technique. If we balance V'^2 with V^4 in Eq. (14), we find as $N = 1$. The values of unknowns A_0, A_1, B_1 are calculated as:
SET 1:

$$A_0 = \frac{4+6n}{9n^2\beta_1},\ \ A_1 = \pm\frac{4+6n}{9n^2\beta_1},\ \ B_1 = 0,\ \ \lambda_1 = -\frac{9n^2(1+3n)\beta_1^2}{4(2+3n)^2},\ \ w = k^2 - \frac{4}{9n^2}.$$

Dark soliton solutions can be founded for **SET 1** as

$$f_4(x,t) = e^{i(kx-wt)}\left[\pm\frac{4+6n}{9n^2\beta_1}\tanh(\Upsilon) + \frac{4+6n}{9n^2\beta_1}\right]^{\frac{1}{3n}}. \tag{30}$$

4.2 Method-II (MAE) technique

In this subsection, we will find soliton solutions of Eq. (3) via MAE technique. Since N is founded as 1, Eq. (24) takes the following form

$$V(\Upsilon) = p_0 + q_1 w^G + r_1 w^{-G}, \tag{31}$$

where p_0, q_1 and r_1 are constants to be determined by Inserting Eq. (31) in the Eq. (11). Then assemble all the coefficients of the powers of $w^{G(\xi)}$ and put them equal to zero. This leads to construction of set of algebraic equations. Upon solving the obtained system gives the value of arbitrary parameters p_0, q_1 and r_1, which are summarized in the following sets of solutions as

SET 1:

$$\begin{aligned} &p_0 = 0, \quad q_1 = \frac{\sigma}{(1-2n)\gamma_1}, \quad r_1 = 0, \quad \alpha = 0, \quad \beta = \frac{n}{2n-1}, \\ &\lambda_1 = -\frac{(1-2n)^2(1+2n)\gamma_1^2}{4n^2}, \quad w = \frac{1+(4-8n)(k^2+\alpha_1)}{4-8n} \end{aligned} \tag{32}$$

SET 2:

$$\begin{aligned} &p_0 = \frac{1+n}{2n^2\gamma_1}, \quad q_1 = \frac{(1+n)\sigma}{2n^2\gamma_1}, \quad r_1 = \frac{(1+n)\alpha}{2n^2\gamma_1}, \quad \beta = 1, \\ &\lambda_1 = -\frac{n^2(1+2n)\gamma_1^2}{(1+n)^2}, \quad w = k^2 - \alpha_1 + \frac{4\alpha\sigma - 1}{4n^2}. \end{aligned} \tag{33}$$

The solutions corresponding to **SET 1:** are evaluated below.

Type 2:

When $\beta^2 - 4\alpha\sigma > 0$ and $\sigma \neq 0$ then

$$f_5(x,t) = e^{i(kx-wt)}\left[\frac{n(1+\tanh\left[\frac{n\xi}{4n-2}\right])}{2(1-2n)^2\gamma_1}\right]^{\frac{1}{2n}} \tag{34}$$

or

$$f_6(x,t) = e^{i(kx-wt)}\left[\frac{n(1+\coth\left[\frac{n\xi}{4n-2}\right])}{2(1-2n)^2\gamma_1}\right]^{\frac{1}{2n}} \tag{35}$$

The solutions corresponding to **SET 2:** are evaluated below.

Type 1:

When $\beta^2 - 4\alpha\sigma < 0$ and $\sigma \neq 0$ then

$$f_7(x,t) = e^{i(kx-wt)}\left[\frac{(1+n)(-1+4\alpha\sigma)\sec\left[\frac{1}{2}\sqrt{-1+4\alpha\sigma}\xi\right]^2}{4n^2\gamma_1\left(-1+\sqrt{-1+4\alpha\sigma}\tan\left[\frac{1}{2}\sqrt{-1+4\alpha\sigma}\xi\right]\right)}\right]^{\frac{1}{2n}} \tag{36}$$

or

$$f_8(x,t) = e^{i(kx-wt)}\left[-\frac{(1+n)(-1+4\alpha\sigma)\csc\left[\frac{1}{2}\sqrt{-1+4\alpha\sigma}\xi\right]^2}{4n^2\gamma_1\left(1+\sqrt{-1+4\alpha\sigma}\cot\left[\frac{1}{2}\sqrt{-1+4\alpha\sigma}\xi\right]\right)}\right]^{\frac{1}{2n}} \tag{37}$$

Type 2:

When $\beta^2 - 4\alpha\sigma > 0$ and $\sigma \neq 0$ then

$$f_9(x,t) = e^{i(kx-wt)}\left[\frac{(1+n)(1-4\alpha\sigma)\,\mathrm{sech}\left[\frac{1}{2}\sqrt{1-4\alpha\sigma}\xi\right]^2}{4n^2\gamma_1\left(1+\sqrt{1-4\alpha\sigma}\tanh\left[\frac{1}{2}\sqrt{1-4\alpha\sigma}\xi\right]\right)}\right]^{\frac{1}{2n}} \tag{38}$$

or

$$f_{10}(x,t) = e^{i(kx-wt)}\left[-\frac{(1+n)(1-4\alpha\sigma)\,\mathrm{csch}\left[\frac{1}{2}\sqrt{1-4\alpha\sigma}\xi\right]^2}{4n^2\gamma_1\left(1+\sqrt{1-4\alpha\sigma}\coth\left[\frac{1}{2}\sqrt{1-4\alpha\sigma}\xi\right]\right)}\right]^{\frac{1}{2n}} \tag{39}$$

We wil find soliton solutions of Eq. (4) via MAE procedure.

SET 1:

$$\begin{aligned} &p_0 = 0, \quad q_1 = \frac{(3n+2\beta)\sigma}{9n^2\beta_1}, \quad r_1 = 0, \quad \alpha = 0, \quad \lambda_1 = -\frac{9n^2(1+3n)\beta_1^2}{(3n+2\beta)^2}, \\ &w = \frac{9n^2k^2 + 3n(-1+\beta)\beta - \beta^2}{9n^2} \end{aligned} \tag{40}$$

SET 2:

$$\begin{aligned} &p_0 = \frac{2+3n}{9n^2\beta_1}, \quad q_1 = \frac{(2+3n)\sigma}{9n^2\beta_1}, \quad r_1 = \frac{(2+3n)\alpha}{9n^2\beta_1}, \quad \beta = 1, \\ &\lambda_1 = -\frac{9n^2(1+3n)\beta_1^2}{(2+3n)^2}, \quad w = \frac{-1+9n^2k^2+2\alpha\sigma}{9n^2}. \end{aligned} \tag{41}$$

The solutions corresponding to **SET 1:** are evaluated below.

Type 2:

When $\beta^2 - 4\alpha\sigma > 0$ and $\sigma \neq 0$ then

$$f_{11}(x,t)=e^{i(kx-wt)}\left[\frac{\beta(3n+2\beta)(1+\tanh\left[\frac{\beta\xi}{2}\right])}{18n^2\beta_1}\right]^{\frac{1}{3n}} \tag{42}$$

or

$$f_{12}(x,t)=e^{i(kx-wt)}\left[\frac{\beta(3n+2\beta)(1+\coth\left[\frac{\beta\xi}{2}\right])}{18n^2\beta_1}\right]^{\frac{1}{3n}} \tag{43}$$

The solutions corresponding to **SET 2:** are evaluated below.
Type 1:
When $\beta^2-4\alpha\sigma<0$ and $\sigma\neq 0$ then

$$f_{13}(x,t)=e^{i(kx-wt)}\left[\frac{(2+3n)(-1+4\alpha\sigma)\sec\left[\frac{1}{2}\sqrt{-1+4\alpha\sigma}\xi\right]^2}{18n^2\beta_1\left(-1+\sqrt{-1+4\alpha\sigma}\tan\left[\frac{1}{2}\sqrt{-1+4\alpha\sigma}\xi\right]\right)}\right]^{\frac{1}{3n}} \tag{44}$$

or

$$f_{14}(x,t)=e^{i(kx-wt)}\left[-\frac{(2+3n)(-1+4\alpha\sigma)\csc\left[\frac{1}{2}\sqrt{-1+4\alpha\sigma}\xi\right]^2}{18n^2\beta_1\left(1+\sqrt{-1+4\alpha\sigma}\cot\left[\frac{1}{2}\sqrt{-1+4\alpha\sigma}\xi\right]\right)}\right]^{\frac{1}{3n}} \tag{45}$$

Type 2:
When $\beta^2-4\alpha\sigma>0$ and $\sigma\neq 0$ then

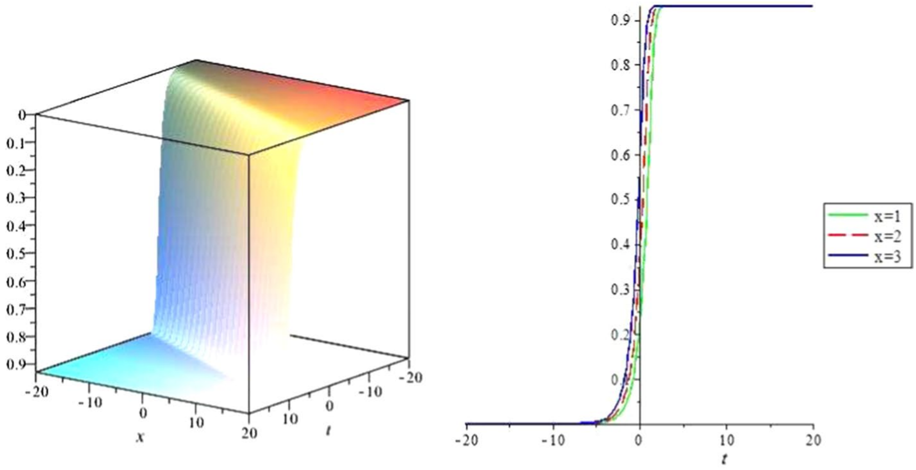
$$f_{15}(x,t)=e^{i(kx-wt)}\left[\frac{(2+3n)(1-4\alpha\sigma)\,\mathrm{sech}\left[\frac{1}{2}\sqrt{1-4\alpha\sigma}\xi\right]^2}{18n^2\beta_1\left(1+\sqrt{1-4\alpha\sigma}\tanh\left[\frac{1}{2}\sqrt{1-4\alpha\sigma}\xi\right]\right)}\right]^{\frac{1}{3n}} \tag{46}$$

or

$$f_{16}(x,t)=e^{i(kx-wt)}\left[-\frac{(2+3n)(1-4\alpha\sigma)\,\mathrm{csch}\left[\frac{1}{2}\sqrt{1-4\alpha\sigma}\xi\right]^2}{18n^2\beta_1\left(1+\sqrt{1-4\alpha\sigma}\coth\left[\frac{1}{2}\sqrt{1-4\alpha\sigma}\xi\right]\right)}\right]^{\frac{1}{3n}} \tag{47}$$

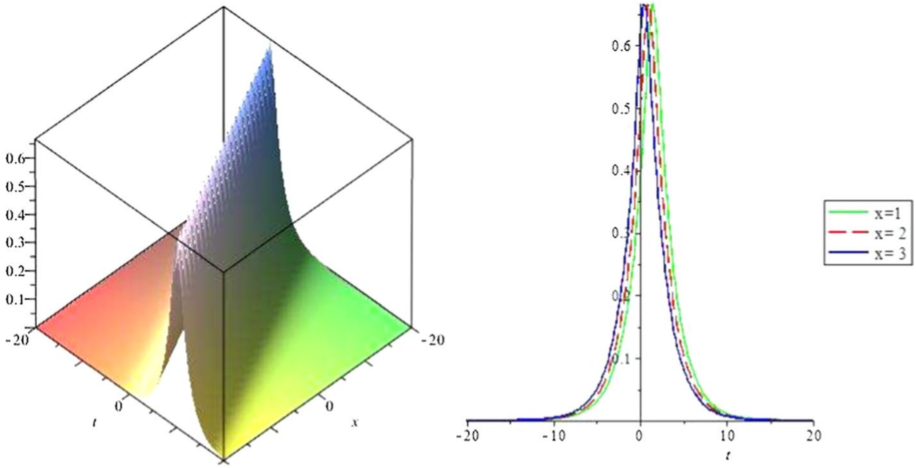
5 Graphical illustrations

In this section, we provide the graphical illustrations of few of the determined solutions. It is important to mention here that explicit and consistent wave solutions are extracted by applying two different reliable schemes. Figures 1, 2, 3 and 4 represents the 3D and 2D



(a) **(b)**

Fig. 1 **a** is 3D plot of Eq. (29) for $n = 2$, **b** is 2D line plots of Eq. (29) with respect x



(a) **(b)**

Fig. 2 **a** is 3D plot of Eq. (30) for $n = 2$, **b** is 2D line plots of Eq. (30) with respect x

plots of Eqs. (29), (30), (35) and (44) respectively by taking $\gamma_1 = 2$, $k = 1$, $w = 2$, $\alpha = 1$, $\sigma = 1$ and $\beta_1 = 2$.

6 Conclusion

In this work, the trigonometric function solutions, hyperbolic function solutions, and rational function solutions have been analyzed to investigate the propagation of pulses in optical fiber. These results are beneficial and useful in optical fibers and nonlinear

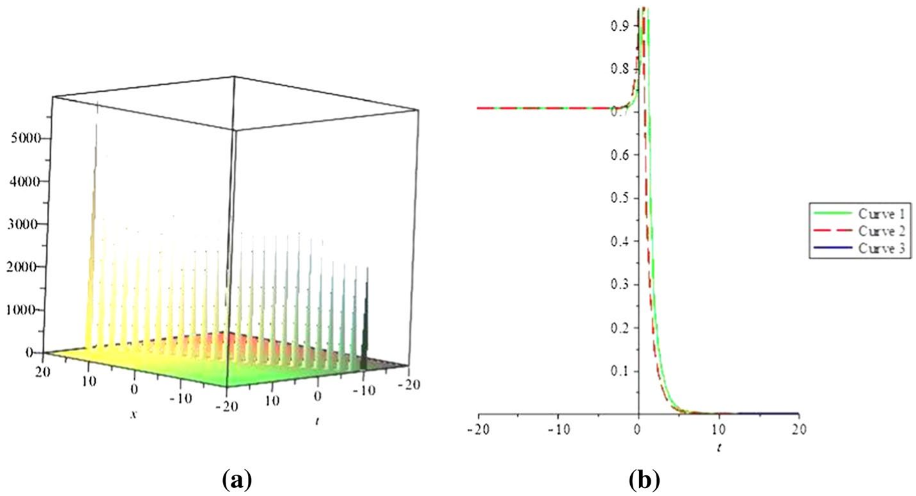


Fig. 3 **a** is 3D plot of Eq. (35) for $n = 2$, **b** is 2D line plots of Eq. (35) with respect x

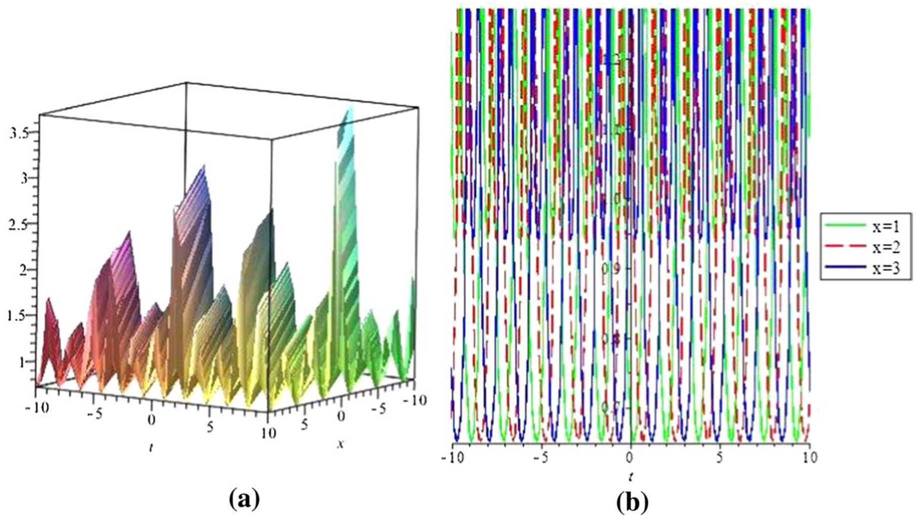


Fig. 4 **a** is 3D plot of Eq. (44) for $n = 2$, **b** is 2D line plots of Eq. (44) with respect x

wave phenomena. We have successfully implemented the SGE and the MAE technique with the help of computerized symbolic computation. The outcomes of the present manuscript affirmed the capacity of the schemes in handling a broad diverseness of NPDEs. Moreover, for future works, super nonlinear, quasi-periodic, chaotic, and solitonic waves can be founded.

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Declarations

Conflict of interest The authors have not disclosed any competing interests.

References

Akbulut, A., Tascan, F.: Trivial conservation laws and solitary wave solution of the fifth order Lax equation. Chaos Solitons Fractals **100**, 1–6 (2017a)

Akbulut, A., Tascan, F.: Application of conservation theorem and modified extended tanh-function method to (1+1)-dimensional nonlinear coupled Klein-Gordon-Zakharov equation. Chaos Solitons Fractals **104**, 33–40 (2017b)

Akbulut, A., Kaplan, M., Kaabar, M.K.A.: New conservation laws and exact solutions of the special case of the fifth-order KdV equation. J. Ocean Eng. Sci. (2021a). https://doi.org/10.1016/j.joes.2021.09.010

Akbulut, A., Tascan, F., Ozel, E.: Trivial conservation laws and solitary wave solution of the fifth order Lax equation. Partial Differ. Equ. Appl. Math. **4**, 100101 (2021b)

Akbulut, A., Islam, S.R., Rezazadeh, H., Tascan, F.: Obtaining exact solutions of nonlinear partial differential equations via two different methods. Int. J. Mod. Phys. B **36**(05), 2250041 (2022)

Akinyemi, L., Rezazadeh, H., Shi, Q.H., Inc, M., Khater, M.M.A., Ahmad, H., Jhangeer, A., Akbar, M.A.: New optical solitons of perturbed nonlinear Schrodinger-Hirota equation with spatio-temporal dispersion. Res. Phys. **29**(7), 104656 (2021)

Aksoy, E., Kaplan, M., Bekir, A.: Exponential rational function method for space-time fractional differential equations. Waves Random Complex Media **262**, 142–151 (2016)

Alquran, M., Krishnan, E.V.: Applications of sine-Gordon expansion method for a reliable treatment of some nonlinear wave equations. Nonlinear Stud. **23**(4), 639–649 (2016)

Durur, H., Kurt, A., Tasbozan, O.: New travelling wave solutions for KdV6 equation using sub equation method. Appl. Math. Nonlinear Sci. **5**(1), 455–460 (2020)

Hosseini, K., Pouyanmehr, R., Ansari, R.: Multiple complex and real soliton solutions to the new integrable (2+1) dimensional Hirota-Satsuma-Ito equation. Comput. Sci. Eng. **1**(2), 91–97 (2021)

Hosseini, K., Akbulut, A., Baleanu, D., Salahshour, S.: The Sharma-Tasso-Olver-Burgers equation: its conservation laws and kink solitons. Commun. Theor. Phys. **74**, 025001 (2022)

Inc, M., Aliyu, A.I., Yusuf, A., Baleanu, D.: Optical solitons for Biswas-Milovic model in nonlinear optics by sine-Gordon equation method. Optik **157**, 267–274 (2018)

Inc, M., Az-Zo'bi, E.A., Jhangeer, A., Rezazadeh, H., Ali, M.N., Kaabar, M.K.A.: New soliton solutions for the higher-dimensional non-local Ito Eequation. Nonlinear Eng. **10**, 374–384 (2021)

Khater, M., Attia, R.A.M., Lu, D.: Modified auxiliary equation method versus three nonlinear fractional biological models in present explicit wave solutions. Math. Comput. Appl. **24**(1), 1 (2019)

Kumar, S.: Some new families of exact solitary wave solutions of the Klein-Gordon-Zakharov equations in plasma physics. Pramana **95**, 1–15 (2021)

Ma, W.X., Osman, M.S., Arshed, S., Raza, N., Srivastava, H.M.: Practical analytical approaches for finding novel optical solitons in the single-mode fibers. Chin. J. Phys. **72**, 475–486 (2021)

Mirzazadeh, M., Akbulut, A., Tascan, F., Akinyemi, L.: A novel integration approach to study the perturbed Biswas-Milovic equation with Kudryashov's law of refractive index. Optik **252**, 168529 (2022)

Osman, M.S., Baleanu, D., Tariq, K. Ul-Haq., Kaplan, M., Younis, M., Rizvi, S.T.R.: Different types of progressive wave solutions via the 2D-chiral nonlinear Schrödinger equation. Front. Phys. **8**, 215 (2020)

Raza, N., Seadawy, A.R., Kaplan, M., Butt, A.R.: Symbolic computation and sensitivity analysis of nonlinear Kudryashovs dynamical equation with applications. Phys. Scr. **96**, 105216 (2021)

Sun, Y.L., Chen, J., Ma, W.X., Yu, J.P., Khalique, C.M.: Further study of the localized solutions of the (2+1)-dimensional B-Kadomtsev-Petviashvili equation. Commun. Nonlinear Sci. Numer. Simul. **107**, 106131 (2022)

Wazwaz, A.M.: The tanh method: solitons and periodic solutions for the Dodd-Bullough-Mikhailov and the Tzitzeica-Dodd-Bullough equations. Chaos Solitons Fractals **25**(1), 55–63 (2005)

Yasar, E., Yildirim, Y., Yasar, E.: New optical solitons of space time conformable fractional perturbed Gerdjikov-Ivanov equation by sine-Gordon equation method. Res. Phys. **9**, 1666–1672 (2018)