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A novel approach for the temperature dependence of the saturated signal power in EDFAs

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Abstract. The temperature dependence of the saturated signal power for the $^4I_{13/2} \rightarrow ^4I_{15/2}$ transition in erbium-doped fiber amplifiers (EDFAs) pumped at 980 nm and 1480 nm pumping wavelengths within a temperature range from −20 to 60°C are investigated by a novel approach. This approach is based on the temperature dependence of the saturated signal power. The influence of pump excited-state absorption (ESA) is inserted into the rate equations so as to investigate a generally situation. For 1480 nm pumping regime, it is seen that the saturated signal power increases more quickly than that of 980 nm pumping regime, with the increasing temperature. The variation in the saturated signal power with temperature is nearly constant at 980 nm pumping regime. In addition, the population inversion with respect to the increasing normalized signal power is examined and it is seen that it is independent temperature for 980 nm but it strongly depends on temperature for 1480 nm especially at lower normalized signal powers.

Key words: EDFAs, ESA, relative population inversion, saturated signal power, temperature dependence

1. Introduction

Erbium-doped fiber amplifiers (EDFAs) operating in the saturated signal power (SSP) regime have been widely used in optical networks and optical communication systems due to their low crosstalk and low noise figure (Dai *et al.* 1997; Sun *et al.* 1997). The performance analysis of these amplifiers depends on theoretical model to be used, such as the rate or the propagation equations. In addition, compositional studies of different types of Er:glass as well as theoretical modeling are necessary to investigate the performance of EDFAs, because the optimization of the amplifier parameters such as pumping efficiency and saturation power is becoming an important issue. Another important subject for EDFAs comes from pump exited-state absorption (ESA) process that can place a significant limit on the efficiency of Er^{3+}-doped amplifiers pumped at 980 and 1480 nm. In fact, there are many more energy levels in the Er^{3+}-ion, and the effects of pump or signal ESA are also possible. Owing to these effects, either a pump or signal photon is absorbed by an erbium ion in an excited state and it can be promoted much greater energy states (Becker *et al.* 1999).

A model of amplification in the presence of pump ESA includes a population rate equation for a fourth level that is the upper level for the excited-state transition. In principle, with regard to solve the rate equation systems, one consider a steady-state situation, where the rates of change of the populations are set to zero and then the saturated signal powers can be obtained in terms of the pump powers for some given fiber parameters. In some works (Desurvire and Giles 1989; Horowitz *et al.* 1999), the saturation effect of a signal, operating at the signal wavelength of nearly 1530 nm, in EDFAs is only related to pump powers, but the temperature dependence is not included. Consequently, the temperature factor has to be added into the relevant equations.

In this work, we investigate the basic rate equations for the four-level amplification model of Er^{3+}, because of the presence of excited-state absorption effect at several useful pump wavelengths. We ignore the amplified spontaneous emission for simplicity. In the theoretical approach, we first obtain the relative population difference for the four-level system and derive again in terms of the maximum population difference it. After that, the temperature dependence of the saturated signal power for both 980 nm and 1480 nm pumping configurations is investigated at the temperature range from −20 to 60°C. An expression relating the relative population difference to the signal power normalized by the saturated signal power is also derived. In this novel theoretical approach, we use a silica-base Er^{3+}-doped fiber for both 980 nm pumping and 1480 nm pumping regimes.

2. Mathematical model

We consider simplified energy diagram of a four-level amplification system with ESA at the pump wavelength, as shown in Fig. 1.

The rate equations for the populations N_1, N_2, N_3 and N_4 of levels 1, 2, 3 and 4, respectively, are given in the following matrix form:

$$\begin{pmatrix} \dot{N}_1 \\ \dot{N}_2 \\ \dot{N}_3 \\ \dot{N}_4 \end{pmatrix} = \begin{pmatrix} -S_{12} - R_\mathrm{p} & A_{21} + S_{21} & R_\mathrm{p} & 0 \\ S_{12} & -A_{21} - S_{21} - R'_\mathrm{p} & A_{32} & R'_\mathrm{p} \\ R_\mathrm{p} & 0 & -A_{32} - R_\mathrm{p} & A_{43} \\ 0 & R'_\mathrm{p} & 0 & -A_{43} - R'_\mathrm{p} \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{pmatrix}, \quad (1)$$

with $N_1 + N_2 + N_3 + N_4 = N$. Here, $\dot{N}_i (i = 1, 2, 3$ and $4)$ denotes the derivatives as to the time, R_p and R'_p are the pumping rates corresponding to ground state absorption and ESA which take place between levels 1–3, and 2–4, respectively, S_{12} and S_{21} are the stimulated emission rates between level 1 and level 2, respectively, and A_{ij} represent the total spontaneous decay rates (radiative and nonradiative) from level i to level j. Steady-state

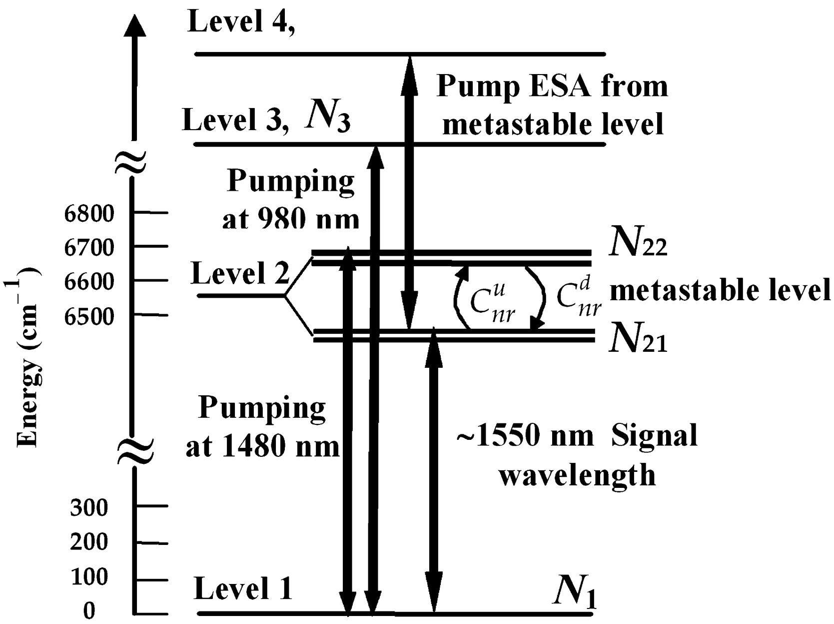

Fig. 1. Four-level amplification system in the different pumping regimes.

solutions of Equation (1) give the relative population difference $\Delta N/N = (N_2 - N_1)/N$ so as to determine the saturated signal power of an amplifier. In that case, the relevant difference is obtained as follows (see for details, Desurvire and Giles 1989);

$$\frac{\Delta N}{N} = \frac{R_p\tau + S_{12}\tau\ (1+\varepsilon R_p\tau) - \left[1 + S_{21}\tau + R'_p\tau(1-k)\right]\ (1+\varepsilon R_p\tau) + kA_{43}\tau}{\left[1 + S_{21}\tau + R'_p\tau(1-k)\right]\ (1+2\varepsilon R_p\tau) + (1+k)\left[(1+\varepsilon R_p\tau)S_{12}\tau + R_p\tau\right] + kA_{43}\tau(\varepsilon S_{12}\tau - 1)}. \quad (2)$$

Let $\tau = 1/A_{21}$, $\varepsilon = A_{21}/A_{32}$ and $k = R'_{\mathrm{p}}/(R'_{\mathrm{p}} + A_{43})$. As seen from Equation (2), the population difference is at its maximum when $S_{12}\tau = 0$ or $S_{21}\tau = 0$. Thus, the maximum population difference is derived by means of Equation (2) and under the condition of $S_{12}\tau = 0$:

$$\left(\frac{\Delta N}{N}\right)_{\max} = \frac{R_{\mathrm{p}}\tau - \left[1 + R'_{\mathrm{p}}\tau(1-k)\right]\ (1+\varepsilon R_{\mathrm{p}}\tau) + kA_{43}\tau}{\left[1 + R'_{\mathrm{p}}\tau(1-k)\right]\ (1+2\varepsilon R_{\mathrm{p}}\tau) + (1+k)R_{\mathrm{p}}\tau - kA_{43}\tau}. \quad (3)$$

Inserting Equation (3) into Equation (2) gives

$$\frac{\Delta N}{N} = \left(\frac{\Delta N}{N}\right)_{\max} \frac{1 + \frac{S_{12}\tau\ (1+\varepsilon R_{\mathrm{p}}\tau)(1-\eta)}{R_{\mathrm{p}}\tau - \left[1+R'_{\mathrm{p}}\tau(1-k)\right]\ (1+\varepsilon R_{\mathrm{p}}\tau) + kA_{43}\tau}}{1 + \frac{S_{12}\tau\left[\eta(1+2\varepsilon R_{\mathrm{p}}\tau) + (1+k)(1+\varepsilon R_{\mathrm{p}}\tau) + k\varepsilon A_{43}\tau\right]}{\left[1+R'_{\mathrm{p}}\tau(1-k)\right](1+2\varepsilon R_{\mathrm{p}}\tau) + (1+k)R_{\mathrm{p}}\tau - kA_{43}\tau}}, \quad (4)$$

where the equality of $S_{21} = \eta S_{12}$ is used. The last equation gives the relative population difference in terms of the pumping, stimulated emission and relevant decay rates, and can be applied for the silica, fluoride, tellurite and some other fluoro-silicate-based EDFAs (Jha *et al.* 2000; Jarabo and Alvarez 2001; Chung *et al.* 2004). In our opinion, Equation (4) can be solved with an attractive simplification arises from realization that in most practical cases, for instance germanosilicate fiber doped with Er^{3+} ions (Morkel and Laming 1989) and this solution can be used for the 980 nm or 1480 nm pump wavelengths in the absence of pump ESA. In our case, we will use a silica-based Er^{3+}-doped fiber amplifier for the either pump wavelengths, ignoring the pump ESA.

Let us now introduce the effect of temperature into the population inversion of an EDFA, under the saturated signal power regime for both 980 nm and 1480 nm pumping configurations. In general, the stimulated absorption rate, S_{12}^{sat}, which corresponds to the saturated signal power P_s^{sat}, is given by $S_{12}^{sat} = \Gamma_{12}\sigma_{12}P_s^{sat}/h\nu_{12}A_{12}$ (Desurvire 1994). Here σ_{12} is the signal absorption cross section between first and second (intermediate or metastable) levels, $h\nu_{12}$ is the photon energy for the ${}^4I_{15/2} \rightarrow {}^4I_{13/2}$ transition, Γ_{12} is the mode overlap or the confinement factor at the signal wavelength λ_{12} and A_{12} is the effective cross-sectional area of the fiber core. 980 nm pumping regime is treated as a three-level amplification system while 1480 nm pumping regime is considered as a two-level amplification system.

According to the modified rate equation model which taken the temperature into account (Berkdemir and Özsoy 2005, submitted), for 980 nm pumping regime, the population of third and fourth levels is nearly equal to zero and thus the total population N consists of Er^{3+}-ions in the remaining levels, i.e. $N = N_1 + N_2$. But, for the 1480 nm pumping regime, the population of the second level is supposed to take form $N_2 = N_{21} + N_{22}$, where the sublevel populations N_{21} and N_{22} are filled with Er^{3+}-ions coming from the ground state via the absorbed signal and pump powers, respectively.

The population sharing of Er^{3+}-ions between the two-sublevels within the ${}^4I_{13/2}$ energy state is governed by the Boltzmann's distribution, for maintaining a constant population at the thermal equilibrium. The thermalization process occurring between sublevels is represented by the non-radiative rates C_{nr}^{u} (transition to the upper sublevel) and C_{nr}^{ℓ} (transition to the lower sublevel). Thus, under the conditions of overall thermal equilibrium, we have the well-known relation $\beta = N_{22}/N_{21} = C_{nr}^{u}/C_{nr}^{\ell} = e^{(-\Delta E_2/k_B T)}$, where k_B is Boltzmann's constant, and $\Delta E_2 = E_{22} - E_{21}$ is the energy difference between the relevant sublevels. Consequently, total population of the two-level system can be written as $N = N_1 + N_{21}(1+\beta)$. In this study, for the two-level amplification system, it is assumed that the β parameter takes different values for different temperatures and a value of 200 cm^{-1} is used for the energy interval ΔE_2 between N_{22} and N_{21} at the temperature range from -20 to 60°C.

Based on the above arguments, the rate equations are modified for three and two level amplification systems, which represent EDFAs pumped by 980 nm and 1480 nm, respectively, and the relations for the relative population difference are given as (Berkdemir and Özsoy 2005a, b, submitted.):

$$\left(\frac{\Delta N}{N}\right)_{980} = \frac{R_p\tau + S_{12}\tau(1-\eta) - 1}{R_p\tau + S_{12}\tau(1+\eta) + 1}, \tag{5a}$$

$$\left(\frac{\Delta N}{N}\right)_{1480} = \frac{R_p\tau(1-\beta) + S_{12}\tau(1-\eta) - 1}{R_p\tau(1+2\beta) + S_{12}\tau(1+\beta+\eta) + 1}. \tag{5b}$$

Note that if $\beta = 0$, Equation (5b) reduces to Equation (5a). Now, the maximum population difference $\Delta N_{\max}$, which is required for the saturated signal power, can be derived for both pumping configurations. The population difference has its maximum when $S_{12} = 0$:

$$\left(\frac{\Delta N}{N}\right)_{\max} = \frac{R_p\tau - 1}{R_p\tau + 1}, \quad \text{for 980 nm pumping} \tag{6a}$$

$$\left(\frac{\Delta N}{N}\right)_{\max} = \frac{R_p\tau(1-\beta) - 1}{R_p\tau(1+2\beta) + 1} \quad \text{for 1480 nm pumping} \tag{6b}$$

Where R_p refers to both pump absorption and pump emission rates ($R_p = \Gamma_p\sigma_p P_p / h\nu_p A_p; P_p$ is the pump power), and τ is the lifetime of second level. If ΔN equals to $\Delta N_{\max}/2$, then P_s becomes the saturated signal power P_s^{sat} and is given by

$$\left(P_s^{sat}\right)_{980} = b_s\left(\frac{\left(b_p P_p\right)^2 - 1}{b_p P_p\,(3\eta - 1) - (3-\eta)}\right), \tag{7a}$$

$$\left(P_s^{sat}\right)_{1480} = b_s\left(\frac{\left[b_p P_p(1+2\beta) + 1\right]\left[b_p P_p(1-\beta) - 1\right]}{b_p P_p\left[3\eta(\beta+1) - \beta(\beta+4) - 1\right] + \eta - \beta - 3}\right). \tag{7b}$$

Here η is the ratio of the signal emission and absorption cross sections, i.e., $\eta = \sigma_{21}(\lambda_s)/\sigma_{12}(\lambda_s)$ and obtained by using the McCumber relation (McCumber 1964; Zech 1995), $b_p = \tau\,\Gamma_p\sigma_p / h\,\nu_p A_p$ and $b_s = h\,\nu_s A_s / \tau\,\Gamma_s\sigma_{12}$. Now, Equations (7a) and (7b) can be employed to investigate how the saturated signal power changes with the launched pump power in the temperature range from -20 to 60°C.

By considering the system has three levels for all pumping wavelengths (Iizuka 2002), the relative population difference in terms of the saturated signal power is given as below:

$$\frac{\Delta N}{N} = \frac{\Delta N_{\max}}{N} \frac{1}{1 + P_s / P_s^{\mathrm{sat}}} \tag{8}$$

In the case of pumping at 980 nm, for a large normalized pump power, the maximum obtainable relative population difference is 1 from Equation (6a), but from Equation (6b) for 1480 nm it changes with the parameter β. Under the assumption that number of levels is different for different pump wavelengths, the following relations are obtained by considering Equations (7) and (8).

$$\left(\frac{\Delta N}{N}\right)_{980} = \left(\frac{\Delta N_{\max}}{N}\right)_{980} \cdot \left\{1 + \frac{2(P_p b_p \eta - 1) \times F}{P_p b_p c_p + F \times (1 - \eta)(P_p b_p + 1)}\right\}^{-1}, \tag{9}$$

$$\left(\frac{\Delta N}{N}\right)_{1480} = \left(\frac{\Delta N_{\max}}{N}\right)_{1480} \cdot \left\{1 + \frac{(2 + \beta)\left[P_p b_p (\eta - \beta) - 1\right] \times F}{P_p b_p d_p + F \times (1 - \eta)[P_p b_p (1 + 2\beta) + 1]}\right\}^{-1}, \tag{10}$$

where $c_p = [3\eta - 1] - (3 - \eta)$, $d_p = [3\eta(\beta + 1) - \beta(\beta + 4) - 1] + \eta - \beta - 3$ and $F = P_s / P_s^{\mathrm{sat}}$.

3. Results and discussion

In this saturated signal power calculations, we take the fiber parameter values used in Table 1 for 980 nm pumping regime from the study of Liu (Liu *et al.* 1995) and those in Table 2 for 1480 nm from the study of Desurvire (Desurvire *et al.* 1990). The values calculated for η at −20, 20 and 60°C for 980 nm pumping regime are 1.110, 1.095 and 1.084, respectively. The values of η and β at the relevant temperatures for 1480 nm are obtained as 1.217, 1.185, 1.161 and 0.33, 0.38, 0.43, respectively.

In the case of 980 nm pumping regime, Fig. 2 shows the saturated signal power as a function of launched pump power at the temperature values of −20, 20 and 60°C. This figure shows that the saturated signal power is proportional to not only the pump power but also to the temperature. If the temperature is increased from −20 to 60°C, the saturated signal powers increase almost the same amount for all the launched pump powers, and it can be seen that its temperature dependence is almost linear.

Table 1. Fiber parameter values used in our model calculations for a silica-base Er^{3+}-doped fiber amplifier at 980 nm pumping regime (from reference Liu *et al.* 1995)

Parameters	Values	Parameters	Values
σ_{12}	5.25×10^{-25} m^2	$A_{p,s}$	19.6×10^{-12} m^2
σ_{21}	5.75×10^{-25} m^2	$\Gamma_{p,s}$	0.5
σ_p	2.0×10^{-25} m^2	N	5.8×10^{24} m^{-3}
τ	11 ms	λ_s	1558 nm
L	12 m	λ_p	980 nm

Table 2. Fiber parameter values used in our model calculations for an aluminosilicate EDFA at 1480 nm pumping regime (from reference Desurvire *et al.* 1990)

Parameters	Values	Parameters	Values
σ_{12}	5.94×10^{-25} m^2	$A_{p,s}$	14.6×10^{-12} m^2 ; 28.3×10^{-12} m^2
σ_{21}	7.04×10^{-25} m^2	$\Gamma_{p,s}$	0.5; 0.3
σ_p	2.0×10^{-25} m^2	N	2.2×10^{24} m^{-3}
τ	10 ms	λ_s	1531 nm
L	38.5 m	λ_p	1480 nm

Figure 3, which is for 1480 nm pumping regime, shows a different behavior than those of Fig. 2, i.e., slops of the curves are not equal and higher for increasing temperatures. This difference is due to β and hence due to the temperature dependence of saturated signal power. The dependent of the saturated signal power on temperature is again almost linear.

The relative population difference vs. the normalized signal power is plotted in Fig. 4 for both pumping regimes. The uppermost curve is related to Equation (7), which assumes that the system has three levels for all wavelengths, the next curve belongs to 980 nm pumping, for all of the temperatures, and the other curves are connected with 1480 nm pumping for the

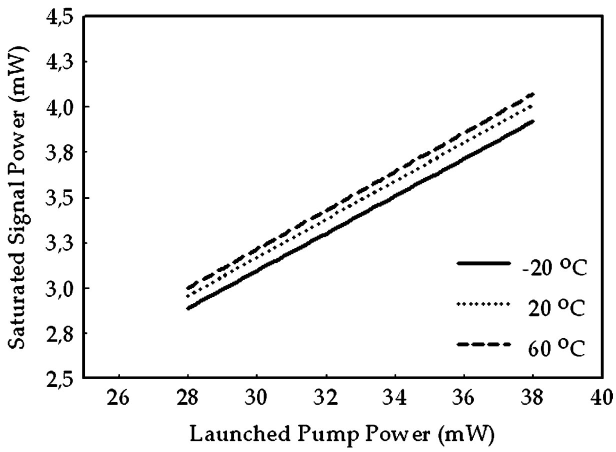

Fig. 2. The dependence of the saturated signal power to the launched pump power in the related temperature values for 980 nm pumping regime.

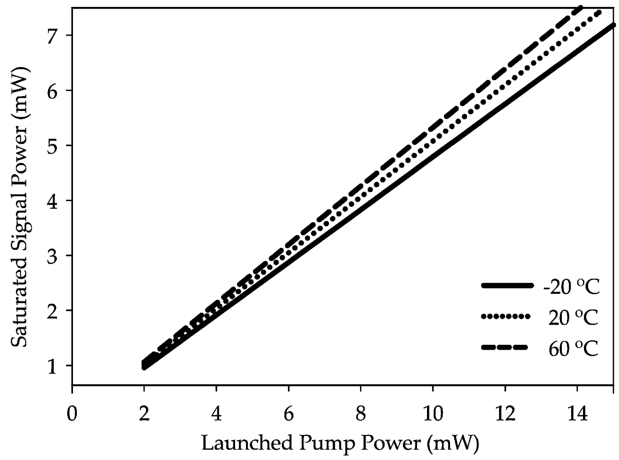

Fig. 3. The dependence of the saturated signal power to the launched pump power in the related temperature values for 1480 nm pumping regime.

temperatures of −20, 20 and 60°C, respectively. It is seen from all of the curves that, the relative population difference gradually decreases when the saturated signal power increases. For the pumping at 1480 nm, an increase in the temperature results in a decrease in the relative population difference, especially at lower normalized signal powers, but at 980 nm, it is not influenced from the temperature changes.

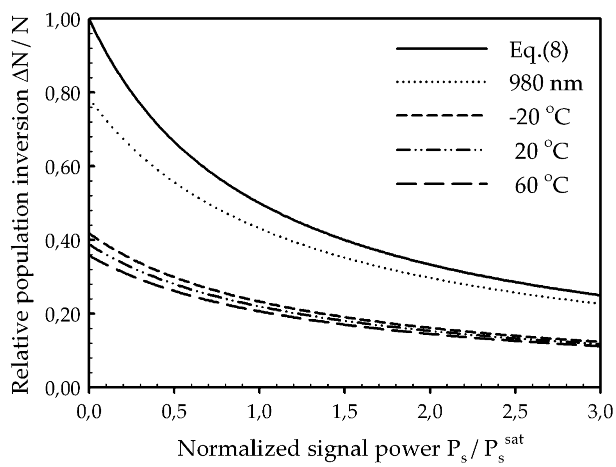

Fig. 4. The comparison of the relative population difference with the normalized signal power in the relevant temperature values for both pumping regimes. The upper curve is related to Equation (8) and its lower curve is corresponded to the case of 980 nm pumping regime. The rest curves are related to the case of 1480 nm at −20, 20 and 60°C, respectively.

4. Conclusion

We have used a new approach to describe the temperature dependence of the saturated signal power for EDFAs operated at 980 nm and 1480 nm pumping regimes, and analyzed the results for the temperature range from -20 to $60°C$. The expressions for the relative population difference have also been derived in terms of the parameters including the temperature and the saturated signal power for both pumping regimes. In this approach the main point is that the saturated signal power is connected not only to the pumping powers but also to the temperature. It is seen that, for 1480 nm pumping regime the saturated signal power more quickly increases than that of 980 nm pumping regime, with the increasing temperature. On the other hand, at 980 nm pumping regime, the variation of signal power saturated is nearly constant for all the pump powers within the relevant temperature range. Furthermore, the population inversion with respect to normalized signal power is almost independent temperature for 980 nm but it strongly depends on temperature and increases with decreasing temperature for 1480 nm pumping. In our theoretical calculations, we ignore the effect of pump ESA for both pumping regimes by reason of the simplicity.

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