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New hyperbolic structures for the conformable time-fractional variant bussinesq equations

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Abstract In this work, exact analytical solutions for the time fractional variant bussinesq equations are constructed in the sense of the newly devised fractional derivative called conformable fractional derivative. Using wave transformation, we converted the problem under consideration to an ordinary differential equation and then employed the modified extended tanh expansion method for hyperbolic function solutions. The Mathematica software is used throughout for the solution of the system of algebraic equations obtained along the way and also for the graphical illustrations, respectively.

Keywords Bussinesq equations · Variant bussinesq equations · Conformable fractional derivative

Mathematics Subject Classification Primary 35C07 · 35R11 · 35Q53

1 Introduction

Differential equations featuring fractional orders are of great importance in many sciences and engineering fields since they best describe physical situations. This importance is what makes numerous researchers to devise many definitions among which the newly proposed definition by Khalil et al. (2014). Further, many researchers utilized this fractional derivative and other well-known fractional derivatives alongside some analytical methods to solve many solitary wave problems such as the variant bussinesq equations (Inan et al. 2017)

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$$\begin{cases} u_t + u_x v + v_x u + v_{xxx} = 0, \\ v_t + u_x + v v_x = 0. \end{cases} \tag{1}$$

Other analytical methods used in constructing traveling wave solutions include the Kudryashov method (Kudryashov 2012), the G'/G-expansion method (Bekir and Guner 2013), the collocation based methods (Raslan et al. 2016a, b, 2017c; Talaat 2016), the modified extended tanh method (Khalid et al. 2016), $tan(\phi(\xi)/2)$-expansion method (Raslan et al. 2017b), the Sine-Gordon method (Manafian and Lakestani 2016a; Bulut et al. 2017), the $\exp(-\phi(\xi))$ (Baskonus et al. 2017) and other analytical methods among others (Islam et al. 2014; Guner et al. 2015a, b; Manafian et al. 2015; Shukri and Al-Khaled 2010; Nuruddeen et al. 2018; Nuruddeen 2017a; Nuruddeen et al. 2017; Alquran et al. 2017; Inc et al. 2017a, b; Hosseini et al. 2017; Younis et al. 2016; Islam et al. 2015; Wazwaz 2017; Bakodah et al. 2017a, b; Banaja et al. 2016; Khalid et al. 2017, 2018; Nuruddeen 2017b; Lakestani and Manafian 2017; Manafian 2017; Manafian and Lakestani 2016b; Manafian et al. 2017a, b; Sindi and Manafian 2017; Manafian et al. 2017c). However, in this article, the time fractional variant bussinesq equations would be considered in the sense of the newly devised conformable fractional derivative definition. The considered problem would then be solved by employing the modified extended tanh expansion method after recasting the problem to an ordinary differentiation equation via wave transformation. The Mathematica software would be used in the solution of the system of algebraic equations obtained and also in the graphical illustrations of the solution.

2 Preliminaries

Definition 1 Let $u : [0, \infty) \to \mathbb{R}$ be a function. The α's order conformable derivative of u is defined by

$$D_t^\alpha(u(t)) = \lim_{\epsilon \to 0} \frac{u(t + \epsilon t^{1-\alpha}) - u(t)}{\epsilon}, \tag{2}$$

for all $t > 0$ and $\alpha \in (0, 1)$. Further, the following theorems gives some properties of conformable derivative:

Theorem 1 *Let $\alpha \in (0, 1)$ and suppose $u(t)$ and $v(t)$* are *α-differentiable at $t > 0$. Then*

- (i) $D_t^\alpha(t^c) = ct^{c-\alpha}$, *for all $c \in \mathbb{R}$.*
- (ii) $D_t^\alpha(a) = 0$, *a for all constant function $u(t) = a$.*
- (iii) $D_t^\alpha(au(t)) = aD_t^\alpha(u(t))$, *for all a constant.*
- (iv) $D_t^\alpha(au(t) + bv(t)) = aD_t^\alpha u(t) + bD_t^\alpha v(t)$, *for all $a, b \in \mathbb{R}$.*
- (v) $D_t^\alpha(v(t)u(t)) = v(t)D_t^\alpha(u(t)) + u(t)D_t^\alpha(v(t))$.
- (vi) $D_t^\alpha\left(\frac{u(t)}{v(t)}\right) = \frac{v(t)D_t^\alpha u(t) - u(t)D_t^\alpha v(t)}{v^2(t)}, \quad v(t) \neq 0.$
- (vii) *If, in addition to $u(t)$ differentiable, then $D_t^\alpha u(t) = t^{1-\alpha}\frac{du}{dt}$.*

Definition 2 Let $\alpha \in (0, 1)$ such $u(t)$ is differentiable and also α-differentiable. Let $v(t)$ be a function defined in the range of $u(t)$ also differentiable, then

$$D_t^\alpha(u(t) \circ v(t)) = t^{1-\alpha}v'(t)u'(v(t)).$$

See also [1].

3 Analysis of the method

We present the modified extended tanh expansion method by considering the following nonlinear fractional differential equation of the form:

$$G(u, D_t^{\alpha}u, D_x^{\alpha}u, D_{tt}^{2\alpha}u, D_{xx}^{2\alpha}u, D_t^{\alpha}D_x^{\alpha}u, \ldots) = 0, \quad 0<\alpha<1, \tag{3}$$

where α is order of the derivative of the function $u = u(x, t)$. Also, we use the wave transformation

$$u(x, t) = U(\xi), \quad \xi = ax + b\frac{t^{\alpha}}{\alpha}, \tag{4}$$

where a and b are nonzero constants. Substitution of wave transformation (4) into (3), we obtain an ordinary differential equation of the form

$$P(U, U', U'', \ldots) = 0, \tag{5}$$

where, $'$ is a derivative w.r.t ξ. Further, the solution is assumed to be of the finite series of the form:

$$U(\xi) = a_0 + \sum_{n=1}^{n=N}\left(a_n\Phi^n(\xi) + \frac{b_n}{\Phi^n(\xi)}\right), \tag{6}$$

where $a_0, a_n, b_n, n = 1, 2, \ldots N$ are nonzero constants to be computed; where N is a positive integer determined by balancing the highest order derivative with the highest nonlinear terms in the equation, and $\Phi(\xi)$ satisfies the Riccati differential equation:

$$\Phi'(\xi) = d + \Phi^2(\xi), \tag{7}$$

where d is a constant. Further, the Riccati differential equation in (7) has solutions of the form:

(i) If $d<0$, then

$$\Phi(\xi) = -\sqrt{-d}\tanh(\sqrt{-d}\xi),$$
$$\Phi(\xi) = -\sqrt{-d}\coth(\sqrt{-d}\xi).$$

(ii) If $d = 0$, then

$$\Phi(\xi) = -\frac{1}{\xi}.$$

(iii) If $d > 0$, then

$$\Phi(\xi) = \sqrt{d}\tan(\sqrt{d}\xi),$$

$$\Phi(\xi) = -\sqrt{d}\cot(\sqrt{d}\xi).$$

Therefore, substituting equation (6) and its necessary derivatives into (5) gives a polynomial in $\Phi(\xi)$. Collecting coefficients of the obtained polynomials and subsequently setting each one to zero, we will get a set of over-determined algebraic equations for $a_0, a_n, b_n (n = 1, 2, \ldots)$, and b with the aid of symbolic computation using Mathematica.

Finally, solving the algebraic equations and the above possible solutions of Raccati equation into (5), we obtain the solution of Eq. (3).

4 Application

We consider the conformable time fractional variant bussinesq equations version of (1) the form

$$\begin{cases} D_t^{\alpha}u + u_x v + v_x u + v_{xxx} = 0, \\ D_t^{\alpha}v + u_x + v v_x = 0. \end{cases} \tag{8}$$

Employing the wave transformation, Eq. (8) becomes an ordinary differential equation:

$$\begin{cases} bU' + aU'V + aV'U + a^3V''' = 0, \\ bV' + aU' + aVV' = 0. \end{cases} \tag{9}$$

Now, balancing Eq. (9) using the homogeneous balancing method, we get $N_1 = 2$; and $N_2 = 1$, Thus, Eq. (8) has a solution of the form:

$$\begin{cases} U(\xi) = a_0 + a_1\Phi(\xi) + a_2\Phi^2(\xi) + \dfrac{b_1}{\Phi(\xi)} + \dfrac{b_2}{\Phi^2(\xi)}, \\ V(\xi) = c_0 + c_1\Phi(\xi) + \dfrac{d_1}{\Phi(\xi)}. \end{cases} \tag{10}$$

where from (7)

$$\Phi'(\xi) = d + \Phi^2(\xi), \ \textit{and} \ \Phi''(\xi) = 2\Phi(\xi)(d + \Phi^2(\xi)). \tag{11}$$

Then, putting the values of Eq. (10) and their necessary derivatives alongside Eq. (11) into (9); collecting the coefficients of same degree of $\Phi(\xi)$ and thereafter setting them to zero, we get the following algebraic equations:

$$\begin{aligned}
& -2bb_2 - 2ab_2c_0 - 2ab_1d_1 = 0, \\
& 2bda_2 + 2ada_2c_0 + 2ada_1c_1 = 0, \\
& -2bdb_2 - 2adb_2c_0 - 2adb_1d_1 = 0, \\
& 2ba_2 + 2aa_2c_0 + 2aa_1c_1 = 0, 6a^3c_1 + 3aa_2c_1 = 0, \\
& ba_1 + aa_1c_0 + 8a^3dc_1 + aa_0c_1 + 3ada_2c_1 + aa_2d_1 = 0, \\
& -bdb_1 - adb_1c_0 - adb_2c_1 - 8a^3d^2d_1 - ada_0d_1 - 3ab_2d_1 = 0, \\
& ada_1 - ab_1 + bdc_1 + adc_0c_1 - bd_1 - ac_0d_1 = 0, 2ada_2 + adc_1^2 = 0, \\
& aa_1 + bc_1 + ac_0c_1 = 0, 2aa_2 + ac_1^2 = 0, -6a^3d^3d_1 - 3adb_2d_1 = 0, \\
& -2adb_2 - add_1^2 = 0, -adb_1 - bdd_1 - adc_0d_1 = 0, -2ab_2 - ad_1^2 = 0, \\
& bda_1 - bb_1 + ada_1c_0 - ab_1c_0 + 2a^3d^2c_1 + ada_0c_1 - ab_2c_1 - 2a^3dd_1 - aa_0d_1 + ada_2d_1 = 0.
\end{aligned}$$

Solving the above system via Mathematica software, we get the following:

Case 1

$$a_0 = -2a^2d, \quad a_1 = 0, \quad a_2 = -2a^2, \quad b_1 = 0, \quad b_2 = 0,$$
$$c_0 = -\frac{b}{a}, \quad c_1 = \pm 2a, \quad d_1 = 0, \quad d = d.$$

Hence, we get the following solutions:

$$\begin{cases} u_{1,2}(x,t) = -2a^2d + 2a^2d\tanh^2(\sqrt{-d}\xi). \\ v_{1,2}(x,t) = -\frac{b}{a} \mp 2a\sqrt{-d}\tanh(\sqrt{-d}\xi). \end{cases} \tag{12}$$

$$\begin{cases} u_{3,4}(x,t) = -2a^2d + 2a^2d\coth^2(\sqrt{-d}\xi). \\ v_{3,4}(x,t) = -\frac{b}{a} \mp 2a\sqrt{-d}\coth(\sqrt{-d}\xi). \end{cases} \tag{13}$$

Case 2

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = -2a^2, \quad b_1 = 0, \quad b_2 = -2a^2d^2,$$
$$c_0 = -\frac{b}{a}, \quad c_1 = \pm 2a, \quad d_1 = \pm 2ad, \quad d = d.$$

Thus, we get the following solutions:

$$\begin{cases} u_{5,6}(x,t) = 2a^2d\tanh^2(\sqrt{-d}\xi) + \frac{2a^2d}{\tanh^2(\sqrt{-d}\xi)}. \\ v_{5,6}(x,t) = -\frac{b}{a} \mp 2a\sqrt{-d}\tanh(\sqrt{-d}\xi) \mp \frac{2ad}{\sqrt{-d}\tanh(\sqrt{-d}\xi)}. \end{cases} \tag{14}$$

$$\begin{cases} u_{7,8}(x,t) = 2a^2d\coth^2(\sqrt{-d}\xi) + \frac{2a^2d}{\coth^2(\sqrt{-d}\xi)}, \\ v_{7,8}(x,t) = -\frac{b}{a} \mp 2a\sqrt{-d}\coth(\sqrt{-d}\xi) \mp \frac{2ad}{\sqrt{-d}\coth(\sqrt{-d}\xi)}. \end{cases} \tag{15}$$

Case 3

$$a_0 = -4a^2d, \quad a_1 = 0, \quad a_2 = -2a^2, \quad b_1 = 0, \quad b_2 = -2a^2d^2,$$
$$c_0 = -\frac{b}{a}, \quad c_1 = \mp 2a, \quad d_1 = \pm 2ad, \quad d = d.$$

Therefore, we get the following solutions:

$$\begin{cases} u_{9,10}(x,t) = -4a^2d + 2a^2d\tanh^2(\sqrt{-d}\xi) + \frac{2a^2d}{\tanh^2(\sqrt{-d}\xi)}, \\ v_{9,10}(x,t) = -\frac{b}{a} \pm 2a\sqrt{-d}\tanh(\sqrt{-d}\xi) \mp \frac{2ad}{\sqrt{-d}\tanh(\sqrt{-d}\xi)}. \end{cases} \tag{16}$$

$$\begin{cases} u_{11,12}(x,t) = -4a^2d + 2a^2d\coth^2(\sqrt{-d}\xi) + \dfrac{2a^2d}{\coth^2(\sqrt{-d}\xi)}, \\ v_{11,12}(x,t) = -\dfrac{b}{a} \mp 2a\sqrt{-d}\coth(\sqrt{-d}\xi) \pm \dfrac{2ad}{\sqrt{-d}\coth(\sqrt{-d}\xi)}. \end{cases} \tag{17}$$

Case 4

$$a_0 = -\,2a^2d, \quad a_1 = 0, \quad a_2 = 0, \quad b_1 = 0, \quad b_2 = -2a^2d^2,$$
$$c_0 = -\frac{b}{a}, \quad c_1 = 0, \quad d_1 = \mp 2ad, \quad d = d.$$

Also, we get the following solutions:

$$\begin{cases} u_{13,14}(x,t) = -2a^2d + \dfrac{2a^2d}{\tanh^2(\sqrt{-d}\xi)}, \\ v_{13,14}(x,t) = -\dfrac{b}{a} \pm \dfrac{2ad}{\sqrt{-d}\tanh(\sqrt{-d}\xi)}. \end{cases} \tag{18}$$

$$\begin{cases} u_{15,16}(x,t) = -2a^2d + \dfrac{2a^2d}{\coth^2(\sqrt{-d}\xi)}, \\ v_{15,16}(x,t) = -\dfrac{b}{a} \pm \dfrac{2ad}{\sqrt{-d}\coth(\sqrt{-d}\xi)}. \end{cases} \tag{19}$$

Case 5

$$a_0 = a_1 = 0, \quad a_2 = -2a^2, \quad b_1 = b_2 = 0, \quad c_0 = -\frac{b}{a}, \quad c_1 = \pm 2a, \quad d_1 = 0, \quad d = 0.$$

Finally, we get the following solutions:

$$\begin{cases} u_{17,18}(x,t) = -\dfrac{2a^2}{\xi^2}, \\ v_{17,18}(x,t) = -\dfrac{b}{a} \pm \dfrac{2a}{\xi}. \end{cases} \tag{20}$$

It is also important to note that in the cases $1-4$ above, we made use of $d<0$ to get those solutions from Eq. (7). However, when d is taken >0; these solutions containing hyperbolic functions are expected to change to trigonometric function solutions as explained in Sect. 3. Also, ξ in Cases 1–5 is given by:

$$\xi = ax + b\frac{t^\alpha}{\alpha}. \tag{21}$$

5 Graphical illustrations of the solutions

In this section, we give the graphical illustrations of the conformable time-fractional variant bussinesq equations at different time levels with different values of α: $u(x, t)$ is given in Fig. 1 while their corresponding solutions of $v(x, t)$ are given in Fig. 2, respectively.

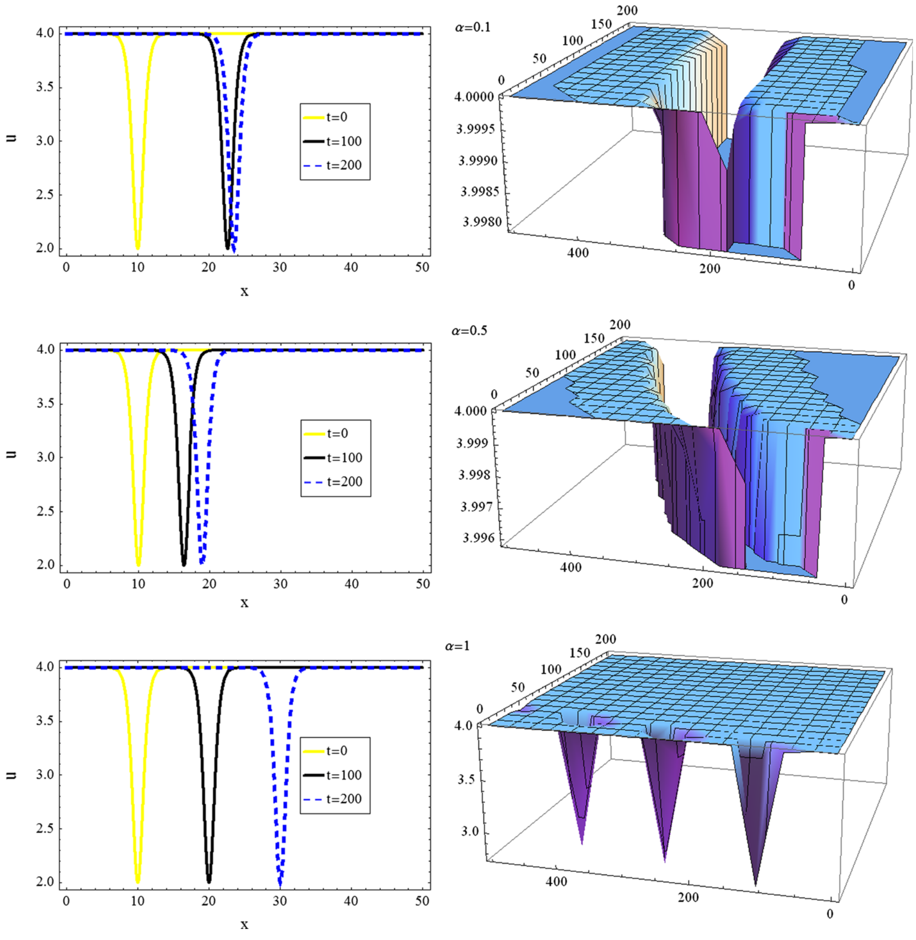

Fig. 1 Profiles of $u(x, t)$ at different time levels

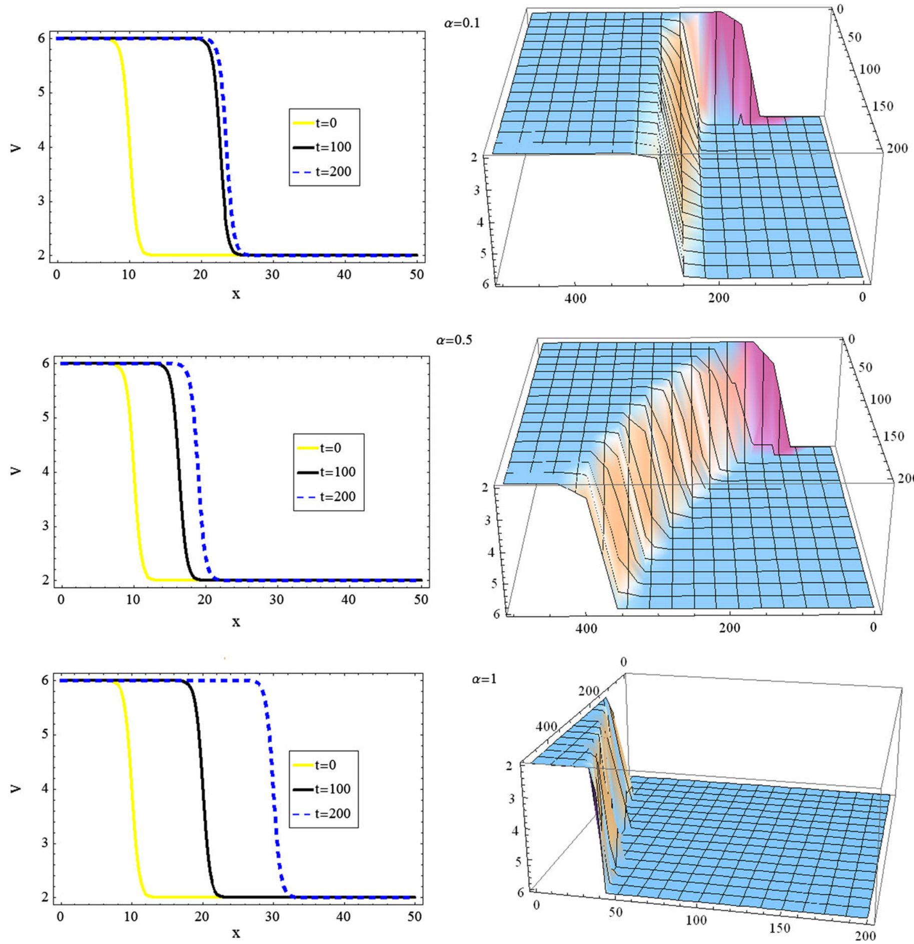

Fig. 2 Profiles of $v(x, t)$ at different time levels

6 Conclusion

In conclusion, various exact hyperbolic function solutions for the time fractional variant bussinesq equations are constructed using the modified extended tanh expansion method in the sense of the newly devised fractional derivative called the conformable fractional derivative. We made use of the wave transformation to convert the problem to an ordinary differentiation equation. The Mathematica software is used for the solution of the system of algebraic equations and also for the graphical illustrations, respectively.

References

Alquran, M., Jaradat, H.M., Syam, M.I.: Analytical solution of the time-fractional Phi-4 equation using modified power series method. Nonlinear Dyn. **90**, 2525–2529 (2017)

Bakodah, H.O., Al Qarni, A.A., Banaja, M.A., Zhou, Q., Moshokoa, S.P., Biswas, A.: Bright and dark thirring optical solitons with improved Adomian decomposition method. Int. J. Light Electron Opt. **130**, 1115–1123 (2017a)

Bakodah, H.O., Al-Zaid, N.A., Mirzazadeh, M., Zhou, Q.: Decomposition method for solving Burgers' equation with Dirichlet and Neumann boundary conditions. Int. J. Light Electron Opt. **130**, 1339–1346 (2017b)

Banaja, M., Al Qarni, A.A., Bakodah, H.O., Zhou, Q., Moshokoa S., Biswas, A.: Bright and dark solitons in cascaded system by improved Adomian decomposition scheme. Int. J. Light Electron Opt. (2016). https://doi.org/10.1016/j.ijleo.2016.11.125

Baskonus, H.M., Sulaiman , T.A., Bulut, H.: New solitary wave solutions to the (2+1)-dimensional Calogero–Bogoyavlenskii-Schiff and the Kadomtsev-Petviashvili hierarchy equations. Indian J Phys **91**, 1237–1243 (2017)

Bekir, A., Guner, O.: Exact solutions of nonlinear fractional differential equations by G'/G-expansion method. China Phys. B **22**, 110202 (2013)

Bulut, H., Sulaiman, T.A., Erdogan, F., Baskonus, H.M.: On the new hyperbolic and trigonometric structures to the simplified MCH and SRLW equations. Eur. Phys. J. Plus. **132**, 350 (2017)

Guner, O., Bekir, A., Cevikel, A.C.: A variety of exact solutions for the time fractional Cahn–Allen equation. Eur. Phys. J. Plus. **130**, 146–58 (2015a)

Guner, O., Bekir, A., Bilgil, H.L.: A note on exp-function method combined with complex transform method applied to fractional differential equations. Adv. Nonlinear Anal. **4**, 201–208 (2015b)

Hosseini, K., Ayati, Z., Ansari, R.: New exact solutions of the Tzitzéica type equations arising in nonlinear optics using a modified version of the improved $tan(\phi(\xi)/2)$-expansion method. Opt. Quantum Electron. **49**, 273 (2017)

Inan, I.E.: $tan(F(\xi)/2)$-expansion method for traveling wave solutions of the variant bussinesq equations. Acilim Metodu, Karaelmas Fen ve Muh. Derg. **7**, 201–206 (2017)

Inc, M., Yusuf, A., Aliyu, A.I.: Dark optical and other soliton solutions for the three different nonlinear schrodinger equations. Opti. Quantum **49**, 11 (2017a)

Inc, M., Yusuf, A., Aliyu, A.I., Baleanu, D.: Optical soliton solutions for the higher-order dispersive cubic-quintic nonlinear Schrodinger equation. Superlattice Microstruct. **112**, 164–179 (2017b)

Islam, M.H., Khan, K., Akbar, M.A., Salam, M.A.: Exact traveling wave solutions of modified KdV–Zakharov–Kuznetsov equation and viscous Burgers equation. SpringerPlus. **3**, 105 (2014)

Islam, M.T., Akbar, M.A., Azad, M.A.K.: A rational (G/G)-expansion method and its application to modified KdV-Burgers equation and the (2+1)-dimensional Boussineq equation. Nonlinear Stud. **6**, 4 (2015)

Khalid, K.A., Nuruddeen, R.I.: Analytical treatment for the conformable space-time fractional Benney–Luke equation via two reliable methods. Int. J. Phys. Res. **5**, 109–114 (2017)

Khalid, K.A., Raslan, K.R., Talaat, S.E.: Non-polynomial spline method for solving coupled Burger's equations. Comput. Methods Differ. Equ. **3**, 218–230 (2016)

Khalid, K.A., Nuruddeen, R.I., Raslan, K.R.: New structures for the space-time fractional simplified MCH and SRLW equations. Chaos, Solitons Fractals **106**, 304–309 (2018)

Khalil, R., Al Horani, M., Yousef, A., Sababheh, M.: A new definition of fractional derivative. J. Comput. Appl. Math. **265**, 65–701 (2014)

Kudryashov, N.A.: One method for finding exact solutions of nonlinear differential equations. Commun. Nonlinear Sci. Numer. Simul. **17**(6), 2248–2253 (2012)

Lakestani, M., Manafian, J.: Application of the ITEM for the modified dispersive water-wave system. Opt. Quantum Electron. **49**, 128 (2017)

Manafian, J.: Application of the ITEM for the system of equations for the ion sound and Langmuir waves. Opt. Quantum Electron. **49**, 17 (2017)

Manafian, J., Lakestani, M.: Application of $tan(\phi(\xi)/2)$-expansion method for solving the Biswas-Milovic equation for Kerr law nonlinearity. Optik. **127**, 2040–2054 (2016a)

Manafian, J., Lakestani, M.: Dispersive dark optical soliton with Tzitzeica type nonlinear evolution equations arising in nonlinear optics. Opt. Quantum Electron. **48**, 116 (2016b)

Manafian, J., Eslami, M., Kordy, M.: The functional variable method for solving the fractional Korteweg-de Vries equations and the coupled Korteweg–De Vries equations. Pramana J. Phys. **85**, 583–92 (2015)

Manafian, J., Jalali, J., Ranjbaran, A.: Applications of IBSOM and ETEM for solving a discretion electrical lattice. Opt. Quantum Electron. **49**, 406 (2017a)

Manafian, J., Bolghar, P., Mohammadalian, A.: Abundant soliton solutions of the resonant nonlinear Schrodinger equation with time-dependent coefficients by ITEM and He's semi-inverse method. Opt. Quantum Electron. **49**, 322 (2017b)

Manafian, J., Foroutan, M., Guzali, A.: Application of the ETEM for obtaining optical soliton solution for the Lakshmann–Porsezian–Daniel model. Eur. Phys. J. Plus. **132**, 494 (2017c)

Nuruddeen, R.I.: Elzaki decomposition method and its applications in solving linear and nonlinear Schrodinger equations. Sohag J. Math. **4**, 2 (2017a)

Nuruddeen, R.I.: Multiple soliton solutions for the (3+1) conformable space-time fractional modified Korteweg-de-Vries equations. Ocean J. Eng. Sci. (2017b). https://doi.org/10.1016/j.joes.2017.11.004

Nuruddeen, R.I., Nass, A.M.: Exact solutions of wave-type equations by the Aboodh decomposition method. Stoch. Model Appl. **21**, 23–30 (2017)

Nuruddeen, R.I., Muhammad, L., Nass, A.M., Sulaiman, T.A.: A review of the integral transforms-based decomposition methods and their applications in solving nonlinear PDEs. Palest. J. Math. **7**, 262–280 (2018)

Raslan, K.R., Talaat, S.E., Khalid, K.A.: Collocation method with quintic B-spline method for solving Hirota-Satsuma coupled KDV equation. Int. J. Appl. Math. Res. **5**, 123–31 (2016a)

Raslan, K.R., Talaat, S.E., Khalid, K.A.: Collocation method with quintic B-spline method for solving the Hirota equation. J. Abstr. Comput. Math. **1**, 1–12 (2016b)

Raslan, K.R., Khalid, K.A., Shallal, M.A.: The modified extended tanh method with the Riccati equation for solving the space-time fractional EW and MEW equations. Chaos, Solitons Fractals **103**, 404–409 (2017a)

Raslan, K.R., Talaat, S.E., Khalid, K.A.: Exact solution of space-time fractional coupled EW and coupled MEW equations. Eur. Phys. J. Plus. **132**, 319 (2017b)

Raslan, K.R., Talaat, S.E., Khalid, K.A.: New exact solution of coupled general equal width wave equation using sine-cosine function method. J. Egypt Math. Soc. **25**, 350–354 (2017c)

Shukri, S., Al-Khaled, K.: The extended tan method for solving systems of nonlinear wave equations. Appl. Math. Comput. **217**, 5 (2010)

Sindi, C.T., Manafian, J.: Soliton solutions of the quantum Zakharov-Kuznetsov equation which arises in quantum magneto-plasmas. Eur. Phys. J. Plus. **132**, 67 (2017)

Talaat, S.E., Raslan, K.R., Khalid, K.A.: Collocation method with cubic B-splines for solving the generalized long wave equation. Int. J. Num. Method Appl. **15**, 39–59 (2016)

Wazwaz, A.: A variety of soliton solutions for the Boussinesq-Burgers equation and the higher-order Boussinesq-Burgers equation. Filomat. **31**, 831–840 (2017)

Younis, M., Cheemaa, N., Mahmood, S.A., Rizvi, S.T.R.: On optical solitons: the chiral nonlinear Schrodinger equation with perturbation and Bohm potential. Opt. Quantum Electron. **48**, 542 (2016)