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ORIGINAL PAPER

# Adaptive coordinated control of networked non-affine nonlinear systems with a non-autonomous nonlinear leader

**Yi Dong · Rongrong Gu**

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**Abstract** This paper considers the coordinated control problem of the multi-agent system modeled by a class of time-varying non-affine nonlinear systems and a non-autonomous nonlinear leader system with an input, capable of adjusting the reference signal in real time. We first propose a dynamic nonlinear observer to estimate the state of the active leader system, which is fully distributed without any prior knowledge of the graph and the input. Then by further employing a special group of time-varying functions, an observer-based adaptive control law is constructed to asymptotically track the nonlinear reference signals, which are allowed to be arbitrarily different for each follower system. Our control law can be applied to solve the leader-following synchronization and formation problems of multiple non-affine and affine nonlinear systems, and the whole design is robust to actuator fault, time-varying uncertain parameters and the external disturbances, whose upper bounds can be arbitrarily large and unknown.

**Keywords** Nonlinear systems · Multi-agent system · Adaptive control

Y. Dong (✉) · R. Gu
College of Electronic and Information Engineering, Shanghai Research Institute for Intelligent Autonomous Systems, Tongji University, Shanghai 200092, China
e-mail: yidong@tongji.edu.cn

R. Gu
e-mail: rongronggu@tongji.edu.cn

## 1 Introduction

Non-affine systems capture the dynamics of a large group of nonlinear systems including the helicopters [1], missile systems [2] and Brusselator model for the supply of reservoir chemicals [3], etc. Various methods have been proposed to solve the stabilization and tracking control of a non-affine system. Ren et al. [4] constructed the uncertainty and disturbance estimator-based control law to asymptotically track a stable linear reference model, while [3, 5] designed barrier Lyapunov functions-based adaptive control laws to handle the full state constraints after transforming the non-affine system into a strict-feedback structure. An adaptive neural output-feedback control was proposed in [6] by combining Levant's differentiator for the estimation of the unbailable states. Zhang et al. [7] considered an adaptive neural control for an unknown non-affine system with input deadzone, and Wu et al. [8] designed a scalarly virtual parameter adaptation approach to achieve the prescribed performance.

Motivated by practical scenarios such as the coordinated control of heterogeneous automatic guided vehicle in the unmanned warehouse, the tracking control of missiles with multiple targets, the cooperative control of non-affine helicopters and so on, many efforts have also been devoted to the coordinated control problems of the non-affine multi-agent systems in many scenarios. For the leaderless case, Fan et al. [9] studied the stabilization and consensus problem of non-affine multi-agent systems with output constraints and partially unknown control directions by incorporating error transformation technique. Qin et al. [10] proposed a neural network-based adaptive consensus protocol to achieve the synchronization with a bounded residual error. In the leader-following case, the desired reference signals can usually be classified into two groups. The first one is represented by the bounded unmodeled reference signal as in [11, 12], which designed the neural network based approach with guaranteed performance to make sure the synchronization errors are semiglobally, uniformly, and ultimately bounded. In the second group, the reference signal is generated by a leader system, e.g., Wang et al. [13] considered the reference signals generated from a lower-triangular nonlinear system, and based on the scaling function, developed a conceptually new and structurally simple control law to achieve the zero-error tracking.

In the framework of output regulation [14, 15], both the reference signal and external disturbances are generated by a so-called exosystem. When extended into the cooperative output regulation problem, the exosystem is regarded as the leader system, heterogeneous with follower systems. In order to design control methods in distributed fashion, a variety of observers have been investigated based on the local information. In particular, Cai et al. [16] and Su and Huang [17] designed distributed observers for a linear leader system and solved the synchronization problem of linear heterogeneous multi-agent systems, and Dong and Chen [18] proposed a nonlinear observer for a nonlinear autonomous leader system. Motivated by the practical circumstances such as reaching a desirable consensus value or avoiding hazardous obstacles [19, 20], the attention is paid to tracking an active leader system with an input. For example, Hong et al. [21] and Hu and Feng [22] considered the second-order linear leader system and local dynamic observers were proposed to estimate the state of the leader system. Li et al. [19] and Lv et al. [23] studied general linear leader system and the adaptive control technique was employed for the observer design when the input of the leader system was bounded and unknown, while the case of multiple active linear leaders was considered in [20], which could cooperatively generate safe trajectories and avoid dynamic obstacles.

This paper further considers an active nonlinear leader system with an unknown input to generate a group of arbitrary trajectories for networked non-affine nonlinear systems. Technically, it is essentially required to deal with an information coupled general time-varying nonlinear system with high orders whose desired trajectories can be adjusted at any time, and as a result, the reference is only attainable in real time and its time derivatives are unknown since the input is unknown. In order to overcome these difficulties, we first design a continuous nonlinear distributed observer to accurately estimate the state of the active nonlinear leader system, different from the linear/nonlinear autonomous leader system in [16–18] or the active linear leader system in [20–22]. Second, by incorporating a special class of time-varying functions, a distributed control law based on the nonlinear observer is designed for networked systems with time-varying and arbitrarily large parametric uncertainties and external disturbances without any information of the input. It is also fully distributed in nature in the sense that it depends on the state of itself and neighboring agents without the global information of the graph. Finally, the whole design is not only robust to input–output nonlinearity such as hysteresis and actuator faults, but also capable of achieving the asymptotic tracking of the non-autonomous leader system, instead of guaranteeing the tracking error ultimately uniformly bounded. It is applicable in leader-following synchronization and formation problems for multi-agent systems in both non-affine and affine forms.

The rest of this paper is organized as follows. In Sect. 2, we will formulate the problem. In Sects. 3 and 4, the distributed observer and control design will be presented, which will be illustrated by an example in Sect. 5. Finally, we will close this paper in Sect. 6 with some concluding remarks. Throughout the paper, the following notations are used. $||\cdot||$ represents the Euclidean norm of a vector or the 2-norm of a matrix. For $x_i \in \mathbb{R}$, $\bar{x}_i = [x_1, \quad \ldots, \quad x_i]^T$.

## 2 Problem formulation

Consider a multi-agent system composed of a nonlinear leader system and a group of followers modeled by non-affine systems as follows,

$$\begin{aligned} \dot{x}_{ji} &= f_{ji}(\bar{x}_{j+1,i}), \quad j = 1, \ldots, n-1, \; n \geq 2 \\ \dot{x}_{ni} &= f_{ni}(\bar{x}_{ni}, \kappa_i(u_i, t), w_i(t)), \quad i = 1, \ldots, N \\ e_i &= x_{1i} - x_{0i}(\eta_0) \end{aligned} \tag{1}$$

where $\bar{x}_{ji} \in \mathbb{R}^j$, $j = 1, \ldots, n$, $i = 1, \ldots, N$, are the states, $w_i(t) \in \mathbb{R}^{n_w}$ represents the unknown and bounded time-varying parameters or/and external disturbances, $\kappa_i : \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ represents the input–output characteristics of the actuator, $e_i \in \mathbb{R}$ is the tracking error for agent $i$, and $\eta_0 \in \mathbb{R}^{n_0}$ is the state of a non-autonomous nonlinear leader system described by

$$\dot{\eta}_0 = f_0(\eta_0) + u_0(t) \tag{2}$$

where $u_0(t)$ is the control input. All functions $f_0(\cdot)$, $f_{ji}(\cdot)$, $j = 1, \ldots, n$, $i = 1, \ldots, N$, are assumed to be globally defined, sufficiently smooth and satisfy $f_{ji}(0) = 0$, $j = 1, \ldots, n-1$, $f_{ni}(0, 0, w_i(t)) = 0$. The input–output characteristics of the actuator satisfy

$$\kappa_i(u_i, t) = \pi_i(t) u_i(t) + \phi_i(t) \tag{3}$$

where $\pi_i(t)$ and $\phi_i(t)$ are time-varying functions. (3) is a generalized form to model the input nonlinearity such as hysteresis, dead-zone and backlash [9].

The communication network of the multi-agent systems (1) and (2) is denoted by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{0, 1, \ldots, N\}$ is the node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. Define a subgraph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ of $\mathcal{G}$ where $\bar{\mathcal{V}} = \{1, \ldots, N\}$, and $\bar{\mathcal{E}} = \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ is obtained from $\mathcal{E}$ by removing all the edges between the node 0 and the nodes in $\bar{\mathcal{V}}$. Each element $(j, i) \in \mathcal{E}$ represents the edge from agent $j$ to agent $i$. For $i, j \in \mathcal{V}$, $a_{ii} = 0$, $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Associated with $\mathcal{G}$, define the Laplacian matrix $H = [h_{ij}]_{i,j=1}^N$ where $h_{ii} = \sum_{j=0}^N a_{ij}$ and $h_{ij} = -a_{ij}$ for $i \neq j$, $i, j = 1, \ldots, N$.

The objective is to design a distributed control law for each non-affine nonlinear system (1) to track the reference signal from (2) and guarantee that the tracking errors asymptotically converge to zero. Then the coordinated control problem can be described as follows.

**Problem 1** Given the multi-agent system composed of N followers given by (1) and an active nonlinear leader (2), and the corresponding graph $\mathcal{G}$, find a distributed control law such that the trajectory of the closed-loop system from any initial conditions exists and is bounded for $t \geq 0$, and the asymptotic tracking can be achieved, i.e.,

$$\lim_{t \to \infty} e_i(t) = 0, \quad i = 1, \ldots, N \tag{4}$$

***Remark 1*** The coordinated control problem considered here generalizes the leader-following consensus problem in the sense that an input is introduced in the nonlinear non-autonomous leader system (2) to adjust the desired trajectory for each follower system in real time, and each subsystem is modeled by time-varying non-affine nonlinear system (1), whose desired trajectories $x_{0i}(\eta_0)$ are further allowed to be arbitrarily different, motivated by more practical scenarios including cooperative control of multiple AGVs, missiles, helicopters, etc. Thus the technique can also be applied to solve the formation problem. At the same time, those control laws depending on the virtual consensus error $e_{vi} = \sum_{j=0}^N a_{ij}(x_{1i} - x_{1j})$ for the purpose of reaching consensus such as [9, 11, 12], cannot be applied here since the virtual error $e_{vi}$ will not converge to zero if the reference signals are different for each agent. These generalizations also make the internal model based methods in [18, 24] for cooperative output regulation problem infeasible.

For the solvability of the problem, we need the following assumptions on the reference and the graph.

**Assumption 1** The input $u_0(t)$ and the state $\eta_0(t)$ in (2) are bounded for all $t \geq 0$, but their upper bounds are unknown. It is also assumed that $x_{0i}(\eta_0)$ is globally defined, sufficiently smooth and bounded for $t \geq 0$.

**Assumption 2** The graph $\mathcal{G}$ is connected in the sense that each node $i = 1, \ldots, N$, is reachable from the node 0 and $\bar{\mathcal{G}}$ is undirected. Laplacian matrix $H$ is unavailable to all agents.

***Remark 2*** Same as the references considering the input in the leader system in [19, 20, 23], $u_0(t)$ is assumed to be bounded. Also same as references for non-affine systems in [5, 13, 18], it is required that the reference to track is bounded. In Assumption 2, we require the graph is undirected in order to design a fully distributed control law, i.e. the global information of the graph represented by $H$ is unknown. Our method is also applicable to a general directed graph if the upper bounds of $u_0$, $\eta_0$ and $H$ are available to all agents, detailed in Remark 4 in what follows.

We end this section by providing a useful lemma.

**Lemma 1** [25] *For any scalar positive function $\upsilon(t) : \mathbb{R}_{\geq 0} \to \mathbb{R}^+$ and any variable $z \in \mathbb{R}$, the following inequality holds*

$$0 \leq |z| < \upsilon(t) + \frac{z^2}{\sqrt{z^2 + \upsilon^2(t)}}.$$

## 3 Nonlinear observer design

In this section, we propose a distributed nonlinear observer for the non-autonomous nonlinear leader. For this purpose, define a class of time-varying positive functions $\upsilon_j(t), j = 1, \ldots, 4$, satisfying

$$\int_0^t \upsilon_j(\tau)d\tau \leq \bar{\upsilon}_j, \quad |\dot{\upsilon}_i(t)| \leq \underline{\upsilon}_j \tag{5}$$

where $\bar{\upsilon}_j > 0$ and $\underline{\upsilon}_j > 0$. The nonlinear observer takes the following form:

$$\dot{\eta}_i = f_0(\eta_i) - \frac{\hat{\vartheta}_i \eta_{vi}}{\sqrt{\upsilon_1^2(t) + ||\eta_{vi}||^2}} - \hat{k}_i \rho_i(\eta_{vi})\eta_{vi} - c_{0i}\eta_{vi} \tag{6a}$$

$$\dot{\hat{\vartheta}}_i = c_{1i}\left(\frac{||\eta_{vi}||^2}{\sqrt{\upsilon_1^2(t) + ||\eta_{vi}||^2}} - \upsilon_2(t)\hat{\vartheta}_i\right) \tag{6b}$$

$$\dot{\hat{k}}_i = c_{2i}\Big(\rho_i(\eta_{vi})||\eta_{vi}||^2 - \upsilon_2(t)\hat{k}_i\Big), \; i = 1, \ldots, N \tag{6c}$$

where $\eta_{vi} = \sum_{j=0}^N a_{ij}(\eta_i - \eta_j)$, $i = 1, \ldots, N$, $c_{ji} > 0$, $j = 0, 1, 2$, and $\rho_i(\cdot) \geq 1$ is some smooth function determined later.

We construct a lemma to show the property of the observer (6).

**Lemma 2** *Under Assumption 2, consider the system (2) and (6). The solution of the closed-loop system exists and is bounded, and*

$$\lim_{t\to\infty} \Big(\eta_i(t) - \eta_0(t)\Big) = 0, \quad i = 1, \ldots, N. \tag{7}$$

***Proof*** Let $\bar{\eta}_i = \eta_i - \eta_0$, $i = 0, 1, \ldots, N$. Define $\bar{\vartheta}_i = \vartheta - \hat{\vartheta}_i$, $\bar{k}_i = k_i - \hat{k}_i$, $i = 1, \ldots, N$, for some constants $\vartheta > 0$ and $k_i > 0$. The closed-loop system of (2) and (6) becomes, for $i = 1, \ldots, N$,

$$\begin{aligned}
\dot{\bar{\eta}}_i &= \bar{f}_0(\bar{\eta}_i, \eta_0) + \bar{\alpha}_i(t) - u_0(t), \quad \eta_{vi} = \sum_{j=0}^N a_{ij}(\bar{\eta}_i - \bar{\eta}_j) \\
\dot{\bar{\vartheta}}_i &= c_{1i}\left(\frac{||\eta_{vi}||^2}{\sqrt{\upsilon_1(t)^2 + ||\eta_{vi}||^2}} - \upsilon_2(t)\hat{\vartheta}_i\right) \\
\dot{\bar{k}}_i &= c_{2i}\Big(\rho_i(\eta_{vi})||\eta_{vi}||^2 - \upsilon_2(t)\hat{k}_i\Big)
\end{aligned} \tag{8}$$

where $\bar{\alpha}_i(t) = -\frac{\hat{\vartheta}_i\eta_{vi}}{\sqrt{\upsilon_1^2(t)+||\eta_{vi}||^2}} - \hat{k}_i\rho_i(\eta_{vi})\eta_{vi} - c_{0i}\eta_{vi}$ and $\bar{f}_0(\bar{\eta}_i, \eta_0) = f_0(\bar{\eta}_i + \eta_0) - f_0(\eta_0)$. Let $\bar{\eta} =$ col $(\bar{\eta}_1, \ldots, \bar{\eta}_N)$ and $\bar{\eta}_v =$ col $(\bar{\eta}_{v1}, \ldots, \bar{\eta}_{vN})$. Note that $\eta_v = (H \otimes I_{n_0})\bar{\eta}$. Under Assumption 2, by Lemma 1 in [17], all the nonzero eigenvalues of $H$ have positive real parts and $H$ is nonsingular. Since the graph $\bar{\mathcal{G}}$ is undirected, $H$ is positive definite. Define

$$W = \frac{1}{2}\bar{\eta}^T(H \otimes I_{n_0})\bar{\eta} + \sum_{i=1}^N\left(\frac{1}{2c_{1i}}\bar{\vartheta}_i^2 + \frac{1}{2c_{2i}}\bar{k}_i^2\right). \tag{9}$$

Along the trajectory of (8), the time derivative of $W$ satisfies,

$$\begin{aligned}
\dot{W} &= \sum_{i=1}^N\left(\eta_{vi}^T\dot{\bar{\eta}}_i - \frac{1}{c_{1i}}\bar{\vartheta}_i\dot{\hat{\vartheta}}_i - \frac{1}{c_{2i}}\bar{k}_i\dot{\hat{k}}_i\right) \\
&= \sum_{i=1}^N\left(\eta_{vi}^T\bar{f}_0(\bar{\eta}_i, \eta_0) + \eta_{vi}^T\bar{\alpha}_i - \eta_{vi}^T u_0(t) - \frac{1}{c_{1i}}\bar{\vartheta}_i\dot{\hat{\vartheta}}_i - \frac{1}{c_{2i}}\bar{k}_i\dot{\hat{k}}_i\right)
\end{aligned}$$

Let $F_0(\bar{\eta}, \eta_0) = \sum_{i=1}^N \eta_{vi}^T\bar{f}_0(\bar{\eta}_i, \eta_0)$. Note that $\bar{f}_0(\cdot)$ is sufficiently smooth and satisfies $\bar{f}_0(0, \eta_0) = 0$. By Lemma 7.8 in [15], there exists some smooth function $\rho_i(\cdot) \geq 1$ such that, for any bounded signal $\eta_0$, $||F_0(\bar{\eta}, \eta_0)|| = ||F_0((H^{-1} \otimes I_{n_0}) \quad \eta_v, \eta_0)|| \quad \leq \sum_{i=1}^N$ $k_i\rho_i(\eta_{vi}) \; ||\eta_{vi}||^2$ with $k_i$ unknown since the upper bounds of $\eta_0$ and $H$ are unknown to all agents. Since

$u_0(t)$ is bounded with unknown upper bound, there exists some unknown constant $\vartheta > 0$ such that $||u_0(t)|| \leq \vartheta$. By Lemma 1, we can obtain,

$$\eta_{vi}^T u_0(t) \leq \vartheta \upsilon_1(t) + \frac{\vartheta ||\eta_{vi}||^2}{\sqrt{\upsilon_1^2(t) + ||\eta_{vi}||^2}} \tag{10}$$

Then,

$$\begin{aligned}\dot{W} \leq & \sum_{i=1}^{N} \Big( - c_{0i} ||\eta_{vi}||^2 + \bar{k}_i \rho_i(\eta_{vi}) ||\eta_{vi}||^2 \\ & - \frac{1}{c_{2i}} \bar{k}_i \dot{\hat{k}}_i \\ & - \frac{1}{c_{1i}} \bar{\vartheta}_i \dot{\hat{\vartheta}}_i + \frac{\bar{\vartheta}_i ||\eta_{vi}||^2}{\sqrt{\upsilon_1^2(t) + ||\eta_{vi}||^2}} + \vartheta \upsilon_1(t) \Big) \\ = & \sum_{i=1}^{N} \Big( - c_{0i} ||\eta_{vi}||^2 + \vartheta \upsilon_1(t) \\ & + \upsilon_2(t) \bar{\vartheta}_i \hat{\vartheta}_i + \upsilon_2(t) \bar{k}_i \hat{k}_i \Big)\end{aligned}$$

Note that

$$\begin{aligned}\upsilon_2(t) \bar{\vartheta}_i \hat{\vartheta}_i &= \upsilon_2(t)(\bar{\vartheta}_i \vartheta - \bar{\vartheta}_i^2) \leq \frac{1}{4} \upsilon_2(t) \vartheta^2 \\ \upsilon_2(t) \bar{k}_i \hat{k}_i &= \upsilon_2(t)(\bar{k}_i k_i - \bar{k}_i^2) \leq \frac{1}{4} \upsilon_2(t) k_i^2\end{aligned} \tag{11}$$

Then,

$$\begin{aligned}\dot{W} \leq & \sum_{i=1}^{N} (-c_{0i} ||\eta_{vi}||^2 + \vartheta \upsilon_1(t) \\ & + \frac{1}{4} \upsilon_2(t) \vartheta^2 + \frac{1}{4} \upsilon_2(t) k_i^2).\end{aligned}$$

By the property of $\upsilon_j(t), j = 1, 2$, in (5), we can obtain,

$$\begin{aligned}W(t) \leq & W(0) + \sum_{i=1}^{N} \Big( \vartheta \int_0^t \upsilon_1(\tau) d\tau \\ & + \frac{1}{4} \vartheta^2 \int_0^t \upsilon_2(\tau) d\tau + \frac{1}{4} k_i^2 \int_0^t \upsilon_2(\tau) d\tau \Big) \\ \leq & W(0) + W_m\end{aligned} \tag{12}$$

where $W_m = \sum_{i=1}^{N} (\vartheta \bar{\upsilon}_1 + \frac{1}{4}(\vartheta^2 + k_i^2) \bar{\upsilon}_2)$, implying $\bar{\eta}_i$, $\bar{\vartheta}_i$, $\bar{k}_i$, $\hat{\vartheta}_i$ and $\hat{k}_i$, $i = 1, \ldots, N$, are bounded. Since $\eta_0$ is bounded for $t \geq 0$, $\eta_i$ is bounded for $t \geq 0$. From (8), we can also obtain $\dot{\bar{\eta}}_i$ is bounded for $t \geq 0$, which implies $\ddot{W}(t)$ is bounded together with (5). By Barbalat's Lemma in [26], we can obtain $\lim_{t \to \infty} \eta_{vi}(t) = 0$. Thus, under Assumption 2, (7) can be achieved. □

***Remark 3*** The design idea for (6a) is to compensate the nonlinear term $f_0(\eta_0)$ by the first term, counteract the effect of the input $u_0$ and $F_0(\bar{\eta}, \eta_0)$ by the second and third terms, respectively, and the last term is for the consensus purpose. And then we design the adaptive update laws (6b) and (6c) to generate enough force not only to make sure the control parameters do not depend on the global information of the network, such as the Laplacian matrix $H$, but also to allow the upper bounds of $u_0(t)$ and $\eta_0(t)$ to be arbitrarily large and unknown to all agents. As a result, (6) can accurately estimate the state $\eta_0$ from the nonlinear leader system (2), as opposed to the linear one in [16–18]. At the time, it allows the existence of the input $u_0$ to adjust the course in real time, which is also more general than those in [20, 24] and promises more applications.

## 4 Distributed control design

Benefiting from the design of the nonlinear observer (6) for the non-autonomous leader system (2), we can then propose a control law in a distributed form for the heterogeneous multi-agent system systems composed of (1) and (2).

### 4.1 System transformation

First, transform each of the non-affine nonlinear systems (1) into the strict-feedback system. Define a set of variables $s_{ji} \in \mathbb{R}, j = 1, \ldots, n, i = 1, \ldots, N$, with $s_{1i} = x_{1i} = b_{1i}(\bar{x}_{1i})$.

$$
\begin{aligned}
\dot{s}_{1i} &= f_{1i}(\bar{x}_{2i}) = b_{2i}(\bar{x}_{2i}) \triangleq s_{2i}, \quad i = 1, \ldots, N \\
\dot{s}_{2i} &= \frac{\partial b_{2i}(\bar{x}_{2i})}{\partial x_{1i}} f_{1i}(\bar{x}_{2i}) \\
&\quad + \frac{\partial b_{2i}(\bar{x}_{2i})}{\partial x_{2i}} f_{2i}(\bar{x}_{3i}) = b_{3i}(\bar{x}_{3i}) \triangleq s_{3i} \\
\dot{s}_{ji} &= \sum_{k=1}^{j} \frac{\partial b_{ji}(\bar{x}_{ji})}{\partial x_{ki}} f_{ki}(\bar{x}_{k+1,i}) \\
&= b_{j+1,i}(\bar{x}_{j+1,i}) \triangleq s_{j+1,i}, \quad j = 3, \ldots, n-1 \\
\dot{s}_{ni} &= \sum_{k=1}^{n-1} \frac{\partial b_{ni}(\bar{x}_{ni})}{\partial x_{ki}} f_{ki}(\bar{x}_{k+1,i}) \\
&\quad + \frac{\partial b_{ni}(\bar{x}_{ki})}{\partial x_{ni}} f_{ni}(\bar{x}_{ni}, \kappa_i, w_i)
\end{aligned} \tag{13}
$$

By the mean value theorem, there exists $\lambda_i \in (0, 1)$ such that

$$
\begin{aligned}
& f_{ni}(\bar{x}_{ni}, \kappa_i, w_i) \\
&\quad = f_{ni}(\bar{x}_{ni}, 0, w_i) + \frac{\partial f_{ni}(\bar{x}_{ni}, \kappa_i, w_i)}{\partial \kappa_i}\Big|_{\kappa_i = \lambda_i \kappa_i} \kappa_i
\end{aligned}
$$

It is also noted that from (13),

$$
\begin{aligned}
& \frac{\partial b_{2i}(\bar{x}_{2i})}{\partial x_{2i}} = \frac{\partial f_{1i}(\bar{x}_{2i})}{\partial x_{2i}} = g_{1i}(\bar{x}_{2i}) \\
& \frac{\partial b_{j+1,i}(\bar{x}_{j+1,i})}{\partial x_{j+1,i}} \\
&\quad = \frac{\partial b_{ji}(\bar{x}_{ji})}{\partial x_{ji}} \frac{\partial f_{ji}(\bar{x}_{j+1,i})}{\partial x_{j+1,i}} \\
&\quad = \prod_{k=1}^{j} g_{ki}(\bar{x}_{k+1,i}), \quad j = 2, \ldots, n-1
\end{aligned}
$$

where

$$
\begin{aligned}
g_{ji}(\bar{x}_{j+1,i}) &= \frac{\partial f_{ji}(\bar{x}_{j+1,i})}{\partial x_{j+1,i}}, \ j = 1, \ldots, n-1 \\
g_{ni}(\bar{x}_{ni}, \kappa_i, w_i) &= \frac{\partial f_{ni}(\bar{x}_{ni}, \kappa_i, w_i)}{\partial \kappa_i}, \ i = 1, \ldots, N
\end{aligned}
$$

As a result, we can obtain the following strict-feedback system,

$$
\begin{aligned}
\dot{s}_{ji} &= s_{j+1,i}, \quad j = 1, \ldots, n-1, \quad i = 1, \ldots, N \\
\dot{s}_{ni} &= \bar{f}_i(\bar{x}_{ni}, w_i) + g_i(\bar{x}_{ni}, \kappa_i, w_i)\kappa_i
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
\bar{f}_i(\bar{x}_{ni}, w_i) &= \sum_{k=1}^{n-1} \frac{\partial b_{ni}(\bar{x}_{ni})}{\partial x_{ki}} f_{ki}(\bar{x}_{k+1,i}) \\
&\quad + \frac{\partial b_{ni}(\bar{x}_{ni})}{\partial x_{ni}} f_{ni}(\bar{x}_{ni}, 0, w_i) \\
g_i(\bar{x}_{ni}, \kappa_i, w_i) &= \frac{\partial b_{ni}(\bar{x}_{ni})}{\partial x_{ni}} g_{ni}(\bar{x}_{ni}, \lambda_i \kappa_i, w_i) \\
&= g_{ni}(\bar{x}_{ni}, \lambda_i \kappa_i, w_i) \prod_{k=1}^{n-1} g_{ki}(\bar{x}_{k+1,i})
\end{aligned} \tag{15}
$$

For the solvability of the coordinated control problem, we also need the following assumption to guarantee the controllability of the nonlinear systems, similar to [2, 3, 9, 11, 13], etc.

**Assumption 3** The signs of $g_{ji}(\cdot)$, $j = 1, \ldots, n$, $i = 1, \ldots, N$, are known, and assumed to be positive without loss of generality. And it is further assumed that $g_{ji}(\cdot) \geq \underline{g}_{ji}$, $\pi_i(t) \geq \underline{\pi}_i$, and $|\phi_i(t)| \leq \bar{\phi}_i$ for $\underline{g}_{ji} > 0$, $\underline{\pi}_i > 0$ and $\bar{\phi}_i \geq 0$.

### 4.2 Controller design

In this subsection, a distributed controller is proposed to achieve the asymptotic tracking of the non-autonomous leader system, which takes the following form,

$$
\begin{aligned}
z_{1i} &= s_{1i} - x_{0i}(\eta_i), \\
z_{ji} &= s_{ji} - \alpha_{j-1,i}, \quad j = 2, \ldots, n \\
\alpha_{1i} &= -p_{1i} z_{1i} - \frac{\hat{m}_{1i} z_{1i}}{\sqrt{\upsilon_3^2(t) + z_{1i}^2}}, \quad i = 1, \ldots, N \\
\dot{\hat{m}}_{1i} &= \gamma_{1i} \left( \frac{z_{1i}^2}{\sqrt{\upsilon_3^2(t) + z_{1i}^2}} - \upsilon_4(t) \hat{m}_{1i} \right) \\
\alpha_{ji} &= -p_{ji} z_{ji} - z_{j-1,i} \\
&\quad - \frac{\hat{m}_{ji} l_{ji}^2 z_{ji}}{\sqrt{\upsilon_3^2(t) + l_{ji}^2 z_{ji}^2}} + \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial s_{ki}} s_{k+1,i} \\
&\quad + \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial \hat{m}_{ki}} \dot{\hat{m}}_{ki}, \ j = 2, \ldots, n-1
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\dot{\hat{m}}_{ji} =&\gamma_{ji}\left(\frac{l_{ji}^2 z_{ji}^2}{\sqrt{\upsilon_3^2(t)+l_{ji}^2 z_{ji}^2}}\right.\\
&\left.-\upsilon_4(t)\hat{m}_{ji}\right),\quad j=2,\ldots,n-1\\
l_{2i} &= \left|\frac{\partial\alpha_{1i}}{\partial\upsilon_3(t)}\right| + \left\|\frac{\partial\alpha_{1i}}{\partial\eta_i}\right\|\\
l_{ji} &= \left|\frac{\partial\alpha_{j-1,i}}{\partial\upsilon_3(t)}\right| + \left|\frac{\partial\alpha_{j-1,i}}{\partial\upsilon_4(t)}\right| + \left\|\frac{\partial\alpha_{j-1,i}}{\partial\eta_i}\right\|,\\
&j=3,\ldots,n\\
\alpha_{ni} &= -p_{ni}z_{ni} - z_{n-1,i} - \hat{\theta}_i^T\xi_i\\
\dot{\hat{\theta}}_i &= \gamma_{ni}(z_{ni}\xi_i - \upsilon_4(t)\hat{\theta}_i)\\
\bar{\xi}_i &= \left[\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1,i}}{\partial s_{ki}}s_{k+1,i} + \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1,i}}{\partial\hat{m}_{ki}}\dot{\hat{m}}_{ki}l_{ni}\ \mathrm{sign}\ (z_{ni})\right]^T\\
\xi_i &= \left[\frac{\varrho_i^2 z_{ni}}{\sqrt{\upsilon_4^2(t)+\varrho_i^2 z_{ni}^2}}\quad \bar{\xi}_i^T\right]^T\\
u_i &= -\frac{\alpha_{ni}^2 z_{ni}}{\sqrt{\upsilon_3^2(t)+\alpha_{ni}^2 z_{ni}^2}} - \frac{1}{\underline{\pi}_i}\bar{\phi}_i\ \mathrm{sign}\ (z_{ni})
\end{aligned}
\tag{16}
$$

where constant $\gamma_{ji} > 0, j = 1,\ldots,n, i = 1,\ldots,N$, and $\varrho_i = \varrho_i(\bar{x}_{ni})$ is a smooth positive function determined by (26) in what follows.

Now we can present the main theorem for the solvability of Problem 1.

**Theorem 1** *Under Assumptions 1 and 3, the coordinated control problem of multi-agent systems with follower systems given by (1) and a nonlinear non-autonomous leader system (2) can be solved by the observer-based control law (16) and (6).*

***Proof*** The proof includes n steps. Define $\bar{m}_{ji} = m_{ji} - \hat{m}_{ji},\ j = 1,\ldots,n,\ i = 1,\ldots,N,$ for some constant $m_{ji} > 0$.

In the 1-st step, define

$$V_{1i} = \frac{1}{2}z_{1i}^2 + \frac{1}{2\gamma_{1i}}\bar{m}_{1i}^2 \tag{17}$$

Note that

$$\dot{z}_{1i} = s_{2i} - \dot{x}_{0i}(\eta_i) = z_{2i} + \alpha_{1i} - \dot{x}_{0i}(\eta_i) \tag{18}$$

Then the time derivative of $V_{1i}$ satisfies

$$\dot{V}_{1i} = z_{1i}z_{2i} + z_{1i}\alpha_{1i} - z_{1i}\dot{x}_{0i}(\eta_i) - \frac{1}{\gamma_{1i}}\bar{m}_{1i}\dot{\hat{m}}_{1i}$$

In the proof of Lemma 2, we have shown that $\eta_i$ and $\dot{\bar{\eta}}_i$, $i = 1,\ldots,N$, are bounded for $t \geq 0$. From (2), $\dot{\eta}_0$ is bounded since $\eta_0$ and $u_0(t)$ are bounded and $f_0(\cdot)$ is smooth, and thus $\dot{\eta}_i$ is bounded, but the upper bound is unknown. Under Assumption 1, $\dot{x}_{0i}(\eta_i)$ is bounded with unknown upper bound, i.e., $|\dot{x}_{0i}(\eta_i)| \leq m_{1i}$ for $m_{1i} > 0$. By Lemma 1,

$$z_{1i}\dot{x}_{0i}(\eta_i) \leq |z_{1i}|m_{1i} \leq m_{1i}\upsilon_3(t) + \frac{m_{1i}z_{1i}^2}{\sqrt{\upsilon_3^2(t)+z_{1i}^2}}.$$

Then we can obtain,

$$
\begin{aligned}
\dot{V}_{1i} \leq\ & z_{1i}z_{2i} - p_{1i}z_{1i}^2 - \frac{\hat{m}_{1i}z_{1i}^2}{\sqrt{\upsilon_3^2(t)+z_{1i}^2}} + m_{1i}\upsilon_3(t)\\
&+ \frac{m_{1i}z_{1i}^2}{\sqrt{\upsilon_3^2(t)+z_{1i}^2}} - \frac{1}{\gamma_{1i}}\bar{m}_{1i}\dot{\hat{m}}_{1i}\\
\leq\ & z_{1i}z_{2i} - p_{1i}z_{1i}^2 + m_{1i}\upsilon_3(t)\\
&+ \bar{m}_{1i}\left(-\frac{1}{\gamma_{1i}}\dot{\hat{m}}_{1i} + \frac{z_{1i}^2}{\sqrt{\upsilon_3^2(t)+z_{1i}^2}}\right)\\
\leq\ & z_{1i}z_{2i} - p_{1i}z_{1i}^2 + m_{1i}\upsilon_3(t) + \frac{1}{4}\upsilon_4(t)m_{1i}^2
\end{aligned}
$$

Define for $j = 2,\ldots,n-1,\ i = 1,\ldots,N,$

$$\bar{V}_{ji} = \frac{1}{2}z_{ji}^2 + \frac{1}{2\gamma_{ji}}\bar{m}_{ji}^2,\quad V_{ji} = V_{j-1,i} + \bar{V}_{ji}. \tag{19}$$

Now we claim that

$$
\begin{aligned}
\dot{V}_{j-1,i} \leq\ & z_{j-1,i}z_{ji} - \sum_{k=1}^{j-1}p_{ki}z_{ki}^2\\
&+ \upsilon_3(t)\sum_{k=1}^{j-1}m_{ki} + \frac{1}{4}\upsilon_4(t)\sum_{k=1}^{j-1}m_{ki}^2
\end{aligned}
\tag{20}
$$

To verify (20), in the $j$-th step, $j = 2,\ldots,n-1$, note that $\alpha_{j-1,i} = \alpha_{j-1,i}(\bar{s}_{j-1,i}, \hat{m}_{1i},\ldots,\hat{m}_{j-1,i},\eta_i,\upsilon_3(t),\upsilon_4(t))$ with $\bar{s}_{j-1,i} = [\,s_{1i}\quad\ldots\quad s_{j-1,i}\,]^T$. Then,

$$
\begin{aligned}
\dot{z}_{ji} &= \dot{s}_{ji} - \dot{\alpha}_{j-1,i}\\
&= s_{j+1,i} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial s_{ki}} s_{k+1,i} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial \hat{m}_{ki}} \dot{\hat{m}}_{ki}\\
&\quad - \frac{\partial \alpha_{j-1,i}}{\partial \eta_i} \dot{\eta}_i - \frac{\partial \alpha_{j-1,i}}{\partial \upsilon_3(t)} \dot{\upsilon}_3(t) - \frac{\partial \alpha_{j-1,i}}{\partial \upsilon_4(t)} \dot{\upsilon}_4(t)\\
&= z_{j+1,i} + \alpha_{ji} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial s_{ki}} s_{k+1,i}\\
&\quad - \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial \hat{m}_{ki}} \dot{\hat{m}}_{ki} - \frac{\partial \alpha_{j-1,i}}{\partial \eta_i} \dot{\eta}_i\\
&\quad - \frac{\partial \alpha_{j-1,i}}{\partial \upsilon_3(t)} \dot{\upsilon}_3(t) - \frac{\partial \alpha_{j-1,i}}{\partial \upsilon_4(t)} \dot{\upsilon}_4(t)
\end{aligned} \tag{21}
$$

Since $\dot{\eta}_i, i = 1, \ldots, N$, is bounded for $t \geq 0$, there exists some constant $\eta_{Mi} > 0$ such that $||\dot{\eta}_i|| \leq \eta_{Mi}$. From (5), $|\dot{\upsilon}_3(t)| \leq \underline{\upsilon}_3$ and $|\dot{\upsilon}_4(t)| \leq \underline{\upsilon}_4$. Let

$$
\begin{aligned}
m_{2i} &= \max\{\underline{\upsilon}_3, \eta_{Mi}\},\\
m_{ji} &= \max\{\underline{\upsilon}_3, \underline{\upsilon}_4, \eta_{Mi}\},\ j = 3, \ldots, n-1.
\end{aligned} \tag{22}
$$

From the definition of $l_{ji}$ in (16), we can obtain that, along the trajectory of (21), the time derivative of $\bar{V}_{ji}$ satisfies,

$$
\begin{aligned}
\dot{\bar{V}}_{ji} &= z_{ji} z_{j+1,i} + z_{ji} \alpha_{ji}\\
&\quad - \frac{1}{\gamma_{ji}} \bar{m}_{ji} \dot{\hat{m}}_{ji} - z_{ji} \Big( \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial s_{ki}} s_{k+1,i}\\
&\quad + \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial \hat{m}_{ki}} \dot{\hat{m}}_{ki} \Big)\\
&\quad - z_{ji} \Big( \frac{\partial \alpha_{j-1,i}}{\partial \eta_i} \dot{\eta}_i + \frac{\partial \alpha_{j-1,i}}{\partial \upsilon_3(t)} \dot{\upsilon}_3(t) + \frac{\partial \alpha_{j-1,i}}{\partial \upsilon_4(t)} \dot{\upsilon}_4(t) \Big)\\
&\leq z_{ji} z_{j+1,i} + z_{ji} \alpha_{ji}\\
&\quad - \frac{1}{\gamma_{ji}} \bar{m}_{ji} \dot{\hat{m}}_{ji} + |z_{ji}| l_{ji} m_{ji}\\
&\quad - z_{ji} \Big( \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial s_{ki}} s_{k+1,i} + \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial \hat{m}_{ki}} \dot{\hat{m}}_{ki} \Big)
\end{aligned}
$$

By Lemma 1,

$$
|z_{ji}| l_{ji} m_{ji} \leq m_{ji} \upsilon_3(t) + \frac{m_{ji} l_{ji}^2 z_{ji}^2}{\sqrt{\upsilon_3^2(t) + l_{ji}^2 z_{ji}^2}}.
$$

Then,

$$
\begin{aligned}
\dot{\bar{V}}_{ji} &\leq z_{ji} z_{j+1,i} - p_{ji} z_{ji}^2 - z_{ji} z_{j-1,i} + m_{ji} \upsilon_3(t)\\
&\quad + \bar{m}_{ji} \Big( -\frac{1}{\gamma_{ji}} \dot{\hat{m}}_{ji} + \frac{l_{ji}^2 z_{ji}^2}{\sqrt{\upsilon_3^2(t) + l_{ji}^2 z_{ji}^2}} \Big)\\
&= z_{ji} z_{j+1,i} - p_{ji} z_{ji}^2 - z_{ji} z_{j-1,i} + m_{ji} \upsilon_3(t)\\
&\quad + \frac{1}{4} \upsilon_4(t) m_{ji}^2,
\end{aligned}
$$

and from (20),

$$
\begin{aligned}
\dot{V}_{ji} &\leq z_{ji} z_{j+1,i} - p_{ji} z_{ji}^2 - z_{ji} z_{j-1,i}\\
&\quad + m_{ji} \upsilon_3(t) + \frac{1}{4} \upsilon_4(t) m_{ji}^2\\
&\quad + z_{j-1,i} z_{ji} - \sum_{k=1}^{j-1} p_{ki} z_{ki}^2\\
&\quad + \upsilon_3(t) \sum_{k=1}^{j-1} m_{ki} + \frac{1}{4} \upsilon_4(t) \sum_{k=1}^{j-1} m_{ki}^2\\
&= z_{ji} z_{j+1,i} - \sum_{k=1}^{j} p_{ki} z_{ki}^2 + \upsilon_3(t) \sum_{k=1}^{j} m_{ki}\\
&\quad + \frac{1}{4} \upsilon_4(t) \sum_{k=1}^{j} m_{ki}^2,
\end{aligned}
$$

which verifies the claim.

In the $n$-th step, define $g_{0i} = \min\{\prod_{j=1}^{n} \underline{g}_{ji}, \underline{\pi}_i \prod_{j=1}^{n} \underline{g}_{ji}\}$. It is noted that under Assumption 3, from (15),

$$
g_i(\bar{x}_{ni}, \kappa_i, w_i) \geq \underline{g}_{ni} \prod_{k=1}^{n-1} \underline{g}_{ki} \geq g_{0i} > 0 \tag{23}
$$

Define

$$
\bar{V}_{ni} = \frac{1}{2 g_{0i}} z_{ni}^2 + \frac{1}{2 \gamma_{ni}} \bar{\theta}_i^T \bar{\theta}_i, \quad V_{ni} = \bar{V}_{ni} + V_{n-1,i} \tag{24}
$$

where $\bar{\theta}_i = \theta_i - \hat{\theta}_i$ for $\hat{\theta}_i \in \mathbb{R}^3$ and some constant vector $\theta_i$. Note that $\alpha_{n-1,i} = \alpha_{n-1,i}(\bar{s}_{n-1,i}, \hat{m}_{1i}, \ldots, \hat{m}_{n-1,i}, \eta_i, \upsilon_3(t), \upsilon_4(t))$. Then,

$$\begin{aligned}\dot{z}_{ni} =& \dot{s}_{ni} - \dot{\alpha}_{n-1,i}\\ =& \bar{f}_i(\bar{x}_{ni}, w_i) + g_i(\bar{x}_{ni}, \kappa_i, w_i)\kappa_i\\ &- \Big(\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1,i}}{\partial s_{ki}} s_{k+1,i} + \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1,i}}{\partial \hat{m}_{ki}} \dot{\hat{m}}_{ki}\Big)\\ &- \Big(\frac{\partial \alpha_{n-1,i}}{\partial \eta_i}\dot{\eta}_i + \frac{\partial \alpha_{n-1,i}}{\partial \upsilon_3(t)}\dot{\upsilon}_3(t) + \frac{\partial \alpha_{n-1,i}}{\partial \upsilon_4(t)}\dot{\upsilon}_4(t)\Big)\end{aligned} \tag{25}$$

Then along the trajectory of (25), the time derivative of $\bar{V}_{ni}$ is given by,

$$\begin{aligned}\dot{\bar{V}}_{ni} =& \frac{1}{g_{0i}} z_{ni}\bar{f}_i(\bar{x}_{ni}, w_i) + \frac{g_i(\bar{x}_{ni}, \kappa_i, w_i)}{g_{0i}} z_{ni}\kappa_i\\ &- \frac{1}{g_{0i}} z_{ni}\Big(\sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial s_{ki}} s_{k+1,i} + \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1,i}}{\partial \hat{m}_{ki}} \dot{\hat{m}}_{ki}\Big)\\ &- \frac{1}{g_{0i}} z_{ni}\Big(\frac{\partial \alpha_{n-1,i}}{\partial \eta_i}\dot{\eta}_i + \frac{\partial \alpha_{n-1,i}}{\partial \upsilon_3(t)}\dot{\upsilon}_3(t)\\ &+ \frac{\partial \alpha_{n-1,i}}{\partial \upsilon_4(t)}\dot{\upsilon}_4(t)\Big) - \frac{1}{\gamma_{ni}}\bar{\theta}_i^T \dot{\hat{\theta}}_i\end{aligned}$$

From (15), under Assumption 3,

$$\begin{aligned}|\bar{f}_i(\bar{x}_{ni}, w_i)| \le& \Big|\sum_{k=1}^{n-1} \frac{\partial b_{ni}(\bar{x}_{ki})}{\partial x_{ki}} f_{ki}(\bar{x}_{k+1,i})\Big|\\ &+ \Big|\frac{\partial b_{ni}(\bar{x}_{ni})}{\partial x_{ni}} f_{ni}(\bar{x}_{ni}, 0, w_i)\Big|\end{aligned}$$

Note that $w_i(t)$ is bounded. There exists a positive constant $\hat{w}_i$ such that $||w_i(t)|| \le \hat{w}_i$. Define $\Sigma_i = \{w_i(t) : ||w_i(t)|| \le \hat{w}_i\}$. Since $f_{ni}(\bar{x}_{ni}, 0, w_i)$ is sufficiently smooth with $f_{ni}(0, 0, w_i) = 0$ for all $w_i(t) \in \Sigma_i$, by Lemma 7.8 in [15], there exist smooth functions $\bar{\varrho}_{ji} : \mathbb{R} \to \mathbb{R}$, $j = 1, \ldots, n$, satisfying $\bar{\varrho}_{ji}(0) = 0$, such that

$$|f_{ni}(\bar{x}_{ni}, 0, w_i)| \le \sum_{j=1}^{n} \bar{c}_{ji}\bar{\varrho}_{ji}(x_{ji}) \le \bar{c}_i\bar{\varrho}_i(\bar{x}_{ni})$$

where $\bar{c}_i = \max_{j=1,\ldots,n}\{\bar{c}_{ji}\}$ and $\bar{\varrho}_i(\bar{x}_{ni}) = \sum_{j=1}^{n} \bar{\varrho}_{ji}(x_{ji})$. As a result,

$$\begin{aligned}|\bar{f}_i(\bar{x}_{ni}, w_i)| \le& \Big|\sum_{k=1}^{n-1} \frac{\partial b_{ni}(\bar{x}_{ki})}{\partial x_{ki}} f_{ki}(\bar{x}_{k+1,i})\Big|\\ &+ \Big|\frac{\partial b_{ni}(\bar{x}_{ni})}{\partial x_{ni}}\Big|\bar{c}_i\bar{\varrho}_i(\bar{x}_{ni})\\ \le& \tilde{c}_{ni}\varrho_i(\bar{x}_{ni})\end{aligned} \tag{26}$$

where $\tilde{c}_{ni} = \max\{1, \bar{c}_i\}$, and $\varrho_i(\bar{x}_{ni}) = |\sum_{k=1}^{n-1} \frac{\partial b_{ni}(\bar{x}_{ki})}{\partial x_{ki}} f_{ki}(\bar{x}_{k+1,i})| + \bar{\varrho}_i(\bar{x}_{ni})|\frac{\partial b_{ni}(\bar{x}_{ni})}{\partial x_{ni}}|$. Thus,

$$\begin{aligned}\frac{1}{g_{0i}} z_{ni}\bar{f}_i(\bar{x}_{ni}, w_i) \le& \frac{\tilde{c}_{ni}}{g_{0i}}|z_{ni}|\varrho_i(\bar{x}_{ni})\\ \le& c_{ni}\upsilon_3(t) + \frac{c_{ni}\varrho_i^2(\bar{x}_{ni})z_{ni}^2}{\sqrt{\upsilon_3^2(t) + \varrho_i^2(\bar{x}_{ni})z_{ni}^2}}\end{aligned}$$

where $c_{ni} = \frac{\tilde{c}_{ni}}{g_{0i}}$. Let $u_i = u_{1i} + u_{2i}$ with $u_{1i} = -\frac{1}{\underline{\pi}_i}\bar{\phi}_i$ sign $(z_{ni})$ and $u_{2i} = -\frac{\alpha_{ni}^2 z_{ni}}{\sqrt{\upsilon_3^2(t) + \alpha_{ni}^2 z_{ni}^2}}$. From (23),

$$\begin{aligned}&\frac{g_i(\bar{x}_{ni}, \kappa_i, w_i)}{g_{0i}} z_{ni}(\pi_i(t)u_{1i} + \phi_i(t))\\ &\le -\frac{g_i(\bar{x}_{ni}, \kappa_i, w_i)\pi_i(t)}{g_{0i}\underline{\pi}_i} z_{ni}\bar{\phi}_i \text{ sign } (z_{ni})\\ &\quad + \frac{g_i(\bar{x}_{ni}, \kappa_i, w_i)}{g_{0i}}|z_{ni}|\bar{\phi}_i\\ &= \frac{g_i(\bar{x}_{ni}, \kappa_i, w_i)|z_{ni}|\bar{\phi}_i}{g_{0i}}(1 - \frac{\pi_i(t)}{\underline{\pi}_i}) \le 0\\ &\frac{g_i(\bar{x}_{ni}, \kappa_i, w_i)\pi_i(t)}{g_{0i}} z_{ni}u_{2i} - z_{ni}\alpha_{ni}\\ &= -\frac{g_i(\bar{x}_{ni}, \kappa_i, w_i)\pi_i(t)}{g_{0i}} \frac{\alpha_{ni}^2 z_{ni}^2}{\sqrt{\upsilon_3^2(t) + \alpha_{ni}^2 z_{ni}^2}}\\ &\quad + \frac{\alpha_{ni}^2 z_{ni}^2}{\sqrt{\upsilon_3^2(t) + \alpha_{ni}^2 z_{ni}^2}} + v_3(t)\\ &= \Big(1 - \frac{g_i(\bar{x}_{ni}, \kappa_i, w_i)\pi_i(t)}{g_{0i}}\Big)\frac{\alpha_{ni}^2 z_{ni}^2}{\sqrt{\upsilon_3^2(t) + \alpha_{ni}^2 z_{ni}^2}}\\ &\quad + v_3(t) \le v_3(t)\\ &\bar{\theta}_i^T\hat{\theta}_i = \bar{\theta}_i^T\theta_i - \bar{\theta}_i^T\bar{\theta}_i \le ||\theta_i||^2\\ &\quad + \frac{1}{4}||\theta_i||^2 - ||\bar{\theta}_i||^2 \le \frac{1}{4}||\theta_i||^2\end{aligned} \tag{27}$$

Let $\theta_i = \begin{bmatrix} c_{ni} & -\frac{1}{g_{0i}} & \frac{m_{ni}}{g_{0i}} \end{bmatrix}$. Then from (27),

$$
\begin{aligned}
\dot{V}_{ni} \le\; & c_{ni}\upsilon_3(t) + \frac{c_{ni}\varrho_i^2(\bar{x}_{ni})z_{ni}^2}{\sqrt{\upsilon_4^2(t) + \varrho_i^2(\bar{x}_{ni})z_{ni}^2}} \\
& - \frac{1}{\gamma_{ni}}\bar{\theta}_i^T\dot{\hat{\theta}}_i + \frac{g_i(\bar{x}_{ni},\kappa_i,w_i)}{g_{0i}}z_{ni}u_i \\
& + \frac{m_{ni}}{g_{0i}}l_{ni}|z_{ni}| \\
& - \frac{1}{g_{0i}}z_{ni}\Big(\sum_{k=1}^{j-1}\frac{\partial\alpha_{j-1,i}}{\partial s_{ki}}s_{k+1,i} + \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1,i}}{\partial\hat{m}_{ki}}\dot{\hat{m}}_{ki}\Big) \\
=\; & c_{ni}\upsilon_3(t) + z_{ni}\theta_i^T\xi_i - \frac{1}{\gamma_{ni}}\bar{\theta}_i^T\dot{\hat{\theta}}_i \\
& + \frac{g_i(\bar{x}_{ni},\kappa_i,w_i)}{g_{0i}}z_{ni}(\pi_i(t)u_{1i} + \phi_i(t)) \\
& + \frac{g_i(\bar{x}_{ni},\kappa_i,w_i)}{g_{0i}}z_{ni}\pi_i(t)u_{2i} - z_{ni}\alpha_{ni} + z_{ni}\alpha_{ni} \\
=\; & c_{ni}\upsilon_3(t) + z_{ni}\bar{\theta}_i^T\xi_i \\
& - \frac{1}{\gamma_{ni}}\bar{\theta}_i^T\dot{\hat{\theta}}_i - z_{ni}z_{n-1,i} - p_{ni}z_{ni}^2 + \upsilon_3(t) \\
\le\; & (c_{ni}+1)\upsilon_3(t) + \frac{1}{4}v_4(t)||\theta_i||^2 - z_{ni}z_{n-1,i} - p_{ni}z_{ni}^2
\end{aligned}
$$

As a result,

$$
\begin{aligned}
\dot{V}_{ni} \le\; & (c_{ni}+1)\upsilon_3(t) + \frac{1}{4}v_4(t)||\theta_i||^2 \\
& - z_{ni}z_{n-1,i} - p_{ni}z_{ni}^2 \\
& + z_{n-1,i}z_{ni} - \sum_{k=1}^{n-1}p_{ki}z_{ki}^2 + \upsilon_3(t)\sum_{k=1}^{n-1}m_{ki} \\
& + \frac{1}{4}\upsilon_4(t)\sum_{k=1}^{n-1}m_{ki}^2 \\
=\; & -\sum_{k=1}^{n}p_{ki}z_{ki}^2 + V_{Mi}(\upsilon_3(t) + \upsilon_4(t))
\end{aligned} \tag{28}
$$

where $V_{Mi} = \max\{c_{ni} + 1 + \sum_{k=1}^{n-1}m_{ki},\ \frac{1}{4}(||\theta_i||^2 + \sum_{k=1}^{n-1}m_{ki}^2)\}$. Then,

$$
\begin{aligned}
V_{ni}(t) \le\; & V_{ni}(0) + V_{Mi}\int_0^t(\upsilon_3(\tau) + \upsilon_4(\tau))d\tau \\
\le\; & V_{ni}(0) + V_{Mi}(\bar{\upsilon}_3 + \bar{\upsilon}_4)
\end{aligned} \tag{29}
$$

which implies $\bar{z}_{ji}, j = 1,\ldots,n$, $\bar{m}_{ji}, j = 1,\ldots,n-1$, $\bar{\theta}_i$, $i = 1,\ldots,N$ are bounded for $t \ge 0$, and thus $\hat{m}_{ji}$, $\dot{\hat{m}}_{ji}$, $\hat{\theta}_i$, $j = 1,\ldots,n-1$, are bounded.

It remains to verify (4). For this purpose, we need to show that $\dot{z}_{ji}$, $j = 1,\ldots,n$, $i = 1,\ldots,N$, are bounded for $t \ge 0$. It also includes n steps.

In the 1-st step, since $z_{1i}$ and $x_{0i}(\eta_i)$ are bounded, $s_{1i} = z_{1i} + x_{0i}(\eta_i)$ and $\alpha_{1i}$ are bounded, implying $l_{2i}$ is bounded. From (18), together with the fact that $\dot{x}_{0i}(\eta_i)$ is bounded, $\dot{z}_{1i}$ is bounded.

In the 2-nd step, $s_{2i} = z_{2i} + \alpha_{1i}$ is bounded since $z_{2i}$ and $\alpha_{1i}$ are bounded. Since $\alpha_{1i}$ is sufficiently smooth, $\dot{\alpha}_{1i}$ is also bounded since $s_{1i}, s_{2i}, \dot{\hat{m}}_{1i}, \dot{\eta}_i, \dot{\upsilon}_3(t), \dot{\upsilon}_4(t)$ are bounded, and thus from (16), $\alpha_{2i}$ and $l_{3i}$ are bounded. From (21), $\dot{z}_{2i}$ is bounded since $z_{3i}$, $\alpha_{2i}$, and $\dot{\alpha}_{1i}$ are bounded.

Repeat the procedure, we can conclude that $s_{ji}$, $\dot{\alpha}_{j-1,i}$, $\alpha_{ji}$, $l_{j+1,i}$ and $\dot{z}_{ji}$, $j = 2,\ldots,n-1$, are bounded.

In the $n$-th step, it can be verified that $s_{ni} = z_{ni} + \alpha_{n-1,i}$ is bounded. Since $s_{ji}$, $j = 1,\ldots,n$, $i = 1,\ldots,N$, is bounded, from (13) and the fact that functions $f_{ji}(\cdot)$ are sufficiently smooth, $\bar{x}_{ni}$ is bounded, implying $\varrho_i(\bar{x}_{ni})$ and $\bar{f}_{ni}(\bar{x}_{ni},\kappa_i,w_i)$ are bounded. And thus from (16), $\xi_i$ is bounded, which leads to the boundedness of $\alpha_{ni}$ and $u_i$. As a result, $\dot{z}_{ni}$ is bounded for $t \ge 0$ from (25).

From the boundedness of $\dot{z}_{1i},\ldots,\dot{z}_{ni}$ and (5), we can conclude $\ddot{V}_{ni}$ is bounded for $t \ge 0$, implying $\dot{V}_{ni}$ is uniformly continuous for $t \in [0,\infty)$, and from (28), $\lim_{t\to\infty}\dot{V}_{ni}(\tau)d\tau$ exists and is finite. Then by Barbalat's lemma in [26], we can obtain $\lim_{t\to\infty}z_{ji} = 0$, which implies $\lim_{t\to\infty}e_i = z_{1i} = 0$. □

***Remark 4*** Dynamic gains $\hat{k_i}$, $\hat{\vartheta}_i$ in (6b), (6c), and $\hat{m}_{ji}$, $\hat{\theta}_i$ in (16) are designed to generate enough force to dominate large and unknown actuator fault, time-varying uncertain parameters and the external disturbances. If we relax Assumptions 1 and 2 by allowing the upper bounds of $u_0$, $\eta_0$ and $H$ are all known as in [13, 18], (6) can be reduced into

$$
\dot{\eta}_i = f_0(\eta_i) - \frac{\vartheta\eta_{vi}}{\sqrt{\upsilon_1^2(t) + ||\eta_{vi}||^2}} - k_i\rho_i(\eta_{vi})\eta_{vi} - c_{0i}\eta_{vi} \tag{30}
$$

and our control design (16) based on (30) is applicable to a directed graph.

## 5 Example

In this section, we consider a multi-agent system composed of 6 follower systems given by (32) and a non-autonomous system (31), which generates the reference signal to track and can be adjusted at any time by changing the input $u_0(t)$. The leader system is in the following form,

$$\dot{\eta}_0 = \begin{bmatrix} \eta_{20} \\ -\eta_{10} + (1-\eta_{10}^2)\eta_{20} \end{bmatrix} + u_0(t) \tag{31}$$

where $\eta_0 = [\eta_{10} \quad \eta_{20}]^T$. And as in [2], each follower system is modeled by

$$\begin{aligned} \dot{x}_{1i} &= x_{1i} + x_{2i} + \frac{1}{5}x_{2i}^3 \\ \dot{x}_{2i} &= \frac{1}{2}x_{1i}x_{2i} + \kappa_i + \frac{1}{4}\kappa_i^{\frac{5}{3}}, \quad i = 1, \ldots, N \end{aligned} \tag{32}$$

where $\kappa_i(u_i, t) = \pi_i(t)u_i + \phi_i(t)$ with

$$\pi_i(t) = \begin{cases} 1, & t < 15; \\ 0.5, & \text{otherwise} . \end{cases} \qquad \phi_i(t) = 0.1\sin(t)$$

Let $s_{1i} = x_{1i}$ and $s_{2i} = x_{1i} + x_{2i} + \frac{1}{5}x_{2i}^3$. Then the non-affine system (32) can be transformed into (14) where

$$\bar{f}_i(\bar{x}_i, w_i) = x_{1i} + x_{2i} + \frac{1}{5}x_{2i}^3 + \frac{1}{2}(1 + \frac{3}{5}x_{2i}^2)x_{1i}x_{2i}$$

$$g_i(\bar{x}_i, \kappa_i, w_i) = (1 + \frac{3}{5}x_{2i}^2)(1 + \frac{5}{12}(\lambda_i\kappa_i)^{\frac{2}{3}}) \geq 1$$

It can be verified that Assumption 3 is satisfied.

The communication network of the multi-agent systems (31) and (32) is described by the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, shown in Fig. 1, which satisfies Assumption 2.

We first provide the simulation results for the nonlinear observer. For the purpose of simulation, define $u_0(t) = 0$ for $t \in [0, t_1)$ and for $t \geq t_1$, $u_0(t) = \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}$ with $t_1 = 10$.

By Lemma 1, the observer for the non-autonomous leader system can be designed by (6) where

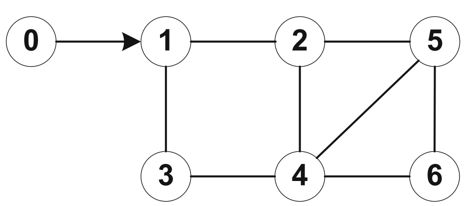


**Fig. 1** The communication network $\mathcal{G}$

$$\begin{aligned} &\eta_i = [\eta_{1i} \quad \eta_{2i}]^T, \quad c_{01} = 2, \quad c_{1i} = c_{2i} = 1 \\ &\rho_i(\eta_{vi}) = 1 + ||\eta_{vi}||^2, \quad \upsilon_1(t) = \upsilon_2(t) = 0.5e^{-0.1t} \end{aligned} \tag{33}$$

For the purpose of simulation, we provide the initial condition for (6) as follows,

$$\begin{aligned} &\eta_1(0) = [2 \quad 3]^T, \ \hat{\vartheta}_1 = 1, \ \hat{k}_1 = 1, \\ &\eta_2(0) = [-2 \quad -3]^T, \ \hat{\vartheta}_2 = 2, \ \hat{k}_3 = 3 \\ &\eta_3(0) = [0 \quad -1]^T, \ \hat{\vartheta}_3 = 3, \ \hat{k}_3 = 1, \\ &\eta_4(0) = [3 \quad 0]^T, \ \hat{\vartheta}_4 = 4, \ \hat{k}_4 = 2 \\ &\eta_5(0) = [-1 \quad -2]^T, \ \hat{\vartheta}_5 = 7, \ \hat{k}_5 = 2, \\ &\eta_6(0) = [2 \quad 0]^T, \ \hat{\vartheta}_6 = 5, \ \hat{k}_6 = 1 \end{aligned} \tag{34}$$

The performance of the observer can be found in Figs. 2, 3. It can be observed that the estimation errors asymptotically converge to zero even though the course is adjusted at $t = 10$. Figure 3 displays the adaptive update laws $\hat{\vartheta}_i(t)$, $\hat{k}_i(t)$ for (6), $i = 1, \ldots, 6$.

If the leader system is an autonomous one as [24], i.e., $u_0(t) \triangleq 0$ for all $t \geq 0$, by allowing the upper bounds of $\eta_0(t)$ and $H$ known to all agents, the observer (6) is reduced to be the following form

$$\dot{\eta}_i = f_0(\eta_i) - k\rho_i(\eta_{vi})\eta_{vi} \tag{35}$$

for a sufficiently large $k > 0$. The performance of (35) can be found in Fig. 4. Note that (35) can be used to estimate the state of the leader system as in [17, 18],

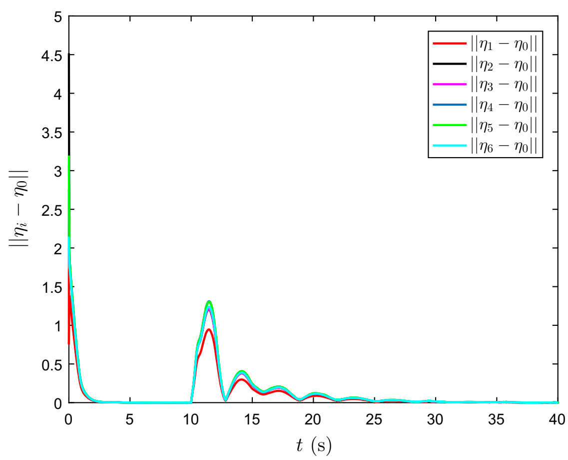


**Fig. 2** Profile of the estimation errors $\eta_i - \eta_0$ for (6), $i = 1, \ldots, 6$

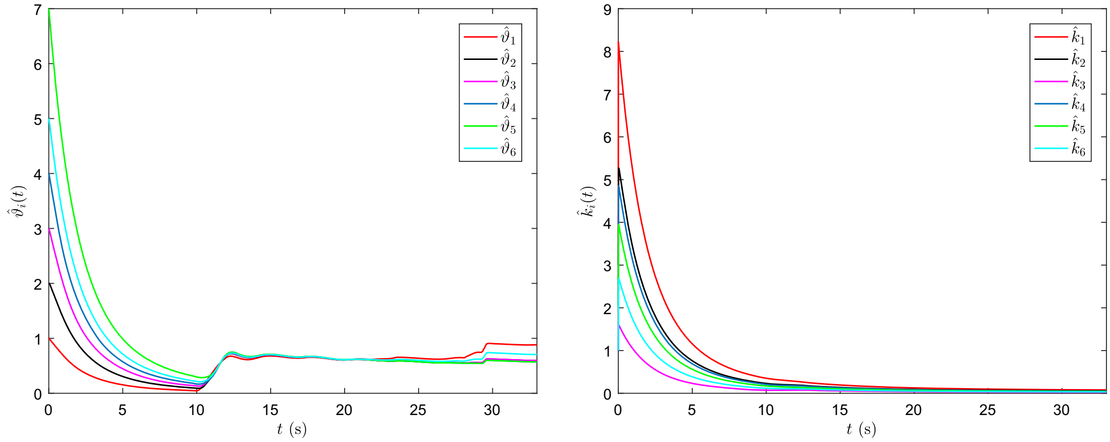


**Fig. 3** Profile of adaptive update laws $\hat{\vartheta}_i(t)$, $\hat{k_i}(t)$ for (6), $i = 1, \ldots, 6$

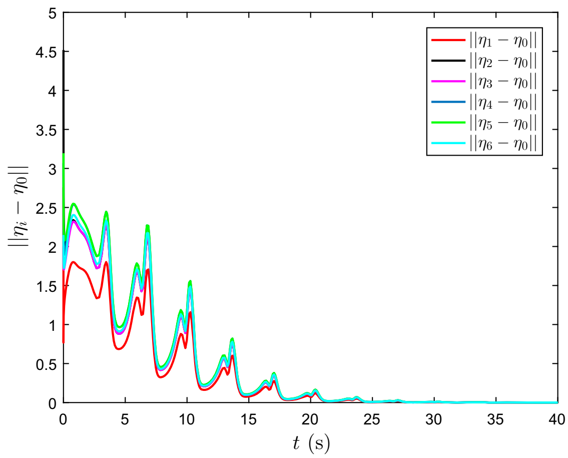


**Fig. 4** Profile of the estimation errors $\eta_i - \eta_0$ for (35), $i = 1, \ldots, 6$

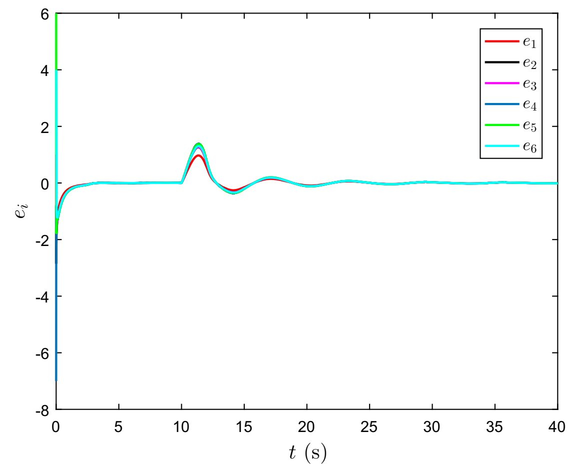


**Fig. 5** Profile of the tracking errors $e_i$, $i = 1, \ldots, 6$

but due to the existence of $u_0$ and the unavailable information of $\eta_0$ and $H$ for all agents, the existing observers, including (35), are inadmissible for estimating the state of (31).

Based on the observer (6), design the distributed control law (16) where

$$p_{1i} = p_{2i} = 10, \;\gamma_{1i} = \gamma_{2i} = 2, \;\upsilon_3(t) = \upsilon_4(t) = 0.5e^{-0.1t}$$

$$\varrho_i = |x_{1i} + x_{2i} + \frac{1}{5}x_{2i}^3| + \frac{1}{2}(1 + \frac{3}{5}x_{2i}^2)|x_{1i}x_{2i}|.$$

The initial values for $x_i$, $\hat{m}_{1i}$ and $\hat{\theta}_i$ are given as follows,

$$\begin{aligned}
&x_1 = [\,0 \quad 1\,]^T, \;\hat{m}_{11} = 1, \;\hat{\theta}_1 = 4, \;x_2 = [\,1 \quad 1\,]^T,\\
&\hat{m}_{12} = 2, \;\hat{\theta}_2 = 3\\
&x_3 = [\,3 \quad 0\,]^T, \;\hat{m}_{13} = 0.1,\\
&\hat{\theta}_3 = 1, \;x_4 = [\,-5 \quad 2\,]^T,\\
&\hat{m}_{14} = 4, \;\hat{\theta}_4 = 6\\
&x_5 = [\,8 \quad -1\,]^T,\\
&\hat{m}_{15} = 2, \;\hat{\theta}_5 = 0.9, \;x_6 = [\,6 \quad 0\,]^T,\\
&\hat{m}_{16} = 3, \;\hat{\theta}_6 = 2
\end{aligned} \tag{36}$$

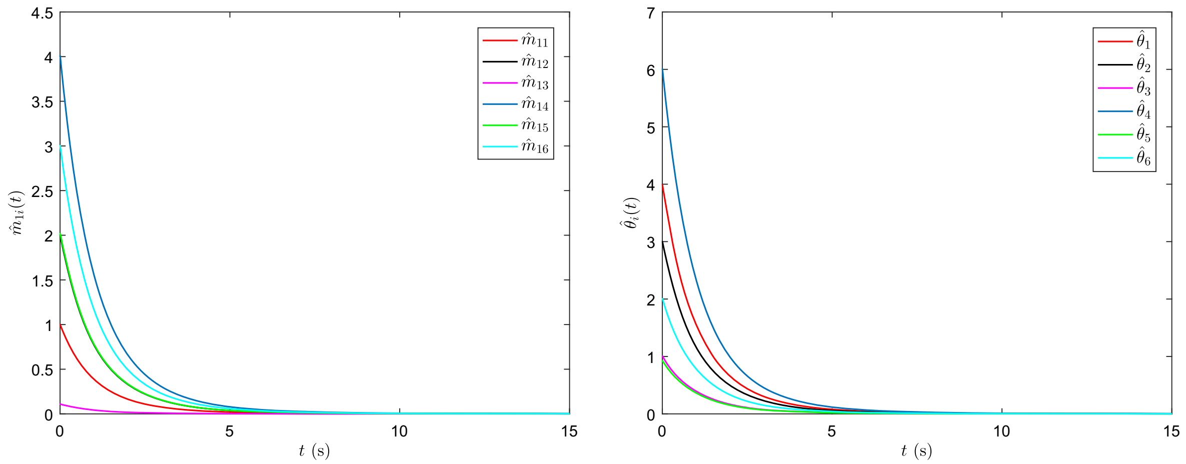


**Fig. 6** Profile of adaptive update laws $\hat{m}_{1i}(t)$ and $\hat{\theta}_i(t)$, $i = 1, \ldots, 6$

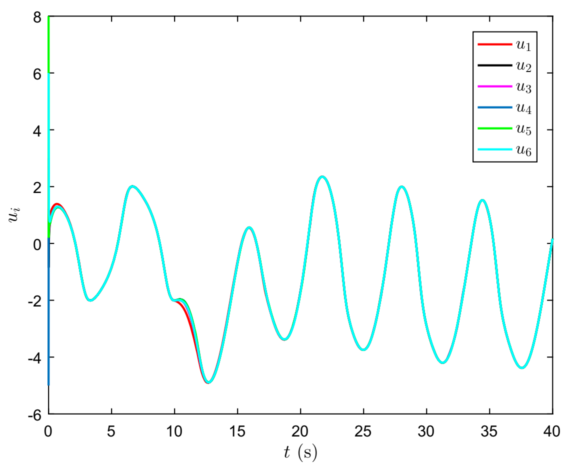


**Fig. 7** Profile of the tracking law $u_i$, $i = 1, \ldots, 6$

Figure 5 displays the tracking error for all agents, which asymptotically converge to zero. Figure 6 shows the adaptive update laws $\hat{m}_{1i}(t)$ and $\hat{\theta}_i(t)$ in (16), $i = 1, \ldots, 6$, and the profile of the tracking law $u_i$, $i = 1, \ldots, 6$, is provided in Fig. 7.

## 6 Conclusion

This paper has proposed a global robust distributed control law for networked nonlinear non-affine systems with parametric uncertainty and external disturbances, which are allowed to be arbitrarily large and unknown. Our control law can not only tolerate a general form of the input–output characteristics including but not limited to actuator fault, hysteresis, etc., but also achieve the asymptotic tracking of the time-varying nonlinear reference signals. The design consists of a dynamic nonlinear observer, which can accurately estimate the state of the non-autonomous nonlinear leader under the condition that the input and its upper bound is unavailable to all agents.

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**Data availability** The manuscript has no associated data.

**Declarations**

**Conflict of interest** The authors have no relevant financial or non-financial interests to disclose.

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