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Novel and diverse soliton constructions for nonlinear space–time fractional modified Camassa–Holm equation and Schrodinger equation

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Abstract
Nonlinear evolution equations of non-integer order have elaborately been taken place in the research field for their importance bearing the significant role to delineate the interior characteristics of nonlinear wonders belonging to the nature. In this present effort, the time and space fractional nonlinear modified Camassa–Holm equation and the Schrodinger equation are disclosed for diverse and innovative wave structures by formulating accurate solutions. Adopting a new wave variable alongside the conformable fractional derivative we attain ordinary differential equations from fractional orders. Thereupon, the execution of directed extended Riccati scheme has brought out further novel results of the supposed models which might be presented in the literature newly. Finally, several physical appearances of the constructed solutions are presented in three dimensional outlines such as kink type, ball shape, compacton, periodic, peakon etc. The recommended technique is appeared as competent, productive and concise, and claimed to be used for further research.

Keywords Extended Riccati scheme · Wave variable translation · Evolution equations of fractional order · Soliton · Accurate solution

Mathematics Subject Classification 35C08 · 35R11

1 Introduction

Nonlinear evolution equations of non-integer order over and above the integer order have drawn the attention of many researchers. These equations have great importance for being the models of amalgamated physical phenomena of nature world. Earlier, a good number of

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knowledge hunters paid their attention to introduce the underlying mechanisms of nonlinearity arise in nature of real world (Oldham and Spanier 1974; Rogers and Shadwick 1982; Ablowitz and Clarkson 1991; Miller and Ross 1993; Podlubny 1999; Wazwaz 2002). Consequently, many mathematicians and physicists have derived various effective and influential methods to examine nonlinear evolution equations for accurate analytic solutions and also numerical solutions. Instantly, we may mention the Adomian decomposition method (Guo 2019), the $\left(G'/G\right)$-expansion method (Bekir and Guner 2013), the fitted fractional reproducing kernel algorithm (Omar 2019), the laplace adomian decomposition method (Shah et al. 2019), the homotopy perturbation method (Golmankhaneh and Baleanu 2011), the improved sub-equation method (Karaagac 2019), the extended simple equation method (Lu et al. 2017), the improved auxiliary equation technique (Islam et al. 2021a), the variational method (Seadawy 2011), the finite element technique (Chu et al. 2021), the q-homotopy analysis transform method (Veeresha et al. 2020), the improved tanh and the rational $\left(G'/G\right)$-expansion techniques (Islam et al. 2021b) Haar wavlet, Adams–Bashforth–Moulton methods and Toufik–Atangana methods (Ghanbari et al. 2020; Kumar et al. 2020a, b) and other mentionable recent studies (Goufo et al. 2020; Wang et al. 2020; Xu et al. 2022; Karthikeyan et al. 2021; Rashid et al. 2022; Shen et al. 2021; Chen et al. 2020a, b; Jahanshahi et al. 2021; Zhou et al. 2021).

In this article, we consider to unravel the nonlinear space and time fractional mCH equation and the space and time fractional Schrodinger equation by means of a new technique. The nonlinear Camassa–Holm equation and its modification of fractional order and also of integer order have been taken into account by many researchers with the aid of different methods to analyze various wave structures (Camassa and Holm 1993; Wazwaz 2006; Alam and Akbar 2015; Ali et al. 2018; Lu et al. 2018; Hassan and Abdelrahman 2018; Islam et al. 2019; Rezazadeh et al. 2019; Zulfiqar and Ahmad 2020). Schrodinger types nonlinear evolution equations are also taken major part in the literature for making visible numerous exact analytic wave solutions by various techniques such as the space–time fractional cubic Schrodinger have been studied by imposing new type F-expansion method (Pandir and Duzgun 2019; Hemida et al. 2012; Saxena and Kalla 2010; Salam et al. 2016; Younis et al. 2017; Rizvi et al. 2017; Alam and Tunc 2020). Many scholars have introduced several techniques along with the general Riccati equation (Naher et al. 2013; Naher and Abdullah 2012a, b; Zayed and Arnous 2013; Malwe et al. 2016; Salathiel et al. 2017; Zhu 2008; Zayed and Al-Nowehy 2017). Being inspired with the above study, we advise a new approach called extended Riccati scheme together with the general Riccati equation to demonstrate the suggested equations. The considered governing models are reduced to ordinary differential equations with the aid of compound wave variable alteration relating to the conformable fractional derivative (Khalil et al. 2014). Other two recently established fractional derivatives are worth mentioning (Alshabanat et al. 2020; Mohammadi et al. 2021). This study is made us relaxed by providing much more distinct and novel wave solutions. The acquired wave solutions might bear the great importance to analyze the interior structure of physical intricate phenomena of nature world and appear in the literature immediately. Khalil et al. have announced the conformable fractional derivative as follows (Khalil et al. 2014):

The conformable derivative of a function $u(x)$ is

$$D_x^{\alpha}(u(x)) = \lim_{\varepsilon\to 0} \frac{u\left(x+\varepsilon x^{1-\alpha}\right) - u(x)}{\varepsilon}, \tag{1}$$

where $x > 0$ and α indicates non-integer number $(0, 1]$. This definition yields some properties as follows:

Theorem 1 *Consider α-differentiable functions $u(x)$ and $v(x)$ at any point $x > 0$ for $\alpha \in (0, 1]$, then*

(i) $D_x^\alpha(x^n) = nx^{n-\alpha}\ \forall n \in \boldsymbol{R}$.
(ii) $D_x^\alpha(\lambda) = 0$, *where λ is any constant.*
(iii) $D_x^\alpha(au(x) + bv(x)) = aD_x^\alpha(u(x)) + bD_x^\alpha(v(x))$ *for all $a, b \in \boldsymbol{R}$.*
(iv) $D_x^\alpha(u(x)v(x)) = u(x)D_x^\alpha(v(x)) + v(x)D_x^\alpha(u(x))$.
(v) $D_x^\alpha(u(x)/v(x)) = \frac{v(x)D_x^\alpha(u(x)) - u(x)D_x^\alpha(v(x))}{v^2(x)}$.
(vi) *if u is differentiable, then* $D_x^\alpha(u)(x) = x^{1-\alpha}\frac{du(x)}{dx}$.

Theorem 2 *If $u(x)$ is both differentiable and α-differentiable with $0 < \alpha \le 1$, then for a differentiable function $v(x)$ defined in the same range of $u(x)$ (Moussa et al. 2021),*

$$D_x^\alpha(u(x) \cdot v(x)) = D_v^\alpha u(v(x))D_x^\alpha v(x).$$

The study bears significant as adopting new competent tool a heap of wave solutions is celebrated for the advised nonlinear partial differential equations. Moreover, distinct wave patterns are displayed in the three-dimensional space such as kink, anti-kink, cuspon, compacton, anti-compacton, bell, anti-bell to depict the interior characteristics of waves.

2 Outline of the scheme

A fractional order nonlinear evolution equation is supposed to be

$$\Xi\left(u, D_t^\alpha u, D_x^\alpha u, D_{tt}^{2\alpha} u, D_{xx}^{2\alpha} u, D_t^\alpha D_x^\alpha u, \dots\dots\dots\right) = 0, \quad 0 < \alpha \le 1 \tag{2.1}$$

where Ξ consists of u alongside its different partial derivatives. The change of wave variable as

$$u = u(x, t) = U(\xi), \quad \xi = \xi(x, t), \tag{2.2}$$

reduces Eq. (2.1) to an ordinary differential equation due to ξ,

$$\Omega\left(U, U', U'', U''', \dots\dots\dots\right) = 0. \tag{2.3}$$

Integrate Eq. (2.3) if permits and set integral constant zero as we seek for soliton solutions. Suppose the solution formula of Eq. (2.1) stands as

$$U(\xi) = \frac{a_0 + \sum_{i=1}^n \left(a_i\phi^i(\xi) + b_i\phi^{-i}(\xi)\right)}{e_0 + \sum_{i=1}^n \left(e_i\phi(\xi)^i + f_i\phi^{-i}(\xi)\right)}, \tag{2.4}$$

where $i = 1, 2, 3, \dots, n$; the involved free constants (2.4) will be found later, imposing homogenous technique provides the value of n and $\phi = \phi(\xi)$ satisfies the Riccati differential equation

$$\phi'(\xi) = \mu + \lambda\phi(\xi) + \nu\phi^2(\xi). \tag{2.5}$$

The solutions of Eq. (2.5) are available in Ref. (Zhu 2008).

Operating Eq. (2.4) alongside Eq. (2.5) in Eq. (2.3) gives a polynomial in ϕ. Solve the algebraic equations formed by setting each coefficient to zero and obtain the values of the unknown parameters present in Eqs. (2.4) and (2.5). Using the parameters values and the solutions of Eq. (2.5) in Eq. (2.4), we acquire the wave solutions of Eq. (2.1).

3 Construction of solutions

This section formulates the solutions to the considered nonlinear space–time fractional evolution equations by means of the advised scheme.

3.1 The space and time fractional nonlinear mCH equation

Consider the equation as

$$D_t^\alpha u + \delta D_x^\alpha u - D_{xxt}^{3\alpha} u + \eta D_x^\alpha u^3 = 0, \delta, \quad (\eta > 0) \in R \tag{3.1.1}$$

where the derivatives are in the sense of conformable fractional derivative. The wave variable

$$u(x,t) = u(\xi), \quad \xi = \frac{kx^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha}. \tag{3.1.2}$$

converts Eq. (3.1.1) into the ODE

$$-cu' + k\delta u' + ck^2 u''' + k\eta\left(u^3\right)' = 0. \tag{3.1.3}$$

Integrating Eq. (3.1.3) and setting the constant of integration zero yields

$$(k\delta - c)u + ck^2 u'' + k\eta u^3 = 0. \tag{3.1.4}$$

Balancing the terms u'' and u^3 produces $n = 1$ and hence the solution (2.4) becomes

$$u = \frac{a_0 + a_1\phi(\xi) + b_1\phi^{-1}(\xi)}{e_0 + e_1\phi(\xi) + f_1\phi^{-1}(\xi)}. \tag{3.1.5}$$

Equation (3.1.5) and Eq. (2.5) forces Eq. (3.1.4) to be a polynomial in $\phi(\xi)$. Collect the coefficients of this polynomial, set them to zero and solve by Maple. Consequently, the following results are obtained:

Set 1: $a_0 = \frac{b_1\lambda}{\mu}, a_1 = \frac{b_1\nu}{\mu}, e_0 = \pm\frac{b_1}{\mu k}\sqrt{\frac{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}{2\delta}},$

$$e_1 = \pm\frac{\lambda b_1}{2\mu^2 k}\sqrt{\frac{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}{2\delta}}, \quad f_1 = 0, \quad c = -\frac{\delta k}{k^2\lambda^2 - 4k^2\mu\nu - 1}, \tag{3.1.6}$$

Set 2: $a_0 = \frac{b_1\lambda}{\mu}, a_1 = \frac{(\lambda^2 - 2\mu\nu)b_1}{2\mu^2}, e_0 = \pm\frac{b_1}{\mu k}\sqrt{\frac{-\eta\left(2k^2\lambda^2 - \delta k^2\mu\nu + 1\right)}{2\delta}},$

$$e_1 = \pm\frac{\lambda b_1}{2\mu^2 k}\sqrt{\frac{-\eta\left(2k^2\lambda^2 - \delta k^2\mu\nu + 1\right)}{2\delta}},\quad f_1 = 0,\quad c = \frac{\delta k}{2k^2\lambda^2 - \delta k^2\mu\nu + 1}. \tag{3.1.7}$$

Set 3: $a_0 = \frac{b_1\lambda}{\mu}$, $a_1 = \frac{\lambda^2 b_1}{4\mu^2}$, $e_0 = \pm\frac{b_1}{2\mu k}\sqrt{\frac{-\eta\left(k^2\lambda^2-4k^2\mu\nu+2\right)}{\delta}}$,

$$e_1 = \pm\frac{\lambda b_1}{4\mu^2 k}\sqrt{\frac{-\eta\left(k^2\lambda^2 - 4k^2\mu\nu + 2\right)}{\delta}},\quad f_1 = 0,\quad c = \frac{2\delta k}{k^2\lambda^2 - 4k^2\mu\nu + 2}, \tag{3.1.8}$$

Set 4: $a_0 = \mp k\left(\lambda e_0 + 2\mu e_1\right)\sqrt{\frac{\delta}{-\eta\left(k^2\lambda^2-4k^2\mu\nu+2\right)}}$, $a_1 = \mp k\lambda e_1\sqrt{\frac{\delta}{-\eta\left(k^2\lambda^2-4k^2\mu\nu+2\right)}}$,

$$b_1 = \pm 2e_0\mu k\sqrt{\frac{-\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu + 2\right)}},\quad f_1 = 0,\quad c = \frac{2\delta k}{k^2\lambda^2 - 4k^2\mu\nu + 2} \tag{3.1.9}$$

Equation (3.1.5) with the aid of Eqs. (3.1.6)–(3.1.9) provides

$$u_1(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{\lambda + \nu\phi(\xi) + \mu\phi^{-1}(\xi)}{2\mu \pm \lambda\phi(\xi)}. \tag{3.1.10}$$

$$u_2(\xi) = \pm k\sqrt{\frac{2\delta}{-\eta\left(2k^2\lambda^2 - 8k^2\mu\nu + 1\right)}} \times \frac{2\lambda\mu + (\lambda^2 - 2\mu\nu)\phi(\xi) + 2\mu^2\phi^{-1}(\xi)}{2\mu \pm \lambda\phi(\xi)}. \tag{3.1.11}$$

$$u_3(\xi) = \pm 2k\sqrt{\frac{\delta}{-\eta\left(k^2\lambda^2 - 4k^2\mu\nu + 2\right)}} \times \frac{4\lambda\mu + \lambda^2\phi(\xi) + 4\mu^2\phi^{-1}(\xi)}{4\mu \pm 2\lambda\phi(\xi)}. \tag{3.1.12}$$

$$u_4(\xi) = \frac{\mp k\sqrt{\frac{\delta}{-\eta\left(k^2\lambda^2-4k^2\mu\nu+2\right)}}\left\{\left(\lambda e_0 + 2\mu e_1\right) \mp \lambda e_1\phi(\xi)\right\} \pm 2e_0\mu k\sqrt{\frac{-\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu+2\right)}}\phi^{-1}(\xi)}{e_0 + e_1\phi(\xi)}. \tag{3.1.13}$$

The expressions (3.1.10)–(3.1.13) along with the solutions of Eq. (2.5) provide analytic wave solutions of Eq. (3.1.1). For simplicity, we display here the outcomes only for expression (3.1.10) as follows:

Solution family 1: When $\phi = \lambda^2 - 4\mu\nu > 0$ and $\lambda\nu \neq 0 (\text{or}\, \mu\nu \neq 0)$,

$$u_{1_1}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\nu\left\{\lambda + \sqrt{\phi}tanh\left(\sqrt{\phi}\xi/2\right)\right\} - \nu\left\{\lambda + \sqrt{\phi}tanh\left(\sqrt{\phi}\xi/2\right)\right\}^2 - 4\mu\nu^2}{4\mu\nu\left\{\lambda + \sqrt{\phi}tanh\left(\sqrt{\phi}\xi/2\right)\right\} \mp \lambda\left\{\lambda + \sqrt{\phi}tanh\left(\sqrt{\phi}\xi/2\right)\right\}^2}. \tag{3.1.14}$$

$$u_{1_2}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\nu\left\{\lambda + \sqrt{\phi}coth\left(\sqrt{\phi}\xi/2\right)\right\} - \nu\left\{\lambda + \sqrt{\phi}coth\left(\sqrt{\phi}\xi/2\right)\right\}^2 - 4\mu\nu^2}{4\mu\nu\left\{\lambda + \sqrt{\phi}coth\left(\sqrt{\phi}\xi/2\right)\right\} \mp \lambda\left\{\lambda + \sqrt{\phi}coth\left(\sqrt{\phi}\xi/2\right)\right\}^2}. \tag{3.1.15}$$

$$u_{1_3}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\nu\left\{\lambda + \sqrt{\phi}\left(tanh\left(\sqrt{\phi}\xi\right) \pm isech\left(\sqrt{\phi}\xi\right)\right)\right\} - \nu\left\{\lambda + \sqrt{\phi}\left(tanh\left(\sqrt{\phi}\xi\right) \pm isech\left(\sqrt{\phi}\xi\right)\right)\right\}^2 - 4\mu\nu^2}{4\mu\nu\left\{\lambda + \sqrt{\phi}\left(tanh\left(\sqrt{\phi}\xi\right) \pm isech\left(\sqrt{\phi}\xi\right)\right)\right\} \mp \lambda\left\{\lambda + \sqrt{\phi}\left(tanh\left(\sqrt{\phi}\xi\right) \pm isech\left(\sqrt{\phi}\xi\right)\right)\right\}^2}. \tag{3.1.16}$$

$$u_{1_4}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\nu\left\{\lambda + \sqrt{\phi}\left(coth\left(\sqrt{\phi}\xi\right) \pm csch\left(\sqrt{\phi}\xi\right)\right)\right\} - \nu\left\{\lambda + \sqrt{\phi}\left(coth\left(\sqrt{\phi}\xi\right) \pm csch\left(\sqrt{\phi}\xi\right)\right)\right\}^2 - 4\mu\nu^2}{4\mu\nu\left\{\lambda + \sqrt{\phi}\left(coth\left(\sqrt{\phi}\xi\right) \pm csch\left(\sqrt{\phi}\xi\right)\right)\right\} \mp \lambda\left\{\lambda + \sqrt{\phi}\left(coth\left(\sqrt{\phi}\xi\right) \pm csch\left(\sqrt{\phi}\xi\right)\right)\right\}^2}. \tag{3.1.17}$$

$$u_{1_5}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{4\lambda\nu\left\{2\lambda + \sqrt{\phi}\left(tanh\left(\sqrt{\phi}\xi/4\right) \pm coth\left(\sqrt{\phi}\xi/4\right)\right)\right\} - \nu\left\{2\lambda + \sqrt{\phi}\left(tanh\left(\sqrt{\phi}\xi/4\right) \pm coth\left(\sqrt{\phi}\xi/4\right)\right)\right\}^2 - 16\mu\nu^2}{8\mu\nu\left\{2\lambda + \sqrt{\phi}\left(tanh\left(\sqrt{\phi}\xi/4\right) \pm coth\left(\sqrt{\phi}\xi/4\right)\right)\right\} \mp \lambda\left\{2\lambda + \sqrt{\phi}\left(tanh\left(\sqrt{\phi}\xi/4\right) \pm coth\left(\sqrt{\phi}\xi/4\right)\right)\right\}^2}. \tag{3.1.18}$$

$$u_{1_6}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\nu\left\{-\lambda + \frac{\sqrt{(A^2+B^2)\phi} - A\sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right)}{Asinh\left(\sqrt{\phi}\xi\right)+B}\right\} + \nu\left\{-\lambda + \frac{\sqrt{(A^2+B^2)\phi} - A\sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right)}{Asinh\left(\sqrt{\phi}\xi\right)+B}\right\}^2 + 4\mu\nu^2}{4\mu\nu\left\{-\lambda + \frac{\sqrt{(A^2+B^2)\phi} - A\sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right)}{Asinh\left(\sqrt{\phi}\xi\right)+B}\right\} \pm \lambda\left\{-\lambda + \frac{\sqrt{(A^2+B^2)\phi} - A\sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right)}{Asinh\left(\sqrt{\phi}\xi\right)+B}\right\}^2}. \tag{3.1.19}$$

$$u_{1_7}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\nu\left\{-\lambda - \frac{\sqrt{(A^2+B^2)\phi} + A\sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right)}{Asinh\left(\sqrt{\phi}\xi\right)+B}\right\} + \nu\left\{-\lambda - \frac{\sqrt{(A^2+B^2)\phi} + A\sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right)}{Asinh\left(\sqrt{\phi}\xi\right)+B}\right\}^2 + 4\mu\nu^2}{4\mu\nu\left\{-\lambda - \frac{\sqrt{(A^2+B^2)\phi} + A\sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right)}{Asinh\left(\sqrt{\phi}\xi\right)+B}\right\} \pm \lambda\left\{-\lambda - \frac{\sqrt{(A^2+B^2)\phi} + A\sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right)}{Asinh\left(\sqrt{\phi}\xi\right)+B}\right\}^2}, \tag{3.1.20}$$

where A and B are non-zero free constants

$$u_{1_8}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\mu cosh\left(\sqrt{\phi}\xi/2\right)\left\{\sqrt{\phi}sinh\left(\sqrt{\phi}\xi/2\right) - \lambda cosh\left(\sqrt{\phi}\xi/2\right)\right\} + \nu\left\{2\mu cosh\left(\sqrt{\phi}\xi/2\right)\right\}^2 + \mu\left\{\sqrt{\phi}sinh\left(\sqrt{\phi}\xi/2\right) - \lambda cosh\left(\sqrt{\phi}\xi/2\right)\right\}^2}{4\mu^2 cosh\left(\sqrt{\phi}\xi/2\right)\left\{\sqrt{\phi}sinh\left(\sqrt{\phi}\xi/2\right) - \lambda cosh\left(\sqrt{\phi}\xi/2\right)\right\} \pm \lambda\left\{2\mu cosh\left(\sqrt{\phi}\xi/2\right)\right\}^2}. \tag{3.1.21}$$

$$u_{1_9}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\mu sinh\left(\sqrt{\phi}\xi/2\right)\left\{\lambda sinh\left(\sqrt{\phi}\xi/2\right) - \sqrt{\phi}cosh\left(\sqrt{\phi}\xi/2\right)\right\} - \nu\left\{2\mu sinh\left(\sqrt{\phi}\xi/2\right)\right\}^2 - \mu\left\{\lambda sinh\left(\sqrt{\phi}\xi/2\right) - \sqrt{\phi}cosh\left(\sqrt{\phi}\xi/2\right)\right\}^2}{4\mu^2 sinh\left(\sqrt{\phi}\xi/2\right)\left\{\lambda sinh\left(\sqrt{\phi}\xi/2\right) - \sqrt{\phi}cosh\left(\sqrt{\phi}\xi/2\right)\right\} \mp \lambda\left\{2\mu sinh\left(\sqrt{\phi}\xi/2\right)\right\}^2}. \tag{3.1.22}$$

$$u_{1_{10}}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\mu cosh\left(\sqrt{\phi}\xi\right)\left\{\sqrt{\phi}sinh\left(\sqrt{\phi}\xi\right) - \lambda cosh\left(\sqrt{\phi}\xi\right) \pm i\sqrt{\phi}\right\} + \nu\left\{2\mu cosh\left(\sqrt{\phi}\xi\right)\right\}^2 + \mu\left\{\sqrt{\phi}sinh\left(\sqrt{\phi}\xi\right) - \lambda cosh\left(\sqrt{\phi}\xi\right) \pm i\sqrt{\phi}\right\}^2}{4\mu^2 cosh\left(\sqrt{\phi}\xi\right)\left\{\sqrt{\phi}sinh\left(\sqrt{\phi}\xi\right) - \lambda cosh\left(\sqrt{\phi}\xi\right) \pm i\sqrt{\phi}\right\} \pm \lambda\left\{2\mu cosh\left(\sqrt{\phi}\xi\right)\right\}^2}. \tag{3.1.23}$$

$$u_{1_{11}}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\mu sinh\left(\sqrt{\phi}\xi\right)\left\{-\lambda sinh\left(\sqrt{\phi}\xi\right) + \sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right) \pm \sqrt{\phi}\right\} + \nu\left\{2\mu sinh\left(\sqrt{\phi}\xi\right)\right\}^2 + \mu\left\{-\lambda sinh\left(\sqrt{\phi}\xi\right) + \sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right) \pm \sqrt{\phi}\right\}^2}{4\mu^2 sinh\left(\sqrt{\phi}\xi\right)\left\{-\lambda sinh\left(\sqrt{\phi}\xi\right) + \sqrt{\phi}cosh\left(\sqrt{\phi}\xi\right) \pm \sqrt{\phi}\right\} \pm \lambda\left\{2\mu sinh\left(\sqrt{\phi}\xi\right)\right\}^2}. \tag{3.1.24}$$

$$u_{1_{12}}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{\begin{array}{c}4\lambda\mu sinh\left(\sqrt{\phi}\xi/4\right)cosh\left(\sqrt{\phi}\xi/4\right)\left\{-2\lambda sinh\left(\sqrt{\phi}\xi/4\right)cosh\left(\sqrt{\phi}\xi/4\right) + 2\sqrt{\phi}cosh^2\left(\sqrt{\phi}\xi/4\right) - \sqrt{\phi}\right\} + \\ \nu\left\{4\mu sinh\left(\sqrt{\phi}\xi/4\right)cosh\left(\sqrt{\phi}\xi/4\right)\right\}^2 \\ +\mu\left\{-2\lambda sinh\left(\sqrt{\phi}\xi/4\right)cosh\left(\sqrt{\phi}\xi/4\right) + 2\sqrt{\phi}cosh^2\left(\sqrt{\phi}\xi/4\right) - \sqrt{\phi}\right\}^2\end{array}}{\begin{array}{c}8\mu^2 sinh\left(\sqrt{\phi}\xi/4\right)cosh\left(\sqrt{\phi}\xi/4\right)\left\{-2\lambda sinh\left(\sqrt{\phi}\xi/4\right)cosh\left(\sqrt{\phi}\xi/4\right) + 2\sqrt{\phi}cosh^2\left(\sqrt{\phi}\xi/4\right) - \sqrt{\phi}\right\} \\ \pm\lambda\left\{4\mu sinh\left(\sqrt{\phi}\xi/4\right)cosh\left(\sqrt{\phi}\xi/4\right)\right\}^2\end{array}}. \tag{3.1.25}$$

Solution family 2: When $\phi = \lambda^2 - 4\mu\nu < 0$ and $\lambda\nu \neq 0(or\mu\nu \neq 0)$,

$$u_{1_{13}}(\xi) = \pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2 - 4k^2\mu\nu - 1\right)}} \times \frac{2\lambda\nu\left[-\lambda + \sqrt{-\phi}\tan\left(\sqrt{-\phi}\xi/2\right)\right] + \nu\left[-\lambda + \sqrt{-\phi}\tan\left(\sqrt{-\phi}\xi/2\right)\right]^2 + 4\mu\nu^2}{4\mu\nu\left[-\lambda + \sqrt{-\phi}\tan\left(\sqrt{-\phi}\xi/2\right)\right] \pm \lambda\left[-\lambda + \sqrt{-\phi}\tan\left(\sqrt{-\phi}\xi/2\right)\right]^2}. \tag{3.1.26}$$

$$u_{1_{14}}(\xi)=\pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu-1\right)}}\times\frac{2\lambda\nu\left[\lambda+\sqrt{-\phi}\cot\left(\sqrt{-\phi}\xi/2\right)\right]-\nu\left[\lambda+\sqrt{-\phi}\cot\left(\frac{\sqrt{-\phi}\xi}{2}\right)\right]^2-4\mu\nu^2}{4\mu\nu\left[\lambda+\sqrt{-\phi}\cot\left(\sqrt{-\phi}\xi/2\right)\right]\mp\lambda\left[\lambda+\sqrt{-\phi}\cot\left(\sqrt{-\phi}\xi/2\right)\right]^2}. \tag{3.1.27}$$

$$u_{1_{15}}(\xi)=\pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu-1\right)}}\times\frac{2\lambda\nu\left[-\lambda+\sqrt{-\phi}\left(\tan\left(\sqrt{-\phi}\xi\right)\pm\sec\left(\sqrt{-\phi}\xi\right)\right)\right]+\nu\left[-\lambda+\sqrt{-\phi}\left(\tan\left(\sqrt{-\phi}\xi\right)\pm\sec\left(\sqrt{-\phi}\xi\right)\right)\right]^2+4\mu\nu^2}{4\mu\nu\left[-\lambda+\sqrt{-\phi}\left(\tan\left(\sqrt{-\phi}\xi\right)\pm\sec\left(\sqrt{-\phi}\xi\right)\right)\right]\pm\lambda\left[-\lambda+\sqrt{-\phi}\left(\tan\left(\sqrt{-\phi}\xi\right)\pm\sec\left(\sqrt{-\phi}\xi\right)\right)\right]^2}. \tag{3.1.28}$$

$$u_{1_{16}}(\xi)=\pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu-1\right)}}\times\frac{2\lambda\nu\left[\lambda+\sqrt{-\phi}\left(\cot\left(\sqrt{-\phi}\xi\right)\pm\csc\left(\sqrt{-\phi}\xi\right)\right)\right]-\nu\left[\lambda+\sqrt{-\phi}\left(\cot\left(\sqrt{-\phi}\xi\right)\pm\csc\left(\sqrt{-\phi}\xi\right)\right)\right]^2-4\mu\nu^2}{4\mu\nu\left[\lambda+\sqrt{-\phi}\left(\cot\left(\sqrt{-\phi}\xi\right)\pm\csc\left(\sqrt{-\phi}\xi\right)\right)\right]\mp\lambda\left[\lambda+\sqrt{-\phi}\left(\cot\left(\sqrt{-\phi}\xi\right)\pm\csc\left(\sqrt{-\phi}\xi\right)\right)\right]^2}. \tag{3.1.29}$$

$$u_{1_{17}}(\xi)=\pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu-1\right)}}\times\frac{4\lambda\nu\left[-2\lambda+\sqrt{-\phi}\left(\tan\left(\sqrt{-\phi}\xi/4\right)-\cot\left(\sqrt{-\phi}\xi/4\right)\right)\right]+\nu\left[-2\lambda+\sqrt{-\phi}\left(\tan\left(\sqrt{-\phi}\xi/4\right)-\cot\left(\sqrt{-\phi}\xi/4\right)\right)\right]^2+16\mu\nu^2}{8\mu\nu\left[-2\lambda+\sqrt{-\phi}\left(\tan\left(\frac{\sqrt{-\phi}\xi}{4}\right)-\cot\left(\frac{\sqrt{-\phi}\xi}{4}\right)\right)\right]\pm\lambda\left[-2\lambda+\sqrt{-\phi}\left(\tan\left(\sqrt{-\phi}\xi/4\right)-\cot\left(\sqrt{-\phi}\xi/4\right)\right)\right]^2}. \tag{3.1.30}$$

$$u_{1_{18}}(\xi)=\pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu-1\right)}}\times\frac{2\lambda\nu\left[-\lambda+\frac{\pm\sqrt{-\phi\left(A^2-B^2\right)}-A\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)}{A\sin\left(\sqrt{-\phi}\xi\right)+B}\right]+\nu\left[-\lambda+\frac{\pm\sqrt{-\phi\left(A^2-B^2\right)}-A\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)}{A\sin\left(\sqrt{-\phi}\xi\right)+B}\right]^2+4\mu\nu^2}{4\mu\nu\left[-\lambda+\frac{\pm\sqrt{-\phi\left(A^2-B^2\right)}-A\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)}{A\sin\left(\sqrt{-\phi}\xi\right)+B}\right]\pm\lambda\left[-\lambda+\frac{\pm\sqrt{-\phi\left(A^2-B^2\right)}-A\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)}{A\sin\left(\sqrt{-\phi}\xi\right)+B}\right]^2}. \tag{3.1.31}$$

$$u_{1_{19}}(\xi)=\pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu-1\right)}}\times\frac{4\lambda\nu\left[-\lambda-\frac{\pm\sqrt{-\phi\left(A^2-B^2\right)}+A\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)}{A\sin\left(\sqrt{-\phi}\xi\right)+B}\right]+\nu\left[-\lambda-\frac{\pm\sqrt{-\phi\left(A^2-B^2\right)}+A\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)}{A\sin\left(\sqrt{-\phi}\xi\right)+B}\right]^2+4\mu\nu^2}{4\mu\nu\left[-\lambda-\frac{\pm\sqrt{-\phi\left(A^2-B^2\right)}+A\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)}{A\sin\left(\sqrt{-\phi}\xi\right)+B}\right]\pm\lambda\left[-\lambda-\frac{\pm\sqrt{-\phi\left(A^2-B^2\right)}+A\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)}{A\sin\left(\sqrt{-\phi}\xi\right)+B}\right]^2},\tag{3.1.32}$$

where A and B are non-zero real parameters such that $A^2-B^2>0$.

$$u_{1_{20}}(\xi)=\pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu-1\right)}}\times\frac{2\lambda\mu\cos\left(\sqrt{-\phi}\xi/2\right)\left\{\sqrt{-\phi}\sin\left(\sqrt{-\phi}\xi/2\right)+\lambda\cos\left(\sqrt{-\phi}\xi/2\right)\right\}-\nu\left\{2\mu\cos\left(\sqrt{-\phi}\xi/2\right)\right\}^2-\mu\left\{\sqrt{-\phi}\sin\left(\sqrt{-\phi}\xi/2\right)+\lambda\cos\left(\sqrt{-\phi}\xi/2\right)\right\}^2}{4\mu^2\cos\left(\sqrt{-\phi}\xi/2\right)\left\{\sqrt{-\phi}\sin\left(\sqrt{-\phi}\xi/2\right)+\lambda\cos\left(\sqrt{-\phi}\xi/2\right)\right\}\mp\lambda\left\{2\mu\cos\left(\sqrt{-\phi}\xi/2\right)\right\}^2}.\tag{3.1.33}$$

$$u_{1_{21}}(\xi)=\pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu-1\right)}}\times\frac{2\lambda\mu\sin\left(\sqrt{-\phi}\xi/2\right)\left\{-\lambda\sin\left(\sqrt{-\phi}\xi/2\right)+\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi/2\right)\right\}+\nu\left\{2\mu\sin\left(\sqrt{-\phi}\xi/2\right)\right\}^2+\mu\left\{-\lambda\sin\left(\sqrt{-\phi}\xi/2\right)+\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi/2\right)\right\}^2}{4\mu^2\sin\left(\sqrt{-\phi}\xi/2\right)\left\{-\lambda\sin\left(\sqrt{-\phi}\xi/2\right)+\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi/2\right)\right\}\pm\lambda\left\{2\mu\sin\left(\sqrt{-\phi}\xi/2\right)\right\}^2}.\tag{3.1.34}$$

$$u_{1_{22}}(\xi)=\pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu-1\right)}}\times\frac{2\lambda\mu\cos\left(\sqrt{-\phi}\xi\right)\left\{\sqrt{-\phi}\sin\left(\sqrt{-\phi}\xi\right)+\lambda\cos\left(\sqrt{-\phi}\xi\right)\pm\sqrt{-\phi}\right\}-\nu\left\{2\mu\cos\left(\sqrt{-\phi}\xi\right)\right\}^2-\mu\left\{\sqrt{-\phi}\sin\left(\sqrt{-\phi}\xi\right)+\lambda\cos\left(\sqrt{-\phi}\xi\right)\pm\sqrt{-\phi}\right\}^2}{4\mu^2\cos\left(\sqrt{-\phi}\xi\right)\left\{\sqrt{-\phi}\sin\left(\sqrt{-\phi}\xi\right)+\lambda\cos\left(\sqrt{-\phi}\xi\right)\pm\sqrt{-\phi}\right\}\mp\lambda\left\{2\mu\cos\left(\sqrt{-\phi}\xi\right)\right\}^2}.\tag{3.1.35}$$

$$u_{1_{23}}(\xi)=\pm 2\mu k\sqrt{\frac{2\delta}{\eta\left(k^2\lambda^2-4k^2\mu\nu-1\right)}}\times\frac{2\lambda\mu\sin\left(\sqrt{-\phi}\xi\right)\left\{-\lambda\sin\left(\sqrt{-\phi}\xi\right)+\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)\pm\sqrt{-\phi}\right\}+\nu\left\{2\mu\sin\left(\sqrt{-\phi}\xi\right)\right\}^2+\mu\left\{-\lambda\sin\left(\sqrt{-\phi}\xi\right)+\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)\pm\sqrt{-\phi}\right\}^2}{4\mu^2\sin\left(\sqrt{-\phi}\xi\right)\left\{-\lambda\sin\left(\sqrt{-\phi}\xi\right)+\sqrt{-\phi}\cos\left(\sqrt{-\phi}\xi\right)\pm\sqrt{-\phi}\right\}\pm\lambda\left\{2\mu\sin\left(\sqrt{-\phi}\xi\right)\right\}^2}.\tag{3.1.36}$$

$$\times\frac{\begin{array}{c}4\lambda\mu sin\left(\sqrt{-\phi}\xi/4\right)cos\left(\sqrt{-\phi}\xi/4\right)\left\{-2\lambda sin\left(\sqrt{-\phi}\xi/4\right)cos\left(\sqrt{-\phi}\xi/4\right)+2\sqrt{-\phi}cos^2\left(\sqrt{-\phi}\xi/4\right)-\sqrt{-\phi}\right\}\\+\nu\left\{4\mu sin\left(\sqrt{-\phi}\xi/4\right)cos\left(\sqrt{-\phi}\xi/4\right)\right\}^2\\+\mu\left\{-2\lambda sin\left(\sqrt{-\phi}\xi/4\right)cos\left(\sqrt{-\phi}\xi/4\right)+2\sqrt{-\phi}cos^2\left(\sqrt{-\phi}\xi/4\right)-\sqrt{-\phi}\right\}^2\end{array}}{\begin{array}{c}8\mu^2 sin\left(\sqrt{-\phi}\xi/4\right)cos\left(\sqrt{-\phi}\xi/4\right)\left\{-2\lambda sin\left(\sqrt{-\phi}\xi/4\right)cos\left(\sqrt{-\phi}\xi/4\right)+2\sqrt{-\phi}cos^2\left(\sqrt{-\phi}\xi/4\right)-\sqrt{-\phi}\right\}\\\pm\lambda\left\{4\mu sin\left(\sqrt{-\phi}\xi/4\right)cos\left(\sqrt{-\phi}\xi/4\right)\right\}^2\end{array}}. \tag{3.1.37}$$

The solutions (3.1.14)–(3.1.37) are followed by $\xi = \frac{kx^\alpha}{\alpha} + \frac{\delta kt^\alpha}{\alpha\left(k^2\lambda^2-4k^2\mu\nu-1\right)}$.

The above effort serves ample traveling wave solutions in accurate form involving many unknown constants to the nonlinear fractional mCH equation among which a few are recorded for making readable. Ali et al. have recently investigated the same equation applying the modified extended tanh tool and found some analytic solutions involving only few parameters (Ali et al. 2018). This comparison ensures the novelty and generality of our achieved outcomes which might be recorded first time in the literature.

3.2 The space and time fractional nonlinear Schrodinger equation

This well-known equation is

$$iD_t^\alpha u + D_x^{2\beta} u + 2|u|^2 u = 0, \quad t > 0, \quad 0 < \alpha, \beta \le 1 \tag{3.2.1}$$

where x and t represent the space and the time variables; the equation occurs in non-linear optics, plasma physics and superconductivity. We introduce a change of wave variable as follows:

$$u(x,t) = e^{i\psi}U(\xi), \quad \psi = \frac{mx^\beta}{\beta} + \frac{kt^\alpha}{\alpha}, \quad \xi = \frac{x^\beta}{\beta} + \frac{ct^\alpha}{\alpha}, \tag{3.2.2}$$

where c, k and m are arbitrary parameters to be calculated later. Equation (3.2.1) with the support of Eq. (3.2.2) attends

$$-\left(k+m^2\right)U + U'' + 2U^3 = 0, \tag{3.2.3}$$

$$cU' + 2mU' = 0, \tag{3.2.4}$$

where the primes in U indicate the order of derivative due to ξ. Equation (3.2.4) produces $m = -\frac{c}{2}$, so that Eq. (3.2.3) becomes

$$-\left(4k+c^2\right)U + 8U^3 + 4U'' = 0. \tag{3.2.5}$$

Balancing the terms U'' and U^3 offers $n = 1$ and hence Eq. (2.4) agrees

$$U(\xi) = \frac{a_0 + a_1\varphi(\xi) + b_1\varphi^{-1}(\xi)}{e_0 + e_1\varphi(\xi) + f_1\varphi^{-1}(\xi)}. \tag{3.2.6}$$

Inserting Eq. (3.2.6) with its indispensable derivatives and Eq. (2.5) into Eq. (3.2.5) makes a polynomial in $\varphi(\xi)$. Assigning like terms of this polynomial to zero and solving them we get the values for $a_0, a_1, b_1, e_0, e_1, f_1, c$ and k as follows:

Set 1: $a_0 = \pm\frac{i}{4}\left(\lambda e_0 + 2f_1\nu\right)$, $a_1 = \pm ie_0\nu$, $b_1 = \pm\frac{i\lambda f_1}{2}$,

$$e_1 = 0,\quad \kappa = \frac{1}{4}\left(8\mu\nu - 2\lambda^2 - c^2\right) \tag{3.2.7}$$

Set 2: $a_0 = \pm\frac{i}{2}\left(\lambda e_0 - 2\nu f_1\right)$, $a_1 = 0$, $b_1 = \mp\frac{i}{2}\left(\lambda f_1 - 2\mu e_0\right)$,

$$e_1 = 0,\quad \kappa = \frac{1}{4}\left(8\mu\nu - 2\lambda^2 - c^2\right) \tag{3.2.8}$$

Set 3: $a_0 = \pm i\nu f_1$, $a_1 = 0$, $b_1 = \pm\frac{i\lambda f_1}{2}$, $e_0 = 0$,

$$e_1 = 0,\quad \kappa = \frac{1}{4}\left(8\mu\nu - 2\lambda^2 - c^2\right) \tag{3.2.9}$$

Set 4: $a_0 = 0$, $a_1 = 0$, $b_1 = \pm\frac{i\left(\lambda^2-4\mu\nu\right)e_0}{4\nu}$,$e_1 = 0$,

$$f_1 = \frac{\lambda e_0}{2\nu},\quad \kappa = \frac{1}{4}\left(8\mu\nu - 2\lambda^2 - c^2\right) \tag{3.2.10}$$

Set 5: $a_0 = \pm\frac{i}{2}\left(\lambda e_0 - 2\nu f_1\right)$, $a_1 = 0$, $b_1 = \mp\frac{i}{2}\left(\lambda f_1 - 2\mu e_0\right)$,

$$e_1 = 0,\quad \kappa = \frac{1}{4}\left(8\mu\nu - 2\lambda^2 - c^2\right) \tag{3.2.11}$$

Set 6: $a_0 = \pm ie_0\lambda$, $a_1 = \pm ie_o\nu$, $b_1 = \pm\frac{ie_0\left(\lambda^2-2\mu\nu\right)}{4\nu}$, $e_1 = 0$,

$$f_1 = \frac{\lambda e_0}{2\nu},\quad \kappa = \frac{1}{4}\left(32\mu\nu - 8\lambda^2 - c^2\right) \tag{3.2.12}$$

Set 7: $a_0 = \pm ie_0\lambda$, $a_1 = \pm ie_o\nu$, $b_1 = \pm ie_o\mu$, $e_1 = 0$,

$$f_1 = \frac{\lambda e_0}{2\nu},\quad \kappa = \frac{1}{4}\left(4\lambda^2 - 16\mu\nu - c^2\right) \tag{3.2.13}$$

Equation (3.2.6) and Eq. (3.2.2) with the aid of Eqs. (3.2.7)–(3.2.13) provides

$$u_1(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + 4e_0\nu\varphi(\xi) + 2\lambda f_1\varphi^{-1}(\xi)}{4\left\{e_0 + f_1\varphi^{-1}(\xi)\right\}}. \tag{3.2.14}$$

$$u_2(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda e_0 - 2\nu f_1\right) - \left(\lambda f_1 - 2\mu e_0\right)\varphi^{-1}(\xi)}{2\left\{e_0 + f_1\varphi^{-1}(\xi)\right\}}. \tag{3.2.15}$$

$$u_3(\xi) = \pm ie^{i\psi}\left\{\frac{\lambda}{2} + \nu\varphi(\xi)\right\}. \tag{3.2.16}$$

$$u_4(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda^2 - 4\mu\nu\right)\varphi^{-1}(\xi)}{4\nu + 2\lambda\varphi^{-1}(\xi)}. \tag{3.2.17}$$

$$u_5(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda e_0 - 2\nu f_1\right) - \left(\lambda f_1 - 2\mu e_0\right)\varphi^{-1}(\xi)}{2\left\{e_0 + f_1\varphi^{-1}(\xi)\right\}}. \tag{3.2.18}$$

$$u_6(\xi) = \pm ie^{i\psi} \times \frac{4\nu\lambda \pm 4\nu^2\varphi(\xi) + \left(\lambda^2 - 2\mu\nu\right)\varphi^{-1}(\xi)}{4\nu + 2\lambda\varphi^{-1}(\xi)}. \tag{3.2.19}$$

$$u_7(\xi) = \pm 2i\nu e^{i\psi} \times \frac{\lambda + \nu\varphi(\xi) + \mu\varphi^{-1}(\xi)}{2\nu + \lambda\varphi^{-1}(\xi)}. \tag{3.2.20}$$

The expressions (3.2.14)–(3.2.20) delivers numerous appropriate analytic traveling wave solutions of Eq. (3.2.1) among which we record here only for the expression (3.2.14) as follows:

Solution family 1: When $\Phi = \lambda^2 - 4\mu\nu > 0$ and $\lambda\nu \neq 0$(or $\mu\nu \neq 0$),

$$u_1^1(\xi) = \pm ie^{i\psi} \times \frac{\begin{array}{c}\left(\lambda e_0 + 2f_1\nu\right) - 2e_0\left(\lambda + \sqrt{\Phi}\tanh\left(\sqrt{\Phi}\xi/2\right)\right) \\ -4\lambda f_1\nu(\lambda + \sqrt{\Phi}\tanh\left(\sqrt{\Phi}\xi/2\right))^{-1}\end{array}}{4\left\{e_0 - 2f_1\nu\left(\lambda + \sqrt{\Phi}\tanh\left(\sqrt{\Phi}\xi/2\right)\right)^{-1}\right\}}. \tag{3.2.21}$$

$$u_1^2(\xi) = \pm ie^{i\psi} \times \frac{\begin{array}{c}\left(\lambda e_0 + 2f_1\nu\right) - 2e_0\left(\lambda + \sqrt{\Phi}\coth\left(\sqrt{\Phi}\xi/2\right)\right) \\ -4\lambda f_1\nu(\lambda + \sqrt{\Phi}\coth\left(\sqrt{\Phi}\xi/2\right))^{-1}\end{array}}{4\left\{e_0 - 2f_1\nu\left(\lambda + \sqrt{\Phi}\coth\left(\sqrt{\Phi}\xi/2\right)\right)^{-1}\right\}}. \tag{3.2.22}$$

$$u_1^3(\xi) = \pm ie^{i\psi} \times \frac{\begin{array}{c}\left(\lambda e_0 + 2f_1\nu\right) - 2e_0\left(\lambda + \sqrt{\Phi}\left(\tanh\left(\sqrt{\Phi}\xi\right) \pm i\operatorname{sech}\left(\sqrt{\Phi}\xi\right)\right)\right) \\ -4\lambda f_1\nu\left(\lambda + \sqrt{\Phi}\left(\tanh\left(\sqrt{\Phi}\xi\right) \pm i\operatorname{sech}\left(\sqrt{\Phi}\xi\right)\right)\right)^{-1}\end{array}}{4\left\{e_0 - 2f_1\nu\left(\lambda + \sqrt{\Phi}\left(\tanh\left(\sqrt{\Phi}\xi\right) \pm i\operatorname{sech}\left(\sqrt{\Phi}\xi\right)\right)\right)^{-1}\right\}}. \tag{3.2.23}$$

$$u_1^4(\xi) = \pm ie^{i\psi} \times \frac{\begin{array}{c}\left(\lambda e_0 + 2f_1\nu\right) - 2e_0\left(\lambda + \sqrt{\Phi}\left(\coth\left(\sqrt{\Phi}\xi\right) \pm \operatorname{csch}\left(\sqrt{\Phi}\xi\right)\right)\right) \\ -4\lambda f_1\nu\left(\lambda + \sqrt{\Phi}\left(\coth\left(\sqrt{\Phi}\xi\right) \pm \operatorname{csch}\left(\sqrt{\Phi}\xi\right)\right)\right)^{-1}\end{array}}{4\left\{e_0 - 2f_1\nu\left(\lambda + \sqrt{\Phi}\left(\coth\left(\sqrt{\Phi}\xi\right) \pm \operatorname{csch}\left(\sqrt{\Phi}\xi\right)\right)\right)^{-1}\right\}}. \tag{3.2.24}$$

$$u_1^5(\xi) = \pm ie^{i\psi} \times \frac{\begin{array}{c}\left(\lambda e_0 + 2f_1\nu\right) - e_0\left(2\lambda + \sqrt{\Phi}\left(\tanh\left(\sqrt{\Phi}\xi/4\right) \pm \coth\left(\sqrt{\Phi}\xi/4\right)\right)\right) \\ -8\lambda f_1\nu\left(2\lambda + \sqrt{\Phi}\left(\tanh\left(\sqrt{\Phi}\xi/4\right) \pm \coth\left(\sqrt{\Phi}\xi/4\right)\right)\right)^{-1}\end{array}}{4\left\{e_0 - 4f_1\nu\left(2\lambda + \sqrt{\Phi}\left(\tanh\left(\sqrt{\Phi}\xi/4\right) \pm \coth\left(\sqrt{\Phi}\xi/4\right)\right)\right)^{-1}\right\}}. \tag{3.2.25}$$

$$u_1^6(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + 2e_0\left(-\lambda + \frac{\sqrt{(A^2+B^2)\Phi} - A\sqrt{\Phi}\cosh\left(\sqrt{\Phi}\xi\right)}{A\sinh\left(\sqrt{\Phi}\xi\right)+B}\right) + 4\lambda f_1\nu\left(-\lambda + \frac{\sqrt{(A^2+B^2)\Phi} - A\sqrt{\Phi}\cosh\left(\sqrt{\Phi}\xi\right)}{A\sinh\left(\sqrt{\Phi}\xi\right)+B}\right)^{-1}}{4\left\{e_0 + 2f_1\nu\left(-\lambda + \frac{\sqrt{(A^2+B^2)\Phi} - A\sqrt{\Phi}\cosh\left(\sqrt{\Phi}\xi\right)}{A\sinh\left(\sqrt{\Phi}\xi\right)+B}\right)^{-1}\right\}}, \tag{3.2.26}$$

$$u_1^7(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + 2e_0\left(-\lambda - \frac{\sqrt{(A^2+B^2)\Phi} + A\sqrt{\Phi}\cosh\left(\sqrt{\Phi}\xi\right)}{A\sinh\left(\sqrt{\Phi}\xi\right)+B}\right) + 4\lambda f_1\nu\left(-\lambda - \frac{\sqrt{(A^2+B^2)\Phi} + A\sqrt{\Phi}\cosh\left(\sqrt{\Phi}\xi\right)}{A\sinh\left(\sqrt{\Phi}\xi\right)+B}\right)^{-1}}{4\left\{e_0 + 2f_1\nu\left(-\lambda - \frac{\sqrt{(A^2+B^2)\Phi} + A\sqrt{\Phi}\cosh\left(\sqrt{\Phi}\xi\right)}{A\sinh\left(\sqrt{\Phi}\xi\right)+B}\right)^{-1}\right\}}, \tag{3.2.27}$$

where AandB are non-zero arbitrary real parameters.

$$u_1^8(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + \frac{8e_0\mu\nu\cosh\left(\sqrt{\Phi}\xi/2\right)}{\sqrt{\Phi}\sinh\left(\sqrt{\Phi}\xi/2\right) - \lambda\cosh\left(\sqrt{\Phi}\xi/2\right)} + \frac{\lambda f_1(\sqrt{\Phi}\sinh(\sqrt{\Phi}\xi/2) - \lambda\cosh(\sqrt{\Phi}\xi/2))}{\mu\cosh(\sqrt{\Phi}\xi/2)}}{4\left\{e_0 + \frac{f_1(\sqrt{\Phi}\sinh(\sqrt{\Phi}\xi/2) - \lambda\cosh(\sqrt{\Phi}\xi/2))}{2\mu\cosh(\sqrt{\Phi}\xi/2)}\right\}}. \tag{3.2.28}$$

$$u_1^9(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) - \frac{8e_0\mu\nu\sinh\left(\sqrt{\Phi}\xi/2\right)}{\lambda\sinh(\sqrt{\Phi}\xi/2) - \sqrt{\Phi}\cosh(\sqrt{\Phi}\xi/2)} - \frac{\lambda f_1(\lambda\sinh(\sqrt{\Phi}\xi/2) - \sqrt{\Phi}\cosh(\sqrt{\Phi}\xi/2))}{\mu\sinh(\sqrt{\Phi}\xi/2)}}{4\left\{e_0 - \frac{f_1(\lambda\sinh(\sqrt{\Phi}\xi/2) - \sqrt{\Phi}\cosh(\sqrt{\Phi}\xi/2))}{2\mu\sinh(\sqrt{\Phi}\xi/2)}\right\}}. \tag{3.2.29}$$

$$u_1^{10}(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + \frac{8e_0\mu\nu\cosh\left(\sqrt{\Phi}\xi\right)}{\sqrt{\Phi}\sinh\left(\sqrt{\Phi}\xi\right) - \lambda\cosh\left(\sqrt{\Phi}\xi\right) \pm i\sqrt{\Phi}} + \frac{\lambda f_1(\sqrt{\Phi}\sinh(\sqrt{\Phi}\xi) - \lambda\cosh(\sqrt{\Phi}\xi) \pm i\sqrt{\Phi})}{\mu\cosh(\sqrt{\Phi}\xi)}}{4\left\{e_0 + \frac{f_1(\sqrt{\Phi}\sinh(\sqrt{\Phi}\xi) - \lambda\cosh(\sqrt{\Phi}\xi) \pm i\sqrt{\Phi})}{2\mu\cosh(\sqrt{\Phi}\xi)}\right\}}. \tag{3.2.30}$$

$$u_1^{11}(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + \frac{8e_0\mu\nu\sinh\left(\sqrt{\Phi}\xi\right)}{-\lambda\sinh(\sqrt{\Phi}\xi)+\sqrt{\Phi}\cosh(\sqrt{\Phi}\xi)\pm\sqrt{\Phi}} + \frac{\lambda f_1(-\lambda\sinh(\sqrt{\Phi}\xi)+\sqrt{\Phi}\cosh(\sqrt{\Phi}\xi)\pm\sqrt{\Phi})}{\mu\sinh(\sqrt{\Phi}\xi)}}{4\left\{e_0 + \frac{f_1(-\lambda\sinh(\sqrt{\Phi}\xi)+\sqrt{\Phi}\cosh(\sqrt{\Phi}\xi)\pm\sqrt{\Phi})}{2\mu\sinh(\sqrt{\Phi}\xi)}\right\}}. \tag{3.2.31}$$

$$u_1^{12}(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + \frac{16e_0\mu\nu\sinh(\sqrt{\Phi}\xi/4)\cosh(\sqrt{\Phi}\xi/4)}{-2\lambda\sinh(\sqrt{\Phi}\xi/4)\cosh(\sqrt{\Phi}\xi/4)+2\sqrt{\Phi}\cosh^2(\sqrt{\Phi}\xi/4)-\sqrt{\Phi}} + \frac{\lambda f_1(-2\lambda\sinh(\sqrt{\Phi}\xi/4)\cosh(\sqrt{\Phi}\xi/4)+2\sqrt{\Phi}\cosh^2(\sqrt{\Phi}\xi/4)-\sqrt{\Phi})}{2\mu\sinh(\sqrt{\Phi}\xi/4)\cosh(\sqrt{\Phi}\xi/4)}}{4\left\{e_0 + \frac{f_1(-2\lambda\sinh(\sqrt{\Phi}\xi/4)\cosh(\sqrt{\Phi}\xi/4)+2\sqrt{\Phi}\cosh^2(\sqrt{\Phi}\xi/4)-\sqrt{\Phi})}{4\mu\sinh(\sqrt{\Phi}\xi/4)\cosh(\sqrt{\Phi}\xi/4)}\right\}}. \tag{3.2.32}$$

Solutions family 2: When $\Phi = \lambda^2 - 4\mu\nu < 0$ and $\lambda\nu \neq 0$(or $\mu\nu \neq 0$),

$$u_1^{13}(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + 2e_0(-\lambda + \sqrt{-\Phi}\tan(\sqrt{-\Phi}\xi/2)) + 4\lambda f_1\nu(-\lambda + \sqrt{-\Phi}\tan(\sqrt{-\Phi}\xi/2))^{-1}}{4\{e_0 + 2f_1\nu(-\lambda + \sqrt{-\Phi}\tan(\sqrt{-\Phi}\xi/2))^{-1}\}}. \tag{3.2.33}$$

$$u_1^{14}(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) - 2e_0(\lambda + \sqrt{-\Phi}\cot(\sqrt{-\Phi}\xi/2)) - 4\lambda f_1\nu(\lambda + \sqrt{-\Phi}\cot(\sqrt{-\Phi}\xi/2))^{-1}}{4\{e_0 - 2f_1\nu(\lambda + \sqrt{-\Phi}\cot(\sqrt{-\Phi}\xi/2))^{-1}\}}. \tag{3.2.34}$$

$$u_1^{15}(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + 2e_0(-\lambda + \sqrt{-\Phi}(\tan(\sqrt{-\Phi}\xi) \pm sec(\sqrt{-\Phi}\xi))) + 4\lambda f_1\nu(-\lambda + \sqrt{-\Phi}(\tan(\sqrt{-\Phi}\xi) \pm \sec(\sqrt{-\Phi}\xi)))^{-1}}{4\{e_0 + 2f_1\nu(-\lambda + \sqrt{-\Phi}(\tan(\sqrt{-\Phi}\xi) \pm \sec(\sqrt{-\Phi}\xi)))^{-1}\}}. \tag{3.2.35}$$

$$u_1^{16}(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) - 2e_0(\lambda + \sqrt{-\Phi}(cot(\sqrt{-\Phi}\xi) \pm csc(\sqrt{-\Phi}\xi))) - 4\lambda f_1\nu(\lambda + \sqrt{-\Phi}(\cot(\sqrt{-\Phi}\xi) \pm \csc(\sqrt{-\Phi}\xi)))^{-1}}{4\{e_0 - 2f_1\nu(\lambda + \sqrt{-\Phi}(\cot(\sqrt{-\Phi}\xi) \pm \csc(\sqrt{-\Phi}\xi)))^{-1}\}}. \tag{3.2.36}$$

$$u_1^{17}(\xi) = \pm ie^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + e_0(-2\lambda + \sqrt{-\Phi}(\tan(\sqrt{-\Phi}\xi/4) - \cot(\sqrt{-\Phi}\xi/4))) + 8\lambda f_1\nu(-2\lambda + \sqrt{-\Phi}(\tan(\sqrt{-\Phi}\xi/4) - \cot(\sqrt{-\Phi}\xi/4)))^{-1}}{4\{e_0 + 4f_1\nu(-2\lambda + \sqrt{-\Phi}(\tan(\sqrt{-\Phi}\xi/4) - \cot(\sqrt{-\Phi}\xi/4)))^{-1}\}}. \tag{3.2.37}$$

$$u_1^{18}(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + 2e_0\left(-\lambda + \frac{\pm\sqrt{-\Phi(A^2-B^2)}-A\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi)}{A\sin(\sqrt{-\Phi}\xi)+B}\right) + 4\lambda f_1\nu\left(-\lambda + \frac{\pm\sqrt{-\Phi(A^2-B^2)}-A\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi)}{A\sin(\sqrt{-\Phi}\xi)+B}\right)^{-1}}{4\left\{e_0 + 2f_1\nu\left(-\lambda + \frac{\pm\sqrt{-\Phi(A^2-B^2)}-A\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi)}{A\sin(\sqrt{-\Phi}\xi)+B}\right)^{-1}\right\}}, \tag{3.2.38}$$

$$u_1^{19}(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + 2e_0\left(-\lambda - \frac{\pm\sqrt{-\Phi(A^2-B^2)}+A\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi)}{A\sin(\sqrt{-\Phi}\xi)+B}\right) + 4\lambda f_1\nu\left(-\lambda - \frac{\pm\sqrt{-\Phi(A^2-B^2)}+A\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi)}{A\sin(\sqrt{-\Phi}\xi)+B}\right)^{-1}}{4\left\{e_0 + 2f_1\nu\left(-\lambda - \frac{\pm\sqrt{-\Phi(A^2-B^2)}+A\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi)}{A\sin(\sqrt{-\Phi}\xi)+B}\right)^{-1}\right\}}, \tag{3.2.39}$$

where $A$and$B$ are non-zero unknown real constants.

$$u_1^{20}(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) - \frac{8e_0\mu\nu\cos(\sqrt{-\Phi}\xi/2)}{\sqrt{-\Phi}\sin(\sqrt{-\Phi}\xi/2)+\lambda\cos(\sqrt{-\Phi}\xi/2)} - \frac{\lambda f_1(\sqrt{-\Phi}\sin(\sqrt{-\Phi}\xi/2)+\lambda\cos(\sqrt{-\Phi}\xi/2))}{\mu\cos(\sqrt{-\Phi}\xi/2)}}{4\left\{e_0 - \frac{f_1(\sqrt{-\Phi}\sin(\sqrt{-\Phi}\xi/2)+\lambda\cos(\sqrt{-\Phi}\xi/2))}{2\mu\cos(\sqrt{-\Phi}\xi/2)}\right\}}. \tag{3.2.40}$$

$$u_1^{21}(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + \frac{8e_0\mu\nu\sin(\sqrt{-\Phi}\xi/2)}{-\lambda\sin(\sqrt{-\Phi}\xi/2)+\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi/2)} + \frac{\lambda f_1(-\lambda\sin(\sqrt{-\Phi}\xi/2)+\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi/2))}{\mu\sin(\sqrt{-\Phi}\xi/2)}}{4\left\{e_0 + \frac{f_1(-\lambda\sin(\sqrt{-\Phi}\xi/2)+\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi/2))}{2\mu\sin(\sqrt{-\Phi}\xi/2)}\right\}}. \tag{3.2.41}$$

$$u_1^{22}(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) - \frac{8e_0\mu\nu\cos(\sqrt{-\Phi}\xi)}{\sqrt{-\Phi}\sin(\sqrt{-\Phi}\xi)+\lambda\cos(\sqrt{-\Phi}\xi)\pm\sqrt{-\Phi}} - \frac{\lambda f_1(\sqrt{-\Phi}\sin(\sqrt{-\Phi}\xi)+\lambda\cos(\sqrt{-\Phi}\xi)\pm\sqrt{-\Phi})}{\mu\cos(\sqrt{-\Phi}\xi)}}{4\left\{e_0 - \frac{f_1(\sqrt{-\Phi}\sin(\sqrt{-\Phi}\xi)+\lambda\cos(\sqrt{-\Phi}\xi)\pm\sqrt{-\Phi})}{2\mu\cos(\sqrt{-\Phi}\xi)}\right\}}. \tag{3.2.42}$$

$$u_1^{23}(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + \frac{8e_0\mu\nu\sin(\sqrt{-\Phi}\xi)}{-\lambda\sin(\sqrt{-\Phi}\xi)+\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi)\pm\sqrt{-\Phi}} + \frac{\lambda f_1(-\lambda\sin(\sqrt{-\Phi}\xi)+\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi)\pm\sqrt{-\Phi})}{\mu\sin(\sqrt{-\Phi}\xi)}}{4\left\{e_0 + \frac{f_1(-\lambda\sin(\sqrt{-\Phi}\xi)+\sqrt{-\Phi}\cos(\sqrt{-\Phi}\xi)\pm\sqrt{-\Phi})}{2\mu\sin(\sqrt{-\Phi}\xi)}\right\}}. \tag{3.2.43}$$

$$u_1^{24}(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) + \frac{16e_0\mu\nu\sin(\sqrt{-\Phi}\xi/4)\cos(\sqrt{-\Phi}\xi/4)}{-2\lambda\sin(\sqrt{-\Phi}\xi/4)\cos(\sqrt{-\Phi}\xi/4)+2\sqrt{-\Phi}\cos^2(\sqrt{-\Phi}\xi/4)-\sqrt{-\Phi}} + \frac{\lambda f_1(-2\lambda\sin(\sqrt{-\Phi}\xi/4)\cos(\sqrt{-\Phi}\xi/4)+2\sqrt{-\Phi}\cos^2(\sqrt{-\Phi}\xi/4)-\sqrt{-\Phi})}{2\mu\sin(\sqrt{-\Phi}\xi/4)\cos(\sqrt{-\Phi}\xi/4)}}{4\left\{e_0 + \frac{f_1(-2\lambda\sin(\sqrt{-\Phi}\xi/4)\cos(\sqrt{-\Phi}\xi/4)+2\sqrt{-\Phi}\cos^2(\sqrt{-\Phi}\xi/4)-\sqrt{-\Phi})}{4\mu\sin(\sqrt{-\Phi}\xi/4)\cos(\sqrt{-\Phi}\xi/4)}\right\}}. \tag{3.2.44}$$

The solutions (3.2.21)–(3.2.44) are followed by $\psi = -\frac{cx^\beta}{2\beta} + \frac{(8\mu\nu-2\lambda^2-c^2)t^\alpha}{4\alpha}$, $\xi = \frac{x^\beta}{\beta} + \frac{ct^\alpha}{\alpha}$.

Solutions family 3: When $\mu = 0$ and $\lambda\nu \neq 0$,

$$u_1^{25}(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) - \frac{4\lambda e_0 d}{d+\cosh(\lambda\xi)-\sinh(\lambda\xi)} - \frac{2f_1\nu}{d}(d+\cosh(\lambda\xi)-\sinh(\lambda\xi))}{4\left\{e_0 - \frac{f_1\nu}{\lambda d}(d+\cosh(\lambda\xi)-\sinh(\lambda\xi))\right\}}, \tag{3.2.45}$$

$$u_1^{26}(\xi) = \pm i e^{i\psi} \times \frac{\left(\lambda e_0 + 2f_1\nu\right) - \frac{4\lambda e_0(\cosh(\lambda\xi)+\sinh(\lambda\xi))}{d+\cosh(\lambda\xi)-\sinh(\lambda\xi)} - \frac{2f_1\nu(d+\cosh(\lambda\xi)-\sinh(\lambda\xi))}{\cosh(\lambda\xi)+\sinh(\lambda\xi)}}{4\left\{e_0 - \frac{f_1\nu(d+\cosh(\lambda\xi)-\sinh(\lambda\xi))}{\lambda(\cosh(\lambda\xi)+\sinh(\lambda\xi))}\right\}}, \tag{3.2.46}$$

where d is an arbitrary constant; $\psi = -\frac{cx^\beta}{2\beta} - \frac{(C^2+2\lambda^2)t^\alpha}{4\alpha}$, $\xi = \frac{x^\beta}{\beta} + \frac{ct^\alpha}{\alpha}$.

Solutions family 4: When $\nu \neq 0$ and $\mu = \lambda = 0$,

$$u_1^{27}(\xi) = \pm i e^{i\psi} \times \frac{f_1\nu - \frac{2e_0\nu}{\nu\xi+c_1}}{2\left\{e_0 - f_1\left(\nu\xi + c_1\right)\right\}}, \tag{3.2.47}$$

where c_1 is an arbitrary constant; $\psi = -\frac{cx^\beta}{2\beta} - \frac{C^2t^\alpha}{4\alpha}$, $\xi = \frac{x^\beta}{\beta} + \frac{ct^\alpha}{\alpha}$.

The above study of space and time fractional nonlinear Schrodinger equation affords plentiful traveling wave solutions in accurate form. The attained solutions are compared with the results reported in the literature. Instantly, Pandir and Duzgan have utilized the new type F-expansion method to disentangle the space–time fractional cubic Schrodinger equation and obtained ten analytic wave solutions (Pandir and Duzgun 2019), the same governing model has been investigated by Hemida et al. using homotopy analysis technique and originated some series solutions (Hemida et al. 2012) while only four solutions to the declared equation have been constructed by Salam et al. (2016). The comparable study certifies that our furnished solutions are further new and general with more free parameters which may be appeared in the literature for the first time.

4 Plot for wave structures of the gained solutions

The solutions of nonlinear evolution equations disclose the underlying structures of natural phenomena. Accordingly, the above constructed solutions to the considered equations are outlined to bring out their physical appearances. The plotted 3-dimensional representations stand for different wave profiles like kink type, singular kink type, cuspon, bell shape, anti-bell shape, compacton, anti-compacton, peakon, periodic etc. We give some of them as follows (Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12):

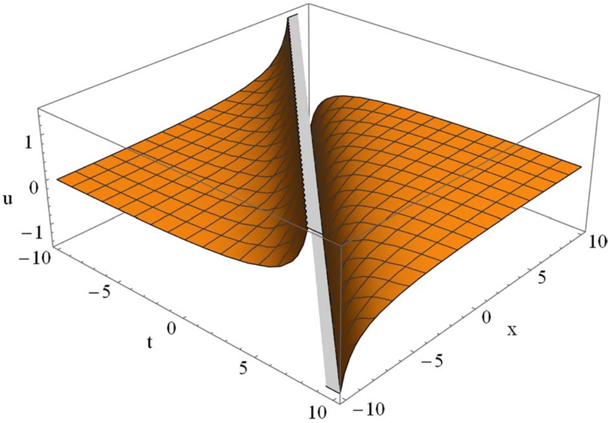


Fig. 1 The singular kink shape soliton of solution (3.1.14) for $\alpha = \nu = \eta = k = 1$, $\lambda = 2.2$, $\delta = c = -1$ and $\mu = 1.2$ within $-10 \le x, t \le 10$

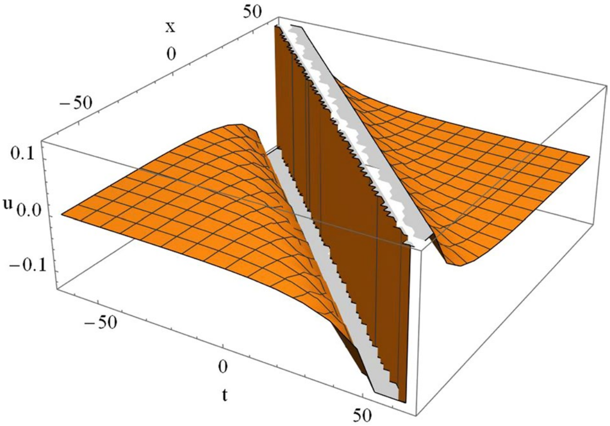


Fig. 2 The kink shape soliton of solution (3.1.17) for $\alpha = \mu = \nu = \eta = k = 1$, $\lambda = 2.001$ and $\delta = c = -1$ within $-65 \le x, t \le 65$

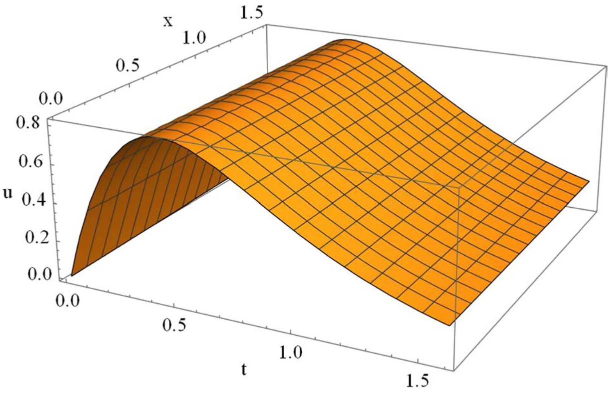


Fig. 3 The compacton soliton of solution (3.1.23) for $\alpha = \mu = \eta = 1$, $\lambda = 3$, $\nu = \delta = k = 2$ and $c = -0.01$ within $0 \le x, t \le 1.6$

5 Conclusions

The objective of the exploration was to investigate appropriate analytic wave solutions to

Fig. 4 The periodic soliton of solution (3.1.30) for $\alpha = \lambda = \mu = \nu = \eta = k = 1$, $\delta = -2$ and $c = -1$ within $-2 \leq \mathrm{x}, \mathrm{t} \leq 4$

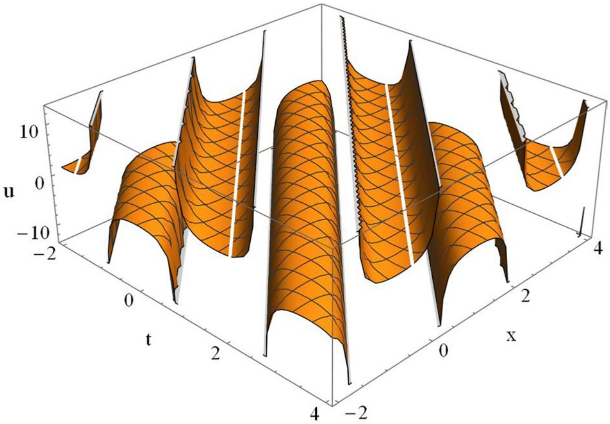


Fig. 5 The peakon soliton of solution (3.2.22) for $\alpha = \beta = \nu = c = k = e_0 = 1$, $\lambda = 1.1$, $\mu = -1$, $m = 2$ and $f_1 = -2$ within $-0.2 \leq \mathrm{x}, \mathrm{t} \leq 0.2$

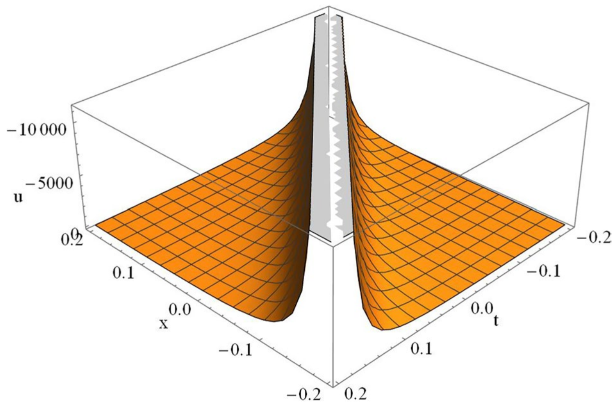
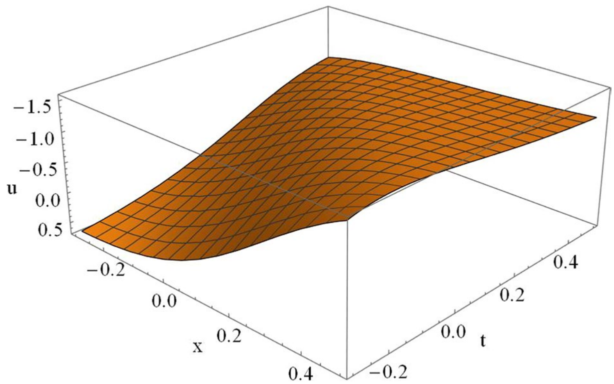


Fig. 6 The kink shape soliton of solution (3.2.23) for $\alpha = \beta = \nu = c = k = 1$, $\lambda = 2$, $\mu = -1$, $m = 0.1$, $e_0 = 0.5$ and $f_1 = -5$ within $-0.3 \leq \mathrm{x}, \mathrm{t} \leq 0.5$



the nonlinear space and time fractional mCH equation the space–time fraction nonlinear Schrodinger equation by utilizing the anticipated extended Riccati scheme. The directed technique has successfully offered attractive and interesting solutions to the suggested equations and has been ensured its high performance. The physical outlines of the achieved

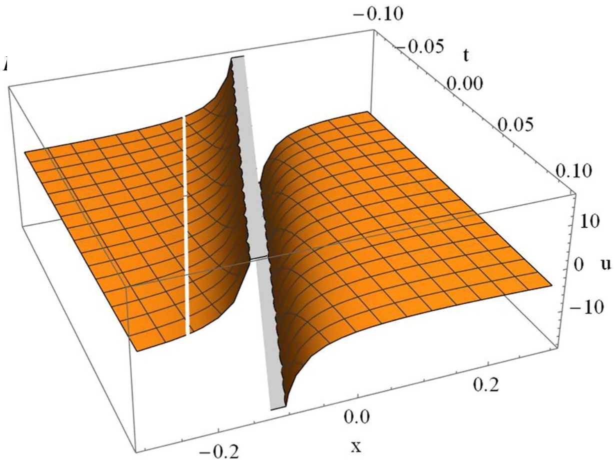


Fig. 7 The singular kink shape soliton of solution (3.2.26) for $\alpha = \beta = \nu = c = m = k = e_0 = f_1 = 1$, $\lambda = 3$, $\mu = -1$ and $A = 2$ within $-0.1 \le x \le 0.1$ and $-0.3 \le t \le 0.3$

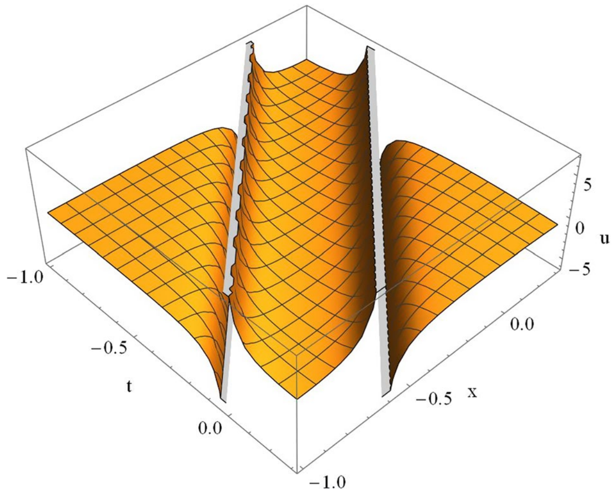


Fig. 8 The periodic soliton of solution (3.2.30) for $\alpha = \beta = \nu = c = m = k = e_0 = 1$, $\lambda = 0.5$, $\mu = -1$ and $f_1 = -0.99$ within $-1 \le x, t \le 0.3$

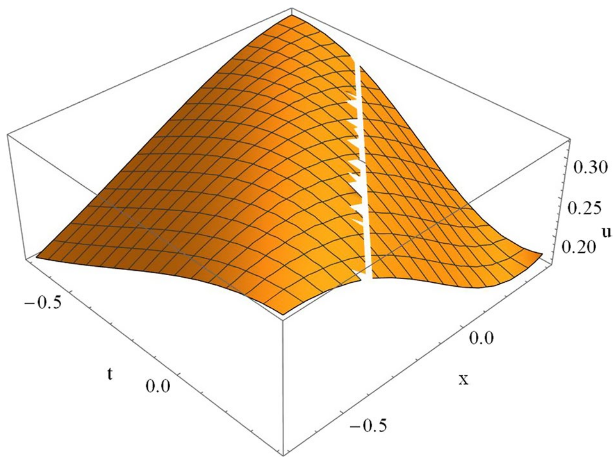


Fig. 9 The compacton soliton of solution (3.2.31) for $\alpha = \beta = \nu = c = m = k = 1$, $\lambda = 0.6$, $\mu = -1$, $e_0 = 0.5$ and $f_1 = 1.62$ within $-0.7 \le x, t \le 0.4$

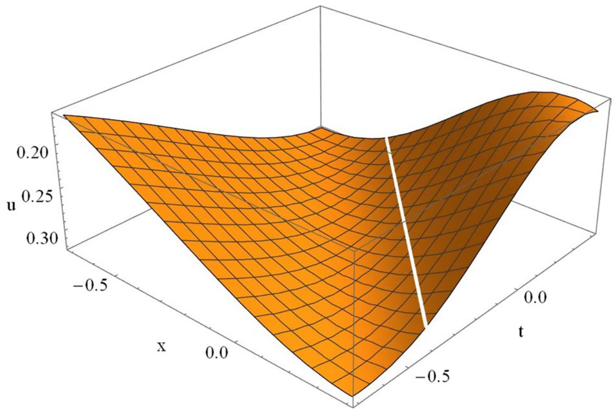


Fig. 10 The anti-compacton soliton of solution (3.2.32) for $\alpha = \beta = \nu = c = m = k = 1$, $\lambda = 0.6$, $\mu = -1$, $e_0 = 0.5$ and $f_1 = 1.62$ within $-0.7 \leq \mathrm{x}, \mathrm{t} \leq 0.4$

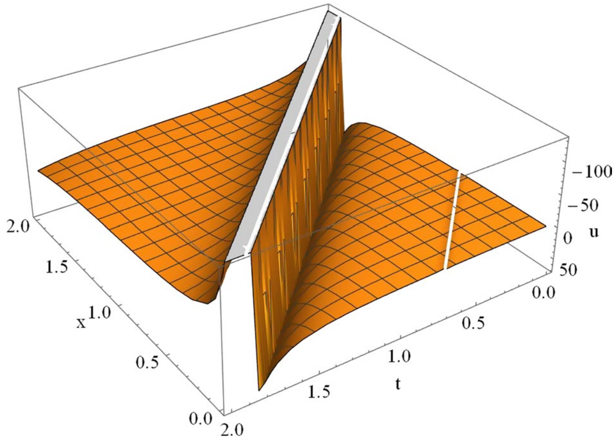


Fig. 11 The singular kink shape soliton of solution (3.2.33) for $\alpha = \beta = \mu = \nu = c = m = k = 1$, $\lambda = 1.1$, $e_0 = 1.2$ and $f_1 = -0.01$ within $0 \leq \mathrm{x}, \mathrm{t} \leq 2$

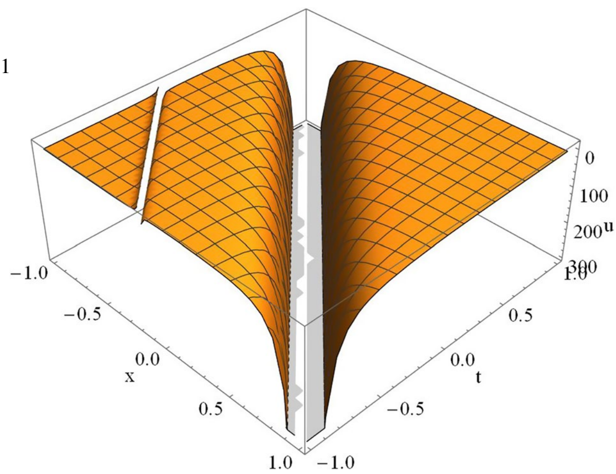


Fig. 12 The anti-bell shape soliton of solution (3.2.34) for $\alpha = \beta = \mu = \nu = c = m = k = f_1 ,= 1$ $\lambda = 1.1$ and $e_0 = 1.2$ within $-1 \leq \mathrm{x}, \mathrm{t} \leq 1$

solutions have been drawn in three-dimensional space which have been appeared in the shapes of kink, anti-kink, bell, anti-bell, compacton, anti-compacton, periodic etc. The obtained solutions comprehend numerous arbitrary parameters and assert to be diverse and novel which might be useful to analyze the underlying characteristics of complex physical phenomena and take significant place in the literature. So far, we have hunted the literature, the acquired solutions have not been reported earlier. This method is straightforward, competent and productive which might be considered for further execution to explore any other nonlinear evolution equations of fractional order arising in numerous fields of engineering science.

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Declarations

Conflict of interest The authors declare that there is no conflict of interests regarding the publication of this paper.

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