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Highly dispersive optical solitons with non-local law of refractive index by Laplace-Adomian decomposition

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Abstract
This paper studies bright highly dispersive optical solitons that are with nonlocal type of nonlinearity. The numerical scheme, adopted in the paper, is Laplace-Adomian method. The analytical results, reported earlier, and the numerical results from the current work, agree with an impressively small error measure.

Keywords Nonlinear Schrödinger equation · Non-local nonlinearity · Higher-order dispersion · Laplace-Adomian decomposition method

1 Introduction

One of the featured concepts that was introduced during 2019 is highly dispersive (HD) optical solitons. This happens when in addition to chromatic dispersion (CD), the effects of inter-modal dispersion (IMD), third-order dispersion (3OD), fourth order dispersion (4OD), fifth order dispersion (5OD) and sixth-order dispersion (6OD) effects are all included. This leads to solitons to the governing nonlinear Schrödinger's equation. There are only four forms of nonlinear refractive index that leads to the retrieval of closed form of a soliton solution (Alshaery et al. 2014; Biswas et al. 2019a, b, c, d, e, f, 2020; Kara et al. 2018; Kohl et al. 2019a, 2020a, b; Kudryashov 2020a, b, c, d; Vega-Guzman et al. 2014; Yanan et al. 2013; Yildirim et al. 2020). These are Kerr law, quadratic-cubic law, non-local

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law and polynomial law. Today's paper will study HD solitons, with nonlocal nonlinear form, in polarization-preserving optical fibers.

In this context, it must be noted that with the effect of higher order dispersions, one will naturally encounter the effect of soliton radiation that will lead to the shedding of energy. However, these effects have been discarded and the solitons only in the discrete regime with bound states have been studied. Our numerical scheme is the Laplace-Adomian algorithm that studies bright optical solitons and the accuracy of the scheme is depicted in the error plots are exhibited with impressive measure. After a quick revisitation of the governing model and the algorithmic scheme, the results are enumerated and displayed.

2 Governing equation

The nonlinear Schrödinger's equation (NLSE) for highly dispersive optical solitons in the presence of a non-local nonlinearity is given by Biswas et al. (2019g, h, i, j), Rehman et al. (2019) and Kohl et al. (2019b):

$$iq_t + ia_1q_x + a_2q_{xx} + ia_3q_{xxx} + a_4q_{xxxx} + ia_5q_{xxxxx} + a_6q_{xxxxxx} + b(|q|^2)_{xx}q = 0, \quad (1)$$

where $q = q(x, t)$ is a complex-valued function of x (space) and t (time). In Eq. (1), the first term stands for linear temporal evolution with $i = \sqrt{-1}$. The next six terms are dispersion terms that make the solitons highly dispersive. These are given by the coefficients of a_k for $1 \leq k \leq 6$ which are inter-modal dispersion (IMD), group velocity dispersion (GVD), third-order dispersion (3OD), fourth-order dispersion (4OD), fifth-order dispersion (5OD) and sixth-order dispersion (6OD) respectively. Finally, b is the coefficient of non-local nonlinearity.

2.1 Bright highly dispersive optical solitons

The bright highly dispersive optical soliton solution to (1) was recently reported in Biswas et al. (2019g), using the extended Jacobi's elliptic function scheme the authors obtain

$$q(x, t) = A\ \mathrm{sech}^2[(x - vt)] \times \exp\{i[-\kappa x + \omega t + \theta_0]\}. \quad (2)$$

In Eq. (2), where v is the speed of the wave, ω is its wave number, κ is the soliton frequency and θ_0 is the phase center constant.

In Biswas et al. (2019g, h), the parameters and constraints for the highly dispersive optical soliton are given by:

(a) The relationship between the system parameters are:

$$\omega = \frac{1260a_1\kappa + b\beta_2^2(25\kappa^6 - 336\kappa^4 + 560\kappa^2 + 768m^3 + 80(7\kappa^2 + 36)m^2 - 16ml_1 + 768)}{1260}, \quad (3)$$

$$a_4 = -\frac{b\beta_2^2(-75\kappa^2 + 112(m + 1))}{1260}, \qquad v = a_1 - 2\kappa(a_2 + 4a_4\kappa^2 + 8a_5\kappa^3), \quad A = \beta_2 m, \quad (4)$$

$$a_2 = -\frac{b\beta_2^2(75\kappa^4 - 672\kappa^2 + 560m^2 - 112(6\kappa^2 + 35)m) + 560)}{1260}, \tag{5}$$

where

$$\beta_2^2 = -\frac{252a_6}{b}, \qquad l_1 = 21\kappa^4 + 245\kappa^2 - 180,$$

and m is any parameter that satisfies the Eqs. (3), (4) and (5).

(b) The constraint condition are

$$a_5 = 6a_6\kappa, \qquad a_3 = \frac{4}{3}\kappa(3a_4 + 5a_5\kappa) \qquad \text{and} \qquad a_6 b < 0. \tag{6}$$

3 Description of the method applied

The aim of this section is to discuss the use of the Laplace-Adomian decomposition method (LADM) and its algorithm to solve the NLSE (1). The present method was first proposed in Adomian (1994) and Khuri (2001).

Let us look for soliton solutions of Eq. (1) in the form $q(x,t) = u(x,t) + iv(x,t)$. Then we can decompose the Eq. (1) in its real and imaginary parts, respectively as

$$u_t = -a_1 u_x - a_2 v_{xx} - a_3 u_{xxx} - a_4 v_{xxxx} - a_5 u_{xxxxx} - a_6 v_{xxxxxx} - 2bv(uu_{xx} + vv_{xx}) \tag{7}$$

$$v_t = -a_1 v_x + a_2 u_{xx} - a_3 v_{xxx} + a_4 u_{xxxx} - a_5 v_{xxxxx} + a_6 u_{xxxxxx} + 2bu(uu_{xx} + vv_{xx}) \tag{8}$$

In order to find analytical approximate solutions for Eq. (1) using LADM, we first rewrite the Eqs. (7) and (8) in the following operator form

$$D_t u = -a_1 D_x^1 u - a_2 D_x^2 v - a_3 D_x^3 u - a_4 D_x^4 v - a_5 D_x^5 u - a_6 D_x^6 v + N_1(u,v) \tag{9}$$

$$D_t v = -a_1 D_x^1 v + a_2 D_x^2 u - a_3 D_x^3 v + a_4 D_x^4 u - a_5 D_x^5 v + a_6 D_x^6 u + N_2(u,v) \tag{10}$$

with initial conditions $u(x,0) = \mathfrak{Re}(q(x,0))$ and $v(x,0) = \mathfrak{Im}(q(x,0))$.

In the equations system (9)–(10) the operator D_t denotes derivative with respect to t, whereas that D_x^j is the j−th order linear differential operator $\frac{\partial^j}{\partial x^j}$ and N_k represents nonlinear differential operators for $k = 1, 2$.

The method consists of first applying the Laplace transform $\mathscr{L}$ to both sides of equations in system (9)–(10) and then by using initial conditions, we have

$$u(x,s) = \frac{u(x,0)}{s} + \frac{1}{s}\mathscr{L}\{-a_1 D_x^1 u - a_2 D_x^2 v - a_3 D_x^3 u - a_4 D_x^4 v - a_5 D_x^5 u - a_6 D_x^6 v + N_1(u,v)\} \tag{11}$$

$$v(x,s) = \frac{v(x,0)}{s} + \frac{1}{s}\mathscr{L}\{-a_1 D_x^1 v + a_2 D_x^2 u - a_3 D_x^3 v + a_4 D_x^4 u - a_5 D_x^5 v + a_6 D_x^6 u + N_2(u,v)\} \tag{12}$$

Now, applying inverse Laplace transform $\mathscr{L}^{-1}$ and initial conditions to system (11) and (12), we get

$$u(x,t) = u(x,0) + \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-a_1D_x^1u - a_2D_x^2v - a_3D_x^3u - a_4D_x^4v - a_5D_x^5u - a_6D_x^6v + N_1(u,v)\}\right] \tag{13}$$

$$v(x,t) = v(x,0) + \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-a_1D_x^1v + a_2D_x^2u - a_3D_x^3v + a_4D_x^4u - a_5D_x^5v + a_6D_x^6u + N_2(u,v)\}\right] \tag{14}$$

Now we represent the unknown functions u and v by an infinite series of the form

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \qquad v(x,t) = \sum_{n=0}^{\infty} v_n(x,t) \tag{15}$$

In addition, the nonlinear terms can be written as

$$N_1(u,v) = -2bv(uu_{xx} + vv_{xx}) = -2b\sum_{n=0}^{\infty} A_n(u_0,u_1,\dots,u_n;v_0,v_1,\dots,v_n) \tag{16}$$

and

$$N_2(u,v) = 2bu(uu_{xx} + vv_{xx}) = 2b\sum_{n=0}^{\infty} B_n(u_0,u_1,\dots,u_n;v_0,v_1,\dots,v_n) \tag{17}$$

where A_n and B_n are the Adomian's polynomials (Rach 1984; Wazwaz 2000), which are defined by

$$A_n(u_0,\dots,u_n;v_0,\dots,v_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N_1\Big(\sum_{i=1}^{\infty}\lambda^i u_i;\sum_{i=1}^{\infty}\lambda^i v_i\Big)\right]_{\lambda=0}, \quad n = 0,1,2,\dots \tag{18}$$

$$B_n(u_0,\dots,u_n;v_0,\dots,v_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N_2\Big(\sum_{i=1}^{\infty}\lambda^i u_i;\sum_{i=1}^{\infty}\lambda^i v_i\Big)\right]_{\lambda=0}, \quad n = 0,1,2,\dots \tag{19}$$

On making the substitution of Eqs. (15) and (16) into Eqs. (13) and (14), we can arrive at

$$\sum_{n=0}^{\infty} u_n = u(x,0) + \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)\sum_{n=0}^{\infty} u_n - (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)\sum_{n=0}^{\infty} v_n - 2b\sum_{n=0}^{\infty} A_n\}\right] \tag{20}$$

$$\sum_{n=0}^{\infty} v_n = v(x,0) + \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)\sum_{n=0}^{\infty} v_n + (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)\sum_{n=0}^{\infty} u_n + 2b\sum_{n=0}^{\infty} B_n\}\right] \tag{21}$$

In general, the recursive relations are given by

$$\begin{cases} u_0(x,t) = \Re e(q(x,0)), \\ u_{n+1}(x,t) = \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)u_n - (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)v_n - 2bA_n\}\right], \quad n \geq 0, \end{cases} \tag{22}$$

$$\begin{cases} v_0(x,t) = \Im m(q(x,0)), \\ v_{n+1}(x,t) = \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)v_n + (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)u_n + 2bB_n\}\right], \quad n \geq 0. \end{cases} \tag{23}$$

With the help of the above procedure first few terms of Adomian polynomials A_n and B_n can be obtained as

$$
\begin{aligned}
A_0 &= v_{0xx}v_0^2 + u_0u_{0xx}v_0,\\
A_1 &= v_{1xx}v_0^2 + u_1u_{0xx}v_0 + u_0u_{1xx}v_0 + 2v_1v_{0xx}v_0 + u_0u_{0xx}v_1,\\
A_2 &= v_{2xx}v_0^2 + u_2u_{0xx}v_0 + +u_1u_{1xx}v_0 + u_0u_{2xx}v_0 + 2v_2v_{0xx}v_0 + 2v_1v_{1xx}v_0 + u_1u_{0xx}v_1 + u_0u_{1xx}v_1\\
&\quad + u_0u_{0xx}v_2 + v_1^2v_{0xx},\\
A_3 &= v_{3xx}v_0^2 + u_3u_{0xx}v_0 + u_2u_{1xx}v_0 + u_1u_{2xx}v_0 + u_0u_{3xx}v_0 + 2v_3v_{0xx}v_0 + 2v_2v_{1xx}v_0 + 2v_1v_{2xx}v_0\\
&\quad + u_2u_{0xx}v_1 + u_1u_{1xx}v_1 + u_0u_{2xx}v_1 + u_1u_{0xx}v_2 + u_0u_{1xx}v_2 + u_0u_{0xx}v_3 + 2v_1v_{0xx}v_2 + v_1^2v_{1xx},\\
A_4 &= v_{4xx}v_0^2 + u_4u_{0xx}v_0 + u_3u_{1xx}v_0 + u_2u_{2xx}v_0 + u_1u_{3xx}v_0 + u_0u_{4xx}v_0 + 2v_4v_{0xx}v_0 + 2v_3v_{1xx}v_0\\
&\quad + 2v_2v_{2xx}v_0 + 2v_1v_{3xx}v_0 + u_3u_{0xx}v_1 + u_2u_{1xx}v_1 + u_1u_{2xx}v_1 + u_0u_{3xx}v_1 + u_2u_{0xx}v_2 + u_1u_{1xx}v_2\\
&\quad + u_0u_{2xx}v_2 + u_1u_{0xx}v_3 + u_0u_{1xx}v_3 + u_0u_{0xx}v_4 + 2v_1v_{0xx}v_2 + 2v_1v_{1xx}v_2 + v_1^2v_{2xx} + v_2^2v_{0xx},\\
A_5 &= v_{5xx}v_0^2 + u_5u_{0xx}v_0 + u_4u_{1xx}v_0 + u_3u_{2xx}v_0 + u_2u_{3xx}v_0 + u_1u_{4xx}v_0 + u_0u_{5xx}v_0 + 2v_5v_{0xx}v_0\\
&\quad + 2v_4v_{1xx}v_0 + 2v_3v_{2xx}v_0 + 2v_2v_{3xx}v_0 + 2v_1v_{4xx}v_0 + u_4u_{0xx}v_1 + u_3u_{1xx}v_1 + u_2u_{2xx}v_1 + u_1u_{3xx}v_1\\
&\quad + u_0u_{4xx}v_1 + u_3u_{0xx}v_2 + u_2u_{1xx}v_2 + u_1u_{2xx}v_2 + u_0u_{3xx}v_2 + u_2u_{0xx}v_3 + u_1u_{1xx}v_3 + u_0u_{2xx}v_3 + u_1u_{0xx}v_4\\
&\quad + u_0u_{1xx}v_4 + u_0u_{0xx}v_5 + 2v_2v_{0xx}v_3 + 2v_1v_{0xx}v_4 + 2v_1v_{1xx}v_3 + 2v_1v_{2xx}v_2 + v_1^2v_{3xx} + v_2^2v_{1xx}.
\end{aligned}
\tag{24}
$$

and

$$
\begin{aligned}
B_0 &= u_{0xx}u_0^2 + u_0v_{0xx}v_0,\\
B_1 &= u_{1xx}u_0^2 + u_1v_{0xx}v_0 + u_0v_{1xx}v_0 + 2u_1u_{0xx}u_0 + u_0v_{0xx}v_1,\\
B_2 &= u_{2xx}u_0^2 + u_2v_{0xx}v_0 + u_1v_{1xx}v_0 + u_0v_{2xx}v_0 + 2u_2u_{0xx}u_0 + 2u_1u_{1xx}u_0 + u_1v_{0xx}v_1 + u_0v_{1xx}v_1\\
&\quad + u_0v_{0xx}v_2 + u_1^2u_{0xx},\\
B_3 &= u_{3xx}u_0^2 + u_3v_{0xx}v_0 + u_2v_{1xx}v_0 + u_1v_{2xx}v_0 + u_0v_{3xx}v_0 + 2u_3u_{0xx}u_0 + 2u_2u_{1xx}u_0 + 2u_1u_{2xx}u_0\\
&\quad + u_2v_{0xx}v_1 + u_1v_{1xx}v_1 + u_0v_{2xx}v_1 + u_1v_{0xx}v_2 + u_0v_{1xx}v_2 + u_0v_{0xx}v_3 + 2u_1u_{0xx}u_2 + u_1^2u_{1xx},\\
B_4 &= u_{4xx}u_0^2 + u_4v_{0xx}v_0 + u_3v_{1xx}v_0 + u_2v_{2xx}v_0 + u_1v_{3xx}v_0 + u_0v_{4xx}v_0 + 2u_4u_{0xx}u_0 + 2u_3u_{1xx}u_0\\
&\quad + 2u_2u_{2xx}u_0 + 2u_1u_{3xx}u_0 + u_3v_{0xx}v_1 + u_2v_{1xx}v_1 + u_1v_{2xx}v_1 + u_0v_{3xx}v_1 + u_2v_{0xx}v_2 + u_1v_{1xx}v_2\\
&\quad + u_0v_{2xx}v_2 + u_1v_{0xx}v_3 + u_0v_{1xx}v_3 + u_0v_{0xx}v_4 + 2u_1u_{0xx}u_2 + 2u_1u_{1xx}u_2 + u_1^2u_{2xx} + u_2^2u_{0xx},\\
B_5 &= u_{5xx}u_0^2 + u_5v_{0xx}v_0 + u_4v_{1xx}v_0 + u_3v_{2xx}v_0 + u_2v_{3xx}v_0 + u_1v_{4xx}v_0 + u_0v_{5xx}v_0 + 2u_5u_{0xx}u_0\\
&\quad + 2u_4u_{1xx}u_0 + 2u_3u_{2xx}u_0 + 2u_2u_{3xx}u_0 + 2u_1u_{4xx}u_0 + u_4v_{0xx}v_1 + u_3v_{1xx}v_1 + u_2v_{2xx}v_1 + u_1v_{3xx}v_1\\
&\quad + u_0v_{4xx}v_1 + u_3v_{0xx}v_2 + u_2v_{1xx}v_2 + u_1v_{2xx}v_2 + u_0v_{3xx}v_2 + u_2v_{0xx}v_3 + u_1v_{1xx}v_3 + u_0v_{2xx}v_3 + u_1v_{0xx}v_4\\
&\quad + u_0v_{1xx}v_4 + u_0v_{0xx}v_5 + 2u_2u_{0xx}u_3 + 2u_1u_{0xx}u_4 + 2u_1u_{1xx}u_3 + 2u_1u_{2xx}u_2 + u_1^2u_{3xx} + u_2^2u_{1xx}.
\end{aligned}
\tag{25}
$$

Using Eqs. (22) and (23) through the LADM method, we obtain the next recursive algorithm for the real and imaginary part of solution $q(x, t)$, respectively.

$$
\begin{aligned}
u_0(x,t) &= \Re e(q(x,0)),\\
v_0(x,t) &= \Im m(q(x,0)),\\
u_1(x,t) &= \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)u_0 - (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)v_0 - 2bA_0\}\right],\\
v_1(x,t) &= \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)v_0 + (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)u_0 + 2bB_0\}\right],\\
u_2(x,t) &= \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)u_1 - (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)v_1 - 2bA_1\}\right],\\
v_2(x,t) &= \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)v_1 + (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)u_1 + 2bB_1\}\right],\\
&\vdots\\
u_n(x,t) &= \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)u_{n-1} - (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)v_{n-\circ} - 2bA_{n-1}\}\right],\\
v_n(x,t) &= \mathscr{L}^{-1}\left[\frac{1}{s}\mathscr{L}\{-(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)v_{n-1} + (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)u_{n-1} + 2bB_{n-1}\}\right].
\end{aligned}
$$

The approach introduced above will illustrated through examples in the following section. All our computations are performed by MATHEMATICA software package (Mangano 2010).

4 Numerical simulations

To illustrate the ability, reliability and the accuracy of the proposed method for find solutions of Eq. (1) in the case of bright highly dispersive optical solitons in the presence of a non-local nonlinearity, some examples are provided. The results reveal that the method is very simple to use and highly efficient.

We now consider the initial condition at $t = 0$ from Eq. (2)

$$q(x,0) = A\,\mathrm{sech}^2(x) \times \exp\{i[-\kappa x + \theta_0]\} = u_0(x,t) + iv_0(x,t). \tag{26}$$

We now perform the simulation of the four cases listed in Table 1 and the results obtained as well as the absolute errors are shown in Figs. 1, 2, 3 and 4.

Table 1 Coefficients of Eq. (1) for bright highly dispersive optical solitons

Cases	a_1	a_2	a_3	a_4	a_5	a_6	b	κ	ω	θ_0	v	N	| Max Error |
I	1.22	0.03	− 0.02	0.60	0.07	− 1.20	1.35	− 0.01	0.22	1.80	2.33	15	2.5×10^{-9}
II	1.32	0.01	0.07	0.28	0.34	1.90	− 1.82	0.03	0.91	1.92	3.15	15	1.5×10^{-9}
III	1.40	0.09	− 0.69	0.35	− 0.30	− 0.05	0.85	1.02	0.36	1.52	2.22	15	5.0×10^{-9}
IV	1.10	0.01	0.14	0.85	0.32	1.35	− 0.75	0.04	0.72	0.34	1.15	15	1.0×10^{-9}

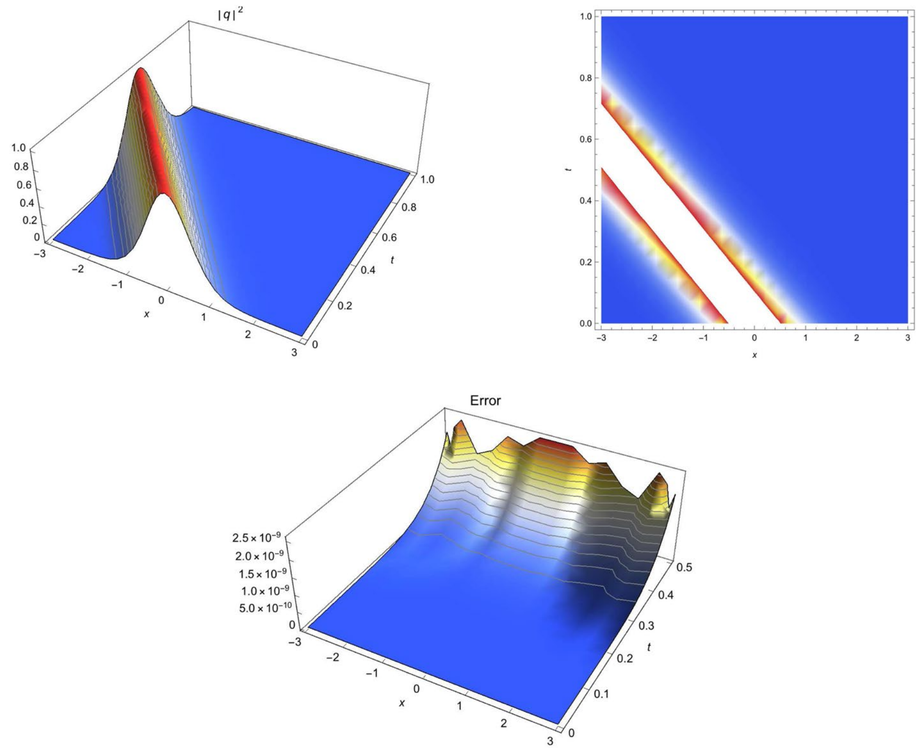

Fig. 1 Numerically computed bright highly dispersive optical soliton (**a**), corresponding density plot (**b**) and absolute error (**c**) for case I

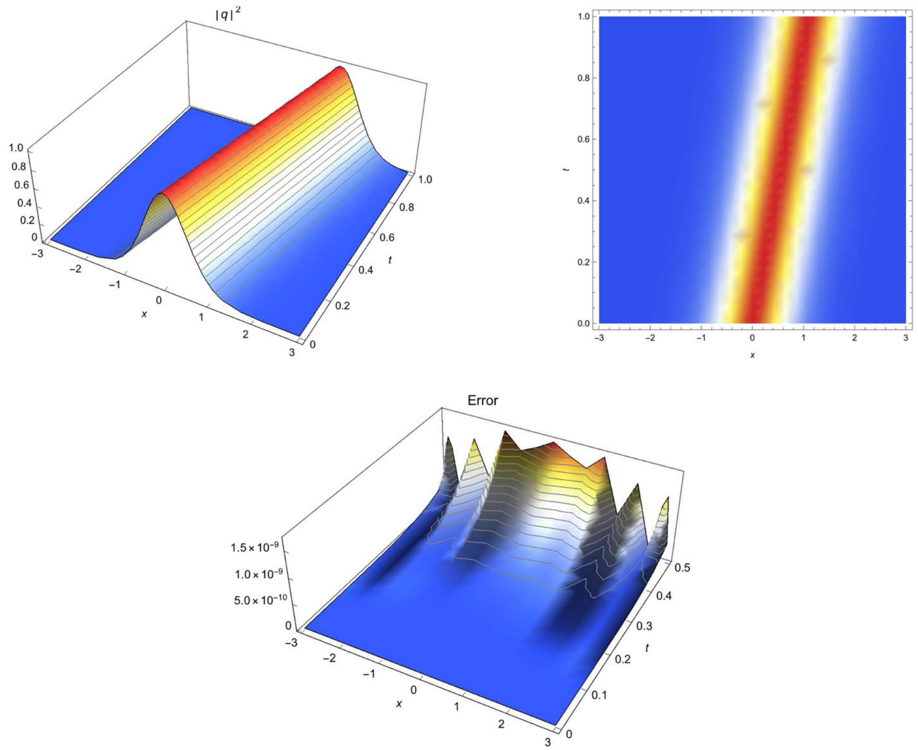

Fig. 2 Numerically computed bright highly dispersive optical soliton (**a**), corresponding density plot (**b**) and absolute error (**c**) for case II

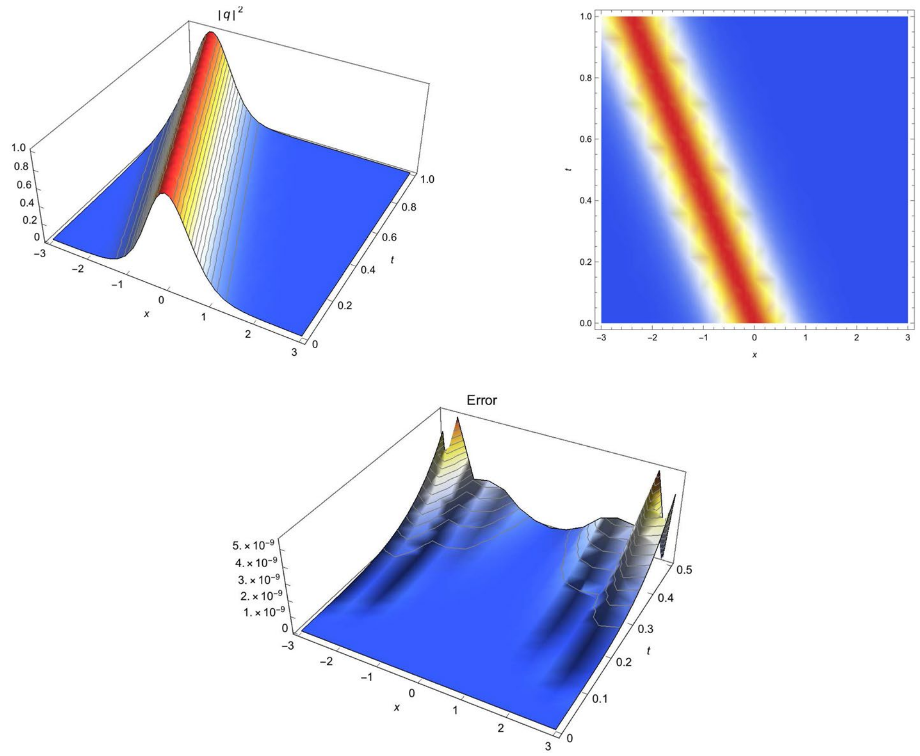

Fig. 3 Numerically computed bright highly dispersive optical soliton (**a**), corresponding density plot (**b**) and absolute error (**c**) for case III

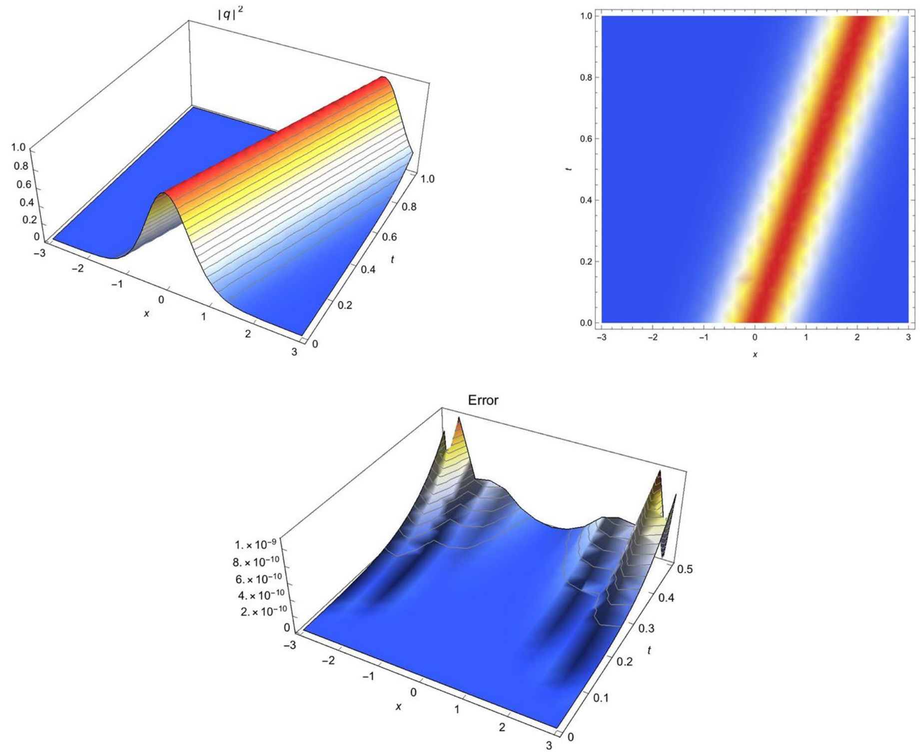

Fig. 4 Numerically computed bright highly dispersive optical soliton (**a**), corresponding density plot (**b**) and absolute error (**c**) for case IV

5 Conclusions

Today's paper retrieved HD bright optical solitons by the aid of LADM. The numerical scheme also yielded the error estimates of the approximations. These error measures stand very impressive. The simulations with bright solitons are exhibited for nonlocal form of nonlinearity. These results will now be extended to birefringent fibers with nonlocal law of nonlinearity where LADM will be implemented to demonstrate the numerics in it. In future, the mode will also be studied with additional forms of nonlinear media in the context of birefringent fibers and these would include Kerr law, quadratic-cubic law and polynomial law. The results of those research activities would be reported with time.

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Compliance with ethical standards

Conflict of interest The authors declare they have no conflict of interest.

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