

# A wide band-pass filter of broad angle incidence based on one-dimensional metallo-dielectric ternary photonic crystal

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**Abstract** A wide band-pass filter has been presented based on one-dimensional metallo-dielectric ternary photonic crystal. ZnS, Ag and MgF<sub>2</sub> are used as the materials of the photonic crystal. The forbidden and allowed bands of the photonic crystal are determined by using the band-edge analysis. The result indicates that the filter has a wide pass band in the visible region and it can block ultraviolet and infrared light. Simulation of the transmission spectra shows that the filter has decent transmittance in the pass band at broad angle incidence ranging from 0° to 70°, which can meet the need of practical use.

**Keywords** One-dimensional metallo-dielectric ternary photonic crystal · Broad angle incidence · Band-pass filter · Band-edge analysis

## 1 Introduction

Photonic crystal, which was proposed by Yablonovitch and John in 1987, is a periodic structure whose lattice constant is comparable to the wavelength of light (Yablonovitch 1987; John 1987). Photonic crystals forbid propagation of photons in a certain range of energies known as photonic band gap, and spontaneous emission of atoms and molecules is inhibited in such a crystal. Compared with two- and three-dimensional photonic crystals, one-dimensional photonic crystals are easier to fabricate and many related devices have been suggested (Scalora et al. 1994a,b; Steinberg and Chiao 1995; Tocci et al. 1995; Winn et al. 1998; Chigrin et al. 1999; Mattarelli et al. 2007).

One of the applications of one-dimensional photonic crystal is filters (Gupta et al. 1997; Lei et al. 1997; Liang et al. 2004; Shen et al. 2008; Yu et al. 2008), but reports on wide band-pass filters of broad angle incidence are rare. At oblique incidence, it's hard to make a filter

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possess a wide pass band, decent transmittance in the pass band and good cut-off property at both sides of the pass band simultaneously. But in this paper, we find that such kind of filter can be achieved by using a metallo-dielectric photonic crystal. Numerical calculation shows its band-pass property and good transmittance in the visible region at broad angle incidence. Such kind of filter is needed in occasions where the visible light is allowed to pass while ultraviolet and infrared light should be blocked, such as the detecting window of spacecrafts, projection displays, sensor protections (Baglio et al. 2001), digital cameras and so on.

## 2 Theoretical model and method

The structure of the filter can be stated as Air/(ABC)<sup>6</sup>/Sub, where (ABC)<sup>6</sup> is the photonic crystal structure containing six periods and Sub means a glass substrate. In each period of the ternary photonic crystal, A, B, C are, respectively referred as 32 nm-thick ZnS layer, 12 nm-thick Ag layer and 58 nm-thick MgF<sub>2</sub> layer. So the lattice constant is considered as  $\Lambda = 102 \text{ nm}$ .

From the theory of optical transfer matrix (Macleod 2001), the characteristic matrix of each single layer can be written specifically as  $\begin{pmatrix} \cos \delta & i \sin \delta / \eta \\ i \eta \sin \delta & \cos \delta \end{pmatrix}$ , where  $\delta$  is the phase depth of each layer,  $\delta = \frac{2\pi n d \cos \theta}{\lambda}$ ,  $n$  is the complex refractive index of the material,  $d$  is the depth of the layer,  $\lambda$  is the wavelength of light in the air, and  $\theta$  is the refractive angle in the layer.  $\eta$  is called the effective refractive index of the layer. For TE mode (s-polarization),  $\eta_s = n \cos \theta$  and for TM mode (p-polarization),  $\eta_p = n / \cos \theta$ .

Set the direction vertical to the surface of the thin-film layer as  $z$  axis. Written in the form of the normal component the wave vector  $k_z, \delta = k_z d$ . The tangential component of the wave vector  $k_y = \frac{2\pi n}{\lambda} \sin \theta$ . In each layer,  $k_y$  is a constant because the refractive angle  $\theta$  satisfies the Snell Law:  $n \sin \theta = n_0 \sin \theta_0$  ( $\theta_0$  is the incident angle in the air and  $n_0 = 1$ ).  $k_y = \frac{2\pi}{\lambda} \sin \theta_0$  and the wave vector  $k = \frac{2\pi n}{\lambda}$ , so the normal component the wave vector  $k_z = \sqrt{k^2 - k_y^2} = \frac{2\pi}{\lambda} \sqrt{n^2 - (\sin \theta_0)^2}$

For a one-dimensional ternary photonic crystal, the characteristic matrix of each period is  $M = M_A M_B M_C = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ , where  $M_A = \begin{pmatrix} \cos k_{1z} d_1 & i \sin k_{1z} d_1 / \eta_1 \\ i \eta_1 \sin k_{1z} d_1 & \cos k_{1z} d_1 \end{pmatrix}$ ,  $M_B = \begin{pmatrix} \cos k_{2z} d_2 & i \sin k_{2z} d_2 / \eta_2 \\ i \eta_2 \sin k_{2z} d_2 & \cos k_{2z} d_2 \end{pmatrix}$  and  $M_C = \begin{pmatrix} \cos k_{3z} d_3 & i \sin k_{1z} d_1 / \eta_1 \\ i \eta_3 \sin k_{3z} d_3 & \cos k_{1z} d_1 \end{pmatrix}$ . So  $\frac{A+D}{2} = \cos k_{1z} d_1 \cos k_{2z} d_2 \cos k_{3z} d_3 - \frac{1}{2} \left( \frac{\eta_3}{\eta_2} + \frac{\eta_2}{\eta_3} \right) \cos k_{1z} \sin k_{2z} d_2 \sin k_{3z} d_3 - \frac{1}{2} \left( \frac{\eta_3}{\eta_1} + \frac{\eta_1}{\eta_3} \right) \sin k_{1z} d_1 \cos k_{2z} d_2 \sin k_{3z} d_3 - \frac{1}{2} \left( \frac{\eta_2}{\eta_1} + \frac{\eta_1}{\eta_2} \right) \sin k_{1z} d_1 \sin k_{2z} d_2 \cos k_{3z} d_3$

The electromagnetic theory of light tells us that a light wave can be expressed as a function with two components (Born and Wolf 2005):  $\Psi(z) = \begin{pmatrix} E(z) \\ H(z) \end{pmatrix}$ . After light passes through one period of the photonic crystal, the wave function satisfies  $\Psi(z) = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \Psi(z + \Lambda)$ .

The Bloch Theorem (Huang and Han 1988) in solid state physics is also applied here. Like the wave function of the electrons in periodic crystal lattice, the wave function of light in a photonic crystal satisfies  $\Psi(z + \Lambda) = \exp(-iK\Lambda) \Psi(z)$ , where  $K$  is the Bloch wave vector. Thus, we obtain the dispersion relation of a one-dimensional photonic crystal:  $\cos(K\Lambda) = \frac{A+D}{2}$   
For TE mode,

**Table 1** Optical constants of Ag thin film

Wavelength (nm)	Refractive index	Extinction coefficient
200	1.13	1.23
300	1.67	0.96
320	1.07	0.32
400	0.08	1.93
500	0.05	2.87
600	0.06	3.75
700	0.08	4.62
800	0.09	5.45
900	0.11	6.22
1,000	0.13	6.83
1,100	0.16	7.59
1,200	0.20	8.34

$$\begin{aligned} \frac{A + D}{2} &= \cos k_{1z}d_1 \cos k_{2z}d_2 \cos k_{3z}d_3 - \frac{1}{2} \left( \frac{k_{3z}}{k_{2z}} + \frac{k_{2z}}{k_{3z}} \right) \cos k_{1z} \sin k_{2z}d_2 \sin k_{3z}d_3 \\ &\quad - \frac{1}{2} \left( \frac{k_{3z}}{k_{1z}} + \frac{k_{1z}}{k_{3z}} \right) \sin k_{1z}d_1 \cos k_{2z}d_2 \sin k_{3z}d_3 \\ &\quad - \frac{1}{2} \left( \frac{k_{2z}}{k_{1z}} + \frac{k_{1z}}{k_{2z}} \right) \sin k_{1z}d_1 \sin k_{2z}d_2 \cos k_{3z}d_3 \end{aligned}$$

For TM mode,

$$\begin{aligned} \frac{A + D}{2} &= \cos k_{1z}d_1 \cos k_{2z}d_2 \cos k_{3z}d_3 - \frac{1}{2} \left( \frac{n_2^2 k_{3z}}{n_3^2 k_{2z}} + \frac{n_3^2 k_{2z}}{n_2^2 k_{3z}} \right) \cos k_{1z} \sin k_{2z}d_2 \sin k_{3z}d_3 \\ &\quad - \frac{1}{2} \left( \frac{n_1^2 k_{3z}}{n_3^2 k_{1z}} + \frac{n_3^2 k_{1z}}{n_1^2 k_{3z}} \right) \sin k_{1z}d_1 \cos k_{2z}d_2 \sin k_{3z}d_3 \\ &\quad - \frac{1}{2} \left( \frac{n_1^2 k_{2z}}{n_2^2 k_{1z}} + \frac{n_2^2 k_{1z}}{n_1^2 k_{2z}} \right) \sin k_{1z}d_1 \sin k_{2z}d_2 \cos k_{3z}d_3 \end{aligned}$$

In the above equations,  $d_1, d_2$  and  $d_3$  are, respectively the depth of the ZnS, Ag and  $MgF_2$  layer in each period, so  $d_1 = 32, d_2 = 12$  and  $d_3 = 58$  nm.  $n_1, n_2$  and  $n_3$  are, respectively the complex refractive index of the three materials (Palik 1998). Table 1 shows the optical constants of Ag thin film at different wavelengths.

The normal components of the wave vector in the ZnS, Ag and  $MgF_2$  layers are, respectively  $k_{1z}, k_{2z}$  and  $k_{3z}$ . Then, we have  $k_{1z} = \frac{2\pi}{\lambda} \sqrt{n_1^2 - (\sin \theta_0)^2}, k_{2z} = \frac{2\pi}{\lambda} \sqrt{n_2^2 - (\sin \theta_0)^2}$  and  $k_{3z} = \frac{2\pi}{\lambda} \sqrt{n_3^2 - (\sin \theta_0)^2}$ .

When  $|\frac{A+D}{2}| < 1$ , the Bloch wave vector is real and light is allowed to propagate in the system. The corresponding regions are called the allowed bands. However, when  $|\frac{A+D}{2}| > 1$ , the Bloch wave vector is a complex number and the Bloch wave is evanescent. These regions are called the forbidden bands or photonic bandgaps. The band edges are given by  $|\frac{A+D}{2}| = 1$ , which we call the band-edge equation. Whether the value of  $|\frac{A+D}{2}|$  is larger than 1 determines the position of forbidden bands. This method is called band-edge analysis (Lekner 2000).

One-dimensional photonic crystals are quite similar to multi-layered coatings, so the transfer matrix method, which is often applied to calculate the reflection and transmission of the thin film layers, can also be used to determine the bandgaps of one-dimensional photonic crystal (Shen et al. 2008; Zhang et al. 2005). The bandgaps are considered as the regions where the transmittance is very low. But one-dimensional photonic crystals are different from multi-layered coatings in the periodicity of their structure. When the band-edge analysis is used, the position of the bandgaps is determined on the basis of the formation mechanism of photonic bandgap instead the transmission value. The band-edge analysis gives a clearer and more concise explanation to the the generation of the bandgap (Liu 2007). It implies that it is the periodicity of the structure that causes the bandgap.

### 3 Results and discussion

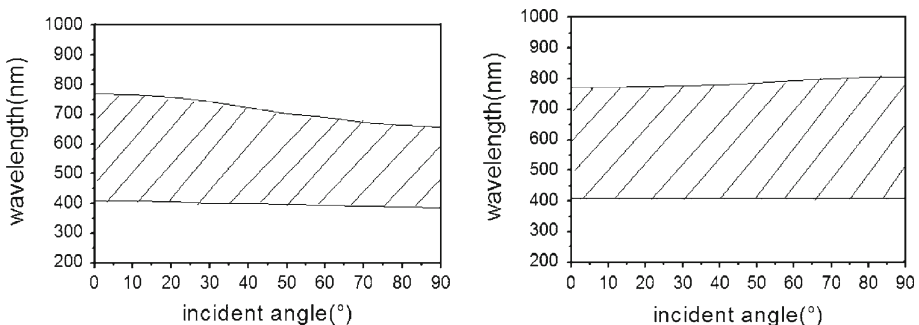
Based on the band-edge analysis, numerical calculation is carried out with the help of MATLAB. The forbidden and allowed bands of the ternary photonic crystal are obtained in the  $\theta_0 - \lambda$  plane, as shown in Fig. 1. The shaded areas are the allowed bands for TE mode in the left half and for TM mode in the right half. The white areas above and below the shaded areas are the forbidden bands.

When light passes through the photonic crystal structure, it comes out from the substrate. So the pass bands of the filter are the allowed bands of the photonic crystal. On the contrary, the cut-off regions of the filter are the forbidden bands of the photonic crystal. Figure 1 clearly shows the band-pass property of the filter. It has a wide pass band in the visible region, but ultraviolet and infrared light cannot propagate through the filter.

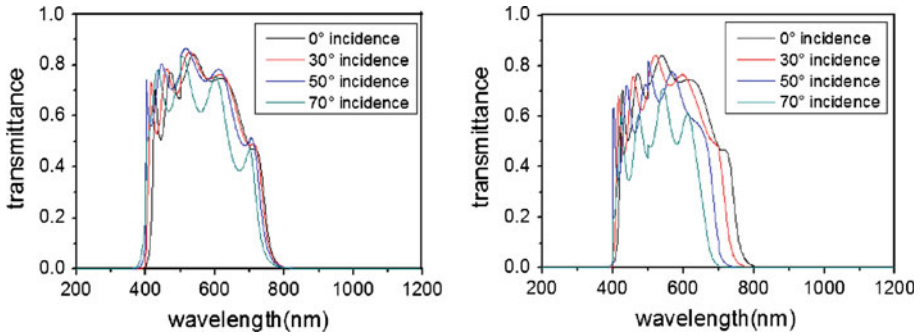
We can also tell from Fig. 1 that the band-edges don't change much with the angle of incidence. Metallo-dielectric photonic crystal is less sensitive to the incident angle and polarization than dielectric photonic crystal, because the combination of metal and dielectric causes a strong resonant tunneling effect in the visible region (Scalora et al. 1998; Bloemer and Scalora 1998).

Then, we calculate the transmittance spectra at wavelengths from 200 to 1,000 nm for TE and TM mode at different angle incidence, as shown in Fig. 2.

The cut-off regions and pass-bands shown in Fig. 2 well agree with the results obtained by the band-edge analysis. The transmittance in the pass band doesn't decrease much when



**Fig. 1** The allowed band (shaded area) and forbidden band (white area) for TE (left) and TM (right) mode in the  $\theta_0 - \lambda$  angle plane



**Fig. 2** p-(left) and s-(right) transmittance at different angle incidence

the incident angle increases. The average value can still reach 60% even at 70° incidence, which can meet the need of practical use.

#### 4 Conclusion

By using band-edge analysis, we have theoretically investigated the forbidden and allowed bands of a ternary photonic crystal using ZnS, Ag and  $\text{MgF}_2$  as materials, based on which a filter of broad angle incidence is presented. The filter is transparent to visible light, and it can also block ultraviolet and infrared light. The simulation of the transmittance spectra has indicated that it has satisfactory transmittance in the pass band at broad angle incidence, which can meet the need of practical use. Such kind of filters simultaneously possess a wide pass band, decent transmittance in the pass band and wide cut-off regions at both sides of the pass band at broad angle incidence. They were rarely reported in previous researches, but they can be used in space technology, projection displays, sensor protections, digital cameras and so on.

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