

# Optimization methods for advanced design of aircraft panels: a comparison

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**Abstract** Advanced nonlinear analyses developed for estimating structural responses for recent applications for the aerospace industry lead to expensive computational times. However optimization procedures are necessary to quickly provide optimal designs. Several possible optimization methods are available in the literature, based on either local or global approximations, which may or may not include sensitivities (gradient computations), and which may or may not be able to resort to parallelism facilities. In this paper Sequential Convex Programming (SCP), Derivative Free Optimization techniques (DFO), Surrogate Based Optimization (SBO) and Genetic Algorithm (GA) approaches are compared in the design of stiffened aircraft panels with respect to local and global instabilities (buckling and collapse). The computations are carried out with software developed for the European aeronautical industry. The specificities of each optimization method, the results obtained, computational time considerations and their adequacy to the studied problems are discussed.

**Keywords** Structural optimization · Sequential convex programming · Derivative-free optimization · Surrogate-based optimization · Aeronautical engineering

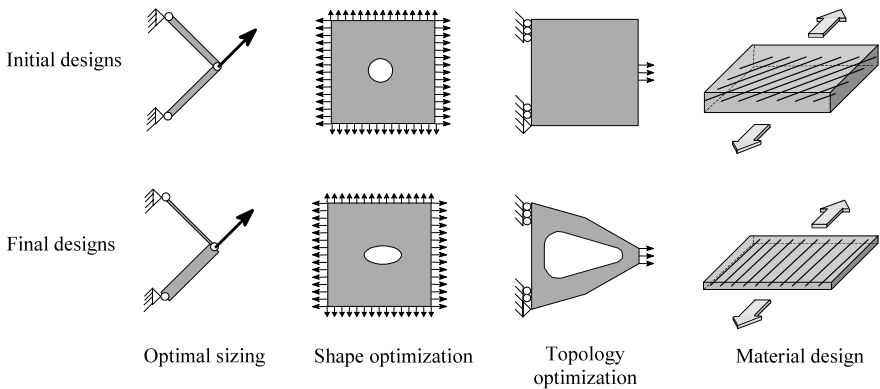
## 1 Introduction

Since the pioneering work of Schmit and Mallet (1963), Schmit and Farschi (1974) in the early seventies, in which the linear behavior of truss structures made of isotropic material under static loading was considered, the complexity of structural optimization problems has increased significantly in two aspects.

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**Fig. 1** Structural optimization problems

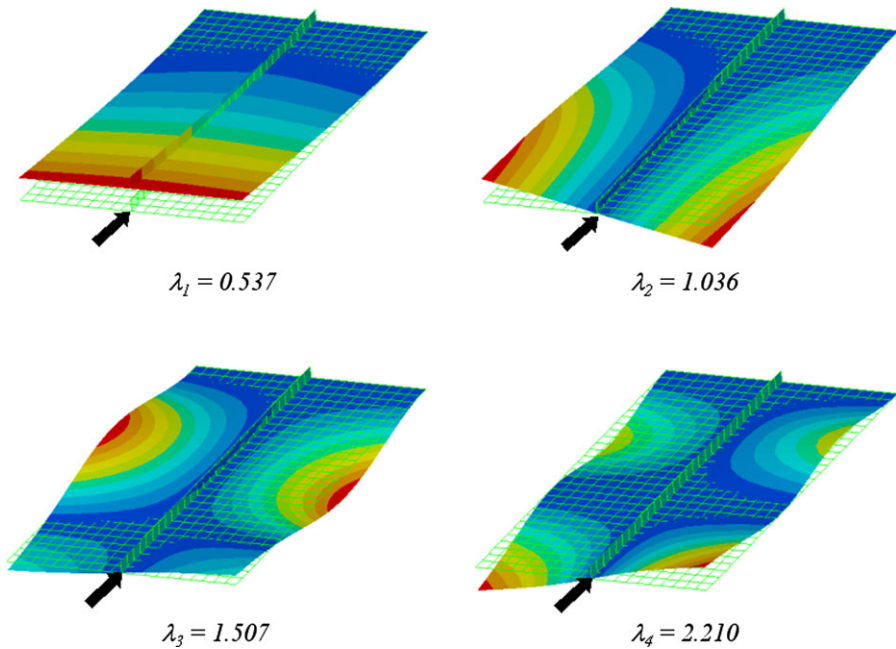
The first concerns the selection of the design variables and, as a consequence, the possibility to investigate not only the optimal size (Fleury 1973) and shape (Braibant and Fleury 1985; Beekers 1991) but also the optimal topology (Bendsoe and Sigmund 2003) and material properties (Sigmund 1995; Pedersen 1991), as depicted in Fig. 1.

Secondly, nonlinearities have been taken into account in the formulation of the optimization problem. Those nonlinear effects can appear at the material (Swan and Kosaka 1997) and/or at the geometrical levels (Buhl et al. 2000) where so-called post-buckling and collapse scenarios are studied. Including such nonlinearities makes the problem much more intricate since both the optimization task and the structural analysis are nonlinear. Computing the sensitivities for the gradient-based optimization methods is more complex. For an overview of the developments carried out in the field, see Saitou et al. (2005) and Arora (1997).

In this paper we consider the optimization of stiffened panels used in aircraft constructions. Linear and nonlinear analyses based on the finite element method are carried out with *SAMCEF* (SAMTECH, [www.samcef.com](http://www.samcef.com)), in order to estimate the buckling loads and the collapse (ultimate) load. These structural analyses are described briefly since they produce the functions appearing in the formulation of the optimization problem. The *BOSS quattro* task manager and optimization tool box is then briefly presented, together with the available optimization methods. These methods are then applied to two industrial test cases on stiffened panels and their efficiency in solving the problems is compared.

## 2 Functions entering the optimization problem

The aircraft panels considered in this paper include a flat skin and one stiffener. Two functions associated with their structural instability (buckling) and collapse (ultimate failure) are defined; both essential in the design of aircraft thin walled panels subjected to compression (Starnes 1980). These two functions are implicit in the design variables. It is the case that they cannot be expressed analytically and can only be evaluated with the finite element approach (Zienkiewicz 1977). They will be used as constraints in the optimization problem formulated later.



**Fig. 2** A stiffened panel buckling under a compressive load. First four modes ( $\Phi_j, j = 1, \dots, 4$ ) are shown together with the associated buckling load factors

### 2.1 Buckling load factors $\lambda_j$

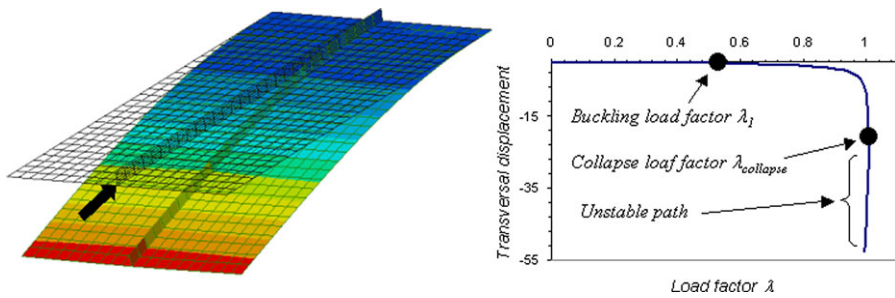
In the finite element formulation, the buckling loads are the eigenvalues  $\lambda$  of the following problem

$$(\mathbf{K} - \lambda_j \mathbf{S})\Phi_j = 0, \quad j = 1, 2, \dots \tag{1}$$

where  $\mathbf{K}$  is the stiffness matrix,  $\mathbf{S}$  is the geometric stiffness matrix, and  $\Phi$  are the eigen-modes (nodal displacements). The  $j$ th buckling load  $\lambda_j$  is the factor by which the applied load must be multiplied for the structure to become unstable with respect to the corresponding eigen-mode  $\Phi_j$ .  $\lambda_1$  is chosen as the lowest buckling load factor. In an optimal design, the buckling loads should be larger than or equal to a prescribed value (say 0.8 or 1.2), meaning that the structure will buckle for a controlled (even desired) proportion of the applied load. Buckling is illustrated in Fig. 2. This type of function is difficult to deal with, since buckling can be local and mode crossing can occur depending on the design variable values.

### 2.2 Collapse load factor $\lambda_{\text{collapse}}$

Even when a stiffened panel buckles, it still can sustain a higher proportion of the applied load. This is observed experimentally (Lanzi and Giavotto 2006) and can be modeled by means of the finite element approach (Fig. 3). To compute the *collapse load*, which is the ultimate load that the structure can support, the analysis method



**Fig. 3** Collapse analysis in a stiffened panel and equilibrium path

must deal with geometric non-linearities. In this case, one is looking for successive equilibrium states for increasing values of the applied load. As can be seen in Fig. 3, a maximum load can be estimated, corresponding to the collapse of the structure where large transversal displacements take place. After this peak, equilibrium is attained only if the load decreases, which results in an unstable configuration. To follow this unstable equilibrium path, Riks' so-called *continuation method* is used (Riks et al. 1996). In an optimal design, the load factor  $\lambda_{\text{collapse}}$  at collapse should be larger than or equal to unity, meaning that the structure can sustain its in-service loading.

The “linearized” buckling analysis based on the eigen-problem (1) is only an approximation of reality. Nonlinear analysis is more accurate since all nonlinear effects are taken into account, so allowing less conservative, lighter optimal designs. The main drawback of a non-linear analysis is that it is more expensive in terms of CPU.

### 2.3 Sensitivity analysis issues

To use gradient-based optimization methods, the first-order derivatives of the structural responses must be available. The sensitivity analyses for buckling and collapse are presented and discussed in Bruyneel et al. (2008) and Colson et al. (2007) and for sake of brevity and clarity the details are not reported here. It is however important to note that the sensitivity of non-linear structural functions, such as the collapse load, is difficult to determine theoretically and to implement in a finite element code. Finally, when a semi-analytical sensitivity analysis is carried out, the data required to compute the derivatives are obtained from the analysis results. This computation is very fast (see Bruyneel et al. 2008 and the references therein for more details).

## 3 Optimization methods compared on aircraft panel optimization

### 3.1 The BOSS quattro optimization toolbox

The computational framework chosen for defining then running the optimization process is *BOSS quattro*, an open application manager for parametric design and optimization (Radovic and Remouchamps 2002). *BOSS quattro* allows a complete integration of the finite element software (e.g. *SAMCEF*) mentioned above for linear and nonlinear finite element analyses. As an application manager, it deals with

the iterative design loop, alternating the structural analyses (including the sensitivity analysis, when needed) and the call to the optimizers. Several optimization methods are available in *BOSS quattro*: SQP, Augmented Lagrangian Method, specific approximation methods dedicated to structural optimization, derivative-free algorithms such as Genetic Algorithms, Surrogate-Based methods, and the more classical response surface capabilities. Since *BOSS quattro* is an open architecture, external optimizers can also be linked to it.

Three specific classes of optimization methods available in *BOSS quattro* are described in this section: Sequential Convex Programming (SCP), Derivative-Free Optimization (DFO) and Surrogate-Based Optimization (SBO). These methods are used to solve problem (2) below, where  $\mathbf{x}$  is the set of design variables,  $w$  denotes the objective function to be minimized, and  $\lambda_j$  ( $j = 1, \dots, m$ ) are the constraints of the problem. In general, besides buckling and collapse loads,  $p$  additional constraints  $g(\mathbf{x})$  can be defined:

$$\begin{aligned} \min w(\mathbf{x}) \\ \lambda_j(\mathbf{x}) &\geq \lambda_j^{\min}, \quad j = 1, \dots, m, \\ g_l(\mathbf{x}) &\leq 0, \quad l = 1, \dots, p, \\ \underline{x}_i &\leq x_i \leq \overline{x}_i, \quad i = 1, \dots, n. \end{aligned} \tag{2}$$

In these three methods, a solution of the problem (2) is obtained by solving approximated problems (3) successively, where the  $\sim$  symbol denotes an approximation of the corresponding function, and  $k$  is the iteration index.

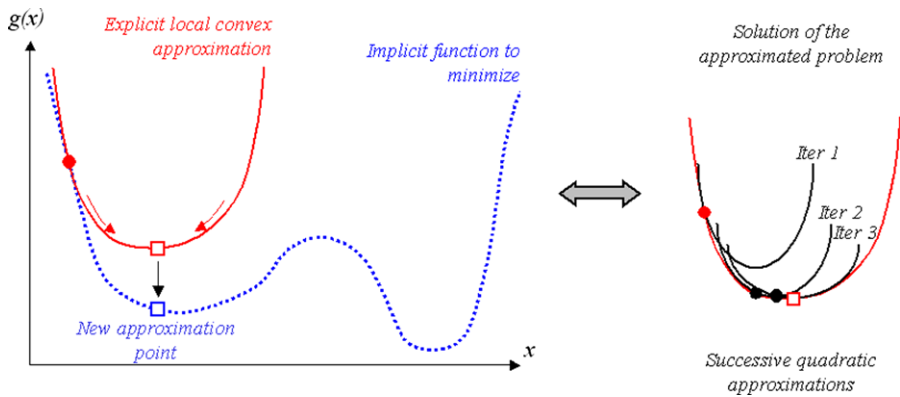
$$\begin{aligned} \min \tilde{w}^{(k)}(\mathbf{x}) \\ \tilde{\lambda}_j^{(k)}(\mathbf{x}) &\geq \lambda_j^{\min}, \quad j = 1, \dots, m, \\ \tilde{g}_l^{(k)}(\mathbf{x}) &\leq 0, \quad l = 1, \dots, p, \\ \underline{x}_i^{(k)} &\leq x_i \leq \overline{x}_i^{(k)}, \quad i = 1, \dots, n. \end{aligned} \tag{3}$$

Sections 3.2 to 3.4 below describe the main features of the SCP, DFO and SBO methods. SCP and SBO methods are original implementations (Remouchamps et al. 2007; Bruyneel 2006) while DFO is an external solver that has been especially linked to *BOSS quattro* for the purpose of a comparison of optimization methods. Note that for the sake of simplicity our descriptions below and the associated pictures are dedicated to *unconstrained* problems only.

The comparison of the methods and the associated results presented in Sect. 4 also involve a standard genetic algorithm (GA), but its implementation follows the classical scheme—as discussed in e.g. Goldberg (1989)—so we did not find interesting to describe it here.

### 3.2 Sequential convex programming (SCP)

The solution of the initial implicit optimization problem (2) is replaced with the solution of a sequence of approximated sub-problems (3) which are explicit in terms



**Fig. 4** Solution procedure for the Sequential Convex Programming approach

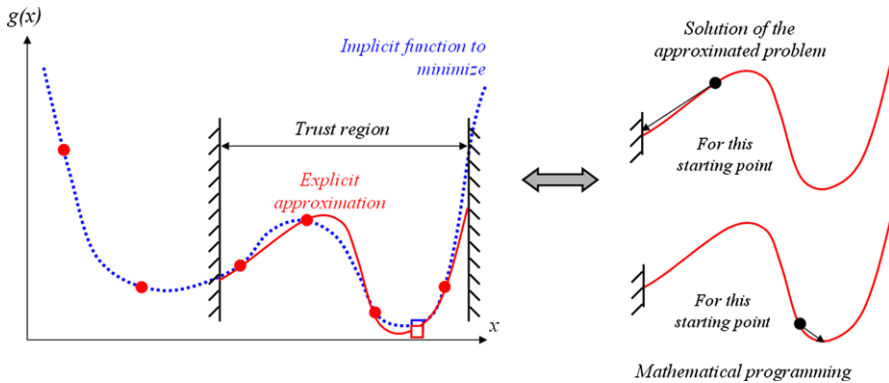
of the design variables. The explicit and convex optimization problem (3) is solved by dedicated methods of mathematical programming, as described in Fleury (1993), based on quadratic approximations solved by a dual approach. Building an approximated problem requires structural and sensitivity analyses (obtained from the finite element method). Solving the related explicit problem no longer necessitates a finite element analysis since the problem is now explicit in terms of the design variables. Each approximation is based on a particular first-order Taylor series expansion. The SCP method used in this paper generates mixed monotonous/non monotonous approximations, depending on the change of sign of the derivatives of the structural responses at two successive iterations (Bruyneel and Fleury 2002). The solution procedure for an unconstrained problem is illustrated in Fig. 4. This method generates successive local approximations, and is therefore more likely to become trapped in a (possible) local optimum. It has proven to be reliable in many structural optimization problems (Bruyneel et al. 2008; Remouchamps et al. 2007; Bruyneel 2006) and usually provides a solution in few iterations (i.e. few structural and sensitivity analyses), irrespective of the size of the problem.

### 3.3 Derivative-free optimization (DFO)

Derivative-free optimization (see Conn et al. 2008 for a recent monograph) aims at solving nonlinear optimization problems based on the function values only. The reason for not using derivatives is that the derivatives of some functions are impossible or very difficult to compute. Moreover their analytical expression may be unknown, which can occur when the values of such a function correspond to the output of some “black box” software, measurements or experiments for instance.

DFO methods are based on the following principle: rather than approximating the missing derivatives of a function, which often proves to be expensive (through e.g. finite difference schemes), the function itself is approximated on the basis of its known values. An improvement in the objective function is then derived from this model.

This concept can be advantageously combined with a trust-region approach (see Conn et al. 2000), as shown in a series of papers (e.g. Conn et al. 1997a, 1997b,



**Fig. 5** Solution procedure for the Derivative-Free Optimization approach

1998): available function values are used to build a polynomial model interpolating the function at those points where it is known, and the model is then minimized within a trust region, yielding a new—potentially good—point. To check this possible improvement and compute trust-region ratios, the function is evaluated at the new point—thus possibly enlarging the interpolation set—and the whole process may be repeated until convergence is achieved (as represented in Fig. 5).

Other components of the method include multivariate interpolation polynomials, namely (quadratic) Newton fundamental polynomials (see e.g. Sauer and Xu 1995), and a suitable strategy for adequately managing the geometry of the interpolation sets. Note also that the procedure may be generalized for handling constrained problems.

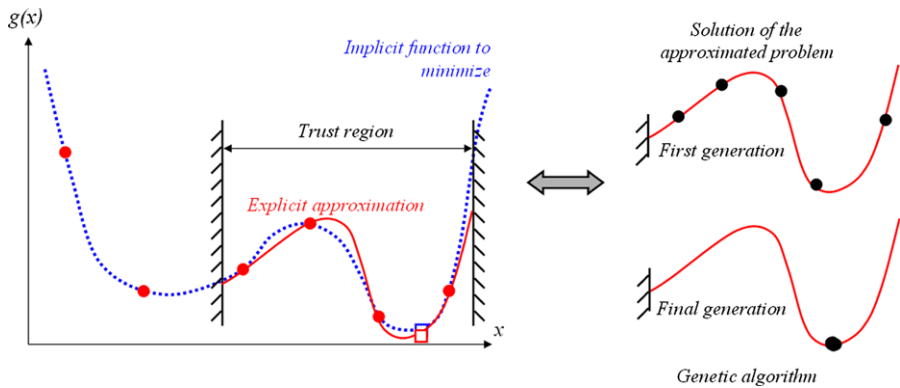
The results presented in Sect. 5 below include those obtained with DFO, an open source Fortran 77 package written by Scheinberg at IBM and available from COIN-OR ([www.coin-or.org](http://www.coin-or.org)). The open nature of *BOSS quattro* as a task manager made it possible to build the necessary procedures for calling DFO as optimizer.

### 3.4 Surrogate-based optimization (SBO)

The third method tested is also the most recent one in *BOSS quattro*. A *surrogate model*—also known as *response surface* or *metamodel*—is essentially a low-definition function that approximates another function. It is expected to be a simpler representation of the original function, less accurate but much cheaper to evaluate. Classical and popular surrogate models are polynomial response surfaces, Kriging, support vector machines and artificial neural networks.

A simple surrogate-based method (called Basic SBO in this text) involves firstly the construction of the surrogate—based on available function values—then its direct optimization using a suitable algorithm. A genetic algorithm is often chosen given the variable nature of possible surrogates and the possible lack of direct derivatives (depending on the nature of the approximation).

The main drawback of such an approach is obviously the fact that the model remains unchanged in the course of the optimization process, hence the idea of an adaptive scheme, where an initial set of function values is first evaluated then used to compute a first surrogate model. This provides responses for an optimizer which



**Fig. 6** Solution procedure for the SBO approach

in turn produces an optimal solution, at least from the point of view of the surrogate. The difference with the basic approach is that the original function is then evaluated at the corresponding point to check the accuracy of the surrogate model at the optimum. The output from this validation step and some possible convergence criteria thereby provide the information necessary to determine whether to stop the whole process or to compute further function values to improve the surrogate. In the latter case, a further loop including the surrogate optimization and the optimum validation is performed. The SBO method we use for the comparisons of Sect. 4 below (see Remouchamps et al. 2007 for more details) features an additional “trust-region mechanism” to ensure that the inner optimizer does not generate points outside the region where the surrogate is valid.

This is summarized on Fig. 6.

An additional comment may be required at this point. While from a pure lexical point of view the SBO method may be considered as a *derivative-free method*, it remains a slightly different approach from the methods of the DFO family discussed above. The major difference between those two classes of algorithms is the fact that in practice SBO may use any type of surrogate, without specific requirements on possible properties (like differentiability), since a genetic algorithm will solve the subproblem.

#### 4 Industrial application: optimal design of stiffened panels

Two applications are considered. The optimization problem to be solved was given in (2), while the structural functions entering the problem were explained in Sect. 2. For both applications, the objective function is related to the weight, which is to be minimized. Both problems include sizing and shape design variables.

The following five approaches are tested and compared:

- Sequential Convex Programming, **SCP** (see Sect. 3.1);
- Derivative Free Optimization, **DFO** (see Sect. 3.2);
- Surrogate-Based Optimization, **SBO** (Sect. 3.3);



- the basic implementation of SBO (**Basic SBO**), which amounts to a single iteration involving just one construction of the surrogate then its use within a genetic algorithm;
- a standard Genetic Algorithm (**GA**), with direct calls to simulation tools, without surrogate.

Two comments are in order at this stage:

- (1) The description and illustration of optimization methods in Sect. 3 above focused on unconstrained problems, for the sake of simplicity. Since both our test cases involve constraints, we use adaptations of the latter methods to the constrained case: SCP uses a dual approach explained in (Bruyneel and Fleury 2002) while the other algorithms use a penalty.
- (2) Our objective function being the weight, in principle it should not be considered as a function without derivatives. However we do so for two reasons. First, the weight is part of the output of the finite-element simulation code, together with the buckling and collapse reserve factors. In an industrial context, this makes life easier since the end-user does not have to code himself analytical function expressions in a dedicated subroutine (which in addition would be problem-dependent). Second, considering the weight as a function with its derivatives available would be useless and inappropriate since the penalty approach used for constrained problems adds the functions without derivatives to the objective function, the latter therefore becoming a more complex function.

Some details about the algorithmic parameters and setup may help understanding the results below:

- For the sole gradient-based method of the panel, namely **SCP**, results are provided for derivatives computed with a semi-analytical approach (**SCP-sa**) as introduced in Sect. 2.3 and through a finite difference scheme (**SCP-fd**). Of course this last solution does not take advantage of the fact that sensitivities may be available directly from the finite-element simulation modules but this manner of proceeding allows a fair comparison with the other approaches, which do not require gradients of the original functions.
- **DFO** is run with the default values for parameters. No initial point is provided, so the first task performed by DFO is to generate two points, being the minimum required for computing a model. Since the Hessian matrix of the quadratic polynomial model is symmetric, a complete model will be obtained after at least  $(n + 1)(n + 2)/2$  evaluations (where  $n$  is the number of design variables), that is 28 and 55 evaluations respectively in the test cases studied below.
- The **Basic SBO** involves firstly the construction of an initial database with a central composite design of experiments (77 points for  $n = 6$  and 531 points for  $n = 9$ ). The buckling and collapse load factors are then approximated by neural networks (5000 iterations for training) while the other functions (analytical expressions for section and aspect ratios) are used “as is” by a genetic algorithm (population of  $n \times 10$  individuals).
- **SBO** uses a Latin hypercube method to generate an initial set of points. Surrogates are neural networks (with 5000 iterations for training) and each iteration allows an enrichment of the database with up to 5 points evaluated in parallel.

– **GA** is used with a population including  $n \times 10$  individuals.

#### 4.1 Z stiffened metallic panel

In this first problem, we want to find the best design of the metallic stiffened panel represented in Fig. 7. Design variables are the lengths and thicknesses of the stiffener profile ( $aft$ ,  $wt$ ,  $fft$  and  $ffw$ ,  $sh$ ) and the thickness of the skin panel ( $st$ ), with the following bounds:  $1.6 \leq aft \leq 4$ ,  $1.6 \leq wt \leq 4$ ,  $2 \leq fft \leq 4$ ,  $8 \leq ffw \leq 26$ ,  $25 \leq sh \leq 55$  and  $1.6 \leq st \leq 8$ .

The objective function  $w$  is the area of the profile of Fig. 7 (skin panel and stiffener). The panel is subjected to axial and compression forces. The buckling and collapse loads must be larger than 1. The first 40 buckling loads are considered in the problem. Besides constraints on buckling and on collapse, some aspect ratios are also considered; the related constraints are expressed as follows:

$$\text{Attached flange (AFR): } 3 \leq \frac{26.8 + wt}{aft} \leq 20.$$

$$\text{Web (WR): } 3 \leq \frac{sh - aft - fft}{wt} \leq 20.$$

$$\text{Free flange (FFR): } 3 \leq \frac{ffw}{fft} \leq 10.$$

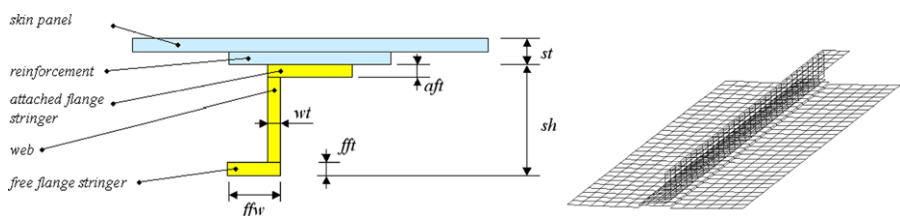
$$\text{Attached flange and skin thicknesses (AFSR): } 1.3 \leq \frac{aft}{st}.$$

As described earlier in Sect. 2, the buckling and collapse loads functions are naturally the most difficult functions to handle here since they are the output of finite-element analyses.

The complete set of results is displayed in Table 1. The solution is presumed to be obtained when, for a feasible design, the relative variation of the design variables or the objective function is first lower than 1%.

A first comment concerns the slight violation of some constraints at the optimal solution (e.g. 0.999 instead of 1 for  $\lambda_{\text{collapse}}$ ): this is largely acceptable in an industrial context, and the same consideration applies for the results of the second test case.

As regards the quality of the solutions, the basic SBO method leads unsurprisingly to the worst solution (the collapse reserve factor is violated by 25%), which simply confirms the need for an adaptive scheme as previously discussed. While SCP, the



**Fig. 7** Stiffened metallic panel and associated design variables

**Table 1** Numerical results for the metallic stiffened panel optimization

	SCP-sa	SCP-fd	DFO	GA	Basic SBO	SBO
Iterations	20	29	132	33	–	39
Structural analyses	20	197	216	1955	77+1	255
Variables at optimum						
sh	47.754	46.299	39.579	45.823	43.823	46.753
aft	3.009	3.097	3.660	3.632	3.623	3.805
fft	2.000	2.096	3.689	2.321	2.878	3.517
ffw	16.182	19.157	11.445	17.388	12.517	10.891
wt	2.137	2.056	3.910	2.014	2.042	2.048
st	2.070	2.059	2.274	2.026	1.600	1.989
Functions at optimum						
Section (weight)	550.890	552.063	655.105	556.791	476.937	553.761
$\lambda_1 (= \min \lambda_j) \geq 1$	1.000	1.000	1.538	1.038	1.513	1.015
$\lambda_{collapse} \geq 1$	0.999	1.013	0.999	1.026	0.757	1.000
AFR	8.951	9.317	8.390	7.931	7.959	3.096
WR	19.999	19.985	8.242	19.794	18.273	19.213
FFR	8.091	9.137	3.102	7.489	4.348	3.123
AFSR	1.453	1.504	1.609	1.792	2.264	1.913

genetic algorithm and SBO provide very similar solutions, DFO cannot decrease the section below 655.1053, which remains high in comparison with other results. This might suggest that it stalled at a local solution, as indicated by the values of the variables at optimum, which are significantly different to those produced by the other approaches. The convergence history for SCP-sa and SCP-fd is different and can be explained by some approximations made in the semi-analytical sensitivity of the buckling loads (Bruyneel et al. 2008).

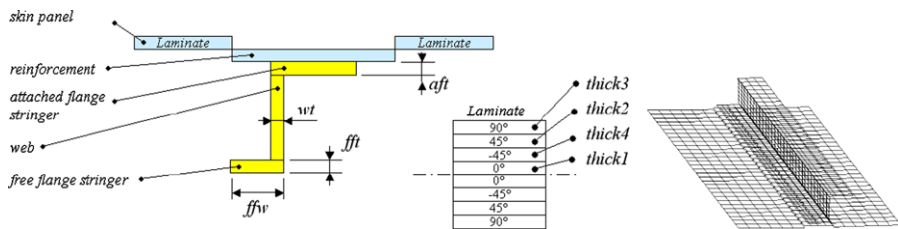
Let us now compare the computational costs of each method. SCP is the cheapest method, followed by SBO, while a standard genetic algorithm is about ten times more expensive. (Note that for the latter method, the optimal solution was actually found after 1800 function evaluations). Using SCP-sa is of course more advantageous, but requires the computation of the semi-analytical derivatives. In terms of pure performance, SCP-fd may launch all runs with perturbed values of variables for finite differences in parallel (which means 6 runs in this case) while the chosen parameters for SBO allow at most 5 runs in parallel (for database enrichment).

#### 4.2 Z-stiffened composite panel

The stiffened panel considered in this problem is illustrated in Fig. 8. It includes a metallic stiffener and a composite skin, linked with specific rivets elements. The structure is subjected to axial and shear forces. The design variables are the lengths and thicknesses of the stiffener profile (*aft*, *wt*, *fft*, *ffw* and *sh*). The skin is made of a [90/45/−45/0]<sub>S</sub> symmetric laminate. The ply thicknesses *thick1*, *thick2*, *thick3* and *thick4*, related to plies oriented at 0, 45, 90 and −45° respectively, are also variable.

**Table 2** Numerical results for the composite stiffened panel optimization

	SCP-sa	SCP-fd	DFO	GA	Basic SBO	SBO
Iterations	22	14	225	30	–	28
Structural analyses	22	131	406	2611	531 + 1	254
Variables at optimum						
sh	35.645	40.803	36.130	36.329	37.463	38.805
aft	1.600	1.600	1.600	1.729	1.809	1.619
fft	4.000	2.000	2.000	3.020	2.070	2.172
ffw	9.199	14.564	20.103	11.297	11.131	13.334
wt	1.600	1.600	1.704	1.815	1.600	1.601
thick1 (0 deg.)	0.389	0.208	0.444	0.341	0.790	0.406
thick2 (45 deg.)	0.363	0.200	0.199	0.234	0.231	0.200
thick3 (90 deg.)	0.204	0.200	0.200	0.232	0.224	0.201
thick4 (−45 deg.)	0.201	0.593	0.979	0.697	0.296	0.612
Functions at optimum						
Mass	0.361	0.333	0.362	0.365	0.368	0.341
$\lambda_1 (= \min \lambda_j) \geq 1$	0.999	1.013	1.771	1.624	1.473	1.518
$\lambda_{collapse} \geq 1.2$	1.199	1.194	1.199	1.252	1.208	1.199



**Fig. 8** Stiffened composite panel and associated design variables

The problem therefore includes 9 design variables with the following bounds:  $1.6 \leq aft \leq 4$ ,  $1.6 \leq wt \leq 4$ ,  $2 \leq fft \leq 4$ ,  $8 \leq ffw \leq 26$ ,  $25 \leq sh \leq 55$ ,  $0.2 \leq thick_1 \leq 2.5$ ,  $0.2 \leq thick_2 \leq 1.25$  and  $0.2 \leq thick_i \leq 1$  ( $i \in \{3, 4\}$ ).

Problem (2) is solved, where  $w$  is the structural weight. The buckling and collapse loads must be larger than 1 and 1.2, respectively. The first 12 buckling loads are considered in the problem. The solution can be assumed when, for a feasible design, the relative variation (over the last two iterations) of either the design variables or the objective function becomes lower than 0.1%.

As for the first test case, the basic SBO produces the worst solution overall. However it is important to notice a much better behavior of the method (no constraint violation at solution). This is probably due to the fact that the central composite design required 531 function evaluations (since there are 9 variables) and thereby produced more accurate response surfaces. SCP-fd yields the best feasible solution with the lowest objective function. The other methods (SCP-sa, DFO, GA and SBO) found local solutions where the buckling reserve factor is not active. Note that DFO per-

forms better than in the previous test case and yields a mass close to that obtained by SCP-sa.

In terms of CPU costs, the trend observed in the previous section is confirmed, with SCP and SBO requiring fewer structural analyses than the other methods. DFO confirms its better performance in the second test case by reducing the gap with SBO, while the standard GA remains much more expensive. The major difference between SCP-sa and the other methods (SCP-fd, DFO, GA and SBO) is that SCP-sa cannot perform more than one function evaluation per iteration and as a consequence it cannot be parallelized.

## 5 Conclusions

This paper focuses on the application of various recent optimization algorithms to solve two industrial optimization problems in the framework of airplane design. These problems may be considered challenging for they involve the output of both linear and nonlinear structural analysis simulation codes in the constraints. The numerical experiments demonstrate initially that classical gradient-based methods remain competitive—even when finite differences are used. SBO performs slightly worse, followed by DFO, GA and basic SBO. However with the increase of the finite element model size—and resulting simulation times—and problems involving higher nonlinearity, we suspect that surrogate-based optimization methods may prove useful. The challenges in that research field lies both in the use of adequate models and in the development of methods able to exploit the maximum amount of information from a limited number of simulations.

Our test cases also suggest that a first possible improvement of the SBO method would be to combine the use of approximations or surrogate models (for complex functions) and the original functions (for simple analytical expressions like aspect ratios). A further advantage of surrogate-based methods is that their exploratory nature makes them particularly well-suited to parallel implementations.

To conclude, for problems where complex non-linear analyses are considered in the design problem, SCP should be used when the derivatives are available. If the sensitivities are not computed, SBO appears to be an interesting alternative.

Finally, since the size of industrial optimization problems is tending to increase (both in terms of number of variables and number of functions), future work will also be dedicated to comparisons involving such larger problems.

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