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Mathematical formulation for transparency and maximum scattering from bilayer MTM-coated PEC cylinders

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Abstract In this paper, a novel formulation interval to design multilayer metamaterial (MTM) radar absorbers and shields in cylindrical structure is presented. In this proposed approach, the interaction of electromagnetic waves by an infinitely long perfect conducting cylinder coated with bilayer MTM is investigated. Transparency and maximum scattering conditions for the structure, under a normally uniform plane wave with transverse electric, transverse magnetic and circular polarizations, are mathematically extracted by using small argument forms of Bessel and Hankel functions. In transparency, the capability of radars is dissipated by the full transmission of the incident waves back to the direction from which they were radiated. On the other hand, scattering maximization by diffuse reflection and making chaff, confuses the radars in target detection. Theoretical formulations are verified by COMSOL which is full wave software. However, our formulations are proved for electrically small cylinders, but simulation results show that they are useful for cylinders with dimensions compared to the wavelength.

Keywords Metamaterial · Transparency · Resonance · COMSOL · Bessel function · Hankel function

1 Introduction

Radar Cross Section (RCS) of an object is a function of signal frequency, polarization, material and dimensions of the object. RCS reduction of aircrafts and missiles is one of the most important issues in military applications interval to achieve transparency. In transparency, capability of radars is dissipated by full transmission of the incident waves back to the direction from which they were radiated. On the other hand, scattering maximization

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by the reflection of waves back to the direction from which they came (i.e. diffuse reflection and making chaff), confused the radars capability in target detection (Oraizi and Abdolali 2008; Irci and Erturk 2007). Recently, MTMs play an important role in designing radar absorbers or shields, because of their unique properties such as negative refraction (Landy et al. 2008; Mohajer-Iravani et al. 2006; Oraizi et al. 2010; Oraizi and Abdolali 2009; Nikooei Tehrani et al. 2013). Also, MTM layers are used interval to enhance the power radiated by electrically small antennas.

So far, several efforts have been made interval to achieve zero reflection from different structures (i.e. planar, cylindrical or spherical structures) which are surrounded by MTM layers (Ahmed and Nagvi 2008; Oraizi and Abdolali 2010; Li and Shen 2003). However, so many applications (e.g., airborne targets) are usually in cylindrical shape and treated as PEC in electromagnetic (EM) solvers. Several papers have proven that conventional materials are not efficient in achieving zero reflection (Alu and Engheta 2005; Oraizi and Abdolali 2008; Oraizi and Abdolali 2008). As mentioned in (Oraizi and Abdolali 2008), RCS reduction occurs in a certain frequency or narrow band width, and in order to have a wideband design, multi-layered structures are required.

According to the above explanation, in the present work, we extend the results to achieve transparency and maximum scattering for cylindrical structure when the core cylinder is particularly PEC and coated with bilayer coating. Our goal is to obtain a simple formula interval to achieve transparency and maximum scattering interval to design multilayer absorbers and shields. It is shown that bilayers composed of common materials are not effective for the two mentioned conditions. But, according to our analysis and formulation, transparency and maximum scattering conditions occur by using MTMs in a fast and simple approach to achieve radar absorbers and shields.

In this paper, transparency and maximum scattering under different polarization waves are studied. In Sect. 2, problem definition and achieving transparency and maximum scattering conditions for TE, TM and circular polarization are achieved. In Sect. 3, numerical results are mentioned. Finally, in Sect. 4, conclusion is discussed.

2 Problem definition and theoretical background

Consider a bilayer MTM-coated PEC cylinder of infinite length. A uniform plane wave is normally incident on the structure which is shown in Fig. 1. The plane wave propagation direction makes an angle $+\varphi_0$ with the $+x$ axis. Transparency and scattering maximization condition under TE, TM and circular polarization are analyzed as follows.

2.1 TE polarization

First, a normally incident uniform plane wave with TE^z polarization is considered, as described by the following field components in the two layers and the free space:

$$\begin{cases} H_{z0} = H_0 \sum_{n=-\infty}^{n=\infty} j^{-n}\left(J_n(k_0\rho) + R_n H_n^{(2)}(k_0\rho)\right)e^{jn(\varphi-\varphi_0)} \\ E_{\varphi 0} = H_0\left(\dfrac{-k_0}{j\omega\varepsilon_0}\right)\sum_{n=-\infty}^{n=\infty} j^{-n}\left(J_n'(k_0\rho) + R_n H_n^{(2)'}(k_0\rho)\right)e^{jn(\varphi-\varphi_0)} \qquad \rho \geq c \\ E_{\rho 0} = H_0\left(\dfrac{1}{j\omega\varepsilon_0}\right)\sum_{n=-\infty}^{n=\infty} j^{-n} jn\left(J_n'(k_0\rho) + R_n H_n^{(2)'}(k_0\rho)\right)e^{jn(\varphi-\varphi_0)} \end{cases} \tag{1}$$

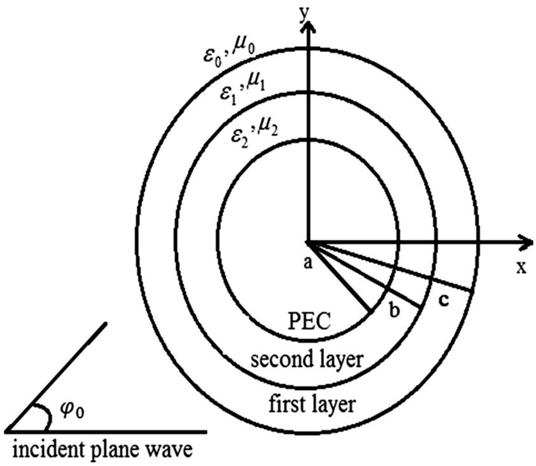

Fig. 1 A cross section of a bilayer MTM-coated PEC cylinder under uniform plane wave incident

$$\begin{cases} H_{zi} = H_0 \sum_{n=-\infty}^{n=\infty} j^{-n}(A_{in}J_n(k_i\rho) + B_{in}Y_n(k_i\rho))e^{jn(\varphi-\varphi_0)} \\ E_{\varphi i} = H_0(\frac{-k_i}{j\omega\varepsilon_i}) \sum_{n=-\infty}^{n=\infty} j^{-n}(A_{in}J'_n(k_i\rho) + B_{in}Y'_n(k_i\rho))e^{jn(\varphi-\varphi_0)} \\ E_{\rho i} = H_0(\frac{1}{j\omega\varepsilon_i}) \sum_{n=-\infty}^{n=\infty} j^{-n}jn(A_{in}J'_n(k_i\rho) + B_{in}Y'_n(k_i\rho))e^{jn(\varphi-\varphi_0)} \end{cases}$$
$$i = 1, 2; (c \le \rho \le b, b \le \rho \le a) respectively \tag{2}$$

Where k_i (i = 0, 1, 2) is the wave number in each space, and R_n, A_{in}, B_{in} (i = 0, 1, 2) are the unknown coefficients which are to be determined from the boundary conditions. At the interface between each space, tangential components of the electric and magnetic fields should be continued. Also, the tangential component of electric field, on the surface of the PEC should be zero. Now, unknown coefficients can be found from a matrix–vector product form as bellow:

$$\begin{bmatrix} R_n \\ A_{1n} \\ B_{1n} \\ A_{2n} \\ B_{2n} \end{bmatrix} = \begin{bmatrix} -H_n^{(2)}(k_0c) & J_n(k_1c) & Y_n(k_1c) & 0 & 0 \\ -H_n^{(2)'}(k_0c) & \varsigma_1 J'_n(k_1c) & \varsigma_1 Y'_n(k_1c) & 0 & 0 \\ 0 & J_n(k_1b) & Y_n(k_1b) & -J_n(k_2b) & -Y_n(k_2b) \\ 0 & J'_n(k_1b) & Y'_n(k_1b) & -\varsigma_2 J'_n(k_2b) & -\varsigma_2 Y'_n(k_2b) \\ 0 & 0 & 0 & J'_n(k_2a) & Y'_n(k_2a) \end{bmatrix}^{-1} \begin{bmatrix} J_n(k_0c) \\ J'_n(k_0c) \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

where $\varsigma_i = \frac{k_i\varepsilon_{i-1}}{k_{i-1}\varepsilon_i} = \frac{\eta_i}{\eta_{i-1}}, i = 1, 2$ and $\eta_i = \sqrt{\frac{\mu_i}{\varepsilon_i}}$ (i = 0, 1, 2) is the wave impedance of each space. The scattering coefficient, R_n, is calculated as:

$$R_n = \varsigma_2(Y_n'(k_2a)J_n'(k_2b) - Y_n'(k_2b)J_n'(k_2a))[\varsigma_1 J_n(k_0c)(J_n(k_1b)Y_n'(k_1c) - Y_n(k_1b)J_n'(k_1c)) \\ + J_n'(k_0c)(J_n(k_1c)Y_n(k_1b) - Y_n(k_1c)J_n(k_1b))] + (J_n(k_1b)Y_n'(k_1c) \\ - Y_n(k_1b)J_n'(k_1c))[\varsigma_1 J_n(k_0c)(J_n'(k_1c)Y_n'(k_1b) - J_n'(k_1b)Y_n'(k_1c)) \\ + J_n'(k_0c)(Y_n(k_1c)J_n'(k_1b) - J_n(k_1c)Y_n'(k_1b))]/\varsigma_2(Y_n'(k_2a)J_n'(k_2b) \\ - Y_n'(k_2b)J_n'(k_2a))[\varsigma_1 H_n^2(k_0c)(J_n(k_1b)Y_n'(k_1c) - Y_n(k_1b)J_n'(k_1c)) \\ + H_n^{2'}(k_0c)(J_n(k_1c)Y_n(k_1b) - Y_n(k_1c)J_n(k_1b))] + (J_n(k_2b)Y_n'(k_2a) \\ - Y_n(k_2b)J_n'(k_2a))[\varsigma_1 H_n^2(k_0c)(J_n'(k_1c)Y_n'(k_1b) - J_n'(k_1b)Y_n'(k_1c)) \\ + H_n^{2'}(k_0c)(Y_n(k_1c)J_n'(k_1b) - J_n(k_1c)Y_n'(k_1b))] \quad (4)$$

As an expression for RCS, the normalized bistatic echo width is found as: (Oraizi and Abdolali 2008; Irci and Erturk 2007)

$$\frac{\sigma}{\lambda} = \frac{2}{\pi}\left|\sum_{n=-\infty}^{n=\infty} R_n e^{jn(\varphi-\varphi_0)}\right|^2 \quad (5)$$

For the monostatic echo width, it is considered $\varphi - \varphi_0 = \pi$.

2.1.1 Transparency condition

The transparency condition for TE^z polarization, is derived by setting the numerator of reflection coefficient in Eq. (4) to zero. In the sub-wavelength limit, assuming $k_0c \ll 1$, $|k_1|b \prec |k_1|c$, $|k_2|a \prec |k_2|b \ll 1$ and utilizing the small argument forms of Bessel and Hankel functions, the following transparency condition is obtained:

$$\varepsilon_1^2\left[\left(\frac{ac}{b^2}\right)^n + \left(\frac{b^2}{ac}\right)^n - \left(\frac{c}{a}\right)^n - \left(\frac{a}{c}\right)^n\right] + \varepsilon_0\varepsilon_1\left[\left(\frac{c}{a}\right)^n + \left(\frac{b^2}{ac}\right)^n - \left(\frac{ac}{b^2}\right)^n - \left(\frac{a}{c}\right)^n\right] \\ + \varepsilon_0\varepsilon_2\left[\left(\frac{ac}{b^2}\right)^n + \left(\frac{c}{a}\right)^n - \left(\frac{b^2}{ac}\right)^n - \left(\frac{a}{c}\right)^n\right] - \varepsilon_1\varepsilon_2\left[\left(\frac{c}{a}\right)^n + \left(\frac{a}{c}\right)^n + \left(\frac{b^2}{ac}\right)^n + \left(\frac{ac}{b^2}\right)^n\right] = 0 \\ for \quad n \neq 0, 1, -1 \quad (6)$$

where n is the index of series summation and a, b and c are PEC cylinder radius and the first and second layer radius respectively. In special cases when $n = 0,\ 1,\ -1$, the ρ component of the electric field and boundary conditions related to it, which have already been presented in Eq. (1) and (2), should be used.

Equation (6) is in form of a quadratic equation, where ε_1 represents an unknown, and a, b, c, n and ε_2 represent numbers, such that the coefficient of ε_1 is not equal to zero. This quadratic equation can be solved by using the quadratic formula. In this way, ε_1 can be extracted and after that ε_2 can be obtained. As it is mentioned in (Irci and Erturk 2007; Alu and Engheta 2005), just dipolar terms (i.e. n = −1, 0, 1) are efficient in cancelation of the incident wave from small PEC cylinders, and higher orders can be neglected or can be considered (both of them lead to equal results). But, as the electrical size of the cylinders increases, in addition to the dipolar terms, high order terms have to be incorporated to cancel the scattering from the PEC core. So, in this paper, large value for n is considered for generality, and thirty is efficient for the value of n. Utilizing large value for n, the quadratic equation in Eq. (6) has two solutions: $\varepsilon_1 = \varepsilon_0$, $\varepsilon_1 = -\varepsilon_2$; In the follow, each answer is explained:

(a) If $\varepsilon_1 = \varepsilon_0$, it means that the outer layer can be ignored. In other words, one layer MTM-coating can be considered. So, ε_2 is calculated by Eq. (6) as follows:

$$\varepsilon_2 = \varepsilon_0 \frac{1 - (\zeta)^{2n}}{1 + (\zeta)^{2n}} \tag{7}$$

Where $\zeta = a/b$, n is the index of series summation and ε_2 is the permittivity of the second layer. As a validation for our formulation, Eq. (7) is equal to the transparency condition for TE^z polarization in (Irci and Erturk 2007). For a specific core coating ratio ζ, the coating permittivity in achieving transparency can be obtained by using Eq. (7). In other words, to find the core coating ratio ζ for a desired coating permittivity, the following equation, which is obtained from Eq. (7), can be used:

$$\zeta = \sqrt[2n]{\frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}} \tag{8}$$

Assuming the limitation $0 < \zeta < 1$ and positive argument for the root in Eq. (8) because of the even degree, the permittivity of the second layer must be $0 \prec \varepsilon_2 \prec \varepsilon_0$.

(b) If $\varepsilon_1 = -\varepsilon_2$, Eq. (7) becomes simplified to:

$$\varepsilon_1 = \varepsilon_0 \frac{(\gamma)^{2n} - 1}{(\gamma)^{2n} + 1} \tag{9}$$

Or in other words:

$$\gamma = \sqrt[2n]{\frac{\varepsilon_0 + \varepsilon_1}{\varepsilon_0 - \varepsilon_1}} \tag{10}$$

where $\gamma = \frac{ac}{b^2}$. For a specific γ value, interval to achieve permittivity of layers, Eq. (9) is used, and vice versa Eq. (10) is used. In fact suitable formula is dependent on our design. Considering positive argument for the root in Eq. (10), permittivity of the first layer must be $-\varepsilon_0 \prec \varepsilon_1 \prec \varepsilon_0$. On the other hand, two conditions can be considered for γ, which γ is given in Eq. (10). The first is $0 \prec \gamma \prec 1$ and the second is $\gamma \succ 1$. According to $0 \prec \gamma \prec 1$, the proper choice for ε_1 has to be $-\varepsilon_0 \prec \varepsilon_1 \prec 0$ and because of the first condition (i.e. $\varepsilon_1 = -\varepsilon_2$), the permittivity of the second layer has to be $0 \prec \varepsilon_2 \prec \varepsilon_0$. According to $\gamma \succ 1$, the proper choice for ε_1 has to be $0 \prec \varepsilon_1 \prec \varepsilon_0$ and because of the first condition (i.e. $\varepsilon_1 = -\varepsilon_2$), the permittivity of the second layer has to be in $-\varepsilon_0 \prec \varepsilon_2 \prec 0$.

2.1.2 Maximum scattering (resonance) condition

Maximum scattering condition can be achieved by setting the denominator of the reflection coefficient to zero. In the sub-wavelength limit, assuming: $k_0 c \ll 1$, $|k_1|b \prec |k_1|c$, $|k_2|a \prec |k_2|b \ll 1$ and utilizing the small argument forms of Bessel and Hankel functions, the following resonance condition is obtained:

$$\varepsilon_1^2\left[\left(\frac{c}{a}\right)^n+\left(\frac{a}{c}\right)^n-\left(\frac{ac}{b^2}\right)^n-\left(\frac{b^2}{ac}\right)^n\right]+\varepsilon_0\varepsilon_1\left[\left(\frac{c}{a}\right)^n+\left(\frac{b^2}{ac}\right)^n-\left(\frac{ac}{b^2}\right)^n-\left(\frac{a}{c}\right)^n\right]$$
$$+\varepsilon_0\varepsilon_2\left[\left(\frac{c}{a}\right)^n-\left(\frac{a}{c}\right)^n+\left(\frac{ac}{b^2}\right)^n-\left(\frac{b^2}{ac}\right)^n\right]+\varepsilon_1\varepsilon_2\left[\left(\frac{c}{a}\right)^n+\left(\frac{b^2}{ac}\right)^n+\left(\frac{ac}{b^2}\right)^n+\left(\frac{a}{c}\right)^n\right]=0$$
$$for \quad n\neq 0,1,-1 \tag{11}$$

As it was mentioned already, specific cases i.e. n = 0, 1, −1 can be achieved by using E_ρ and boundary conditions related to it. Like analysis steps in transparency, Eq. (11) is a quadratic equation. Two answers for this quadratic equation is:$\varepsilon_1 = -\varepsilon_0$, $\varepsilon_1 = -\varepsilon_2$; In the follow, each answer is explained:

(a) If $\varepsilon_1 = -\varepsilon_0$, Eq. (11) becomes simplified to:

$$\varepsilon_2 = \varepsilon_0\frac{(\zeta)^{2n}-1}{(\zeta)^{2n}+1} \tag{12}$$

Or in other words:

$$\zeta = \sqrt[2n]{\frac{\varepsilon_0+\varepsilon_2}{\varepsilon_0-\varepsilon_2}} \tag{13}$$

Where $\zeta = a/b$. Assuming the limitation of $0 < \zeta < 1$ and positive argument for the root in Eq. (13), the appropriate choice for ε_2 has to be$-\varepsilon_0 \prec \varepsilon_2 \prec 0$.

(b) If $\varepsilon_1 = -\varepsilon_2$, equation in (11) become simplified to:

$$\varepsilon_1 = \varepsilon_0\frac{1-(\gamma)^{2n}}{1+(\gamma)^{2n}} \tag{14}$$

Or in other expression:

$$\gamma = \sqrt[2n]{\frac{\varepsilon_0-\varepsilon_1}{\varepsilon_0+\varepsilon_1}} \tag{15}$$

where $\gamma = \frac{ac}{b^2}$. Assuming positive argument for the root in Eq. (15), permittivity of the first layer must be $0 \prec \varepsilon_1 \prec \varepsilon_0$. Like before, two consideration for γ can be considered, which are $0 \prec \gamma \prec 1$ and $\gamma \succ 1$. According to $0 \prec \gamma \prec 1$, the proper choice for ε_1 has to be in $0 \prec \varepsilon_1 \prec \varepsilon_0$ and because of the first condition (i.e. $\varepsilon_1 = -\varepsilon_2$), the permittivity of the second layer has to be $-\varepsilon_0 \prec \varepsilon_2 \prec 0$. According to $\gamma \succ 1$, the proper choice for ε_1 has to be- $\varepsilon_0 \prec \varepsilon_1 \prec 0$ and because of the first condition (i.e. $\varepsilon_1 = -\varepsilon_2$), the permittivity of the second layer has to be $0 \prec \varepsilon_2 \prec \varepsilon_0$.

Note, by setting $\varepsilon_1 = \varepsilon_2$, in Eq. (11), the permittivity of each layer becomes:

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_0\frac{(a/c)^{2n}-1}{(a/c)^{2n}+1} \tag{16}$$

The above equation is equal to the maximum scattering condition in one layer MTM-coating in (Irci and Erturk 2007), and can be used for the validation of our formulation. In Table 1, a summary of transparency and scattering maximization formulation is presented.

Table 1 Transparency and maximum scattering conditions for bilayer MTM-coating PEC cylinder under uniform plane wave by TE polarization

	Limitation of γ	Limitation of ε_1 and ε_2	Magnitude of ε_1 and ε_2
Transparency	No limitation	$\begin{cases}\varepsilon_1 = \varepsilon_0 \\ 0 < \varepsilon_2 < \varepsilon_0\end{cases}$	$\varepsilon_2 = \varepsilon_0 \frac{1-(\zeta)^{2n}}{1+(\zeta)^{2n}}$
	$0 \prec \gamma \prec 1$	$\begin{cases}-\varepsilon_0 < \varepsilon_1 < 0 \\ 0 < \varepsilon_2 < \varepsilon_0\end{cases}$	$\varepsilon_1 = \varepsilon_0 \frac{(\gamma)^{2n}-1}{(\gamma)^{2n}+1} = -\varepsilon_2$
	$\gamma \succ 1$	$\begin{cases}0 < \varepsilon_1 < \varepsilon_0 \\ -\varepsilon_0 < \varepsilon_2 < 0\end{cases}$	$\varepsilon_1 = \varepsilon_0 \frac{(\gamma)^{2n}-1}{(\gamma)^{2n}+1} = -\varepsilon_2$
Scattering maximization	No limitation	$\begin{cases}\varepsilon_1 = -\varepsilon_0 \\ -\varepsilon_0 < \varepsilon_2 < 0\end{cases}$	$\varepsilon_2 = \varepsilon_0 \frac{(\zeta)^{2n}-1}{(\zeta)^{2n}+1}$
	$0 \prec \gamma \prec 1$	$\begin{cases}0 < \varepsilon_1 < \varepsilon_0 \\ -\varepsilon_0 < \varepsilon_2 < 0\end{cases}$	$\varepsilon_1 = \varepsilon_0 \frac{1-(\gamma)^{2n}}{1+(\gamma)^{2n}} = -\varepsilon_2$
	$\gamma \succ 1$	$\begin{cases}-\varepsilon_0 < \varepsilon_1 < 0 \\ 0 < \varepsilon_2 < \varepsilon_0\end{cases}$	$\varepsilon_1 = \varepsilon_0 \frac{1-(\gamma)^{2n}}{1+(\gamma)^{2n}} = -\varepsilon_2$

2.2 TM polarization

In this section, a TM^z polarized uniform plane wave is considered. The electric and magnetic fields in TM^z polarization can be obtained utilizing duality theorem in Eqs. (1) and (2). But duality theorem is not effective for achieving transparency and resonance conditions in TM^z polarization from TE^z polarization, unless the PEC cylinder is replaced by a perfect magnetic conductor (PMC) cylinder, as is mentioned in (Irci and Erturk 2007). For the PEC cylinder, which is our goal in this paper, a separate analysis should be done, as presented in this paper. Now, unknown coefficients can be found from a matrix–vector product form, which is obtained by utilizing boundary conditions, as bellow:

$$\begin{bmatrix} R_n \\ A_{1n} \\ B_{1n} \\ A_{2n} \\ B_{2n} \end{bmatrix} = \begin{bmatrix} -H_n^{(2)}(k_0c) & J_n(k_1c) & Y_n(k_1c) & 0 & 0 \\ -H_n^{(2)'}(k_0c) & \varsigma_1 J_n'(k_1c) & \varsigma_1 Y_n'(k_1c) & 0 & 0 \\ 0 & J_n(k_1b) & Y_n(k_1b) & -J_n(k_2b) & -Y_n(k_2b) \\ 0 & J_n'(k_1b) & Y_n'(k_1b) & -\varsigma_2 J_n'(k_2b) & -\varsigma_2 Y_n'(k_2b) \\ 0 & 0 & 0 & J_n(k_2a) & Y_n(k_2a) \end{bmatrix}^{-1} \begin{bmatrix} J_n(k_0c) \\ J_n'(k_0c) \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

where $\varsigma_i = \frac{k_i \mu_{i-1}}{k_{i-1}\mu_i} = \frac{\eta_{i-1}}{\eta_i}, i = 1,2$, The scattering coefficient, R_n, is calculated as:

$$
\begin{aligned}
R_n = {} & \varsigma_2(Y_n'(k_2a)J_n'(k_2b) - Y_n'(k_2b)J_n'(k_2a))[\varsigma_1 J_n(k_0c)(J_n(k_1b)Y_n'(k_1c) - Y_n(k_1b)J_n'(k_1c)) \\
& + J_n'(k_0c)(J_n(k_1c)Y_n(k_1b) - Y_n(k_1c)J_n(k_1b))] + (J_n(k_2b)Y_n(k_2a) \\
& - J_n(k_2a)Y_n(k_2b))[\varsigma_1 J_n(k_0c)(J_n'(k_1c)Y_n'(k_1b) - J_n'(k_1b)Y_n'(k_1c)) \\
& + J_n'(k_0c)(Y_n(k_1c)J_n'(k_1b) - J_n(k_1c)Y_n'(k_1b))]/\varsigma_2(Y_n'(k_2a)J_n'(k_2b) \\
& - Y_n'(k_2b)J_n'(k_2a))[\varsigma_1 H_n^2(k_0c)(J_n(k_1b)Y_n'(k_1c) - Y_n(k_1b)J_n'(k_1c)) \\
& + H_n^{2'}(k_0c)(J_n(k_1c)Y_n(k_1b) - Y_n(k_1c)J_n(k_1b))] + (J_n(k_2b)Y_n(k_2a) \\
& - J_n(k_2a)Y_n(k_2b))[\varsigma_1 H_n^2(k_0c)(J_n'(k_1c)Y_n'(k_1b) - J_n'(k_1b)Y_n'(k_1c)) \\
& + H_n^{2'}(k_0c)(Y_n(k_1c)J_n'(k_1b) - J_n(k_1c)Y_n'(k_1b))]
\end{aligned} \tag{18}
$$

2.2.1 *Transparency condition*

The transparency condition is derived by setting the numerator of reflection coefficient in Eq. (18) equal to zero. So:

$$
\begin{aligned}
& \mu_1^2\left[\left(\frac{ac}{b^2}\right)^n - \left(\frac{b^2}{ac}\right)^n + \left(\frac{c}{a}\right)^n - \left(\frac{a}{c}\right)^n\right] - \mu_0\mu_1\left[\left(\frac{c}{a}\right)^n + \left(\frac{b^2}{ac}\right)^n + \left(\frac{ac}{b^2}\right)^n + \left(\frac{a}{c}\right)^n\right] \\
& \quad + \mu_0\mu_2\left[\left(\frac{ac}{b^2}\right)^n - \left(\frac{c}{a}\right)^n + \left(\frac{b^2}{ac}\right)^n - \left(\frac{a}{c}\right)^n\right] + \mu_1\mu_2\left[\left(\frac{c}{a}\right)^n - \left(\frac{a}{c}\right)^n + \left(\frac{b^2}{ac}\right)^n - \left(\frac{ac}{b^2}\right)^n\right] = 0 \\
& \quad \mathit{for} \quad n \neq 0, 1, -1
\end{aligned} \tag{19}
$$

In special cases when $n = 0,\ 1,\ -1$, the ρ component of the magnetic field and boundary conditions related to it, should be used. Equation (19) is a quadratic equation, and two obtained answers are: $\mu_1 = \mu_0$, $\mu_1 = -\mu_2$. In what follows, each answer is explained.

(a) If $\mu_1 = \mu_0$, it means that the outer layer can be ignored. In other words, one layer MTM-coating can be considered. So, μ_2 is calculated by Eq. (19) as:

$$\mu_2 = \mu_0 \frac{1 + \zeta^{2n}}{1 - \zeta^{2n}} \tag{20}$$

Or in other words:

$$\zeta = \sqrt[2n]{\frac{\mu_2 - \mu_0}{\mu_2 + \mu_0}} \tag{21}$$

where ζ is a/b and n is the index of series summation. Assuming the limitation $0 < \zeta < 1$ and positive argument for the root in Eq. (21) because of the even degree, the permeability of the second layer must be $\mu_2 \succ \mu_0$.

(b) if $\mu_1 = -\mu_2$, Eq. (21) becomes simplified to:

$$\mu_1 = \mu_0 \frac{\gamma^{2n} + 1}{\gamma^{2n} - 1} \tag{22}$$

Or in other words:

$$\gamma = \sqrt[2n]{\frac{\mu_1 + \mu_0}{\mu_1 - \mu_0}} \tag{23}$$

where $\gamma = \frac{ac}{b^2}$. Assuming positive argument for the root in Eq. (23), will results in two states. The first is when the numerator and the denominator of the fraction under the root in Eq. (23) should be negative simultaneously and the second is when they should be positive simultaneously, and finally the results lead to $\mu_1 \prec -\mu_0$ or $\mu_1 \succ \mu_0$, respectively. Now, two states for γ are considered as $\gamma \succ 1$ and $0 \prec \gamma \prec 1$.

If $0 \prec \gamma \prec 1$, since the argument of the root in Eq. (23) is positive, we have:

$$\left|\frac{\mu_1 + \mu_0}{\mu_1 - \mu_0}\right| < 1 \tag{24}$$

So the proper choice for permeability of layers become: $\mu_1 \prec -\mu_0$, $\mu_2 \succ \mu_0$. But if $\gamma \succ 1$, we have:

$$\left|\frac{\mu_1 + \mu_0}{\mu_1 - \mu_0}\right| > 1 \tag{25}$$

The above equation leads to $\mu_1 \succ \mu_0$ and $\mu_2 \prec -\mu_0$.

2.2.2 *Maximum scattering (resonance) condition*

The resonance condition, is derived by setting the denominator of reflection coefficient in Eq. (18) to zero. So:

$$\begin{aligned}
&\mu_1^2\left[-\left(\frac{ac}{b^2}\right)^n+\left(\frac{b^2}{ac}\right)^n-\left(\frac{c}{a}\right)^n+\left(\frac{a}{c}\right)^n\right]-\mu_0\mu_1\left[\left(\frac{c}{a}\right)^n+\left(\frac{b^2}{ac}\right)^n+\left(\frac{ac}{b^2}\right)^n+\left(\frac{a}{c}\right)^n\right]\\
&\quad+\mu_0\mu_2\left[\left(\frac{ac}{b^2}\right)^n+\left(\frac{b^2}{ac}\right)^n-\left(\frac{c}{a}\right)^n-\left(\frac{a}{c}\right)^n\right]+\mu_2\mu_1\left[-\left(\frac{c}{a}\right)^n-\left(\frac{b^2}{ac}\right)^n+\left(\frac{ac}{b^2}\right)^n+\left(\frac{a}{c}\right)^n\right]=0\\
&\quad \mathit{for}\quad n\neq 0,1,-1
\end{aligned} \tag{26}$$

As it was mentioned already, n = 0, 1, −1 are achieved by H_ρ and related boundary conditions to it. Solving the quadratic equation in Eq. (26), leads to: $\mu_1 = -\mu_0$, $\mu_1 = -\mu_2$

(a) If $\mu_1 = -\mu_0$, Eq. (26) becomes simplified to:

$$\mu_2 = \mu_0 \frac{\varsigma^{2n} - 1}{\zeta^{2n} + 1} \tag{27}$$

In other words:

$$\zeta = \sqrt[2n]{\frac{\mu_2 + \mu_0}{\mu_2 - \mu_0}} \tag{28}$$

Assuming positive argument of the root in (28), the value of μ_2 has to be $\mu_2 \prec -\mu_0$.

(b) If $\mu_1 = -\mu_2$, Eq. (26) becomes simplified to:

$$\mu_1 = \mu_0 \frac{1 + \gamma^{2n}}{1 - \gamma^{2n}} \tag{29}$$

Or in other words:

$$\gamma = \sqrt[2n]{\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}} \tag{30}$$

Table 2 Transparency and maximum scattering conditions in bilayer MTM-coating PEC cylinders under uniform plane wave by TM polarizaion

	Limitation of γ	Limitation of μ_1 and μ_2	Magnitude of μ_1 and μ_2
Transparency	Not limitation	$\begin{cases}\mu_1 = \mu_0 \\ \mu_2 > \mu_0\end{cases}$	$\mu_2 = \mu_0 \frac{1+(\zeta)^{2n}}{1-(\zeta)^{2n}}$
	$0 \prec \gamma \prec 1$	$\begin{cases}\mu_1 < -\mu_0 \\ \mu_2 > \mu_0\end{cases}$	$\mu_1 = \mu_0 \frac{(\gamma)^{2n}+1}{(\gamma)^{2n}-1} = -\mu_2$
	$\gamma \succ 1$	$\begin{cases}\mu_1 > \mu_0 \\ \mu_2 < -\mu_0\end{cases}$	$\mu_1 = \mu_0 \frac{(\gamma)^{2n}+1}{(\gamma)^{2n}-1} = -\mu_2$
Scattering maximization	Not limitation	$\begin{cases}\mu_1 = \mu_0 \\ \mu_2 < -\mu_0\end{cases}$	$\mu_2 = \mu_0 \frac{(\zeta)^{2n}+1}{(\zeta)^{2n}-1}$
	$0 \prec \gamma \prec 1$	$\begin{cases}\mu_1 > \mu_0 \\ \mu_2 < -\mu_0\end{cases}$	$\mu_1 = \mu_0 \frac{1+(\gamma)^{2n}}{1-(\gamma)^{2n}} = -\mu_2$
	$\gamma \succ 1$	$\begin{cases}\mu_1 < -\mu_0 \\ \mu_2 > \mu_0\end{cases}$	$\mu_1 = \mu_0 \frac{1+(\gamma)^{2n}}{1-(\gamma)^{2n}} = -\mu_2$

Assuming positive argument for the root in Eq. (30), will results in two states. The first case is when the numerator and the denominator of the fraction under the root in Eq. (30) should be negative simultaneously and the second is when they should be positive at the same time, and finally the results lead to $\mu_1 \prec -\mu_0$ and $\mu_1 \succ \mu_0$ respectively. Now, two states for γ are considered as $\gamma \succ 1$ and $0 \prec \gamma \prec 1$.

If $0 \prec \gamma \prec 1$, assuming the positive argument of the root in Eq. (30) leads to:

$$\left|\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}\right| < 1 \tag{31}$$

The above equation leads to: $\mu_1 \succ \mu_0$ and $\mu_2 \prec -\mu_0$.If $\gamma \succ 1$, then we have:

$$\left|\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}\right| > 1 \tag{32}$$

Which leads to $\mu_1 \prec -\mu_0$ and $\mu_2 \succ \mu_0$.

In Table 2, transparency and resonance conditions for TM^z polarization are tabulated.

2.3 Circular polarization

Most of the radars have circular polarization due to the difficulty of circular polarization recognition. So, circular analysis is very important and very difficult. As it is mentioned in (Oraizi and Abdolali 2008), desired conditions (i.e. transparency or maximum scattering conditions) in circular polarization are achieved when they are accomplished in both TE and TM polarizations. Therefore, the combination of Tables 1 and 2 lead to transparency and maximum scattering conditions in circular polarization.

Table 3 Desired and obtained ε_{r2} interval to achieve transparency and maximum scattering under uniform plane wave by TE polarization using Eqs. (10) and (15) respectively when $0 \prec \gamma \prec 1$

γ	Maximum scattering condition					Transparency condition		
	a/c	b/c	ε_{r1}	Desired ε_{r2}	Obtained ε_{r2}	ε_{r1}	Desired ε_{r2}	Obtained ε_{r2}
0.30	0.1686	0.75	0.8349	−0.8349	−0.8420	−0.8349	0.8349	0.8340
0.40	0.2250	0.75	0.7241	−0.7241	0.7300	−0.7241	0.7241	0.7250
0.45	0.2531	0.75	0.6632	−0.6632	−0.6680	−0.6632	0.6632	0.6650
0.60	0.3375	0.75	0.4706	−0.4706	−0.4750	−0.4706	0.4706	0.4690
0.80	0.4500	0.75	0.2195	−0.2195	−0.2200	−0.2195	0.2195	0.2100

Table 4 Desired and obtained ε_{r2} interval to achieve transparency and maximum scattering under uniform plane wave by TE polarization using Eqs. (10) and (15) respectively when $\gamma \succ 1$

γ	Maximum scattering condition					Transparency condition		
	a/c	b/c	ε_{r1}	Desired ε_{r2}	Obtained ε_{r2}	ε_{r1}	Desired ε_{r2}	Obtained ε_{r2}
1.10	0.6188	0.75	−0.0940	0.0940	0.0930	0.0940	−0.0940	−0.0940
1.15	0.6469	0.75	−0.1389	0.1389	0.1360	0.1389	−0.1389	−0.1390
1.17	0.6581	0.75	−0.1557	0.1557	0.1530	0.1557	−0.1557	−0.1560
1.20	0.6750	0.75	−0.1803	0.1803	0.1840	0.1803	−0.1803	−0.1790

3 Simulation results

In the previous section, transparency and maximum scattering conditions for bilayer MTM-coated PEC cylinders under uniform plane wave with TE, TM and circular polarization are extracted. It is assumed cylinder dimensions are very small compared to the wave length. Therefore, in simulations, we work with electrically small cylinders covered with our proposed MTM layers. So, the outer radius of MTM-coating is considered as $\lambda/100$(λ is wavelength) and the frequency is 1 GHz in the simulation, except for the last two sections where frequency is changed.

In TE polarization, as it has been proved already, transparency and maximum scattering conditions are dependent on the ratio of γ. For different γ values, where transparency or resonance is desired to be observed, the corresponding permittivity of the first and second layers in $b/c = 0.75$ can be analytically found using Eqs. (10) and (15). The results from equations (desired results) and the simulation (obtained results), are tabulated in Tables 3 and 4 for $0 \prec \gamma \prec 1$ and $\gamma \succ 1$, respectively. Also, the normalized mono static echo widths are depicted according to ε_{r2} for some γ values, in Fig. 2a–d. One can see that transparency or resonance is indeed obtained at the desired ε_{r2} values. Also, it can be deduced from mentioned tables, figures and formulas, that by reducing the total thickness of MTM layer, the magnitude of the permittivity of layers is decreased for $0 \prec \gamma \prec 1$ and vice versa for $\gamma \succ 1$.

As the next step, the normalized mono static echo width according to the ratio of a/c in different γ values and different permittivity of the first and second layers is depicted in Fig. 3a and b. The vertical line shows $\gamma = 1$. As it can be seen in Fig. 3a, for $\varepsilon_{r1} \prec 0$ and $\varepsilon_{r2} \succ 0$, transparency occurs when $0 \prec \gamma \prec 1$ and as the γ becomes larger than unit,

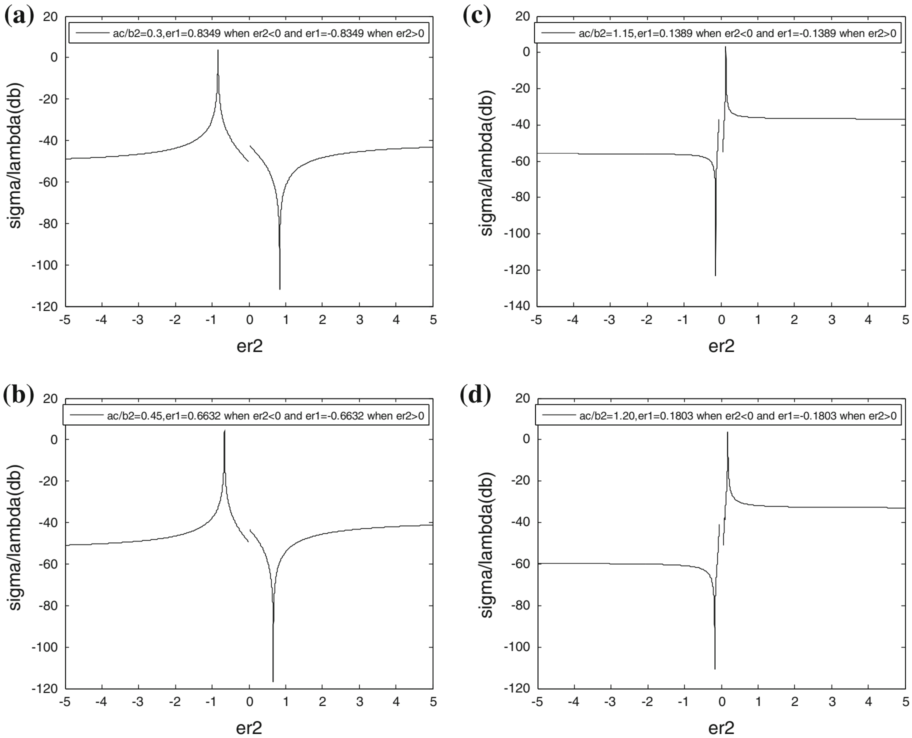

Fig. 2 Normalized monostatic echo width of a bilayer MTM-coated PEC cylinder in TE polarization according to ε_{r2}. **a** $0 \prec \gamma \prec 1$, $\varepsilon_{r1} = 0.8349$ when $\varepsilon_{r2} < 0$, $\varepsilon_{r1} = -0.8349$ when $\varepsilon_{r2} > 0$, **b** $0 \prec \gamma \prec 1$, $\varepsilon_{r1} = 0.6632$ when $\varepsilon_{r2} < 0$, $\varepsilon_{r1} = -0.6632$ when $\varepsilon_{r2} > 0$, **c** $\gamma \succ 1$, $\varepsilon_{r1} = 0.1389$ when $\varepsilon_{r2} < 0$, $\varepsilon_{r1} = -0.1389$ when $\varepsilon_{r2} > 0$ and **d** $\gamma \succ 1$, $\varepsilon_{r1} = 0.1803$ when $\varepsilon_{r2} < 0$, $\varepsilon_{r1} = -0.1803$ when $\varepsilon_{r2} > 0$

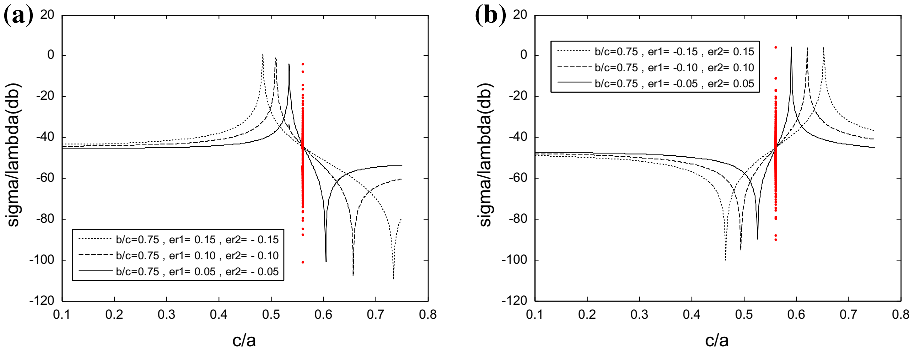

Fig. 3 Normalized monostatic echo width of a bilayer MTM-coated PEC cylinder in TE polarization according to the total thickness of MTM layer. **a** Transparency occurs for $0 \prec \gamma \prec 1$ and maximum scattering occurs for $\gamma \succ 1$ and **b** Transparency occurs for $\gamma \succ 1$ and maximum scattering occurs for $0 \prec \gamma \prec 1$. The vertical lines show $\gamma = 1$

Table 5 Desired and obtained μ_{r2} interval to achieve transparency and maximum scattering under uniform plane wave by TM polarization using Eqs. (23) and (30) respectively when $0 \prec \gamma \prec 1$

γ	Maximum scattering condition					Transparency condition		
	a/c	b/c	μ_{r1}	Desired μ_{r2}	Obtained μ_{r2}	μ_{r1}	Desired μ_{r2}	Obtained μ_{r2}
0.40	0.2250	0.75	1.3810	−1.3810	−1.3810	−1.3810	1.3810	1.3810
0.55	0.3094	0.75	1.8674	−1.8674	1.8890	−1.8674	1.8674	1.8900
0.70	0.39375	0.75	2.9216	−2.9216	−2.9300	−2.9216	2.9216	2.9270
0.85	0.4781	0.75	6.2072	−6.2072	−6.2333	−6.2072	6.2072	6.2000
0.95	0.5344	0.75	19.5128	−19.5128	−19.6200	−0.19.5128	19.5128	19.4600

Table 6 Desired and obtained μ_{r2} interval to achieve transparency and maximum scattering under uniform plane wave by TM polarization using Eqs. (23) and (30) respectively when $\gamma \succ 1$

γ	Maximum scattering condition					Transparency condition		
	a/c	b/c	μ_{r1}	Desired μ_{r2}	Obtained μ_{r2}	μ_{r1}	Desired μ_{r2}	Obtained μ_{r2}
1.05	0.5906	0.75	−20.5122	20.5122	20.4500	20.5122	−20.5122	−20.5400
1.15	0.6469	0.75	−7.2016	7.2016	7.1400	7.2016	−7.2016	−7.1950
1.17	0.6581	0.75	−6.4215	6.4215	6.3280	6.4215	−6.4215	−6.4110
1.20	0.6750	0.75	−5.5455	5.5455	5.4810	5.5455	−5.5455	−5.5300

instead of transparency, maximum scattering occurs. In Fig. 3b, for $\varepsilon_{r1} \succ 0$ and $\varepsilon_{r2} \prec 0$, maximum scattering occurs when $0 \prec \gamma \prec 1$ and as the γ becomes larger than unit, instead of maximum scattering, transparency occurs. The figures show the sensitivity of RCS to γ, which are explained in our formulations.

In what follows, all the previous calculations are repeated for TM polarization. In TM polarization, like TE polarization, transparency and resonance conditions are dependent on the ratio of γ. So, the corresponding permeability of the first and second layers in $b/c = 0.75$ can be analytically found using Eqs. (23) and (30), for γ values between 0 and 1 and γ values larger than unit, where transparency or resonance is desired to be observed. The results from equations (desired results) and the simulations (obtained results) are tabulated in Tables 5 and 6. Also, the normalized mono static echo widths are depicted according to μ_{r2} for some γ values, in Fig. 4a–d. As it can be seen, transparency and maximum scattering occur in desired values of μ_{r2}. As another result, it can be deduced, that by reducing the total thickness of MTM layer, the magnitude of layers permeability increases for $0 \prec \gamma \prec 1$ and vice versa for $\gamma \succ 1$.

Now, lossy MTMs are considered for MTM layers. Drude and Lorentz's models are two common dispersion models for the permittivities and permeabilities in MTMs that are inherently dispersive medium (Irci and Erturk 2007; Oraizi and Abdolali 2008). According to these two models, MTMs have a small loss near their plasma frequency. In this consideration, permeability, permittivity, wave number and wave impedance can be expressed in polar form. Now, transparency and resonance conditions are compared for lossless MTM coating and lossy MTM coating which are used for the structure in Fig. 1. According to (Irci and Erturk 2007), for loosy MTMs, permittivity and permeability are complex quantities. Obtained results are shown in Fig. 5a and b for transparency and resonance, respectively. As it can be seen, there is an ohmic sensitivity for transparency and resonance

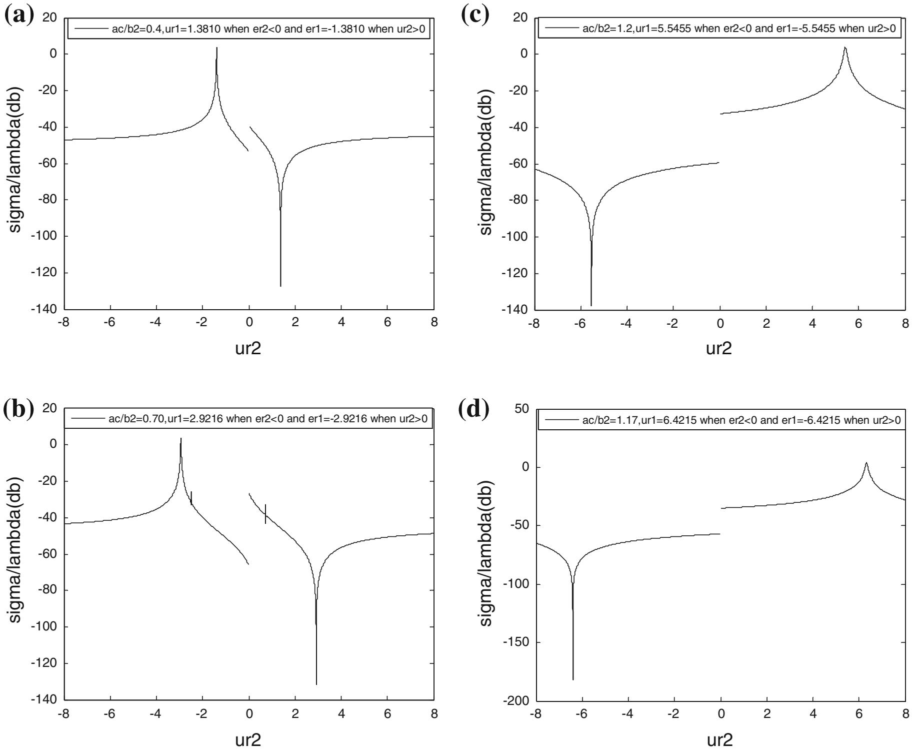

Fig. 4 Normalized monostatic echo width of a bilayer MTM-coated PEC cylinder in TM polarization according to μ_{r2}, **a** $0 \prec \gamma \prec 1$, $\mu_{r1} = 1.3810$ when $\mu_{r2} < 0$ and $\mu_{r1} = -1.3810$ when $\mu_{r2} > 0$, **b** $0 \prec \gamma \prec 1$, $\mu_{r1} = 2.9216$ when $\mu_{r2} < 0$ and $\mu_{r1} = -2.9216$ when $\mu_{r2} > 0$, **c** $\gamma \succ 1$, $\mu_{r1} = 5.5455$ when $\mu_{r2} < 0$ and $\mu_{r1} = -5.5455$ when $\mu_{r2} > 0$ **d** $\gamma \succ 1$, $\mu_{r1} = 6.4215$ when $\mu_{r2} < 0$ and $\mu_{r1} = -6.4215$ when $\mu_{r2} > 0$

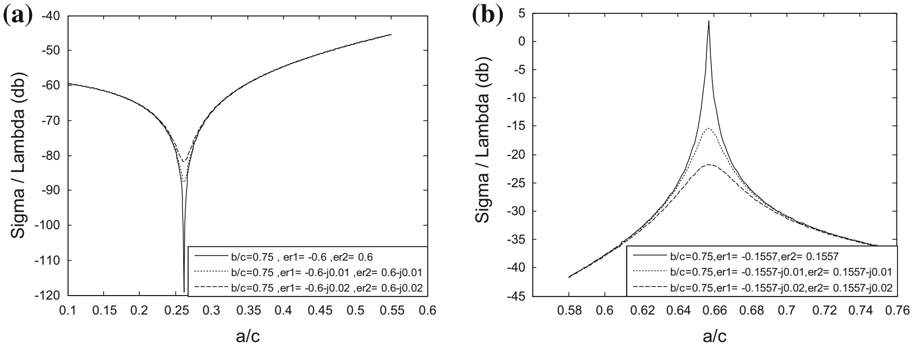

Fig. 5 Effects of ohmic losses on normalized mono static echo width for **a** having transparency and **b** having resonance

conditions. Despite of the decrease in the monostatic echo width for resonance (or increase in monostatic echo width for transparency) due to ohmic losses, metamaterial coating lead to resonance (or transparency) in desired γ values according to our formulation.

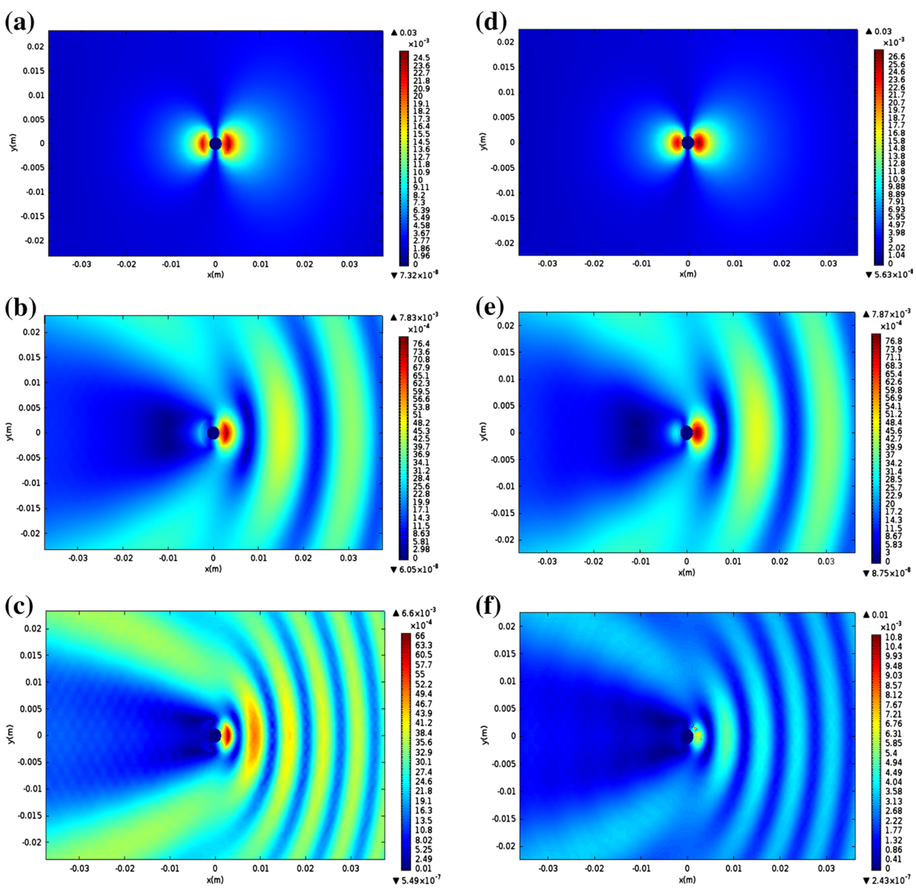

Fig. 6 Contour plot of the total magnetic field around the PEC cylinder in TE polarization for one layer MTM coated PEC by $\varepsilon_{r1} = -0.6632$, $\mu_{r1} = 1$ and $a/b = 0.45$ in **a** f = 1 GHz, **b** f = 10 GHz, **c** f = 20 GHz, and for bilayer MTM-coated PEC cylinder by $\varepsilon_{r1} = 0.2195$, $\varepsilon_{r2} = -0.2195$, $\mu_{r1} = \mu_{r2} = 1$, $a/c = 0.45$, $b/c = 0.75$, in **d** f = 1 GHz, **e** f = 10 GHz and **f** f = 20 GHz

As the final simulation, one layer MTM-coating and bilayer MTM- coating PEC cylinder are simulated by COMLSOL which is a full wave software and the contour plot of the axial component of the total magnetic and electric field around the PEC cylinder is depicted in Figs. 6 and 7 for TE and TM polarizations, respectively, in different frequencies. In all simulations, it is considered that for one layer MTM-coating, it be considered that $a/b = 0.45$ and for bilayer MTM-coating, it be considered that $a/c = 0.45$ (i.e. one layer and bilayer coatings have equal thicknesses for simple comparison).

As it can be seen in Fig. 6a and d, when the frequency is equal to 1 GHz, the contour plot of the axial component of the total magnetic field is the same as a y-directed dipole. In this case, reflected waves exist in the direction from which the waves come. But in physics, backscattering is the reflection of waves back to the direction from which they come, which is a diffuse reflection due to scattering. As the frequency increases to 10 GHz in Fig. 6b and e, the reflection waves in the incident direction decrease and backscattering in different directions occurs which is desirable. When the frequency is considered equal to

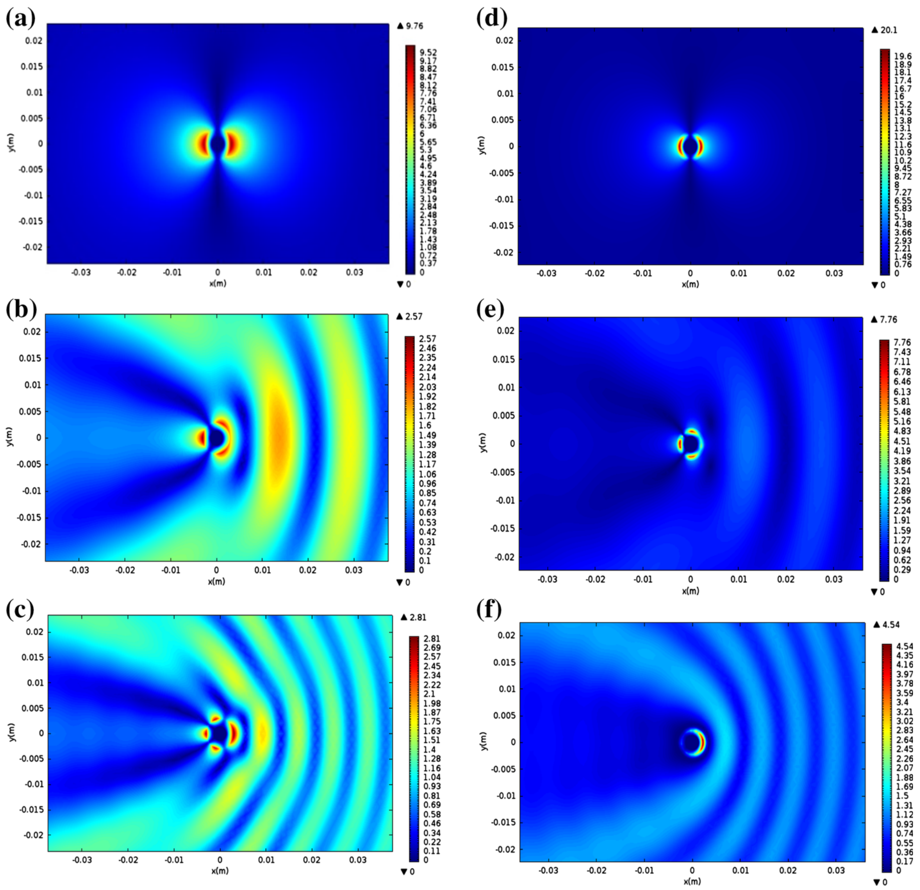

Fig. 7 Contour plot of the axial component of the total electric field outside the PEC cylinder in TM polarization for one layer MTM coated PEC by $\mu_{r1} = -1.5078$, $\varepsilon_{r1} = 1$ and a/b = 0.45 in **a** f = 1 GHz, **b** f = 10 GHz, **c** f = 20 GHz, and for bilayer MTM-coated PEC cylinder by $\mu_{r1} = 4.5556$, $\mu_{r2} = -4.5556$, $\varepsilon_{r1} = \varepsilon_{r2} = 1$, $a/c = 0.45$, $b/c = 0.75$ in **d** f = 1 GHz, **e** f = 10 GHz and **f** f = 20 GHz

20 GHz, backscattering and diffuse reflection can be seen easily. All the mentioned results occur in both one layer and bilayer MTM-coating, but field changes in bilayer MTM-coating are smoother and better than one layer MTM-coating and in bilayer MTM coating, smaller parameters are needed in achieving maximum scattering. As it has been mentioned already, all of our formulations were performed for cylinders with very small dimensions compared to the wavelength. But, according to Fig. 6, our formulation can be used even when the dimensions are comparable to the wavelength, however the magnitude of the contour plot of magnetic field decreases.

Note that all the figures in larger frequencies represent Huygens principle. Actually, the cylinders act as apertures which are located in the X–Y plane, in which the source is located in the -X axis. When the light is propagated from the aperture, the penumbra occurs. The fact is shown in Fig. 8. By increasing the number of cylinders, Huygens principle is represented. As it can be seen, when the number of cylinders increases, the backscattering improves and as the cylinders get closer to each other, the results become

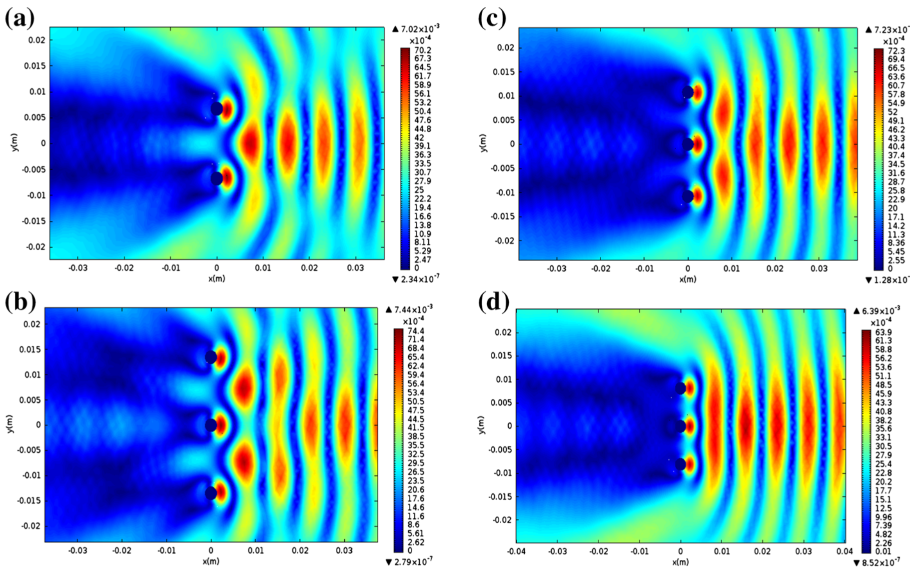

Fig. 8 Contour plot of the axial component of the total magnetic field outside the PEC cylinders in TE polarization for bilayer MTM coated PEC by $\varepsilon_{r1} = 0.2195$, $\varepsilon_{r2} = -0.2195$, $\mu_{r1} = \mu_{r2} = 1$, $a/c = 0.45$, $b/c = 0.75$ in **a** two cylinders with 10a space, **b** three cylinders with 10a space, **c** three cylinders with 8a space, **d** three cylinders with 6a space

better. All the operations for Fig. 6 are performed for TM polarization, but in this case, contour plot of the axial component of the total electric field is presented. As it can be seen, bilayer MTM coatings have better results and Huygens principle is represented.

4 Conclusion

In this paper, bilayer MTM-coating small PEC cylinders are considered, interval to achieve transparency and resonance. In transparency, capability of radars is dissipated by full transmission of the incident waves back to the direction from which they were radiated. On the other hand, scattering maximization by the reflection of waves back to the direction from which they came (i.e. diffuse reflection and making chaff), confused the radars capability in target detection. Also, making resonance is used interval to enhance the power radiated by electrically small antennas. According to our mathematical and physical analysis, simple formulas are presented in order to achieve transparency and maximum scattering conditions in a bilayer MTM coating PEC cylinder design as a good radar absorber and shield. Numerical and simulation results proved accuracy of our formulations. Transparency and resonance conditions are strongly dependent on the structure geometry (thickness) and electric and magnetic parameters. However our formulations are proved for the cylinders which are electrically small, but the formulas are also true for the cylinders which have dimensions comparable to the wavelength. The fact is deduced from the full wave simulation with COMSOL which is full wave software. Our efforts on this topic are now concentrated to considering oblique incident wave and non simple MTMs for coating layers by increasing the number of cylinders, interval to achieve a more general formulation for transparency and maximum scattering conditions.

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