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A non-linear goal programming model and solution method for the multi-objective trip distribution problem in transportation engineering

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Abstract Trip distribution is one of the important stages in transportation planning model, by which decision-makers can estimate the number of trips among zones. As a basis, the gravity model is commonly used. To cope with complicated situations, a multiple objective mathematical model was developed to attain a set of conflict goals. In this paper, a goal programming model is proposed to enhance the developed multiple objective model to optimize three objectives simultaneously, i.e. (1) maximization of the interactivity of the system, (2) minimization of the generalized costs and (3) minimization of the deviation from the observed year. A genetic algorithm (GA) is developed to solve the proposed non-linear goal programming model. As with other genetic algorithms applied to real-world problems, the GA procedure contains representation, initialization, evaluation, selection, crossover, and mutation. The modification of crossover and mutation to satisfy the doubly constraints is described. A set of Hong Kong data has been used to test the effectiveness and efficiency of the proposed mode. Results demonstrate that decision-makers can find the flexibility and robustness of the proposed model by adjusting the weighting factors with respect to the importance of each objective.

Keywords Transportation systems · Trip distribution · Multiple criteria decision making · Genetic algorithm

1 Introduction

Over the past years, a number of models are developed to distribute commuter's trips, freight or information among origins and destinations (Mozolin et al. 2000). The es-

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timation of trips distributed among origins and destinations is very important in the real world. For instance, safety organizations analyze work-related traffic accidents based on trip distribution data (Salminen 2000). In Finland, 50% of accidental deaths were associated with traffic accidents between 1975–1994. In addition, transportation agencies use the number of trips distributed among areas to estimate emissions produced by vehicles (Lin and Niemeier 1998).

In the strategic transport planning, it was studied that better transport systems will facilitate the better environmental performance for intra-city and inter-city communication (Bruton 1985). Transportation planning studies have been conducted in major metropolitan areas for 40 years and have been used to analyze commuter demand and to forecast transport system inventories. An important problem in transportation planning studies is predicting the number of commuters in each given origin-destination (OD) pair. This is a pure trip distribution problem, and can be defined as follows: when the information about the number of trips generated from origin zone i, O_i, $i = 1, 2, \ldots, n$, and attracted to destination zone j, D_j, $j = 1, 2, \ldots, n$ is available, the distribution process links up the generations and attractions to form a trip matrix, T_{ij}, which represent the trips started from origin zone i to destination zone j. The summation of trips from zone i is equal to O_i and the summation of trips to zone j is equal to D_j. These are called doubly constraints and mathematically expressed as (Ortuzar and Willumsen 1994):

$$\sum_{j=1}^{n} T_{ij} = O_i, \quad i = 1, 2, \ldots, n, \tag{1}$$

$$\sum_{i=1}^{n} T_{ij} = D_j, \quad j = 1, 2, \ldots, n. \tag{2}$$

Modeling trip distribution problems were initially derived from an analogy with Newton's law of gravitational force between two masses separated by a distance (Casey 1955). Applied to transportation planning, the formulation of the gravity model can be explained as follows. The number of trips between two zones is directly proportional to the number of trips generated from the origin zone and the number of trips attracted to the destination zone, and the number of interchanges is inversely proportional to the spatial separation between two zones. In general, decision-makers always consider a decreasing function called the generalized cost function, $f(c_{ij})$ where c_{ij} is a generalized cost with one or more components (e.g. travel cost, waiting time, parking cost, etc.) which represents the effect of spatial separation between two zones. The common gravity model subjected to the doubly constraints (1) and (2) is derived as follows:

$$T_{ij} = A_i O_i B_j D_j f(c_{ij}) \tag{3}$$

where A_i and B_j are the balancing factors to ensure that (1) and (2) are satisfied, and expressed as:

$$A_i = 1 \Big/ \sum_{j=1}^{n} B_j D_j f(c_{ij}) \tag{4}$$

and

$$B_j = 1 \Big/ \sum_{i=1}^{n} A_i O_i f(c_{ij}). \tag{5}$$

It is noted that A_i and B_j can be found using an iterative process (Ortuzar and Willumsen 1994).

Over the past three decades, various OR/MS techniques are employed to deal with trip distribution problems such as mathematical programming (Hallefjord and Jornsten 1986; Fang and Tsao 1995), heuristic searching methods (Mozolin et al. 2000), and fuzzy sets theory (Teodorovic 1994, 1999). Many studies have been carried out to calibrate and validate appropriate parameters in the gravity model. Easa (1993a) reviewed the state-of-the-art of trip distribution problem in large and small areas. Arasan et al. (1996) developed gravity models with travel deterrence for trips made for different purposes and using different modes of transport. Vaughan (1985) examined the effect of travel and non-linear crowding costs with three criteria on journey-to-work trip distributions. Duffus et al. (1987) investigated the reliability of the gravity model to predict future travel patterns. Easa (1993b) developed a simplified technique based on transferable parameters to estimate OD matrices from traffic counts and other available information; other miscellaneous simplified methods including self-calibrating gravity model, partial matrix techniques, heuristic methods, and facility forecasting techniques. Dinkel and Wong (1984) studied the impact of external trips which are from and to outside the study area on local trips. Toth et al. (1990) investigated the distribution pattern of shopping trips, and stated that the difficulties of estimating the distribution of shopping trips are associated with: (1) a proportion of trips made are a part of a series of linked trips, and (2) the trips have a variety of origins and destinations before and after the shopping center site. Mozolin et al. (2000) compared the performance of neural network formulation to tradition gravity model for trip distribution problem, and concluded that the predictive ability of neural network is worsen than that of maximum-likelihood doubly-constrained gravity model.

Alternatively, a mathematical model was developed by Wilson (1970) based on the principle of maximum entropy to trip distribution problem, in which the interactivity in the system represented by an entropy function is maximized subject to a set of constraints. The difference in the model structure between the entropy-maximization model and the gravity model is that right-hand-side values in the constraints are specified in the former and parameter values in generalized cost function are calibrated in the latter. Nevertheless, the optimal solution to the entropy-maximization model has the same form as that to the gravity model. However, a single-objective mathematical model seems to be insufficient in solving complicated real-world problems such as economic crises, alternative traffic management and policy, unforeseeable travel behavior and so on, although it is found that the mathematical model can reproduce the solutions from the gravity model. Multiple criteria decision making (MCDM) is adopted in the gravity model because, by calibration, parameters are estimated to achieve the “best-fit” matrix replicating the observed values and/or representing the same degree of accessibility/interactivity. Recently, multi-objective decision making has been acknowledged as a useful tool in solving real world problems and is regarded as one of the most important decision making methodologies. Hallefjord and Jornsten (1986) presented a theoretical framework of a multi-objective model for the trip distribution problem with target values and applied this to the problem in Sweden. An entropy function was used to measure deviations from target values. Under this framework, new target values will be reassigned if decision-makers discover that the solution is generally far away from observed real-life situation. However, the limitation of this approach is the difficulty of choosing appropriate and “realistic” target values. Leung and Lai (2002) proposed a goal programming model to solve a multi-objective mode choice problem, by which decision-makers can estimate the number of commuters using each specified model of transport in each given origin-destination pair. Instead of changing the target values, by adjusting the weights with respect to the importance of each objective, decision-makers and transport planners will find the flexibility and robustness of the proposed model.

In this paper, an effective and robust approach, goal programming, is proposed for application to the trip distribution problem with multiple objectives, in which a priori information representing an optimistic guess or estimation of the outcome is given as a set of target values. The objective is to minimize deviations of optimal solution from the target value. Goal programming, one of the powerful mathematical models which deals with multiple objective problems, is an extension of linear programming and used particularly to optimize the problems with a priori set of information given by decision-makers. The advantages of goal programming are that first, decision-makers can vary weighing factors instead of target values according to the importance of objectives. Second, the model can be easily understood and manipulated by specialists or non-experienced planners, and many developed software and meta-heuristic algorithms can successfully solve linear and non-linear goal programming formulations in reasonable computational time (Gen and Cheng 1997). Moreover, due to the complexity of the proposed model, a genetic algorithm is developed to solve the proposed non-linear goal programming model.

After this introductory section, three mathematical programming models presented by Hallefjord and Jornsten (1986) are described fully in the next section. The general idea and the flexibility of goal programming in solving multi-objective problems are described and the application of this approach to the trip distribution problem with multiple objectives is developed in Sect. 3. In Sect. 4, a genetic algorithm is expressed to solve the developed non-linear mathematical model. A set of Hong Kong data is used to test the effectiveness and efficiency of the goal programming model in Sect. 5. Conclusions are given in last section.

2 Model formulation

Hallefjord and Jornsten (1986) presented three single objective mathematical programming (SOMP) models for the trip distribution problem. In this section, these models are described briefly and the integrated MP model with three objectives, which are formulated in three individual models, is discussed. All models are constrained by doubly constraints (1) and (2), and non-negativity constraint, $T_{ij} \geq 0$, to ensure that T_{ij} are non-negative for all i, j.

2.1 SOMP trip distribution model 1: maximum entropy

In the 1970s, Wilson (1970) developed a SOMP model based on entropy maximization to solve the trip distribution problem, in which the most probable set of T_{ij}'s maximizes the total number of system states, called entropy. Wilson (1970) showed that the solution to an optimization model based on the principle of maximum entropy is identical to that based on the gravity model in (3). The entropy-maximization model maximizes the interactivity in the system formulated in the entropy function and has the following form:

$$\text{Max} \quad z_1(T_{ij}) = -\sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij} \ln T_{ij}. \tag{6}$$

Apart from doubly constraints, this model is also constrained by a cost constraint:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} T_{ij} = C_0, \tag{7}$$

where C_0 is the total generalized cost among all OD pairs in the system.

The advantage of the entropy-maximization method is its ability to help in constructing models to represent complex phenomena and, to a lesser extent, its use in the interpretation of the equation. Applying entropy-maximization to the transport field, the mathematical model is flexible and robust in anticipating future trips. Although the conditions change over time, the transfer and possible change of the right-hand-side values may be handled and understood more easily by the decision-makers than a transfer of and changes in the parameter values, since right-hand-side values have a physical interpretation. Decision-makers can also conveniently take their own experience into account in the planning process.

2.2 SOMP trip distribution model 2: transportation problem

As well as adopting Newton's law of gravitational force and principle of maximum entropy in developing the trip distribution model, other formulations will be modeled (Wilson 1970; Erlander 1981). The classical transportation problem is concerned with the delivery of goods or products from source supplies to customers so as to meet the ordered demand (Hitchcock 1941). Decision-makers have to find the cheapest path to transport the goods in terms of minimal transportation cost. As long as the transportation cost is linear, the problem can be easily solved using the simplex method. To be applied to trip distribution problem, source supplies can be considered to be generation trips, customer demands attraction trips, and transportation costs generalized costs. The objective of this model is to minimize total generalized costs.

$$\text{Min} \quad z_2(T_{ij}) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} T_{ij}. \tag{8}$$

Since the objective in the transportation problem is to minimize total transportation costs, the cheapest pair among source-destination pairs will be fully assigned with the amounts of goods as large as possible, whilst the expensive pair will be considered later. Hence, the resultant matrix is a all-or-nothing matrix. In order to spread the trips evenly and smoothly without changing the objective, an entropy function is added to measure the accessibility in the system.

$$-\sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij} \ln T_{ij} \geq H_0. \tag{9}$$

2.3 SOMP trip distribution model 3: information theory

In all kind of forecasting models, observed past data plays an important role. Decision-makers always intend to reproduce similar patterns in the future by minimizing the deviation between the past data and the obtained future solution. Econometric experts, who are given a set of past data, construct a linear regression model based on the concept of ordinary least squares to forecast the impact of a dependent variable on the change of other independent variables. Information theory has been developed to measure the variable with regard to the deviation from observed data. This is as easy as to incorporate the available data in the entropy function into the objective function. The third SOMP for trip distribution, supposing that the observed past data, T_{ij}^0 are given, has the form:

$$\text{Min} \quad z_3(T_{ij}) = \sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij} \ln \frac{T_{ij}}{T_{ij}^0}. \tag{10}$$

The set of constraints of model 3 is identical to that of model 1. The objective function in (10) is a kind of entropy function but with a different expression from that given in (6).

2.4 Multi-objective mathematical programming (MOMP) trip distribution model

Three mathematical programming models have been developed separately and seem to be able to determine the distribution trips in the system. A single-objective model only considers one objective at a time. However, model builders always want to develop a model which can consider the real-life situation with multiple aspects. To achieve this, a multi-objective model has been considered, with three objectives as discussed previously are:

$$\begin{cases} \text{Objective (1):} \quad \text{Max} \quad z_1(T_{ij}) = -\sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij} \ln T_{ij}, \\ \text{Objective (2):} \quad \text{Min} \quad z_2(T_{ij}) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} T_{ij}, \\ \text{Objective (3):} \quad \text{Min} \quad z_3(T_{ij}) = \sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij} \ln \frac{T_{ij}}{T_{ij}^0} \end{cases} \tag{11}$$

subject to doubly constraints (1) and (2) and non-negativity constraints.

It is clearly noted that objective (1), which serves to optimize total efficiency objective of society, and (2), which serves to optimize total accessibility objective of the individuals, are conflict. Moreover, two entropy functions, objective functions (1) and (3), present in the objective to measure the interactivity in the system because, sometimes, the source of base year observations are missing or uncertain. It is necessary for T_{ij} to take the smallest value possible due to the maximization of entropy, while T_{ij} to take the closest value to T_{ij}^0 possible due to the minimization of deviations from observation data. Finally, objectives (1) and (3) are also conflict.

3 Goal programming approach

3.1 The structure of goal programming

To generate some "good" efficient solutions instead of all efficient points, Hallefjord and Jornsten (1986) used a set of weights to measure the objectives. They used an entropy target point approach to manage a large number of objectives. An alternative approach to managing the deviations between target value and realized solution is to use goal programming (GP) with weights. The explicit definition of GP was given by Charnes and Cooper (1961) and is commonly used to achieve a set of conflict objectives as closely as possible (Rifai 1994). Tamiz et al. (1998) reviewed the state-of-the-art of current development of GP. Many researchers and practitioners are increasingly aware of the presence of multiple objectives in real-life problems (Vincke 1992). With fast computational growth, both linear and non-linear GP can be solved using well-developed software or artificial intelligent such as simulated annealing, genetic algorithms and so on. Moreover, GP is more direct and flexible in manipulating different scenarios by adjusting either target values or weights.

As opposed to linear programming, which directly optimizes objectives, GP attempts to minimize the deviations between target values and the optimal solution. The original objective is re-formulated as a goal constraint with a target value (goal) of the objective and two auxiliary variables. Two auxiliary variables are called positive deviation d^+ and negative deviation d^-, which measure the over-achievement and under-achievement with respect to this target value respectively. It is noted that, in optimization formulation, we have $d_i^+ \cdot d_i^- = 0$ for all i. The weighted goal programming (WGP) assigns weights to the unwanted deviations according to their relative importance indicated by decision-maker's preference, and the sum of all of the deviations between the goals and their aspired level are minimized as an Archimedean sum. The mathematical formulation of a WGP with p objectives has the following form:

$$\text{Min} \quad z = \sum_{i=1}^{p} (w_i^+ d_i^+ + w_i^- d_i^-) \tag{12}$$

s.t.

$$f_i(x) - d_i^+ + d_i^- = b_i, \quad i = 1, 2, \ldots, p, \tag{13}$$

$$g_j(x) \le 0, \quad j = 1, 2, \ldots, q, \tag{14}$$

$$d_i^+, d_i^- \ge 0, \quad i = 1, 2, \ldots, p \tag{15}$$

where

$f_i(x)$: goal constraint i.

$g_j(x)$: system constraint j.

$d_i^+ = \begin{cases} f_i(x) - b_i, & \text{if } f_i(x) > b_i, \\ 0, & \text{otherwise.} \end{cases}$: positive deviation i.

$d_i^- = \begin{cases} b_i - f_i(x), & \text{if } f_i(x) < b_i, \\ 0, & \text{otherwise.} \end{cases}$: negative deviation i.

b_i: target value for objective i.

w_i^+ = the weight attached to positive deviation i from goal i.

w_i^- = the weight attached to negative deviation i from goal i.

3.2 Goal programming model for the trip distribution problems

The GP approach to the multi-objective trip distribution problem is explained as follows. The first step is to formulate all objectives to goal constraints with deviations and target values. The goal constraint (1) re-transformed from the objective (1) will become:

$$-\sum_{i=1}^{n} \sum_{j=1}^{n} T_{ij} \ln T_{ij} - d_1^+ + d_1^- = H_0 \tag{16}$$

where H_0 is the target value of maximum entropy with respect to constraint (9).

The goal constraint (2) re-transformed from the objective (2) will become:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} T_{ij} - d_2^+ + d_2^- = C_0 \tag{17}$$

where C_0 is the target value of total generalized costs with respect to constraint (7).

The goal constraint (3) re-transformed from the objective (3) will become:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij} \ln \frac{T_{ij}}{T_{ij}^{0}} - d_3^{+} + d_3^{-} = E_0 \tag{18}$$

where E_0 is the target value of minimum deviations from observation data, and ideally is equal to zero.

$$\text{For all constraints (16–18), we have } d_i^{+}, d_i^{-} \geq 0 \text{ for all } i. \tag{19}$$

The objective function in GP for a multi-objective trip distribution problem considering all three objectives will become:

$$\sum_{k=1}^{3} (w_k^{+} d_k^{+} + w_k^{-} d_k^{-}) \tag{20}$$

s.t.

$$\sum_{j=1}^{n} T_{ij} = O_i, \quad i = 1, 2, \ldots, n, \tag{21}$$

$$\sum_{i=1}^{n} T_{ij} = D_j, \quad j = 1, 2, \ldots, n, \tag{22}$$

$$-\sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij} \ln T_{ij} - d_1^{+} + d_1^{-} = H_0, \tag{23}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} T_{ij} - d_2^{+} + d_2^{-} = C_0, \tag{24}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij} \ln \frac{T_{ij}}{T_{ij}^{0}} - d_3^{+} + d_3^{-} = E_0, \tag{25}$$

$$T_{ij} \geq 0, \quad i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, n, \tag{26}$$

$$d_k^{+}, d_k^{-} \geq 0, \quad k = 1, 2, 3. \tag{27}$$

4 Genetic algorithm

It can be seen that the proposed goal programming model contains a non-linear constraint, involves a numerous amount of variables and is not as simple as the traveling salesman problem with 0-1 integer interpretation. Reeves (1995) stated that the genetic algorithm (GA) is a versatile approach to solving hard optimization problems. Genetic algorithms have been receiving great attention and have also been successfully applied in many research fields in the last decade (Fogel 1995; Michalewicz 1994). In this study, let $\{t_{ij}\}$ be the chromosome to solution of the goal programming model. Choosing an appropriate chromosome representation of candidate solutions for the problem is the foundation for applying a GA to the optimization problems here. To obtain the complete solution, it is firstly necessary to run several GA iterations with the following input data and parameters.

/* Input data and parameters */

O_i = number of trips generated from zone i, $i \in I = \{1, 2, \ldots, n\}$;
D_j = number of trips attracted to zone j, $j \in J = \{1, 2, \ldots, n\}$;
pop_size = number of chromosomes;
a = value used in evaluation function;
P_c = parameter used in crossover operation; and
P_m = parameter used in mutation operation.

Similar to most genetic algorithms, the proposed GA approach incorporates the following procedures listed in the main module.

/* Main Module */

Procedure 0. Representation.
Procedure 1. Initialize *pop_size* chromosomes.
Procedure 2. Evaluate the function for selection.
Procedure 3. Select the chromosomes based on fitness value and record the best solution.
Procedure 4. Update the chromosomes using crossover operations.
Procedure 5. Update the chromosomes using mutation operations.
Procedure 6. Repeat the 3rd to 5th procedures for given iterations.

/* Procedure 0 Representation */

The objective in the model is to determine the matrix with minimal objective value with respect to three deviations subject to a set of constraints. The matrix T corresponding to the k chromosome can be represented as follows:

$$T_k = \begin{bmatrix} t_{11}^k & t_{12}^k & \cdots & t_{1n}^k \\ t_{21}^k & t_{22}^k & \cdots & t_{2n}^k \\ \cdots & \cdots & t_{ij}^k & \cdots \\ t_{n1}^k & t_{n2}^k & \cdots & t_{nn}^k \end{bmatrix} = \{t_{ij}^k\}, i \in I = \{1, 2, \ldots, n\},\ j \in J = \{1, 2, \ldots, n\}$$

and optimal solution T^* with optimal value.

/* Procedure 1 Initialization Population */

The feasible solution $\{t_{ij}\}$ can be found by the following formulation:

$$t_{ij} = O_i D_j \Big/ \sum_{i \in I} O_i \quad \left(\text{or } t_{ij} = O_i D_j \Big/ \sum_{j \in J} D_j\right).$$

Proof:
Because

$$\sum_{i \in I} O_i = \sum_{j = J} D_j,$$

we get

$$\sum_{j \in J} t_{ij} = O_i, \quad \forall i \in I$$

and

$$\sum_{i \in I} t_{ij} = D_j, \quad \forall j \in J.$$

Although $\{t_{ij}\}$ satisfies the doubly constraints, it may not be an optimal solution. So, for randomly generated initial matrices $\{t_{ij}^k\}$, $k = 1, 2, \ldots, pop\_size$, the following steps are carried out.

Input: $O_i, i \in I; D_j, j \in J.$
Step 1. Set $k \leftarrow 0$.
Step 2. Initialize $t_{ij} \leftarrow 0, i \in I, j \in J$.
Step 3. Set all numbers from 1 to $n \cdot n$ as unvisited.
Step 4. Select an unvisited random number u from 1 to $n \cdot n$ and set it as visited.
Step 5. Calculate corresponding row and column.

$$i \leftarrow \lfloor (u-1)/n \rfloor + 1,$$
$$j \leftarrow (u-1) \bmod n + 1.$$

Step 6. Assign available trips to t_{ij} as follows:
$t_{ij} \leftarrow \min\{O_i, D_j\}$.
Step 7. Update data.

$$O_i \leftarrow O_i - t_{ij},$$
$$D_j \leftarrow D_j - t_{ij}.$$

Step 8. If all numbers are not visited, then go to Step 4.
Step 9. If $k < pop\_size$, then set $k \leftarrow k + 1$ and go to Step 3.
Output: Matrices $T_k = \{t_{ij}^k\}$, $k = 1, 2, \ldots, pop\_size$.

/* Procedure 2 Evaluation Function */

Let $f(T_k)$, $k = 1, 2, \ldots, pop\_size$, denote the fitness value at the current generation which will be assigned a probability of reproducing to each chromosome. The chromosomes that have a higher fitness value will have a higher chance of producing children. This process is presented in Procedure 2 and 3.

Input: Parameter a.
Step 1. Calculate evaluation function as follows:

$$eval_k = a(1-a)^{k-1}, k = 1, 2, \ldots, pop\_size.$$

Step 2. Set $g_0 \leftarrow 0$,

$$g_k \leftarrow \sum_{i=1}^{k} eval_i, k = 1, 2, \ldots, pop\_size.$$

Output: $g_k, k = 0, 1, 2, \ldots, pop\_size.$

/* Procedure 3 Selection Process */

The objective of selection process is to select chromosome to be new population. The selection process proceeds by spinning the roulette wheel *pop_size* times.

Input: Matrices $T_k = \{t_{ij}^k\}, k = 1, 2, \ldots, pop\_size$.

Step 1. Calculate the objective value as fitness value for each chromosome.
Fitness value $f(T_k) = \sum_{l=1}^{3}(w_l^+ d_l^+ + w_l^- d_l^-)$
where

$$d_1^+ = \begin{cases} -\sum_{i\in I}\sum_{j\in J} t_{ij}\ln t_{ij} - H_0, & \text{if } -\sum_{i\in I}\sum_{j\in J} t_{ij}\ln t_{ij} > H_0, \\ 0, & \text{otherwise.} \end{cases}$$

$$d_1^- = \begin{cases} H_0 + \sum_{i\in I}\sum_{j\in J} t_{ij}\ln t_{ij}, & \text{if } -\sum_{i\in I}\sum_{j\in J} t_{ij}\ln t_{ij} < H_0, \\ 0, & \text{otherwise.} \end{cases}$$

$$d_2^+ = \begin{cases} \sum_{i\in I}\sum_{j\in J} c_{ij}t_{ij} - C_0, & \text{if } \sum_{i\in I}\sum_{j\in J} c_{ij}t_{ij} > C_0, \\ 0, & \text{otherwise.} \end{cases}$$

$$d_2^- = \begin{cases} C_0 - \sum_{i\in I}\sum_{j\in J} c_{ij}t_{ij}, & \text{if } \sum_{i\in I}\sum_{j\in J} c_{ij}t_{ij} < C_0, \\ 0, & \text{otherwise.} \end{cases}$$

$$d_3^+ = \begin{cases} \sum_{i\in I}\sum_{j\in J} t_{ij}\frac{t_{ij}}{t_{ij}^0} - E_0, & \text{if } \sum_{i\in I}\sum_{j\in J} t_{ij}\frac{t_{ij}}{t_{ij}^0} > E_0, \\ 0, & \text{otherwise.} \end{cases}$$

$$d_3^- = \begin{cases} E_0 - \sum_{i\in I}\sum_{j\in J} t_{ij}\frac{t_{ij}}{t_{ij}^0}, & \text{if } \sum_{i\in I}\sum_{j\in J} t_{ij}\frac{t_{ij}}{t_{ij}^0} < E_0, \\ 0, & \text{otherwise} \end{cases}$$

where H_0, C_0 and E_0 are the target value of $-\sum_{i\in I}\sum_{j\in J} t_{ij}\ln t_{ij}$, $\sum_{i\in I}\sum_{j\in J} c_{ij}t_{ij}$ and $\sum_{i\in I}\sum_{j\in J} t_{ij}\ln\frac{t_{ij}}{t_{ij}^0}$ respectively.

Step 2. Sort the fitness value in ascending order such that

$$f(T_1') < f(T_2') < \cdots < f(T_{pop\_size}').$$

Step 3. Record the optimal solution.
If $f(T_1') <$ *optimal value*,
then *optimal value* $\leftarrow f(T_1')$ and $T^* \leftarrow T_1'$.

Step 4. Spin roulette wheel.
The chromosomes are generally selected on a fitness basis (the better the fitness value, the higher the chance of it being chosen).

Step I. $k = 1$.
Step II. Generate a random number $u \in (0, g_{pop\_size}]$.
Step III. Select T_k' if $g_{k-1} < u < g_k$.
Step IV. If $k < pop\_size$, then go to Step II.

Step 5. Preserve the optimal solution.
Replace the worse chromosome by T^* as follows:

$$T_{pop\_size} = T^*.$$

Output: Matrices $T_k'' = \{t_{ij}^k\}, k = 1, 2, \ldots, pop\_size$.

/* Procedure 4 Crossover Operation */

A parameter P_c is defined as the probability of crossover among chromosomes. This probability gives the expected number $P_c \cdot pop\_size$ of chromosomes during the crossover operation.

Input: Matrices $T_k^{''} = \{t_{ij}^k\}$, $k = 1, 2, \ldots, pop\_size$ and P_c.

The crossover is performed in three steps:

Step 1. Select matrices as parents for crossover operation.

Input: Matrices $T_k^{''} = \{t_{ij}^k\}$, P_c.
Step I. $k = 1$.
Step II. Generate a random number $u \in [0, 1]$.
Step III. Select $T_k^{''}$ if $u \le P_c$.
Step IV. If $k < pop\_size$, then go to Step II.
Output: Matrices $T_k^{'''} = \{t_{ij}^k\}$, $k = 1, 2, \ldots, pop\_size$.
Remark: If the total selected matrices is odd, then unselect the last selected matrix.

Step 2. Operate crossover.

Step I. Select two matrices, e.g. $T_1^{'''}$ and $T_2^{'''}$, from selected matrices set.
Step II. Create two temporary matrices $D = \{d_{ij}\}$ and $R = \{r_{ij}\}$ as follows:

$$d_{ij} = \lfloor (t_{ij}^1 + t_{ij}^2)/2 \rfloor,$$
$$r_{ij} = (t_{ij}^1 + t_{ij}^2) \bmod 2.$$

Step III. Divide matrix R into two matrices $R_1 = \{r_{ij}^1\}$ and $R_2 = \{r_{ij}^2\}$ such that $R = R_1 + R_2$

$$\sum_{j \in J} r_{ij} = \sum_{j \in J} r_{ij}^1 + \sum_{j \in J} r_{ij}^2, \quad \forall i \in I.$$
$$\sum_{i \in I} r_{ij} = \sum_{i \in I} r_{ij}^1 + \sum_{i \in I} r_{ij}^2, \quad \forall j \in J.$$

It can be seen that there are too many possible ways to divide R into R_1 and R_2 while satisfying the above conditions.
It is noted that r_{ij}, r_{ij}^1 and r_{ij}^2 are either 0 or 1, where $i \in I$, $j \in J$.
R_1 and R_2 can be probably found by the following steps:

Step i. $$O_i' = \sum_{j \in J} r_{ij}^1 = \left\lfloor \frac{1}{2} \sum_{j \in J} r_{ij} \right\rfloor, \quad \forall i \in I.$$
$$D_j' = \sum_{i \in I} r_{ij}^1 = \left\lfloor \frac{1}{2} \sum_{i \in I} r_{ij} \right\rfloor, \quad \forall j \in J.$$
Step ii. Initialize $r_{ij}^1 \leftarrow 0$, $i \in I$, $j \in J$.
Step iii. Set all numbers from 1 to $n \cdot n$ as unvisited.
Step iv. Select an unvisited random number u from 1 to $n \cdot n$ and set it as visited.

Step v. Calculate corresponding row and column.

$$i \leftarrow \lfloor (u-1)/n \rfloor + 1,$$
$$j \leftarrow (u-1) \bmod n + 1.$$

Step vi. If $O_i' > 0$, $D_j' > 0$ and $r_{ij} \neq 0$, then $r_{ij}^1 \leftarrow 1$.

Step vii. Update data.

$$O_i' \leftarrow O_i' - r_{ij}^1,$$
$$D_j' \leftarrow D_j' - r_{ij}^1,$$
$$r_{ij}^2 \leftarrow r_{ij} - r_{ij}^1.$$

Step viii. If all numbers are not visited, then go to Step iv; However, one should be careful since experience shows that the final matrices R_1 and R_2 may not satisfy the doubly constraints. Step ii is required to reproduce R_1 and R_2 until the doubly constraints are satisfied.

Step 3. Replace two selected parents by two offspring.

Two selected parents are replaced by two new offspring T_1''' and T_2''' as follows:

$$T_1''' \leftarrow D + R_1,$$
$$T_2''' \leftarrow D + R_2.$$

If all selected matrices for the crossover operation are not operated, then go to Step 2.

Output: Matrices $T_k''' = \{t_{ij}^k\}$, $k = 1, 2, \ldots, pop\_size$.

/* Procedure 5 Mutation Operation */

A parameter P_m is defined as the probability of mutation operation. This probability gives the expected number $P_m \cdot pop\_size$ of chromosomes during the mutation operation.

Input: Matrices $T_k''' = \{t_{ij}^k\}$, $k = 1, 2, \ldots, pop\_size$ and P_m.

The mutation is performed in three steps:

Step 1. Select matrices as parents for mutation operation.

Input: Matrices $T_k''' = \{t_{ij}^k\}$, P_m.

Step I. $k = 1$.

Step II. Generate a random number $u \in [0, 1]$.

Step III. Select T_k''' if $u \leq P_m$.

Step IV. If $k < pop\_size$, then go to Step II.

Output: Matrices $T_k'''' = \{t_{ij}^k\}$, $k = 1, 2, \ldots, pop\_size$.

Step 2. Operate mutation.

Step I. Select a matrix, e.g. T_1'''', from selected matrices set.

Step II. Randomly select rows and columns to create a $p \times q$ sub-matrix $Y = \{y_{\alpha\beta}\}$ where $\alpha \in \{i_1, i_2, \ldots, i_p\} \subseteq I$ and $2 \leq p \leq n$, and $\beta \in \{j_1, j_2, \ldots, j_q\} \subseteq J$ and $2 \leq q \leq n$.

Step III. $O''_\alpha = \sum_{\forall\beta} y_{\alpha\beta}$,

$D''_\beta = \sum_{\forall\alpha} y_{\alpha\beta}$.

Step IV. As an initialization matrix.

Step i. Initialize $y_{\alpha\beta} \leftarrow 0$,

$\alpha \in \{i_1, i_2, \ldots, i_p\}$,
$\beta \in \{j_1, j_2, \ldots, j_q\}$.

Step ii. Set all numbers from 1 to $p \cdot q$ as unvisited.

Step iii. Select an unvisited random number u from 1 to $p \cdot q$ and set it as visited.

Step iv. Calculate the corresponding row and column;

$\alpha \leftarrow \lfloor (u-1)/n \rfloor + 1$,
$\beta \leftarrow (u-1) \bmod n + 1$.

Step v. Assign available trips to $t_{\alpha\beta}$ as follows:

$t_{\alpha\beta} \leftarrow \min\{O''_\alpha, D''_\beta\}$.

Step vi. Update data.

$O''_\alpha \leftarrow O''_\alpha - t_{\alpha\beta}$,
$D''_\beta \leftarrow D''_\beta - t_{\alpha\beta}$.

Step vii. If all numbers are not visited, then go to Step iii.

Output: Matrices $T''''_k = \{t^k_{ij}\}$, $k = 1, 2, \ldots, pop\_size$.

5 Application to Hong Kong

The proposed GP model for multi-objective trip distribution discussed in the previous sections has been tested on Hong Kong data, and we report the test results in this section. A rather small set of data is used to illustrate the model, in which the study area was aggregated into 12 major districts and trips were made by workers with similar economic backgrounds. The trip matrix in the observed year contains 12 districts, D1 to D12, which primarily cover a part of Hong Kong. Decision-makers often concentrate on travel between work and place of residence, since this is believed to constitute a large part of private travel. Moreover, work journeys are easier to predict than, for instance, shopping tours or leisure trips.

The observed data for the past year are given in Table 1. It is noted that D1 and D2 shared a large proportion of trips, whilst D11 and D12 produced a smaller number of inter-district trips. This is because D1 and D2 are in the central business district (CBD) area which generate a large amount of working trips. D11 and D12 are located in more remote areas.

Table 1 Observed data in the past year

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12
D1	1543	1579	841	935	584	2112	268	710	59	56	32	6
D2	1937	3587	1054	1007	879	2211	287	732	60	256	33	8
D3	346	305	202	327	123	495	104	268	22	10	11	7
D4	769	698	675	946	302	1255	391	1037	78	28	26	8
D5	1245	1646	766	791	545	1370	267	678	55	48	24	10
D6	396	361	206	275	113	494	84	222	16	15	10	2
D7	474	600	429	769	194	906	1008	1745	127	13	97	24
D8	549	615	520	1029	227	983	859	1802	135	11	61	17
D9	350	450	293	548	143	661	572	1106	211	9	45	20
D10	361	860	180	229	146	376	99	221	21	136	7	4
D11	643	801	451	791	272	1028	888	1567	140	20	2885	1023
D12	144	203	122	189	61	297	201	374	33	2	749	479

Total entropy $= -\sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij}\ln T_{ij} = -491\,916$; total generalized cost $= \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}T_{ij} = 635\,640$

Table 2 Generalized cost (HK$; US$ = HK$7.8)

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12
D1	5	6	7	7	6	8	10	9	9	10	16	16
D2	7	5	9	9	6	9	11	10	11	8	17	18
D3	6	7	5	6	6	7	8	7	7	10	14	15
D4	7	8	7	5	7	8	8	7	7	11	14	14
D5	6	6	8	8	4	9	10	9	10	9	16	17
D6	6	7	7	7	7	6	9	8	8	10	15	15
D7	11	13	11	10	11	12	4	6	7	15	10	11
D8	10	11	9	8	9	10	6	5	6	13	12	13
D9	11	13	11	10	11	12	7	7	4	15	13	13
D10	11	9	13	13	10	13	15	14	15	7	21	22
D11	19	20	19	18	19	20	12	15	13	23	6	8
D12	20	22	20	19	20	21	14	17	15	25	9	4

The generalized cost is given in Table 2. Since D11 and D12 are in rural areas, the generalized costs from and to these areas are relatively high compared with other districts. This is consistent with the trip matrix in the observed year.

It is noted that there exists trips and generalized cost from one origin to itself because some commuters make trips start from and end at the same district, and that travel cost, waiting cost, etc. are incurred to make such trips.

Table 3 shows the total number of trips produced from and attracted to 12 districts in future year. These trips can be estimated by regression analysis model or cross-classification model (Ortuzar and Willumsen 1994).

Table 3 Estimated trips in future year

	Future year	
	From (O_i)	To (D_j)
D1	11485	8040
D2	12866	15177
D3	1965	6299
D4	8948	11532
D5	11532	4459
D6	3369	17474
D7	5925	6484
D8	7938	9950
D9	2993	2281
D10	5614	3977
D11	14879	4936
D12	4572	1477
Total	92086	92086

Table 4 Maximization of entropy

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12
D1	1018	1862	785	1453	552	2197	799	1221	284	507	619	188
D2	1128	2146	876	1611	634	2408	910	1395	314	562	675	207
D3	169	328	133	250	89	388	132	212	49	82	103	30
D4	782	1495	612	1130	429	1699	620	949	224	382	480	146
D5	1000	1904	797	1448	564	2168	826	1239	285	506	611	184
D6	301	552	232	428	158	639	238	364	83	142	181	51
D7	511	984	399	747	290	1109	425	653	143	250	321	93
D8	693	1313	534	969	385	1523	555	869	196	345	428	128
D9	258	492	202	375	144	569	211	332	72	131	161	46
D10	488	932	380	704	269	1077	395	607	139	232	300	91
D11	1298	2416	1039	1842	724	2818	1059	1613	377	644	809	240
D12	394	753	310	575	221	879	314	496	115	194	248	73

Total entropy = −626 128; total generalized cost = 996 300

For future years, optimal trip matrices are compared to three "extreme models", which are extreme model (1)—the maximum entropy solution (Table 4); extreme model (2)—the minimum cost solution (Table 5); and extreme model (3)—the minimum deviations from the observed year (Table 6). The extreme model (1), maximization of entropy, is to maximize $-\sum\sum T_{ij}\ln T_{ij}$. The extreme model (2), minimization of generalized cost, is to minimize $\sum\sum c_{ij}T_{ij}$. The extreme model (3), minimization of deviations from observation data, is to minimize $\sum\sum T_{ij}\ln(T_{ij}/T_{ij}^0)$. It is noted that all these three extreme models are subject to doubly constraints (1) and (2), and non-negativity constraints only. Hence, the set of

Table 5 Minimization of generalized cost

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12
D1	6488	0	1121	0	0	3876	0	0	0	0	0	0
D2	0	12866	0	0	0	0	0	0	0	0	0	0
D3	0	0	1965	0	0	0	0	0	0	0	0	0
D4	0	0	0	8948	0	0	0	0	0	0	0	0
D5	1552	674	917	0	4459	3930	0	0	0	0	0	0
D6	0	0	0	0	0	3369	0	0	0	0	0	0
D7	0	0	0	0	0	0	0	5925	0	0	0	0
D8	0	0	1206	214	0	3373	0	3145	0	0	0	0
D9	0	0	159	312	0	117	0	124	2281	0	0	0
D10	0	1637	0	0	0	0	0	0	0	3977	0	0
D11	0	0	236	794	0	1673	6484	756	0	0	4936	0
D12	0	0	695	1264	0	1136	0	0	0	0	0	1477

Total entropy = −769 942; total generalized cost = 679 390

Table 6 Minimization of deviations from observed data

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12
D1	1460	2095	950	1440	756	3148	363	717	153	369	30	4
D2	1392	3637	915	1182	863	2543	306	569	124	1308	23	4
D3	222	272	155	343	101	500	96	185	38	43	7	3
D4	801	1039	836	1597	428	2036	597	1150	230	201	27	6
D5	1373	2645	1030	1430	835	2388	433	817	174	373	26	8
D6	440	564	269	495	167	860	139	260	49	113	11	2
D7	325	583	351	849	178	982	997	1278	243	62	65	12
D8	465	751	535	1439	262	1300	1073	1663	326	62	52	10
D9	171	307	167	427	94	504	405	576	287	28	21	6
D10	465	1574	278	472	247	758	185	304	75	1243	9	4
D11	735	1329	617	1448	419	1828	1495	1897	460	157	3579	915
D12	191	381	196	410	109	627	395	534	122	18	1086	503

Total entropy = −643 316; total generalized cost = 841 802

constraints for each of extreme models is different from that of each single objective model introduced in Sect. 2.

Owing to the non-linearity of the models, a genetic algorithm (GA) is developed to solve this hard optimization problem efficiently. To obtain the complete solution, it is necessary to run several GA iterations with the parameters consisting of the number of chromosomes, the probability used in crossover operations and the probability used in mutation operations (Gen and Cheng 1997). The solutions of the three extreme models run by C++ on Pentium III 600 MHz personal computers are shown in Tables 4–6. The total entropy obtained in the extreme model (1) is −626 128 which

Table 7 Two-objective model

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12
D1	3783	1823	1370	1043	287	3149	1	15	1	13	0	0
D2	870	8726	319	242	502	2034	0	9	0	164	0	0
D3	112	53	817	232	24	716	0	9	1	1	0	0
D4	342	167	918	5233	71	2132	2	76	4	3	0	0
D5	2109	2851	760	591	3342	1799	1	23	1	55	0	0
D6	161	78	161	125	12	2825	0	5	0	2	0	0
D7	83	15	226	472	17	523	1744	2786	58	1	0	0
D8	103	49	748	1589	59	1783	106	3426	72	3	0	0
D9	69	12	179	384	14	431	71	856	976	1	0	0
D10	129	1295	47	36	73	301	0	1	0	3732	0	0
D11	187	92	503	1057	39	1192	3843	2320	992	2	4633	19
D12	92	16	251	528	19	589	716	424	176	0	303	1458

Total entropy = −703 988; total generalized cost = 711 809

is the highest value of total entropy in a given set of numerical data. Total generalized cost obtained in the extreme model (2) is 679390 which is the lowest value of generalized cost in a given set of numerical data. It can be checked that total entropy obtained from extreme model (3) is smaller than −626 128 (the highest value of total entropy obtained from extreme model (1)), and total generalized cost is greater than 679 390 (the lowest value of total generalized cost obtained from extreme (2)).

In Table 7, the simple two-objective model, using objectives (1) and (2) presented in (11), with the target values obtained from the extreme models (1) and (2), $(H_0, C_0) = (-626\,128, 679\,390)$, is chosen. This trip matrix, excluding the minimization of deviations from observed data, is used for comparison with the matrix from three-objective model.

In a given set of numerical data, the highest value of total entropy from extreme model (1) is −626 128, and the lowest value of total generalized cost from extreme model (2) is 679 390. As discussed in Sect. 3, w_1^+ and w_2^- will be zero when the highest target value of objective 1 and the lowest target value of objective 2 are chosen. Moreover, it is assumed that $w_3^+ = w_3^-$ in this stage. The three-objective model with target values $(H_0, C_0, E_0) = (-626\,128, 679\,390, 0)$ is listed in Table 8. Initially, equal weighting factors are assigned to each of the three objectives, indicating that the importance of all three objectives is equal.

It is noted that in D12 the degree of dispersion is smaller than that generally observed in past year. Instead of changing the target values of E_0 or other target values preformed by Hallefjord and Jornsten (1986) in the proposed goal programming, it is simply to re-assign a new set of weight factors through the consideration of different parties from transport agencies and decision-makers to make a new solution with more dispersion. The suggested weighting factors are $(w_1^-, w_2^+, w_3^+, w_3^-) = (0.6, 0.1, 0.3, 0.3)$, indicating the importance of interactivity in the system. The results are shown in Table 9.

Table 8 Three-objective model with $(w_1^-, w_2^+, w_3^+, w_3^-) = (1/3, 1/3, 1/3, 1/3)$

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12
D1	2560	2249	1214	1353	582	3251	20	140	18	98	0	0
D2	1232	6501	577	602	835	2333	15	100	8	663	0	0
D3	192	157	424	348	70	684	9	62	8	11	0	0
D4	631	524	1027	3572	240	2357	63	442	53	39	0	0
D5	1868	3122	936	1013	2066	2105	27	181	14	200	0	0
D6	357	294	268	335	70	1950	8	58	7	22	0	0
D7	155	92	254	603	60	633	1761	2200	158	8	1	0
D8	225	207	626	1567	146	1472	493	2963	222	17	0	0
D9	103	62	164	400	41	423	234	837	724	5	0	0
D10	254	1557	114	137	165	464	4	26	2	2891	0	0
D11	345	339	508	1188	138	1297	3178	2424	882	20	4350	210
D12	118	73	187	414	46	505	672	517	185	3	585	1267

Total entropy = −667 289; total generalized cost = 746 914

Table 9 Three-objective model with $(w_1^-, w_2^+, w_3^+, w_3^-) = (0.6, 0.1, 0.3, 0.3)$

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12
D1	1452	2310	934	1535	664	2653	416	821	181	436	68	15
D2	1321	3507	873	1314	814	2623	401	789	154	988	67	15
D3	196	295	186	299	99	487	94	187	39	64	15	4
D4	801	1200	760	1667	413	1999	516	1017	211	274	72	18
D5	1326	2498	913	1440	904	2305	463	890	171	536	69	17
D6	363	552	267	456	153	949	138	285	56	122	23	5
D7	376	568	364	782	195	990	964	1166	220	114	154	32
D8	516	816	576	1247	299	1481	838	1562	293	162	121	27
D9	206	319	193	410	106	544	339	535	214	61	51	15
D10	495	1462	317	535	291	956	186	354	71	913	26	8
D11	761	1285	697	1403	401	1863	1681	1820	528	255	3395	790
D12	227	365	219	444	120	624	448	524	143	52	875	531

Total entropy = −638 885; total generalized cost = 857 493

6 Conclusion

In this paper, a GP model was presented to solve a multi-objective trip distribution problem in which three objectives with target values are optimized simultaneously. The three objectives are the maximization of interactivity in the system, the minimization of generalized costs, and the minimization of deviations from the observed year. Decision-makers can pre-specify target values for goal constraints to ensure that the trip matrix is more realistic. The example shows that, by adjusting weighting factors in the objective function corresponding to deviations from the goals, the goals' priority can be quantified. Decision-makers may find the proposed GP model is more flexible and comprehensive than the standard gravity model or entropy-maximization model in terms of multiple decision-making given the trade-off of a degree of dispersion with respect to the importance of goals.

The proposed model is a hard combinatorial problem and is not solved as easily as linear programming or even integer programming. A genetic algorithm is proposed to solve the problem and is seen to be quite successful and effective. The robustness of the model is illustrated by a set of Hong Kong data, and the genetic algorithm's efficacy is also demonstrated by the way in which the solution changes in adjusting the importance of the objectives. The modification of the genetic operation is successful because the doubly constraints are still satisfied. It is obvious that not only transportation problems, which are widely used to demonstrate how genetic algorithm operate can be successfully solved, but also other real-world problems, denoted by a matrix form, can be found by following this modified genetic algorithm.

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