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New optical solitons based on the perturbed Chen-Lee-Liu model through Jacobi elliptic function method

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Abstract
In this study, we investigate the perturbed Chen-Lee-Liu equation that represents the propagation of an optical pulse in plasma and optical fiber. The Jacobi elliptic function technique is used for this purpose. As a result, we obtain some new solitary wave solutions such as the Jacobi elliptic function, dark-bright, trigonometric, exponential, hyperbolic, periodic, and singular soliton solutions. To express the pulse propagation of the generated solutions, specific values for the free parameters under conditions are also given.

Keywords Perturbed Chen-Lee-Liu model · Jacobi elliptic function technique · Analytical solutions

1 Introduction

Nonlinear partial differential equations are employed to examine the features of many physics models. One of these equations is the Schrödinger type equation. Such equation has a crucial role in fields such as mathematical physics, optic, plasma, and fiber-optic telecommunications engineering. Exact solutions of nonlinear Schrödinger's equation have an important role in the applied mathematics Ali et al. (2020); Eslami et al. (2014); He (2020); Zhang et al. (2017); Gao et al. (2020a, 2020b). There are several techniques have been developed to extract exact solutions for nonlinear partial differential equations such as the bilinear neural network method Zhang and Bilige (2019); Zhang et al. (2021, 2020), the Kudryashov method Zafar et al. (2022); Alquran et al. (2021); Srivastava et al. (2020); Sulaiman et al. (2021), the extended rational sin-cos and sinh-cosh methods Cinar et al. (2021), the sine-Gordon expansion method Ali et al. (2020a, 2020b); Fahim et al. (2021); Abdul Kayum et al. (2020), the unified method and its generalized technique Abdel-Gawad and Osman (2015); Abdel-Gawad et al. (2016); Osman and Abdel-Gawad (2015), the extended simple equation method Khater et al. (2021), the (G'/G)-expansion

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method Ismael et al. (2020); Durur (2020); Ekici et al. (2016), the Hirota bilinear method Abdulkadir Sulaiman and Yusuf (2021), the unified auxiliary equation method Zayed and Shohib (2019); Zayed et al. (2021), F-expansion method Biswas et al. (2019); Yıldırım (2021); Karaman (2021),the generalized exponential rational function method Khodadad et al. (2021), and other techniques Ali et al. (2020); Baskonus et al. (2018); Zhang et al. (2021a, 2021b); Ali et al. (2021); Rehman et al. (2020); Ali et al. (2020); Ozdemir et al. (2021). Additionally, many authors started up to employ the Jacobi elliptic function method Kurt (2019); Alquran and Jarrah (2019); Ali (2011); Ghanbari et al. (2021); Lü (2005); Zayed and Alurrfi (2015).

In this manuscript, we consider the perturbed Chen-Lee-Liu (CLL) model Ozdemir et al. (2021)

$$i\psi_t + \alpha\psi_{xx} + i\beta|\psi|^2\psi_x = i[\gamma\psi_x + \mu(|\psi|^{2n}\psi)_x + \delta(|\psi|^{2n})_x\psi], \tag{1}$$

where γ is the coefficient of inter-modal dispersion, μ and δ symbolize coefficients of self-steepening for short pulses and nonlinear dispersion, respectively. Additionally, the coefficients of the group velocity dispersion and the nonlinearity are refereed by α and β, respectively. We mention that n indicates the density for the complex wave function. The CLL equation drives solitons propagation dynamics in nonlinear optical fibers, and it also has applications in solitons cooling, optical couplers, meta-materials, and optoelectronic devices.

In this study, we investigate Eq. (1) at $n = 1$ which reads

$$i\psi_t + \alpha\psi_{xx} + i\beta|\psi|^2\psi_x = i[\gamma\psi_x + \mu(|\psi|^2\psi)_x + \delta(|\psi|^2)_x\psi]. \tag{2}$$

We establish a variety of optical solutions with the help of the Jacobi elliptic function method to the perturbed Chen-Lee-Liu equation, which depicts the propagation of an optical pulse through plasma and optical fiber. In Yıldırım et al. (2020), the Riccati method has been employed. Sardar subequation method used to find solitary wave solutions Esen et al. (2021). Zhang et al. have investigated qualitative analysis and the bifurcation method in Zhang et al. (2011). Apart from these, many studies have been made and continue to be done for the Chen-Lee-Liu equation Biswas (2018); Biswas et al. (2018); Triki et al. (2018); Yıldırım (2019). Akbar and others studied the Chen-Lee-Liu model via using different solutions functions with the help of the Jacobi elliptic functions Akinyemi et al. (2021). Kudryashov found general solutions by using different methods with elliptic function approach Kudryashov (2019).

The main goal of this paper is implementing firstly the Jacobi elliptic function method to obtain new solutions with different wave structures for Eq. (2).

This study is organized as follows, introduction is given in Sect. 1. In Sect. 2, we devoted ourselves to presenting the Jacobi elliptic functions method. In Sect. 3, we studied the new exact solutions of the perturbed Chen-Lee-Liu equation by applying the described method. The figures of the constructed solutions are drawn by the degenerate states of the Jacobi elliptic functions for $m \to 0$ and $m \to 1$. The conclusion of this study is presented in Sect. 4.

2 Description of the Jacobi elliptic function method

The nonlinear partial differential equation is expressed as:

$$P(\psi, \psi_x, \psi_t, \psi_{xx}, \psi_{tt}, \psi_{tx}, \cdots) = 0, \tag{3}$$

where P is a polynomial function containing $\psi(x,t)$ and its partial derivatives.

By taking the transformation

$$\psi(x,t) = U(v)e^{i\phi(x,t)}, v = x - \rho t, \phi(x,t) = -kx + wt + \eta, \tag{4}$$

Eq.(3) becomes a nonlinear ordinary differential equation (NODE) as follows

$$F(U', U'', U''', \cdots) = 0. \tag{5}$$

Here, $U(v)$, $\phi(x,t)$, ρ, k, w and η stand for the amplitude competent, phase function, speed of the soliton, frequency, wave number, and phase, respectively. Then, The following steps are followed to construct the solutions:

Step 1 The solution of the NODE is as follows:

$$U(v) = g_0 + \sum_{i=1}^{D} (\frac{z(v)}{1+z(v)^2})^{i-1} \left(g_i \frac{z(v)}{1+z(v)^2} + f_i \frac{1-z(v)^2}{1+z(v)^2} \right), \tag{6}$$

where g_i, and f_i are constants ($g_D \neq 0$ or $f_D \neq 0$). The $z(v)$ function is expressed as:

$$z'(v) = \sqrt{s + cz^2(v) + rz^4(v)}, \tag{7}$$

where s, c and r are constants.

Step 2 The value of D is defined by the balance principle which depends on the comparison between the highest-order linear term and the non-linear term in Eq. (5).

Step 3 Substituting Eq. (6) along with Eq. (7) into Eq. (5), we get a polynomial in $z(v)$. Afterthat, we obtain a system of algabraic equations by setting the coefficients of $z^b(v)$, $b = 0, \cdots, 7$ equal to zero. We solve the obtained system with the help of mathematica software to find the unknown parameters.

Step 4 The general solutions of Eq. (6) are as follows according to the conditions of s, c and r in Table 1:

In Table 2, we express to change hyperbolic and trigonometric functions for $m \to 0$ and $m \to 1$ of the Jacobi elliptic functions. Thereby, solutions are structure of trigonometric and hyperbolic functions.

3 Applications

In this section, we apply the Jacobi elliptic function method to Eq.(2). Firstly, by inserting Eq. (4) into Eq.(2)

$$\begin{aligned} &-i\rho U' - wU + \alpha U'' - 2k\alpha iU' - \alpha k^2 U + i\beta U^2 U' \\ &+ \beta k U^3 - i\gamma U' - \gamma k U - 3i\mu U^2 U' - \mu k U^3 - 2i\delta U^2 U' = 0, \end{aligned} \tag{8}$$

is attain. We express the following parts of Eq.(8) as follows. The real part is

$$(-w - \alpha k^2 - \gamma k)U + \alpha U'' + k(\beta - \mu)U^3 = 0, \tag{9}$$

Table 1 Jacobi elliptic functions

No.	s	c	r	$z(v)$
1	1	$-1-m^2$	m^2	$sn(v)$
2	$1-m^2$	$2m^2-1$	$-m^2$	$cn(v)$
3	m^2	$-1-m^2$	1	$ns(v)$
4	$-m^2$	$-1+2m^2$	$1-m^2$	$nc(v)$
5	$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	$ns(v) \mp cs(v)$
6	$\frac{1-m^2}{4}$	$\frac{1+m^2}{2}$	$\frac{1-m^2}{4}$	$nc(v) \mp sc(v)$ or $\frac{cn(v)}{1 \mp sn(v)}$
7	$\frac{1}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$\frac{sn(v)}{1 \mp dn(v)}$
8	1	$2-m^2$	$1-m^2$	$sc(v)$
9	$1-m^2$	$2-m^2$	1	$cs(v)$
10	m^2-1	$2-m^2$	-1	$dn(v)$
11	$\frac{m^4}{4}$	$\frac{m^2-2}{2}$	$\frac{1}{4}$	$ns(v) \mp ds(v)$
12	$\frac{1}{4}$	$\frac{1+m^2}{2}$	$\frac{(1-m^2)^2}{4}$	$\frac{sn(v)}{dn(v) \mp cn(v)}$

Table 2 Jacobi elliptic functions for $m \to 0$ and $m \to 1$

		$m \to 0$	$m \to 1$			$m \to 0$	$m \to 1$
1	$sn(v)$	$sin(v)$	$tanh(v)$	7	$dc(v)$	$sec(v)$	1
2	$cn(v)$	$cos(v)$	$sech(v)$	8	$nc(v)$	$sec(v)$	$cosh(v)$
3	$dn(v)$	1	$sech(v)$	9	$sc(v)$	$tan(v)$	$sinh(v)$
4	$cd(v)$	$cos(v)$	1	10	$ns(v)$	$csc(v)$	$coth(v)$
5	$sd(v)$	$sin(v)$	$sinh(v)$	11	$ds(v)$	$csc(v)$	$csch(v)$
6	$nd(v)$	1	$cosh(v)$	12	$cs(v)$	$cot(v)$	$csch(v)$

and the imaginary part is

$$(-\rho - 2k\alpha - \gamma)U' + (\beta - 3\mu - 2\delta)U^2U' = 0. \tag{10}$$

Then, we set the coefficients of the components of the imaginary part equal to zero, we get $\rho = -2k\alpha - \gamma$ and $\beta = 3\mu + 2\delta$. Considering these two constraints in the real part, we get

$$(-w - \alpha k^2 - \gamma k)U + \alpha U'' + 2k(\mu + \delta)U^3 = 0. \tag{11}$$

By using the balance principle, we get $D = 1$. Considering Eq. (6), we may express the solution of Eq. (11) as below:

$$U(v) = g_0 + g_1 \frac{z(v)}{1 + z(v)^2} + f_1 \frac{1 - z(v)^2}{1 + z(v)^2}. \tag{12}$$

If Eq. (12) is substituted in Eq. (11), we find the solutions by taking into account the equation system obtained for the following conditions by performing the necessary operations (Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9).

Result 1: Considering the case $s = 1$, $r = m^2$, $c = -1 - m^2$ and $z(v) = sn(v, m)$.

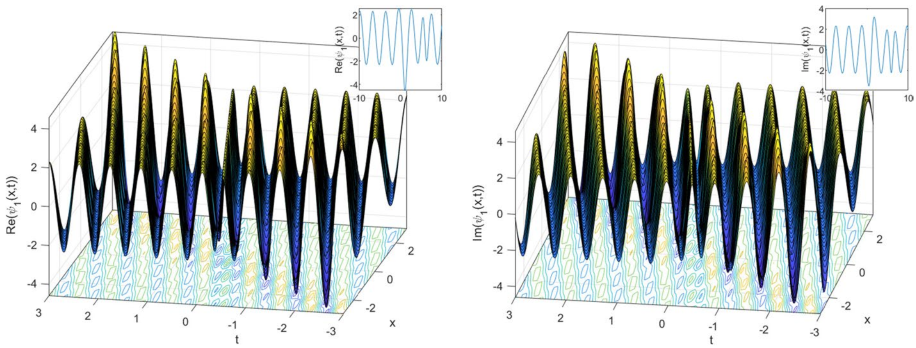

Fig. 1 Dark-bright solitary wave solution of Eq. (13) for values of $\alpha = -1$, $k = 2$, $\eta = 0.05$, $\delta = 0.01$,$\gamma = -2$,$f_1 = 2.3$, $w = -2\alpha - k^2\alpha - k\gamma$, $\mu = \frac{\alpha - kf_1^2\delta}{kf_1^2}$, $\beta = 3\mu + 2\delta$, and $\rho = -2k\alpha - \gamma$

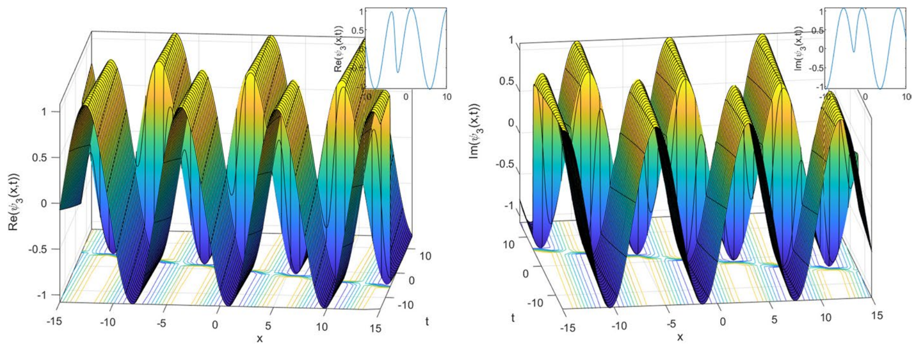

Fig. 2 Singular soliton solution of Eq. (15)for values of $\alpha = -0.25$, $k = 0.7$, $\eta = 0.9$,$\mu = 1.2$, $\delta = 0.02$, $\gamma = 3$, $w = -8\alpha - k^2\alpha - k\gamma$, $\beta = 3\mu + 2\delta$, and $\rho = -2k\alpha - \gamma$

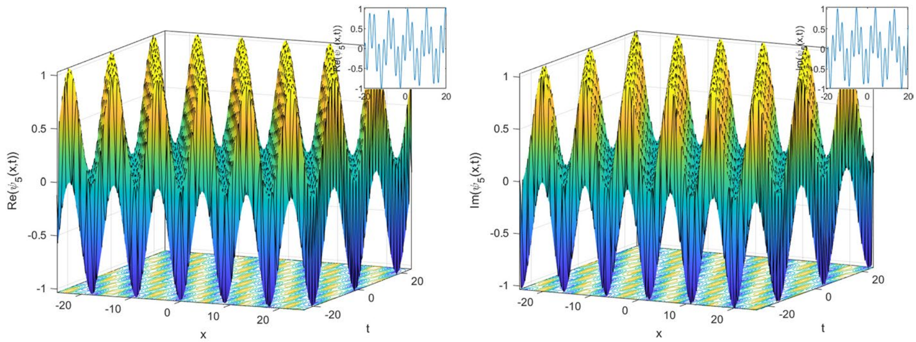

Fig. 3 Periodic solution of Eq. (17) for values of $\alpha = -0.6$, $k = -1.7$, $\eta = 1.9$, $\mu = 0.75$, $\delta = -\mu$, $\gamma = 0.04$, $g_1 = 0.3$, $f_1 = 1.02$, $w = -\alpha - k^2\alpha - k\gamma$, $\beta = 3\mu + 2\delta$, and $\rho = -2k\alpha - \gamma$

When $g_0 = 0$, $g_1 = \mp 2if_1$, $f_1 = f_1$, $\mu = \frac{\alpha - kf_1^2\delta}{kf_1^2}$ and $w = -2\alpha - k^2\alpha - k\gamma$. For $m \to 1$, we have $z(v) \to \tanh(v)$ and the general solution is given by

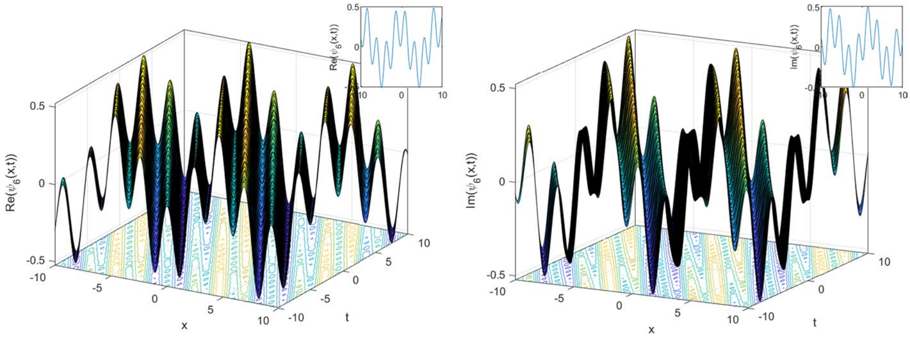

Fig. 4 Periodic solution of Eq. (18) for values of $\alpha = -0.06$, $k = -1.7$, $\eta = 1.2$, $\mu = 0.75$, $\delta = -\mu$, $\gamma = 0.4$, $g_1 = 0.3$, $f_1 = 0.5$, $w = -\alpha - k^2\alpha - k\gamma$, $\beta = 3\mu + 2\delta$ and $\rho = -2k\alpha - \gamma$

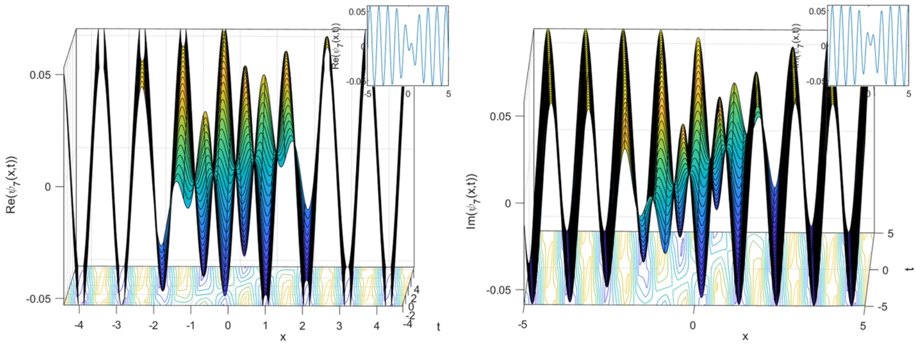

Fig. 5 Hyperbolic function solutions of Eq. (19) for values of $\alpha = 0.06$, $k = -5.7$, $\eta = 1.5$, $\mu = 1$, $\delta = 2.09$,$\gamma = 0.4$, $w = -2\alpha - k^2\alpha - k\gamma$, $\beta = 3\mu + 2\delta$, and $\rho = -2k\alpha - \gamma$

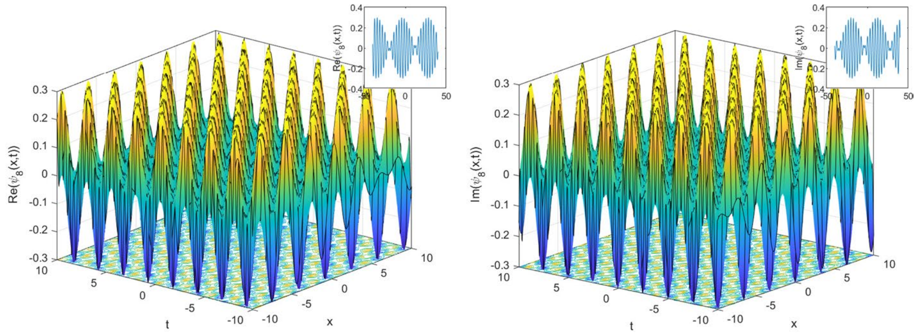

Fig. 6 Periodic solution of Eq. (20) for values of $\alpha = 2.06$, $k = 0.1$, $\eta = 5$, $\mu = -\delta$, $\delta = 0.25$, $\gamma = 2.6$, $f_1 = 0.3$, $w = -2\alpha - k^2\alpha - k\gamma$, $\beta = 3\mu + 2\delta$, and $\rho = -2k\alpha - \gamma$

$$\psi_1(x,t) = (-if_1 \tanh(2x - 2\rho t) + f_1 \operatorname{sech}(2x - 2\rho t))e^{i(-kx+wt+\eta)}, \tag{13}$$

which is a dark-bright solitary wave solution.

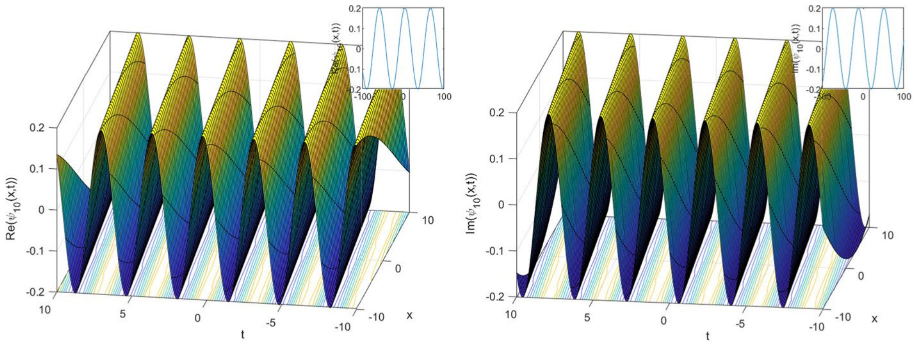

Fig. 7 Exponential solution of Eq. (22) for values of $\alpha = 1.06$, $k = 0.1$, $\eta = 0.1$, $\mu = \frac{-\alpha - kf_1^2\delta}{kf_1^2}$, $\delta = 1$, $\gamma = 2.6$, $f_1 = 0.2$, $w = 2\alpha - k^2\alpha - k\gamma$, $\beta = 3\mu + 2\delta$, and $\rho = -2k\alpha - \gamma$

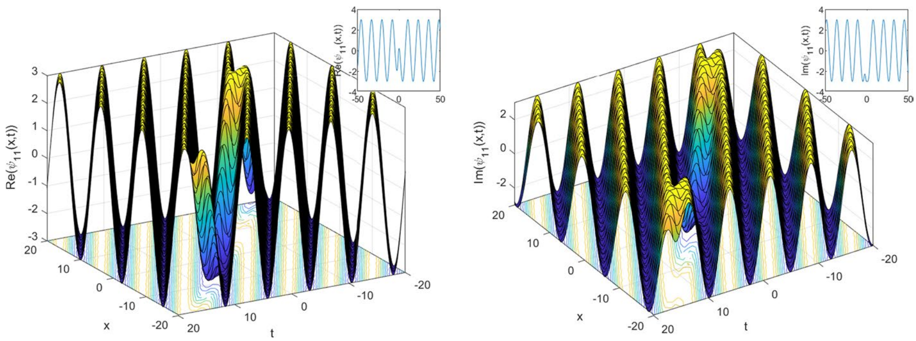

Fig. 8 Singular solitary wave solution of Eq. (23) for values of $\alpha = 1$, $k = 0.5$, $\eta = 1.5$, $\mu = \frac{\alpha - kf_1^2\delta}{kf_1^2}$, $\delta = 0.3$, $\gamma = 0.2$, $f_1 = 3$, $w = \frac{1}{2}(-\alpha - 2k^2\alpha - 2k\gamma)$, $\beta = 3\mu + 2\delta$ and $\rho = -2k\alpha - \gamma$

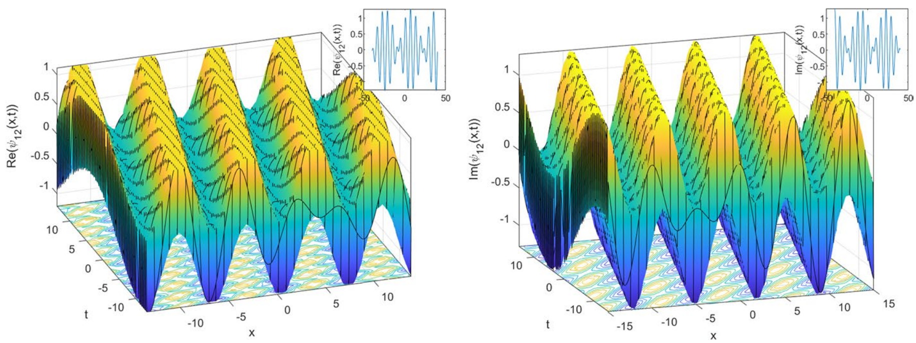

Fig. 9 Trigonometric function solution of Eq. (24) for values of $\alpha = 6.02$, $k = 0.1$, $\eta = 1.05$, $\mu = -\delta$, $\delta = 0.3$, $\gamma = 5.2$, $f_1 = 1$, $w = -\alpha - k^2\alpha - k\gamma$, $\beta = 3\mu + 2\delta$, and $\rho = -2k\alpha - \gamma$

Result 2: For $s = 1 - m^2$, $r = -m^2$, $c = -1 + 2m^2$ and $z(v) = cn(v, m)$.
When $g_0 = \frac{f_1}{2}$, $g_1 = 0$, $f_1 = f_1$, $m = \frac{1}{2}$, $\mu = \frac{\alpha - kf_1^2\delta}{kf_1^2}$, and $w = \frac{1}{2}(5\alpha - 2k^2\alpha - 2k\gamma)$, we have

$$\psi_2(x,t)=\left(\frac{f_1}{2}+\frac{f_1(1-(cn(x-\rho t))^2)}{(1+(cn(x-\rho t))^2)}\right)\mathrm{e}^{i(-kx+wt+\eta)}. \tag{14}$$

Result 3: Considering the case $s=m^2, r=1, c=-1-m^2$ and $z(v)=ns(v,m)$,

When $g_0=0$, $g_1=\mp\frac{4\sqrt{\alpha}}{\sqrt{-k\delta-k\mu}}$, $f_1=0$, $\mu=\frac{\alpha-kf_1^2\delta}{kf_1^2}$ and $w=-8\alpha-k^2\alpha-k\gamma$. For $m\to 1$, we have $z(v)\to\coth(v)$ and the general solution is given by

$$\psi_3(x,t)=\left(\mp\frac{4\sqrt{\alpha}}{\sqrt{-k\delta-k\mu}}\frac{\coth(x-\rho t)}{(1+(\coth(x-\rho t))^2)}\right)\mathrm{e}^{i(-kx+wt+\eta)}, \tag{15}$$

which represents a singular soliton solution under conditions $\alpha k(\delta+\mu)<0$.

Result 4: For $s=-m^2, r=1-m^2, c=-1+2m^2$ and $z(v)=nc(v,m)$.

$g_0=\frac{f_1}{2}$, $g_1=0$, $f_1=f_1$, $m=\frac{\sqrt{3}}{2}$, $\mu=\frac{\alpha-kf_1^2\delta}{kf_1^2}$ and $w=\frac{1}{2}(-5\alpha-2k^2\alpha-2k\gamma)$, we have

$$\psi_4(x,t)=\left(\frac{f_1}{2}+\frac{f_1(1-(nc(x-\rho t))^2)}{(1+(nc(x-\rho t))^2)}\right)\mathrm{e}^{i(-kx+wt+\eta)}. \tag{16}$$

Result 5: For $s=\frac{1}{4}, r=\frac{1}{4}, c=\frac{1-2m^2}{2}$ and $z(v)=ns(v,m)\mp cs(v,m)$.

When $g_0=0$, $g_1=g_1$, $f_1=f_1$, $\mu=-\delta$ and $w=-\alpha-k^2\alpha-k\gamma$. For $m\to 0$, we have $z(v)\to\csc(v)\mp\cot(v)$. Thus, we get

$$\psi_5(x,t)=\left(g_1\frac{\cot(v)\mp\csc(v)}{1+(\cot(x-\rho t)\mp\csc(x-\rho t))^2}+f_1\frac{(1-(\cot(x-\rho t)\mp\csc(x-\rho t))^2)}{(1+(\cot(x-\rho t)\mp\csc(x-\rho t))^2)}\right)\mathrm{e}^{i(-kx+wt+\eta)}, \tag{17}$$

which is a periodic solution.

Result 6: For $s=\frac{1-m^2}{4}, r=\frac{1-m^2}{4}, c=\frac{1+m^2}{2}$ and $z(v)=nc(v,m)\mp sc(v,m)$.

When $g_0=0$, $g_1=g_1$, $f_1=f_1$, $\mu=-\delta$ and $w=-\alpha-k^2\alpha-k\gamma$. For $m\to 0$, $z(v)\to\sec(v)\mp\tan(v)$ and the solution is given by

$$\psi_6(x,t)=\left(g_1\frac{\sec(v)\mp\tan(v)}{1+(\sec(v)\mp\tan(v))^2}+f_1\frac{(1-(\sec(v)\mp\tan(v))^2)}{(1+(\sec(v)\mp\tan(v))^2)}\right)\mathrm{e}^{i(-kx+wt+\eta)}, \tag{18}$$

which is a periodic solution.

Result 7: For $s=\frac{1}{4}, r=\frac{m^2}{4}, c=\frac{m^2-2}{2}$ and $z(v)=\frac{sn(v,m)}{1\mp dn(v,m)}$.

When $g_0=0$, $g_1=\frac{\mp 2\sqrt{\alpha}}{\sqrt{-k\delta-k\mu}}$, $f_1=0$, $\mu=-\delta$ and $w=-2\alpha-k^2\alpha-k\gamma$. For $m\to 1$, $z(v)\to\frac{\tanh(v)}{1\mp\mathrm{sech}(v,m)}$, we have

$$\psi_7(x,t)=\left(\frac{\mp 2\sqrt{\alpha}}{\sqrt{k\delta-k\mu}}\frac{\sinh(x-\rho t)}{1\mp\sinh(x-\rho t)}\right)\mathrm{e}^{i(-kx+wt+\eta)}, \tag{19}$$

which is a hyperbolic function solution under conditions $\alpha k(\delta+\mu)<0$.

Result 8: For $s=1, r=1-m^2, c=2-m^2$ and $z(v)=sc(v,m)$.

When $g_0=0$, $g_1=0$, $f_1=f_1$, $\mu=-\delta$ and $w=-4\alpha-k^2\alpha-k\gamma$. For $m\to 0$, $z(v)=\to\tan(v)$ and we get

$$\psi_8(x,t)=\left(f_1\frac{1-(\tan(x-\rho t))^2}{1+(\tan(x-\rho t))^2}\right)\mathrm{e}^{i(-kx+wt+\eta)}, \tag{20}$$

which is a periodic solution.

Result 9: For $s = 1 - m^2, r = 1, c = 2 - m^2$ and $z(v) = cs(v, m)$.

$g_0 = 0$, $g_1 = 0$, $f_1 = f_1$, $\mu = -\delta$ and $w = -4\alpha - k^2\alpha - k\gamma$. For $m \to 0$, $z(v) \to \cot(v)$ and we get

$$\psi_9(x,t) = \left(f_1 \frac{1 - (\cot(x-\rho t))^2}{1 + (\cot(x-\rho t))^2}\right) e^{i(-kx+wt+\eta)}, \tag{21}$$

which is a periodic solution.

Result 10: For $s = m^2 - 1, r = -1, c = 2 - m^2$ and $z(v) = dn(v, m)$.

When $g_0 = 0$, $g_1 = \mp 2if_1$, $f_1 = f_1$, $\mu = \frac{-\alpha - kf_1^2\delta}{kf_1^2}$ and $w = 2\alpha - k^2\alpha - k\gamma$. For $m \to 0$, $z(v) \to 1$. Thus, we have

$$\psi_{10}(x,t) = (\mp if_1) e^{i(-kx+wt+\eta)}, \tag{22}$$

which is an exponential function solution.

Result 11: For $s = \frac{m^4}{4}, r = \frac{1}{4}, c = \frac{m^2-2}{2}$ and $z(v) = ns(v,m) \mp ds(v,m)$.

When $g_0 = 0$, $g_1 = \mp 2if_1$, $f_1 = f_1$, $\mu = \frac{\alpha - kf_1^2\delta}{kf_1^2}$ and $w = \frac{1}{2}(-\alpha - 2k^2\alpha - 2k\gamma)$. For $m \to 1$, $z(v) = \coth(v) \mp \mathrm{csch}(v)$. So, we have

$$\psi_{11}(x,t) = \left(\mp 2if_1\left(\frac{\coth(x-\rho t) \mp \csc h(x-\rho t)}{1 + (\coth(v) \mp \mathrm{csch}(v))^2}\right) + f_1 \frac{1 - (\coth(x-\rho t) \mp \mathrm{csch}(x-\rho t))^2}{1 + (\coth(c-\rho t) \mp \mathrm{csch}(x-\rho t))^2}\right) e^{i(-kx+wt+\eta)}, \tag{23}$$

which represents a singular solitary wave solution.

Result 12: For $s = \frac{1}{4}, r = \frac{(1-m^2)^2}{4}, c = \frac{1+m^2}{2}$ and $z(v) = \frac{sn(v)}{dn(v) \mp cn(v)}$.

When $g_0 = 0$, $g_1 = 0$, $f_1 = f_1$, $\mu = -\delta$ and $w = -\alpha - k^2\alpha - k\gamma$. For $m \to 1$, $z(v) \to \frac{\sin(v)}{1 \mp \cos(v)}$ and we have

$$\psi_{12}(x,t) = \left(f_1 \frac{1 - \left(\frac{\sin(x-\rho t)}{1 \mp \cos(x-\rho t)}\right)^2}{1 + \left(\frac{\sin(x-\rho t)}{1 \mp \cos(x-\rho t)}\right)^2}\right) e^{i(-kx+wt+\eta)}, \tag{24}$$

which is a trigonometric function solution.

4 Conclusions

In this article, we have found several novel solutions to the perturbed Chen-Lee-Liu equation by using the Jacobi elliptic function method. These solutions are Jacobi elliptic function, dark-bright, trigonometric, exponential, hyperbolic, periodic, and singular soliton solutions. The constraint conditions are determined to vouch the existence of valid solutions. For some values of free parameters, the 2D and 3D graphs to some of the obtained solutions are depicted. The obtained results can be effective in interpreting the physical meaning of this nonlinear system. The Jacobi elliptic function method is a powerful mathematical technique which can be utilized to acquire the analytical solutions to different complex nonlinear mathematical models.

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References

Abdel-Gawad, H.I., Osman, M.: On shallow water waves in a medium with time-dependent dispersion and nonlinearity coefficients. J. Adv. Res. **6**(4), 593–599 (2015)

Abdel-Gawad, H.I., Tantawy, M., Osman, M.S.: Dynamic of DNA's possible impact on its damage. Math. Methods Appl. Sci. **39**(2), 168–176 (2016)

Abdul Kayum, M., Ali Akbar, M., Osman, M.S.: Stable soliton solutions to the shallow water waves and ion-acoustic waves in a plasma. Waves in Random and Complex Media (2020). https://doi.org/10.1080/17455030.2020.1831711

Abdulkadir Sulaiman, T., Yusuf, A.: Dynamics of lump-periodic and breather waves solutions with variable coefficients in liquid with gas bubbles. Waves in Random and Complex Media (2021). https://doi.org/10.1080/17455030.2021.1897708

Akinyemi, L., Ullah, N., Akbar, Y., Hashemi, M.S., Akbulut, A., Rezazadeh, H.: Explicit solutions to nonlinear Chen-Lee-Liu equation. Modern Phys Letts B **35**(25), 2150438 (2021)

Ali, A.T.: New generalized Jacobi elliptic function rational expansion method. J. Comput. Appl. Math. **235**(14), 4117–4127 (2011)

Ali, K.K., Wazwaz, A.M., Osman, M.S.: Optical soliton solutions to the generalized nonautonomous nonlinear Schrödinger equations in optical fibers via the sine-Gordon expansion method. Optik **208**, 164132 (2020)

Ali, K.K., Osman, M.S., Abdel-Aty, M.: New optical solitary wave solutions of Fokas-Lenells equation in optical fiber via Sine-Gordon expansion method. Alexandria Eng. J. **59**(3), 1191–1196 (2020)

Ali, K.K., Yilmazer, R., Yokus, A., Bulut, H.: Analytical solutions for the (3+1)-dimensional nonlinear extended quantum Zakharov-Kuznetsov equation in plasma physics. Phys. A: Stat. Mech. Appl. **548**, 124327 (2020)

Ali, K.K., Yilmazer, R., Baskonus, H.M., Bulut, H.: Modulation instability analysis and analytical solutions to the system of equations for the ion sound and Langmuir waves. Physica Scripta **95**(6), 065602 (2020)

Ali, K.K., Yilmazer, R., Yokus, A., Bulut, H.: Analytical solutions for the (3+ 1)-dimensional nonlinear extended quantum Zakharov-Kuznetsov equation in plasma physics. Phys. A Stat. Mech. Appl. **548**, 124327 (2020)

Ali, K.K., Yilmazer, R., Bulut, H., Aktürk, T., Osman, M.S.: Abundant exact solutions to the strain wave equation in micro-structured solids. Modern Phys. Letts. B **35**(26), 2150439 (2021)

Alquran, M., Jarrah, A.: Jacobi elliptic function solutions for a two-mode KdV equation. J. King Saud Univ.-Sci. **31**(4), 485–489 (2019)

Alquran, M., Sulaiman, T.A., Yusuf, A.: Kink-soliton, singular-kink-soliton and singular-periodic solutions for a new two-mode version of the Burger-Huxley model: applications in nerve fibers and liquid crystals. Opt. Quantum Electron **53**(5), 227 (2021)

Baskonus, H.M., Sulaiman, T.A., Bulut, H.: On the exact solitary wave solutions to the long-short wave interaction system. In ITM Web of Conferences (22, 01063). EDP Sciences (2018)

Biswas, A.: Chirp-free bright optical soliton perturbation with Chen-Lee-Liu equation by traveling wave hypothesis and semi-inverse variational principle. Optik **172**, 772–776 (2018)

Biswas, A., Ekici, M., Sonmezoglu, A., Alshomrani, A.S., Zhou, Q., Moshokoa, S.P., Belic, M.: Chirped optical solitons of Chen-Lee-Liu equation by extended trial equation scheme. Optik **156**, 999–1006 (2018)

Biswas, A., Sonmezoglu, A., Ekici, M., Alshomrani, A.S., Belic, M.R.: Optical solitons with Kudryashov's equation by F-expansion. Optik **199**, 163338 (2019)

Cinar, M., Onder, I., Secer, A., Yusuf, A., Sulaiman, T.A., Bayram, M., Aydin, H.: The analytical solutions of Zoomeron equation via extended rational sin-cos and sinh-cosh methods. Physica Scripta **96**(9), 094002 (2021)

Durur, H.: Different types analytic solutions of the (1+ 1)-dimensional resonant nonlinear Schrödinger's equation using (G'/G)-expansion method. Modern Phys. Letts. B **34**(03), 2050036 (2020)

Ekici, M., Mirzazadeh, M., Sonmezoglu, A., Zhou, Q., Moshokoa, S.P., Biswas, A., Belic, M.: Dark and singular optical solitons with Kundu-Eckhaus equation by extended trial equation method and extended (G'/G)-expansion scheme. Optik **127**(22), 10490–10497 (2016)

Esen, H., Ozdemir, N., Secer, A., Bayram, M.: On solitary wave solutions for the perturbed Chen-Lee-Liu equation via an analytical approach. Optik **245**, 167641 (2021)

Eslami, M., Mirzazadeh, M., Vajargah, B.F., Biswas, A.: Optical solitons for the resonant nonlinear Schrödinger's equation with time-dependent coefficients by the first integral method. Optik **125**(13), 3107–3116 (2014)

Fahim, M.R.A., Kundu, P.R., Islam, M.E., Akbar, M.A., Osman, M.S.: Wave profile analysis of a couple of (3+1)-dimensional nonlinear evolution equations by sine-Gordon expansion approach. Journal of Ocean Engineering and Science (2021). https://doi.org/10.1016/j.joes.2021.08.009
Gao, W., Ismael, H.F., Husien, A.M., Bulut, H., Baskonus, H.M.: Optical soliton solutions of the cubic-quartic nonlinear Schrödinger and resonant nonlinear Schrödinger equation with the parabolic law. Appl. Sci. **10**(1), 219 (2020)
Gao, W., Ismael, H.F., Bulut, H., Baskonus, H.M.: Instability modulation for the (2+ 1)-dimension paraxial wave equation and its new optical soliton solutions in Kerr media. Physica Scripta **95**(3), 035207 (2020)
Ghanbari, B., Kumar, S., Niwas, M., Baleanu, D.: The Lie symmetry analysis and exact Jacobi elliptic solutions for the Kawahara-KdV type equations. Results Phys. **23**, 104006 (2021)
He, J.H.: Variational principle and periodic solution of the Kundu-Mukherjee-Naskar equation. Results Phys **17**, 103031 (2020)
Ismael, H.F., Bulut, H., Baskonus, H.M.: Optical soliton solutions to the Fokas-Lenells equation via sine-Gordon expansion method and $(m + (G'/G))$-expansion method. Pramana **94**(1), 35 (2020)
Karaman, B.: The use of improved-F expansion method for the time-fractional Benjamin-Ono equation. Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas, 115(3), 1-7 (2021)
Khater, M., Jhangeer, A., Rezazadeh, H., Akinyemi, L., Akbar, M.A., Inc, M., Ahmad, H.: New kinds of analytical solitary wave solutions for ionic currents on microtubules equation via two different techniques. Optical Quantum Electron **53**(11), 609 (2021)
Khodadad, F.S., Mirhosseini-Alizamini, S.M., Günay, B., Akinyemi, L., Rezazadeh, H., Inc, M.: Abundant optical solitons to the Sasa-Satsuma higher-order nonlinear Schröinger equation. Optical Quantum Electron **53**(12), 702 (2021)
Kudryashov, N.A.: General solution of the traveling wave reduction for the perturbed Chen-Lee-Liu equation. Optik **186**, 339–349 (2019)
Kurt, A.: New periodic wave solutions of a time fractional integrable shallow water equation. Appl Ocean Res **85**, 128–135 (2019)
Lü, D.: Jacobi elliptic function solutions for two variant Boussinesq equations. Chaos Solitons Fractals **24**(5), 1373–1385 (2005)
Osman, M.S., Abdel-Gawad, H.I.: Multi-wave solutions of the (2+ 1)-dimensional Nizhnik-Novikov-Veselov equations with variable coefficients. Euro. Phys. J. Plus **130**(10), 215 (2015)
Ozdemir, N., Esen, H., Secer, A., Bayram, M., Sulaiman, T.A., Yusuf, A., Aydin, H.: Optical solitons and other solutions to the Radhakrishnan-Kundu-Lakshmanan equation. Optik **242**, 167363 (2021)
Ozdemir, N., Esen, H., Secer, A., Bayram, M., Yusuf, A., Sulaiman, T.A.: Optical Soliton Solutions to Chen Lee Liu model by the modified extended tanh expansion scheme. Optik **245**, 167643 (2021)
Rehman, S.U., Yusuf, A., Bilal, M., Younas, U., Younis, M., Sulaiman, T.A.: Application of (G'/G^2)-expansion method to microstructured solids, magneto-electro-elastic circular rod and (2+ 1)-dimensional nonlinear electrical lines. Math. Eng. Sci. Aerospace **11**(4), 789–803 (2020)
Srivastava, H.M., Baleanu, D., Machado, J.A.T., Osman, M.S., Rezazadeh, H., Arshed, S., Günerhan, H.: Traveling wave solutions to nonlinear directional couplers by modified Kudryashov method. Physica Scripta **95**(7), 075217 (2020)
Sulaiman, T.A., Yusuf, A., Tchier, F., Inc, M., Tawfiq, F.M.O., Bousbahi, F.: Lie-Bäcklund symmetries, analytical solutions and conservation laws to the more general (2+ 1)-dimensional Boussinesq equation. Results Phys. **22**, 103850 (2021)
Triki, H., Hamaizi, Y., Zhou, Q., Biswas, A., Ullah, M.Z., Moshokoa, S.P., Belic, M.: Chirped dark and gray solitons for Chen-Lee-Liu equation in optical fibers and PCF. Optik **155**, 329–333 (2018)
Yıldırım, Y., Biswas, A., Asma, M., Ekici, M., Ntsime, B.P., Zayed, E.M., Moshokoa, S.P., Alzahrani, A.K., Belic, M.R.: Optical soliton perturbation with Chen-Lee-Liu equation. Optik **220**, 165177 (2020)
Yıldırım, Y.: Optical solitons to Chen-Lee-Liu model with trial equation approach. Optik **183**, 849–853 (2019)
Yıldırım, Y.: Optical solitons with Biswas-Arshed equation by F-expansion method. Optik **227**, 165788 (2021)
Zafar, A., Shakeel, M., Ali, A., Akinyemi, L., Rezazadeh, H.: Optical solitons of nonlinear complex Ginzburg-Landau equation via two modified expansion schemes. Optic Quantum Electron **54**(1), 5 (2022)
Zayed, E.M.E., Alurrfi, K.A.E.: A new Jacobi elliptic function expansion method for solving a nonlinear PDE describing the nonlinear low-pass electrical lines. Chaos Solitons Fractals **78**, 148–155 (2015)
Zayed, E.M.E., Shohib, R.M.A.: Solitons and Other Solutions for Two Higher-Order Nonlinear Wave Equations of KdV Type Using the Unified Auxiliary Equation Method. Acta Physica Polonica A **136**(1), 33–41 (2019)

Zayed, E.M., Gepreel, K.A., Shohib, R.M., Alngar, M.E., Yıldırım, Y.: Optical solitons for the perturbed Biswas-Milovic equation with Kudryashov's law of refractive index by the unified auxiliary equation method. Optik **230**, 166286 (2021)

Zhang, R.F., Bilige, S.: Bilinear neural network method to obtain the exact analytical solutions of nonlinear partial differential equations and its application to p-gBKP equation. Nonlinear Dyn. **95**(4), 3041–3048 (2019)

Zhang, Z.Y., Liu, Z.H., Miao, X.J., Chen, Y.Z.: Qualitative analysis and traveling wave solutions for the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity. Phys. Letts. A **375**(10), 1275–1280 (2011)

Zhang, Y.S., Guo, L.J., Chabchoub, A., He, J.S.: Higher-order rogue wave dynamics for a derivative nonlinear Schrödinger equation. Romanian J. Phys. **62**, 102 (2017)

Zhang, R.F., Bilige, S., Liu, J.G., Li, M.: Bright-dark solitons and interaction phenomenon for p-gBKP equation by using bilinear neural network method. Physica Scripta **96**(2), 025224 (2020)

Zhang, R.F., Li, M.C., Albishari, M., Zheng, F.C., Lan, Z.Z.: Generalized lump solutions, classical lump solutions and rogue waves of the (2+ 1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada-like equation. Appl. Math. Comput. **403**, 126201 (2021)

Zhang, R., Bilige, S., Chaolu, T.: Fractal solitons, arbitrary function solutions, exact periodic wave and breathers for a nonlinear partial differential equation by using bilinear neural network method. J. Syst. Sci. Complexityq **34**(1), 122–139 (2021)

Zhang, R.F., Li, M.C., Yin, H.M.: Rogue wave solutions and the bright and dark solitons of the (3+ 1)-dimensional Jimbo-Miwa equation. Nonlinear Dyn. **103**(1), 1071–1079 (2021)