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Optical solitons in nematic liquid crystals with Kerr and parabolic law nonlinearities

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Abstract
In this work, a study is carried out to analyze nematicons in liquid crystals in the presence of Kerr and parabolic law nonlinearity. Exp($-\phi(\xi)$)-Expansion method is incorporated in this regard. Solutions obtained include hyperbolic, periodic and rational solutions along with their combo type solutions in both cases of nonlinearity and their existence is guaranteed by the constraints retrieved during the process.

Keywords Spacial optical solitons · Liquid crystals · Nematicons · Kerr law nonlinearity · Parabolic law nonlinearity

1 Introduction

In optics, spatial optical solitons in nematic liquid crystals (NLC), also known as nematicons, is a well-established topic now and it has been addressed in many books and a large number of scientific articles. Spatial optical solitons form a specific class, as optics in space is characterized by diffraction rather than dispersion, beam size rather than pulse duration, one or two transverse dimensions rather than one in the temporal domain. A lot of work has been done recently on solitons and, especially, spacial solitons due to their importance and vast applications, for instance, see Zhou et al. (2013), Ekici et al. (2017a), Raza et al. (2017), Raza and Javid (2018a, b), Javid and Raza (2018), Raza and Zubair (2018), Zubair et al. (2018), Liu

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et al. (2018, 2019a, b), Zhang et al. (2018), Zhou et al. (2018), Yu et al. (2018, 2019), Yang et al. (2018), Bulut et al. (2018), Rezazadeh et al. (2018a, b), Biswas et al. (2018) and Tabiryan et al. (1986) and references therein. Nematic liquid crystals (NLC) are a class of dielectric media possessing a number of mechanisms of optical nonlinearity such as thermal, electronic and photo refractive nonlinearities (Khoo and Wu 1993; Simoni 1997; Assanto 2012). However, their re-orientational nonlinearity became their signature. NLC are supposed to be an ideal test bed to explore nonlinear optical phenomena and light manipulation (Peccianti and Assanto 2012; Assanto and Karpierz 2009) because of their unusual nonlinearity and negligible absorption. Nematicons are stable and robust self-trapped non-diffractive light rays in NLC. The importance of their study arises due to their optically nonlinear, non-resonant, non-local and saturable response which enables light to self-confine and guide additional optical signals. Nematicons are being utilized as an ideal case for applications in all optical information processing. The name 'nematicon' was invented by Assanto in 2003 and used thereafter (Conti et al. 2004). The term was first used in Garcìa-Reimbert et al. (2006a) as part of its title and, since then, a large number of theoretical, experimental and numerical results have been obtained in many papers and conferences and paved the way to form a body of literature on this topic (Garcìa-Reimbert et al. 2006b; Minzoni et al. 2007; Sciberras et al. 2014; Savescu et al. 2015; Kavitha et al. 2013, 2014; Ekici et al. 2017b). Nematicons have also been recently studied by extended trial equation method (He and Wu 2006).

Keeping in mind this extensive study of nematicons in optics and other fields, we employed Exp($-\phi(\xi)$)-Expansion method in this article for the investigation of nematicons with Kerr law nonlinearity as well as Parabolic law nonlinearity. Exp($-\phi(\xi)$)-Expansion method is a powerful method to determine the travelling wave solutions with wide applications in optics and other fields. The method was first proposed by He and Abdou (2007) and further studied systematically afterwards (Noor et al. 2008; Navickas et al. 2010; Ebaid 2012; Bekir and Boz 2008). In the recent years, the method proved to be a powerful method for determining travelling wave solutions of a number of nonlinear PDEs (Akbar and Ali 2011; Kamruzzaman and Akbar 2014; Ekici et al. 2017c; Ravi et al. 2017; Arnous et al. 2017). The method maintained its pace and we obtained quite promising results in this article.

The rest of the article is organized such that in Sect. 2, the method is briefly explained and in Sect. 3, it is applied successfully to the governing equations of nematicons to determine different solutions for Kerr law nonlinearity and Parabolic law nonlinearity in Sects. 3.1 and 3.2 respectively. The article has concluding discussion in Sect. 4 which summarizes the whole article in a nutshell.

2 Exp($-\phi(\xi)$)-Expansion method

In this section, the Exp($-\phi(\xi)$)-Expansion method is summarized step by step as follows:

Step 1 Let us consider a nonlinear PDE given by

$$f\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial t^2}, \frac{\partial^2 u}{\partial x^2}\right) = 0. \tag{1}$$

Step 2 Applying the wave transformation $u(x,t) = F(\varepsilon)$, where $\varepsilon = x - lt$ to (1), we get a nonlinear ODE given by

$$g(F, F', F'', \ldots) = 0, \tag{2}$$

where prime ($'$) indicates the derivative w.r.t. ε.

Step 3 Assume that (2) has a truncated series form given by

$$F(\varepsilon) = \sum_{n=0}^{N} a_n(\exp(-\varphi(\varepsilon)))^n, \tag{3}$$

where $a_n, n = 0, 1, ..., N(a_n \neq 0)$ are both constants to be determined and $\varphi(\varepsilon)$ is given by the following expressions for different cases:

Case 1 When $\lambda^2 - 4\nu > 0$ and $\nu \neq 0$, then

$$\varphi_1(\varepsilon) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\nu}\tanh(\frac{\sqrt{\lambda^2-4\nu}}{2}(\varepsilon + C)) - \lambda}{2\nu}\right).$$

Case 2 When $\lambda^2 - 4\nu > 0$, $\nu = 0$ and $\lambda \neq 0$, then

$$\varphi_2(\varepsilon) = -\ln\left(\frac{\lambda}{\cosh(\lambda(\varepsilon + C) + \sinh(\lambda(\varepsilon + C)) - 1}\right).$$

Case 3 When $\lambda^2 - 4\nu < 0$ and $\nu \neq 0$, then

$$\varphi_3(\varepsilon) = \ln\left(\frac{\sqrt{4\nu - \lambda^2}\tan(\frac{\sqrt{4\nu-\lambda^2}}{2}(\varepsilon + C)) - \lambda}{2\nu}\right).$$

Case 4 When $\lambda^2 - 4\nu > 0$, $\nu \neq 0$ and $\lambda \neq 0$, then

$$\varphi_4(\varepsilon) = \ln\left(-\frac{2\lambda(\varepsilon + C) + 4}{\lambda^2(\varepsilon + C)}\right).$$

Case 5 When $\lambda^2 - 4\nu = 0$, $\nu = 0$ and $\lambda = 0$, then

$$\varphi_5(\varepsilon) = \ln((\varepsilon + C)).$$

The following condition is also satisfied:

$$\varphi'(\varepsilon) = \exp(-\varphi(\varepsilon)) + \nu \exp \varphi(\varepsilon)) + \lambda.$$

Step 4 The value of N is determined by balancing the highest order nonlinear term with the highest order derivative of F in (2).

Step 5 Substituting (3) in (2) gives

$$P(\exp(-\varphi(\varepsilon))) = 0. \tag{4}$$

Comparison of different terms in above equation gives the system of nonlinear equations whose solution yields a number of exact solutions for (1).

3 Governing system

The following coupled system of equations describe the dynamics of nematicons (Simoni 1997; Arnous et al. 2017):

$$\imath(q)_t + \alpha q_{xx} + \beta \vartheta q = 0, \tag{5}$$

$$\gamma \vartheta_{xx} + \chi \vartheta + aF(|q|^2) = 0, \tag{6}$$

where $q(x, t)$ stands for the wave profile, $\vartheta(x, t)$ represents the tilt angle of the liquid crystal molecule. Furthermore, the first term in (5) accounts for the temporal evolution of nematicons and the second term stands for the group velocity dispersion. Functional F will be chosen for the type of nonlinearity to be studied and $\alpha, \beta, \gamma, \chi, a$ are real numbers.

The solution of the above system of equations (5) and (6), chosen in phase-amplitude format, is given by

$$q(x, t) = Q(\xi)e^{\imath\Phi(x,t)}, \quad \xi = k(x - ut), \tag{7}$$

and

$$\vartheta(x, t) = P(\xi). \tag{8}$$

The phase $\Phi(x, t)$ is defined by

$$\Phi(x, t) = -\kappa x + \omega t + \theta, \tag{9}$$

where κ is soliton frequency, θ is a phase constant and ω gives the wave number of soliton. Inserting (7) and (8) in (5) and (6) and further splitting the resulting equation in real and imaginary parts yields

$$\alpha k^2 Q'' - (\alpha\kappa^2 + \omega)Q + \beta PQ = 0, \tag{10}$$

$$\gamma k^2 P'' + \chi P + aF(Q^2) = 0, \tag{11}$$

$$-kuQ' - 2\alpha k\kappa Q' = 0, \tag{12}$$

$$u = -2\alpha\kappa, \tag{13}$$

where u represents the speed of the soliton.

In the next subsections, nematicons shall be investigated in the presence of Kerr and parabolic law nonlinearity for the functional F.

3.1 Kerr law nonlinearity

The Kerr law nonlinearity arises when $F(q) = q$. In this case, (6) simplifies to

$$\gamma \vartheta_{xx} + \chi \vartheta + a(|q|^2) = 0, \tag{14}$$

and (11) reduces to

$$\gamma k^2 P'' + \chi P + \alpha Q^2 = 0. \tag{15}$$

Balancing Q'' with PQ and P'' with Q^2 in Eqs. (10) and (15) gives $N = 2$ and $M = 2$. Resultantly, we get

$$Q(\xi) = \alpha_0 + \alpha_1 \exp(-\varphi(\xi)) + \alpha_2 \exp(-2\varphi(\xi)), \quad \alpha_2 \neq 0, \tag{16}$$

$$P(\xi) = \beta_0 + \beta_1 \exp(-\varphi(\xi)) + \beta_2 \exp(-2\varphi(\xi)), \quad \beta_2 \neq 0. \tag{17}$$

Substituting Eqs. (16), (17) and required derivatives of $P(\xi)$ and $Q(\xi)$ in Eqs. (10) and (15) and equating the coefficients of each power of $Q(\xi)$ to zero, we get a system of nonlinear algebraic equations given by

$$
\begin{aligned}
&ak^2(\lambda\mu\alpha_1 + 2\alpha_2\mu^2) - (w + a\kappa^2)\alpha_0 = 0,\\
&-(w + a\kappa^2)\alpha_1 + ak^2(6\lambda\mu\alpha_2 + 2\alpha_1\mu + \lambda^2\alpha_1) + \beta\alpha_0\beta_1 + \beta\alpha_1\beta_0 = 0,\\
&-(w + a\kappa^2)\alpha_2 + ak^2(4\lambda^2\alpha_2 + 3\lambda\alpha_1 + 8\alpha_2\mu) + \beta\alpha_0\beta_2 + \beta\alpha_1\beta_1 + \beta\alpha_2\beta_0 = 0,\\
&ak^2(10\lambda\alpha_2 + 2\alpha_1) + \beta\alpha_1\beta_2 + \beta\alpha_2\beta_1 = 0,\\
&6ak^2\alpha_2 + \beta\alpha_2\beta_2 = 0,\\
&\gamma k^2(\lambda\mu\beta_1 + 2\beta_2\mu^2) + \chi\beta_0 + a\alpha_0^2 = 0,\\
&\chi\beta_1 + \gamma k^2(6\beta_2\lambda\mu + 2\beta_1\mu + \lambda^2\beta_1) + 2a\alpha_0\alpha_1 = 0,\\
&\chi\beta_2 + \gamma k^2(4\beta_2\lambda^2 + 3\beta_1\lambda + 8\beta_2\mu) + a(2\alpha_0\alpha_2 + \alpha_1^2) = 0,\\
&\gamma k^2(10\beta_2\lambda + 2\beta_1) + 2a\alpha_1\alpha_2 = 0,
\end{aligned}
$$

and

$$6\gamma k^2\beta_2 + a\alpha_2^2 = 0.$$

Solving above system of equations, we have the following cases and accordingly different forms of hyperbolic solutions.

Case 1

$$
w = -\frac{\alpha(\chi + \gamma\kappa^2)}{\gamma},\ \ \alpha_0 = \pm\frac{1}{24}\left(\frac{\beta a\alpha_1^2 + 36\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right),\ \ \alpha_1 = \alpha_1,\ \ \alpha_2 = \pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}
$$

$$
\beta_0 = -\frac{1}{24}\frac{a\beta\alpha_1^2 + 36\alpha k^2\chi}{k^2\beta\gamma},\quad \beta_1 = \pm\frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}},\quad \beta_2 = -\frac{6\alpha k^2}{\beta}.
$$

Now, the exact traveling wave solutions of the system along with the condition $\lambda^2 - 4\mu > 0$ (where $\mu \neq 0$) are as follows:

$$
\begin{aligned}
q_{1,2}(x,t) =& \frac{1}{24}\left(\frac{\beta a\alpha_1^2 + 36\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right) + \alpha_1\left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x + 2\alpha\kappa t) + C)) + \lambda}\right)\\
&\pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}\left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x + 2\alpha\kappa t) + C)) + \lambda}\right)^2 \times e^{i(-\kappa x + \varpi t + \theta)},\\
\vartheta_{1,2}(x,t) =& -\frac{1}{24}\frac{a\beta\alpha_1^2 + 36\alpha k^2\chi}{k^2\beta\gamma} \pm \frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}\left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x + 2\alpha\kappa t) + C)) + \lambda}\right)\\
&-\frac{6\alpha k^2}{\beta}\left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x + 2\alpha\kappa t) + C)) + \lambda}\right)^2 \times e^{i(-\kappa x + \varpi t + \theta)}.
\end{aligned}
$$

For the constraint conditions $\lambda^2 - 4\mu > 0, \mu = 0$ and $\lambda \neq 0$, another combined form of hyperbolic solutions are obtained as

$$
\begin{aligned}
q_{3,4}(x,t) =& \frac{1}{24}\left(\frac{\beta a\alpha_1^2 + 36\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right) \\
&+ \alpha_1\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C)) + \sinh(\lambda(k(x+2\alpha\kappa t)+C)) - 1}\right) \\
&\pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C)) + \sinh(\lambda(k(x+2\alpha\kappa t)+C)) - 1}\right)^2 \\
&\times e^{i(-\kappa x+\varpi t+\theta)}, \\
\vartheta_{3,4}(x,t) =& -\frac{1}{24}\frac{a\beta\alpha_1^2 + 36\alpha k^2\chi}{k^2\beta\gamma} \\
&\pm \frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C)) + \sinh(\lambda(k(x+2\alpha\kappa t)+C)) - 1}\right) \\
&- \frac{6\alpha k^2}{\beta}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C)) + \sinh(\lambda(k(x+2\alpha\kappa t)+C)) - 1}\right)^2 \\
&\times e^{i(-\kappa x+\varpi t+\theta)}.
\end{aligned}
$$

Periodic solutions for the constrained conditions $\lambda^2 - 4\mu < 0, \mu \neq 0$ emerged during the calculations as follows:

$$
\begin{aligned}
q_{5,6}(x,t) =& \pm\frac{1}{24}\left(\frac{\beta a\alpha_1^2 + 36\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right) + \alpha_1\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C)) + \lambda}\right) \\
&\pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tanh(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C)) + \lambda}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)}, \\
\vartheta_{5,6}(x,t) =& -\frac{1}{24}\frac{a\beta\alpha_1^2 + 36\alpha k^2\chi}{k^2\beta\gamma} \pm \frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C)) + \lambda}\right) \\
&- \frac{6\alpha k^2}{\beta}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C)) + \lambda}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)}.
\end{aligned}
$$

The constrained conditions $\lambda^2 - 4\mu = 0, \mu \neq 0$ and $\lambda \neq 0$ exist to hold the rational solutions given by

$$q_{7,8}(x,t) = \pm \frac{1}{24}\left(\frac{\beta a\alpha_1^2 + 36\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right) + \alpha_1\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right) \pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)},$$

$$\vartheta_{7,8}(x,t) = -\frac{1}{24}\frac{a\beta\alpha_1^2 + 36\alpha k^2\chi}{k^2\beta\gamma} \pm \frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right) - \frac{6\alpha k^2}{\beta}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)}.$$

Also, another form of rational solution is obtained for $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$ as follows:

$$q_{9,10}(x,t) = \pm \frac{1}{24}\left(\frac{\beta a\alpha_1^2 + 36\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right) + \alpha_1\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2 \pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)},$$

$$\vartheta_{9,10}(x,t) = -\frac{1}{24}\frac{a\beta\alpha_1^2 + 36\alpha k^2\chi}{k^2\beta\gamma} \pm \frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2 - \frac{6\alpha k^2}{\beta}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)}.$$

Case 2

$$w = -\frac{\alpha(\chi+\gamma\kappa^2)}{\gamma}, \quad \alpha_0 = \pm\frac{1}{24}\left(\frac{\beta a\alpha_1^2 - 12\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right), \quad \alpha_1 = \alpha_1, \quad \alpha_2 = \pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}$$

$$\beta_0 = \frac{1}{24}\frac{-a\beta\alpha_1^2 + 12\alpha k^2\chi}{k^2\beta\gamma}, \quad \beta_1 = \pm\frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}, \quad \beta_2 = -\frac{6\alpha k^2}{\beta}.$$

In this case, the exact traveling wave solutions of the system for $\lambda^2 - 4\mu > 0, \mu \neq 0$ are as follows:

$$
\begin{aligned}
q_{1,2}(x,t) = &\pm \frac{1}{24}\left(\frac{\beta a\alpha_1^2 - 12\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right) + \alpha_1\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)\\
&\pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)},\\
\vartheta_{1,2}(x,t) = &\frac{1}{24}\frac{-a\beta\alpha_1^2 + 12\alpha k^2\chi}{k^2\beta\gamma} \pm \frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)\\
&- \frac{6\alpha k^2}{\beta}\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)}.
\end{aligned}
$$

For the constraint conditions $\lambda^2 - 4\mu > 0, \mu = 0$ and $\lambda \neq 0$, another combined form of hyperbolic solutions is calculated as follows:

$$
\begin{aligned}
q_{3,4}(x,t) = &\pm \frac{1}{24}\left(\frac{\beta a\alpha_1^2 - 12\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right)\\
&+ \alpha_1\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C)) + \sinh(\lambda(k(x+2\alpha\kappa t)+C)) - 1}\right)\\
&\pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C)) + \sinh(\lambda(k(x+2\alpha\kappa t)+C)) - 1}\right)^2\\
&\times e^{i(-\kappa x+\varpi t+\theta)},\\
\vartheta_{3,4}(x,t) = &\frac{1}{24}\frac{-a\beta\alpha_1^2 + 12\alpha k^2\chi}{k^2\beta\gamma}\\
&\pm \frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C)) + \sinh(\lambda(k(x+2\alpha\kappa t)+C)) - 1}\right)\\
&- \frac{6\alpha k^2}{\beta}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C)) + \sinh(\lambda(k(x+2\alpha\kappa t)+C)) - 1}\right)^2\\
&\times e^{i(-\kappa x+\varpi t+\theta)}.
\end{aligned}
$$

For $\lambda^2-4\mu<0, \mu\neq 0$, periodic solutions are given by

$$q_{5,6}(x,t)=\pm\frac{1}{24}\left(\frac{\beta a\alpha_1^2-12\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right)+\alpha_1\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)$$
$$\pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tanh(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)},$$
$$\vartheta_{5,6}(x,t)=\frac{1}{24}\frac{-a\beta\alpha_1^2+12\alpha k^2\chi}{k^2\beta\gamma}\pm\frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)$$
$$-\frac{6\alpha k^2}{\beta}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)}.$$

Rational solutions for the constrained conditions $\lambda^2-4\mu=0, \mu\neq 0$ and $\lambda\neq 0$, are calculated as follows:

$$q_{7,8}(x,t)=\pm\frac{1}{24}\left(\frac{\beta a\alpha_1^2-12\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right)+\alpha_1\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)$$
$$\pm 6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)},$$
$$\vartheta_{7,8}(x,t)=\frac{1}{24}\frac{-a\beta\alpha_1^2+12\alpha k^2\chi}{k^2\beta\gamma}\pm\frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)$$
$$-\frac{6\alpha k^2}{\beta}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)}.$$

Again, another form of rational solution is obtained for $\lambda^2-4\mu=0, \mu=0, \lambda=0$ given below.

$$q_{9,10}(x,t)=\pm\frac{1}{24}\left(\frac{\beta a\alpha_1^2-12\alpha k^2\chi}{a\beta k^2\sqrt{\frac{\alpha\gamma}{a\beta}}}\right)+\alpha_1\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2$$
$$\pm\,6k^2\sqrt{\frac{\alpha\gamma}{a\beta}}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)},$$
$$\vartheta_{9,10}(x,t)=\frac{1}{24}\frac{-a\beta\alpha_1^2+12\alpha k^2\chi}{k^2\beta\gamma}\pm\frac{\alpha\alpha_1}{\sqrt{\frac{\alpha\gamma}{a\beta}}}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2$$
$$-\,\frac{6\alpha k^2}{\beta}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)}.$$

All these solutions are valid if $a\alpha\beta\gamma\geq 0$.

3.2 Parabolic law nonlinearity

This type of nonlinearity occurs when $F(s)=c_1s+c_2s^2$. In this case, (6) reduces to

$$c\theta_{xx}+\lambda\theta+\alpha(c_1|q|^2+c_2|q|^4)=0, \tag{18}$$

and (11) simplifies to

$$ck^2Q''+\lambda Q+\alpha(c_1P^2+c_2P^4)=0. \tag{19}$$

Balancing P'' with PQ and Q'' with P^4 in Eqs. (10) and (19) gives $N=1$ and $M=2$. Thus, we get

$$Q(\xi)=\alpha_0+\alpha_1\exp(-\varphi(\xi)),\quad \alpha_1\neq 0, \tag{20}$$

$$P(\xi)=\beta_0+\beta_1\exp(-\varphi(\xi))+\beta_2\exp(-2\varphi(\xi)),\quad \beta_2\neq 0. \tag{21}$$

Substituting (20), (21) and required derivatives of $P(\xi)$ and $Q(\xi)$ in Eqs. (10) and (19) and equating the coefficients of each power of $Q(\xi)$ to zero, we obtain a system of nonlinear algebraic equations given by

$$\begin{aligned}
\alpha k^2\lambda\mu\alpha_1+\beta\alpha_0\beta_0-(w+\alpha\kappa^2)\alpha_0 &= 0,\\
\alpha k^2(2\alpha_1\mu+\lambda^2\alpha_1)+\beta\alpha_0\beta_1+\beta\alpha_1\beta_0-(w+\alpha\kappa^2)\alpha_1 &= 0,\\
3\alpha k^2\lambda\alpha_1+\beta\alpha_0\beta_2+\beta\alpha_1\beta_1 &= 0,\\
2\alpha k^2\alpha_1+\beta\alpha_1\beta_2 &= 0,\\
\gamma k^2(\lambda\mu\beta_1+2\beta_2\mu^2)+\chi\beta_0+a(c_1\alpha_0^2+c_2\alpha_0^4) &= 0,\\
\chi\beta_2+\gamma k^2(4\lambda^2\beta_2+3\lambda\beta_1+8\beta_2\mu)+a(c_1\alpha_1^2+6c_2\alpha_0^2\alpha_1^2) &= 0,\\
ck^2(10\lambda\beta_2+2\beta_1)+4ac_2\alpha_0\alpha_1^3 &= 0,
\end{aligned}$$

and

$$6ck^2\beta_2+ac_2\alpha_1^4 = 0.$$

Solving the above system of equations, we have following cases and accordingly different forms of hyperbolic solutions.

Case 1

$$\alpha_0 = 0, \ \alpha_1 = k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}, \ \beta_0 = -\frac{1}{16}\frac{\chi a c_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2 - c_2\alpha\chi^2 - 3\gamma a c_1^2\beta}{c_2\gamma\beta\chi},$$

$$\beta_1 = 0, \ \beta_2 = -\frac{2\alpha k^2}{\beta}, \ \varpi = \frac{1}{16}\frac{\chi a c_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2 - 3c_2\alpha\chi^2 - 16c_2\alpha\kappa^2\chi c + 3\gamma a c_1^2\beta}{c_2\gamma\chi}.$$

In this case, exact traveling wave solutions for $\lambda^2 - 4\mu > 0, \mu \neq 0$ are

$$q_{1,2}(x,t) = -k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)\times e^{i(-\kappa x+\varpi t+\theta)},$$

$$\vartheta_{1,2}(x,t) = \frac{1}{16}\frac{-\chi a c_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2 + c_2\alpha\chi^2 + 3\gamma a c_1^2\beta}{c_2\gamma\beta\chi} - \frac{2\alpha k^2}{\beta}\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)}.$$

For the constraint conditions $\lambda^2 - 4\mu > 0, \mu = 0$ and $\lambda \neq 0$, another combined form of hyperbolic solutions is obtained as follows:

$$q_{3,4}(x,t) = k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C))+\sinh(\lambda(k(x+2\alpha\kappa t)+C))-1}\right)\times e^{i(-\kappa x+\varpi t+\theta)},$$

$$\vartheta_{3,4}(x,t) = \frac{1}{16}\frac{-\chi a c_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2 + c_2\alpha\chi^2 + 3\gamma a c_1^2\beta}{c_2\gamma\beta\chi} - \frac{2\alpha k^2}{\beta}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C))+\sinh(\lambda(k(x+2\alpha\kappa t)+C))-1}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)}.$$

Periodic solutions obtained for the constrained conditions $\lambda^2 - 4\mu < 0, \mu \neq 0$ are given by

$$q_{5,6}(x,t) = -k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)\times e^{i(-\kappa x+\varpi t+\theta)},$$

$$\vartheta_{5,6}(x,t) = \frac{1}{16}\frac{-\chi a c_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2 + c_2\alpha\chi^2 + 3\gamma a c_1^2\beta}{c_2\gamma\beta\chi} - \frac{2\alpha k^2}{\beta}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)}.$$

The constrained conditions $\lambda^2 - 4\mu = 0, \mu \neq 0$ and $\lambda \neq 0$ exist to hold the rational solution given by

$$q_{7,8}(x,t) = -k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)\times e^{i(-\kappa x+\varpi t+\theta)},$$

$$\vartheta_{7,8}(x,t) = \frac{1}{16}\frac{-\chi a c_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2 + c_2\alpha\chi^2 + 3\gamma a c_1^2\beta}{c_2\gamma\beta\chi} - \frac{2\alpha k^2}{\beta}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)}.$$

As before, another form of rational solution for $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$ is obtained as follows:

$$q_{9,10}(x,t) = -k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)\times e^{i(-\kappa x+\varpi t+\theta)},$$

$$\vartheta_{9,10}(x,t) = \frac{1}{16}\frac{-\chi a c_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2 + c_2\alpha\chi^2 + 3\gamma a c_1^2\beta}{c_2\gamma\beta\chi} - \frac{2\alpha k^2}{\beta}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2 \times e^{i(-\kappa x+\varpi t+\theta)}.$$

Case 2

$$w = \frac{1}{16}\frac{\chi a c_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2 - 3c_2\alpha\chi^2 - 16c_2\alpha\kappa^2\chi c + 3\gamma a c_1^2\beta}{c_2\gamma\chi}, \quad \alpha_0 = \alpha_0, \quad \alpha_1 = k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}},$$

$$\beta_0 = -\frac{1}{48}\frac{1}{c_2\gamma\beta\chi}8c_2^2\chi\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}a\beta\alpha_0^2 + 3\chi a c_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2 - 3c_2\alpha\chi^2 - 9\gamma a c_1^2\beta,$$

$$\beta_1 = -\frac{4\alpha k\alpha_0}{b\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}}, \quad \beta_2 = -\frac{2\alpha k^2}{\beta}.$$

In this case, exact traveling wave solutions for $\lambda^2 - 4\mu > 0, \mu \neq 0$, are

$$q_{1,2}(x,t) =\alpha_0 - k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)\times e^{i(-\kappa x+\varpi t+\theta)},$$

$$\begin{aligned}\vartheta_{1,2}(x,t) =& -\frac{1}{48}\frac{1}{c_2\gamma\beta\chi}8c_2^2\chi\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}a\beta\alpha_0^2+3\chi ac_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2-3c_2\alpha\chi^2-9\gamma ac_1^2\beta\\ &+\frac{4\alpha k\alpha_0}{b\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}}\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)\\ &-\frac{2\alpha k^2}{\beta}\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)}.\end{aligned}$$

For the constraint conditions $\lambda^2-4\mu>0, \mu=0$ and $\lambda\neq 0$, another combined form of hyperbolic solutions are obtained as follows:

$$q_{3,4}(x,t) = \alpha_0 + k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C))+\sinh(\lambda(k(x+2\alpha\kappa t)+C))-1}\right)\times e^{i(-\kappa x+\varpi t+\theta)},$$

$$\begin{aligned}\vartheta_{3,4}(x,t) =& -\frac{1}{48}\frac{1}{c_2\gamma\beta\chi}8c_2^2\chi\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}a\beta\alpha_0^2+3\chi ac_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2-3c_2\alpha\chi^2-9\gamma ac_1^2\beta\\ &-\frac{4\alpha k\alpha_0}{b\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C))+\sinh(\lambda(k(x+2\alpha\kappa t)+C))-1}\right)\\ qquad &-\frac{2\alpha k^2}{\beta}\left(\frac{\lambda}{\cosh(\lambda(k(x+2\alpha\kappa t)+C))+\sinh(\lambda(k(x+2\alpha\kappa t)+C))-1}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)}.\end{aligned}$$

Periodic solutions are determined for the constrained conditions $\lambda^2-4\mu<0, \mu\neq 0$ as follows:

$$q_{5,6}(x,t) =\alpha_0 - k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)\times e^{i(-\kappa x+\varpi t+\theta)},$$

$$\begin{aligned}\vartheta_{5,6}(x,t) =& -\frac{1}{48}\frac{1}{c_2\gamma\beta\chi}8c_2^2\chi\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}a\beta\alpha_0^2+3\chi ac_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2-3c_2\alpha\chi^2-9\gamma ac_1^2\beta\\ &+\frac{4\alpha k\alpha_0}{b\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)\\ &-\frac{2\alpha k^2}{\beta}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(k(x+2\alpha\kappa t)+C))+\lambda}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)}.\end{aligned}$$

The constrained conditions $\lambda^2-4\mu=0, \mu\neq 0$ and $\lambda\neq 0$ exist to give the rational solution given below:

$$
\begin{aligned}
q_{7,8}(x,t) &= \alpha_0 - k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)\times e^{i(-\kappa x+\varpi t+\theta)},\\
\vartheta_{7,8}(x,t) &= -\frac{1}{48}\frac{1}{c_2\gamma\beta\chi}8c_2^2\chi\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}a\beta\alpha_0^2+3\chi ac_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2-3c_2\alpha\chi^2\\
&- 9\gamma ac_1^2\beta-\frac{4\alpha k\alpha_0}{b\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)\\
&- \frac{2\alpha k^2}{\beta}\left(\frac{\lambda^2((k(x+2\alpha\kappa t)+C)}{2\lambda((k(x+2\alpha\kappa t)+C)+4}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)}.
\end{aligned}
$$

Again, another form of rational solution is obtained for $\lambda^2-4\mu=0, \mu=0, \lambda=0$ as follows:

$$
\begin{aligned}
q_{9,10}(x,t) &= \alpha_0 - k\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)\times e^{i(-\kappa x+\varpi t+\theta)},\\
\vartheta_{9,10}(x,t) &= -\frac{1}{48}\frac{1}{c_2\gamma\beta\chi}8c_2^2\chi\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}a\beta\alpha_0^2+3\chi ac_1\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}\beta c_2-3c_2\alpha\chi^2\\
&- 9\gamma ac_1^2\beta-\frac{4\alpha k\alpha_0}{b\sqrt[4]{\frac{12\alpha\gamma}{a\beta c_2}}}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2\\
&- \frac{2\alpha k^2}{\beta}\left(\frac{1}{(k(x+2\alpha\kappa t)+C)}\right)^2\times e^{i(-\kappa x+\varpi t+\theta)}.
\end{aligned}
$$

All the results presented in this subsection are valid if the condition $ac_2\alpha\beta\gamma\geq 0$ holds.

Remark The results retrieved in this paper are different from those presented in Simoni (1997) and Kavitha et al. (2014).

4 Conclusion

Over the past two decades, the intense theoretical and experimental study of nematicons has considerably improved the understanding of light localization in re-orientational non-local media. Driven by this quest, this article produced hyperbolic, periodic and rational solutions of nematicon equations with Kerr law and parabolic law nonlinearities. Some combined solutions also emerged. This extraction of solutions is helped with the versatile

Exp($-\phi(\xi)$)-Expansion method. The symbolic computations are done on the MAPLE software. The validity conditions are listed along with the solutions. The results produced here are novel and form a great little addition to the existing literature.

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