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Effect of atmospheric conditions and aperture averaging on capacity of free space optical links

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Abstract We evaluate the capacity of a free space optical link under the influence of various atmospheric conditions and turbulence levels. Also the effect of aperture averaging on the capacity is investigated. Results show that in weather conditions such as fog and haze, the capacity decreases significantly. Increasing the receiver aperture causes an increase in the capacity but the rate at which the capacity increases reduces with larger diameters and finally the capacity saturates.

Keywords Aperture averaging · Channel capacity · Free space optical (FSO) communication · Gamma-gamma distribution

1 Introduction

Free space optics (FSO) has emerged as a promising and commercially viable technology for next generation wireless applications ranging from short-range wireless communication links to last-mile links, going all the way up to laser communications in outer-space links. It provides a preferable alternative to wireless technologies like radio frequency (RF) and millimeter wave wireless systems by offering the potential of high bandwidth capacity over unlicensed optical wavelength, cost effectiveness, portability, lesser power and immunity to electromagnetic interference.

In the practical implementation of FSO systems, the atmospheric channel conditions play a crucial role to determine the system performance. The particulate matter like aerosols, fog etc. and the molecules present in the atmosphere cause optical extinction of the signal-carrying laser beam due to scattering and absorption. The attenuation caused is exponential in nature and limits the practical range of FSO devices to several kilometers (Ghassemlooy et al. 2012). Another challenge facing FSO communications is the optical turbulence caused

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by both spatial and temporal random fluctuations of refractive index due to inhomogeneities in the temperature, pressure, and wind speed along the optical propagation path through the channel (Zhu and Kahn 2002). Turbulence induced effects include scintillations, phase-front distortions, beam spreading and beam wander (Majumdar and Ricklin 2007).

The ergodic channel capacity of a FSO link with on-off keying under weak turbulence has been calculated considering log-normal intensity fading statistics (Li and Uysal 2003; Laourine et al. 2009). For turbulence conditions varying from weak to strong, the capacity was computed by Anguita et al. (2005) using the gamma-gamma distribution model for scintillation. This analysis was performed for a point receiver and a plane wave propagation model. Nistazakis et al. (2009) calculated the capacity of a terrestrial FSO link, modeling weak to moderate turbulence by the log-normal distribution and moderate to strong turbulence by the gamma-gamma distribution for varying receiver diameters. Various turbulence models such as the Generalized-K and I-K distributions have also been used to determine the average channel capacity and outage probability over various turbulence strengths (Bithas et al. 2006; Sandalidis and Tsiftsis 2008; Nistazakis et al. 2011). Further, the capacity has also been calculated under the combined effect of turbulence, pointing errors and beam wander (Farid and Hranilovic 2007; Yongxiong et al. 2010; Liu et al. 2010). The performance of the system under the influence of fog and pointing errors in the absence of turbulence was evaluated by Kedar and Arnon (2003). Lee et al. (2012) studied the effect of aperture averaging on the error probability and capacity of a FSO link in the presence of turbulence and pointing errors for a Gaussian wave approximation of the propagating signal. However, the effect of weather conditions and atmospheric turbulence when present simultaneously, has not been considered in any of the above studies. In this paper, we have studied the combined effect of atmospheric loss due to various weather conditions such as very clear air, drizzle, haze, fog etc. along with turbulence induced channel fading on the channel capacity based on a spherical wave model appropriate for a small-aperture source (Andrews et al. 2001). The atmospheric loss is determined by the Beer-Lambert law. For the atmospheric turbulence, gamma-gamma distribution valid for weak to strong turbulence is considered. Further, the influence of aperture averaging on the average capacity of an optical link is examined.

The remainder of the paper is organized as follows. Section 2 describes the considered channel model in detail. In Sect. 3, analytical expression for the channel capacity taking into account the atmospheric conditions and turbulence is derived. Simulation results are presented and discussed in Sect. 4. The paper concludes with Sect. 5 giving a summary of the results obtained.

2 Channel model

A single-input single-output (SISO) FSO link is considered with intensity modulation/direct detection (IM/DD) using On-Off Keying (OOK) signaling scheme due to its ease of implementation. The channel is assumed to be memoryless, stationery and ergodic, with independent and identically distributed intensity statistics. The signal at the detector output y is given by

$$y = h\mathbb{R}x + n \tag{1}$$

where $x \in \{0, 2P_t\}$ is the transmitted signal intensity with P_t being the average transmitted power, n is the additive white gaussian noise (AWGN) with variance σ_n^2, $\mathbb{R}$ the detector responsivity and h the channel state that models the random attenuation experienced by the

Table 1 Visibility and attenuation coefficients for various atmospheric conditions at 1,550 nm wavelength

Atmospheric condition	Visibility (km)	Attenuation (dB/km)
Very clear air	50.0	0.0647
Clear air/drizzle	20.0	0.2208
Haze	6.0	0.7360
Light fog	2.0	4.2850
Moderate fog	0.6	25.5160

propagating laser beam in the atmospheric channel due to scintillation (h_a) and atmospheric attenuation/path loss (h_l), given by $h = h_l h_a$ (Lee et al. 2012).

The path loss can be quantified by the Beer-Lambert law (Farid and Hranilovic 2007) as

$$h_l = e^{-\sigma z} \tag{2}$$

where z is the propagation distance and σ the attenuation coefficient which can be calculated from the link visibility (i.e., the meteorological visual range or the distance where the image contrast drops to 2 % of what it would be if the object were nearby) using the Kim model (Naboulsi et al. 2004). The values of the attenuation coefficient for various atmospheric conditions are given in Table 1 for a wavelength of 1,550 nm.

The intensity fluctuations probability density function (pdf), $f_{h_a}(h_a)$ is modeled as a gamma-gamma distribution (Andrews et al. 2001) due to its tractability in calculations and validity for all turbulence scenarios.

$$f_{h_a}(h_a) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} h_a^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta}) \tag{3}$$

where $K_{\alpha-\beta}(.)$ is the Bessel function of the second order and the aperture averaged values of α and β for a spherical wave are given by

$$\alpha = \left(exp\left[\frac{0.49\beta_o^2}{(1 + 0.18d^2 + 0.56\beta_o^{12/5})^{7/6}} \right] - 1 \right)^{-1}, \tag{4}$$

$$\beta = \left(exp\left[\frac{0.51\beta_o^2 \left(1 + 0.69\beta_o^{12/5}\right)^{-5/6}}{(1 + 0.90d^2 + 0.62d^2\beta_o^{12/5})} \right] - 1 \right)^{-1} \tag{5}$$

where $\beta_o^2 = 0.5C_n^2 k^{7/6} z^{11/6}$ is the spherical wave Rytov variance, $k = 2\pi/\lambda$ the optical wave number, λ the wavelength, C_n^2 the refractive index structure parameter and $d = \sqrt{kD^2/4z}$ with D being the receiver aperture diameter.

Since the path loss h_l is deterministic (refer Eq. (2)), the pdf of the channel state h can be written as

$$f_h(h) = \left| \frac{d}{dh}\left(\frac{h}{h_l}\right) \right| f_{h_a}\left(\frac{h}{h_l}\right) \tag{6}$$

Expressing the Bessel function in Eq. (3) in terms of Meijer-G function (Adamchik and Marichev 1990) i.e., $K_v(2\sqrt{x}) = \frac{1}{2} G_{2,0}^{0,2}\left(x \,\middle|\, \begin{matrix} \cdot & \cdot \\ \frac{v}{2} & \frac{-v}{2} \end{matrix} \right)$ and using the modified Eqs. (3) and (6) we get

$$f_h(h) = \frac{(\alpha\beta)^{\frac{\alpha+\beta}{2}} h^{\frac{\alpha+\beta}{2}-1}}{\Gamma(\alpha)\Gamma(\beta) h_l^{\frac{\alpha+\beta}{2}}} G_{0,2}^{2,0}\left(\alpha\beta \frac{h}{h_l} \middle| \begin{matrix} \cdot & \cdot \\ \frac{\alpha-\beta}{2} & \frac{\beta-\alpha)}{2} \end{matrix}\right) \quad (7)$$

3 Capacity evaluation

Channel capacity refers to the maximum achievable data rate that can be reliably communicated between the transmitter and the receiver. It is an important performance metric from the perspective of information theory while designing a FSO system. The capacity is considered to be a random variable as it varies with SNR which is a function of the random variable h. The average channel capacity is given by

$$C = \int_0^\infty B \log_2(1 + SNR(h)) f_h(h) \mathrm{d}h \quad (8)$$

where B is the channel bandwidth.

For OOK signaling, the instantaneous received SNR is given by γh^2 where $\gamma = P_t^2 \mathbb{R}^2/\sigma_n^2$ (Majumdar and Ricklin 2007). Substituting for $f_h(h)$ from Eqs. (7) in (8) and solving the integral using the Meijer-G expression for log function (Adamchik and Marichev 1990) i.e., $log_2(1+x) = \frac{1}{ln2} G_{2,2}^{1,2}\left(x \middle| \begin{matrix} 1, 1 \\ 1, 0 \end{matrix}\right)$, we obtain an expression for channel capacity as

$$C = \frac{2^{\alpha+\beta} B}{4\pi ln2 \Gamma(\alpha)\Gamma(\beta)} G_{6,2}^{1,6}\left(\frac{16\gamma h_l{}^2}{(\alpha\beta)^2} \middle| \begin{matrix} 1, 1, \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}, \frac{1-\beta}{2}, 1-\frac{\beta}{2} \\ 1, 0 \end{matrix}\right) \quad (9)$$

The above equation can be used to evaluate the capacity in presence of turbulence only by taking $h_l{}^2 = 1$.

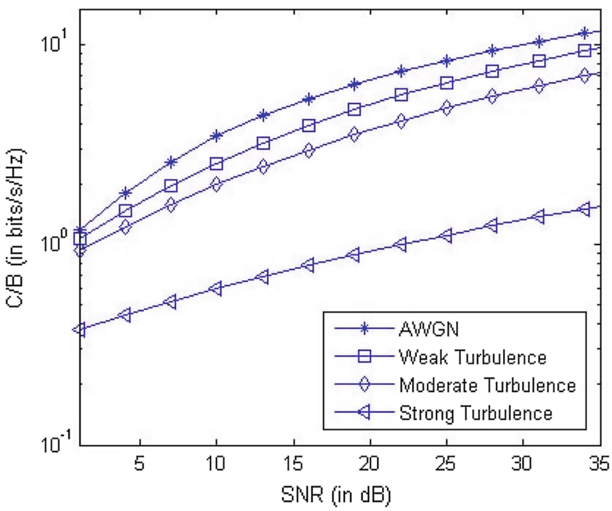

Fig. 1 Average channel capacity C/B versus average electrical SNR in presence of turbulence only

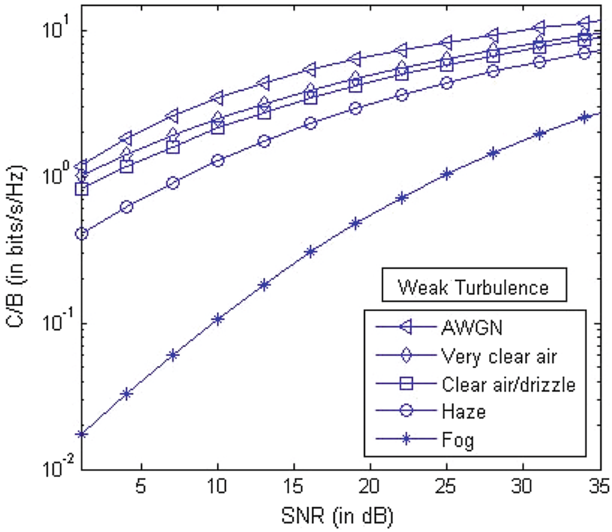

Fig. 2 Average channel capacity C/B versus average electrical SNR under various weather conditions for weak turbulence, $C_n^2 = 8.4 \times 10^{-15} \mathrm{m}^{\frac{-2}{3}}$

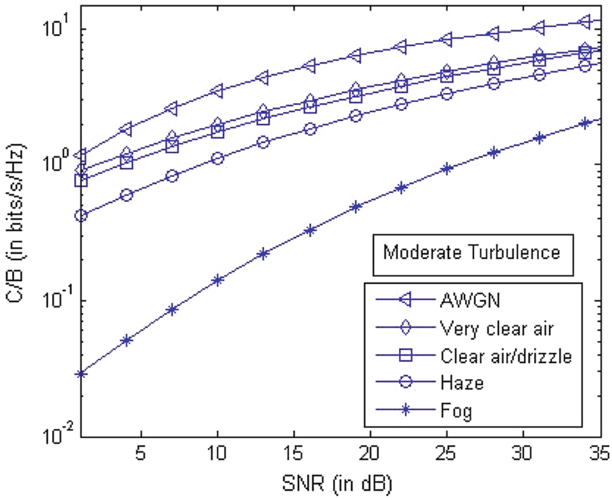

Fig. 3 Average channel capacity C/B versus average electrical SNR under various weather conditions for moderate turbulence, $C_n^2 = 1.7 \times 10^{-14} \mathrm{m}^{\frac{-2}{3}}$

4 Results and discussion

Figure 1 illustrates the average channel capacity C/B in terms of the SNR under various turbulence strengths for a link of length 5 km operating at a wavelength of 1,550 nm. We use the values of $C_n^2 \left(\mathrm{in\, m}^{\frac{-2}{3}}\right)$ as 8.4×10^{-15}, 1.7×10^{-14} and 5×10^{-14} to represent weak, moderate and strong turbulence conditions, respectively (Nistazakis et al. 2009). As expected the average capacity decreases as the turbulence strength goes from weak to strong.

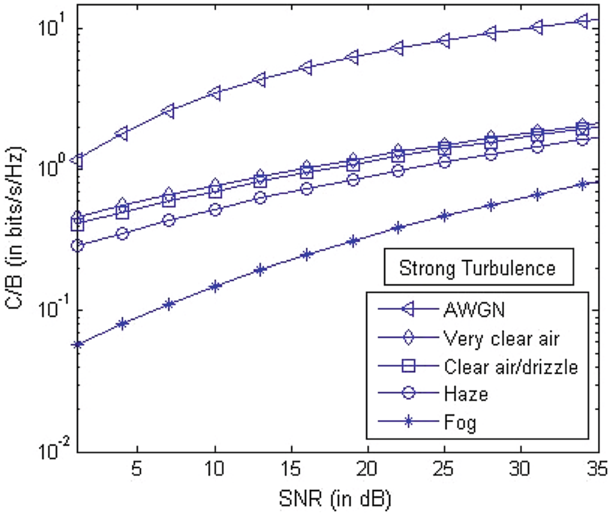

Fig. 4 Average channel capacity C/B versus average electrical SNR under various weather conditions for strong turbulence, $C_n^2 = 5 \times 10^{-14} \mathrm{m}^{\frac{-2}{3}}$

Table 2 Percentage decrease in capacity for various weather conditions

Weather condition	Percentage decrease in capacity		
	Weak turbulence	Moderate turbulence	Strong turbulence
Clear air/drizzle	7.42	6.99	6.29
Haze	28.19	26.39	23.43
Fog	80.05	74.56	66.49

Table 3 Variation of capacity with aperture averaging

Receiver aperture	Average channel capacity (in bits/s/Hz)		
diameter (in cm)	Weak turbulence	Moderate turbulence	Strong turbulence
0*	7.96	5.95	1.32
2	8.21	6.65	3.22
4	8.69	7.84	6.36
6	9.08	8.65	8.08
8	9.34	9.11	8.86
10	9.50	9.36	9.24
12	9.61	9.52	9.45
14	9.69	9.62	9.57
16	9.74	9.69	9.66
18	9.78	9.74	9.72
20	9.80	9.77	9.76

* Point receiver

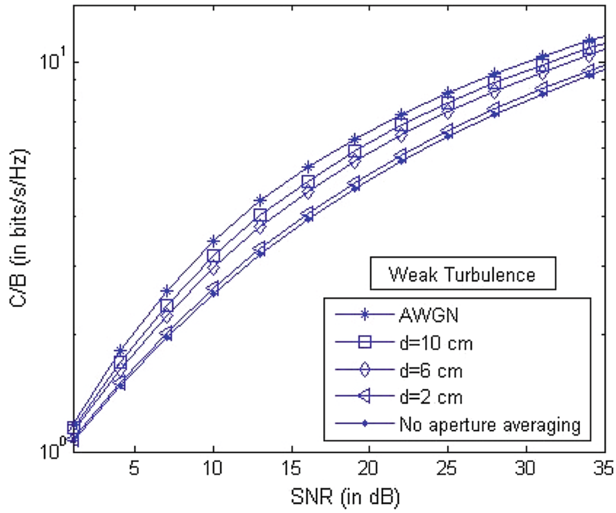

Fig. 5 Average channel capacity C/B versus the average electrical SNR for different receiver aperture diameters in weak turbulence, $C_n^2 = 8.4 \times 10^{-15} \text{m}^{\frac{-2}{3}}$

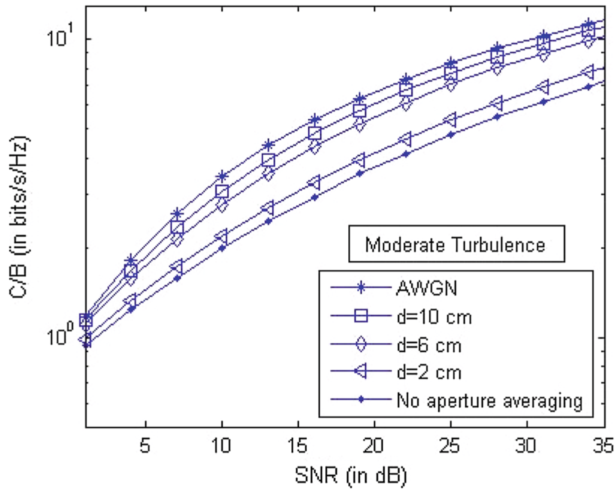

Fig. 6 Average channel capacity C/B versus the average electrical SNR for different receiver aperture diameters in moderate turbulence, $C_n^2 = 1.7 \times 10^{-14} \text{m}^{\frac{-2}{3}}$

For a given turbulence level (weak, moderate and strong), effect of weather conditions on the channel capacity is depicted in Figs. 2, 3, and 4, respectively. The percentage decrease in the capacity in presence of drizzle, haze and fog from the very clear weather condition for various levels of turbulence are given in Table 2. It is seen that the capacity decreases significantly in the presence of fog as compared to haze and drizzle. Also these weather conditions have a greater degrading effect on the capacity in the case of weak turbulence as compared to the moderate and strong turbulence regimes.

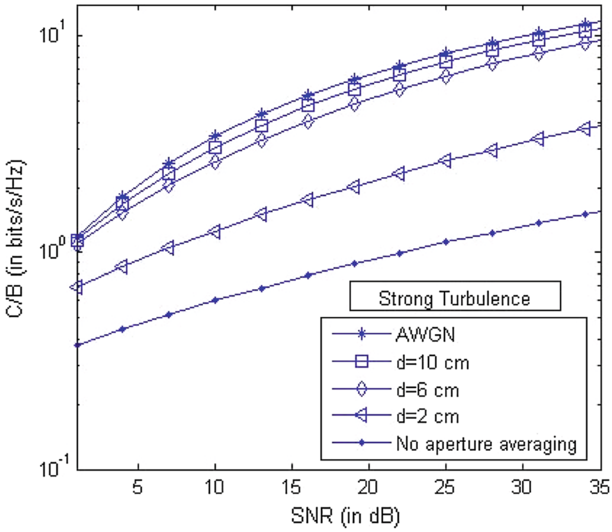

Fig. 7 Average channel capacity C/B versus the average electrical SNR for different receiver aperture diameters in strong turbulence, $C_n^2 = 5 \times 10^{-14} \mathrm{m}^{\frac{-2}{3}}$

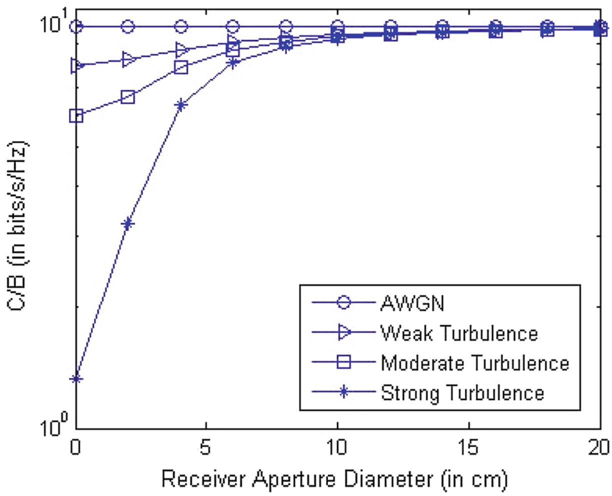

Fig. 8 Average channel capacity C/B versus the receiver aperture diameter

Also the impact of aperture averaging (increasing the size of the receiver aperture D to average out the signal fluctuations Khalighi et al. 2009) on the capacity under very clear weather conditions can be studied from Eqs. (4), (5) and (7). Figures 5, 6, and 7 show the effect of increasing the receiver aperture diameter on the capacity in weak, moderate and strong turbulent regimes, respectively. These figures are used to plot the variation of average capacity C/B with aperture diameter in weak, moderate and strong turbulence for an average electrical SNR of 30 dB as shown in Fig. 8. The capacity in AWGN at a SNR of 30 dB is 9.97 bits/s/Hz. It is observed that the value of C/B increases significantly with the initial increase in receiver aperture diameter and tends to saturate as the diameter is further increased. This is clearly seen from Table 3 which lists average channel capacity as the receiver diameter is increased. It can be seen that the percentage increase in the capacity falls below 3 % for a diameter of 6 cm in case of weak turbulence, 8 cm for moderate turbulence and 10 cm for strong turbulence. It is observed that the effect of aperture averaging in increasing the capacity is more pronounced in the strong turbulence regime.

5 Conclusions

The capacity of a FSO link considering impairments caused by various weather conditions and atmospheric turbulence is evaluated. It is observed that the capacity decreases as the turbulence strength goes from weak to strong. Also it is seen that the presence of various weather conditions such as fog, haze, drizzle etc. have a detrimental effect on the capacity of the FSO link. The worst degradation in the capacity is caused by the presence of fog in the atmosphere, followed by haze and then drizzle. This trend is followed irrespective of the turbulence strength. The effect of aperture averaging on the capacity is demonstrated and it is seen that increase in receiver diameter greatly enhances the capacity, the greatest improvement being in the case of strong turbulence. But as the diameter is increased beyond 6, 8 and 10 cm in the case of weak, moderate and strong turbulence respectively, the increase in capacity drops below 3 %. It implies that the capacity has become nearly constant and reached to a saturated value. The results obtained are quite significant in the design of a FSO link under practical conditions.

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