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On new complex soliton structures of the nonlinear partial differential equation describing the pulse narrowing nonlinear transmission lines

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Abstract The present paper studies the pulse narrowing nonlinear transmission lines equation, describing pulse narrowing in the field of communication engineering. More precisely, the pulse narrowing nonlinear transmission line equation is solved analytically using the recently developed techniques viz the modified Kudraysov method, the sine-Gordon equation expansion method and the extended sinh-Gordon equation expansion method. As a result, a wide range of dark, bright, dark–bright, singular or combined singular and optical soliton solutions for the pulse narrowing nonlinear transmission lines equation is formally obtained. All solutions have been verified back into its corresponding equation with the aid of maple package program.

Keywords Pulse narrowing nonlinear transmission lines · Kirchhoffs current and Kirchhoffs voltage laws · Mathematical methods · Soliton and optical solutions

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1 Introduction

The study of the nonlinear dynamics of electrical transmission line equation are used in the numerous applied field like as distributing cable television signals, connecting radio transmitters and receivers with their antennas, computer network connections and high-speed computer data buses, trunk lines routing calls between telephone switching centers (Afshari and Hajimiri 2005; Zayed and Alurrfi 2015a, b; Malwe et al. 2014, 2016; El-Borai et al. 2017; Younis et al. 2015; Younis and Ali 2014; Kengne and Lakhssassi 2015; Kengne et al. 2007). In communication engineering, a electrical transmission line is a specialized medium or other structure designed to carry alternating current of radio frequency (El-Borai et al. 2017). More precisely, currents with a frequency high enough that their wave nature must be taken into account their natural phenomena (El-Borai et al. 2017). The nonlinear transmission lines (NLTLs) also provide a useful way to check how the nonlinear excitations behave inside the nonlinear medium and to model the exotic properties of new systems (Sekulic et al. 2012; Pelap and Faye 2005). Nonlinear systems are generic in the mathematical representation of physical phenomena. In order to understand the mechanisms of those physical phenomena which can be described by nonlinear evolution equations (NLEEs), it is necessary to explore their solutions and properties. So, the effort in finding exact solitary wave solutions of nonlinear evolution equations by means of different schemes has grown rapidly in recent years which is one of the most excited advances of nonlinear science and theoretical physics. These exact solutions are important to understand the mechanism of the complicated nonlinear physical phenomena. Thus, it is very important to know the theory of the special waves called solitons. Solitons play an imperative role in many physical systems and it appears in various forms like as kink, pulse, envelope, bright, breather, dark and many others. A soliton is a localized wave form that travels along the system with constant velocity and undeformed shape. Researchers in physics and engineering have proved that solitons are extremely interesting to due their localized and stable nature in applied field like nonlinear optics, plasma physics, communication engineering, ocean engineering, fluid mechanics and so on. Physically, signal shaping means changing certain features of incoming signals, such as the frequency content, pulse width, and amplitude. At the present time, there are many influential integration schemes have developed and utilized for seeking the exact and approximated solutions of these NLEEs, such as the $\left(\frac{G'}{G}\right)$-expansion method, extended $\left(\frac{G'}{G}\right)$-expansion method, auxiliary equation method, new auxiliary equation method, new Jacobi elliptic function expansion method, modified Kudryashov method, the improved extended tanh-function method, the generalized Riccati equation mapping method, the sine-Gordon equation expansion method, the extended sinh-Gordon equation method, exp function method, semi-inverse variational principle, and many more (Afshari and Hajimiri 2005; Zayed and Alurrfi 2015a, b; Malwe et al. 2014, 2016; El-Borai et al. 2017; Younis et al. 2015; Younis and Ali 2014; Kengne and Lakhssassi 2015; Kengne et al. 2007; Sekulic et al. 2012; Pelap and Faye 2005; Lu et al. 2017, 2018; Hosseini et al. 2017a, b, 2018; Kumar et al. 2017, 2018; Yan and Zhang 2001; Bulut et al. 2017a, b, 2018; Khater et al. 2000, 2003, 2018; Seadawy and Lu 2016; Seadawy et al. 2017a, b, 2017; Seadawy 2016; Ma and Lee 2009; Ma and Zhu 2012; Ma and Fuchssteiner 1996; Ma and Zhou 2018; Zhang and Ma 2017; Zhao and Ma 2017; Yang et al. 2017; Ma et al. 2017; Baskonus 2016; Baskonus and Sulaiman 2017; Baskonus et al. 2017).

Many integration schemes including auxiliary equation method, new Jacobi elliptic function expansion method, the Kudryashov method, the $\left(\frac{G'}{G}\right)$-expansion method, the improved extended tanh-function method and the generalized Riccati equation mapping method was executed and to generate explicit solitary wave solutions of the electrical transmission lines equation (NETLEs) in the past (Afshari and Hajimiri 2005; Zayed and Alurrfi 2015a, b; Malwe et al. 2014, 2016; El-Borai et al. 2017). The NETLEs are very convenient tools to study the propagation of electrical solitons which can propagate in the form of voltage waves in nonlinear dispersive media. Afshari and Hajimiri (2005), Zayed and Alurrfi (2015a, b) solved the electrical transmission lines equation using first-order linear approximation and Malwe et al. (2014, 2016) and El-Borai et al. (2017) used the second order curve fitting for the diode characteristics. However, the main aim of this study is to produce new dark, bright, dark–bright, singular or combined singular and optical soliton solutions of the pulse narrowing nonlinear transmission lines equation using three efficient distinct methods, known as the modified Kudryashov method, the sine-Gordon equation expansion method and the extended sinh-Gordon equation expansion method. Under investigation in this work, we consider the nonlinear PDE describing pulse narrowing nonlinear transmission lines equation with first-order linear approximation (Afshari and Hajimiri 2005; Zayed and Alurrfi 2015a, b):

$$\frac{\delta^2 V(x,t)}{\delta t^2} - \frac{1}{LC_0}\frac{\delta^2 V(x,t)}{\delta x^2} - \frac{b_1}{2}\frac{\delta^2 V^2(x,t)}{\delta t^2} + \frac{\delta^2}{12LC_0}\frac{\delta^4 V(x,t)}{\delta x^4} = 0. \tag{1}$$

where $V(x,\ t)$ is the voltage pulse, and C_0, L, b_1 are all constants. The physical details of the derivation of Eq. (1) is elaborated in Afshari and Hajimiri (2005) using the Kirchhoffs current law and Kirchhoffs voltage law, which are omitted here for simplicity. It is well-known (Afshari and Hajimiri 2005) that Eq. (1) has the solution:

$$V(x,y) = \frac{3\left(v^2 - v_0^2\right)}{b_1 v^2} sech^2\left[\sqrt{\frac{3\left(v^2 - v_0^2\right)}{v_0}}\left(\frac{x - vt}{\delta}\right)\right] \tag{2}$$

where v is the propagation velocity of the pulse and $v_0 = 1/(LC_0)$, provided that $v > v_0$.

Afshari and Hajimiri (2005) and Zayed and Alurrfi (2015a) solved the Eq. (1) by using the new Jacobi elliptic function expansion method, auxiliary equation method and the generalized projective Riccati equations method respectively. The obtained solutions of Eq. (1) expressed by kink and anti-kink solitons, bell (bright) and anti-bell (dark) solitary wave solutions, hyperbolic solutions and trigonometric solutions.

The remainder of the paper is organized as follows. A brief discussion modified Kudryashov method is presented and its application in Sect. 2. Section 3 and its subsections deal with the description of the sine-Gordon equation expansion method and its application are discussed. Algorithm of the extended sinh-Gordon equation expansion method and its application are also discussed in Sect. 4. Finally, we draw a concluding remark about the generated results in Sect. 5.

2 Modified Kudryashov method for solving the pulse narrowing nonlinear transmission lines equation

We consider the modified Kudryashov method as a new problem-solving technique to obtain new exact soliton solutions of nonlinear differential equations which used in mathematical physics.

2.1 Description of modified Kudryashov method

Now, we give a brief description of the modified Kudryashov method to find new exact soliton solutions for a given nonlinear partial differential equation. A general form of nonlinear partial differential equation can be written as

$$F(V, V_x, V_t, V_{xx}, V_{xt}, V_{tt}, \ldots) = 0. \tag{3}$$

where F is a polynomial function with respect to some functions or specified variables, which contains nonlinear terms and highest order derivatives of the $V(x, t)$ and the function $V = V(x,t)$ is unknown. The main steps are as follows:

Step-1 By introducing the transformation $V(x,t) = V(\xi)$where $\xi = (x - vt)$, converts Eq. (3) to the following nonlinear ordinary differential equation

$$G(V,\ V^{'},\ V^{''}, \ldots) = 0. \tag{4}$$

where G is a polynomial of V and its derivatives and the superscripts indicate the ordinary derivatives with respect to ξ.

Step-2 Let us assume that the solution $U(\xi)$of the nonlinear Eq. (4) can be presented as

$$V(\xi) = a_0 + \sum_{i=1}^{N} a_i Q^i(\xi). \tag{5}$$

where the arbitrary constants $a_i(i = 1, 2, \ldots, N)$ are determined latter but $a_N \neq 0$ and is a positive integer, which can be determined by using balancing principle on Eq. (4) and satisfies the following new ansatz equation

$$Q^{'}(\xi) = \left(Q^2(\xi) - Q(\xi)\right) ln(a). \tag{6}$$

where $a \neq 0, 1$ and the general solution of the Eq. (6) is

$$Q(\xi) = \frac{1}{1 + da^{\xi}}.$$

Step-3 By inserting Eq. (5) along with Eq. (6) into Eq. (4) and equating the coefficients of powers of $Q^i(\xi)$ to zero, we get a system of algebraic equations in parameters $a_0,\ a_1$ and v. Setting the obtained values in Eq. (5), finally generates new exact solutions for the Eq. (3).

2.2 Solution of the pulse narrowing nonlinear transmission lines equation

By considering the traveling wave transformation as:

$$V(x,t) = V(\xi), \quad \xi = (x - vt) \tag{7}$$

where k and v are arbitrary constant to be determined later. Using the transformation Eq. (7), the pulse narrowing nonlinear transmission lines Eq. (1) can be reduced to a nonlinear ordinary differential equation as below

$$\delta^2 c_0^2 V^{''} - 12\left(v^2 - c_0^2\right) V + 6 b_1 v^2 V^2 = 0. \tag{8}$$

Now balancing between $V^{''}$ and V^2, we obtain $N = 2$. Then, assuming solution of the Eq. (8) is

$$V(\xi) = a_0 + a_1 Q(\xi) + a_2 Q^2(\xi). \tag{9}$$

By substituting Eq. (9) along with its first and second derivatives into Eq. (8) and comparing the terms in the resulting equation, a nonlinear system is gained which by solving it, we determined the following sets:

Set 1 $a_0 = 0$, $a_1 = \frac{12\delta^2(ln(a))^2}{\left(12+\delta^2(ln(a))^2\right)b_1}$, $a_2 = -\frac{12\delta^2(ln(a))^2}{\left(12+\delta^2(ln(a))^2\right)b_1}$, and $v = \pm\frac{1}{2}c_0\sqrt{\frac{\delta^2(ln(a))^2}{3}+4}$.

Now, the following new exact traveling wave solutions to the pulse narrowing nonlinear transmission lines equation is extracted:

$$V_{1,2}(x,t) = \frac{12\delta^2(ln(a))^2}{\left(12+\delta^2(ln(a))^2\right)b_1}\left(\frac{1}{1+da^{\left(x\mp\frac{1}{2}c_0t\sqrt{\frac{\delta^2(ln(a))^2}{3}+4}\right)}} - \frac{1}{\left(1+da^{\left(x\mp\frac{1}{2}c_0t\sqrt{\frac{\delta^2(ln(a))^2}{3}+4}\right)}\right)^2}\right). \tag{10}$$

Set 2 $a_0 = \frac{2\delta^2(ln(a))^2}{\left(12+\delta^2(ln(a))^2\right)b_1}$, $a_1 = -\frac{12\delta^2(ln(a))^2}{\left(12+\delta^2(ln(a))^2\right)b_1}$, $a_2 = \frac{12\delta^2(ln(a))^2}{\left(12+\delta^2(ln(a))^2\right)b_1}$, and $v = \pm\frac{1}{2}c_0\sqrt{\frac{-\delta^2(ln(a))^2}{3}+4}$.

Now, the following new exact traveling wave solutions to the pulse narrowing nonlinear transmission lines equation is determined:

$$V_{3,4}(x,t) = \frac{2\delta^2(ln(a))^2}{\left(12+\delta^2(ln(a))^2\right)b_1}\left(1 - \frac{6}{1+da^{\left(x\mp\frac{1}{2}c_0t\sqrt{\frac{\delta^2(ln(a))^2}{3}+4}\right)}} + \frac{6}{\left(1+da^{\left(x\mp\frac{1}{2}c_0t\sqrt{\frac{\delta^2(ln(a))^2}{3}+4}\right)}\right)^2}\right). \tag{11}$$

3 Sine-Gordon expansion approach for solving the pulse narrowing nonlinear transmission lines equation

The sine-Gordon equation expansion method (Kumar et al. 2017; Yan and Zhang 2001) is one of the most efficient techniques for seeking the exact solutions of various forms of nonlinear differential equations. The fundamental of the sine-Gordon expansion approach can be summarized as follows:

3.1 The fundamental factors of the sine-Gordon approach

Consider the following sine-Gordon equation

$$u_{xx} - u_{tt} = m^2 \sin(u), \tag{12}$$

where $u = u(x,t)$ and m is a constant. By using the transformation $u(x,t) = U(\xi)$ where $\xi = \mu(x - ct)$, we obtain Eq. (12) in form of the following nonlinear ordinary differential equation:

$$U^{''} = \frac{m^2}{\mu^2(1-c^2)}\sin(U). \tag{13}$$

Multiplying U' on the both sides of Eq. (13) and integrating it once gives

$$\left[\left(\frac{U}{2}\right)^{'}\right]^2 = \frac{m^2}{\mu^2(1-c^2)}\sin^2\left(\frac{U}{2}\right) + K, \tag{14}$$

where K is an integration constant.

By setting $K = 0$, $\frac{U}{2} = w(\xi)$, and $\frac{m^2}{\mu^2(1-c^2)} = p^2$ in Eq. (14), we obtain

$$w' = p\ \sin(w). \tag{15}$$

If we take $p = 1$ in Eq. (15), we find

$$w' = \sin(w). \tag{16}$$

This is a simplified form of the sine-Gordon equation. Therefore, Eq. (16) has the following solutions:

$$\sin(w) = \operatorname{sech}(\xi), \quad \cos(w) = \tanh(\xi) \tag{17}$$

and

$$\sin(w) = i\operatorname{csch}(\xi), \quad \cos(w) = \coth(\xi). \tag{18}$$

Now, consider a nonlinear partial differential equation as

$$F(V, V_x, V_t, V_{xx}, V_{xt}, V_{tt}, V_{xxx}, \ldots) = 0. \tag{19}$$

Consider the following transformation:

$$V(x,t) = V(\xi), \quad \xi = \mu(x - ct). \tag{20}$$

Implementing the transformation of Eq. (20) into Eq. (19), then Eq. (20) converted to the following nonlinear ordinary differential equation

$$G(V, V^{'}, V^{''}, \ldots) = 0, \tag{21}$$

where G is a polynomial of V and its derivatives and the superscripts indicate the ordinary derivatives with respect to ξ. If possible, we should integrate Eq. (21) term by term one or more times.

Now, we use the following transformation

$$V(w) = \sum_{j=1}^{N} \cos^{j-1}(w)\left[B_j \sin(w) + A_j \cos(w)\right] + A_0. \tag{22}$$

It is assumed that the solution $V(\xi)$ of the nonlinear Eq. (22) along with Eqs. (17) and (18) can be presented as follows

$$V(\xi) = \sum_{j=1}^{N} \tanh^{j-1}(\xi)\left[B_j \operatorname{sech}(\xi) + A_j \tanh(\xi)\right] + A_0, \tag{23}$$

and

$$V(\xi)=\sum_{j=1}^{N}\coth^{j-1}(\xi)\left[iB_j\mathrm{csch}(\xi)+A_j\coth(\xi)\right]+A_0. \tag{24}$$

By determining the value of N using the homogeneous balance principle, substituting the value of N into Eq. (21) and putting the result into the reduced ordinary differential equation using Eq. (16) give a nonlinear algebraic system. Equating the coefficients of $\sin^j(w)$ and $\cos^j(w)$ equal to zero and solving the acquired system give the values of A_j, B_j, μ and c. Finally, after substituting the values of A_j, B_j and c into Eqs. (23) and (24), we can retrieve the solitary wave solutions of Eq. (19).

3.2 Solution of the pulse narrowing nonlinear transmission lines equation

To determine the parameter N, we balance the linear terms of highest order in Eq. (7) with the highest order nonlinear terms and we obtain $N=2$. As a result, Eqs. (22) and (23) takes the sine-Gordon expansion approach in the finite expansion form

$$V(\xi)=B_1\mathrm{sech}(\xi)+B_2\tanh(\xi)\mathrm{sech}(\xi)+A_1\tanh(\xi)+A_2\tanh^2(\xi)+A_0, \tag{25}$$

and

$$V(\xi)=iB_1\mathrm{csch}(\xi)+iB_2\coth(\xi)\mathrm{csch}(\xi)+A_1\coth(\xi)+A_2\coth^2(\xi)+A_0, \tag{26}$$

and so from Eq. (21)

$$U(w)=B_1\sin(w)+B_2\sin(w)\cos(w)+A_1\cos(w)+A_1\cos^2(w)+A_0, \tag{27}$$

where either A_2 or B_2 may be zero, but both A_2 and B_2 cannot be zero simultaneously.

By substituting Eq. (27) into Eq. (8) and using some mathematical operations, we arrive at a nonlinear algebraic system. Solving the resulting system with the help of symbolic computation package, results in:

Set 1 $A_0=-\frac{\delta^2}{(\delta^2-3)b_1}$, $A_1=0$, $A_2=\frac{3\delta^2}{(\delta^2-3)b_1}$, $B_1=0$, $B_2=0$ and $v=\pm c_0\sqrt{-\frac{\delta^2}{3}+1}$.

Therefore, we substitute the values of Set 1-1 into Eqs. (25) and (26), we generate the following new solitary wave solutions for the pulse narrowing nonlinear electric transmission lines equation:

$$V_{5,6}(x,t)=\frac{\delta^2}{(\delta^2-3)b_1}\left(3\tanh^2\left(\mathrm{x}\mp \mathrm{c_0t}\sqrt{-\frac{\delta^2}{3}+1}\right)-1\right). \tag{28}$$

and

$$V_{7,8}(x,t)=\frac{\delta^2}{(\delta^2-3)b_1}\left(3\coth^2\left(\mathrm{x}\mp \mathrm{c_0t}\sqrt{-\frac{\delta^2}{3}+1}\right)-1\right). \tag{29}$$

Set 2 $A_0=\frac{3\delta^2}{(\delta^2-3)b_1}$, $A_1=0$, $A_2=-\frac{3\delta^2}{(\delta^2-3)b_1}$, $B_1=0$, $B_2=0$ and $v=\pm c_0\sqrt{\frac{\delta^2}{3}+1}$.

Therefore, we substitute the values of Set 2 into Eqs. (25) and (26), we find the following new solitary wave solutions for the pulse narrowing nonlinear electric transmission lines equation:

$$V_{9,10}(x,t) = -\frac{3\delta^2}{(\delta^2-3)b_1}\left(\tanh^2\left(\mathrm{x} \mp \mathrm{c_0 t}\sqrt{\frac{\delta^2}{3}+1}\right)-1\right). \tag{30}$$

and

$$V_{11,12}(x,t) = -\frac{3\delta^2}{(\delta^2-3)b_1}\left(\coth^2\left(\mathrm{x} \mp \mathrm{c_0 t}\sqrt{\frac{\delta^2}{3}+1}\right)-1\right). \tag{31}$$

Set 3-1 $A_0 = -\frac{4\delta^2}{(\delta^2-12)b_1}$, $A_1 = 0$, $A_2 = \frac{6\delta^2}{(\delta^2-12)b_1}$, $B_1 = 0$, $B_2 = \frac{6i\delta^2}{(\delta^2-12)b_1}$ and $v = \pm c_0\sqrt{-\frac{\delta^2}{3}+4}$.

Therefore, we substitute the values of Set 3-1 into Eqs. (25) and (26), we produce the following new solitary wave solutions for the pulse narrowing nonlinear electric transmission lines equation:

$$\begin{aligned} V_{13,14}(x,t) = \frac{\delta^2}{(\delta^2-12)b_1}\Bigg(&-4+6\tanh^2\left(\mathrm{x} \mp \mathrm{c_0 t}\sqrt{-\frac{\delta^2}{3}+4}\right) \\ &+6\mathrm{i}\tanh\left(x \mp \mathrm{c_0 t}\sqrt{-\frac{\delta^2}{3}+4}\right)\mathrm{sech}\left(x \mp \mathrm{c_0 t}\sqrt{-\frac{\delta^2}{3}+4}\right)\Bigg). \end{aligned} \tag{32}$$

and

$$\begin{aligned} V_{15,16}(x,t) = \frac{\delta^2}{(\delta^2-12)b_1}\Bigg(&-4+6\coth^2\left(\mathrm{x} \mp \mathrm{c_0 t}\sqrt{-\frac{\delta^2}{3}+4}\right) \\ &-6\coth\left(\mathrm{x} \mp \mathrm{c_0 t}\sqrt{-\frac{\delta^2}{3}+4}\right)\mathrm{csch}\left(\mathrm{x} \mp \mathrm{c_0 t}\sqrt{-\frac{\delta^2}{3}+4}\right)\Bigg). \end{aligned} \tag{33}$$

Set 3-2 $A_0 = -\frac{4\delta^2}{(\delta^2-12)b_1}$, $A_1 = 0$, $A_2 = \frac{6\delta^2}{(\delta^2-12)b_1}$, $B_1 = 0$, $B_2 = -\frac{6i\delta^2}{(\delta^2-12)b_1}$ and $v = \pm c_0\sqrt{-\frac{\delta^2}{3}+4}$.

Therefore, we substitute the values of Set 3-2 into Eqs. (25) and (26), we extract the following new solitary wave solutions for the pulse narrowing nonlinear electric transmission lines equation:

$$\begin{aligned} V_{17,18}(x,t) = \frac{\delta^2}{(\delta^2-12)b_1}\Bigg(&-4+6\tanh^2\left(\mathrm{x} \mp \mathrm{c_0 t}\sqrt{-\frac{\delta^2}{3}+4}\right) \\ &-6\mathrm{i}\tanh\left(x \mp \mathrm{c_0 t}\sqrt{-\frac{\delta^2}{3}+4}\right)\mathrm{sech}\left(x \mp \mathrm{c_0 t}\sqrt{-\frac{\delta^2}{3}+4}\right)\Bigg). \end{aligned} \tag{34}$$

and

$$V_{19,20}(x,t) = \frac{\delta^2}{(\delta^2-12)b_1}\left(-4+6\coth^2\left(x \mp c_0 t\sqrt{-\frac{\delta^2}{3}+4}\right)\right.$$
$$\left.+6\coth\left(x \mp c_0 t\sqrt{-\frac{\delta^2}{3}+4}\right)\operatorname{csch}\left(x \mp c_0 t\sqrt{-\frac{\delta^2}{3}+4}\right)\right). \tag{35}$$

Set 4-1 $A_0 = \frac{6\delta^2}{(\delta^2-12)b_1}$, $A_1 = 0$, $A_2 = -\frac{6\delta^2}{(\delta^2-12)b_1}$, $B_1 = 0$, $B_2 = \frac{6i\delta^2}{(\delta^2-12)b_1}$ and $v = \pm c_0\sqrt{\frac{\delta^2}{3}+4}$.

Therefore, we substitute the values of Set 4-1 into Eqs. (25) and (26), we obtain the following new solitary wave solutions for the pulse narrowing nonlinear electric transmission lines equation:

$$V_{21,22}(x,t) = \frac{6\delta^2}{(\delta^2-12)b_1}\left(1-\tanh^2\left(x \mp c_0 t\sqrt{\frac{\delta^2}{3}+4}\right)\right.$$
$$\left.+i\tanh\left(x \mp c_0 t\sqrt{\frac{\delta^2}{3}+4}\right)\operatorname{sech}\left(x \mp c_0 t\sqrt{\frac{\delta^2}{3}+4}\right)\right). \tag{36}$$

and

$$V_{23,24}(x,t) = \frac{6\delta^2}{(\delta^2-12)b_1}\left(1-\coth^2\left(x \mp c_0 t\sqrt{\frac{\delta^2}{3}+4}\right)\right.$$
$$\left.-\coth\left(x \mp c_0 t\sqrt{\frac{\delta^2}{3}+4}\right)\operatorname{csch}\left(x \mp c_0 t\sqrt{\frac{\delta^2}{3}+4}\right)\right). \tag{37}$$

Set 4-2 $A_0 = \frac{6\delta^2}{(\delta^2-12)b_1}$, $A_1 = 0$, $A_2 = -\frac{6\delta^2}{(\delta^2-12)b_1}$, $B_1 = 0$, $B_2 = -\frac{6i\delta^2}{(\delta^2-12)b_1}$ and $v = \pm c_0\sqrt{\frac{\delta^2}{3}+4}$.

Therefore, we substitute the values of Set 4-2 into Eqs. (25) and (26), we receive the following new solitary wave solutions for the pulse narrowing nonlinear electric transmission lines equation:

$$V_{25,26}(x,t) = \frac{6\delta^2}{(\delta^2-12)b_1}\left(1-\tanh^2\left(x \mp c_0 t\sqrt{\frac{\delta^2}{3}+4}\right)\right.$$
$$\left.-i\tanh\left(x \mp c_0 t\sqrt{\frac{\delta^2}{3}+4}\right)\operatorname{sech}\left(x \mp c_0 t\sqrt{\frac{\delta^2}{3}+4}\right)\right). \tag{38}$$

and

$$V_{27,28}(x,t) = \frac{6\delta^2}{(\delta^2-12)b_1}\left(1-\coth^2\left(\mathrm{x} \mp \mathrm{c_0 t}\sqrt{\frac{\delta^2}{3}+4}\right)\right.$$
$$\left. + \coth\left(\mathrm{x} \mp \mathrm{c_0 t}\sqrt{\frac{\delta^2}{3}+4}\right)\mathrm{csch}\left(\mathrm{x} \mp \mathrm{c_0 t}\sqrt{\frac{\delta^2}{3}+4}\right)\right). \tag{39}$$

4 The fundamental aspects of the extended sinh-Gordon equation expansion method

Consider the following sine-Gordon equation

$$u_{xt} = m\sinh(u), \tag{40}$$

where $u = u(x,t)$ and m is a constant. Introducing the transformation $u(x,t) = U(\xi)$ where $\xi = k(x-\mu t)$, reduces Eq. (40) to the following nonlinear ordinary differential equation:

$$U'' = -\frac{m}{k^2\mu}\sinh(U). \tag{41}$$

Multiplying U' on the both sides of Eq. (41) and integrating it once gives

$$\left[\left(\frac{U}{2}\right)'\right]^2 = -\frac{m}{k^2\mu}\sinh^2\left(\frac{U}{2}\right) + p, \tag{42}$$

where p is an integration constant.

By setting $\frac{U}{2} = w(\xi)$, and $-\frac{m}{k^2\mu} = q$ in Eq. (42), we obtain

$$w' = \sqrt{p + q\sinh^2(w)}. \tag{43}$$

For different values of parameters p and q, Eq. (43) possess the following set of solutions:

Case-I When we take $p = 0$ and $q = 1$, Eq. (43) becomes

$$w' = \sinh(w). \tag{44}$$

This is a simplified form of the sinh-Gordon equation. Simplifying Eq. (44), the following solutions are obtained:

$$\sinh(w) = \pm i\mathrm{sech}(\xi), \cosh(w) = -\tanh(\xi) \tag{45}$$

and

$$\sinh(w) = \pm\mathrm{csch}(\xi), \cosh(w) = -\coth(\xi). \tag{46}$$

where $i = \sqrt{-1}$ represent an imaginary number.

Case-II When we take $p = 1$ and $q = 1$, Eq. (43) becomes

$$w' = \cosh(w). \tag{47}$$

This is also a simplified form of the sinh-Gordon equation. Simplifying Eq. (47), the following solutions are obtained:

$$\sinh(w) = \tan(\xi), \cosh(w) = \pm\sec(\xi) \tag{48}$$

and

$$\sinh(w) = -\cot(\xi), \cosh(w) = \pm\csc(\xi). \tag{49}$$

Now, consider a nonlinear partial differential equation as

$$F(V, V_x, V_t, V_{xx}, V_{xt}, V_{tt}, V_{xxx}, \ldots) = 0. \tag{50}$$

Consider the following transformation:

$$V(x,t) = V(\xi), \xi = k(x - \mu t). \tag{51}$$

Implementing the transformation of Eq. (51) into Eq. (50), then Eq. (50) converted to the following nonlinear ordinary differential equation

$$G(V, V^{'}, V^{''}, \ldots) = 0, \tag{52}$$

where G is a polynomial of V and its derivatives and the superscripts indicate the ordinary derivatives with respect to ξ. If possible, we should integrate Eq. (52) term by term one or more times.

Now, we use the following transformation

$$V(w) = \sum_{j=1}^{N} \cosh^{j-1}(w)\left[B_j \sinh(w) + A_j \cosh(w)\right] + A_0. \tag{53}$$

It is assumed that the solution $V(\xi)$ of the nonlinear Eq. (53) along with Eq. (44), Eq. (45) and Eq. (46) can be presented as follows

$$V(\xi) = \sum_{j=1}^{N} (-\tanh(\xi))^{j-1}\left[\pm iB_j \mathrm{sech}(\xi) - A_j \tanh(\xi)\right] + A_0, \tag{54}$$

and

$$V(\xi) = \sum_{j=1}^{N} (-\coth(\xi))^{j-1}\left[\pm B_j \mathrm{csch}(\xi) - A_j \coth(\xi)\right] + A_0, \tag{55}$$

Similarly, it is supposed that the solution $V(\xi)$ of the nonlinear Eq. (53) along with Eqs. (47), (48) and (49) can be presented as follows

$$V(\xi) = \sum_{j=1}^{N} (\pm\sec(\xi))^{j-1}\left[B_j \tan(\xi) \pm A_j \sec(\xi)\right] + A_0, \tag{56}$$

and

$$V(\xi) = \sum_{j=1}^{N} (\pm\csc(\xi))^{j-1}\left[-B_j \cot(\xi) \pm A_j \csc(\xi)\right] + A_0, \tag{57}$$

By determining the value of N using the homogeneous balance principle, substituting the value of N into Eq. (53) and putting the result into the reduced ordinary differential equation using Eqs. (45) and (46) give a nonlinear algebraic system. Equating the coefficients of $\sinh^j(w)$ and $\cosh^j(w)$ equal to zero and solving the acquired system give the values of A_j, B_j, μ and k. Finally, after substituting the values of A_j, B_j, μ and k into Eqs. (54), (55), (56) and (57), we can retrieve the solitary wave solutions of Eq. (53).

Remarks For the similar techniques of the sine-Gordon equation expansion method, we do not show the solutions elaborately for the extended sinh-Gordon equation expansion method. To the best of our knowledge all the solutions mentioned above have not been reported so far by other authors in the literature. All solutions have been verified by putting back into original equation via the symbolic software maple and found them correct.

5 Discussion of results

The modified Kudrayshov method, the sine-Gordon expansion method and the extended sinh-Gordon equation expansion method are utilized to look for new closed form complex hyperbolic and trigonometric function solution, especially dark, bright, dark–bright, singular or combined singular and optical soliton solutions for the pulse narrowing nonlinear transmission lines equation are acquired. The pulse narrowing nonlinear transmission lines equation have studied several researchers in past (Zayed and Alurrfi 2015a, b). Zayed and Alurrfi (2015a, b) only found kink and anti-kink solitons, bell (bright) and anti-bell (dark) solitary wave solutions, hyperbolic solutions and trigonometric solutions by utilizing the generalized projective Riccati equations method, the Jacobi elliptic expansion method and the auxiliary equation method. But, in our case, we employed three distinct integration schemes viz modified Kudryashov method, sine-Gordon expansion equation method and extended sinh-Gordon equation expansion method for seeking new soliton and other solutions and found some different solutions from Zayed and Alurrfi (2015a) and Zayed and Alurrfi (2015b). In our case, combined soliton solutions are totally new and have different physical significance.

6 Conclusion

In this article, we have solved the nonlinear PDE describing the pulse narrowing nonlinear transmission lines using three distinct integration schemes such as modified Kudryashov method, sine-Gordon expansion equation method and extended sinh-Gordon equation expansion method. As a results, a series of dark, bright, dark–bright, singular or combined singular and optical soliton solutions for the pulse narrowing nonlinear transmission lines equation is formally extracted. The extracted results emphasized the power of executed methods are robust and effective than other methods for acquiring new and more general soliton and other solutions in the field of mathematical physics and engineering.

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