Optimization and Engineering (2019) 20:1227–1248
https://doi.org/10.1007/s11081-019-09455-0

RESEARCH ARTICLE

Multiperiod optimization model for oilfield production planning: bicriterion optimization and two-stage stochastic programming model

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Received: 5 November 2018 / Revised: 27 June 2019 / Accepted: 2 July 2019 / Published online: 19 July 2019

Abstract

In this work, we present different tools of mathematical modeling that can be used in oil and gas industry to help improve the decision-making for field development, production optimization and planning. Firstly, we formulate models to compare simultaneous multiperiod optimization and sequential single period optimization for the maximization of net present value and the maximization of total oil production over long term time horizons. This study helps to identify the importance of multiperiod optimization in oil and gas production planning. Further, we formulate a bicriterion optimization model to determine the ideal compromise solution between maximization of the two objective functions, the net present value (NPV) and the total oil production. To account for the importance of hedging against uncertainty in the oil production, we formulate a two-stage stochastic programming model to compute an improved expected value of NPV and total oil production for uncertainties in oil prices and productivity indices.

Keywords Bicriterion optimization · Mixed-integer Nonlinear programming · Multiperiod optimization · Nonlinear programming · Oilfield production planning · Stochastic programming

List of symbols

Constants

c	Linear cost for first stage decisions
$disc_t$	discount factor for NPV at time t
gcc_t	Cost of gas compression at time t
MO_w	Maximum amount of oil that can be produced from a well

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PI_w	Productivity index of well w
pg_t	Gas price at time t
po_t	Oil price at time t
S	Number of scenarios
Sep	Maximum amount of liquid that can be separated in the separator
TH	Time horizon
wtc_t	Cost of water treatment at time t

Sets

s	Scenarios {1…., S}
t	Time {1, 2…, TH}
w	Wells {well 1, well 2, …}

Functions

$g(x, y_s)$	Stochastic model constraints
$GOR_{w,t}$	Gas oil ratio of a well w at time t, which is a function (f_{GOR}) of cumulative oil produced
$Pr_{w,t}$	Pressure of well w at time t, which is a function (f_{PR})of cumulative oil produced
$WCT_{w,t}$	Water cut of well w at time t, which is a function (f_{WCT}) of cumulative oil produced
$\psi_s(x, y_s)$	Second stage problem

Variables

CC_t	Total cost associated with gas compression and water treatment at time t
CR_t	Total revenue generated from oil and gas production at time t
NPV	Net present value
$rL_{w,t}$	Liquid produced from well w at time t
$Rorc_{w,t}$	Cumulative oil produced form well w at time t
$ro_{w,t}$	Oil produced from well w at time t
$rg_{w,t}$	Gas produced from well w at time t
TrL_t	Total liquid produced at time t
Trg_t	Total gas produced at time t
Tro_t	Total oil produced at time t
Trw_t	Total water produced at time t
x	First stage decision variables
y_s	Second stage decision variables
Z	Total oil production

1 Introduction

Hydrocarbon production development is a complex process that encompasses physics of multiphase fluid flow from reservoir, wells, pipelines and processing equipment as well as infrastructure management for drilling and operation under safe conditions. In a reservoir, production is often constrained by the reservoir petrophysics and back-pressure as a result of flow characteristics of fluids in the pipelines and ability of surface facilities to handle fluid, as well as safety and economic considerations (Lake et al. 2007). Determination of optimal hydrocarbon production requires planning at several time horizons from 1 year up to a specific time for the lifespan of the reservoir (Gunnerud and Foss 2010). In many oil and gas industries the planning and decision-making takes places for a single time period, but this may result in suboptimal solutions (Verma 2015). For a better decision-making optimization over a time horizon, simultaneous multiperiod optimization is a better approach. In contrast to sequential single period optimization, the multiperiod model is capable to provide optimal solutions that take into account the decision variables over a long-time span and are not subject to short-term changes in the parameters related to hydrocarbon production from the reservoirs.

Production and injection well positioning, planning and surface network optimization are some aspects of optimization in oil and gas field planning. The optimization of the aspects mentioned above impacts the capital investment and profit generation in the oil production facilities. Several studies have focused on production planning of oilfields, such as the work done by Gunnerud and Foss (2010), Camponogara et al. (2010) and Kosmidis et al. (2005). These studies consider the production planning optimization problem considering the constraints for pressure drop in the oil wells, surface networks and gas lift in the oil wells. Furthermore, the aforementioned studies were built assuming steady state conditions for the reservoir as the changes in the reservoir properties with time are slower compared to the changes in the production systems. In our work, we address a subset of the production planning problem taking into account the optimization of oil production. Also, the research works mentioned above have used models of varying complexity, but they have focused on a single objective function, i.e., the maximization of total oil production for the oilfields and these studies aimed at short term optimization of oil production. Tavallali et al. (2013) proposed a comprehensive model for well placement and oil production planning the model maximizes NPV for a time period of 230 days. Jansen et al. (2009) in their study on real-time reservoir management use Kalman filter as a type of surrogate model and have shown NPV improvement by varying the time horizon for up to 4 years. In our study, we aim to highlight that the maximization of total oil production is not the same as the maximization of net present value. We also present a comparison between multiperiod optimization and sequential single period optimization with the focus to demonstrate optimization of oil production for a longer time horizon of around 20 years. As we aim to build models for long term planning, we use a simplified model.

Isebor and Durlofsky (2014), address a general oilfield planning and production optimization problem. In their work, they consider a bi-objective optimization problem for maximization of net present value and maximization of total oil production. They present an algorithm to approximate the Pareto curve by handling the bi-objective problem as a single objective optimization problem. In their study, they have used a Single Objective Product Formulation (SOPF) (Audet et al. 2008) and applied it to the case of optimization of two objective functions. SOPF requires identification of a reference point “r” in the objective space. More points are generated in the objective space such that the distance between the new solution points and reference point are maximized. These points identify the subsets of Pareto front and by varying the position of r different portions of the Pareto front are generated. The study applies SOPF in such a way that it combines the two-objective optimization in a format that maximizes the objective space with the reference point, and a Pareto approximation is generated. Furthermore, the compromise solution is approximated from the Pareto curve generated using the SOPF. In our study, we present a method to obtain the Pareto curve for the maximization of the two objective functions and obtain the exact compromise solution. Details of the formulation to determine the ideal compromise solution for bicriterion optimization are explained in "Appendix" of the paper.

In this paper, we address an integrated production planning problem from reservoir to surface. In order to address problems with long time horizons (e.g. 20 years) we develop a simplified production model. The simultaneous multiperiod problem is optimized to determine the production profiles of the oil wells for the maximization of the net present value (NPV) and the maximization of the total oil production over the time horizon. It is shown that maximizing NPV and maximizing total oil production are not equivalent, and hence there is a trade-off between them. We therefore consider a bicriterion optimization of the two objectives, the NPV and the total oil production. Using the approach by Grossmann et al. (1982) we obtain the set of Pareto optimal (or trade-off) solutions and the ideal compromise solution for the oil production planning problem.

There is uncertainty associated with parameters that affect the revenue generation and oil production. Hence, to optimize the production from oil wells the uncertainty in parameters such as prices and productivity indices, should be considered for optimization. For instance, Harding et al. (1996) developed a stochastic search technique for production scheduling to optimize the net present value. Sahinidis (2004) and Grossmann et al. (2016) present an overview of modeling techniques such as stochastic programming and robust optimization, for process systems that involve uncertainty in variables. In this study, we present a two-stage stochastic programming model. The stochastic model is based on the impact of uncertainty in oil prices and productivity indices on the optimization on the objective functions.

The outline of this paper is as follows. First, we describe the background of the research work done on oilfield planning and production modeling. Second, we formulate a simultaneous multiperiod nonlinear programming model for a production planning problem. In addition, we propose a simplified 5 oil well model and solve the problem to optimize the net present value and total oil production over a time horizon of 20 years. Based on the solutions of the multiperiod NLP problem we generate a Pareto curve with the ϵ-constrained method and present a direct approach to find the ideal compromise solution. Finally, we describe a two-stage stochastic programming formulation to improve the expected NPV and oil production by accounting for the uncertainty in the oil prices and productivity indices.

2 Background

Oil production from a reservoir involves different steps: exploration, appraisal, development and production. At each step, decisions are made that affect the overall performance of the oil field. Gupta and Grossmann (2012), proposed a simultaneous multiperiod Mixed-integer Nonlinear Programming (MINLP) model for optimal planning of offshore oil and gas infrastructure. Refer to Grossmann (2002) for a review on MINLP. In their model formulation, they considered the three components oil, gas and water. The nonlinear behavior of the reservoir is approximated by third or higher order polynomials. The model is reformulated as a mixed-integer linear programming (MILP) model. We consider the model proposed by Gupta and Grossmann (2012) and Gupta (2013), as reference for our model development. We should note that significant work has been done to capture the effect of pressure drop in the oil well and surface network, e.g. mechanistic correlations (Mukherjee and Brill 1999), empirical correlations (Mukherjee and Brill 1999) and Gilbert curve (Mukherjee and Brill 1999; Gilbert 1954). The models build in the aforementioned studies compute the pressure in oil wells for single and multiphase fluid flow, but in contrast to our study these models are restricted to short time horizon.

Several process operations are associated with the oil production from a reservoir. It involves complex fluid flow with multiple components, water, oil and gas flowing in the pipelines together. The exact prediction of the properties of this multiphase fluid flow is difficult. Hence, the fluid flow is approximated by single-phase flow or two-phase flow. Before the start of production, the reservoir has a shut-in pressure that corresponds to the maximum pressure of the reservoir. The reservoir pressure is high enough such that no external assistance is required to carry the fluid to the surface. However, during the life of the field the reservoir pressure steadily decreases and requires artificial methods such as gas lift to sustain economic production at operating conditions. The wellhead pressure controls the fluid flow in the oil well by adjusting the bottomhole pressure. The difference between the reservoir pressure and bottomhole pressure prevents the entry of sand particles into the well (Mukherjee and Brill 1999).

The total liquid produced from a reservoir is determined from the Inflow Performance Relationship (IPR). IPR determines the functional relationship between the production rate from the reservoir and the Bottom Hole Flowing Pressure (BHFP). IPR is derived from an approximation of Darcy's law (Mukherjee and Brill 1999; Darcy 1856; Muskat 1936) for single-phase liquid flow and it is used to determine the total production rate. This IPR correlation depends on the productivity index, bottomhole pressure of well and the reservoir pressure. The approximated formulation of Darcy's law (IPR) (Szilas 1975) is represented as follows,

$$J = \mathrm{q_o} / (\mathrm{p_r} - \mathrm{p_{wf}}) \tag{1}$$

where, J: Productivity index, p_r, p_{wf}: average reservoir pressure, bottom hole flowing pressure, q_o: oil flowrate into the well.

The liquid production depends on the bottomhole pressure and the productivity index, which is the capability of a well to produce oil. The factors that affect the productivity index are reservoir drainage area, pay zone thickness, effective permeability of the formation of oil, well length, fluid velocity and well completion method. The productivity index formulation in Eq. 1 can be used under steady condition of the reservoir for a short time horizon. The value of PI changes with the oil production. The fluid produced in the well is directed to a separator, which is an equipment that separates gas, oil and water from the fluid produced from the reservoir. The capacity of the separator limits the production from the oil well it is a physical design constraint in the model. The oil and gas obtained from the separator is sent to the downstream processing. Further, as the oil is extracted from the wells, water oil ratio (water cut), gas oil ratio and reservoir pressure vary nonlinearly as a function of the cumulative oil recovered from the wells. The water oil ratio, gas oil ratio and pressure correlations are obtained from surface characterization and dynamic modeling studies.

In this paper, the sets of pressure, cumulative offtake, GOR (gas oil ratio), WCT (water cut) curves have been extracted from an off-line numerical reservoir simulation because directly integrating such software adds a significant level of complexity (Mundhada 2016). In this study, for the sake of simplicity, we do not consider pressure drop models for the oil wells. This assumption eliminates the complexity that would arise from using multiphase fluid flow models for pressure drop. The main factors that impact the oil production are reservoir pressure and bottomhole pressure. These two pressures determine the amount of liquid that can be produced based on IPR. Hence without using pressure drop model for the oil well we can estimate the liquid that can be produced from the oil well which is separated into oil, water and gas in the separator.

3 Problem statement

Given is a reservoir with a set of oil wells, $w = \{\text{well } 1, \text{well } 2\ldots\}$. The oil production problem is considered for a given time horizon of TH years. The pressure profiles, gas oil ratio and water cut have a linear or polynomial correlation with the cumulative oil produced from the respective wells. These correlations are obtained from actual data from oilfields and are functions of the cumulative oil produced from the wells. It is also assumed that the wells do not interact with each other. The productivity indices are fixed for the wells and do not vary with time. Further, the maximum cumulative liquid that can be produced from all the wells is limited by the capacity of the separator to handle processing of liquid into oil, gas and water. The revenue generation from the oil production depends on the selling prices of oil and gas. The cost of the oil production depends on the compression cost for the gas and the processing cost of water. The cost and price parameters are fixed for the model. Fluctuation in the oil prices, productivity indices takes place over the course of production from the oil wells. The uncertainty of the parameters is used to develop a stochastic model.

To determine the optimum oil production from the wells, two objective functions can be formulated. First, maximization of the net present value (NPV) for the oil wells. This objective function optimizes the oil production subject to increase in the capital generation. The capital generated from oil production is computed from the revenue and cost associated with the oil production. Second, maximization of the total oil production from the wells. This objective function increases the oil production form each well but does not focus on the optimization of the capital generation from oil production. The model is solved for a time horizon of TH years with the objective to maximize the net present value of production. Further, the simplified model is also solved to maximize the total oil production from all the wells over the long-term time horizon of TH years.

The time horizon of TH years is discretized into intervals Δt of 1 year time periods, where t = {1, 2…TH}. It is assumed that the surface pipeline network is already established, and the reservoir depletes at a natural rate. The liquid flow from the oil wells is combined and transferred to the separator, which is assumed to be of fixed capacity. It is also assumed that there is no pressure drop in the oil wells and the surface network. Further, in the IPR correlation the bottomhole pressure is assumed to be zero. The constraint equations formulated for oil and gas production are nonlinear yielding a multiperiod nonlinear programming model. This multiperiod NLP model is solved using the global optimization solver BARON (Sahinidis, 1996) and local NLP solvers such as CONOPT (Drud 1994), SNOPT (Gill et al. 2002, 2005).

3.1 Multiperiod nonlinear programming (NLP) model

The simultaneous multiperiod NLP model has objective functions to either maximize the NPV or maximize total oil production (Awasthi 2017) over the long-term time horizon of TH years (see Nomenclature section). The initial investment for oil field planning is not included in the objective functions since it is constant and is paid up-front.

The cumulative oil produced $Rorc_{w,t}$ for each well w for the time period t is computed by summing up the oil production $ro_{w,t}$ over time for each well until time t.

$$Rorc_{w,t} = \sum_{tau=1}^{t} (ro_{w,tau}) \quad \forall t,\, w \tag{2}$$

The water cut $WCT_{w,t}$, gas oil ratio $GOR_{w,t}$ and pressure variation $Pr_{w,t}$ for each well at time t is computed using the empirical correlations (linear equations or polynomial correlations) of the cumulative oil production $Rorc_{w,t}$ (Gupta and Grossmann 2012). These correlations of pressure variation (f_{PR}), and water cut (f_{WCT}) gas oil ratio (f_{GOR}) are obtained using data from geological studies of reservoir.

$$Pr_{w,t} = f_{PR}\left(Rorc_{w,t}\right) \quad \forall t,\, w \tag{3}$$

$$WCT_{w,t} = f_{WCT}\left(Rorc_{w,t}\right) \quad \forall t,\, w \tag{4}$$

$$GOR_{w,t} = f_{GOR}\left(Rorc_{w,t}\right) \quad \forall t,\, w \tag{5}$$

The liquid produced $rL_{w,t}$ from the oil wells is determined by using the productivity indices PI_w and the pressure correlation $Pr_{w,t}$ (Eq. 3) (Szilas 1975). Given is the inflow performance relationship mentioned earlier (Eq. 1) with the assumption that the bottomhole pressure (BHP) is zero,

$$rL_{w,t} = PI_w * Pr_{w,t} \quad \forall t, w \tag{6}$$

The oil produced $ro_{w,t}$ is computed from the total liquid produced (Eq. 6) from the oil wells and the water cut correlation (Eq. 4).

$$ro_{w,t} = rL_{w,t} * \left(1 - WCT_{w,t}\right) \quad \forall t, w \tag{7}$$

The gas produced $rg_{w,t}$ is calculated from the gas oil ratio (Eq. 5) and the oil produced from each well $ro_{w,t}$.

$$rg_{w,t} = ro_{w,t} * GOR_{w,t} \quad \forall t, w \tag{8}$$

The total liquid produced TrL_t, total oil produced Tro_t and total gas produced Trg_t and total water produced Trw_t are computed by summing up the production of liquid, oil, gas and water over the wells w.

$$TrL_t = \sum_w \left(rL_{wt}\right) \quad \forall t \tag{9}$$

$$Trg_t = \sum_w \left(rg_{wt}\right) \quad \forall t \tag{10}$$

$$Tro_t = \sum_w \left(ro_{wt}\right) \quad \forall t \tag{11}$$

$$Trw_t = \sum_w \left(rw_{wt}\right) \quad \forall t \tag{12}$$

The model has constraint on the total liquid TrL_t which is limited by the maximum separator capacity (Sep). In addition, there is a constraint on the total oil produced from each well (maximum oil produced MO_w).

$$TrL_t \le Sep \quad \forall t \tag{13}$$

$$ro_{w,t} \le MO_w \quad \forall t, w \tag{14}$$

The NPV depends on the revenue CR_t and cost CC_t associated with the oil production. The revenue is generated from the price po_t of total oil produced Tro_t, and the price pg_t of total gas produced Trg_t at each time interval from the wells. The cost is calculated from cost gcc_t for the compression of gas produced $Trgt$ and the cost of wtc_t treatment of water produced Trw_t during oil production.

$$CR_t = \Delta t * \left(po_t * Tro_t + pg_t * Trg_t\right) \quad \forall t \tag{15}$$

$$CC_t = \Delta t * \left(gcc_t * Trg_t + wtc_t * Trw_t\right) \quad \forall t \tag{16}$$

As discussed earlier, two objective functions can be used for the optimization of the oil production. First, the maximization of the NPV that depends on the revenue CR_t and cost CC_t associated with oil production which is discounted over time.

Second, the maximization of the total oil production Z that is the sum of the total oil produced Tro_t summed over the time horizon.

$$max.\ NPV = \sum_t \left(disc_t\ *\ \left(CR_t - CC_t\right)\right) \tag{17}$$

$$max.\ Z = \sum_t (Tro_t) \tag{18}$$

Subject to the constrains (2)–(16).

These two optimization problem correspond to NLPs since constraints (3)–(5), (7) and (8) are nonlinear. The nonlinear model can be solved using local NLP solvers CONOPT (Drud 1994), SNOPT (Gill et al. 2002, 2005)) or global NLP solver BARON (Sahinidis 1996) to optimize the oil production subject to maximization of NPV and maximization of total oil production.

4 Numerical results

The NLP model (2)–(17) and (2)–(16) and (18) are solved and three studies are performed using the simplified multiperiod NLP models over a 20 year time horizon:

(i) The multiperiod models are solved for maximization of the NPV and maximization of total oil production for all time periods simultaneously and compared to the case where the model is solved sequentially for each time period (sequential multiperiod optimization).

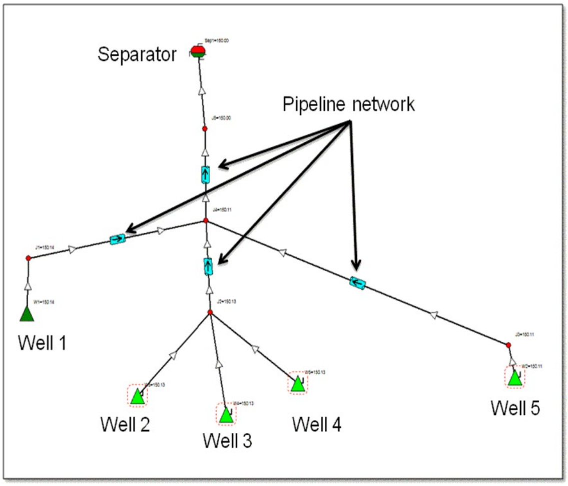
(ii) The bicriterion optimization problem (Eqs. 17 and 18) is solved to estimate the Pareto curve (see "Appendix") between the net present value and the total oil production to determine the optimal tradeoffs between the two objective

(iii) functions. Further, a model is formulated to find the ideal compromise solution between the two objective functions.

(iv) The simultaneous multiperiod model is assumed to have uncertainty in the oil prices and productivity indices. A two-stage stochastic model is developed and solved for the two objective functions subject to uncertainty in oil prices, productivity index.

The simplified multiperiod NLP model is solved for a case of five wells, Well = {well 1, well 2, well 3, well 4, well 5} over a time horizon of 20 years. The time horizon is discretized into 1-year time periods. The separator capacity is fixed to 8000 stock tank barrel per day (stb/day). Figure 1 shows the network of five oil wells. The oil prices in the model range from 28 USD per barrels to 84 USD per barrels and the gas prices are in the range 0.65 USD per MMBTU to 1.3 USD per MMBTU. The

Fig. 1 Oil wells network



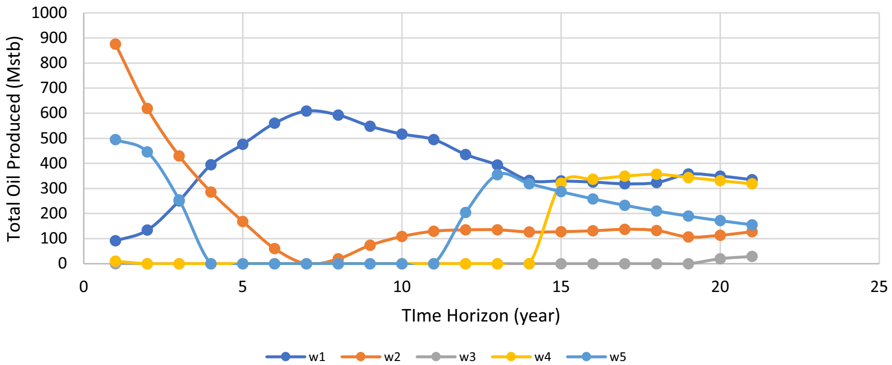


Fig. 2 Production profiles for maximization of Net present value over 20 years

rate of return to compute the NPV for the multiperiod model is 10%. The productivity indices are also assumed to be constant for the wells and are in the range of 1 stb/(day/psi) to 5 stb/(day/psi).

The multiperiod nonlinear programming model is solved using the global NLP solver BARON (Sahinidis 1996) and the local NLP solvers CONOPT (Drud 1994) and SNOPT (Gill et al. 2002, 2005). The results of the global solver and local solvers were the same, but the computational time required were quite different.

4.1 Five well production model

(a) Oil production for the maximization of NPV:

The multiperiod NLP model with an objective function to maximize the NPV yields an NPV of 6401.8 Million USD and a total oil production of 18.5 Mil-

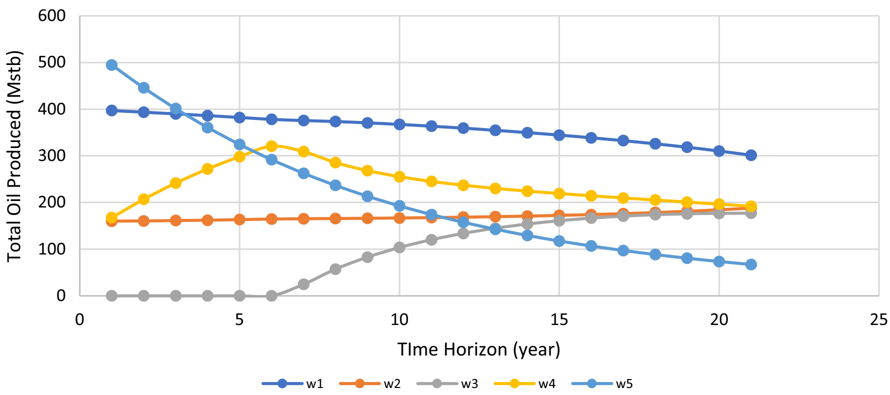


Fig. 3 Production profiles for maximization of total oil production over 20 years

lion stock tank barrels for a time horizon of 20 years. The model consists of 1115 equations with 1073 variables. The global optimization solver BARON (Sahinidis 1996) is used to solve the problem with maximum time limit of 1000 CPU seconds. The model was also solved using CONOPT (Drud 1994) (0.374 CPU seconds) and SNOPT (Gill et al. 2002, 2005) (0.437 CPU seconds). The results of the local solvers are the same as that of BARON (Sahinidis 1996). The production profiles of the five wells are shown in Fig. 2.

(b) Oil production for maximization of total oil production:
The multiperiod NLP model with an objective function to maximize the total oil production over a time horizon of 20 years yields a total oil production of 22.56 Million stock tank barrels and an NPV of 6361.3 Million USD for a time horizon of 20 years. The model consists of 1115 equations with 1073 variables. The global optimization solver BARON (Sahinidis 1996) is used to solve the problem with maximum time limit of 1000 CPU seconds. The model was also solved using CONOPT (Drud 1994) (0.203 CPU seconds) and SNOPT (Gill et al. 2002, 2005) (0.608 CPU seconds). The results of the local solvers are the same as that of BARON. The wells oil production profiles are shown in Fig. 3.

4.2 Comparison of single period optimization (SP) and multiperiod optimization (MP)

The nonlinear programming model is solved successively for the sequential single period optimizations over the time horizon of 20 years and compared to the simultaneous multiperiod optimization for the two objective functions in Eqs. 17 and 18.

Case a. Maximization of net present value over 20 years. The results of the multiperiod optimization model (MP) and single period model (SP) to optimize the NPV over the time horizon of 20 years yields the results shown in Table 1.

The results of simultaneous multiperiod and sequential single period optimization show that the total NPV value for 20-year time period (6401.8 MUSD) is greater

Table 1 Results of multiperiod vs single period optimization for max. NPV

	NPV_MP (MUSD)	Total oil_MP (MMstb)	NPV_SP (MUSD)	Total oil_SP (MMstb)	Δ NPV (MUSD)	Δ Oil Production (MMstb)
Total	6401.8	18.504	6400.5	18.863	1.35	−0.359

Table 2 Results of multiperiod vs single period optimization for max. Total oil production

	Total oil_MP (MMstb)	NPV_MP (MUSD)	Total oil_SP (MMstb)	NPV_SP (MUSD)	Δ NPV (MUSD)	Δ Oil Production (MMstb)
Total	22.563	6361.3	22.203	6368.0	−6.75	0.315

than the NPV summation for single period optimization for a period of 20 years (6400.5 MUSD). The model has a gain of 1.35 million USD (0.021%) for the multiperiod optimization. In addition, the total oil produced is greater for the single period optimization by 0.359 MMstb (1.93%). This shows that simultaneous multiperiod optimization yields higher NPV than sequential single period optimization.

Case b. Maximization of total oil production over 20 years. The results of multiperiod optimization model (MP) and single period model (SP) to optimize the total oil production over the time horizon of 20 years yields the results shown in Table 2.

The results of the multiperiod and sequential single period cases for the objective function to maximize total oil production for a time horizon of 20 years, show that the total oil produced is greater for the multiperiod optimization by 0.315 MMstb, i.e., multiperiod optimization has 1.6% more oil production rate than single period. Furthermore, the total NPV value for multiperiod optimization (6361.3 MUSD) is less than the summed NPV of single period optimization (6368.05 MUSD), which shows a gain of 6.75 million USD (0.12%).

For both the cases we see improvement of the simultaneous multiperiod vs the sequential single period. However, there are small differences in values because we are using a reduced model that does not account for depletion. Next, we perform a case study for different rates of return and oil prices. This study helps to understand the impact of different rates of return and oil prices on the NPV value.

We consider five cases to study the effect rate of return and oil prices has on the two objective functions. The cases are as follow:

- BC: Base case with 10% rate of return.
- LI: Low rate of return 5%
- HI: High rate of return 15%
- DO: Oil prices in decreasing order for base case
- IO: Oil price are in increasing order for base case

Table 3 shows the results for simultaneous multiperiod and sequential single period optimization for maximization of NPV for the five cases BC, LI, HI, DO, IO. The

Table 3 Results of max NPV

Case	NPV MP (MUSD)	Total oil produced MP (MMstb)	NPV SP (MUSD)	Total oil produced SP (MMstb)	Δ NPV (MUSD)	Δ Oil Production (MMstb)
BC	6401.84	18.5	6400.49	18.86	1.35	−0.36
LI	9254.67	18.27	9250.91	18.86	3.766	−0.6
HI	4843.16	18.65	4842.73	18.86	0.427	−0.21
DO	6664.86	18.81	6662.88	18.53	1.982	0.274
IO	6430.18	19.73	6421.27	20.48	8.91	−0.76

Table 4 Results of max total oil production

Case	NPV MP (MUSD)	Total oil Produced MP (MMstb)	NPV SP (MUSD)	Total oil Produced SP (MMstb)	Δ NPV (MUSD)	Δ Oil Production (MMstb)
BC	6361	22.56	6368	22.2	−6.5	0.36
LI	9199	22.56	9201	22.2	−1.43	0.36
HI	4810	22.56	4820	22.2	−9.69	0.36
DO	6627	22.56	6644	22.2	−17.4	0.36
IO	6406	22.56	6406	22.2	0.063	0.36

case IO in which the oil prices are in increasing order in the range from 28 USD per barrel to 84 USD per barrel yields the results with the largest difference in the NPV between SP and MP optimization (8.91 MUSD).

Table 4 shows the results for multiperiod optimization and sequential single period optimization for the maximization of the total oil production for the cases BC, LI, HI, DO and IO. The difference in the total oil production between multiperiod and single period optimization is same for the five cases. Since the objective at maximizing the total oil production is not affected by the interest rates or prices. On the other hand, the NPV value is better for multiperiod optimization than the single period (0.063 MUSD) for IO.

The results from the case study indicate the case in which the oil prices are in increasing order from 28 USD per barrel to 84 USD per barrel yield the largest differences. Single period optimization is a myopic approach; hence it computes a lower NPV for the case IO. Multiperiod optimization considers the overall time horizon and gives better results than single period optimization.

4.3 Bicriterion optimization model

A bicriterion optimization study of the five well model was performed for maximizing NPV and maximizing total oil production for the time horizon of 20 years. We first maximize each objective individually. The results for the two cases are shown in Table 5.

Table 5 Results of max NPV and max total oil production

	Maximization of NPV		Maximization of oil
NPV (MUSD)	6401.8	NPV (MUSD)	6361.3
Total oil (MMstb)	18.50	Total Oil (MMstb.)	22.56

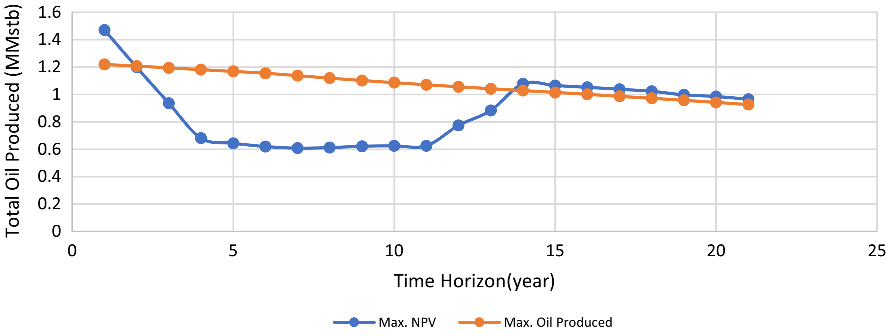


Fig. 4 Oil production for the two cases over a time horizon of 20 years

The total oil production for the two cases are plotted in Fig. 4.

In the case of maximization of NPV, as shown in Fig. 4, the total oil produced first decreases and then increases. In contrast, for the case where the objective function is the maximization of the total oil production, all wells start producing from the beginning and the total oil produced steadily decreases over the time horizon. To generate the Pareto optimal solutions the reduced oil production model is solved with the objective function to maximize the total oil production with a constraint on NPV value. Six different cases were formulated by imposing constraint on the maximum value of NPV. The model formulation for the bicriterion optimization is as follows.

$$\begin{aligned} &max\,Total\,oil\,production \\ &s.t. \quad NPV \leq \in \\ &\quad Constraints \end{aligned} \tag{19}$$

where the values at ϵ are selected within the interval min NPV $\leq \epsilon \leq$ max NPV

The results of the Pareto analysis are shown in Table 6, the results clearly indicate that as the NPV increases the total oil production decreases and vice versa. This behavior leads to a set of Pareto optimal solutions. Figure 5 shows the oil production profiles for the six different cases that are displayed in Table 6.

Figure 6 shows the Pareto optimal curve for maximization of NPV and maximization of total oil production. The figure shows that as the total oil produced increase from 18 to 22.5 MMstb, the change in the values of NPV is lowest in the beginning, whereas as the total oil production approaches 22 MMstb the NPV

Table 6 Results for Pareto analysis

Cases	NPV (MUSD)	Total oil produced (MMstb)
Max oil	6361	22.563
Case1	6377	21.971
Case2	6385	21.367
Case3	6393	20.562
Case4	6397	19.978
Max NPV	6401	18.504

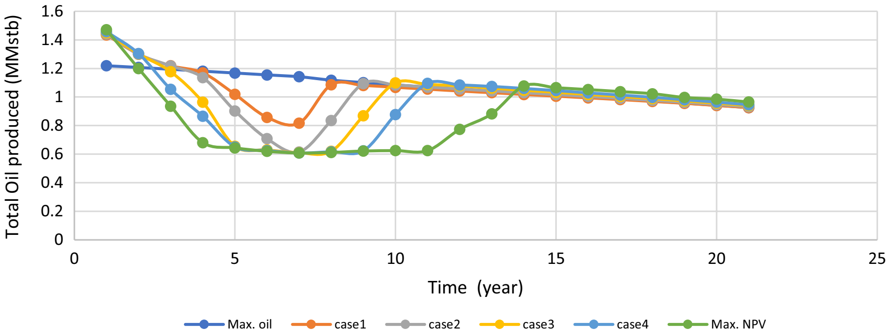


Fig. 5 Total oil production for different values of NPV

values decrease by a large magnitude for a small variation in the total oil production values.

In the graph of Fig. 6, point A represents the max NPV and min oil produced, while point B

represents the max oil produced and min NPV. The Pareto curve shown in Fig. 6 corresponds to a multiobjective optimization problem with the two objective functions:

(a) Maximization of NPV
(b) Maximization of total oil produced.

The utopia point represents the point where we have both maximum NPV and maximum oil production (see Table 5). The problem is also formulated and solved to obtain the ideal compromise solution between the NPV and the total oil production (see "Appendix"). The ideal compromise solution is the closest point on the Pareto optimal curve to the utopia point calculated using a norm (e.g. Euclidean norm). As shown in Fig. 7 and Table 7, the ideal compromise solution is given by a value 6393.759 Million USD for NPV, which is 0.13% less than the maximum NPV. The total oil production is 20.559 MMstb, which is 9% less than the maximum value for the total oil production from the oil well.

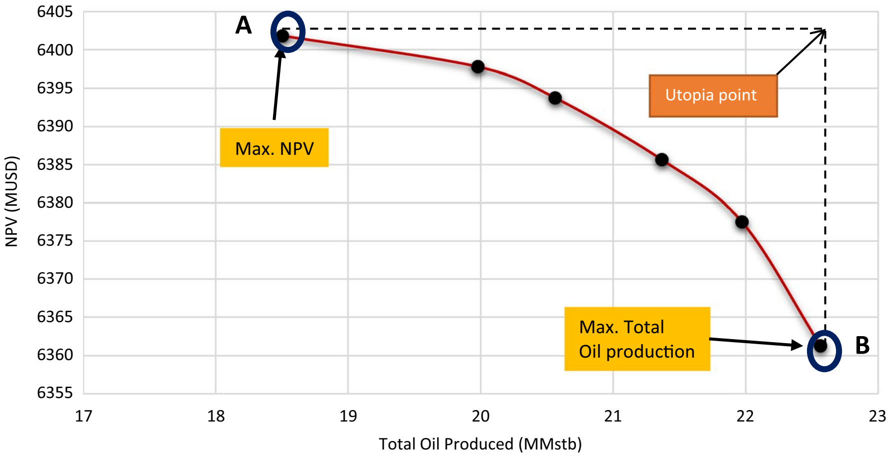


Fig. 6 Pareto curve for NPV and total oil production maximization

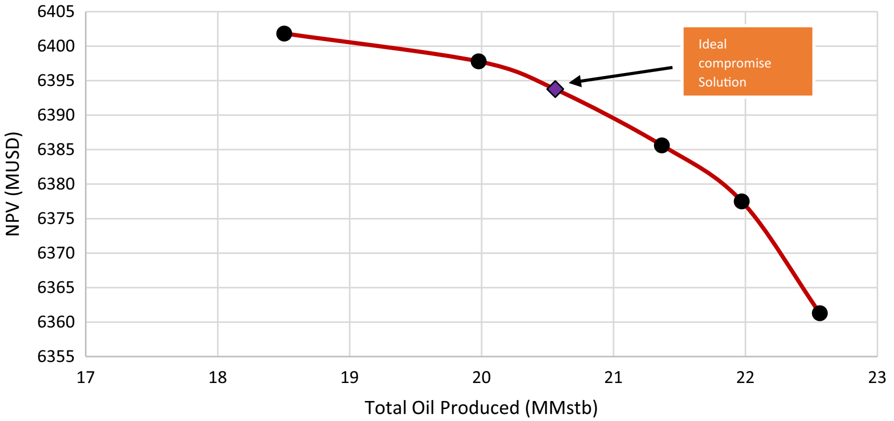


Fig. 7 Ideal compromise solution for the Pareto analysis

Table 7 Results of bicriterion optimization

	Maximize NPV	Ideal compromise solution	Maximize total oil
NPV (MUSD)	6401.843	6393.759	6361.297
Total oil (MMstb)	18.504	20.559	22.563

The total oil production profile for the three cases of maximization of NPV, ideal compromise solution and maximization total oil production for the reduced model are shown in Fig. 8.

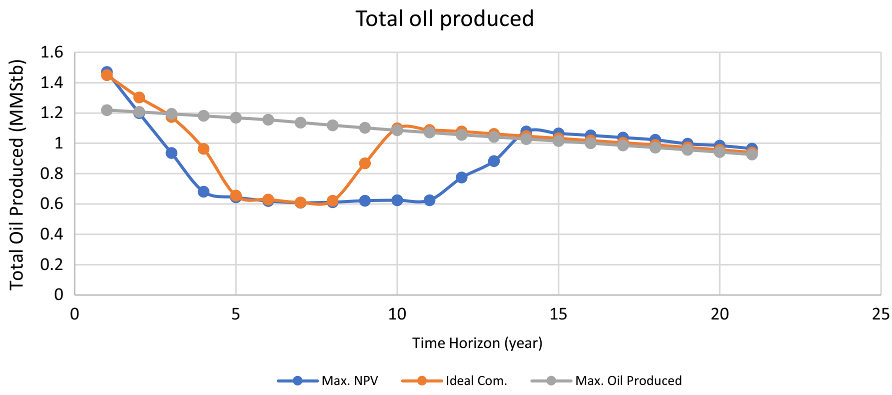


Fig. 8 Total oil produced over 20 years

4.4 Two-stage stochastic model

Uncertainty in the parameters associated with oil production can be handled using stochastic programming (Birge and Louveaux 2011), which is used for long term production planning problems. We formulate a two-stage stochastic programming model to optimize the expected value for the production model given uncertainties in the oil prices and productivity indices. In a two-stage stochastic programming (Birge and Louveaux 2011) we have two sets of decisions variables, first stage variables are the here and now decision variables that are decided before the uncertainty is realized, and the second stage variables are the recourse action decisions based on the realization of uncertainty.

A two-stage stochastic programming model (Birge and Louveaux 2011) is formulated for a simultaneous deviation of $\pm 20\%$ in the oil prices and productivity indices. The multiperiod NLP model (2)–(18) is modified for the formulation of a stochastic model. The first stage variables for the stochastic model are taken to be the selection of 3 oil wells from a total set of 5 oil wells. The selection of oil wells is chosen as the first stage variables because in oil production industry the selection of wells is a design decision. Hence, understanding how oil production can be optimized when uncertainty in productivity indices and oil prices takes place for a set of selected wells. This helps in making decisions as to which wells should be operated. Binary variables for the selection of wells are added to the model as the first stage decision variables. The second stage variables that perform the corrective actions are the oil productions. Three scenarios are considered for both oil prices and productivity indices, low, medium and high. This two-stage stochastic programming model is formulated as a Mixed-integer Nonlinear programming (MINLP) model. The MINLP model is solved for the case of maximization of NPV and the case of maximization of total oil production. The probability of the scenarios for oil prices and productivity indices are {0.25, 0.5 and 0.25} for the pessimistic, nominal and optimistic cases respectively. The MINLP model (5 binary variables, 10,634 continuous variables and 11,764 constraints) is solved using the SBB solver (Bussieck and Drud 2001) in 0.047 CPU seconds and the results are shown in Table 8.

Table 8 Values of the deterministic and stochastic model

	Max. NPV		Max. Total Oil Produced	
	Deterministic	Stochastic	Deterministic	Stochastic
NPV (MUSD)	6455.44	6511.34	6186.86	5932.75
Total Oil Produced (MMstb)	15.77	16.28	19.46	18.79

Table 9 Value of stochastic solution (VSS) results

Objective function	Stochastic	Expected stage 1	VSS	VSS%
Max. NPV (MUSD)	6511.34	6456.36	54.987	0.85
Max. Total Oil Produced (MMstb)	19.43	18.78	0.656	3.5

The formulation for the two-stage stochastic model for s = 1,2….S scenarios is given below (Grossmann et al. (2016)).

$$\begin{aligned} max \quad & NPV = c^T x + \sum_s \psi_s\left(x, y_s\right) \\ s.t. \quad & g\left(x, y_s\right) \leq 0 \quad \forall s = 1, 2 \ldots S \end{aligned} \tag{20}$$

where x are the first stage decisions (selection of wells), while y_s are the second stage recourse decisions (oil production rate) for each scenario. Here we assume linear cost $c^T x$ for the selection of wells, and nonlinear cost for recourse and nonlinear model constraints.

The value of stochastic solution (VSS) is computed (Birge and Louveaux 2011) to compare the deterministic and the stochastic solutions. VSS is the difference between the optimal solutions of the two-stage stochastic model to the solution obtained by solving the two-stage model with the first stage variables fixed to the values of the optimal solution obtained for the deterministic problem with expected values of the parameters.

Based on the results mentioned in Table 9, the VSS for the case of maximization of NPV is 54.99 MUSD (0.85%), while for maximization of total oil production is 0.66 MMstb (3.5%). Hence, for the scenarios considered for the oil well model, the two-stage stochastic model provides an improved solution and gives a better NPV (for Max. NPV) and total oil production (for Max. Total Oil Produced) values compared to the deterministic solution.

5 Conclusion

This paper has described a simultaneous multiperiod NLP model to determine the optimal solutions for oil production planning. In this study, we solve a simplified model for production from oil wells. The results of the multiperiod NLP model determines the production profiles of the oil wells as shown in Figs. 2 and 3. We formulate a case study for different rates of return and oil prices to compare the objective function values for simultaneous multiperiod and sequential single period optimization (Tables 3 and 4). The study mentioned in Sect. 4.3, clearly shows that maximization of net present value or maximization of total oil production does not determine the best tradeoff between these objectives. The best tradeoff solution is an ideal compromise solution that is closest to the utopia point where both objectives are at their maximum, where the objectives are at their maximum. This is an important finding, as in industries maximization of either of the objective functions mentioned above is considered as the best solution. It is also shown that the simultaneous multiperiod model provides significantly improved solutions compared to the case where successive single-period problems are solved sequentially. Finally, the model is solved to optimize the oil production for uncertainty in oil prices and productivity indices. The results of the VSS for the two-stage stochastic model are tabulated in Table 9. The values of stochastic solution show that the results of the two-stage stochastic model are better than the deterministic solution with expected values of the parameters. Hence, considering the two-stage stochastic model improves the solution for both objective functions.

Acknowledgements The authors acknowledge financial support from Total and from the Center of Advanced Process Decision-Making at Carnegie Mellon.

Appendix

Bicriterion optimization

The Pareto analysis of conflicting objective functions results in the formulation of a bicriterion optimization problem. The bicriterion optimization yields tradeoff solutions between two objective functions such as the net present value and the total oil production. To generate the Pareto cure the ϵ—constrained method (Hanes et al. 1975) is used. In this method one of the objective functions is optimized subject to a constraint ϵ on the other objective function. Further, after obtaining the Pareto curve one can determine the ideal compromise solution. In this case, the resulting Pareto front clearly shows that as the NPV value is increased the total oil production is decreased as shown in Fig. 9.

The ideal solution would be the one in which we obtain the maximum NPV and maximum total oil production. This point is denoted as the utopia point (Freimer and Yu 1976; Yu 1973) as it has the best value for the objective, but it is infeasible. The ideal compromise solution corresponds to the point in the Pareto curve that has the shortest distance to the Utopia point.

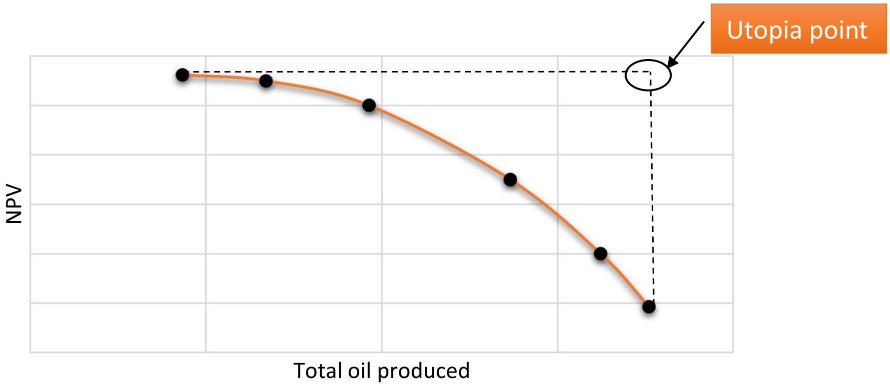


Fig. 9 Pareto curve of NPV vs Total oil production

Model

Let f1: Net present value, f2: total oil production. (Grossmann et al. 1982)

- The utopia point in Fig. 10 corresponds to $[f1^U, f2^U]$, the maximum of both variables, where superscript U represent upper bound.
- An ideal compromise solution can be obtained by finding the point on the curve closest to utopia point i.e. minimizing the distance (δ_p) for a norm p, where:

$$\delta_p = \left[\left(f1^U - f1\right)^p + \left(f2^U - f2\right)^p\right]^{1/p} \quad 1 \le p \le \infty \tag{21}$$

- The variables f1 and f2 are scaled from zero to one.
- After scaling of the functions to f1′ and f2′. The utopia point for scaled variables is (1, 1).

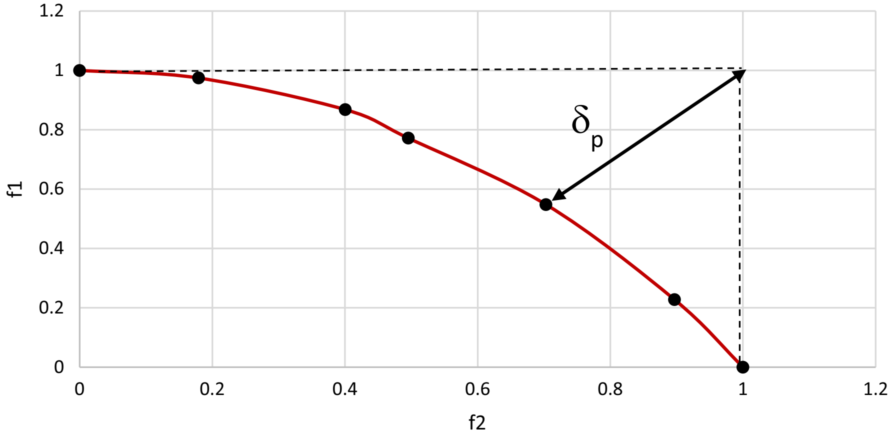


Fig. 10 Scaled Net present value (f1) vs Total oil produced (f2)

- The Euclidean norm p = 2 is considered for minimizing the fractional deviations 1 − f1′ and 1 − f2′.
- To obtain the ideal compromise solution. For $p = 2$, solve,

$$\min \left(\left(1 - f1'\right)^2 + \left(1 - f2'\right)^2 \right)^{1/2}, \tag{22}$$

s.t. Constraints.

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