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RESEARCH ARTICLE

Uncertainty analysis of structural response under a random impact

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Abstract
The problem of the vulnerability of structures facing impacts came to the front line of the scientific scene in the last decades. Structural impacts usually present dangerous potential hazards, e.g. domino accident. Deterministic models are not sufficient for reliability analysis of impacted structures. Uncertainty of the environmental conditions and material properties have to be taken into account. The proposed research is devoted to the analysis of structural behavior under a variable impact loading. Bernoulli beam model is used as a structural model of a pipe or load-bearing element, while the contact -force formulation for impact is studied to simulate the wide range of possible types of debris. Model sensitivity is studied first. The influence of the impact force, its duration, position on structural behavior, as well as beam material are then considered. Uncertainty analysis of a single impact and several impacts are next considered. The obtained insights can provide the guidelines for modeling the structure under the debris loading taking into account the uncertainties. It was shown that determinist simulations of structural reliability under the impact are not enough. The impactors that can be considered "safe" based on the deterministic simulation can affect structural integrity if their characteristics are slightly varying.

Keywords Impact · Rigid · Soft · Sensitivity · Uncertainty

1 Introduction

To design impact resistant structures, efforts have to be made in developing reliability analysis and design methods. Research efforts, stimulated by industrial needs, are still required to achieve this goal. Zhu et al. (2015) have analyzed oil vapor explosion accident, various causes that led to the explosion, high casualties and severe damages. They mentioned that debris usually presents dangerous potential hazards, e.g.

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domino accident. Statistical distribution of debris (Zhu et al. 2015; Van der Voort and Weerheijm 2013) shows that the attention has to be paid to a variable impactor modeling. Among possible affected structures, pipelines or load-bearing beams can play a major role in the domino effect. This consideration defines the object of the present research. Prediction of a debris impact on pipes and beams needs a detailed understanding of impact phenomena, structural and material response of pipes, and, of course, uncertainties consideration. Structural design by safety factors using nominal values without considering uncertainties may lead to designs that are either unsafe or too conservative and thus not efficient.

Nevertheless, works concerning variability, reliability or uncertainty remain rare. Regarding the specific analysis of pipes and beams, we may find works on the same topics, analogously limited to deterministic situations.

Numerous experimental and numerical studies of an impact on a pipe or beam have been carried out. But very few studies concern the stochasticity of the systems. Existing studies are considering structure geometry, material and impactor velocity uncertainties. Among them is the study of Wagner et al. (2017) concerning a robust design criterion for axially loaded cylindrical shells. Alizadeh et al. (2016) studied pipe conveying fluid with stochastic structural and fluid parameters. Li and Liu (2003) studied a phenomenon of elastic-plastic beam dynamics keeping the geometrical and material parameters of a beam fixed and applying uncertainty on the pressure amplitude, i.e. impact characteristic. Riha et al. (2006) proposed a model to predict the penetration depth of a projectile in a two- and three-layer target both deterministically and probabilistically. Material properties and projectile velocity are considered in the probabilistic analysis. Antoine and Batra (2015) have analyzed the influence of impact velocity, laminate material and geometrical parameters on the plate response during the low-velocity impact by a rigid hemispherical-nosed cylinder. Wang and Morgenthal (2018) conducted the reliability analysis of bridge piers impacted by barge. Here barge mass, impact velocity, oblique impact angle, water elevation, material properties of pier members and soil are random parameters. Fyllingen et al. (2007) conducted stochastic numerical and experimental tests of square aluminum tubes subjected to axial crushing. Uncertain parameters considered were a geometry of a tube (extrusion length and wall thickness) and impact velocity of the impactor. Lönn et al. (2010) presented an approach to robust optimization of an aluminum extrusion with quadratic cross-section subjected to axial crushing taking into account the geometry uncertainty. Villavicencio and Soares (2011) showed that the dynamic response of a clamped steel beams struck transversely at the center by a mass is very sensitive to the way in which the supports are modeled. So the boundary conditions of a structure are to be studied as well.

The authors try to fill in the existing gap and conduct an analysis that takes into account all the variability of the dangerous debris, and thus allows the realistic assessment of risks during Domino effects. In light of the above, the present research evaluates the probability of a failure of a structure (beam or pipe) under an impact load when impactor characteristics are unknown. Several parameters are taking into account: a material of a structure, characterized by the Young modulus, position of an impact on a structure and special attention is given to impactor characteristics. Not only its velocity and mass are considered, but also the material of an impactor. For this, a simplified model of an impacted beam, suitable for stochastic simulations, is proposed. An impact is introduced into the model by its contact-force history as a pulse of sinusoidal shape. First, the sensitivity of a model versus loading parameters and beam material is studied. Then a dynamic response of a beam to multiple impacts is considered.

2 Numerical model of a beam under variable impactors

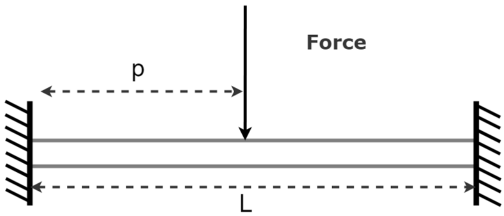
2.1 Structure modeling

A finite-element model of an elastic Bernoulli beam (Labuschagne et al. 2009) was implemented in Matlab with Newmark time integration scheme (Lindfield and Penny 2019).

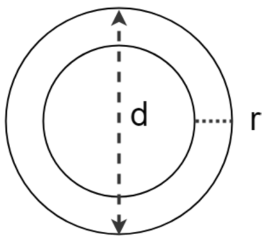
The perfectly clamped hollow cylindrical steel beam was considered, see Fig. 1. A choice of a cross-section was defined by the possible application in the pipe failure analysis. Obtained results are valid for other symmetrical cross-sections. The characteristics are the length $L = 1$ m, the diameter $d = 0.1$ m, the thickness $r = 0.02$ m, Young modulus $E = 2.158e11$ Pa, density $\rho = 7966\ kg/m^3$ and yield stress $\sigma_y = 2.5e8$ Pa.

2.2 Limit state criterion

Depending on the failure cause different limit states are associated with the structural elements of the linear pipeline parts (Timashev and Bushinskaya 2016). Parameters such as degradation of strength under impact force and loss of pipe material plasticity can be taken into account for impacted pipes risk assessment. In the present study, an elastic limit is chosen for the analyses as a serviceability limit state. Being too conservative for the design of most pipes due to the capacities of the elastic-plastic range, it can be reasonable for the dangerous sites where a Domino effect is highly possible. According to these considerations, only stresses in the elastic domain are calculated and stresses with $\sigma > \sigma_y$ are considered as a failure.



a) General view



b) Cross-section

Fig. 1 Beam subjected to a single impact

2.3 Impactor modeling

The article aims to analyze the structural response of a beam under the impact of arbitrary debris, with uncertain mass, velocity, and material. For uncertainty analysis a lot of computations have to be made. Developing a complete model of beam and impactor, and remodeling of an impactor for each iteration will demand high computational cost. We decided to introduce the impact of an object by its contact-force history. The question of a pulse shape for a contact-force history arises. The impact of a rigid object on a rigid structure provokes a sinusoidal shape of a contact-force history, see works of Abrate (2001). Perera et al. (2016) gave an idea of the realistic contact force estimation for solid debris impact. The model presented in this paper enables the value of the peak contact force generated by the impact of a piece of debris to be predicted. Results of calculations employing the derived relationships have been verified by comparison with experimental results across a wide range of impact scenarios. The observed contact load has a sinusoidal shape as well. Grady (1988) studied numerically and experimentally structural response under the impact of a soft projectile. He pointed out that the pulse is not symmetrical sinus: force reaches the maximum early in the contact interval and then decreases slowly till the contact end. Nevertheless, we adopted the hypothesis that the contact force history can be described with a sinusoidal pulse. Following Youngdahl (1971; 1970), we defined it by two correlation parameters: a pulse amplitude and a pulse duration:

$$\text{load} = (\pi - 2)h \sin(2\omega t) \tag{1}$$

where $0 < t < \frac{\pi}{2\omega}$, $\omega = \frac{\pi-2}{l}$, amplitude h and duration l.

Duration time is the main difference between rigid and soft impacts, as formulated in Andreaus and Casini (2000). A comprehensive overview of soft impact models is done by Abrate (2016) for three types of soft projectile - liquid, bird, hailstone. Thus rigid and soft impact impulses are introduced into the model by considering the pulse of different time duration. Summarizing, the material, mass and velocity of the impactor are introduced into the model by the sinusoidal contact-force history with corresponding amplitude and duration, see Fig. 2. The developed numerical model of a beam under an impact is valid for the case of elastic material, so the impactor's characteristics have to be in the ranges which will induce stresses in the beam that will not exceed the yield stress $\sigma_y = 2.5e8$ Pa. The parametric study was conducted with varying amplitude and duration. Obtained maximum stresses σ_x for each pair of parameters $(amplitude, duration) = (h, l)$ are presented in Fig. 3 for an impact at the midspan of the beam for an impact close to one end of the beam. Only values of stresses in the elastic domain are marked with color. The normal stresses for impact position at $p = 0.1$ m are smaller than for $p = 0.5$ as expected.

The input parameters of the system that are considered to keep it in elastic domain are chosen following the values in Fig. 3 and are given in Table 1.

To give an idea of what are the impactors that can provoke such stresses and that are used in the present study, we followed the study of Shivakumar et al. (1985) and calculated mass and velocity of steel and rubber impactors that can provide the maximum values in Table 1. Parameters of possible impactors are given in Table 2.

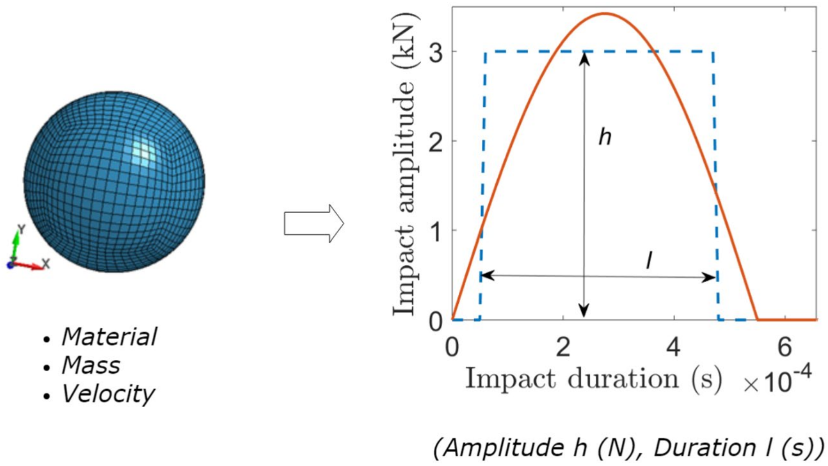


Fig. 2 Modeling of an impactor by corresponding contact-force history: load = $(\pi - 2)h\sin(2\omega t)$

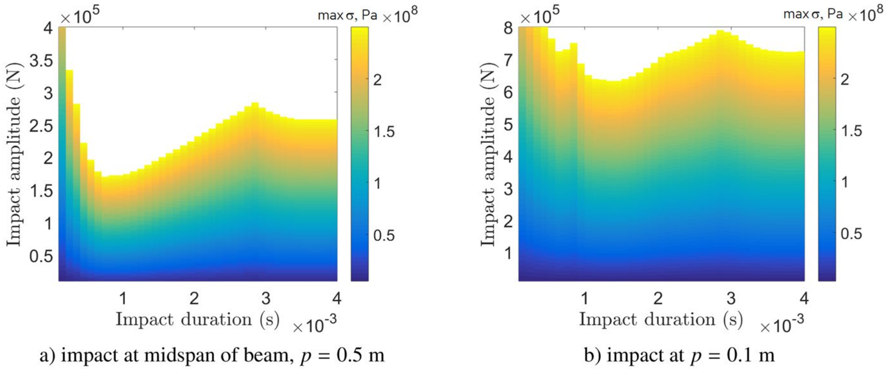


Fig. 3 Maximum stresses during the impact. Stresses values in an elastic domain are given by a colorbar. The white area corresponds to a plasticity domain

Table 1 Values of contact-force history parameters used for the analysis

Input parameters	Min	Max
Amplitude h (N)	0	15e4
Duration l (ms)	0.01	4
Position p (m)	0.05	0.5

2.4 Dynamic analysis

Natural frequencies of the considered beam were calculated to analyze the dynamic response under an impact and evaluate the influence of a contact-force history shape.

Table 2 Impactors for which the analysis is valid. Approximate values of mass and velocity for rigid and soft impactors, calculated according to Shivakumar et al. (1985)

Material	From			To
Rigid	Mass	1 kg	↗	10 kg
	Velocity	9 m/s	↘	1 m/s
Soft	Mass	1 kg	↗	10 kg
	Velocity	1 m/s	↗	10 m/s

Values of the first three natural frequencies and corresponding modal participations are given in Table 3. Comparing natural frequencies of the beam and frequencies, excited by considered contact-force histories (Fig. 4) it is shown that beam natural frequencies are excited by an impact with duration from 0.56 to 0.01 ms. Impacts of duration from 0.56 to 4 ms cannot induce natural vibrations of this beam. Influence of the first natural frequency is observed in Fig. 3: stresses are rising when $l = 0.56$ ms. Other modes have lower modal participation index. Thus, the present research will use the sinusoidal shape for the contact-force history.

3 Sensitivity analysis of impactor and pipe characteristic

The sensitivity analysis has two main purposes. The first is to identify the input variables that have a strong influence on the output of the model. These variables have to be determined precisely to improve the accuracy of the model. The second is to identify, on the contrary, the input variables that have less influence on the output. It is then not necessary to have precise values for these variables. Thus, for models with a large number of input variables, the sensitivity analysis allows determining

Table 3 Natural frequencies and corresponding modal participations of the considered beam

Natural frequency	Modal participation (%)
650.27 Hz	60
1756.3 Hz	1
3340.4 Hz	16

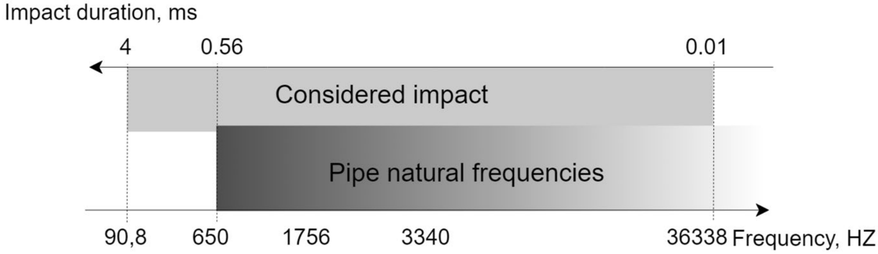


Fig. 4 Modeling of an impactor by corresponding contact-force history

variables that have a significant impact on output and simplifies the model by neglecting the precision for input variables with small influence.

3.1 Parameters considered in sensitivity study

The study is concentrated on the stochastic nature of the impact. The characteristics of the impact that will be studied are the impact force, duration and position. The variability of the structure material will be considered through variation of the Young modulus E.

Parameters are assumed to be independent and have uniform distribution in the limits given in Table 1 for impulse characteristics. Considering Young modulus, the material of a pipe is assumed to be known, but it can vary slightly due to manufacturing or aging. Thus, E has uniform distribution over the range $[1.95e11; 2.2e11]$ Pa. The sample of size $N = 1400$ is obtained using Latin hypercube sampling (LHS) (McKay et al. 2000).

The output parameters of the model that are influenced by the impact characteristics and are important for the evaluation of the structural integrity are the maximum beam deflection w_{max} and maximum stresses σ_{max} together with deflection w_p and stresses σ_p at the impact position.

All the numerical calculations presented in the article were conducted with the help of Matlab programs and in-build functions and were run on the computer with the following parameters: Intel(R) Core(TM) i5-6440HQ CPU 2.60GHz, RAM 8 Go.

3.2 Sobol method of sensitivity analysis

Sensitivity analysis is a study of how the variation in the output of a model (numerical or otherwise) depends, qualitatively or quantitatively, on the model inputs. The concept of using variance as an indicator of the importance of an input parameter is the basis for many variance-based sensitivity analysis methods. The method of Sobol (2001) is one of the most established and widely used methods and is capable of computing the Total Sensitivity Indices (TSI), which measure the main effects of a given parameter and all the interactions (of any order) involving that parameter. Sobol's method uses the decomposition of the model output function $y = f(x)$ into summands of variance using combinations of input parameters in increasing dimensionality. To determine the sensitivity of the output to the variation of an input parameter, an input factor space, $\Omega^n = (\mathbf{x}|0 \le x_i \le 1;\ i = 1, ..., n)$ is defined, where n is the number of variables. For a given model f linking input parameters $\mathbf{x} = (x_1, ...x_n)$ to a scalar output $y = f(\mathbf{x})$, there exists a unique partition of f so that

$$y = f\left(x_1, x_2, ...x_n\right) = f_0 + \sum_{i=1}^{n} f_i\left(x_i\right) + \sum_{i} \sum_{i<j} f_{ij}\left(x_i, x_j\right) + \cdots + f_{1...n}\left(x_1, \ldots, x_n\right) \tag{2}$$

where f_0 is the mean of f, provided that each function f_I for a given set of indices $I = i_1, \dots, i_n$, involved in the decomposition has zero mean over its range of variation:

$$\int_0^1 f_I(x_I)dx_I = 0. \tag{3}$$

The terms can be defined as:

$$f_0 = E(Y), \tag{4}$$

$$f_i(X_i) = E(Y|X_i) - f_0, \tag{5}$$

$$f_{ij}(X_i, X_j) = E(Y|X_i, X_j) - f_0 - f_i - f_j. \tag{6}$$

Assuming that $f(x)$ is square integrable, the total variance D of $f(\mathbf{x})$ is defined to be

$$D = \int_{\Omega^n} (f^2(\mathbf{x}) - f_0^2)d\mathbf{x}. \tag{7}$$

The partial variance is therefore the variance of f_I

$$D_I = \int_0^1 f_I^2(x_I)dx_I. \tag{8}$$

and the sensitivity index relative to the set I is expressed as the ratio of the variance of the function f_I to the total variance of the model:

$$S_I = \frac{D_I}{D}. \tag{9}$$

Another important sensitivity measure for a given parameter i is the total sensitivity index S_{Itot}, defined as the sum of the indices of all sets of parameters I to which i belongs

$$S_{Itot} = 1 - \frac{D_{\sim I}}{D}. \tag{10}$$

where $D_{\sim I}$ is the partial variance of all the parameters except of I. The first-order index represents the share of the output variance that is explained by the considered parameter alone. Most important parameters, therefore, have a high index, but a low one does not mean the parameter has no influence, as it can be involved in interactions. The total index is a measure of the share of the variance that is removed from the total variance when the considered parameter is fixed to its reference value. Therefore parameters with low S_{Itot}, can be considered as non-influential.

In practice, Sobol's method is relatively easy to implement using Monte Carlo based integration. Sobol's first order and total effect sensitivity indices can be implemented by expressing equation (9) in a discrete form following the procedure described in Saltelli et al. (2008). First, two matrices of data has to be generated, $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$, $i = 1, ..n$, $j = 1, ..N$. After this a matrix $\mathbf{C_i}$ has to be formed by all columns of **B** except the ith column, which is taken from **A**. The model output for all the input values in the sample matrices **A**, **B**, and $\mathbf{C_i}$ is computed to obtain three vectors of model outputs $y_A = f(\mathbf{A})$, $y_B = f(\mathbf{B})$, $y_C = f(\mathbf{C_i})$. With:

$$f_0^2 = \left(\frac{1}{N} \sum_{j=1}^{N} y_A^{(j)} \right)^2, \tag{11}$$

first-order sensitivity indices are estimated as follows:

$$S_i = \frac{y_A \cdot y_{C_i} - f_0^2}{y_A \cdot y_A - f_0^2} = \frac{\left(\frac{1}{N} \sum_{j=1}^{N} y_A^{(j)} y_{C_i}^{(j)} - f_0^2 \right)}{\left(\frac{1}{N} \sum_{j=1}^{N} y_A^{(j)} y_A^{(j)} - f_0^2 \right)}. \tag{12}$$

Similarly, the method total-effect indices are calculated as follows:

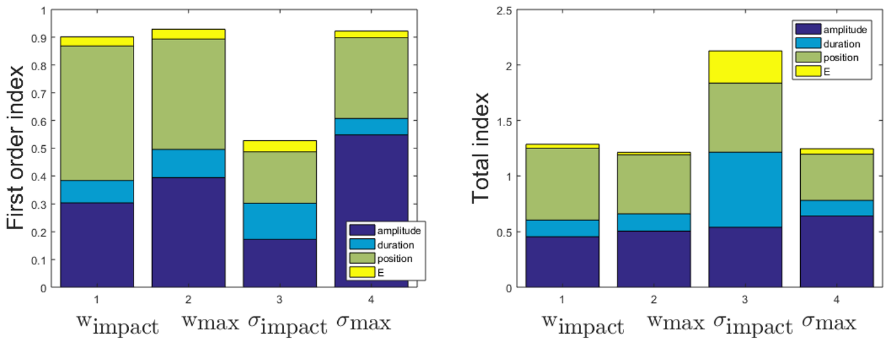
$$S_{Itot} = 1 - \frac{y_B \cdot y_{C_i} - f_0^2}{y_A \cdot y_A - f_0^2} = 1 - \frac{\left(\frac{1}{N} \sum_{j=1}^{N} y_B^{(j)} y_{C_i}^{(j)} - f_0^2 \right)}{\left(\frac{1}{N} \sum_{j=1}^{N} y_A^{(j)} y_A^{(j)} - f_0^2 \right)}. \tag{13}$$

3.3 Results

Following the algorithm presented in the previous section, numerical results for sensitivity analysis of an elastic beam are obtained.

First order effects and total effects of the stresses and beam deflection due to changes in loading characteristics were calculated. Numerical values are presented in the Fig. 5. It can be seen, that the variation of Young modulus doesn't play a major role in the obtained values. On the contrary, the position of the impact relative to the boundary conditions and impact amplitude strongly influence the beam response. Impact duration variation also affects the results. Comparing first order and total sensitivity indices we can say, that input parameter Young Modulus E has no interaction with other parameters, as values of the indices are the same. On the contrary, duration, amplitude and impact position are not independent parameters, as a total index is greater than first order index, meaning that there are indices of higher orders. This interaction is more important for stresses under the impact position.

According to the analysis, we will not take into account the changes in the beam material during the following studies but will consider the uncertainties of impactors: amplitude, duration and impact position.



a) First order effects b) Total order effects

Fig. 5 Sensitivity indices using Sobol's method calculated for such input parameters (from the bottom to the top): amplitude (dark blue), duration (light blue), position (green), and E Young modulus (yellow). Here w_{max} is the maximum beam deflection, σ_{max} is maximum stresses, w_p and σ_p are deflection and stresses at the impact position correspondingly. (Color figure online)

4 Uncertainty analysis of pipe response under point impact

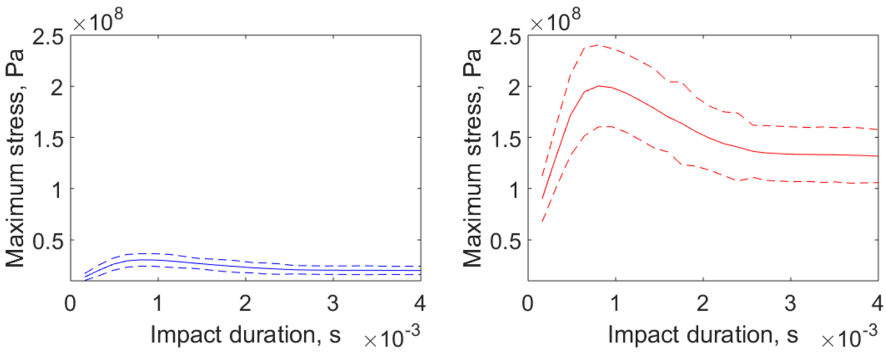
This section is devoted to the uncertainty analysis of the structural response of an impacted beam. To begin with, a normal distribution of the impact pulse parameters have been considered to estimate the possible error of deterministic simulation. In the following, the realistic situation that can lead to the Domino effect is modeled. The possible values of impactor properties are unknown and the aim is to compute the structural response. In this case, the uniform distribution of the impulse characteristics is used.

4.1 Stresses variability under single impact

In reality, all the parameters used for structural design or the reliability calculations are cursed by uncertainties. The deterministic design thus can give the solution that may not be able to meet the demands. Currently, the structure vulnerability evaluation against the impact is conducted through deterministic simulations. See for example the report of the French National Institute for Industrial Environment and Risks (INERIS) (2000) where different projectiles are listed.

To evaluate the influence of the uncertainty of the projectile characteristics, a stochastic analysis is done, that is based on the deterministic calculations, given in Fig. 3a. Impulse amplitude h and duration l have a normal distribution around mean values with a standard deviation of 20%. 50 samples for each mean value are obtained with LHC algorithm.

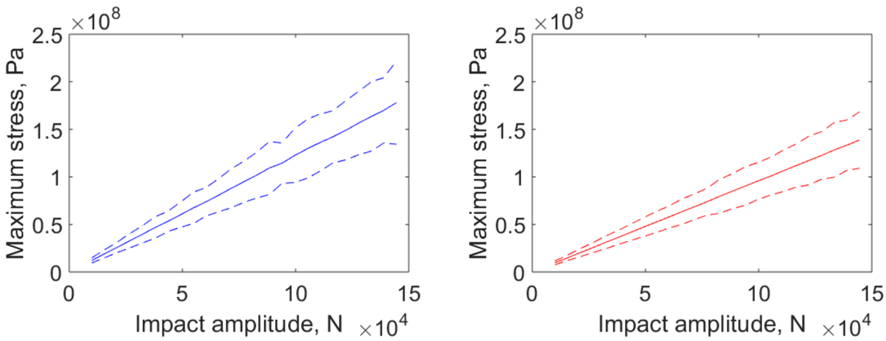
Some results are presented in Figs. 6 and 7 for the characteristic cases. Obtained stresses also have the normal distribution around the mean value with a standard deviation from 30 % for rigid impact (short duration) to 18 % for soft impact (long duration), meaning that the deterministic analysis of rigid impacts with high amplitudes can give 30 % of error.



a) amplitude $h = 2e4$ N

b) amplitude $h = 1.4e5$ N

Fig. 6 Mean and standard deviation of stresses with variable duration for different amplitude



a) duration $l = 0.001$ s, rigid impact

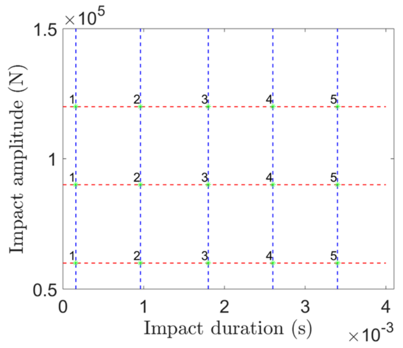
b) duration $l = 0.0032$ s, soft impact

Fig. 7 Mean and standard deviation of stresses with variable amplitude for different duration

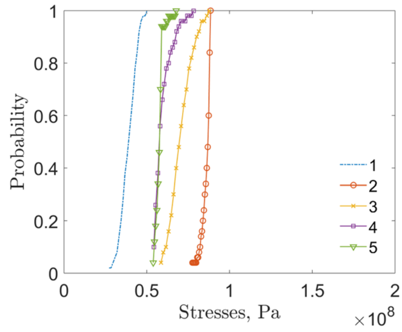
Cumulative distribution functions (CDF's) are calculated. It can be seen, that for any amplitude, all durations give the same standard deviation and thus uncertainty character (Figs. 8, 9). On the contrary, for a given duration the higher the amplitude is, the higher the standard deviation of the stresses is (Figs. 10, 11).

4.2 Stresses variability under multiple impacts

Situations of a single impact, two impacts with a different interval between them and three impacts with variable time were considered. Impulse characteristics, such as impact amplitude, duration and position (see Table 1) are considered as random variables with uniform distribution using Latin hypercube sampling (LHS). The sample size for single impact is $N = 1000$, for two impacts $N = 2000$ and for three impacts $N = 3000$. Obtained stresses distributions are presented in Figs. 12 and 13. In red are marked the number of tests when stresses exceeded the plastic limit σ_y.

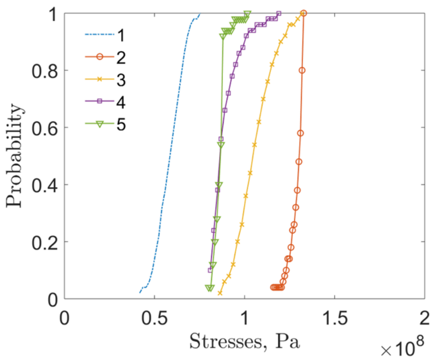


a) The layout of CDF studies for three amplitudes (red horizontal lines).

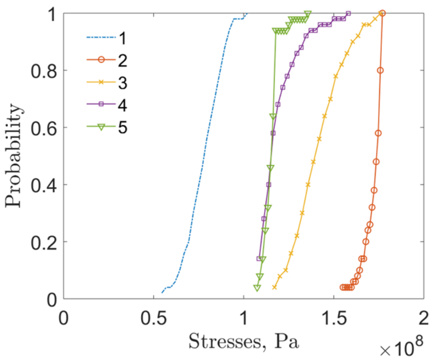


b) amplitude h = 6e4 N

Fig. 8 Cumulative distribution functions (CDF's) of stresses for a single impact at the mid-span of a beam. Value of amplitude is fixed, and duration has a normal distribution around a mean value. Mean values of duration are marked with 1, 2, 3, 4, 5



a) amplitude h = 9e4 N

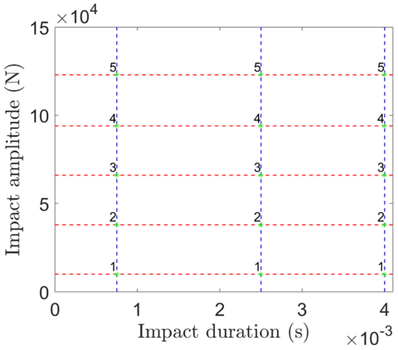


a) amplitude h = 12e4 N

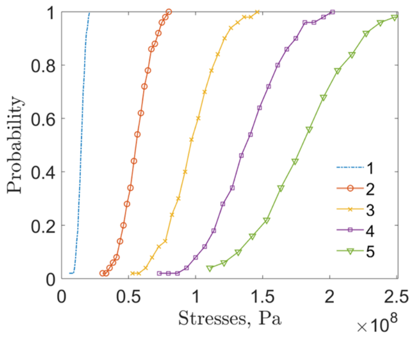
Fig. 9 Cumulative distribution functions (CDF's) of stresses for a single impact at the mid-span of a beam. Value of amplitude is fixed, and duration has a normal distribution around a mean value. Mean values of duration are marked with 1, 2, 3, 4, 5

Figure 14 represents the Cumulative Distribution Functions (CDF) of the stresses for one, two and three safe impactors. The same tendance can be noted: more impactors are falling on the pipe, the higher is the damage probability.

According to Fig. 14, when the interval between two impacts is 0.001 s less than 4% of impact can provoke plastic deformations. If the impacts will occur almost simultaneously with an interval of 0.0005 s, this number increases up to 12%. And in the case of three impacts, it becomes almost 20%. Figure 15 presents mean and median values of maximum stresses for these cases of multiple impacts and also demonstrates the tendency of increasing the stresses with increasing of impactors. Thus even if the impactor's characteristics do not provoke the plastic deformation in

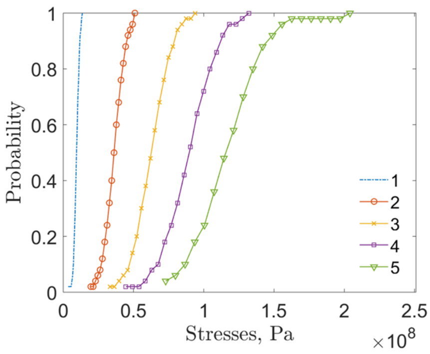


a) The layout of CDF studies for three durations (blue vertical lines).

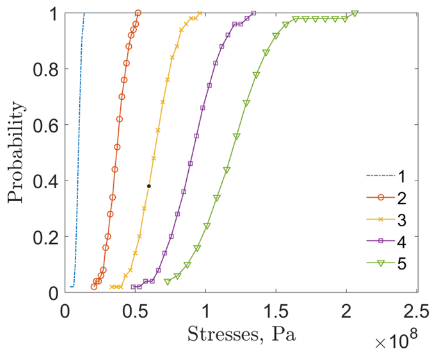


b) $l = 0.00075$ s

Fig. 10 Cumulative distribution functions (CDF's) of stresses for a single impact at the mid-span of a beam. Value of duration is fixed, and amplitude has a normal distribution around a mean value. Mean values of the duration are marked with 1, 2, 3, 4, 5



a) $l = 0.0025$ s



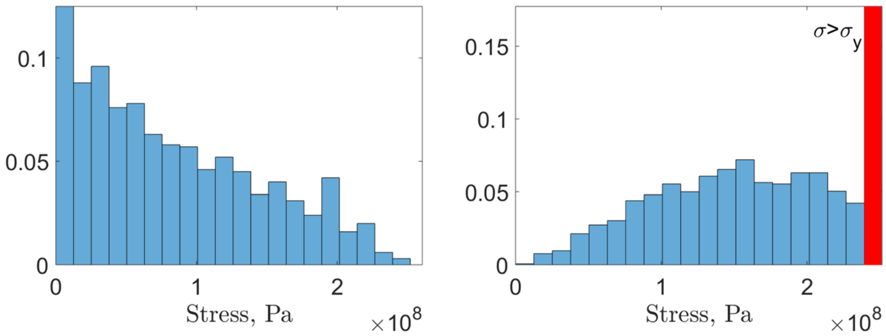
a) $l = 0.004$ s

Fig. 11 Cumulative distribution functions (CDF's) of stresses for a single impact at the mid-span of a beam. Value of duration is fixed, and amplitude has a normal distribution around a mean value. Mean values of the duration are marked with 1, 2, 3, 4, 5

the case of single impact, two and more impactors of the same size and velocity can damage the impacted structure.

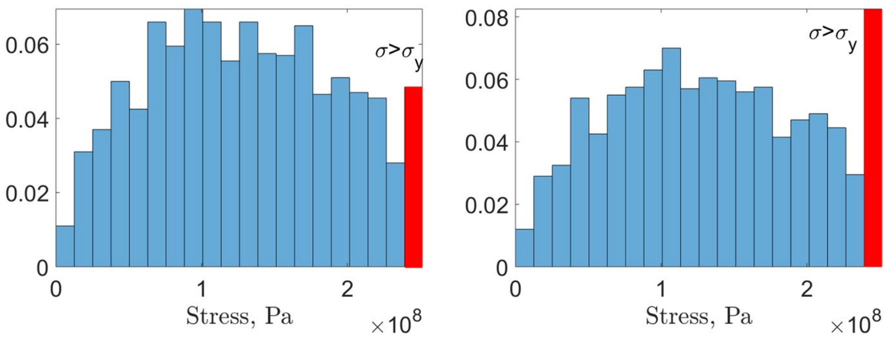
5 Conclusion

During industrial accidents, the debris after explosion can impact elongated structural elements (e.g. pipelines or beams) and provoke the Domino effect. Risk analysis needs to consider the uncertain nature of the system parameters, notably the



a) single impact b) three impacts with the interval of 0.001 s

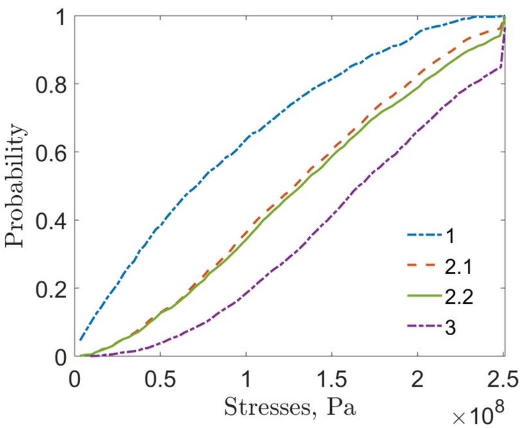
Fig. 12 Probability of possible stresses under single and three impacts



a) 0.001 s b) 0.0005 s

Fig. 13 Probability of possible stresses under two impacts with different intervals

Fig. 14 Cumulative distribution functions (CDF's) of stresses distribution. (1) corresponds to the case of one impact, (2.1) to the two impacts with an interval of 0.001 s, (2.2) to the two impacts with an interval of 0.0005 s and (3) corresponds to the three impacts



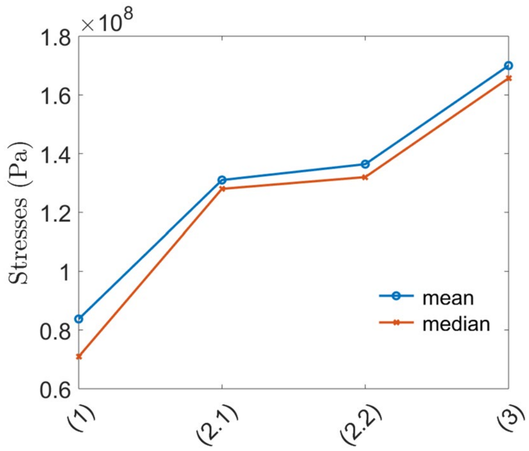


Fig. 15 Mean and median values of maximum stresses for multiple impactors. (1) corresponds to the case of one impact, (2.1) to the two impacts with an interval of 0.001 s, (2.2) to the two impacts with an interval of 0.0005 s and (3) corresponds to the three impacts

material, mass, and velocity of the debris. To answer this need, our paper proposes a stochastic analysis of structural beam response under a random impact.

A steel pipeline is simulated with the Bernoulli beam model; debris impacts are introduced in the system as impulses of sinusoidal shape. Amplitude and duration of a contact-force history represent the changes in the material, velocity, and mass. The proposed approach for impactor simulation saves time for complex numerical analysis and provides insights into the pipeline's failure origins.

In the sensitivity study, first and total Sobol indices were calculated. It was shown, that the impactor properties (amplitude and duration) and impact position are more important than the structure material variation for structural dynamic response.

The elastic limit was chosen for the serviceability limit state. Impactors that do not lead to damage in deterministic calculations were considered for the uncertainty quantification.

Stochastic analysis for a normal distribution of amplitude and duration around mean values with a standard deviation of 20% showed that the variation in amplitude gives up to 30% of error. A study of multiple impactors shows an increase in failure probability up 20% in the case of three consecutive "safe" impacts. This analysis shows the need to consider not only big/heavy impactors or impactors with high velocity. Relatively small and slow impactors can cause plastic deformations and lead to rupture of a pipe and the following domino effect. Further studies with detailed 3D modeling are ongoing to detect failure modes for different kinds of impactors.

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