

# Optical, electrical, mechanical properties of  $Pr<sup>3+</sup>$  and Yb<sup>3+</sup> **doped phosphate glasses**

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## **Abstract**

The rare earth  $Pr<sup>3+</sup>$  and  $Yb<sup>3+</sup>$  doped phosphate glass were prepared by quenching method using electrical muful furnace. The optical band gap of rare earth  $Pr<sup>3+</sup>$  and  $Yb<sup>3+</sup>$  doped phosphate glass were studied. The dielectric permittivity, dielectric loss and electrical conductivity of the prepared samples have been studied using LCR meter with diferent frequencies at room temperature. The Vickers hardness number, brittle index, yield strength, fracture toughness of the rare earth  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses were also calculated. The nonlinear optical parameters such as third order nonlinear coefficient and nonlinear refractive index were estimated using Z-scan method. The magnetic property of the  $Pr<sup>3+</sup>$  and  $Yb<sup>3+</sup>$  doped phosphate glass was studied using VSM analysis. The electro chemical properties of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glass electrodes in 5 M KOH electrolyte were studied using cyclic voltammetry, Galvanostatic charge–discharge and electrochemical impedance spectroscopy. The specifc capacitance, energy density and capacity retention of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses were calculated.

**Keywords** Phosphate glass · Rare earth · Optical nonlinearity · Nonlinear susceptibility · Vickers hardness · Refractive index

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# **1 Introduction**

Glass is an inorganic solid material, which is transparent at room temperature. Glass is a name given all amorphous materials which can be obtained by lowering the temperature of melt, independently of its chemical composition ant its temperature range of solidifcation. When a liquid is cooled to solid state, there is a possibility of getting two outputs (i) glassline materials (ii) amorphous materials. In glassy materials, the constituent particles are arranged in a random manner and lack the long range periodic glass structure.

Eg: Glass, Rubber and Plastic. The amorphous materials show short range periodicity. In past decades, peoples used glasses for making ornaments with the unawareness of its properties.

But in present century glasses attain prior position in both optical and communication feld with its emerging trends in latest technologies. In our daily life, glasses are used for numerous applications. For example, glass components are used in medical diagnostic equipments. Normally pure silica is being used as the raw material for glass preparation.

Due to the high melting point of silica and other disadvantages many researchers have changed the basic components of making glass and focused on the preparation of non-silicate glasses (network glasses, oxide glasses) like borate glasses, phosphate glasses, alumina silicate glasses, etc.

The three components of oxide glasses are glass former, intermediate and modifer. Instead of silica, one can choose oxides of elements as glass formers. This oxides exhibit intermediate electro negativity, which forms glass of their own. Eg:  $B_2O_3$ ,  $P_2O_5$ etc. Intermediates do not form glass itself but helps in the formation of glasses. The cation of intermediate is capable of entering into the glass network and occupies the position of glass former. Eg: Zn, Ti. The Modifer (Eg: Li, Na) forms glass, only when it mixed in correct proportion with glass former. By the addition of modifer, the continuous glass network is disrupted.

The famous scientist Goldschmidt framed some rules to select a glass former. They are (i) Glass forming oxides are those for which the ratio of ionic radii of anions and cations lies in 0.2 to 0.4 (ii) 4 anions are surrounded to each cation. Anions should be situated at corners, ie.triangular confguration is must. Later Zachariasen agrees this with some modifcations:

(i) Oxygen coordination should be less normally 3 or 4.

(ii) Sample contains high percentage of cation which are surrounded by oxygen tetra-hedral or trigonal (Shelby [2020](#page-26-0)).

 In the present work we have concentrated on phosphate glasses. In phosphate glass,  $P_2O_5$  is considered as glass former. The phosphate glass is a class of optical glass composed of metaphosphate of various metals. While comparing to borate glasses, in phosphate glasses oxygen is non-bridging.

The Phosphate glasses exhibits in sheet form and consists of layers of oxygen polyhedral with weak Vaan der Waals force of attraction between the layers. The rigidity of the structure can be modifed by the addition of alkali or alkaline earth elements (Sharaf et al. [2008](#page-26-1)). The Phosphate glasses are more superior than silicate glasses in practical conditions due to their low melting point high refractive index and good rare earth solubility. While comparing with others, they are high resistance to hydrofuoric acid. They show low resistance to chemical corrosion. The phosphate glasses are well suited for doping with various elements like transition metals and rare earth elements.

The Rare earth elements are taken as dopants in prepared samples. The lanthanides are known as Rare earth elements. These elements are not occur commonly in earth and difcult to separate each other. Usually RE elements are trivalent. Lanthanides are d-block elements. Among the seventeen RE elements, Praseodymium and Ytterbium are chosen as dopants.

The Praseodymium is used to make specialized yellow glasses for glass welders. On the basis of this concept, the  $Pr^{3+}$  doped phosphate glasses were synthesized and characterized.

Ytterbium is more reactive than other lanthanides. Ytterbium is used for making lasers. The primary application of ytterbium doped glass lasers is in material processing like welding, cutting etc. The ytterbium doped glasses fnds use as active medium for lasers. The ytterbium is popular for their NIR luminescence, while  $Pr<sup>3+</sup>$  show transitions in the NIR region. In this present paper have been reported the linear and Nonlinear optical, mechanical and electrical properties of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses.

## **2 Sample preparation**

Glasses are formed through several processes but the most common method is melt quenching. In this technique, the raw material is heated to molten state and then rapidly cooled to solid state. All the raw materials are melted at ambient temperature.

All the raw materials (Sigma Aldrich, 99.99%, purity) were weighted in correct proportion based on the batch formula, grinded and mixed using electric mortar for 15 min to get fne powder mixture. The prepared samples were taken in crucible and kept in a furnace which was maintained at 1400 °C for 4–5 h. After continuous monitoring, the melt is transferred to another furnace for annealing when melting process gets over. The as prepared  $Yb^{3+}$  and  $Pr^{3+}$  doped phosphate glasses are shown in Fig. [1a](#page-2-0), b.

<span id="page-2-0"></span>

**Fig. 1** As prepared (a)  $Yb^{3+}$  and (b)  $Pr^{3+}$  doped phosphate glass

# **3 Characterization**

#### **3.1 Fluorescence emission spectra**

The fluorescence spectra of rare earth  $Pr<sup>3+</sup>$  and  $Yb<sup>3+</sup>$  doped phosphate glass have been recorded by using time correlated Single Photon counting instrument (purchased from Horiba Jobin Yvon, USA, NJ). The Fluorescence spectroscopy deals with electronic and vibrational states. The energy level  ${}^{3}H_{4}$  is ground state of  $Pr^{3+}$ . In order to record the emission spectra, the radiations have been excited to 390 nm.Several weak intense emission bands appeared in the graph. Four peaks observed in the range 480–560 nm region are distinct.

The sharp features are absent in the optical spectra of amorphous materials. The transitions such as  ${}^{3}P_{0}$ - ${}^{3}H_{4}$ ,  ${}^{3}P_{0}$ - ${}^{3}H_{5}$ ,  ${}^{1}P_{1}$ - ${}^{3}H_{5}$ ,  ${}^{3}P_{0}$ - ${}^{3}H_{6}$  and their wavelengths are 485,510,540 and 560 nm respectively. An emission band at 485 nm attains comparatively more intense peak.

The two transitions like  ${}^{3}P_{0}$ - ${}^{3}H_{5}$ ,  ${}^{1}P_{1}$ - ${}^{3}H_{5}$  occurred in 520–560 nm range and their infuence in total fuorescence is very small (Sharma et al. [2000;](#page-26-2) Rupa et al. [2018](#page-26-3)). The fluorescence spectra of and  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glass is shown in Fig. [2](#page-3-0) and [3.](#page-4-0) The absorption and emission spectra were drawn in same graph with black and red color respectively.

Normally Yb can be used as an acceptor because it exhibits only two energy levels  ${}^{2}F$  $_{7/2}$  and <sup>2</sup> F<sub>5/2</sub>. The level <sup>2</sup>F<sub>7/2</sub> is the ground energy level and <sup>2</sup> F<sub>5/2</sub> is excited level. The energy of excited Yb ions are very close to the band gap of silicon and its luminescence quantum efficiency is maximum in NIR region. On account of these properties, Yb can be used in memory devices, tunable lasers and also as industrial catalyst. The broad absorption and emission of  $Yb^{3+}$  ions are due to electronic transitions involving the Stark sublevels. From the measurement, the emission spectra of  $Yb^{3+}$  doped phosphate glass shows maximum intensity at 978 nm which indicates the  ${}^{2}F_{7/2}$ — ${}^{2}F_{5/2}$  transition.

The Optical properties of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses were explained by UV–Vis spectroscopy. The resultant absorption spectrum recorded with Perkin Elmer



<span id="page-3-0"></span>**Fig. 2** Fluorescence emission spectra of  $Pr<sup>3+</sup>$  doped Phosphate glasses



<span id="page-4-0"></span>**Fig. 3** Fluorescence emission spectra of  $Yb^{3+}$ doped Phosphate glasses

Lambda 1050 spectrophotometer, in the range of 200–800 nm and  $Pr<sup>3+</sup>$  doped phosphate glass is shown in Fig. [4.](#page-4-1)

The number of peaks with diferent wavelength are seen which is happens due to the electronic transition between the inner orbits of RE ions (Elisa et al. [2013\)](#page-26-4). Also it doesn't show any sharp absorption edge, which supports the characteristics of glassy state. The absorption bands of  $Pr^{3+}$  are seen at 437 nm, 467 nm, 480 nm and 560 nm respectively. The corresponding transitions are from ground state  ${}^{3}H_4$  to  ${}^{2}P_3$ ,  ${}^{3}P_1$ ,  ${}^{1}P_0$ ,  ${}^{1}D_2$  respectively. The earlier H. Zhang et al. has reported the similar transitions in their studies (Zhang et al. [1999\)](#page-27-0). Similar absorption curve of  $Yb^{3+}$  doped phosphate glass are depicted in Fig. [5.](#page-5-0)

The excitation spectra of Yb does not shown any single sharp peak because the energy levels become split into small lines. The two 4f levels can split into 4stark lines



<span id="page-4-1"></span>**Fig. 4** UV –Visible spectrum of  $Pr<sup>3+</sup>$  doped Phosphate glasses

<span id="page-5-0"></span>

for ground and 3 stark lines for excited state. Generally to attain this kind of spectral splitting in room temperature is tedious.

In low temperature measurements, the interaction between Yb ions with lattice vibrations are considered. The more vibrational transitions occur in the spectra due the interaction between electron and phonon. The Yb ions are located in various crystallographic sites because of the different values of the absorption coefficients (IR region). The stark lines are discriminated using Greek letters. The graph shows an excitation at 236 nm, which denotes Y-lines, another peak at 242 nm denotes Alpha line.

The spectra shows a large number of weak transitions also, which indicates the presence of impurities. Due to the difculty of getting absorption spectra for powder samples in Infra-Red range, normally we can neglect the lowest part of the spectra.

The less absorption has been observed in the visible region and confrms that the quality of the glass is good. The required properties for NLO activity are minimum absorption and low cut-off wavelength.

The electrons are excited from a flled band to an empty one by photon absorption and shows a marked increases in the absorption coefficient.

The optical absorption coefficient  $(\alpha)$  was calculated using the following relation (Krishnakumar and Nagalakshmi [2005;](#page-26-5) Yakuphanoglu and Erten [2005\)](#page-27-1).

$$
\alpha = \frac{2.3026 \log\left(\frac{1}{T}\right)}{t} \tag{1}
$$

where T is the transmittance and d is the thickness of the glass. The various other optical constants were calculated using the following theoretical formulae. The energy dependence of the absorption coefficient suggests the occurrence of direct band gap and hence it obeys the relation for high photon energy.

$$
(\alpha h v)^2 = A (E_g - h v) \tag{2}
$$

where  $E_{\sigma}$  is the optical band gap and A is a constant.



<span id="page-6-0"></span>**Fig. 6** Photon energy verses  $( \alpha h v)^2$  of  $Pr^{3+}$  doped phosphate glass



<span id="page-6-1"></span>**Fig. 7** Photon energy verses  $(αhυ)<sup>2</sup>$  of  $Yb<sup>3+</sup>$  doped phosphate glass

The variation of  $(\alpha ho)^2$  versus photo energy in the fundamental absorption region are plotted in the Figs. [6](#page-6-0), [7](#page-6-1) for  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses. E<sub>g</sub> is examined by the extrapolation of the linear part. The optical band gap of  $Pr^{3+}$  and  $\gamma_{D}^{s}$  doped phosphate glasses were found to be 3.6 eV and 3.0 eV respectively.

The extinction coefficient is obtained in terms of the absorption coefficient,



<span id="page-7-0"></span>**Fig. 8** Photon energy verses Extinction coefficient of  $Pr<sup>3+</sup>$  doped phosphate glass



<span id="page-7-1"></span>**Fig. 9** Photon energy verses extinction coefficient of  $Yb^{3+}$  doped phosphate glass

$$
K = \frac{\alpha \lambda}{4\pi} \tag{3}
$$

The photon energy versus extinction coefficients of  $Pr<sup>3+</sup>$  and Yb<sup>3+</sup> doped phosphate glasses are shown in Figs. [8](#page-7-0) and [9](#page-7-1) respectively. The extinction coefficients of  $Pr<sup>3+</sup>$  and  $Yb^{3+}$  doped phosphate glasses were found to be 1.107161  $\times$  10<sup>-5</sup> and 8.412 $\times$  10<sup>-5</sup> at 3.5 eV respectively.

The reflectance in terms of the of absorption coefficient is derived,

$$
R = \frac{\exp(-\alpha t) \pm \sqrt{\exp(-\alpha t)T - \exp(-3\alpha t)T + \exp(-2\alpha t)T^2}}{\exp(-\alpha t) + \exp(-2\alpha t)T}
$$
(4)

The glass refractive index has a main dependency on two kinds of transmissions, they are.

- (i) Anion and cations that are bridging and non-bridging in the UV region.
- (ii) The vibrations of the lattice in glass networks in the region of infrared.

The linear refractive index is given by

$$
n = -(R+1) \pm 2^9 \frac{\sqrt{R}}{(R-1)}
$$
 (5)

Figures [10](#page-8-0) and [11](#page-9-0) show the photon energy verses refractive Index of  $Pr^{3+}$  and  $Yb^{3+}$ doped phosphate glasses. The refractive index (n) of  $Pr<sup>3+</sup>$  and  $Yb<sup>3+</sup>$  doped phosphate glasses were estimated and found to be 1.54 and at 1.4833 eV respectively.

From the optical constants, electric susceptibility  $(\chi_c)$  can be calculated according to the following relation.

$$
\chi_c = \frac{n^2 - k^2 - \epsilon_o}{4\pi} \tag{6}
$$

where  $\varepsilon_o$  is the dielectric constant. The value of electric susceptibility ( $\chi_c$ ) of Pr<sup>3+</sup> and  $Yb<sup>3+</sup>$ doped phosphate glasses were calculated and found to be 0.1654 and 0.3390 at  $λ = 600$  nm.

Also the complex dielectric constant is related to the refractive index and the extinction coefficient as



<span id="page-8-0"></span>**Fig. 10** Photon energy verses refractive Index of  $Pr<sup>3+</sup>$  doped phosphate glass

![](_page_9_Figure_2.jpeg)

<span id="page-9-0"></span>**Fig. 11** Photon energy verses refractive index of  $Yb<sup>3+</sup>$  doped phosphate glass

$$
\varepsilon_{\rm c} = \varepsilon_{\rm r} + \varepsilon_{\rm i} \tag{7}
$$

The value of real dielectric constants of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses are found to be  $3.0779 \times 10^{-5}$  and 6.  $4957 \times 10^{-5}$  and imaginary dielectric constants are 0.0059 and 0.0044 at  $\lambda$  = 600 nm respectively, where the real and imaginary dielectric constant are

$$
\varepsilon_r = n^2 - K^2 \tag{8}
$$

$$
\varepsilon_i = 2nK \tag{9}
$$

The optical conductivity is a measure of the frequency response of the material when irradiated with light (Lucarelli et al. [2002](#page-26-6); Ugwu et al. [2009\)](#page-26-7).,which is given by

$$
\sigma_{op} = \frac{\text{enc}}{4\pi} \tag{10}
$$

where c is the velocity of light. The electrical conductivity can also be estimated by optical method using the relation

$$
\sigma_e = \frac{2\lambda\sigma_{op}}{4\pi} \tag{11}
$$

Figure [12](#page-10-0) and [13](#page-10-1) shows the variation of the electrical conductivity as a function of photon energy for the rare earth  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glass samples respectively.

The extinction coefficient changes accordance with that photon energy suggest the extinction coefficient decreases with energy and inverse dependence with E.

Figures [14,](#page-11-0) [15](#page-11-1) show the variation of the optical conductivity as a function of photon energy for the rare earth  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glass sample respectively. It can

![](_page_10_Figure_2.jpeg)

<span id="page-10-0"></span>**Fig. 12** Photon energy verses Electrical conductivity of  $Pr<sup>3+</sup>$  doped phosphate glass

![](_page_10_Figure_4.jpeg)

<span id="page-10-1"></span>**Fig. 13** Photon energy verses electrical conductivity of  $Yb^{3+}$  doped phosphate glass

be seen clearly that the optical conductivity directly depends on the absorption coefficient and the refractive index of the material.

The high transmission, low absorbance, low refectance and low refractive index of rare earth  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses in the UV–Vis region make the material a prominent one for anti-refection coating in solar thermal devices and nonlinear optical application.

![](_page_11_Figure_2.jpeg)

<span id="page-11-0"></span>**Fig. 14** Photon energy verses Optical conductivity of  $Pr<sup>3+</sup>$  doped phosphate glass

![](_page_11_Figure_4.jpeg)

<span id="page-11-1"></span>**Fig. 15** Photon energy verses optical conductivity of  $Yb^{3+}$  doped phosphate glass

The low extinction value (10<sup>-3</sup>) and electrical conductivity (10<sup>2</sup> $\Omega$ cm<sup>-1</sup>) show the semiconducting nature of the material.

The high magnitude of optical conductivity confrms the presence of very high photo response nature of the material.

This makes the material more prominent for device applications in information processing and computing. The lower dielectric constant and the higher optical response suggest the better conversion efficiency of the material.

#### **3.2 Dielectric studies**

Rare earth  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glass of desired size were selected for dielectric measurements. Silver paint was applied on opposite faces of cut and polished rare earth  $Pr<sup>3+</sup>$  and  $Yb<sup>3+</sup>$  doped phosphate glass to make a capacitor for investigation. The samples were mounted on a sample holder made of stainless steel. The dielectric measurements were carried out at the frequency range from 100 Hz to 5 MHz at various temperatures with the help of an impedance analyser. Then the dielectric constant of the sample as the function of frequency and temperature were further automated using a computer for data recording, storage and analysis.

The values of capacitance  $(C_p)$  and loss (tan  $\delta$ ) were obtained from the dielectric measurement. Other parameters such as dielectric constant and  $\varepsilon_r$  are calculated using the formula

$$
\varepsilon_{\rm r} = \frac{C_{\rm p}t}{\varepsilon_0 A} \tag{12}
$$

where t is the thickness of the sample,  $\varepsilon_0$ —permittivity of free space. A- The area of the sample, *f*—the frequency of applied field. Studies of temperature and frequency dependence of dielectric properties unveil useful information about structural changes, defect behaviour, and transport phenomena.

Figure [16](#page-12-0) shows the frequency dependency of dielectric constant and dielectric loss for  $Pr<sup>3+</sup>$  doped phosphate glass. The dielectric constant is maximum at low frequencies and decreases further with increasing frequency. It is evident from the measurement,

![](_page_12_Figure_9.jpeg)

<span id="page-12-0"></span>**Fig. 16** Frequency verses dielectric constant and dielectric loss of Pr<sup>3+</sup>doped phosphate glass

that the frequency of the electric feld and the natural frequency of bound charges are equal. It results in the oscillation of the molecules with high energy.

At higher frequencies, the dielectric constant decreases due to the natural frequency of the bounded charge, moreover, at low frequencies all mechanisms such as space charge, orientation, ionic and electric polarization are operative, hence the  $\varepsilon_r$  is maxi-mum (Valente et al. [2011;](#page-27-2) Sadhukhan et al. [1999](#page-26-8)).

Figure [17](#page-13-0) shows the frequency dependency of dielectric constant and dielectric loss for  $Yb^{3+}$  doped phosphate glass. The dielectric constant is minimum at higher frequencies as all the mechanisms mentioned above do not follow the applied electric feld. At this stage, the frequency of the applied electric feld is not equal to the natural frequency of bounded charge. It is not an indication for spontaneous polarization.

The dielectric loss also exhibits similar behaviour for both as prepared glasses. Increase of dielectric loss at low frequencies is attributed to the oscillation of dipoles. At higher frequencies, all the polarization mechanism are not operative, hence energy need not to be spent rotate dipoles, therefore, dielectric loss is also minimum (Raja et al. [2004](#page-26-9)).

#### **3.3 Third order nonlinear study**

Measurement of optical nonlinearity was carried out by Single beam Z-scan technique with nanosecond laser pulses to measure rapidly both non-linear absorption (NLA), non-linear refraction (NLR), Nonlinear changes in index  $(\Delta n)$  and changes in absorption ( $\Delta \alpha$ ) of phosphate glasses. To measure these parameter, 2 mm thick sample was analysed using 300 ns TEA  $CO<sub>2</sub>$  laser pulses having energy of 0.85 mJ.

The transmittance of the sample through the aperture is monitored in the far feld as a function of the position 'Z' of the nonlinear sample in the vicinity of the linear optics

![](_page_13_Figure_9.jpeg)

<span id="page-13-0"></span>**Fig. 17** Frequency verses dielectric constant and dielectric loss of  $Yb^{3+}$ doped phosphate glass

focal position. The required scan range in the experiment depends on the beam parameters and the sample thickness L. Critical parameter is the diffraction length ' $Z_0'$ ' of the focused beam defined as  $\pi \omega_0^2 / \lambda$  for a Gaussian beam where  $\omega_0$  is the focal spot size.

From the closed aperture,  $\Delta T_{p-v} = T_p - T_v$ .

where  $T_p$  and  $T_v$  are the normalized peak and valley transmittances.

The empirically determined relation between the induced phase distortion,  $\Delta \Phi_0$ , and  $\Delta T_{p-v}$ for the third-order nonlinear refractive process in the absence of NLA is (Sheik-Bahae et al. [1989](#page-26-10)).

$$
\Delta T_{p-v} \cong 0.406(1-s)^{0.25} |\Delta \phi| \tag{13}
$$

where  $\Delta \phi = \frac{2\pi}{\lambda} n_2 I_o L_{eff}$ <br>With,  $L_{eff} = (1 - e^{-\alpha L})/\alpha$ , and  $S = 1 - \exp\left(\frac{-2r_a^2}{\omega_a^2}\right)$ ) , is the transmittance of the aperture in the absence of a sample.  $\Delta \Phi_0$  and  $I_0$  are the on-axis (r=0), peak (t=0) nonlinear phase shift and the irradiance with the sample at the focus (Z=0) respectively. The sign of  $\Delta\Phi_0$  and hence n<sub>2</sub> is determined from the relative positions of the peak and valley with Z.

The above relation is accurate to within  $\pm 3\%$  for  $\Delta T_{p-v}$  < 1. As an example, if the induced optical path length change due to the nonlinearity is  $\lambda$ /250,  $\Delta T_{p-v} \approx 1\%$  for an aperture transmittance of  $S=0.5$ . Use of  $S=0.5$  is a good compromise between having a large signal which averages possible beam non-uniformities, thus reducing background signals, and loss of sensitivity. Figure  $18. Z/Z<sub>o</sub>$  $18. Z/Z<sub>o</sub>$  verses normalized transmittance for closed aperture and open aperture of  $Pr^{3+}$  phosphate glass. The nonlinear refractive index  $(n_2)$  and nonlinear absorption coefficient ( $\beta$ ) are given by (Mojdehi et al. [2013\)](#page-26-11),

$$
n_2 = \frac{\Delta \phi}{K I_{o L_{eff}}} \tag{14}
$$

![](_page_14_Figure_11.jpeg)

<span id="page-14-0"></span>**Fig. 18**  $Z/Z_0$  verses normalized transmittance for closed aperture and open aperture of  $Pr^{3+}$  phosphate glass

$$
\beta = \frac{2\sqrt{2}\Delta T}{I_o L_{\text{eff}}}
$$
\n(15)

where  $\Delta T$  is the difference between the normalized transmittance at  $z=0$  and  $z \to \infty$  of the open aperture Z-scan.  $I_0$  is the radiation intensity at the focal point, which is 5 W/cm<sup>2</sup>. Figure [19](#page-15-0) shows Z/Zo verses normalized transmittance for closed aperture and open aperture of  $Yb^{3+}$  phosphate glass. The NLO absorptive property of rare earth doped phosphate glasses indicate a positive sign for the nonlinear refraction and exhibits self-defocusing behaviour.

Then the effective third-order NLO refractive index nonlinear refractive index  $(n_2)$  and nonlinear absorption coefficient (β) of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses are estimated and listed in Table [1](#page-16-0).

The real and imaginary parts of the third order nonlinear susceptibility  $\chi$ 3 are defined as,

$$
Re \chi^3 = \frac{10^{-4} (\varepsilon_o C^2 n_o^2)}{3.14} \text{esu}
$$
 (16)

$$
Im \chi^3 = \frac{10^{-2} (\varepsilon_o C^2 n_o^2 \cdot \lambda \cdot \beta)}{4\pi^2} \, \text{esu} \tag{17}
$$

where  $\varepsilon_0$  is the vacuum permittivity, n<sub>o</sub> is the linear refractive index of the sample and c is the velocity of light in vacuum. Thus, we can easily obtain the absolute value of  $\chi^3$  by the following formula [32].

$$
\chi^3 = \sqrt{\left(Re\,\chi^3\right)^2 + \left(Im\,\chi^3\right)^2} \, \text{esu} \tag{18}
$$

The third order susceptibility  $(\chi^3)$  of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses are found to be  $11.3073 \times 10^{-8}$  $11.3073 \times 10^{-8}$  esu and  $11.8078 \times 10^{-8}$  esu respectively. Table 1 shows the

![](_page_15_Figure_11.jpeg)

<span id="page-15-0"></span>**Fig. 19**  $Z/Z_0$  verses normalized transmittance for closed aperture and open aperture of Yb<sup>3+</sup> phosphate glass

<span id="page-16-0"></span>![](_page_16_Picture_88.jpeg)

experimental results of the Z-scan technique for prepared glasses. Z-scan measurement is important parameter because it is used to estimate and control refractive index gratings, which are signifcant for some laser based dynamic holography (Shanmugavelu and Venkatramu [2014](#page-26-12)).

#### **3.4 Micro‑hardness study**

Hardness is the resistance offered by the glass for localized plastic deformation. This property is basically related to the glass structure of the material and the electronic factors operating to make the structure stable.

The micro hardness has been measured along the plane of the rare earth  $Pr^{3+}$  and  $Yb^{3+}$ doped phosphate glass at room temperature using Leitz–Wetzler hardness tester ranging from 10 to 200 g. The corresponding indentation length were measured. The Vickers micro hardness number  $(H_v)$  at each load was calculated using the relation (Sangwal [2009](#page-26-13)).

$$
H_v = 1.8544 \text{ P/d}^2 (\text{Kg/mm}^2)
$$
 (19)

where,  $H_v$  is the Vickers hardness number, P is the applied load and d is the average diagonal length of the indentation mark. The microhardness number varied with respect to applied load.

Figures [20](#page-17-0), [21](#page-18-0) represent load verses. hardness number and yield strength of  $Pr<sup>3+</sup>$  and  $Yb<sup>3+</sup>$  doped phosphate glass. It is seen that the hardness value increases with increasing load values up to 100 g, which is known as reverse indentation size effect (RISE). The crack was formed beyond 100 g. it is due to high stress required for homogeneous nucleation of dislocations in the small dislocation free region indented.

The yield strength of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses also increased with increasing load. The yield strength refer to the stress at which the materials begin to deform plastically and depends on the Meyer's index (n). It is calculated using the relation,

![](_page_17_Figure_10.jpeg)

<span id="page-17-0"></span>Fig. 20 Load verses hardness number and yield strength of  $Pr<sup>3+</sup>$  doped phosphate glass

![](_page_18_Figure_2.jpeg)

<span id="page-18-0"></span>**Fig. 21** Load verses hardness number and yield strength of  $Yb<sup>3+</sup>$  doped phosphate glass

$$
\sigma_{y} = (H_{v}/3) \text{ kg/mm}^2 \tag{20}
$$

The work hardening coefficient value (*n*) of the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses were determined from a plot of log P versus log d using Mayer's relation,  $P = k_1 d^n$  (Onitsch [1956\)](#page-26-14) and it shows a straight line as shown in Figs. [22](#page-19-0) and [23.](#page-19-1) According to Onitsch, 'n' should lie between 1 and 1.6 for hard materials and above 1.6 for soft materials. The n value of both glasses are above 1.6. Hence  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses confrmed that both material belong to soft materials category.

The elastic stiffness constant (C11) for  $Pr<sup>3+</sup>$  and Yb<sup>3+</sup> doped phosphate glasses were calculated using Wooster's empirical formula C11 =  $H^{7/4}$ . Figures [24](#page-20-0) and [25](#page-20-1) show that stiffness constants of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses varied with applied load. From the measurement, the stiffness constants of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses are increased upto 100 g. It represent that the binding force between the ions are very strong. The analysis based on the fracture toughness of  $Pr<sup>3+</sup>$  and  $Yb<sup>3+</sup>$  doped phosphate glasses. if  $c/a \ge 2.5$  (i.e. the crack system is median or half penny) where a is the length of the half diagonal of the indent. For  $c/a < 2.5$  i.e. the crack system is Palmqvist type. The result suggest that nature of crake is median.

The fracture toughness of the material was given by the relation (Anstis et al. [1981](#page-26-15)),

$$
K_c = P/\beta_0 C^{3/2} (MPa m^{-1/2})
$$
 (21)

where P is an applied load in kg, c is the crack length measured and  $\beta_0$  is taken as 7 for Vickers's indenter. Figures [26](#page-21-0) and [27](#page-21-1) show load verses fracture toughness and brittle index of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses. Brittle index of the  $Pr^{3+}$  and  $Yb^{3+}$ doped phosphate glasses were calculated using the relation (Lawn and Marshall [1979](#page-26-16)),

$$
B_i = H_v / K_c m^{-1/2}
$$
 (22)

![](_page_19_Figure_2.jpeg)

<span id="page-19-0"></span>**Fig. 22**  $\log P$  verses  $\log d$  of  $Pr^{3+}$  doped phosphate glass

![](_page_19_Figure_4.jpeg)

<span id="page-19-1"></span>**Fig. 23**  $\log P$  verses  $\log d$  of  $Yb^{3+}$  doped phosphate glass

It is observed that fracture toughness increases with increasing load and brittle index decreased with respect to load. The fracture induced brittleness and afect the mechanical behaviour of the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses. Both glasses have good brittle index.

![](_page_20_Figure_2.jpeg)

<span id="page-20-0"></span>**Fig. 24** Load verses stiffness constant and  $c/a$  of  $Pr<sup>3+</sup>$  doped phosphate glass

![](_page_20_Figure_4.jpeg)

<span id="page-20-1"></span>**Fig. 25** Load verses stiffness constant and  $c/a$  of  $Yb^{3+}$  doped phosphate glass

## **3.5 Vibrating sample magnetometer study**

The magnetic property of the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses were studied using VSM analysis at room temperature and hysteresis curves are shown in Fig. [28.](#page-22-0) From the measurement, the magnetization M linearly depends on applied feld.The maximum values of magnetization of occurred at  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses were found to be  $2.72 \times 10^{-2}$  emu/g and  $2.68 \times 10^{-2}$  emu/g respectively. While decreasing the field, the M slightly decreases with decreasing H and shows a rentivity  $1.134 \times 10^{-3}$  emu/g and  $6.952 \times 10^{-4}$  emu/g for Pr<sup>3+</sup> and Yb<sup>3+</sup> doped phosphate glasses at H=0.

![](_page_21_Figure_2.jpeg)

<span id="page-21-0"></span>**Fig. 26** Load verses fracture toughness and brittle index of  $Pr<sup>3+</sup>$  doped phosphate glass

![](_page_21_Figure_4.jpeg)

<span id="page-21-1"></span>Fig. 27 Load verses fracture toughness and brittle index of Yb<sup>3+</sup> doped phosphate glass

The magnetization decreases after saturation. This may be due to decrease of spin exchange interaction at higher applied feld. For magnetic feld of opposite sign, the variation of M shows anomalous property. The M suddenly drops to zero under small magnetic feld which denotes a frst order metamagnetic transition (Benila et al. [2017](#page-26-17); J. Elberin Mary Theras, D. Kalaivani, J. Arul Martin Mani, D. Jayaraman, V. Joseph [2016](#page-26-18)). The coercivity of the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses are observed to be quite remarkable even though the magnetization is weak and it is found to be 207 G, 205 G respectively. The area of the loop is very steep, so the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses belong to soft magnetic material type.

![](_page_22_Figure_2.jpeg)

<span id="page-22-0"></span>**Fig. 28** Hysteresis curves of  $Yb^{3+}$  and  $Pr^{3+}$ doped phosphate glass

### **3.6 Electrochemical properties**

The Electrochemical properties were carried out for  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses with diferent scan rate in 5 M KOH electrolyte.

The specifc capacitances of the cyclic voltammetry measurement (CCV) of the electrodes are calculated by using the relation (Gomez and Kalu [2013](#page-26-19); Xiang et al. [2013\)](#page-27-3):

$$
C_{CV} = 1/mv\Delta v f \, ddV \tag{23}
$$

where I is the response current (A),  $v$  is the scan rate (mV s-1), m is the mass of the active materials (g), and  $\Delta V$  is the potential window (V).

The specific capacitance of the galvanostatic charge–discharge measurements  $(C_{GCD})$  of the electrodes was calculated using the equation:

$$
C_{GCD} = I\Delta t / m\Delta V \tag{24}
$$

where I is the discharge current (A),  $\Delta t$  is the discharge time, m is the mass of the active materials (g), and  $\Delta V$  is the potential window (V).

The energy density (E) and power density ( P) for the electrodes were calculated by the following equations (Khajonrit et al. [2018](#page-26-20); Yan et al. [2012](#page-27-4)):

$$
E ( Wh / Kg) = CGCD \Delta V^2 / 7.2
$$
 (25)

$$
P(W/Kg) = E \times 3600 / t \tag{26}
$$

where  $C_{GCD}$  is the specific capacitance calculated from charge–discharge measurement (F/g),  $\Delta V$  is the potential window and t is the discharge time (s). Current density versus specific capacitance of the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses as shown in Fig. [29.](#page-23-0)

The measurements show that the specific capacitance of the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses decreases with increasing current density. Figure [30](#page-24-0) shows the energy density versus power density of the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses.

 $Yb^{3+}$  doped phosphate glasses have more specific capacitance and power density than  $Pr<sup>3+</sup>$  doped phosphate glasses. The power density decreases with increase in energy density. The V-I curves of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glass was drawn from CV analysis and shown in Fig.  $31$  and  $32$  respectively. The oxidatiom peak potential of  $Yb^{3+}$  doped phosphate glass is more than  $Pr^{3+}$  doped phosphate glass at the scan rate of 80 mV/s.

The nyquist plots drawn between real and imaginary parts of the impedance of  $Pr<sup>3+</sup>$  and  $Yb<sup>3+</sup>$  doped phosphate glasses and shown in Fig. [33](#page-25-1). From the measurement, the imaginary impedance increases with respect to real impedance. The end of the curve represent the resistance of the  $Yb^{3+}$  and  $Pr^{3+}$  doped phosphate glasses at a particular temperature. In general, the glass are ionic or mixed ionic conductor. This resistance may afect the conductivity of the glasses which include network rigidity, network connectivity, the charge, size and mobility of the modifier ions. So the electrochemical performance of the  $Yb^{3+}$ doped phosphate is more than  $Pr^{3+}$  doped phosphate glass. This result suggest that the  $Pr^{3+}$ and  $Yb^{3+}$  doped phosphate glass have a potential for development as electrode materials in super capacitor application.

# **4 Conclusion**

The rare earth  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glass were prepared by quenching method using electrical mufulfurnace. The optical parameters such as optical band gap, optical conductivity, refractive index and extinction coefficient of rare earth  $Pr<sup>3+</sup>$  and  $Yb^{3+}$  doped phosphate glass were estimated through Uv–Visible spectroscopy analysis. The dielectric permittivity, dielectric loss and electrical conductivity of the prepared

![](_page_23_Figure_7.jpeg)

<span id="page-23-0"></span>**Fig. 29** Current density verses Specific capacitance of the  $Pr<sup>3+</sup>$  and  $Yb<sup>3+</sup>$  doped phosphate glasses

![](_page_24_Figure_2.jpeg)

<span id="page-24-0"></span>**Fig. 30** Energy density verses Power density of the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses

![](_page_24_Figure_4.jpeg)

<span id="page-24-1"></span>**Fig. 31** Potential applied verses Current for  $Pr<sup>3+</sup>$  doped phosphate glass

samples have been studied using LCR meter with diferent frequencies. The results suggest that the dipoles formed in the glass containing high frequency value are polarized more easily.

The Vickers hardness number, brittle index, yield strength, fracture toughness of the prepared glasses were calculated. The nonlinear optical properties such as third order nonlinear coefficient, nonlinear refractive index were estimated for  $Pr^{3+}$  and  $Yb^{3+}$  doped

![](_page_25_Figure_2.jpeg)

<span id="page-25-0"></span>**Fig. 32** Potential applied verses Current for  $Yb^{3+}$  doped phosphate glass

![](_page_25_Figure_4.jpeg)

<span id="page-25-1"></span>**Fig. 33** The Nyquist plot of  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses

phosphate glass using Z-scan technique. These studies prove that these type of glasses can be used in nonlinear optical applications.

The coercivity and retentivity of the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glasses were calculated. The hysteresis loop suggest that the material belong to soft magnetic material category. The energy density (E) and power density (P) of the  $Pr^{3+}$  and  $Yb^{3+}$  doped phosphate glass were calculated.

# **Compliance with ethical standards**

**Confict of interest** The authors declare that they have no confict of interest.

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