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REVIEW

A review on nonlinear energy sinks: designs, analysis and applications of impact and rotary types

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Abstract Dynamical and structural systems are susceptible to sudden excitations and loadings such as wind gusts, blasts, earthquakes, and others which may cause destructive vibration amplitudes and lead to catastrophic impact on human lives and economy. Therefore, various vibration absorbers of linear and nonlinear coupling dynamics have been widely studied in plenty of publications where some have been applied in real-world practical applications. Firstly, the tuned-mass-damper (TMD), the first well-known linear vibration absorber that has been well-studied in the literature and applied with various structural and dynamical systems, is discussed. The linear vibration absorbers such as TMDs are widely used in real-life small- and large-scale structures due to their robust performance in vibration suppression of the low natural frequency structural modes. However, the TMD performs efficiently at narrowband frequency range where its performance is deteriorated by any changes in the frequency content in the structure and the TMD itself. Therefore, the targeted-energy-transfer mechanism which is found to be achieved by nonlinear energy sinks (NESs) has ignited the interest in passive nonlinear vibration suppression. Unlike TMDs, the NESs are dynamical vibration absorbers that achieve vibration suppression for wide range of frequency-energy levels. Given the very rapid growth in this field and the extensive research studies supporting the robustness of the NESs, this paper presents the different types of NESs and their applications with main emphasis on the rotary-based and impact-based NESs since they are of high impact in the literature due to their strong nonlinear dynamical behavior and robust targeted energy transfer.

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1 Introduction

Variety of fields involve absorption of unwanted energy introduced to small dynamical systems and large-scale structures. These include shock and seismic mitigation in structures and bridges, aircraft flutter suppression, passive energy harvesting, spacecraft vibration reduction, chatter instability isolation, oil and gas drilling instability control, and others. In particular, a highly reliable foundation of civil infrastructures is susceptible to sudden seismic excitations and impulsive loadings such as wind gusts, blasts, collisions, earthquakes, typhoons, extreme waves, heavy traffic and highway loading that put them in danger of damage and destruction. For example, the famous Tacoma Narrows bridge collapsed after four months of its opening on July 1, 1940, as a result of aeroelastic flutter. Although several strategies such as tie-down cables and inclined cable stays were implemented in the design and construction of Tacoma Narrows bridge to reduce its vibration, the bridge could not withstand the unstable oscillations caused by the wind [1–3]. Most recently, Polcevera viaduct, a cable-stayed bridge in Genoa, collapsed on August 14, 2018, killing 43 people and leaving hundreds homeless. Bridges failure including Polcevera viaduct has been thoroughly addressed in [4, 5] for analyzing the most likely scenarios that caused failure of bridges. According to [4], due to the accumulation of fatigue failure and corrosion under high cyclic heavy traffic loading, the collapse of the bridge was most probably started with failure in one of the bridge strands and gradually propagates to the entire system. Therefore, the structural integrity of the bridge has been deteriorated over time which finally caused the bridge to collapse.

Throughout the history, many other bridge collapses were reported. Although the failures may vary (caused by aging, poor construction or maintenance, overload, wind or natural disasters), a high number of fatalities was mostly accompanied with bridge collapses making it vital to maintain the integrity of such structures. Apart from that, buildings and towers as well as bridges and other infrastructure can be critically affected by natural disasters, specifically earthquakes. Earthquakes account for a high percentage of the total deaths due to natural disasters and an estimated total of 1,685,000 officially reported deaths during the twentieth century [6]. Although they cause annual average of economic losses of 18.7 billion USD representing 19% of losses from natural disasters [7], the costs are expected to increase dramatically as more property, that is indeed more valuable, will be affected. Moreover, it was found that the number of fatalities per earthquake is almost directly proportional to the number of collapsed buildings per earthquake [6]. In addition, they are not only destructive because of their high amplitude ground accelerations but also other factors like the frequency content, duration and direction can cause severe damage [8]. Therefore, the rapid suppression of vibrations induced into such structures by impulsive shock vibration or seismic excitations is becoming a high priority requirement of all structural designs to protect them from destruction and eventually save human lives as well as prevent economic losses.

To account for shock and seismic hazards, structures were traditionally designed to have sufficient strength capacity in addition to being allowed to undergo ductile response under loading conditions. Although the resulting yielding dissipates some of the introduced energy, inelastic behavior in civil structures is undesired because it causes residual plastic deformations that require expensive maintenance or replacement. At the same time, it is costly to design a civil structure that remains elastic under all loading conditions as an essentially heavier elastic design is more vulnerable to shock and seismic loadings [9]. Consequently, suppressing the high vibration amplitudes which are induced into structures by external sources is essential to save human and preventing economical losses. Such vibration control systems are categorized as semi-active, passive or hybrid systems.

Figure 1 shows schematic diagrams of passive and active structural control systems. The active control system uses measured excitation and structural response to generate a calculated control signal which is sent to the electrohydraulic or electromechanical actuators to provide the required control forces. One of the most common examples of such systems is the active mass damper system which is composed of a small auxiliary mass coupled with upper floors of the primary structure through an actuator. The recorded accelerometer and displacement measurements from the input excitation and key locations in the primary structure are continuously monitored by a controller which processes the measurements according to a predetermined control algorithm and sends the appropriate control signal to the actuator. The electrohydraulic or electromechanical actuator applies inertial control forces to the auxiliary mass and the primary structure to attenuate the structural response. Obviously, generating the control forces requires large and constant sources of power which is in the order of kilowatts for small-scale structures and megawatts for large-scale ones [10]. The first large-scale application of an active vibration control system in civil structures is the 11-story Kyobashi office building in Tokyo accomplished by Kajima Corporation in 1989 which incorporates an active structural control system having a

Table 1 NESs types and applications

Application	References	NES type	Contribution
Aeroelastic instabilities	Vakakis et al. [44]	Type I and type III	The capability of Type I NES to passively suppress aeroelastic instabilities was investigated. The performance was validated through a set of experimental analysis in wind tunnel tests. Later investigations can be found in [66, 162, 163]
	Vaurigaud et al. [166]	Type I	Analytical study based on complexification methods and multiple scales expansion to investigate the passive control of instability induced by flutter on a long space bridge model by using the concept of limiting phase trajectories
	Ebrahimzade et al. [161]	Type I & Type III	Linear and nonlinear vibration absorbers were compared for enhancing the stability properties and nonlinear behavior of an airfoil model subjected to quasi-steady aerodynamics flow
	Guo et al. [164]	Type I	The suppression of limit cycle oscillations was studied for an NES coupled to an airfoil model with control surface freeplay, which may significantly affect the amplitude of the oscillations obtained by applying the harmonic balance method
	Escudero et al. [167]	Flap-NES	An experimental testing has been presented in [167] and a numerical analysis in [168] for flutter instabilities suppression by employing a flap-NES with 2-DOF pitch–plunge airfoil model
Vortex-induced vibrations (VIV)	Tumkur et al. [169]	Type I	An NES coupled to a varying multiscale residual-based stabilized finite element model of a sprung rigid cylinder constrained to move perpendicularly was capable of reducing the amplitude of limit-cycle oscillations by as much as 75% without extensive optimization of its parameters. Further studies on the topic are discussed in [170, 171]
	Blanchard et al. [91]	Rotary	The dynamics of a sprung cylinder constrained to move in the cross-flow directions coupled to a rotary NES was investigated numerically and analytically using an asymptotic analysis to define the 1:1 resonance captures in the slow-invariant-manifold [91]. In [194], proper orthogonal decomposition is applied to further explain the TET mechanisms
Quenching chatter instability	Gourc et al. [176]	DSVI	Theoretical and experimental analyses of using of NESs to control chatter instability in turning processes were conducted where suppression in the vibration amplitude of the lathe-tool was observed
	Li et al. [177]	DSVI	The activation characteristics was analytically formulated in two attributes: displacement amplitude and effectiveness in broad-frequency range. Numerical and experimental observations were presented, and the design procedure was applied for chatter control
	Nankali et al. [178]	Type I	Numerical and bifurcation analyses were carried to study the NES parameters that can increase the stability margins in machining and conditions for the required suppression mechanism were analyzed using an asymptotic analysis based on complexification averaging technique
Spacecraft	Yang et al. [63]	Type I	From frequency sweeping tests, an equivalent spacecraft model was developed using finite element analysis where it was shown that an accurately designed NES can engage in efficient TET for broadband frequency-energy range without changing the properties of the spacecraft
	Zeng et al. [158]	Bi-stable	Bi-stable NES was used for micro-amplitude vibration suppression in spacecraft applications. The response of the coupled system with the NES was obtained by Harmonic balance method. The optimal parameters of the BNES were obtained by Particle Swarm Optimization algorithm
	Zhang et al. [197]	Hysteretic	An NES with NiTiNOL-steel wire ropes was applied in vibration mitigation for a whole spacecraft system described using a SDOF model. Significant percentage of the received energy by the structure is dissipated by the damping of the NES and the NiTiNOL-steel wire hysteretic action

Table 1 continued

Application	References	NES type	Contribution
Flywheel systems	Sun et al. [179]	Type I	An NES was employed to a micro-vibration environment of laser communication satellites to mitigate harmonic vibration amplitudes under radial disturbance excitation of the flywheel
Rotating machines	Guo et al. [185]	Type I	The NES parameters were numerically optimized to enable passive TET for resonance whirling vibration suppression in rotor systems. The influence of some critical parameters of the system was investigated and the efficiency was compared to that of TMD. This model has been extended to study and mitigate vibrations in rotary systems such as engine crankshafts [181], Jeffcott rotors [182, 183]and flexible bladed discs [180, 187, 188]
	Cao et al. [198]	Multi-stable	A multi-stable NES (MNES) was coupled with a rotor-system to control its torsional vibration. The MNES consists of an NES mass, sinusoidal beam with negative stiffness, cantilever beam, sleeves guide rods and bearings. Prominent energy suppression in both transient and steady-state operations was achieved
	Cao et al. [199]	Rotary	Rotary NES was employed for vibration suppression in a gear shaft system. The numerical simulation and experimental validations showed steady-state vibration suppression rates of 76.4% in the simulation and 81.7% in the experiments
Composite plates	Zhang et al. [190]	Type I	An NES is employed at different locations in a composite laminated plate, and it was found that it can be very efficient to suppress severe vibrations induced during speed wind loadings in supersonic flight
	Chen et al. [191]	Type I	The effect of an incorporated NES on the dynamics of a truss core sandwich plate under harmonic and shock loadings was investigated. Although efficient vibration mitigation was obtained, a significant reduction in performance was observed for severe harmonic loads
Beams	Ahmadabadi et al. [194]	Type I and Type III	Different configurations of Type I NES (grounded and ungrounded) employed to a cantilever beam were investigated for energy suppression under shock excitations. Bifurcations and topological structures of the NNMs were also studied. Further investigations on the topic were conducted in [191] and [192]
	Zhang et al. [200]	Inerter	An inertial NES was employed in [200] for vibration suppression in an elastic beam with asymmetric boundary condition
	Zhang et al. [201]	Inerter	In [201], the inertial NES was placed on the boundary of elastic beam for vibration mitigation of the multi-modal resonance. The optimized inertial NES was found capable to reduce the resonance amplitude by 98%
	Zhang et al. [202]	Type I & Type III	Type I and Type III NESs were applied to a flexible beam structure subjected to varying external energy inputs. From the dynamic response analysis, it was found that the vibration absorption capabilities of the NES were dependent on its location with respect to the primary structure. Hence, more energy absorption was observed when the NES is positioned at the free end. Moreover, Type III NES provided better vibration suppression capability than Type I NES
Wind turbines	Zuo et al. [126]	Track	As part of the practical applications, two track-NESs with track profile that incorporates quadratic and quartic polynomials were investigated in [126] to attenuate the seismic vibration responses of offshore wind turbines in earthquake prone areas
Landing gear	Sanches et al. [203]	Type I	A Type I NES was investigated to suppress vibrations resulting from the shimmy phenomenon, a significant concern of aircraft landing gears design
Piecewise systems	Wang et al. [204]	Type I	A Type-I NES was used for vibration mitigation in a piecewise primary system. A hyperbolic tangent function was employed in modeling the piecewise restoring force. The experimental and numerical simulation response results have shown that the NES is efficient in vibration suppression of the considered piecewise system

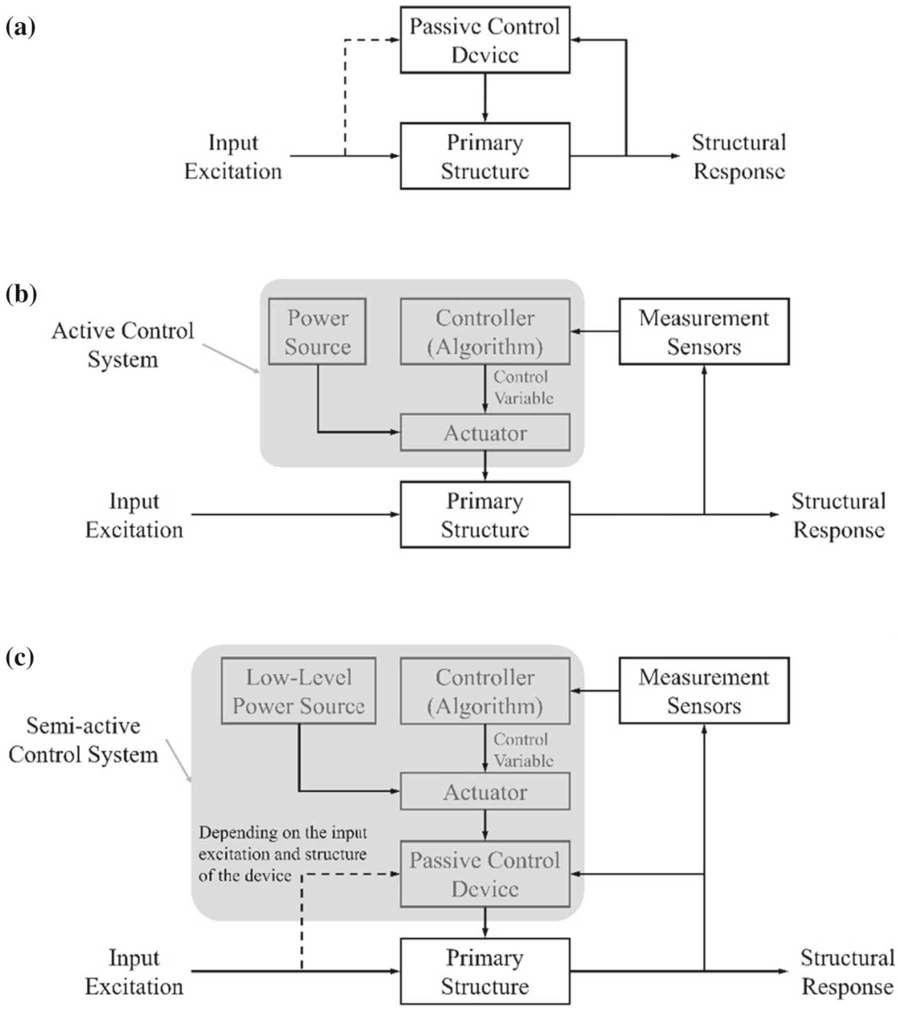

Fig. 1 Block Diagram of **a** passive, **b** active and **c** semi-active control systems [18]

primary four-ton active mass damper for transverse motion and a secondary one-ton active mass damper for torsional motion [9, 11]. More examples and recent advances in active structural control systems can be found in [12–16]. Although they are effective in reducing structural response and are adaptable to a

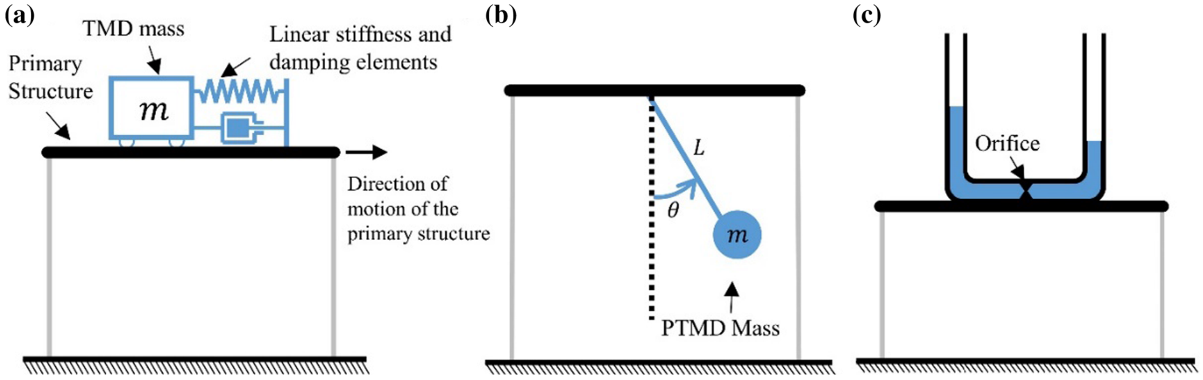

Fig. 2 Schematic illustration of TMD types: **a** traditional TMD, **b** pendulum TMD (PTMD) and **c** tuned liquid damper (TLD)

wide range of operating conditions and structural properties, several challenges arise for their full-scale implementation. In addition to the reliance on large constant power supply, other challenges include reduction of capital and maintenance costs, complexity of system design, huge space requirements, reliability and robustness of the control algorithm and requirement of fast real-time measurement and computation.

On the other hand, passive control systems consist of local attachments that interact with the structural oscillation to localize and dissipate high percentage of the induced input energy, thus reducing the vibration levels in the structure. As shown in Fig. 1, since the control forces are generated through the coupling mechanism between the primary structure and the passive control device, no external power source is required for operation. Input excitations may also contribute to the operation of the passive control devices depending on the nature of the loading and the structure of the control system. In addition, the passive control systems do not require feedback, therefore, they waive the need for sensors, actuators and control computers. Types of passive control systems include tuned mass, liquid, and viscous fluid dampers as well as viscoelastic, friction, metallic yield dampers, and base isolation systems. Given the simple implementation and widespread acceptance by the engineering community, passive control systems have been increasingly implemented in many structures [17]. However, although they increase the energy dissipation capabilities of primary structures for specific cases, their efficiency is significantly affected by changes in the structure properties, usage pattern and stochastic nature of loading conditions.

The semi-active control device [18] can be defined as a modified controllable passive control system which does not induce energy into the primary structure or the control device, but it adjusts the device properties to optimally mitigate the response of the system (cf. Fig. 1) [9]. Therefore, like a passive control system, the control forces are developed through the motion of the primary structure; however, the magnitude of the forces is controlled through feedback from sensors measuring the response of the primary structure as in an active control system. Consequently, it maintains the adaptability of an active control system without requiring large power sources since only a small external power is required. Simultaneously, they offer the energy dissipation efficiency of the passive control devices and more importantly, can remain functional as a passive device in critical events where main power fails such as during earthquakes and typhoons. Semi-active control systems include controllable stiffness vibration absorbers, controllable tuned liquid or fluid dampers, variable orifice dampers, and controllable impact dampers. As an example of operation, the recorded measurements from the response of the structure and input excitation are fed into a controller which sends the required mechanical property such as orifice value, stiffness value or friction coefficient to the actuator. The actuator will, in turn, regulate the property of an otherwise passive device, to optimally reduce the structural vibration response. Although the semi-active control devices are more adaptable than passive devices and can be as efficient as active control ones, they still require a permanent source of power in addition to the added complexity and expense. These aspects led to their limited full-scale implementation [9]. Recent advances and applications can be found in [15–18].

Hybrid control systems consist of a combination of passive, semi-active and active control systems aiming to integrate the beneficial features of the conventional control systems into one. Consequently, higher levels of reliability, efficiency and adaptability are achievable. However, this comes with an added design complexity and increased expense in addition to that the full-scale implementation of the hybrid control system encounters other design constraints such as severe space limitations and instability issues if the system is incorrectly designed [9, 17].

To this end, there are many factors affecting hybrid, active and semi-active control mechanisms. These factors include design complexities, the need for permanent power source, the capital and maintenance costs, and the constraints in application with real-world dynamical structures [9]. Therefore, passive vibration control by nonlinear vibration absorbers rather than linear absorbers has become of substantial interest in the literature and research due to their robustness in vibration suppression at broadband frequency energy fashion. To initiate the discussion on nonlinear energy sinks (NESs), a type of passive control system, we first discuss the tuned mass dampers (TMDs), the most popular passive control device.

2 Tuned mass dampers

Vibration mitigation can be achieved by employing passive linear absorbers like TMDs to rapidly dissipate most of the received energy from seismic or impulsive excitation by the associated structure [19–28]. Tuned Mass Damper (TMD), first introduced by Frahm [19], is the most popular passive linear vibration absorber utilized in structures, particularly in tall buildings and towers [20] and sometimes in aerospace applications [21]. Such a device is referred to as "tuned" because its natural frequency is tuned to a specific structural mode frequency at which most of the induced vibration energy into the structure is captured. Therefore, most of the vibration energy is rapidly received from that mode by the tuned linear absorber where it is immediately dissipated, thus suppressing the vibration of the whole structure. In its simplest and most common form, the so-called traditional TMD consists of a supplemental mass which is linearly coupled with the structure of interest by elastic element and linear viscous damping device. However, due to the difficulty of retuning its natural frequency, other types such as pendulum TMD and tuned liquid column damper, as shown in Fig. 2, are investigated. The pendulum TMD is coupled with the primary structure via a cable or link as shown in Fig. 2b [9]. Upon an excitation, the mass will move creating a force in the opposite direction of motion and thus gradually mitigate the structural vibration response. In addition to being easy to retune the natural frequency of pendulum TMDs by changing the cable length, they require less space for installation compared to the traditional TMDs and can work bi-directionally without any necessary modification. Moreover, it has been found that the nonlinearity in pendulum TMD can be neglected when the angle of rotation is below 9° [22]. In addition, more related analysis to pendulum TMDs can be found in [23–30].

The tuned liquid dampers (TLD) are employed to reduce the vibration effect on structures by tuning the sloshing period of liquid within the container to the fundamental frequency period of the structure [31]. Figure 2c shows a tuned liquid column damper consisting of U-shaped liquid tube, which incorporates an orifice. The motion of the liquid is given a particular natural frequency by tuning the orifice to create an opposite force to external vibrations. The main shortcoming of using TLDs is that the total liquid mass is not involved in the sloshing motion within the container where varying container shapes have to be considered, accordingly. Moreover, the effective mass and damping ratio of the TLD increase with increased excitation amplitude [32]. At harmonic excitation, the sloshing liquid in TLDs exhibits frequency–amplitude dependence, nonlinear behavior at higher-order harmonics, irregular hysteretic loops of irregular shape, chaotic response and jump phenomenon [33]. Further, the sloshing liquids can also undergo either spring hardening or softening effects depending on the excitation amplitudes and depth of the liquid in the container. Spring-hardening effect is observed in shallow-water TLD while spring-softening effect is observed in deep-water TLD at high excitation amplitudes. Additionally, detuned shallow-water TLD was found to perform better than the detuned classic TMD at large-amplitude harmonic as well as stochastic excitations [33]. Another challenge in TLDs application is the difficulty of obtaining an accurate prediction of the nonlinear response of sloshing liquids. Different mathematical methods have been employed for studying the liquid sloshing, some of which include (but are not limited to) the use of simple particle hydrodynamics [34], finite difference method [35] and finite element method [36]. TLDs have been used as vibration suppression dampers for rotating wind turbine blades in [37] and for controlling the lateral tower vibrations of multimegawatt wind turbines in [38]. Furthermore, the TLD was employed for seismic mitigation in [39] where real-time hybrid shaking table was used to study the dynamic response mitigation of building structure attached with TLD.

The Millennium Bridge, also known as the "Wobbly Bridge", in London, presents a perfect example for the importance of vibration absorbers installment to civil structures. It was first opened in June 2000 for pedestrians crossing the river Thames. The simultaneous participation of the pedestrians with the bridge has affected the design vibrational modes and due to unexpected lateral resonant vibration, the bridge was closed for few days after its opening and required major modifications. Out of all the potential solutions, the utilization of passive control system was optimum for this case. As a result, several vertical and horizontal traditional TMDs weighing 1000–2500 kg were installed to suppress the lateral vibration amplitudes of the bridge [40, 41]. Therefore, the bridge was reopened and did not experience significant lateral vibrations since then. Schwedter Straße with four 900 kg TMDs and Britzer Damm with two 520 kg TMDs, made by GERB vibration control systems, are other examples of bridges with successful implementation of passive control systems [41].

Linear vibration absorbers have also been utilized in many tall buildings for the protection against earthquakes, wind and blast excitations. For example, Burj-Al Arab and Emirates Towers in Dubai buildings equipped with eleven 5000 kg and six 1200 kg TMDs respectively [20, 41]. The most famous passive control system is the pendulum TMD installed in Taipei 101 tower. Given the active geographical location, the 101-floor building in Taiwan is prone to strong winds, typhoons and earthquakes. Therefore, huge 728-ton pendulum TMD was suspended from the 92nd floor to the 87th floor [42]. Since its opening in 2004, several earthquakes hit Taiwan with varying magnitudes and several typhoons took place resulting in large number of deaths, yet the tower withstood all the excitations. This proves the importance and efficiency of the employment of vibration absorbers for the protection of civil structures. TMDs and in general, passive vibration absorbers, are not only limited to buildings and bridges, but they are also implemented in many masts, chimneys, cooling towers, drilling equipment, spacecraft structures, etc. A list of some more vibrating structures with TMDs can be found in [20, 41, 43].

Tuning the TMD natural frequency to a certain structural mode frequency is highly important to enable rapid vibration energy absorption and dissipation. Accordingly, high vibration suppression performance is achieved when the selected structural mode is engaged with the TMD at 1:1 strong resonance capture at nearly equivalent frequency content in that mode and the TMD. However, this performance substantially deteriorates when there are frequency fluctuations between the TMD and the associated structure. Consequently, there are several drawbacks regarding the application of TMDs arising from this lack of robustness. Firstly, the natural frequency of the primary system might vary due to unexpected actions, aging, and extreme loading conditions. For instance, the previously mentioned Millennium Bridge experienced a significant variation in the natural frequency of the lateral oscillations due to the unexpected simultaneous swaying of the pedestrians. Since the TMDs are only effective at a certain natural frequency, the solution was obtained by installing TMDs with

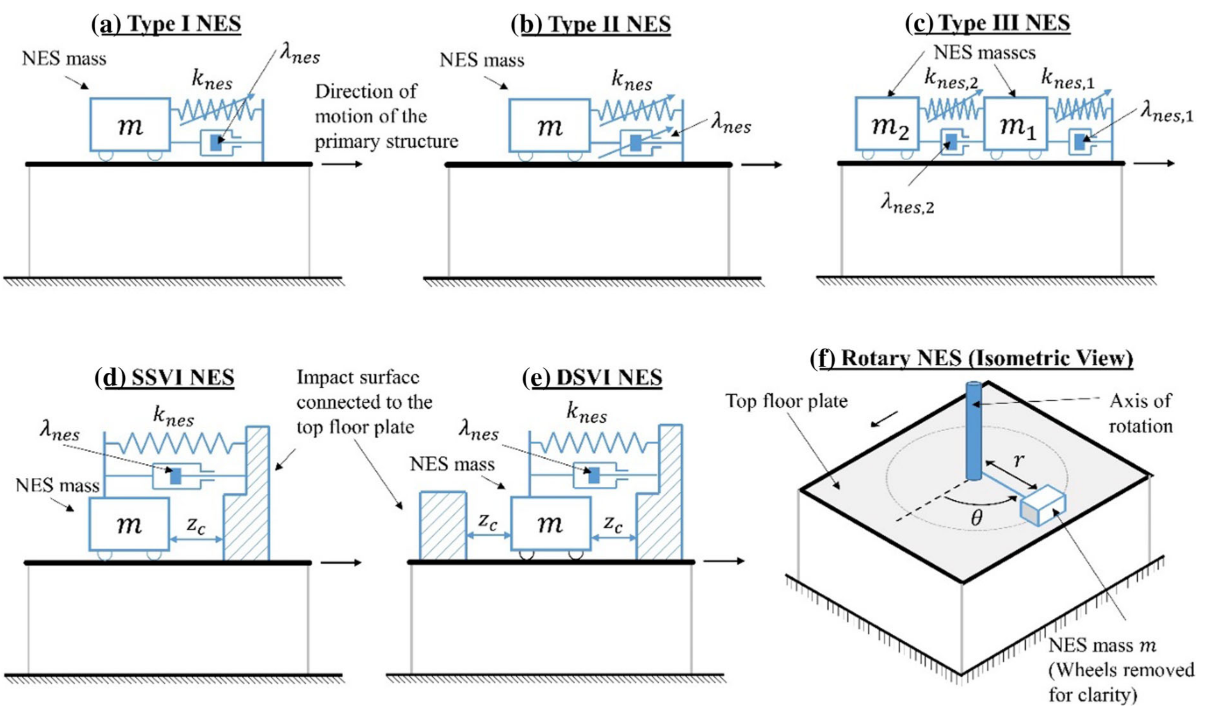

Fig. 3 Schematic illustrations of different types of NESs coupled to a single-story primary structure

different masses (i.e. different natural frequencies) to offer energy dissipation for a range of vibrational excitations [41]. Aging and factors such as settlement, fatigue, creep, humidity, and temperature effects cause changes in the stiffness as well as the mass of the primary structure. This produces slight variations in the natural frequency leading to decreased efficiency of TMDs. Moreover, extreme loading conditions is a huge concern for passive control systems because of two main reasons: (1) multiple modes are usually excited in extreme seismic or shock loading conditions and since the TMDs are only effective for a certain predetermined mode, the response of the other structural modes will not be mitigated (2) large-scale systems have multiple natural frequencies and this makes the efficiency of TMDs limited for applications with such complicated systems. Although the largest amplitude and most severe oscillations occur with the lowest natural frequencies, dissipating oscillations with higher natural frequencies becomes critically important especially with large-scale structures. Even though the structures are designed to keep their elastic properties at various loading conditions, in extreme events such as blasts or earthquakes, the structures are designed to yield to allow for more energy dissipation. However, this inelastic behavior causes variations in the design frequencies and therefore significantly decreasing the TMD efficiency. Moreover, several estimations and idealizations are performed when computed the actual natural frequency of the real-world structures. However, these result in uncertainties in initial natural frequency estimation where a consequence of which is a detuned ineffective TMD [9].

In conclusion, it is an inevitable necessity that we need to absorb induced vibrational energy from primary structures via energy dissipaters to protect them from damage. Since the induced energy can come in different forms, magnitude and frequency, passive linear vibration absorbers, which depend largely on the natural frequency of the system, are not the best solution. Therefore, this limitation ignited the interest in employing nonlinear passive dissipation systems to achieve rapid and efficient vibration suppression in broadband frequency-energy fashion. Accordingly, the TET mechanism in nonlinear vibration absorber has been developed and thoroughly studied in the past two decades.

3 Targeted energy transfer and nonlinear energy sinks

The ultimate goal of vibration absorbers is to protect structural dynamical systems from destructive vibration amplitudes by passive transfer and dissipation of unwanted energy induced into such systems. The process of energy transfer from a primary dynamical structure to an attached recipient dynamical system in irreversible manner is called targeted energy transfer (TET) [44]. This process results from the nonlinear interaction caused by the nonlinear coupling of the NES with the structure which leads to an efficient passive energy transfer. However, even for structural modes with broadband frequencies, the nonlinear energy interactions can still occur in nonlinear dynamical systems due to single or multiple internal resonances [44–47]. Unlike linear vibration absorbers, which function at narrowband frequency, the global dynamical behavior of the linear system is substantially altered by the nonlinear attachment. This is because the nonlinear attachment possesses frequency-energy dependence property which makes it capable to nonlinearly function at broadband frequency-energy fashion to transfer and dissipate energy through multiple resonance captures with the linear system. Such nonlinear attachment is well known as a nonlinear energy sink (NES). Consequently, the NES is regarded as a passive structural control device.

An efficient TET process by NESs should have certain properties including:

- The energy should be transferred to the NES attachment in a nearly one-way irreversible manner (i.e., minimum energy should be pumped back to the primary structure).
- Rapid transfer and dissipation of large amounts of energies out of the primary system.
- The TET should occur over wide ranges and types of impulsive or seismic excitation conditions.
- The added mass should be relatively light weight compared to the primary structure mass.
- The efficiency of the TET process should be robust to changes in the dynamical system parameters.

In the past few decades, TET has become of substantial interest in shock mitigation and energy harvesting fields. The concept of using nonlinear energy sinks for TET was first introduced by Vakakis and Gendelman in 2001 where they showed analytically that properly designed NESs can perform one-way irreversible passive and effective TET in the considered systems for broadband frequency ranges [48–51]. Consequently, extensive investigations have been conducted in the literature up-to-date to apply various existing or new kinds of NESs in different engineering applications. The NES is a relatively new class of passive structural control devices which can be defined as a light-weighted nonlinearly coupled attachment of which the mass should be much smaller than the associated structure mass. It is aimed to be applied to passively mitigate the induced vibration into the associated structure by external impulsive, shock or seismic loading. The recent literature has focused on introducing and develop various efficient NESs of robust performance to achieve TET at broadband frequency-energy manner. The NES must possess an essentially nonlinear coupling element with the associated structure to alter its global dynamics where passive energy transfer can take place. Unlike the single-mode single-frequency tuning linear absorber, the NES is transferring energy at broadband frequency range.

Although linear vibration absorbers, such as TMDs, are of long-term application in vibration suppression, their performance is significantly affected by slight changes in the primary structure's natural frequencies. Given that variations in the frequency content of the primary structures are common, possibly due to temperature or humidity variations, fatigue damage, aging and design imperfections, the use of NESs, which are effective over a relatively broad range of the energy-frequency domain is gaining popularity.

An important outcome of Vakakis and Gendelman in 2001 [48–51] was that the NESs are frequency-independent and thus can be excited at any frequency if the amplitude is sufficient. Many works related to the enhancement and application of NESs stemmed from this frequency-energy dependence. Three years later, Panagopoulos et al. studied the use of multiple degrees-of-freedom (DOF) NES in [52] to prove irreversibility of TET using NESs. Given the promising efficiency and performance of the NESs, Gourdon and Lamarque extended the application of NESs to seismically excited primary structures in [53] and the first experimental investigation of the NESs was reported in 2005 in [54]. The NESs can be classified according to the type of the nonlinear coupling mechanism with the targeted linear structure.

3.1 Stiffness-based NESs

This kind of NES employs a purely cubic stiffness in the NES coupling force with the linear structure. This force incorporates an essentially nonlinear cubic stiffness in these NESs which can be realized by geometric nonlinearity. Therefore, the cubic stiffness NES force can be generated by coupling the NES to the structure by two opposite linear springs that deform in the transverse direction to the NES motion. Linear or nonlinear dissipative viscous damping was employed with this kind of NESs. Some of the stiffness-based NESs can be in single or multiple degrees of freedom. There are many kinds of stiffness-based NES such as Type I, Type II and Type III. The nonlinear cubic restoring force and a linear viscous damping element are employed in Type I as shown in the schematic illustrations in Fig. 3a. This type has been extensively studied numerically, analytically and experimentally in the literature [44, 50, 51, 54–59]. It has also been applied to models of shear buildings [60, 61], aerospace structures [62] and spacecraft structures [63]. The presence of the purely nonlinear coupling element with absence of linear stiffness component enables the NES to be engaged in multiple resonance captures on harmonic and subharmonic resonance backbones of periodic oscillation in the frequency-energy plot [44]. The linear damping element was replaced by a nonlinear one in Type II NES, which was first studied in [59, 64, 65], as shown in Fig. 3b. In Type III NES, two coupled Type I NESs are employed where the bottom one is coupled to the linear structure [59, 66, 67] (cf. Fig. 3c). This modification poses challenges in obtaining the desired dynamic characteristics of the device as more system parameters have to be optimized. In Type I, II and III, the high performance is only achieved at or near the input energy used for optimization of the NES parameters which limits their application. Therefore, stiffness-based NESs performance has been enhanced by employing hardening and softening in the nonlinear coupling element [68], employing unsymmetrical nonlinear coupling force [69, 70], or by including negative linear or nonlinear stiffness content in the coupling nonlinear force [71–77]. This has resulted in substantial enhancement in the performance of the NES even at high intensity impulsive loadings [73].

The frequency energy plots (FEPs) analysis has a substantial role on revealing the underlying nonlinear

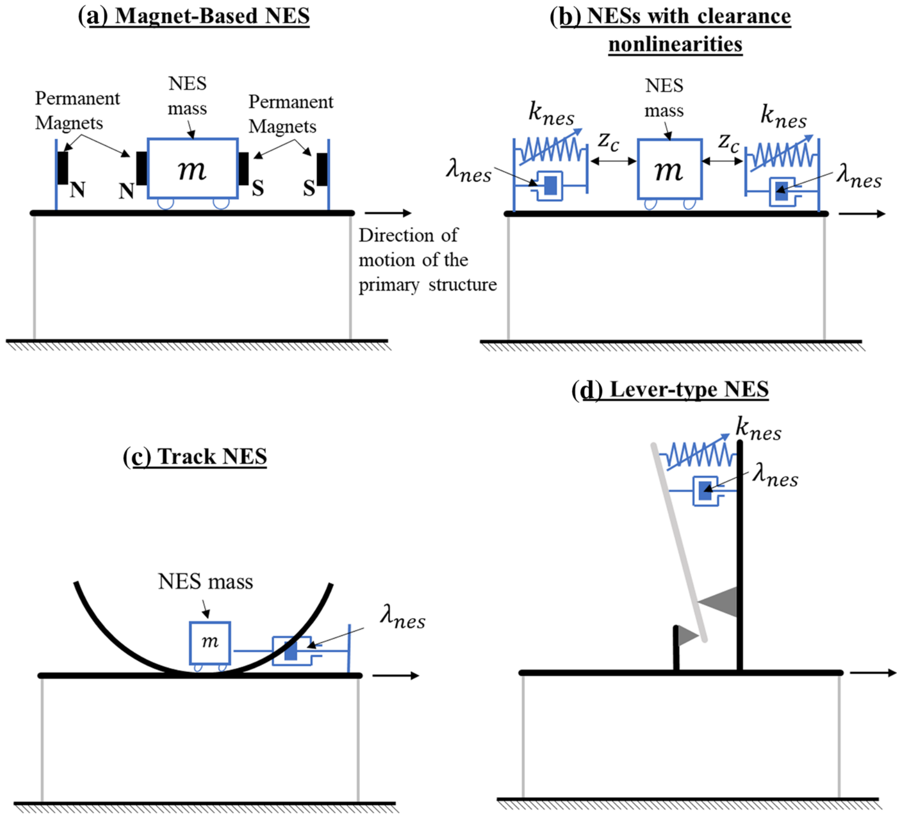

Fig. 4 Schematic illustrations of other types of NESs coupled to a single-story primary structure

dynamical signature of the NES. The periodic motion of stiffness-based structure-NES systems has been extensively studied on the harmonic backbone curves of nonlinear normal modes (NNMs) and their associated branches/tongues of subharmonic and super harmonic periodic motions in the FEPs [44, 45, 50, 55, 57, 58, 78–83]. The FEPs accompanied with the superimposed wavelet frequency spectrum content have confirmed the existence of multiple resonance captures of the NES and structure oscillations on the FEP backbones and their associated subharmonic tongues. The resonance captures in the FEPs has explained the passive energy absorption and dissipation which is achieved by the nonlinear action of the NES. Recently, another kind of energy plots have revealed the nonlinear modal-damping variations in the structure-NES systems in [84]. The methods developed in [84] were based on the analytical analysis that was introduced in [85–87]. Therefore, strongly nonlinear behavior of the modal damping content was discovered in the stiffness-based structure-NES systems in which linear damping content is usually incorporated in physical coordinates.

3.2 Rotary NESs

The Rotary NES (RNES) is inertially coupled with primary structure which generates the nonlinear coupling action. In Fig. 3f, the RNES mass is linked with a primary structure by a rigid arm that rotates in horizontal plane [88–92]. The performance of the RNES is either comparable or slightly outperforming that of Type I, Type II and Type III NESs [88]. Similar to stiffness-based NESs, the periodic motion and the TET mechanism have been studied on the FEPs in [89, 90]. These studies were limited to SDOF, and 2-DOF linear dynamical systems attached with rotary NES where the nonlinear normal modes (NNMs) backbone branches have been obtained via numerical simulation. The TET was observed taking place between the RNES and the linear structure responses by multiple resonance captures on the harmonic backbones and their subharmonic branches on the obtained FEP.

3.3 Impact-based NESs

The two common types of vibro-impact NESs (VI NESs) are the symmetric double-sided impact NES and the asymmetric single-sided impact NES as shown in Fig. 3e and d, respectively. In impact-based NESs, weak linear stiffness is incorporated to restore the motion between the consecutive impacts. The energy dissipation in the VI NESs is achieved by their linear viscous damping and the inelastic collisions with the rigid barriers. The nonlinear coupling force is generated by the consecutive impacts of the VI NES with particular floor of the structure. In the symmetric double-sided vibro-impact (DSVI) NES, the rigid barriers are fixed to the primary structure in opposite directions to the NES mass motion to restrict its oscillation within the impact clearance [60, 61, 93–105]. However, in the asymmetric single-sided vibro-impact (SSVI) NES, the impacts take place in one direction of the NES mass oscillation [106–110]. Among these vibro-impact NESs and other existing NESs, the SSVI type has been found of robust performance in vibration suppression even at severe input energies [107]. Numerical and experimental investigations were conducted in [106, 109, 110] to verify the robustness of the SSVI NESs in shock mitigation in large-scale 9-story structure subjected to real blast loading in [110]. Consequently, impact-based (or vibro-impact) NESs, when properly designed, can act as the most effective NESs for shock and seismic energy absorption and dissipation even at severe loadings. In addition, the efficiency of SSVI NES was found to be improved with proper selection and optimization of the impact coefficient of restitution [111]. The periodic orbits at NNMs on the FEPs and the TET mechanisms have been thoroughly studied in [103, 104, 112] for linear oscillators attached to the symmetric DSVI NES. The FEP backbones and several symmetrical and unsymmetrical resonance subharmonic branches have been obtained for the SDOF linear system in [103] and the 3-DOF linear system in [104] when they were attached to the SSVI NES. In addition, the damped transitions during the nonlinear action of the SSVI NES under shock or seismic loading have been analyzed for TET by superimposing the frequency spectrum content obtained by wavelet transform on the FEPs. It was observed in these studies that the optimum TET is achieved at the excitation of high energy impulsive orbits. In addition, different TET mechanisms between the linear system and the SSVI NES were analyzed. In [112], a detailed FEP analysis using the piecewise numerical continuation method has been also performed with a SDOF system attached with SSVI NES.

3.4 Other kinds or modified NESs

The NESs are not limited to the aforementioned categories. However, variety of related, modified or other types of NESs have been also analytically, numerically, and experimentally studied for vibration suppression for various applications. These recent, stiffness-based related or modified NESs can be categorized according to their design and the type of nonlinear coupling force as the following:

1. Magnet-based NES [113–117]: In this type of NES, the nonlinear magnetic repulsive force of permanent magnets is used as an alternative to the nonlinear spring elements to generate tunable symmetric and asymmetric nonlinear coupling force with the linear structure [113, 114], as illustrated in Fig. 4a. In addition, the magnets are capable to be arranged in transverse directions to generate a bistable nonlinear coupling force as in [116]. This later design was applied for efficient seismic vibration control. The magnet-based NES was also employed in [117] to suppress the resonance vibration whirl amplitudes of a rotor subjected to unbalance force excitation.
2. NESs with clearance nonlinearities [118–126]**:** This type of NES is coupled to the primary structure through non-smooth piecewise nonlinearities with optimizable clearances and stiffness characteristics. In [118], the clearance zone separates two regions of nonzero linear stiffness values whereas in [119], the clearance zone separates the zero-stiffness region within the clearance zone from the nonzero stiffness elements that were coupled with the linear structure at the clearance boundaries, as shown in Fig. 4b. The periodic motion of the considered piecewise NES in [119] has been recently studied in [120] for understanding the nonlinear dynamical behavior of this kind on the FEP. Consequently, the TET mechanism by the piecewise NES was verified on the FEP via multiple strong resonance captures on

the in phase and out of phase backbone curves. The concept of clearance nonlinearity was also employed in the NES coupling force in [121] for electroacoustic wave energy harvesting. In [122] two clearance zones piecewise NES was proposed to nearly approximate the cubic stiffness force effect, where this configuration has been proven to be efficient for vibration suppression at moderate periodic excitation in the considered system. Another configuration of multiple stiffness clearance zones called tri-stable piecewise NES was considered in [123]. This NES design was attached to a cantilever beam for vibration suppression of the transient and steady-state excitations. Within the non-smooth nonlinearity concept, the cubic NES stiffness coupling force is subjected to discontinuity at the clearance zone boundaries in [124]. Therefore, the cubic stiffness coefficients and the corresponding nonlinear coupling forces at the clearance boundaries are not of similar values. This modification resulted in efficient reduction in resonance amplitudes of the excitation. A piecewise spring was incorporated in coupling the NES mass with linear oscillator in [125] to restrict the excitation amplitude of the NES. The numerical and experimental results have proven that the piecewise stiffness and the clearance are the main factors that affect the vibration suppression of the LO. In certain cases, the damping of the NES was negatively affected by the presence of the additional piecewise spring. The NES parameters were optimized by using the particle swarm optimization technique [126]. In [127], an encapsulated type of NES (E-NES) was employed to reduce the vibration of a 2-DOF system. The E-NES consists of piecewise nonlinearity in the form of combination of piecewise spring and piecewise damping. Significant vibration suppression in the 2-DOF linear system was achieved by the E-NES.

3. Track NESs [128–134]: The track NES was first studied in [128] where the NES mass moves along a symmetric curved shape track as seen from Fig. 4c. Therefore, the NES oscillatory motion on the track creates a reaction force that represents the NES coupling force with the primary structure. Accordingly, the efficiency of this design was numerically analyzed with a 2-story physical structure that is subjected to impulsive and seismic excitation loads. In later studies, the robustness of an optimized version of the track NES has been experimentally verified with a 2-story lab structure subjected to an impulsive and seismic excitation in [129] and with 5-story steel frame subjected to five earthquake waves of different frequency contents in [130]. Moreover, an analytical technique based on the harmonic balance method was utilized in [131] to analyze the frequency-energy dependence of the track NES when the structure-NES system is subjected to impulsive and harmonic excitations. The track NES was also modified by adding an impact barrier at one side of the special shape curved track to incorporate the single-sided vibro-impact (SSVI) effect which further enhanced the efficiency of shock mitigation [132]. In a recent study in [133] the synergetic effect of the SSVI NES with a track NES was tested with a model of 32-story high-rise building where the improved track NES with SSVI NES has outperformed the track NES at structural stiffness variations as well as the input energy strength.
4. NES with inerter [135–140]: In the inerter NES, the NES mass is either replaced by an inerter or accompanied with it to achieve a significant reduction in the NES mass which maintains or improves the optimized efficiency of vibration suppression [135, 136]. The NES mass in [137] is kept and attached to an inerter of small mass fraction of the original linear system. When the inerter mass is equal to the NES mass, the vibration response of the structure has been considerably reduced compared to the case of using the NES without inerter. In recent studies, the inerter NES has been further investigated with a suspension system by using a quasi-zero stiffness in [138], by using an asymmetric nonlinear energy sink in [139] and for suppressing torsional vibration in [140].
5. Lever-type NES [141–144]: The lever-type NES (LNES) was firstly proposed in [141] for effective suppression of harmonic excitation resonance amplitudes in 2-DOF dynamical system. The study in [141] has employed harmonic balance and numerical simulation methods to verify the efficacy of the proposed LNES. Detailed bifurcation analysis on Poincaré maps and frequency response plots of the LNES with harmonically

excited SDOF spring-mass system has been performed in [142] to understand the nonlinear dynamical signature of the LNES. The observed bifurcations were mainly the saddle-node and Neimark–Sacker bifurcations [143, 144]. The concept of the lever-type NES is illustrated in Fig. 4d.

6. Variable stiffness NES [145, 146]: The time-periodic stiffness has been employed in the coupling NES force in [145] rather than the traditional coupling nonlinear elements. This type of NES was employed with harmonically excited 2-DOF spring-mass system to strongly modulate the resonance amplitudes. In [146], the variable stiffness NES is aimed to be adaptable to the effect of changes in harmonic excitation amplitudes and the surrounding temperature.
7. NESs with nonlinear damping [147–149]: The nonlinear geometric damping has been firstly applied with NESs in [64, 65]. However, the nonlinear damping and combined stiffness components are incorporated in the NES in [147] for studying the dynamical behavior of single and 2-DOF systems attached with this NES. It was concluded that incorporating nonlinear damping in the considered NES has enhanced the vibration mitigation performance compared with the case of only incorporating a linear damping. In addition, the 2-DOF NES was found generating additional strongly modulated response (SMR) than that the SDOF NES. This study was an extension to the work in [148] in which the mass distribution of the 2-DOF NES introduced new dynamics compared with the SDOF NES. Moreover, the analysis in [149] has addressed the combined linear and nonlinear damping effect on suppressing the induced vibration by impulsive loading where the performance with combined linear and nonlinear damping NES was found to be more improved compared with either linearly damped or nonlinearly damped NESs. At small impulsive loading, the performance of linearly damped NES is higher than that of the nonlinearly damped NES. However, at high excitation amplitudes, the incorporation of nonlinear damping has improved the performance compared with the linear damping case. In addition, the dissipation capacity of the NES was improved by incorporating the combined damping compared with only employing linear damping case.
8. Hysteretic NES [150–156]: The Hysteretic NES (HNES) incorporates purely hysteretic spring which is accompanied with linear negative stiffness and nonlinear cubic stiffness springs [150]. The Bouc–Wen model was used to express the resultant force obtained by the purely hysteretic spring when accompanied with the linear stiffness spring. The performance of the HNES was found to be enhanced when negative linear stiffness is incorporated [150]. Furthermore, the NES that dissipates energy based on the hysteretic behavior of the arrangement of the NiTiNOL-steel wire ropes was employed in [151]. The restoring force was produced by the short NiTiNOL-steel wires undergoing flexural-tension loading where it has a quasi-linear trend at low excitation amplitudes and pinching effect at high excitation amplitudes [152, 153]. This pinching effect is mainly because of the interaction between the interwire friction and NiTiNOL phase transformation. In [154], the proposed HNES was compared with the classical TMD by attaching them separately on an elasto-plastic steel structure and a masonry building under seismic excitations. It was observed that the classical TMD underwent a detuning effect because the underlying structure had a reduction in its resonance frequency thereby decreasing the effectiveness of the TMD. The HNES was more robust than the linear TMD because it can vary its stiffness, without any external intervention, based on the excitation amplitudes. Moreover, the study on the HNES has been extended to a 20-story concrete frame using Finite Element Methods in [155]. The effectiveness of HNES for shock mitigation was also studied in [156] wherein the robustness of the HNES in vibration suppression was evaluated by keeping two uncertain parameters which are the frequency of linear associated structure and the stiffness of the HNES spring. The authors kept either of the parameters (or both) as uncertain by taking values (of the uncertain parameter) from a normal distribution. By adding uncertainty in the system, the overall robustness of a system can be evaluated based on the reliability index; if the system, with uncertainty, has a reliability index within a certain range, the system can be considered robust. Moreover, it was found

that incorporating a negative linear stiffness in the HNES enhanced the robustness of performance for broadband energy levels. Nevertheless, the classical Type I NES with purely cubic stiffness was found performing better compared with the HNES at some energy levels and uncertain natural frequency of the primary system.

9. Bi-stable NESs [71–77, 157–160]: The dynamical behavior of the bistable NES (BNES) with negative stiffness, which is already addressed in this review in [71–77], was further studied in [157–160]. Accordingly, a dependence on the excitation frequency and amplitude was observed in [157] on the truncated damping and the bifurcation boundary. Moreover, closer to the fundamental resonance frequency, the NES with negative stiffness has better performance capabilities because of the SMRs. Elevating the nonlinear stiffness and lowering the damping were also found improving the vibration suppression performance near to the resonance up to certain threshold values of stiffness and damping, respectively. Once the nonlinear stiffness exceeds or the damping is reduced beyond the threshold values, failure of vibration mitigation was observed. The BNES which is modeled by an axially loaded beam equipped with a concentrated mass was used for micro-amplitude vibration suppression in [158]. Micro-amplitude vibrations are prevalent in spacecraft structures and affect their performance in the form of resolution, stability, and pointing accuracy [159]. The FEP of the considered BNES in [73] is generated in a new study in [160] for further understanding to the Hamiltonian dynamics of this kind of NESs. Several unsymmetrical backbone curves were obtained at low frequency energy levels which are found to be as a unique signature of the BNES nonlinear dynamical behavior. In addition, the TET mechanism by the BNES was verified on the FEP by observing multiple strong resonance captures on the symmetrical and unsymmetrical backbone curves.

4 Applications of NESs

Various types of NESs have been intensively investigated for several engineering applications. The NESs have been applied for aeroelastic flutter suppression in [44, 66, 161–167]. However, incorporating additional vibration absorber mass could not be realistic for an aircraft because the aircraft's weight is a constrained parameter that affects it's functioning. As such, a flap-NES with cubic stiffness was used in [167] and [168] to be integrated on the outboard tip of an aircraft wing without adding any extra mass to the wing. The functioning of the flap-NES is based on its action as a secondary oscillator that controls the aeroelastic behavior of the wing [167] and the base structure is not significantly modified where the flap-NES can take advantage of the aerodynamic damping during its operation.

Similarly, different types of translational NESs [169–171] and rotational NESs [172, 173] [129, 130] were implemented for the purpose of reducing vortex-induced vibrations in sprung cylinder or a cantilever viscoelastic cylinder in [174]. In [175], a three-way coupled fluid–structure-NES system is studied for suppressing fluid–structure interaction induced instabilities. In addition, NESs were used for quenching chatter instability [176–178], and for vibration reduction in different systems, such as spacecraft, flywheel systems [179], rotating machines [180–189], composite plates [190, 191] and beams [71, 77, 192–194]. In [195] and [196], NESs were used for a variety of elastic wave control mechanisms.

A summary of the major contributions in each application is listed in Table 1.

5 Impact-based NESs: design, analysis and applications

Impact-based (or vibro-impact) NESs, when properly designed, can act as effective shock and seismic energy dissipater by absorbing and dissipating most of the received energy by the structure in fast-time scale even at high intensity impulsive loading. Therefore, they have been extensively studied analytically, numerically and experimentally in the last few decades. This section illustrates a comprehensive literature review of the recent works devoted to impact-based NES.

In general, the structural vibration control aims to dissipate excessive vibrational energy from a primary structure using an auxiliary device. Different types of passive damping devices such as metallic yield, viscoelastic, viscous fluid, and tuned liquid dampers have been theoretically and experimentally investigated for vibration mitigation in civil structures but no particular damper was found efficient under all circumstances [14].

Impact between any two or more objects results in energy and momentum transfer and dissipation. Consequently, this ignited interest for developing Vibro-Impact Dampers (VIDs), also known as impact dampers, to be applied to structures to absorb the unwanted vibrations and dissipate the energy via momentum exchange and energy dissipation occurring during the inelastic impacts. Seventy years ago, this idea, initially named as acceleration damper, was first presented by Rusakov and Kharkevich for forced vibration response suppression of a system that is impacting a rigid wall [205] and by Leiber and Jensen to reduce the vibration of the mechanical systems of a tank by employing a moving mass between its walls [206]. Few years later, Arnold further investigated the response of a system with an impact absorber excited by a forced vibration [207]. Following that, extensive studies around VID dynamics and ability to act as a mere damper were carried out. Accordingly, the dynamical behavior of the VIDs was investigated in [208–219] by analyzing the regular and chaotic dynamics and bifurcations using time series, phase trajectories, bifurcation diagrams, Poincare maps, Lyapunov exponents and other methods. Simultaneously, single-unit and multi-unit VIDs were investigated for attenuating vibration levels of structures under forced, periodic, impulsive and seismic excitations [214–216, 219–230]. To highlight their importance, the employment of VIDs to reduce the vibration levels in machine tools was discussed in [225] and VIDs were used to control the structural vibration due to wind excitation in a cable-stayed bridge as illustrated in [226]. The aforementioned works which discuss the design, analysis and investigation of dynamic systems with VIDs considered the steady-state response of the system due to external periodic or stochastic forcing.

Recently, the VIDs are viewed from the perspective of TET which is directly related to the transient response rather than the steady-state one. Therefore, the VID is not viewed as a basic damper, instead the concentration is on the efficiency and rapidness of energy dissipated by the VID and the induced global dynamical changes in the system (energy scattering). In this perspective, the VID is called impact-based or vibro-impact (VI) NES where its ability to achieve TET through passively transferring, localizing and dissipating the induced vibrational energy by impacts, NES damping and the structural modal damping content. Therefore, the focus of most investigations was on the effect of impact-based NES physical parameters such as clearance, mass, stiffness and coefficient of restitution on the energy dissipated rather than the dynamical behavior of the system (i.e., bifurcations, chaos, response regimes, etc.).

Gendelman and Vakakis were the first to introduce the concept of using nonlinear vibration absorbers for TET in 2001 where they showed that properly designed NESs can perform one-way irreversible passive and effective TET in the considered systems for broadband frequency ranges [48–51]. Knowing that nonlinear vibration absorbers are frequency-independent and can be excited at any frequency if the amplitude is sufficient, many works related to the enhancement of nonlinear vibration absorbers are investigated using various types of NESs. However, the non-smooth stiffness nonlinearities were first analyzed four years later by Georgiades et al. in [105] where 2-DOF linear primary system was coupled to two NESs possessing a linear internal restoring spring together with a clearance stiffness. If the clearance stiffness is increased to relatively high value, the vibro-impact limit is achieved.

Plenty of publications have studied VI NESs because they are suitable for applications where shock and seismic elimination at the initial highly energetic cycles of motion is critical. Depending on the impact conditions, VI NESs are classified into two types: double-sided (DS) VI NES (also called symmetric VI NES) and single-sided (SS) VI NES (also called asymmetric VI NES) as previously shown in Fig. 3. Hereafter, the literature review of the works related to the VI NES is presented. Since 2006, the DSVI NES was extensively investigated for seismic excitations as in [60, 61, 104, 231], forced excitations as in [94–96, 98–101, 176, 177, 232] and impulsive excitations as in [93, 94, 98, 102, 103]. The DSVI NES has shown significant energy dissipation and fast effective TET for the different types and magnitudes of excitations. It was also found that the non-smooth vibro-impact nonlinearity causes an intermodal energy transfer which enhances the energy dissipation performance. On the other hand, the SSVI NES concept was first introduced in 2013 by M. AL-Shudeifat et al. in [107] where its performance for shock mitigation for a 2-story linear primary structure was compared numerically and experimentally to DSVI NES. The SSVI NES generated strong vibro-impacts with the primary structure because of the asymmetry in the system caused using one barrier to the NES motion which enabled the NES mass to gain high momentum for consequent impacts. Several important outcomes resulted from that work. Firstly, the SSVI NES is capable to transfer energy through the structural modes where it is dissipated by the inherent modal damping of the structure. Secondly, the strong vibro-impacts take place at early phase of the motion resulted in faster reaction time and therefore rapid energy dissipation. In fact, the SSVI NES was able to dissipate large amounts of induced shock energy in the first and second cycles of motion which indicates that they are applicable to structures vulnerable to different kinds of excitations where there is an inevitable immediate shock mitigation. Thirdly, the SSVI NES was found to be more robust by having effective energy dissipation for broad range of input initial energy levels even at severe shock excitations. As a result, the SSVI NES is more efficient for TET compared to DSVI NES and all other existing types of NESs. Consequently, more works have followed [107] where large-scale structures were numerically and experimentally investigated for seismic mitigations as in [109] and shock mitigations as in [106, 110]. Additionally, the energy transfer mechanism of the SSVI NES was investigated in [98]. In all of these works, the SSVI NES had a fast reaction time and caused transferring and dissipating of significant amount of the received energy by the structure by its structural modal damping. From the literature review conducted, it was found that more researches were devoted to the study of DSVI NES due to its earlier implementation in TET. However, given the efficiency of the SSVI NES, it is a promising area of research, and it is expected to dwarf the TET field in the near future.

5.1 Advantages of VI NESs

In VI NESs, a non-smooth stiffness restoring force is generated which introduces the required essential nonlinearity that enables the VI NES to robustly function under different types and magnitudes of impulsive, periodic and seismic excitations. This is because the NESs are broadband frequency-energy system and are therefore capable of efficient passive irreversible (mainly to the lowest frequency mode) energy transfer. In addition, the VI NESs do not require the employment of nonlinear springs. Therefore, they are the easiest to be implemented compared to all of the other types of NESs. This makes them more suitable for real-life TET related full-scale implementations.

Based on the comprehensive review conducted, it was observed that the VI NESs rapidly react to dissipate a large amount of induced energy into the structure-NES system even at high intensity impulsive loading. This energy dissipation is accomplished by three mechanisms: (1) the NES damping, (2) energy loss during NES inelastic impacts and (3) the structural modal damping. Frequently, the early phase of motion is the most energetic such as in severe shocks and some earthquakes and thus the NES needs to be activated in the first few cycles of motion. The fast-reaction time of the VI NES makes it most suitable for

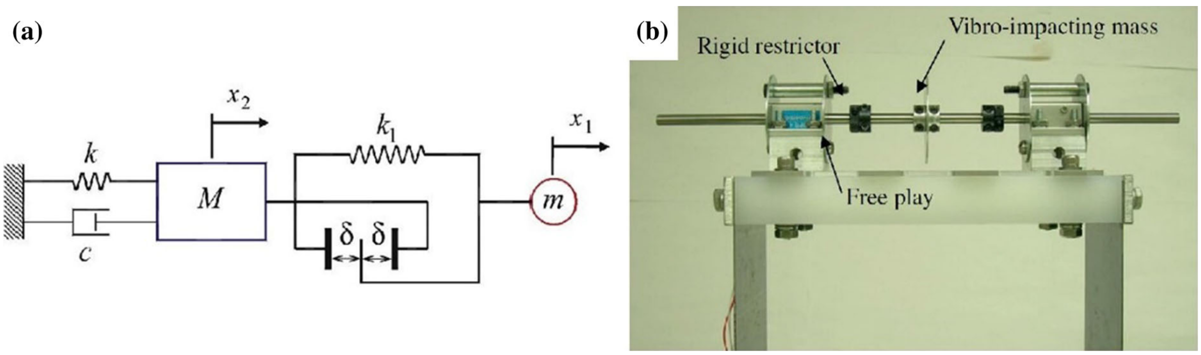

Fig. 5 An example of the first model of DSVI NES: **a** schematic illustration [103] and **b** experimental realization [60]

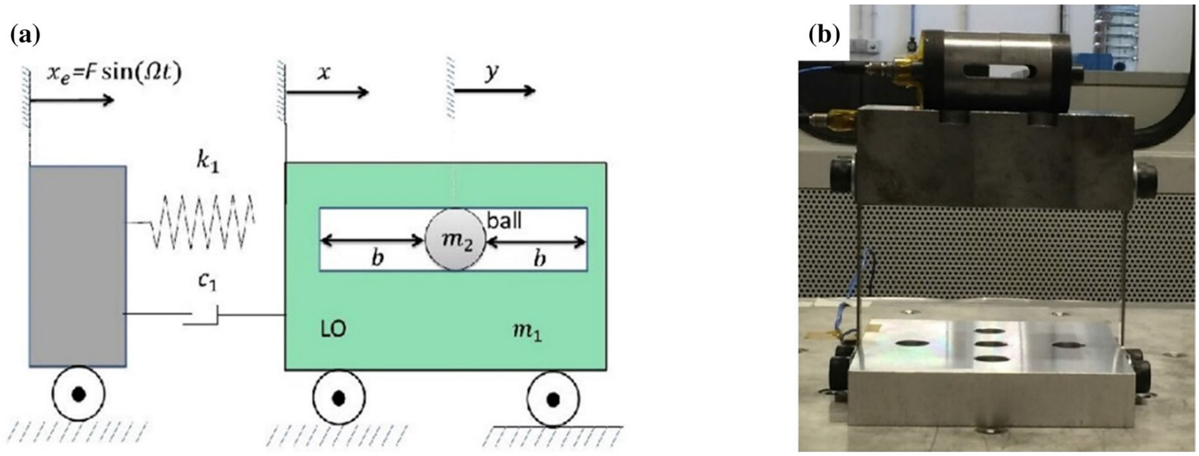

Fig. 6 An example of the second model of DSVI NES: **a** schematic illustration [176] and **b** experimental realization [101]

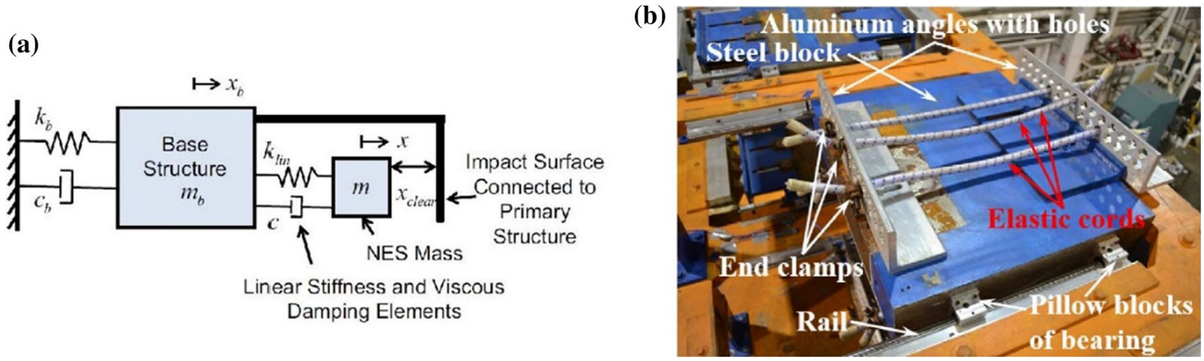

Fig. 7 An example of a modal of SSVI NES: **a** schematic illustration [110] and **b** experimental realization [109]

such applications. The rapidity of the energy transfer is critical in structures because of the severity of the initial few excited cycles of oscillation caused by earthquakes, impulsive loading or blast. Consequently, this distinctive fast-time scale feature has increased the interest in utilizing VI NESs in large-scale structures and test them experimentally against seismic and blast excitations. As demonstrated in [109, 110], the VI NESs have shown a unique performance in the experiments where the highest energy motion in the initial stage was efficiently reduced and therefore the structure was protected.

5.2 Realization and models

The VI NESs are realized by coupling relatively small mass(es) to the high level floor(s) of a structure with linear stiffness spring(s) in addition to rigid barrier(s) to restrict the NES mass(es) oscillation. The rigid barriers cause impacts between the NESs and the associated floors. As a result of the inelastic collision and momentum exchanges, high percentage of the induced energy is rapidly dissipated.

It was found that there are two main schematic models used to realize the DSVI NES. The first model, which is shown in Fig. 5a [103], consists of stiffness element, linear viscous damping, and the rigid walls that constraint the motion of the NES mass on both directions of its oscillation. This model is usually implemented by works which consider the numerical study of the DSVI NES such as in [60, 61, 95, 102–104, 228, 230–232]. Experimental implementation of this model is shown in Fig. 5b for a 3-DOF linear primary small-scale system where the DSVI NES is coupled with the highest floor of the structure [60]. In the second model, the NES is included as a small spherical mass inserted into a longitudinal cavity in the primary mass as shown in

Fig. 6. The NES mass will move freely inside the cavity which will cause inelastic impacts upon reaching the cavity's ends. This model does not involve the stiffness or damping coupling elements and is usually considered by works dealing with the analytical treatment of the DSVI NES such as in [95–101, 176, 177, 232, 233]. Both models are good representations of the DSVI NES; however, the first model is easier to implement experimentally because it does not involve cutting a cavity inside the primary structure's mass. Secondly, the NES parameter optimization is simpler when using the first model because there is no cavity restraint (i.e., clearance and NES mass and size, etc., can be changed easily). Most importantly, the second model does not have a stiffness component which plays a major role in restoring the NES mass to gain significant amount of momentum for the consequent impacts where more energy will be dissipated. On the other hand, it was found that there is one schematic model used to realize the SSVI NES which is shown in Fig. 7a [106–110]. The requirement of a single rigid barrier facilitates the employment of damping and stiffness elements on the opposite direction of the impact barrier. The SSVI NES was implemented in small- and large-scale structures for experimental investigations as shown in Fig. 7b. In general, the realization of SSVI NES is easier in terms of experimental testing than the DSVI NES, however, both are still applicable to dynamical structures.

5.3 Excitation models

Earthquakes, wind and blasts induce seismic, periodic and impulsive excitations, respectively, to small- and large-scale structures. The severity of such excitations varies according to the magnitude of the initial induced vibrational energy. The NESs' primary job is to absorb the energy induced to the primary structures as fast as possible. Most studies such as [94–101, 176, 177, 232] which are related to DSVI NES consider the forced excitation to analytically investigate the dynamical behavior of the system. Moreover, several works, namely [60, 61, 104, 231], are devoted to the implementation of DSVI NES for seismic mitigation in which several time-scaled earthquakes were considered to excite the primary structure numerically and experimentally. In these studies, the DSVI NES showed fast reaction and resonance capture time where it demonstrated the capability of redistributing seismic energy to higher structural modes. Few works ([93, 103]) addressed the employment of DSVI NES for energy dissipation of structures subjected to impulsive loading. This is mainly because the DSVI NES demonstrated limited performance in achieving efficient TET for structures subjected to severe shocks. On the other hand, due to the enhanced performance of the SSVI NES in dissipating shock energy induced into primary structures, several works considered investigating impulsively loaded small- and large-scale dynamical structures coupled to SSVI NESs [106–108, 110]. Moreover, the seismic mitigation of a large system coupled to SSVI NES was also studied in [109]. Given the short time from its invention, the improved performance of the SSVI NES to achieve efficient TET indicates that it is a promising trend and will gain significant popularity in the shock mitigation field in the near future.

5.4 Tested structures

Numerous works considered SDOF primary linear oscillator coupled with DSVI NES [95–101, 176, 177, 232]. The primary goal of these works is to analytically study dynamical behavior of systems possessing non-smooth nonlinearities. In addition, in one of the earliest works, a 2-DOF primary structure coupled with DSVI NES was numerically investigated for seismic mitigation as shown in [104]. In [60] and [61], a 3-DOF linear primary structure was also numerically and experimentally investigated for different time-scaled earthquakes. Structures of 4-, 7- and 10-story models which represent low- to medium-rise structures were coupled with DSVI NES for seismic mitigation in [231] in which the frames were numerically simulated for eight historic scaled earthquakes and it was found that the DSVI NESs were activated in the early stages of motion. Generally speaking, when compared to SSVI NES, the DSVI NES showed less effective TET and due to this limited performance to attenuate structural response to seismic and shock excitations, with the exception of [231], no work considered the implementation of DSVI NES for large-scale structures analytically, numerically or experimentally. On the contrary, the enhanced performance of the SSVI NES which provided efficient TET for wide range of initial impulsive excitations enabled its employment in small- as well as large-scale structures. After the first successful realization of SSVI NES in [107] in which a 2-story linear primary structure was considered, series of publications in which a large-scale 9-story structure coupled with two SSVI NES in the eighth and ninth floors followed [106, 109, 110]. In these large-scale structures, the SSVI NES was able to dissipate significant amount of energy during the highly energetic initial phase of excitation through consecutive inelastic impacts and the redistribution and dissipation of portion of the energy at high frequency modes.

5.5 Analytical treatment

The singular perturbation approach and multiple scale analysis was successfully adopted to analytically describe the transient damped responses of systems coupled with smooth NESs. For VI NESs, the aforementioned approach is not applicable due to the non-smooth nonlinearity present in the system. However, it was enhanced by Gendelman in [93] where the analytical treatment of the transient TET process for the case of impulsive loadings in a linear oscillator coupled with DSVI NES was presented. Using this approach, the slow invariant manifold (SIM) of vibration amplitudes which represents set of stable and unstable fixed points under 1:1 resonance was obtained. This tool can be used to describe the existence of TET and the variations of resonance captures. The analytical study is then followed by several works which considered the case of harmonic forcing applied to the same system [99, 100]. An important aspect of these forced nonlinear systems was presented in [99, 100] where it was observed that the lower stable branch was not involved in the SIM. Since the phase point should eventually exhibit a jump from the SIM, it will not find a lower branch to land on which is not usually the case for smooth NESs where the phase point lands on a lower branch and causes stable SMR. For the case of VI NES, due to the absence of stable lower branch, the system will leave the regime of 1:1 resonance but will be captured again by the SIM after getting energized [99, 100]. It is difficult to know how much the amplitude of the 1:1 resonance oscillations will be or how long it will last before jumping out. However, the phase point will eventually jump and be captured again by the SIM and this will cause intervals of 1:1 oscillations with random amplitude and length [99, 100]. In [234], two viscoelastic dissipative contact models were implemented in the NES which included instantaneous and finite contact (Tsuji [235] and Kuwabara [236]) models. The resulting NES is similar to the one employed in [95, 97, 101]. A 10-story frame structure under two types of earthquake excitation (El-Centro and Kobe) was employed for test purposes. Unlike the finite contact model, it was found out in [234] that the instantaneous contact model was very sensitive to the initial conditions (either in the form of initial displacements and/or velocities). However, both models were found to converge to the same mean values where the finite contact models were acting as a low pass filter which keeps the response away from the nonphysical chaotic states.

In [96], the authors studied analytically and numerically the same SDOF system coupled with DSVI NES by generalizing the proposed approach in [93] through relaxing the 1:1 resonance capture condition. However, the generalization did not capture all the complicated dynamics of the system. Following that, Li et al. in [97] considered the same system where the effect of the route to bifurcations as well as the effect of NES mass, clearance, coefficient of restitution, damping of the primary system and the amplitude and frequency of the forced excitation effects on TET and SIM were investigated considering two impacts per cycle to resemble a 1:1 resonance oscillation. Further study on the dynamics of the same SDOF system followed in [101] where the SMR and constant amplitude responses were analyzed. A continuation of this research work followed in [88] where the optimization of the DSVI NES was viewed from analytical perspective for periodic and transient excitations. More recently, in [94, 232], two parallel DSVI NES masses were inserted into two separate cavities in the primary mass and the system was analytically studied where each NES clearance was optimized to have separate activation levels and it was found that different levels of initial amplitude displacements resulted in separate activation of the two VI NESs. However, the addition of another DSVI NES did not result in increased TET efficiency although it might reduce the impact strengths. In [95], the analytical investigations of DSVI NESs were further studied to calculate the Lyapunov exponents of a VI system, and with some numerical simulations, the chaotic characteristics of the system were analyzed. One direct application of the analytical treatment of systems coupled with DSVI NESs can be found in [176] and [177] where the system was analytically studied to control chatter instability.

On the other hand, despite the fact that the SSVI NES contributes to more efficient energy dissipation for structures subjected to shock loading or seismic excitation, few research works have addressed the analytical treatment of systems coupled to SSVI NES. Although the analytical treatment is essential for understanding the underlying dynamics of systems coupled with nonlinear vibration absorbers, there are few drawbacks that hinder their application. Systems coupled with VI NES possess highly nonlinear and complex dynamics that cannot be captured analytically without making some simplifications which result in loss of important dynamical behavior. Examples of such simplifications include the consideration of 1:1 resonance in [93, 99, 100, 237] where, in fact, this is not usually the case. Another example can be observed from [93] where the simplifications were the main source of the inconsistent results at the initial regime of motion (transient response) between the numerically obtained system's response and the one predicted by the SIM derived from the analytical treatment. Since this inaccuracy is at an early stage of motion, it is very critical to TET analysis where rapid shock and seismic mitigations are required. From the review conducted, it was observed that all the works related to the analytical treatment of VI NESs consider a SDOF linear oscillator. This is again done to simplify the dynamics involved where multi-DOF systems will require more complicated mathematical analysis. Moreover, the reviewed works have not considered stiffness or damping elements in the NES coupling (i.e. all works considered the cavity schematic model of the DSVI NES as previously shown in Fig. 3). These two elements are important for analysis of shock and seismic mitigation because the damping element coupling the NES with the primary system contributes to the energy dissipated and the stiffness element helps in inducing significant restoring momentum into the NES mass which makes the consequent impacts stronger and thus causes more energy dissipation.

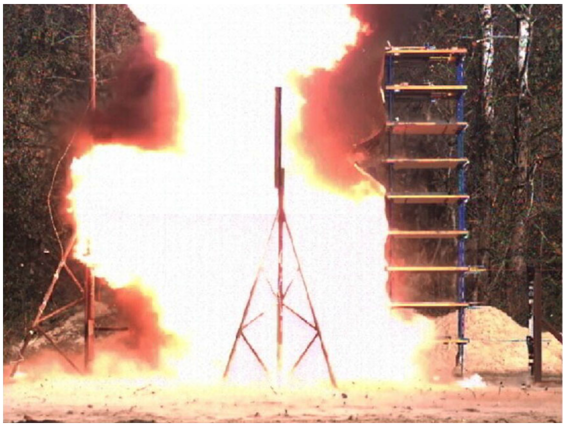

Fig. 8 Large-scale 9-story structure with two SSVI NESs and four Type I NESs under a high intensity blast loading [110]

5.6 Numerical analysis

Based on the review conducted, all research works, either related to DSVI NES or SSVI NES, included numerical studies either to verify the analytical or experimental results or to show that the systems can achieve efficient TET. The numerical analysis is usually carried out using MATLAB where Runge–Kutta methods are applied to solve the governing differential equations of motion. A fundamental research work in this field was presented in [103] where a linear oscillator coupled with a DSVI NES was numerically studied. First, the underlying nonlinear dynamical signature of the Hamiltonian system was studied and depicted in frequency-energy plots. In addition, the damped dynamics and the passive TET mechanism for different NES parameters and excitations were explained via wavelet transform spectra. It was observed that the TET is efficiently achieved when the VI impulsive orbits of the highest energy content are excited. This outcome has provided a main advance for further analytical and numerical investigations in the field.

5.7 Experimental validations

Plenty of works included experimental tests to validate the analytical or numerical investigations of systems coupled with DSVI and SSVI NESs [60, 94, 96–98, 100, 107, 109, 110, 176, 177, 232]. Seismic, periodic and impulsive excitations were employed using appropriate shake tables. Small- as well as large-scale structures were tested in different sized shakers with and without VI NESs to evaluate the capability of energy dissipation. One important observation deduced from the review conducted is that most research works related to SSVI NESs [107, 109, 110] contained experimental evaluations. This is believed to be due to the enhanced performance demonstrated by the SSVI NESs in the numerical simulations. To the author's knowledge, the largest test bed for TET testing is the 11-ton 9-story steel frame structure investigated in [106, 109, 110] for seismic and shock mitigations using SSVI NESs and Type I NESs. A recent experimental work by N. E.Wierschem et al. in [110] considered this large-scale structure for the vibration mitigation of real offset-blast loadings with different intensities as shown in Fig. 8. The 9-story large-scale structure was equipped with one SSVI NES and two Type I NESs in the highest floors. The SSVI NES caused quick modal energy transfer where this portion of energy was dissipated through the modes of the structure. Another important benefit of this redistribution is that the higher frequency structural modes have lower amplitude oscillations and therefore this limits the peak stresses and strains subjected to the structure. The SSVI NESs demonstrated immediate response to impulsive loading where they were activated in the early energy stage of motion to redistribute energy to the higher modes and to Type I NESs which were added to the structure because they activate at a later stage to further enhance energy dissipation. The large structure was able to withstand different intensity blasts and thus this work is considered as a main advancement in the field of experimental investigations of NES systems because it represents the first real blast successful response attenuation for large-scale structures. In [238], experiments were conducted for SDOF linear system coupled with a DSVI NES model which is symmetrically constrained with dissipative and deformable bumpers. Multiple scenarios that can occur during the vibro-impact cycle were classified based on the forward and backward Pseudo Resonance Curves. Some of these scenarios includes the presence of the existence of primary resonance with hysteresis and secondary hysteresis, secondary regular resonance without hysteresis. The purpose of the scenarios is to provide a better understanding of the VI NES.

5.8 Efficiency measures and optimization procedures

This subsection focuses mainly on research works involving the efficiency measures and optimization procedures of VI NESs to achieve efficient energy dissipation. Starting with seismic excitations, a set of eight evaluation criteria introduced in [239, 240] is utilized as quantitative measures of the VI NESs. The first four criteria of the set are related to the maximum absolute floor displacement, inter-story drift, floor acceleration and inertial force at each DOF. The other four criteria of the set are related to the maximum normed values of the same quantities. For all of them, the numerator represents the controlled response (i.e., when the VI NES is activated) and the uncontrolled response was represented by denominator (i.e., when the NES is locked). In all publications in that regard, the evaluation criteria related to the maximum absolute, normed acceleration, and inertial force were not considered in the objective function because the strong ground motion and the strong impacts caused by the NESs induce high accelerations and forces to the associated floors and are therefore inadequate efficiency measures. The remaining four evaluation criteria are involved in the objective function as done in [60, 61, 104, 231]. The only work that considered two evaluation criteria instead of four is [109] which addressed the employment of SSVI NESs for large-scale structures under seismic excitations. It only considered the maximum absolute displacement and inter-story drift. In general, the ultimate goal is to find the NES parameters that will minimize the objective function. That is how the efficiency is measured for systems excited seismically. The systems are then optimized for the NES parameters which are the impact clearance, NES mass ratio, stiffness NES coupling element and the coefficient of restitution.

From the review conducted, it was found that two different methods were followed to get the optimum parameters after specifying some ranges in which the parameters can be varied. In [60, 61, 104], a global optimization method called differential evolution was followed. Details about the method can be found in [241]. The other method followed in [231] is referred to as meta-heuristic approach which was introduced recently in [242]. Both methods provided optimum results for the NES parameters; however, the differential evolution method was found to be more popular due to its simplicity and earlier utilization. Accordingly, [111] considered an optimization approach based on Nelder–Mead Simplex method [243] which gave enhanced results for the mitigation of shock-loaded large-scale primary structures.

For impulsive excitations, the efficiency of the systems is measured by calculating the percentage of energy transferred from the structure and dissipated by the VI NESs. Therefore, it is essential to recall the sources of energy dissipation due to the VI NESs which are:

1. The NES viscous damping.
2. The energy loss caused by the inelastic impacts which is calculated based on kinetic energy change due to impacts.
3. The third source of energy dissipation is the energy scattering and dissipation by the modes of the structure. This dissipated energy by structural modal damping is calculated by defining a set of equivalent modal oscillators where the energy pumped and dissipated by each structural mode can be found.

In [106] and [107], all the previous sources of dissipation were included in the efficiency measure of the TET, however, in [102] and [103], only the first two sources were included. One critical aspect regarding these efficiency measures is that they lack the quantification of how rapid the energy gets dissipated. In most of the previously mentioned publications, enough simulation time was given so that the system settles and the amount of energy dissipated can be measured but the rate of dissipation was not included and therefore one cannot judge the rapidness of the shock mitigation which is vital particularly here where energy dissipation at the initial stages of motion is required. One work by Lee et al. in [103] illustrated an innovative approach to measure the efficiency of the TET for a SDOF system coupled with a DSVI NES. In addition, a logarithmic function of the remaining, instead of dissipated, energy has been considered in [108] as the optimization parameter to account for the energy stored in the NES which may transfer back to the structure.

Similar to seismic excitation works, after determining the efficiency measures, the systems are then optimized for the NES parameters which are the clearance, mass ratio, stiffness element, damping element and coefficient of restitution. From the review conducted, it is observed that the damping in the NES is considered in some works obviously related to the shock mitigation because it represents the major source for energy dissipation in the early phase of motion. Contour plots that consider ranges of two parameters of the NES to present the fraction of the energy dissipated by the VI NES were implemented in [102, 103, 106–108]. Although this method sometimes provides sufficient means to achieve efficient TET, however, nonlinear systems possess very complex dynamics and the NESs have many variable parameters. As a result, since the contour plots consider the change of only two parameters at a time, this optimization might not capture the synergistic effects of the highly nonlinear dynamics. In [110], which presented blast response mitigation in a large-scale real structure, the optimization was carried out in two stages. Firstly, the attached Type-I NESs with the eight floor were optimized to eliminate significant portion of energy from the first torsional mode. The remaining NESs were optimized in a second stage to eliminate significant portion of energy from the lowest frequency lateral oscillation mode. In this work, given the large number of NES parameters to be optimized, a MATLAB gradient-based optimization function called *fmincon* was used in the first stage. The same function along with another MATLAB-based genetic algorithm was used in the second stage. Given the complicated algorithms followed by these methods, they are better than contour plots (double-for loops)

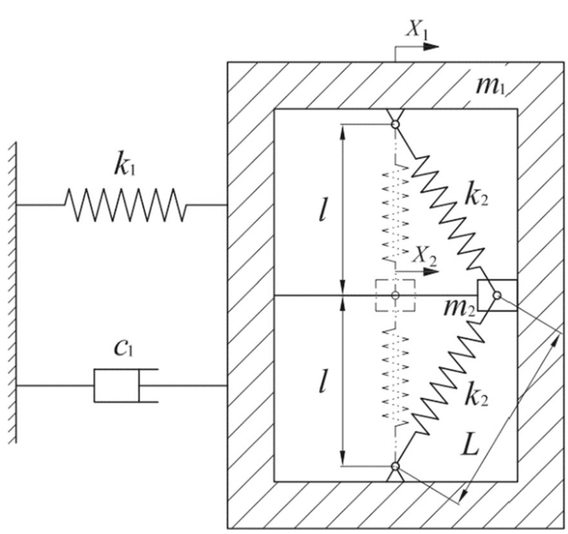

Fig. 9 Schematic diagram of a 2-DOF system coupled with a VI-BNES [253]

for locating the global and local maximums or minimums for multiple input parameters because they involve the synergistic effects between all the NES parameters. Lastly, from the review conducted, it was found that there is one major difference observed between the works that consider seismic excitations and those that consider impulsive excitations. The seismic mitigation efficiency measures are related to the response of the system (maximum displacement of the floors, etc.). On the other hand, works that involved shock mitigation focused more on the energy dissipated through the VI NESs themselves rather than the response of the floors of the systems. This is due to the fact that seismic excitations continuously energize the system and therefore continuous monitoring of the response of the floors and a determination of the maximum response values is important. However, the shock excitation only occurs during the initial stage of motion and what mostly matters is the quick dissipation of that energy.

5.9 Improvements of VI NESs

The concept of low-to-high frequency TET refers to the intermodal energy flow, caused by the nonlinear mechanism of the NES, through the structural modes of the structure of concern. This concept was firstly addressed in the literature in [106] where it was later renamed as “intermodal targeted energy transfer (IMTET)” in [244]. The IMTET can be broadly defined as the case when the energy applied to a system is passively redistributed through the structural modal space by the action of local nonlinearities [245]. It is a non-resonant mechanism because the impacts takes place on fast time scales. Another advantage of the concept is that it eliminates the need of additional NESs or other vibration absorbers thereby reducing the constraints involving the structural design.

The IMTET is a recent development in passive targeted energy transfer where some of the applications of IMTET are addressed here. In [246], a steel beam core structure was placed at suitable position within the 9-story structure where the optimized clearance distribution is incorporated between the original structure and the core structure. It has been found out that the impact nonlinearities enable a significant percentage of energy to be rapidly transferred and dissipated by the high frequency structural modes. However, high acceleration levels could be induced into the structure during the early phase strong impacts. The low-to-high frequency TET was recently addressed in [247] where the performance of the NES in a softening-hardening spring was studied. Accordingly, incorporating SH spring in the NES helped in vibration suppression through low-to-high frequency TET. In [248] a vibro-impact acoustic black hole (VI-ABH) mechanism was proposed for mitigating vibration in flexural beams. The ABH consists of placing a thin viscoelastic coating in the minimum thickness area of the light-weight structure in the form of tapered beams with a wedge profile where the theory of propagating flexural waves is applied. The main drawback of ABH is that it cannot function in low-frequency range [249] since it has a cut-on frequency, below which it cannot function properly [250]. Therefore, ABH functions properly at high frequency levels while it is ineffective at low frequency levels. As such, in [251], Type I NES, BNES, and the SSVI NES were employed in conjunction with ABH to control more modes of operation below the cut-on frequency.

The difference between the traditional VI NES and the Electromagnetic VI NES was investigated in [252]. Accordingly, the inelastic contact between the impact mass and the barriers causes the energy dissipation in the traditional VI NES, whereas in electromagnetic VI NES the electric damping as well as the impacts are the main sources for energy dissipation. In the electromagnetic VI NES, the damping due to impacts contribute up to 30% of the total damping while the remaining damping effect was caused by the linear electric damping. Even though the performance of the electromagnetic VI NES was subpar with that of the traditional NES, it is more efficient than the traditional VI NES in vibration suppression for broadband vibration energy levels. A new NES that combines the characteristics of vibro-impact and bistability in one NES, called VI BNES is shown in Fig. 9 and has been investigated in [253]. The Melnikov method which is used with non-smooth systems was employed for studying the global dynamics of the VI-BNES system. The performance of the VI-BNES was also analyzed in comparison with the BNES and the VI NES where it was claimed that the VI-BNES slightly outperforms the BNES and VI NES.

Although the field of impact damping has long been investigated, there is still huge room for further developments in the area. As an example of a recent

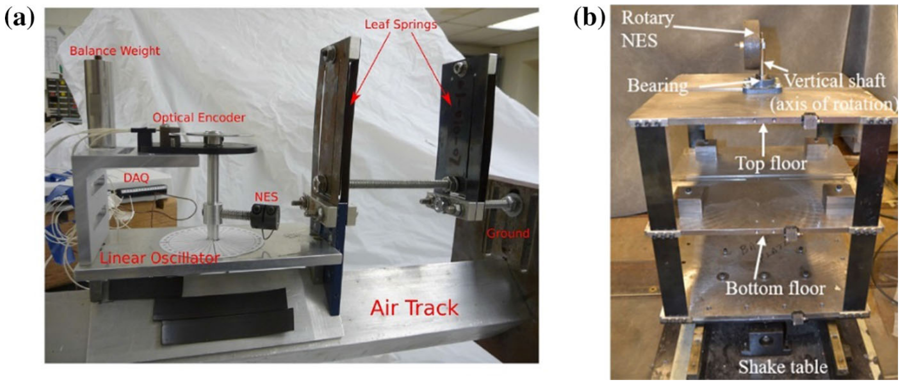

Fig. 10 Experimental realizations of rotary NESs equipped to: **a** a linear oscillator [90] and **b** 2-story structure [88]

breakthrough, AL-Shudeifat developed the single-sided VI NES which significantly outperformed the previous double-sided VI NES and, in fact, is considered the best NES to be applied for shock mitigation in structures [128–132]. A further enhancement to the single-sided VI NES is discussed in [111]. Usually, a coefficient of restitution of 0.7 is used in VI NESs which is nearly equivalent to that of steel-to-steel collision. Accordingly, the work presented in [111] was aimed to optimize the VI NESs parameters by including the coefficient of restitution as one of the objective optimization functions. Consequently, a coefficient of restitution near 0.45 has been found improving the shock mitigation performance in the VI NESs.

Another enhancement is the track single-sided VI NES where the NES mass is forced to move in a pre-optimized track to enhance energy dissipation [128–132]. The performance of the track SSVI NES in [132] was found outperforming the TMD performance in the considered 2-story lab structure subjected to various intensity impulsive and seismic excitation inputs. However, compared with stiffness-based Type I NES, the TMD stays more efficient in seismic mitigation [254]. In the DSVI NES, the linear coupling element was replaced by a nonlinear cubic restoring element in [255] to broaden the efficient energy absorption range of VI NESs. For the same reason, a hybrid system comprising a tuned mass damper and DSVI NESs is proposed in [256] where the TMD is active for low impulsive energies and the VI NES is activated for high energies.

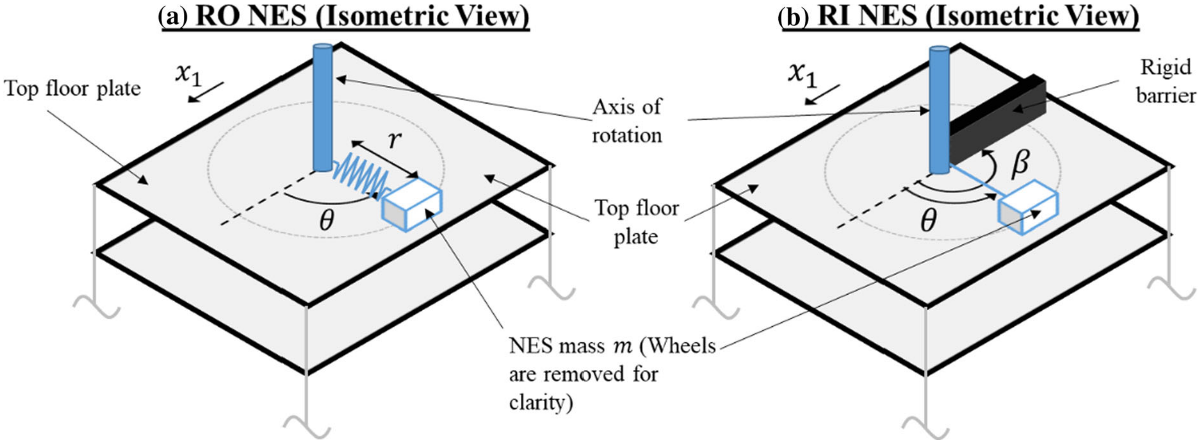

Fig. 11 Schematic diagrams of rotary-oscillatory NES in (**a**) and rotary-impact NES in (**b**)

6 Rotary-based NESs: design, analysis and applications

The rotating NES mass is inertially coupled with the primary structure as shown in Fig. 10 by a horizontal rigid arm. The NES mass rotates in a horizontal plane in which the primary structure also moves. In addition, linear rotational viscous damping is incorporated into the rotary NES. This novel type of NES was first proposed in 2012 by the work of Gendelman et al. [90] where the ability of the eccentric rotator to resonate with any frequency of the linear oscillator, and thus act as an NES, was proven. Given the simplicity and efficacy of the rotary NES demonstrated analytically and experimentally in [90], several works followed with further investigations. In [81], the occurrence of a cascade of resonance captures for a system consisting of linear oscillator and rotary NES was shown. In addition, the correspondence of the resonance captures to the NNMs was extensively studied. More importantly, it was shown how these dynamical mechanisms contribute to TET to further validate the ability of the rotator to act as an NES. Sigalov et al. [92] further studied the NNMs of the same 2-DOF system highlighting the alternate transitions between regular and chaotic dynamics.

The first research to implement the rotary NES to multi-DOF primary structure was done few years later in [88]. The arm radius and torsional damping coefficient were optimized to obtain the maximum possible energy dissipation when the NES was employed to a 2-story real physical structure. In addition, the energy dissipated by the second mode (higher-frequency lower-energy) was also investigated. It was shown that this NES can scatter the induced energy between the modes of the structure through its essential nonlinear coupling. In addition, the rotary NES was proven numerically and experimentally to provide comparable results with the traditional stiffness-based NESs. The two implemented setups of rotary NESs are shown in Fig. 10.

The rotary NES was also implemented for vortex-induced-vibrations (VIV) applications in [89] and [173] where the dynamics of a sprung cylinder constrained to move in the cross-flow directions coupled to a rotary NES was investigated numerically and analytically using an asymptotic analysis to define the 1:1 resonance captures in the slow-invariant-manifold. In [190], proper orthogonal decomposition was applied to further explain the TET mechanisms. In addition, another series of research works has considered the concept of a transitionally coupled two-dimensional model of locally resonant unit-cell incorporating an internal rotator for bidirectional and unidirectional energy channeling [257–259]. Accordingly, reversible and irreversible energy flows have been found to exist between longitudinal and lateral responses due to the coupled property of the NES which leads to efficient passive vibration control of the systems. The analysis was extended to study the following: (1) a quasi-one-dimensional chain of coupled cells that all incorporate internal rotators in [232, 260], (2) the effect of asymmetric potential in [261] and (3) the complex nonlinear mechanism caused by energy channeling in three-dimensional locally resonant unit cell (incorporating a spherical rotator) in [262].

In [263], the rotary NES was modified into a rotary oscillatory NES (RO NES) by employing an elastic coupling arm in the radial direction than the rigid coupling arm. Accordingly, the rotating NES becomes associated with a 2-DOF configuration in which radial oscillation and angular rotation or oscillation are taking place. This structural modification provides the rotating NES with the added capacity of radial oscillation as shown in Fig. 11a in order to achieve more robustness in energy transfer and dissipation. Therefore, the transferred energy dissipation by the RO NES is achieved by the angular and radial damping elements during the coupled angular rotation/oscillation and the radial oscillation. The optimized RO NES was found to significantly achieve enhanced shock mitigation for small- and large size structures compared to the rotary NES. In [264], the rotary NES design was modified by incorporating a vibro-impact to the rotary NES as shown in Fig. 11b to explore the synergetic effect by the resultant rotary-impact NES (RI NES) on performance improvement. The resultant RI NES was found outperforming the original RO NESs in performance and in the inter-modal energy transfer through the structural modes.

In a recent study, the rotary NES's capability to function against vortex-induced vibration in submarine currents has been investigated [265, 266]. The rotary NES is attached inside a cylindrical body positioned horizontally in transverses direction to the vortex-induced current. Therefore, the rotary NES rotates about the axis of the cylinder. It was found that the rotary NES is capable of performing passive energy harvesting of significant amount of cylinder's kinetic energy which also resulted in vibration suppression of cylinder oscillation [266].

7 Conclusions

The state-of-the-art of nonlinear energy sinks (NESs) has rapidly grown in the past two decades. The NES is a dynamical attachment of small mass fraction of the whole mass of the associated structure. The NES function is to mitigate unwanted or damaging vibration amplitudes in different kinds of large-scale structures. The nonlinear coupling force of the NES attachment is essential for efficient TET and dissipation. Most of the received input energy by the structure is rapidly transferred to the NES and dissipated there. Unlike the TMD, the NES transfers and dissipates the received energy by the structure of concern in a broadband frequency-energy fashion. This energy transfer takes place by multiple of resonance captures on the backbone curves and the subharmonic resonance branches in the frequency-energy plot. There are several types of NESs employing different kinds of nonlinear coupling such as elastic nonlinear elements, inertial coupling elements and impact-based coupling nonlinearities. This review article is focusing on the developments on the impact- and rotary-based NESs in comparison with the TMD and other existing types of NESs. Generally, the impact- and rotary-based NESs outperform the TMD and the stiffness-based NESs by their capability in rapid vibration suppression and energy redistribution through the structural modes.

Throughout this review of the literature on VI NESs, several key observations are concluded as follows:

- Because of its nonlinear nature, VI NESs are capable of achieving efficient TET for broadband frequency ranges by their local viscous damping, inelastic collisions and redistribution of input energy to higher structural modes. They possess several advantages when compared to other types of NESs. These include simple implementation, fast reaction time and high performance for a wide range of input energies.
- One disadvantage of VI NESs is that coupling it with a primary structures leads to very complex nonlinear dynamics that is difficult to analyze analytically without making several simplifications.
- The research works that considered the analytical treatment focused on periodically excited SDOF linear oscillators coupled with DSVI NESs. Multi-DOF systems were rarely analytically studied. Moreover, the DSVI NESs considered in those works is employed using the cavity model. Most importantly, the analytical works studied the VI NES from a kinematic viewpoint where it was optimized to achieve resonance captures with the system to engage in TET.
- The analytical treatment of systems coupled with SSVI NES and the study of its underlying Hamiltonian system were not considered in the literature.
- DSVI NESs were mainly applied to dissipate energy due to forced excitations. Some works addressed the DSVI NES for seismic mitigation but very few works addressed it for shock mitigation because of its limited performance therein. On the other hand, SSVI NESs were mainly implemented for shock mitigations given their faster reaction time and efficient TET at the initial stages of motion.
- Another observation deduced from the literature review is that the DSVI NESs are mostly applied for small-scale structures, whereas the high efficiency of the SSVI NESs enabled their implementation to large-scale structures.
- It was observed that most research works which investigated the TET using SSVI NES have implemented an experimental validation. This is mainly due to the fact that numerical simulations of systems with SSVI NES provide promising results to be applied to real life structures.
- It was observed that the seismic mitigation efficiency measures were related to the response of the system (maximum displacement of the floors, etc.) and the shock mitigation efficiency measures were related to the energy dissipated through the VI NES.
- Two main aspects were highlighted from the parameter optimization of the impulsively excited systems. The first is that most works do not include the rate of energy dissipation in the efficiency measures. The second is that most works use

contour plots to find the optimal performance when comparing only two NES parameters. This neglects taking into consideration the potential synergistic effects of the multiple NES parameters induced due to the high nonlinearity in the system.
- Generally, SSVI NESs achieved more efficient TET than DSVI NESs by possessing faster reaction time, better energy redistribution to higher structural modes, being more effective at a wider range of initial impulsive loads and were capable of dissipating energies at severe shocks.

Several key observations have been concluded in regard to rotary NESs. Relatively, fewer works have addressed the use of the rotary NESs compared to VI NESs. However, the efficient performance of the rotary NES gives promising potential for the full-scale implementations mainly due to: (1) they do not incorporate strong vibro-impacts which induce high acceleration levels to the primary structures and (2) they can be easily applied for multi-direction primary structures.

Modifying the rotary NES to rotary-oscillatory and rotary-impact versions has significantly enhanced the vibration mitigation and energy redistribution performance. Similarly, modifying the symmetric impact NES into the single-sided asymmetric vibro-impact NES with a proper selection of the coefficient of restitution has provided NES design that outperforms all existing types of NESs in the literature in energy redistribution and dissipation. In addition, the enhanced impact- based NESs are robust for a wide range of viscous damping degradation compared with the TMD and other types of NESs in the literature. The robustness to damping deterioration in NESs is also a fundamental property for successful NES design and performance. As a final remark, since the literature in frequency-energy analysis is challenging and limited, it will be very attractive research topic for thorough analysis, analytical numerical and experimental, that is expected to reveal more information and findings to the underlying nonlinear dynamical behavior and facilitate the implementation of NESs in real-life structures and applications in the future.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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