



The dual Fourier transform spectra (DFTS): a new nonlinear damage indicator for identification of breathing cracks in beam-like structures

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Abstract Fatigue damage is a common type of structural damage associated with long-term cyclic loading operating on engineering structures. Within the fatigue damage phenomenon, the breathing cracks are basic

and widely occurred forms of fatigue damage in structures. Therefore, the identification and assessment of breathing cracks are very crucial. Some damage identification methods to assess the breathing cracks are available, but they have a common limitation is that they focus on detection of the breathing crack, but they cannot characterize its severity. Hence, the characterization of breathing cracks by creating robust nonlinear damage indicators, that can accurately quantify the degree of nonlinearity of breathing cracks, is of great significance. To this end, a robust nonlinear damage factor is developed by effectively utilizing the Fourier transform in a multi-stage manner. The procedure of creating this factor involves two steps of progressive use of fast Fourier transform (FFT): (i) damage detection, where the secondary peaks of the Fourier spectrum indicate the presence of breathing cracks and imply the dynamic inner interactions of them, and (ii) damage quantification, where the FFT of the Fourier spectrum is implemented and the resulted features that can quantitatively depict the degree of nonlinearity of breathing cracks are utilized. The new method is called the dual Fourier transform spectra (DFTS), where the FFT of the Fourier spectrum is defined as a robust nonlinear damage factor for breathing crack assessment in beam-like structures. Various single and multiple damage scenarios are studied numerically and experimentally after considering noisy conditions. The proposed method demonstrates outstanding performance and can

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be recommended for damage detection and quantification of fatigue-breathing cracks in beam-like structures.

Keywords Fatigue crack · Breathing crack · Multiple cracks · Damage severity · FEA · Experimental tests

1 Introduction

Fatigue-breathing cracks can appear in engineering structures after a long duration of repetitive loading and can develop to severe levels, threatening structural integrity and safety which might lead to structural collapse [1]. The fatigue cracks are dangerous because they are usually negligible compared to the cracked structure, as they cause minor structural changes in the dynamic properties. These cracks open and close continuously according to the cyclic working loads, and thus they are called breathing cracks [2]. The structural stiffness at the damage position is considered to be instantaneous and nonlinear to precisely represent the opening–closing translation of harmonically excited breathing cracks [3].

Compared with the modeling approaches of the open cracks, the modeling of the breathing cracks is more realistic because it leads to nonlinear vibration responses due to the contact between the cracks surfaces, which results in higher harmonics [4]. These higher harmonics could be sensitive indicators of the presence of fatigue damages in structures. Methods concerning the modeling of breathing cracks and their generation of higher harmonics mechanisms were reviewed in [5].

It is well-known that nonlinearity could be exploited for detecting damage that can be realized from several studies reported in the literature. Nguyen [6] presented a damage detection method based on the wavelet spectrum to study the effect of breathing and open cracks on a vehicle-bridge system under a moving vehicle. Andreaus and Baragatti [7] proved that breathing crack increases damping and decreases the natural frequencies, which can be considered as damage indicators. Giannini et al. [8] proposed a nonlinear harmonic identification technique based on responses to the system's nonlinear characteristics.

For detecting the breathing cracks, Boungou et al. [9] investigated the nonlinear model of a breathing crack under low cycle fatigue utilizing cyclostationarity. Habtour et al. [10] presented a nonlinear vibration method for detecting fatigue damage precursors based

on the extraction of nonlinear dynamic characteristics. Huang et al. [11] introduced a novel approach using the nonlinear output frequency response functions to study the cumulative fatigue damage. Lin and Ng [12] studied the higher-order frequency response functions and their application for damage detection and assessment to structures with breathing cracks. Prawin and Rao [13] introduced a nonlinear damage identification method based on the Volterra series applying the adaptive Volterra filter utilizing the nonlinear time history response. In their other research work [14], they examined the adaptive filter algorithm via solving a bilinear oscillator of a beam containing a breathing crack. Prawin et al. [15] presented a novel baseline-free detection algorithm for detecting, localizing, and characterizing fatigue cracks using the singular spectrum analysis. Long et al. [16] proposed a novel stiffness model that reflects the breathing crack partial closure effect to show the stiffness variation in the damaged beam. Avramov and Malyshev [17] derived a nonlinear dynamic model based on the Galerkin method to studied the bifurcations and chaotic forced vibrations of a cantilever beam containing two opposite breathing cracks. Huang et al. [18] proposed a method for crack detection based on the phase diagram properties.

In the forced vibration, the stiffness is changing periodically during the harmonic vibration. Higher harmonics could be an indication of the presence of breathing cracks. Andreaus and Casini [19] introduced an identification method based on analyzing the static deflection of multi-damaged beams. They developed a new damage index based on CWT by calculating the deflection of damaged beams for different crack locations, sizes, and load conditions. The analysis results proved that the damage index does not depend on the mechanical features of beams, and the damage index of one damage also does not depend on either the location or size of other damages of the beams. In other work, Andreaus et al. [20] presented an identification method for crack detection and quantification of static deflection in beams using wavelet analysis. They studied simply supported beams with single and double cracks and open and fatigue cracks of different severities and locations. Xu et al. [21] proposed a new identification method based on nonlinear pseudo-force to detect and locate breathing cracks on beams and analytically investigated the intrinsic force that drives the breathing crack to generate higher harmonics. The influence of higher harmonics has been examined by introducing

several scenarios. Cui et al. [22] studied the effects of a beam's nonlinear characteristics containing a breathing crack under different harmonic loadings using a bispectrum analysis method. The influence of loading conditions, crack depth, location, and noise on the nonlinear dynamic properties was examined. Recently, Prawin and Rao [23] introduced two new indices for breathing crack detection in beam-like structures. The two indices exploited the decrease in the amplitude of the Fourier power spectrum of the linear and nonlinear components due to the energy transformation to the nonlinear harmonic components. The results demonstrated that the linear components of the Fourier power spectrum amplitude could be efficiently analyzed for damage detection of breathing cracks.

Several studies [24–26] have examined the breathing crack behavior of damaged structures when subjected to external dynamic excitation. When the crack is completely closed, the structure is considered an intact structure. On the contrary, when the crack is completely open, the structure is considered a damaged structure. The breathing crack behavior was observed in the form of higher-order harmonics in the frequency spectrum of the structural nonlinear vibration response, which are fractional or integral multiples of the excitation frequency. Higher-order harmonics are generated due to the opening and closing of breathing cracks under external dynamic excitation, which produces tensile and compressive stresses in the cracked area, known as breathing crack behavior. The dynamic response of the structure then becomes nonlinear due to the existence of a breathing crack, which is associated with localized stiffness changes. The local stiffness during the crack opening will decrease due to the tensile stress, which corresponds to a damaged state. Conversely, the local stiffness during the crack closing is restored due to the compressive stress, which corresponds to a healthy state. The applied load to the structure induces a periodic change in the local stiffness between intact and damaged states, which can be utilized to characterize the nonlinear dynamic response. If the change in the local stiffness is large, the generated nonlinearity is significant, which corresponds to a significant crack size. On the other hand, when the change in the local stiffness is insignificant, the generated nonlinearity is small, which occurs in the case of small depths of the crack.

The limitations of existing research can be summarized as the fact that most of the existing methods for

characterizing breathing cracks are based on comparing the first or second amplitudes of superharmonics and sideband components collected from intact and damaged structures. Moreover, these nonlinear harmonics depend on different factors, including crack depth, location, damping amount, as well as the excitation frequencies and magnitudes. Nevertheless, the extraction of sidebands and superharmonics may mislead the damage identification process due to the high noise measurement and environmental fluctuations.

Based on the gaps in the existing research, in this paper the vibrations of beam-like structures containing breathing cracks are analyzed, where different beam models of single and multiple breathing cracks are studied. The main aim of this research is to analyze the effect of crack depth, crack location, damping, excitation frequency and magnitude in addition to noise immunity on the nonlinear dynamic responses of a cantilever beam. Furthermore, in order to provide a complete picture about breathing cracks in beam structures, different beam models, including multiple, opposite, and open cracks, are studied. The numerical study is first conducted using the ABAQUS finite element software, and the acceleration signals are registered. Thereafter, the proposed method of breathing crack identification in beam structures is applied and results are analyzed and discussed. Finally, the numerical simulation results of the proposed method are validated experimentally.

The rest of the paper is organized as follows: (i) the problem formulation of identifying breathing cracks, the typical methods, and their limitations are described in Sect. 2; (ii) Sect. 3 introduces the fundamentals of the proposed approach in which a new concept based on the FFT is proposed; (iii) Sect. 4 presents the application of the new DFTS method for the breathing crack assessment in beam-like structures based on intensive numerical investigations; (iv) by taking the successful numerical results as basis, Sect. 5 presents the experimental setup and validation of the applicability of the proposed concept; (v) finally, the main conclusions of this work are drawn together and presented in Sect. 6.

2 Problem formulation in identifying breathing cracks

2.1 Typical methods to identify breathing cracks

In recent decades, the community have showed a great interest in structural dynamics and their application in SHM by analyzing the dynamic response of structures exposed to excitation forces. Such interest has led to the development of many SHM approaches, most of which are limited to linear structures [27,28]. Various research works assumed that cracks remain open during vibration, and thus a linear system has been considered, in which the dynamic properties of the linear system, such as changes in natural frequencies and mode shapes, are exploited to determine the extent of the damage. However, the linear characteristics of the response are invalid when identifying fatigue crack due to the effect of crack breathing in reality [16].

On the other hand, considering the crack as a breathing crack is more realistic, but many difficulties surround the identification process where the changes in the natural frequencies are between the changes of the healthy beam and the damaged beam, i.e., a fully open crack. Gudmonson [29] studied the effects of continuous crack closure on fractured beam vibrations through experiments. Therefore, the dynamic system is considered nonlinear due to the stiffness changes caused by the opening and closing of the crack.

Over the past few years, some researchers have developed several methods used to study beams containing breathing cracks, taking advantage of nonlinear features. Among these methods, there are analytical methods [30–32] numerical methods [33,34], and experimental methods [35,36]. Such studies have shown that the nonlinear vibration features have a higher sensitivity to breathing cracks, particularly the existence of the so-called sub- and superharmonic resonances [37]. One successful study was reported by Wei and Shang [38] in which they established a mathematical model to simulate bi-linear stiffness based on the Timoshenko beam theory in order to describe the effect of breathing cracks.

2.2 Limitations in existing methods

Nowadays, the industrial facilities have many methods for detecting structural damage. Each method dif-

fers from the others in terms of its advantages, limitations, drawbacks, and applications. However, there is no fully reliable and viable method for detecting different types of damage and structures [39–43]. The application under interest mainly determines the selection of an appropriate damage identification method [44–47]. From this point of view, the identification of damage based on using nonlinear vibrational effects could be very promising for applications such as rotating shafts and beam-like structures. The most important characteristic of nonlinear vibrations is their high sensitivity to the presence of breathing crack. Among the nonlinear effects, the appearance of super- and subharmonic resonances of orders 2/1 and 1/2 are useful for detecting small cracks and indicating the severity of cracks, respectively.

Another important point, when utilizing the nonlinear effects to detect damage, is paying attention to the damping value in the system because the crack propagation increases in the level of damping significantly. Therefore, it suppresses partially or totally the nonlinear effects that will appear. Wong et al. [48] used a simple harmonic balance method to obtain the periodic solutions of a harmonically excited nonlinear oscillator in order to simulate breathing crack. The method works well for a weakly nonlinear system but has some difficulties to deal with large nonlinear system. Such difficulty could be overcome by applying a method called the Incremental Harmonic Balance Method (IHBM). Choi and Lou [49] introduced a combined method based on both harmonic balance and FFT methods to study the dynamic responses of both piece-wise linear and nonlinear systems. In another related study, Chatterjee et al. [50] extended the equivalent linearization method for obtaining periodic responses of piece-wise nonlinear oscillators with harmonic excitation. The method has the advantage of involving super- and subharmonics, providing better findings than the harmonic balance method. Andreaus and Baragatti [36] studied experimentally and numerically the damage detection of a steel cantilever beam containing a transverse breathing crack using the nonlinear characteristics of forced responses. The FFT spectrum of the acceleration signals showed both sub- and superharmonics components of the damaged states, indicating the presence of damage. This method induces many uncertainties; therefore, its use in practice is limited.

The FFT is a common tool that can be used to describe the properties of breathing cracks. There

are some common problems and limitations in the aforementioned methods used to characterize fatigue-breathing cracks. One major problem is that they can detect breathing cracks but not very clearly. For example, the Fourier spectrum method cannot clearly distinguish between several fatigue-breathing cracks. In addition, these methods have a common restriction: they are qualified to indicate the existence of breathing cracks, but they cannot characterize the severities of breathing cracks. Therefore, the present work intends to solve such problems by introducing the new DFTS damage identification approach that is efficiently able to both detect the breathing cracks and determine their severities.

3 Theoretical formulation of the DFTS for breathing crack identification

The Fourier transform is a frequency-based transform broadly utilized in the study of linear systems. Fourier transform breaks up the signal into different sine waves with different frequencies and corresponding amplitudes. However, the sum of different sine waves returns the original signal.

The Fourier transform is a powerful tool used for the analysis of many scientific problems and applications. It converts any signal from the time domain to the frequency domain. The objective of converting data from the time domain to the frequency domain is to extract the required data that cannot be extracted in the time domain. The Fourier transform and its inverse [51,52] of a continuous signal can be defined as:

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft} dt, \tag{1}$$

and

$$x(t) = \int_{-\infty}^{+\infty} X(f)e^{j2\pi ft} df, \tag{2}$$

where $x(t)$ is the time-domain signal, the variable t represents the time, and f is the frequency in units of hertz (Hz). However, when we have a discrete signal, the Fourier transform of the continuous signal is invalid, so the so-called discrete Fourier transform (DFT) is introduced. The Fourier transform of a discrete signal transforms a sequence of N complex numbers $x(n) :=$

$x(0), x(1), \dots, x(N - 1)$ into another sequence of complex numbers, $X(k) := X(0), X(1), \dots, X(N - 1)$, which is given by the following expression as:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi kn}{N}}, \tag{3}$$

The DFT is characterized by the fact that its spectrum is periodic and its period in the frequency domain is f . And the formula for the Inverse Discrete Fourier Transform (IDFT) is defined as:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{\frac{j2\pi kn}{N}}, \tag{4}$$

where n and $k = 0, 1, 2, \dots, (N - 1)$.

The number of computations involved in calculating the DFT is much higher, so in order to overcome this problem, an easy and fast algorithm called the FFT algorithm was introduced. The number of computations is significantly reduced; instead of $O(N^2)$ in the DFT, it is $O(N \log_2 N)$ in the FFT. The difference increases the more the number of samples increases. Thus, the FFT is an important and simple algorithm. Moreover, it has been proven that the DFT of length N could be rewritten as the sum of two DFTs, each of length $N/2$.

The FFT can be deduced from the DFT as follows:

$$S(k) = \sum_{n=0}^{N-1} s(n)W^{nk}, \tag{5}$$

where $W = \exp(-j2\pi/N)$, which is a complex number. Euler's formula for any real number α states that: $e^{\pm j\alpha} = \cos \alpha \pm j \sin \alpha$,

where the exponential is the base of the natural logarithm, j is the imaginary unit, and \cos and \sin are the trigonometric functions cosine and sine, respectively.

The objective of the FFT algorithm is to simplify and perform the calculations in the least time, so Eq. (5) can be rewritten as even and odd summations as follows:

$$S(k) = \sum_{\substack{n=0 \\ n : \text{even}}}^{N-1} s(n)W^{nk} + \sum_{\substack{n=0 \\ n : \text{odd}}}^{N-1} s(n)W^{nk}, \tag{7}$$

where $(n = 2m)$ for even samples and $(n = 2m + 1)$ for odd samples, and $m = 0, \dots, (N/2 - 1)$, by substituting the value of n for even and odd samples into Eq. (7), we get:

$$S(k) = \sum_{m=0}^{N/2-1} s(2m)W^{2mk}$$

$$+ \sum_{m=0}^{N/2-1} s(2m + 1)W^{(2m+1)k}, \tag{8}$$

and

$$S(k) = \sum_{m=0}^{N/2-1} s(2m)W^{2mk} + \sum_{m=0}^{N/2-1} s(2m + 1)W^{2mk}W^k, \tag{9}$$

For a given value of k , W^k is a constant, so it is taken out of the summation as,

$$S(k) = \sum_{m=0}^{N/2-1} s(2m)W^{2mk} + W^k \sum_{m=0}^{N/2-1} s(2m + 1)W^{2mk}, \tag{10}$$

$$S(k) = S_k^e + W^k S_k^o, \tag{11}$$

where S_k^e and S_k^o are the k th components of the DFT of length $N/2$ formed from the even and odd components of the original samples, respectively.

The last equation gives the discrete Fourier transform of the discrete signal by using the FFT algorithm.

A large number of calculations are performed to calculate the Fourier transform for a certain number of samples in the time domain, which is also the same number obtained in the frequency domain.

Now, to introduce a new concept called dual Fourier transform spectra (DFTS), we use the results of FFT obtained from Eq. (11) (i.e., the FFT of the Fourier spectrum).

Let $S(k) = F(b)$. The dual Fourier transform spectra can be defined as:

$$\mathcal{F}(a) = \sum_{b=0}^{R-1} F(b)M^{ab} \tag{12}$$

where a and $b = 0, 1, 2, \dots, (R - 1)$, R is the new number of sampled data resulted from the previous FFT, and $M = \exp(-j2\pi/R)$, which is a complex number. Equation (12) can be rewritten as even and odd summations as follows:

$$\mathcal{F}(a) = \sum_{\substack{b=0 \\ b : \text{even}}}^{R-1} F(b)M^{ab} + \sum_{\substack{b=0 \\ b : \text{odd}}}^{R-1} F(b)M^{ab}. \tag{13}$$

Now, let ($b = 2r$) for even samples and ($b = 2r + 1$) for odd samples, where $r = 0, \dots, (R/2 - 1)$, by substituting the value of b for even and odd samples into Eq. (13), we get:

$$\mathcal{F}(a) = \sum_{r=0}^{R/2-1} F(2r)M^{2ra} + \sum_{r=0}^{R/2-1} F(2r + 1)M^{(2r+1)a}, \tag{14}$$

and

$$\mathcal{F}(a) = \sum_{r=0}^{R/2-1} F(2r)M^{2ra} + \sum_{r=0}^{R/2-1} F(2r + 1)M^{2ra}M^a. \tag{15}$$

For a specific value of (a), M^a is a constant. Therefore, we can take it out of the summation as,

$$\mathcal{F}(a) = \sum_{r=0}^{R/2-1} F(2r)M^{2ra} + M^a \sum_{r=0}^{R/2-1} F(2r + 1)M^{2ra}, \tag{16}$$

where

$$\mathcal{F}(a) = \mathcal{F}_a^e + M^a \mathcal{F}_a^o. \tag{17}$$

Equation (17) gives the dual Fourier transform spectra (DFTS), where \mathcal{F}_a^e and \mathcal{F}_a^o are the even and odd components of the DFTS of length $R/2$, respectively.

In the DFTS method, as will be mentioned in the following sections, it can be deduced that the average energy of frequencies that are integer multiples of the excitation frequency is taken and plotted. The presence of distorted peaks in the DFTS plot detects the crack, and the intensity of these peaks indicates the severity of the crack. Therefore, the DFTS method can be used to detect and quantify breathing cracks in beam-like structures. To this end, the main objective of this work is to develop a method capable of detecting and quantifying single and multiple breathing cracks in beam-like structures. The efficacy of the DFTS method in detecting single and multiple breathing cracks is numerically and experimentally validated in Sects. 4 and 5, respectively.

4 Numerical simulation

This section presents some of the likely damage scenarios to occur in beam-like structures. The nonlinear dynamic characteristics of beam-like structures containing breathing cracks are studied using the finite element (FE) method.

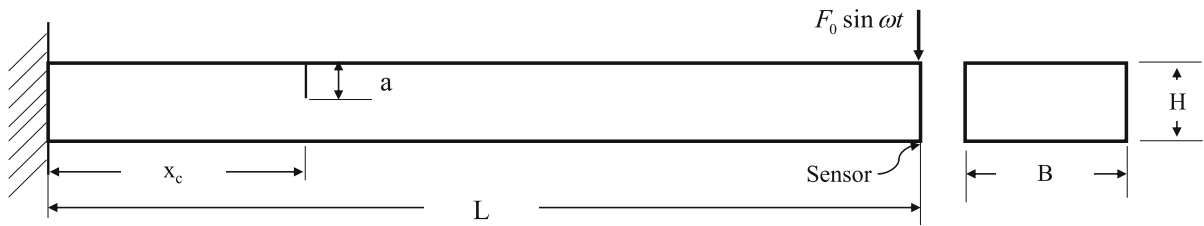


Fig. 1 The geometric drawing of a beam with a fatigue-breathing crack

Table 1 The material properties and dimensions of the beam

E (GPa)	ν	ρ (kg/m ³)	L (mm)	B (mm)	H (mm)
206	0.3	7800	300	25	10

4.1 Modeling of breathing cracks

Figure 1 shows a 3D FE model rectangular section cantilever beam modeled using the ABAQUS FEM software with linear hexahedral elements of type C3D8R. The elements density near the cracks is increased to precisely capture the dynamic responses of the cracks. The geometrical parameters of the beams are identical and chosen as length (L), width (B), and thickness (H). The material properties of the steel which used in the numerical simulation are selected as Young’s modulus (E), mass density (ρ), and Poisson’s ratio (ν), are provided in Table 1. The cantilever beam is excited using harmonic excitation at the tip of the free end, while the sensor is located on the opposite surface of the excitation force. Sensors are used to record the time history of the acceleration responses of the beams. The location and depth of cracks were specified using the dimensionless parameters $q = x_c/L$ and $p = a/H$, respectively.

In this study, characterizing the opening and closing of the breathing cracks are treated as local contact problems. The penalty tangential behavior with a friction coefficient of 0.1 is used in the tangential direction, whereas the hard penalty contact is used in the normal direction. ABAQUS/Standard provides three common contact approaches, general contact, contact pairs, and contact element. In this paper, the contact pairs and the surface-to-surface contact are used. The interactions between the breathing crack surfaces are modeled by considering one of the crack surfaces as a master surface and the second as a slave surface, as illustrated in Fig. 2. Particularly, nodes on the slave surface are not allowed to penetrate the segments of the master surface,

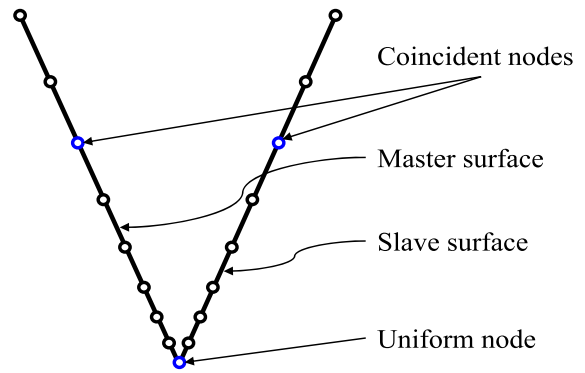


Fig. 2 Crack modeling with a surface-to-surface contact method

but the master nodes are allowed to penetrate the slave surface. During the vibration of beams, three contact states arise of the breathing crack: (1) the crack is fully opened, it means there is no contact between the slave and master surfaces; (2) the crack is fully closed, and all nodes on the slave and master crack surfaces are in contact; (3) the slave and master crack surfaces are in partial contact.

4.2 Deficiencies of Fourier spectrum in representing breathing cracks

It can be noted that there is only one peak in Fig. 3a, while in Figs. 3b–d, higher-order harmonics begin to appear and gradually increase as the depth of the breathing crack increases. However, it is difficult to identify the severity of single and multiple breathing cracks using conventional methods such as FFT. Therefore, in order to solve such deficiency, another method, which is the DFTS, is needed.

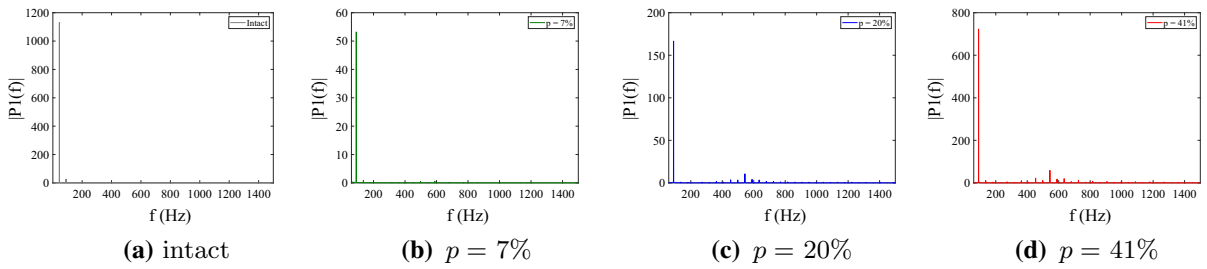


Fig. 3 FFT of intact and cracked cantilever beams at different crack depths

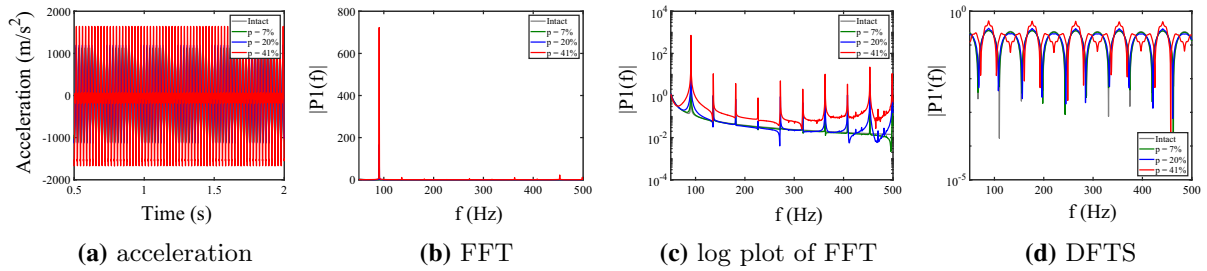


Fig. 4 Effect of damage depth on vibration responses at $q = 0.45$ of the cantilever beam

4.3 Single breathing crack

This section presents the numerical results of analyzing different beam models containing single breathing cracks. The effect of crack depth, crack location, excitation frequency and excitation magnitude, damping, and noise immunity is analyzed.

4.3.1 The effect of crack depth on nonlinear dynamic characteristics

Since crack growing is inevitable throughout the structural life cycle, identification of crack depth is essential. It is crucial to understand the association between the nonlinear dynamic properties and crack depth changes.

The first four natural frequencies of the intact cantilever beam are 90.657, 565.27, 1570.7, and 3044.9 Hz. The cantilever beam is excited at the free end with excitation frequency and magnitude of $\omega = 1/2 f_1$ and $F = 500$ N, respectively. A breathing crack is made at a distance of $q = 0.45$ from the fixed end, and three scenarios of crack depths at $p = 7, 20,$ and 41% are investigated. The three different crack depths are considered to manifest the sensitivity of the proposed method. The numerical simulation results of the intact and damaged beams are shown in Fig. 4.

The numerical time history of the acceleration responses is shown in Fig. 4a, and 4b shows their FFTs. Figure 4c shows the semi-logarithmic plot of the FFTs, and the DFTSs are shown in Fig. 4d.

For the intact beam in Fig. 4b, the FFT plot shows only two high-amplitude peaks; the first represents the excitation frequency (which is excluded from the figure), and the second represents the first fundamental frequency. Figure 4c shows many peaks at higher-order harmonics of $1/2 f_1$. The excitation frequency is 45 Hz, and the peaks are 90, 135, 180, 225, and so on. As the depth of the breathing crack increases, extra higher order-harmonics appear in the Fourier power spectrum. The linear harmonics of the Fourier power amplitude decreases, whereas the amplitude of super-harmonics increases with an increase in the depth of the crack. Generally, regardless of any depth of the crack, the amplitude of higher-order harmonics is an order of magnitude lower than that of the linear harmonic component. Moreover, the amplitude of the Fourier power spectrum for high-order harmonics is very low compared to the fundamental harmonics even for high depth of the crack, which is apparent from Fig. 4c. Regardless of the fundamental harmonics of excitation, the existence of high-order harmonics indicates the bi-linearity of the structure because of breathing crack.

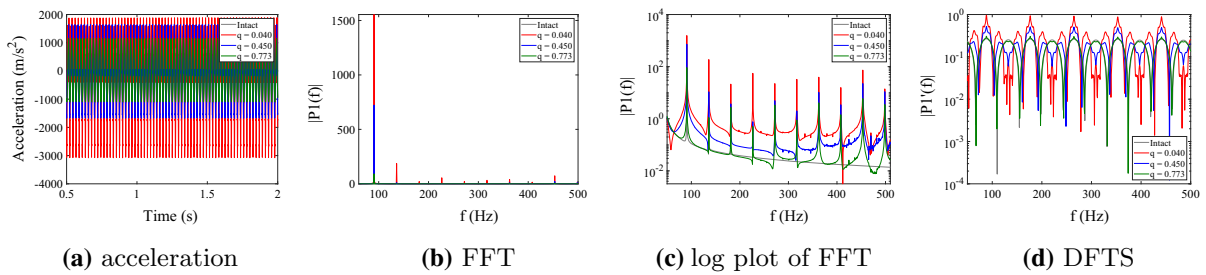


Fig. 5 Effect of damage location on vibration responses at $p = 41\%$ and $q = 0.040, 0.45,$ and 0.773 of the cantilever beam

The results indicate that there is an increase in the amplitude of the nonlinear higher-order harmonics and a decrease in the linear components with the increase in the depth of the crack.

In the previous results, some peaks also exist in the DFTS plot under the intact state. It is most likely attributable to considering the nonlinear geometric effect in the numerical simulation, and this was considered in order to approximate the numerical simulation from the real case.

The physical interpretation of the DFTS method depends on what is the input data. The peaks indicate periodic components of the input data. It is well-known that the output signal in the time domain will be a signal of the same type that carries the nonlinear dynamic characteristics due to the presence of the breathing crack. The nonlinearity induced in the structure leads to what are so-called higher-order harmonics which are multiple integers of the excitation frequency depicted in the frequency domain. Here it is believed that the average of the fundamental and higher-order harmonic frequencies at a specific excitation frequency is taken. Higher-order harmonics appear as peaks of different amplitudes in the DFTS, more pronounced when the content of the nonlinearity of the system is higher. The nonlinearity of the system increases as the crack depth increases, so in such a case, peaks with higher amplitudes will appear.

After analyzing the results, the DFTS method has exhibited excellent performance in almost all cases of different crack depths. The DFTS method shows great potential for detecting and quantifying breathing cracks at different crack depths. More significant distortion of the response appears at higher depths of the crack and decreases as crack depth decreases.

4.3.2 The effect of crack location on nonlinear dynamic characteristics

Crack can occur in different locations of the structure causing a distinct change in the dynamic response. The structural response changes depending on the crack location, as observed when the crack depth changed. Many researchers have demonstrated that changing the crack location leads to a change in the structural response [53]. Therefore, in this paper, the effect of the crack location on the dynamic response is studied to test the capability of the proposed method of detecting damages at different locations. Three different crack locations are considered with the exact locations for all beams. The breathing crack parameters are $p = 40\%$ and $q = 0.040, 0.450,$ and 0.773 . The cantilever beam is excited at the free end with excitation frequency and magnitude of $\omega = 1/2 f_1$ and $F = 500$ N, respectively.

The numerical time history of the acceleration responses is shown in Fig. 5a, and Fig. 5b shows their FFTs. Figure 5c shows the semi-logarithmic plot of the FFTs, and the DFTSs are shown in Fig. 5d.

Figure 5 shows the effect of the crack location at three different locations. It is noticed that a more significant distortion of the signal occurs as the crack approaches the fixed end and vice versa, as illustrated in Fig. 5d. The distortion of the signals demonstrates that the nonlinear dynamic features change according to the crack location. The reason for the changing of the nonlinear dynamic characteristics of structures is due to the change of the local stress field in the beam with the change of the damage location.

The results of the DFTS in all cases of different crack locations are encouraging. The DFTS method shows excellent performance in detecting and quantifying breathing cracks at different crack locations. A more considerable distortion of the response appears at

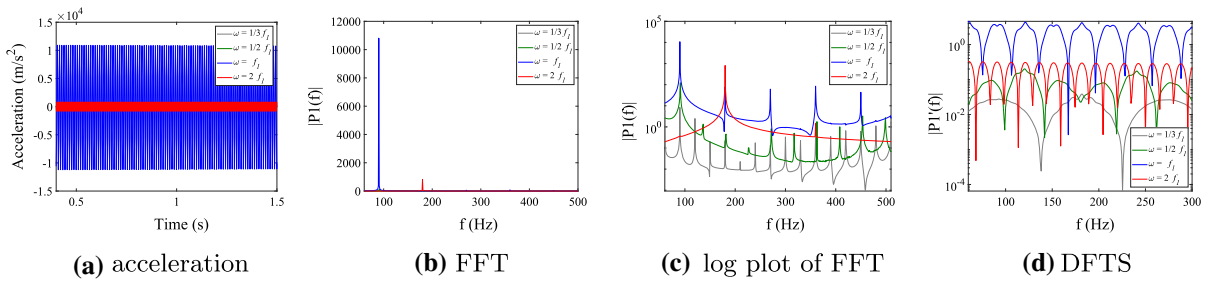


Fig. 6 Effect of harmonic excitation frequency ($F = 100$ N) on vibration responses at $p = 41\%$ and $q = 0.45$ of the cantilever beam

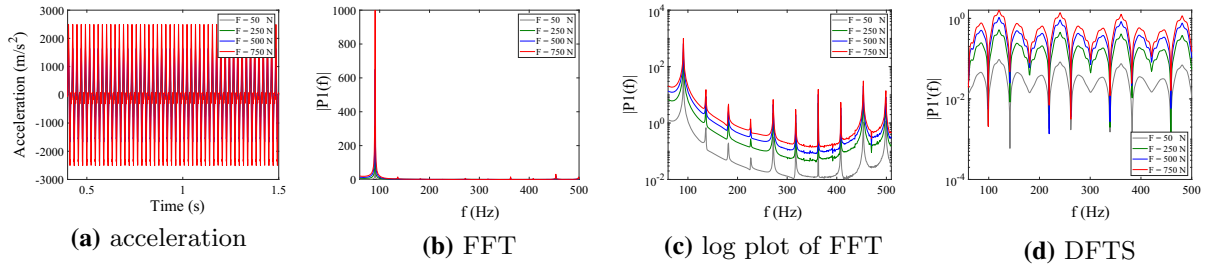


Fig. 7 Effect of harmonic excitation magnitude on vibration responses at $p = 41\%$ and $q = 0.45$ of the cantilever beam

the crack location closer to the fixed end and decreases as the crack location is far from the fixed end.

4.3.3 The effect of harmonic excitation frequency on nonlinear dynamic characteristics

In this subsection, the effect of excitation frequency on nonlinear dynamic characteristics is analyzed. A cantilever beam is excited at the free end with four different excitation frequencies of $\omega = 1/3 f_1, 1/2 f_1, f_1,$ and $2 f_1$ and magnitude of $F = 100$ N. The cantilever beam has one breathing crack made at a distance of $q = 0.45$ with a crack depth of $p = 41\%$ of the total beam depth. The numerical simulation investigation results of the effect of the harmonic excitation frequency are shown in Fig. 6.

Figure 6a represents the time history of the acceleration signals of the damaged beam at three different levels of excitation frequency, Fig. 6b shows their FFTs, Fig. 6c indicates the semi-logarithmic plot of FFT, and Fig. 6d indicates the DFTSs.

Figure 6 shows that the highest amplitude is when $\omega = f_1$. Choosing an excitation frequency close to the fundamental frequency results in a higher amplitude, as shown in Fig. 6b. It can be seen that an excitation frequency equal to the first fundamental frequency has the

highest amplitude. In contrast, an excitation frequency equal to the third first fundamental frequency has the lowest amplitude. This conclusion arises from the fact that at an excitation frequency equals to the first fundamental frequency, the resonance occurs with increased amplitude.

Figure 6c shows many peaks, more peaks at an excitation frequency of $\omega = 1/3 f_1$, and less at excitation frequency of $\omega = 2 f_1$. The reason for this is that the peaks are integer multiples of the excitation frequency.

It is interesting to note that the DFTS method works well in all possible cases of different excitation frequencies. The DFTS method exhibits great prospects for detecting and quantifying breathing cracks at different excitation frequencies. More peaks of the DFTS response appear at a higher level of excitation frequency, and fewer peaks appear at the lower level of the excitation frequency. Therefore, the DFTS method can perform well at any excitation frequency.

4.3.4 The effect of harmonic excitation magnitude on nonlinear dynamic characteristics

In this subsection, the effect of excitation magnitude on nonlinear dynamic characteristics is analyzed. A cantilever beam is excited at the free end with excitation

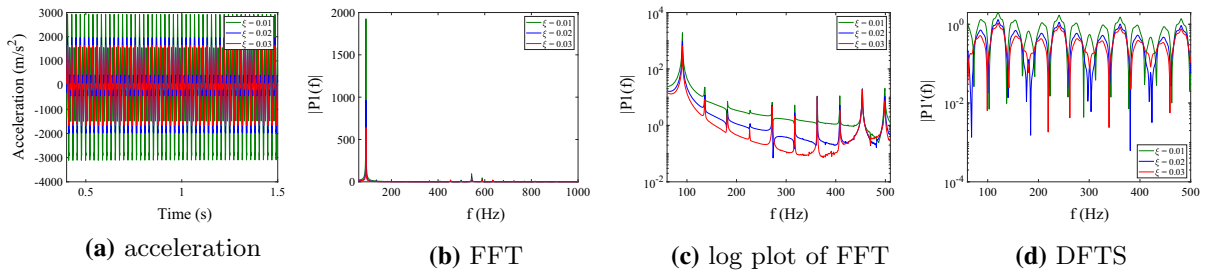


Fig. 8 Effect of the level of damping (ζ) on vibration responses at $q = 0.45$ of the cantilever beam

frequency of $\omega = 1/2f_1$ and four different levels of excitation magnitudes of $F = 50, 250, 500,$ and 750 N. The cantilever beam has one breathing crack made at a distance of $q = 0.45$ from the fixed end with a crack depth of $p = 41\%$.

Figure 7a represents the time history of the acceleration signals of the damaged beam at four different levels of excitation magnitude, Fig. 7b shows their FFTs, Fig. 7c indicates the semi-logarithmic plot of FFT, and Fig. 7d indicates the DFTSs.

It is clear from Fig. 7c and d that when the excitation magnitude increases from 50 N to 750 N, the nonlinear features of the DFTS, the amplitude also increases. The highest amplitude is when $F = 750$ N, and the lowest amplitude is when $F = 50$ N.

Numerical simulation results reveal that the DFTS method offers performance advantages of identifying breathing cracks at different excitation magnitudes. The DFTS method shows great potential for detecting and quantifying breathing cracks at different excitation magnitudes. The number of peaks in the DFTS responses is the same and does not change as the excitation magnitudes change, only amplified. It can be said that the DFTS method is not significantly affected by the change in the excitation magnitude, and it can be used at different levels of excitation magnitude.

4.3.5 The effect of damping on nonlinear dynamic characteristics

The damping level is one of the most important factors to consider when analyzing nonlinear vibrations. Damping plays an important role when diagnosing nonlinear vibrations, as it dramatically affects the shape of the response, especially at super- and subharmonic resonance, and in some cases may suppress it entirely. The fatigue cracks cause a significant increase in struc-

tural damping, so the level of damping must always be known and taken into account [54].

In this subsection, the effect of damping levels on nonlinear dynamic characteristics is analyzed. A cantilever beam is excited at the free end with an excitation frequency and magnitude of $\omega = 1/2f_1$ and $F = 500$ N, respectively. Three damping levels of $\zeta = 0.01, 0.02,$ and 0.03 are considered. The cantilever beam has one breathing crack made at a distance of $q = 0.45$ with $p = 41\%$.

Figure 8 shows the numerical investigation results of the analysis. Figure 8a displays the time history of the acceleration signals of the damaged beam at three different levels of damping, Fig. 8b shows their FFTs, Fig. 8c indicates the semi-logarithmic plot of FFT, and Fig. 8d illustrates the DFTSs.

Figure 8 shows that the highest amplitude occurs when $\zeta = 0.01$, and the lowest amplitude occurs when $\zeta = 0.03$. It can be observed that with increasing the level of damping, the amplitude of the Fourier power spectrum decreases. The peaks are more apparent when the damping ratio is 0.3, as shown in Fig. 8c. Moreover, the signal distortion increases as the damping level increases. Therefore, the selection of the level of damping is essential.

The effectiveness of the DFTS method has been proven in detecting and quantifying breathing cracks in beam-like structures at different levels of damping. The DFTS response shows a higher amplitude at a lower level of damping and lower amplitude at a higher level of damping. The DFTS method performs well despite the differences in the level of damping of analyzed cases.

4.3.6 Noise immunity

In experimental tests, measurement noise affects the quality and sensitivity of the damage identification process [55]. Therefore, noise immunity is an essential property to exhibit the robustness and applicability of the proposed DFTS in the practical identification of damage. For the sake of simulating actual cases, the white Gaussian noise is added to the extracted acceleration time history responses of the different cases so that the noise-contaminated signals could be obtained. The damaged cases include a single breathing crack at $p = 41\%$ and $q = 0.45$. In order to show diverse environmental conditions, five noise levels, i.e., $SNR = 85, 65, 45, 25, \text{ and } 10$, are considered.

Figure 9 shows the effect of noise levels on nonlinear dynamic features. The results demonstrate that proposed method has robust noise immunity. Moreover, the DFTS concept is suitable for the identification of practical damage where measurements are taken under noisy conditions.

Numerical simulation results reveal that the DFTS method works well under different noisy environmental conditions. The ability of the DFTS method to detect and quantify breathing cracks under different noisy conditions is one of the major advantages of this method. The results imply that the DFTS method is competent to detect breathing cracks in different noise environments.

4.4 Multiple breathing cracks

In previous Sect. 4.3, a cantilever beam with single-breathing crack is studied. In this section, multiple cracks in beam-like structures are studied using the concept of DFTS. Numerical simulations are performed to study the effects of multiple, opposite, and mixed (i.e., breathing-open) cracks on nonlinear dynamic properties. Some comparisons are made between several cases. The objective of this study is to emphasize the strength of the proposed concept in detecting multiple cracks.

4.4.1 The effect of multiple cracks on nonlinear dynamic characteristics

Several structures are subjected to high level of vibrations which may influence their integrity and ser-

vice life. These high vibrations result in multiple cracks in the structures [56,57]. In such instances, it is essential to avoid mechanical breakdowns and failures before they occur. One method that can be used to avoid such conditions is to observe the nonlinear dynamic properties of the structures.

Identifying multiple cracks in beam-like structures has become a research focus of increasing interest over the past few years. Therefore, most current studies address sensitivity to detect multiple damages. However, the most current methods are sensitive to measurement noise, reducing their ability to detect multiple damages. Therefore, the proposed DFTS method is used to manifest the effect of multiple breathing cracks on the nonlinear dynamic characteristics to overcome this drawback.

The cantilever beam is excited at the free end with excitation frequency and magnitude of $\omega = 1/2 f_1$ and $F = 500 \text{ N}$, respectively. The breathing cracks depth is set to be $p = 41\%$ at distances of $q = 0.04, 0.207, \text{ and } 0.373$ from the fixed end for the first, second, and third crack, respectively. Three beams with breathing cracks were compared, the first containing one crack, the second containing two cracks, and the third containing three cracks. The comparisons were made in order to show the sensitivity and applicability of the proposed method for detecting multiple breathing cracks in beam-like structures.

Figure 10a represents the time history of the acceleration signals of intact and damaged beams, Fig. 10b shows the FFTs, Fig. 10c indicates the semi-logarithmic plot of FFT, and Fig. 10d indicates the DFTSs of the acceleration signals.

The input signal to a linear system remains without distortion, unlike the nonlinear system that distorts the input signal, i.e., if the input signal is distorted, it means that the system contains a kind of nonlinearity. Therefore, it can be said that the distortion is proportional to the amount of nonlinearity, as it is clear from Fig. 10. The beam containing three cracks has more distortion in the output signal than the beam containing two cracks and so on.

Fourier analysis of the cracked structures in Fig. 10 shows high amplitude in the three-cracks beam and low amplitude in the single-crack beam. It is clear from Fig. 10 that each case has a unique signature. Furthermore, from Fig. 10b and c, it is difficult to distinguish the effect of different damaged cases. Therefore, to characterize this effect, the amplitude of the DFTS is

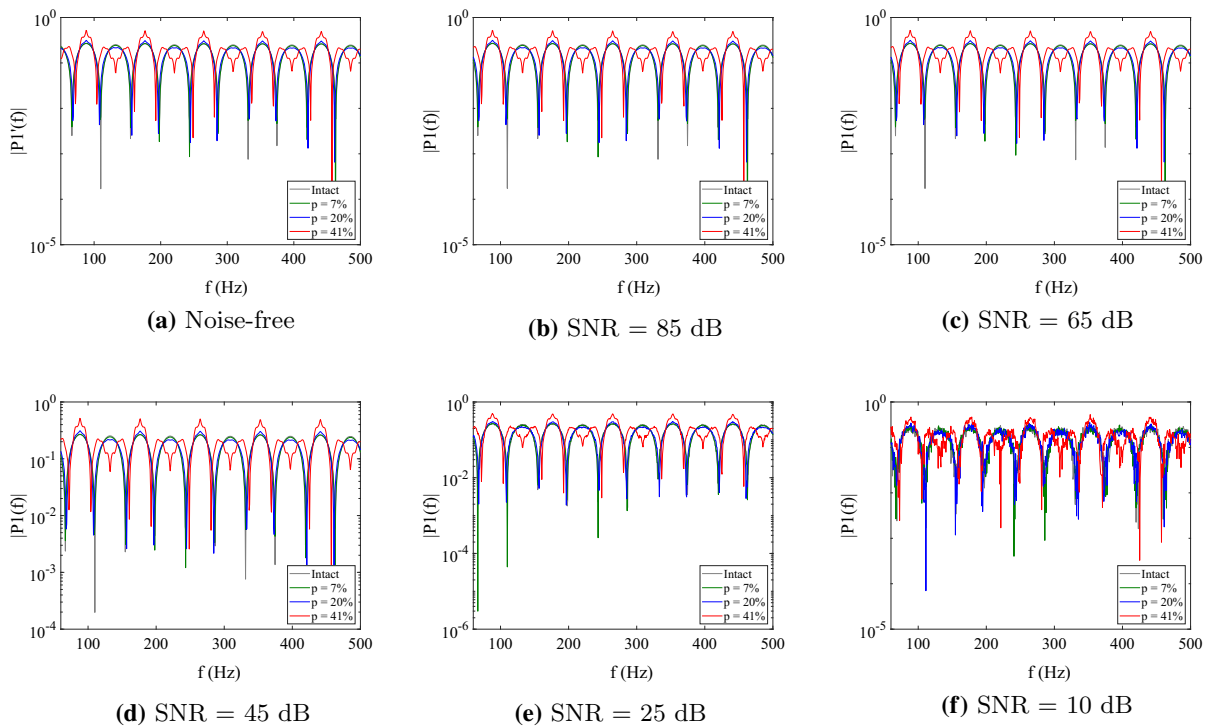


Fig. 9 DFTS of the cantilever beam at different levels of noise

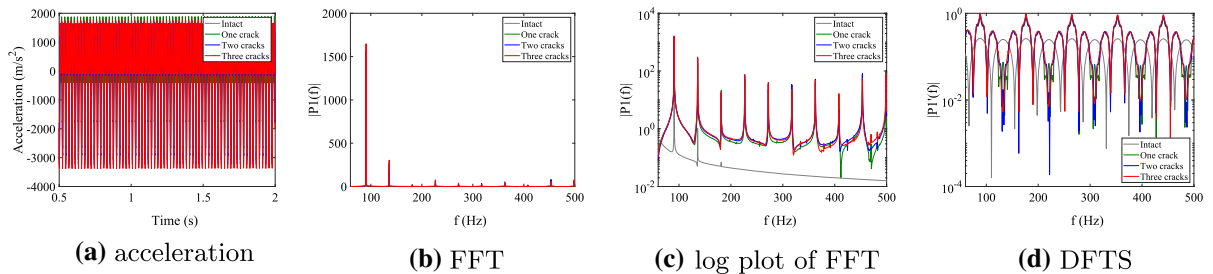


Fig. 10 Effect of multiple cracks on nonlinear dynamic features at $p = 41\%$ and $q = 0.040, 0.207, \text{ and } 0.373$

used as a damage-sensitive feature to indicate the extent of damage, as illustrated in Fig. 10d. One can conclude that the proposed method has the ability to identify multiple breathing cracks in beam-like structures.

The main strength of the DFTS method includes its ability to detect single breathing cracks and can be extended to tackle multiple breathing cracks. Therefore, the DFTS method can be an efficient tool to investigate multiple breathing cracks in beam-like structures.

4.4.2 The effect of opposite cracks on nonlinear dynamic characteristics

Similar to the previous settings, the cantilever beam is excited at the free end with excitation frequency and magnitude of $\omega = 1/2 f_1$ and $F = 500 \text{ N}$, respectively. The comparison is made between two beams, one has two breathing cracks on one side, and the other also has two breathing cracks but on different sides. All breathing cracks in both cases have the same depth of $p = 41\%$ placed at $q = 0.040$ and 0.207 , respectively.

The comparison aims to study the effect of two breathing cracks versus opposite breathing cracks on

the nonlinear dynamic response of structures. Also, to test the performance and ability of the proposed concept in identifying the opposite breathing cracks. Figure 11 shows that the cantilever beam with two breathing cracks on one side has a higher amplitude than the beam with two opposite cracks.

The DFTS method can be used to study various issues associated with breathing cracks in beam-like structures. For example, the DFTS method can also identify breathing cracks in two different surfaces, such as opposite breathing cracks.

4.4.3 The effect of breathing-open cracks on nonlinear dynamic characteristics

As before, the cantilever beam is excited at the free end with excitation frequency and magnitude of $\omega = 1/2 f_1$ and $F = 500$ N, respectively. The comparison is made between two beams, one containing two breathing cracks and the other containing one breathing crack and an open crack. The cracks in both cases have the same depth of $p = 41\%$. The width of the open crack is 2 mm. The two cracks were positioned at $q = 0.04$ and 0.207 , respectively. The aim of the analysis is to study the effect of two breathing cracks versus breathing-open crack on nonlinear dynamic response of structures. Moreover, the results are used to show the ability of the proposed concept to identify different types of damages.

Figure 12a represents the time history of the acceleration signals of intact and damaged beams, Fig. 12b shows their FFTs, Fig. 12c indicates the semi-logarithmic plot of FFT, and Fig. 12d displays the DFTSs.

Figure 12d shows a clear difference between the beam with two breathing cracks and the beam with breathing-open cracks, where the former has the higher amplitude. The higher amplitude in the beam with two breathing cracks is due to the nonlinearity caused by the two breathing cracks. The nonlinearity induced from two breathing cracks is more than that from breathing-open cracks, explained by the nonlinear interaction between the two breathing cracks.

In the view of aforementioned discussion, it is clear that the DFTS method is a flexible tool and applicable even in cases where there are mixed cracks (i.e., breathing and open cracks). The numerical simulation investigation results indicate that the DFTS method is promising and can be used to detect single and multiple

breathing cracks in addition to different cracked beam models.

5 Experimental results and discussion

In order to validate the proposed DFTS method for the identification of breathing cracks in beam-like structures, a verified experimental framework is followed in this work. For developing good breathing crack specimens and by considering the difficulties of obtaining real breathing cracks, some researchers were compelled to construct specimens made up by bonding several pieces together. Prime and Shevitz [58] experimentally investigated a polycarbonate beam containing a breathing crack. The polycarbonate beam was constructed by bonding three pieces together to form a beam with a breathing crack. At the same time, the intact beam was constructed by bonding two pieces. By using the aforementioned experimental approach, Douka and Hadjileontiadis [59] studied the effect of breathing crack on the dynamic behavior of a cantilever beam containing breathing crack both theoretically and experimentally. Prawin and Rao in [55] and [23] considered a cantilever beam with single and multiple breathing cracks constructed by forming two and three pieces using an Araldite epoxy adhesive. By employing the same experimental approach, Prawin et al. [60] proposed a novel method to identify single and multiple breathing cracks based on the zero strain energy nodes concept. Similarly, in their other work [15] they presented a baseline-free algorithm for detecting, localizing, and characterizing breathing cracks using singular spectrum analysis. Hence, in this work, the guidelines and procedures presented in the studies mentioned above are followed to verify the present methods for identifying single and multiple breathing cracks.

In this research, several experiments are performed on steel cantilever beams containing breathing cracks. These experiments aim to validate the proposed DFTS method for identifying breathing cracks. Cantilever beams are formed by bonding two or more pieces of steel with a high-performance structural adhesive. The horizontal surfaces are interconnected, while the surfaces forming the breathing cracks are in contact only. The contact surfaces help simulate the opening and closing behavior of cracks under dynamic loads.

The dimensions of the intact and damaged beams are $300 \text{ mm} \times 25 \text{ mm} \times 10 \text{ mm}$. The dimensions of the

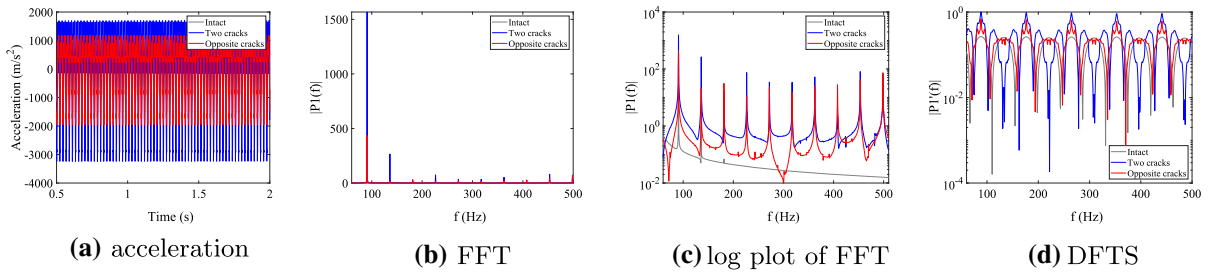


Fig. 11 Effect of opposite cracks on nonlinear dynamic features at $p = 41\%$ and $q = 0.040$ and 0.207

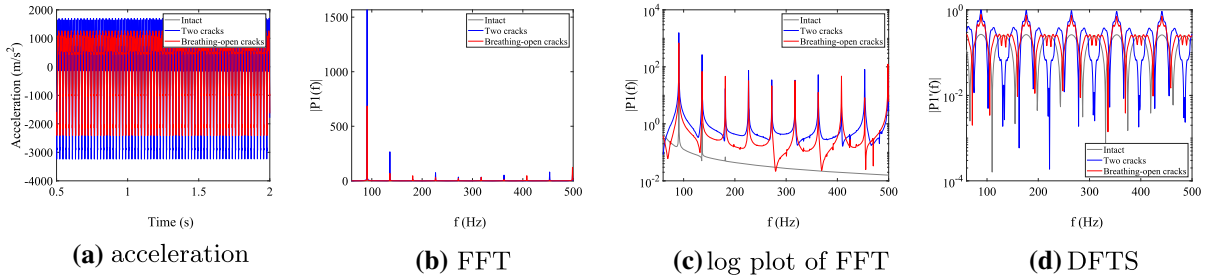


Fig. 12 Effect of breathing-open cracks on nonlinear dynamic features at $p = 41\%$ and $q = 0.040$ and 0.207

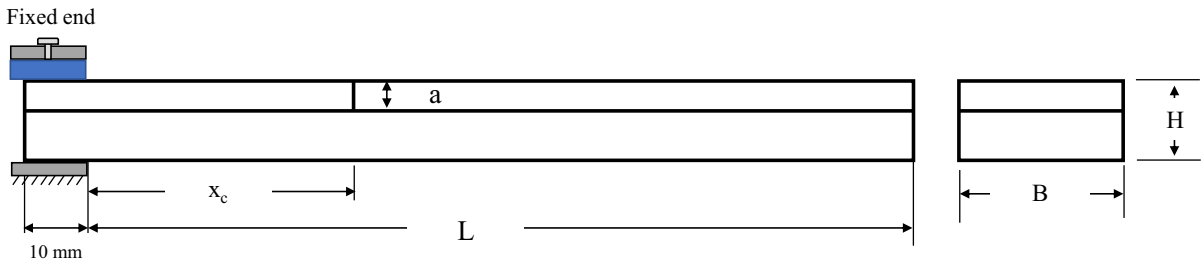


Fig. 13 Sketch of a cantilever beam with a single crack

Fig. 14 Vibration experimental setup of the cantilever beam

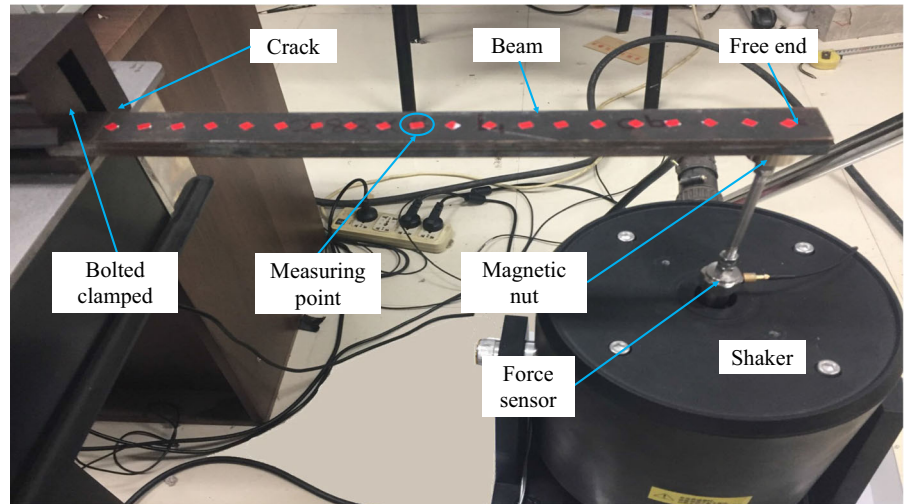


Fig. 15 Experimental setup system for the damage identification of a cantilever steel beam

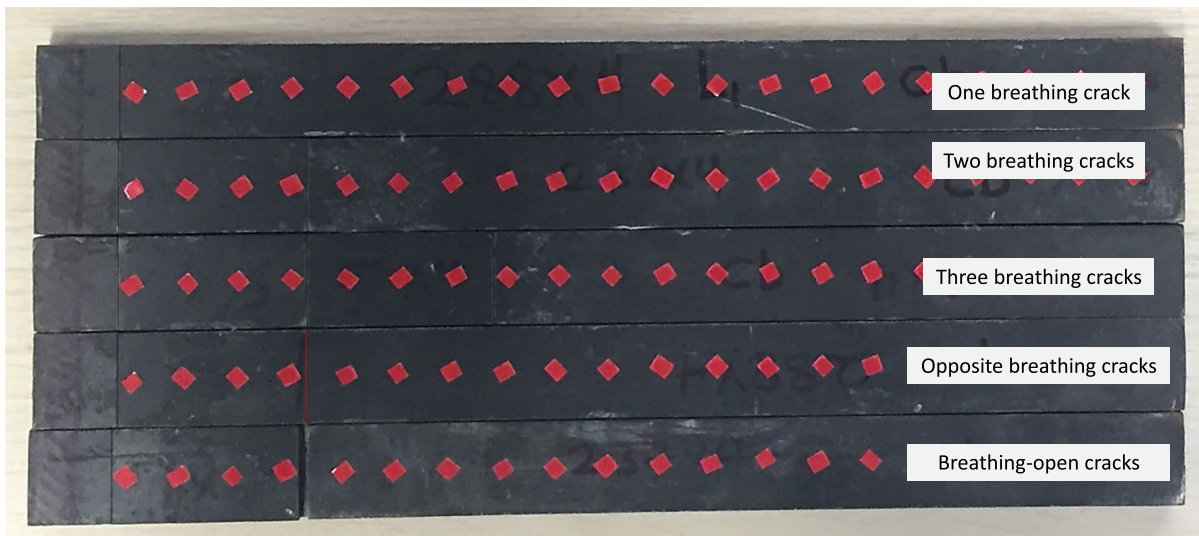
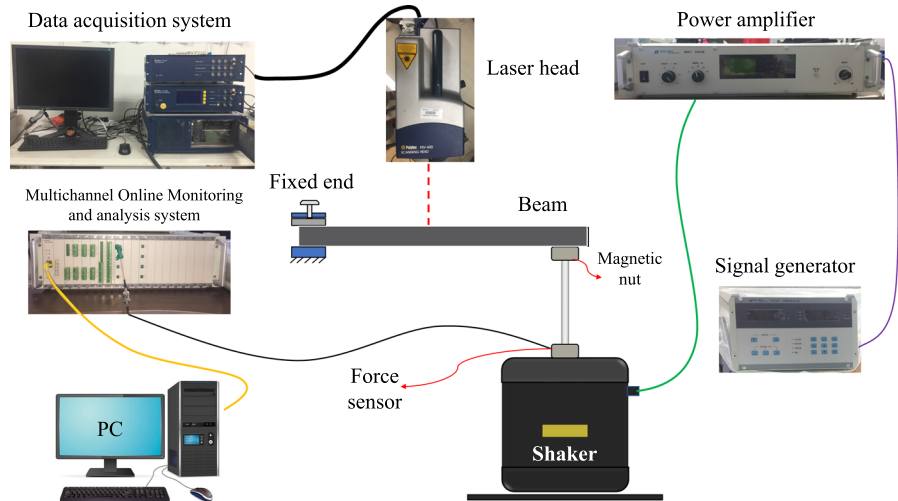


Fig. 16 Tested beams

pieces that formed the cantilever beams determine the depth and location of the breathing cracks, are shown in Fig. 13. The material properties of the steel cantilever beam used in the experiments are selected as Young's modulus $E = 200$ GPa, mass density $\rho = 7300$ kg/m³, and Poisson's ratio $\nu = 0.3$.

The experimental setup is shown in Fig. 14. The specimens are securely fixed to a testing table. A snapshot and a schematic of the most important components of the testing system are shown in Fig. 15. The sweep signal generator (DH1301) generates a sinusoidal signal and sends it to the power amplifier (DH5872). The amplified signal is then sent to an electromechanical

shaker (DH40200) to produce the required excitation. The force is measured using a force sensor (DH5858C) which is attached to the shaker shaft. The force sensor is connected to a multichannel online monitoring and analysis system (DH5972N) connected to a computer. Finally, the data are measured and acquired using a system consisting of a PSV-400 scanning laser head and PSV-400 junction box and an OFV-5000 vibrometer controller. The laser measuring spot is located at $q = 0.48$ from the fixed end. A magnetic nut is used to connect the electromechanical shaker to the cantilever beam. Figure 16 shows a photographic image

Table 2 Damage parameters of the cantilever beams containing breathing cracks

Group	Crack depth (p)	Crack location (q)	Excitation frequency (Hz)	Excitation magnitude (N)
1	20%	0.45	$1/2 f_1$	50
	40%			
2	40%	0.04	$1/2 f_1$	50
		0.45		

Table 3 Harmonic excitation parameters of the cantilever beams with breathing cracks

Group	Excitation frequency (Hz)	Excitation magnitude (N)	Crack depth (p)	Crack location (q)
1	$1/3 f_1$	50	40%	0.45
	$1/2 f_1$			
2	$1/2 f_1$	30	40%	0.45
		50		

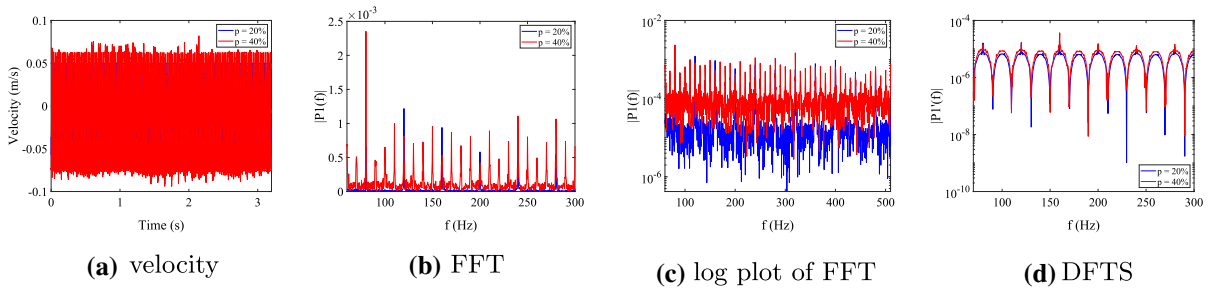


Fig. 17 Effect of damage depth on vibration responses at $q = 0.45$ of the cantilever beam

of the tested cantilever beams utilized to study multiple cracks.

5.1 Single breathing crack

For studying the single breathing crack damage scenario, a cantilever beam with a single breathing crack is analyzed. Different crack and excitation parameters are introduced in these experiments. Table 2 and Table 3 show the crack and the harmonic excitation parameters, respectively.

The bonded test beams may not behave like real fatigued beams and cannot correspond directly to actual field conditions. However, since the main goal is to grasp the nonlinear behavior of breathing cracks rather than accurately quantify the damage, the test beams satisfy the present working purpose. Furthermore, the intact beam has been formed by bonding two pieces together, so the only difference between the intact and cracked beams is limited only around the crack area.

The first four natural frequencies of the intact cantilever beam are 80.63, 495.15, 1212.5, and 2686.7 Hz. The effect of the breathing crack depth, location, and excitation frequency and magnitude on the dynamic response characteristics is shown in Figs. 17, 18, 19, 20, respectively. The time histories of the experimental responses during 3.2 s are shown in Figs. 17, 18, 19, 20a, and Figs. 17, 18, 19, 20b show their frequency spectrums. Figures 17, 18, 19, 20c show the semi-logarithmic plot of the FFTs, and the DFTSs are shown in Figs. 17, 18, 19, 20d.

Similar to the numerical results in Sect. 4, the amplitude of the Fourier spectrum increases with the increase of the crack depth, as shown in Fig. 17. The higher the crack depth, the more numbers of higher-order harmonics appear. Moreover, the amplitude of the linear harmonics decreases, while the amplitude of the superharmonics increases as the crack depth increases. Responses corresponding to the cracked beams show

peaks at integer multiples of the excitation frequency of 40 Hz (i.e., 80, 120, 160, 200, and so on). The presence of higher-order harmonics shows the bi-linearity of the structure due to the nature of the breathing crack.

It can be observed that the dynamic response characteristics change depending on the crack location due to the change of local stress caused by the presence of the crack in that position, as shown in Fig. 18. Similarly, it is also clear from Figs. 19 and 20 that the amplitude increases with the increase in the excitation frequency and magnitude, respectively. Figure 21 shows the normalized power spectral density (PSD) for different beam models for the influence of damage depth, damage location, and excitation frequency and magnitude, respectively.

5.2 Multiple breathing cracks

For analyzing the multiple breathing cracks cases, some experiments are carried out on steel cantilever beams containing multiple cracks. The purpose of these experiments is to study the effect of multiple cracks on the nonlinear dynamic response. The cantilever beams are prepared by bonding many pieces of steel with a high-performance structural adhesive. For example, in order to build a cantilever beam containing three breathing cracks, we need five pieces. The dimensions of the cantilever beams and material properties are the same as in previous Sect. 5.1.

The results are divided into three groups to facilitate comparing different cracked beam models. Table 4 shows the different parameters of crack and excitation used in these experiments.

The effect of the multiple cracks (i.e., multiple breathing cracks, opposite breathing cracks, and breathing-open cracks) on the dynamic responses characteristics is shown in Figs. 22, 23, 24. The time histories of the experimental responses during 3.2 s are shown in Figs. 22, 23, 24a, and Figs. 22, 23, 24b show their frequency spectrums. Figures 22, 23, 24c show the semi-logarithmic plot of the FFTs, and the DFTSs are shown in Figs. 22, 23, 24d.

It is evident from Figs. 22, 23, 24 that each response has a distinctive signature, different in amplitude and the shape of the response. Furthermore, the numerical simulation and experimental results show a similarity between the responses of beams containing two and three breathing cracks, as shown in Fig. 22. The oppo-

site cracks have a lower amplitude than the two breathing cracks on one side, as shown in Fig. 23. Similarly, the breathing-open cracks also have a lower amplitude than the two breathing cracks on one side, as in Fig. 24.

Figure 25 shows the normalized power spectral density (PSD) of the vibration responses of all three cases. The energy of higher-order harmonics is very low compared to linear harmonics, regardless of the depth and number of breathing cracks.

All studied cases show many peaks of integer multiples of the excitation frequency 40 Hz (i.e., 80, 120, 160, and so on). The presence of the peaks is evidence of the bilinear behavior characterized by breathing cracks resulting from continuous opening and closing.

One of the most important characteristics of the DFTS method is that it shows the nonlinearity resulting from the breathing cracks in the form of peaks, the distortion of the peaks increases as the amount of nonlinearity in the system increases. The presence of distorted peaks in the DFTS plot can indicate the crack, and the intensity of these peaks indicates the severity of the crack. Therefore, the DFTS method can be exploited for breathing crack detection and quantification in beam-like structures.

It is clear from the collected numerical simulation and experimental results that the proposed DFTS method is qualified and can identify and quantify both single and multiple breathing cracks in beam-like structures. Therefore, it can be recommended as a powerful damage identification tool and can be adopted for use in industrial applications.

6 Conclusions

In this paper, a novel damage detection and quantification method based on the DFTS damage indicator is proposed. A robust nonlinear damage factor is developed flexibly by applying the FFT in two steps. Firstly, the FFT is applied, and the presence of the secondary peaks of the response serves as indicators of the presence of breathing cracks. Secondly, the FFT of the Fourier spectrum can quantitatively depict the nonlinearity degree of breathing cracks. In the DFTS method, it can be deduced that the average energy of frequencies that are integer multiples of the excitation frequency is taken and plotted. The presence of distorted peaks in the DFTS plot detects the crack, and the intensity of these peaks indicates the sever-

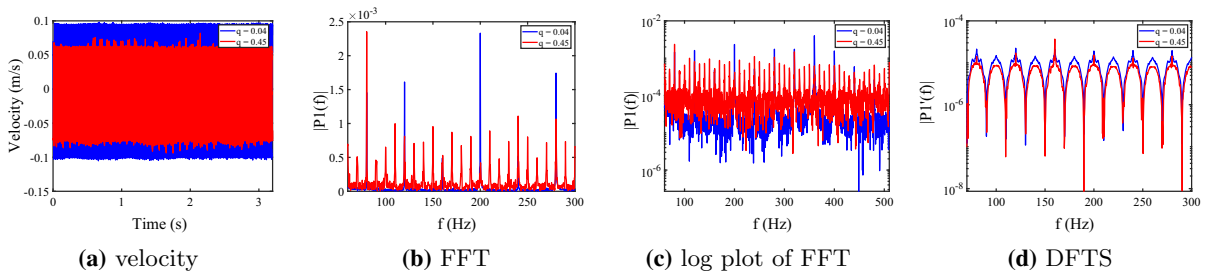


Fig. 18 Effect of damage location on vibration responses at $p = 40\%$ and $q = 0.040$ and 0.450 of the cantilever beam

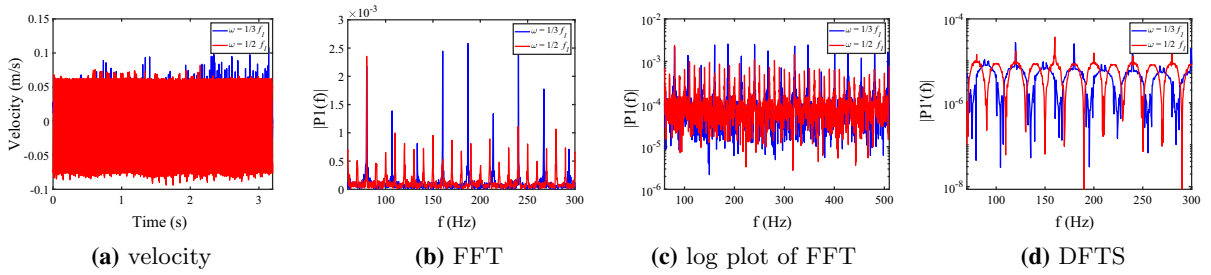


Fig. 19 Effect of harmonic excitation frequency ($F = 50$ N) on vibration responses at $p = 40\%$ and $q = 0.45$ of the cantilever beam

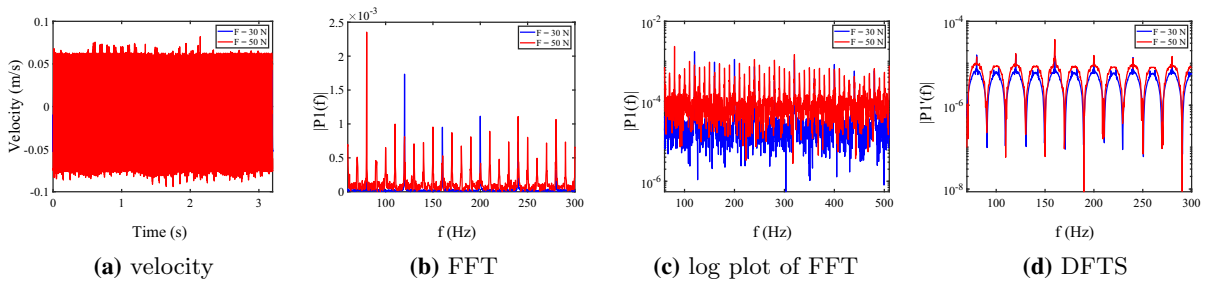


Fig. 20 Effect of harmonic excitation magnitude on vibration responses at $p = 40\%$ and $q = 0.45$ of the cantilever beam

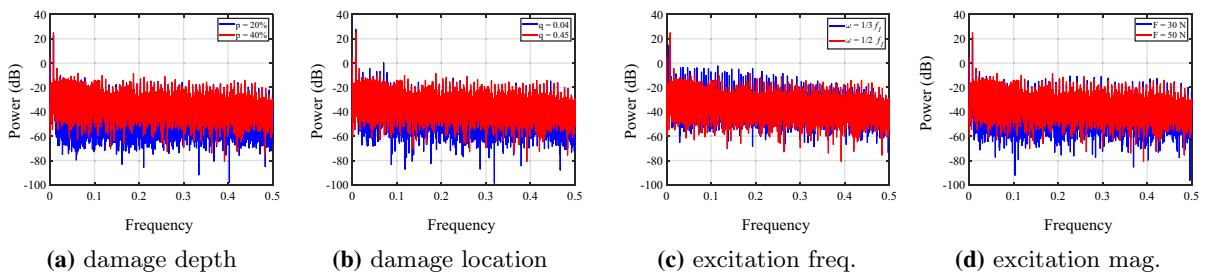


Fig. 21 Power spectral density (PSD) of vibration responses : **a** damage depth, **b** damage location, **c** excitation frequency, **d** excitation magnitude

Table 4 Damage and excitation parameters of beams containing breathing cracks

Model	Crack depth (p%)	Crack location (q)	Excitation frequency (Hz)	Excitation magnitude (N)
One crack	40	0.04	$1/2 f_1$	50
Two cracks	40	0.04 and 0.207	$1/2 f_1$	50
Three cracks	40	0.04, 0.207, and 0.373	$1/2 f_1$	50
Opposite cracks	40	0.04 and 0.207	$1/2 f_1$	50
Breathing-open cracks	40	0.04 and 0.207	$1/2 f_1$	50

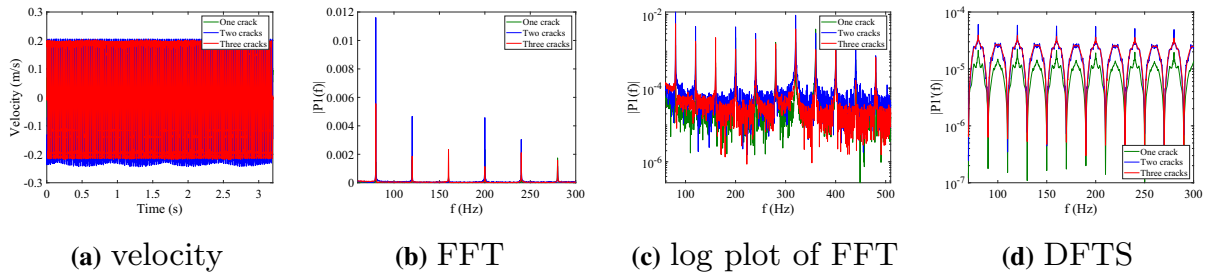


Fig. 22 Vibration responses at $q = 0.040, 0.207,$ and 0.373 of the cantilever beam for the first, second, and third breathing cracks, respectively

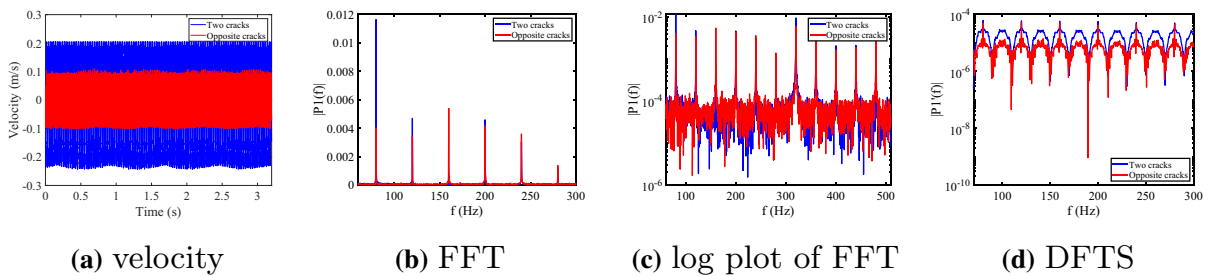


Fig. 23 Vibration responses at $q = 0.040$ and 0.207 of the cantilever beam for the first breathing and opposite cracks, respectively

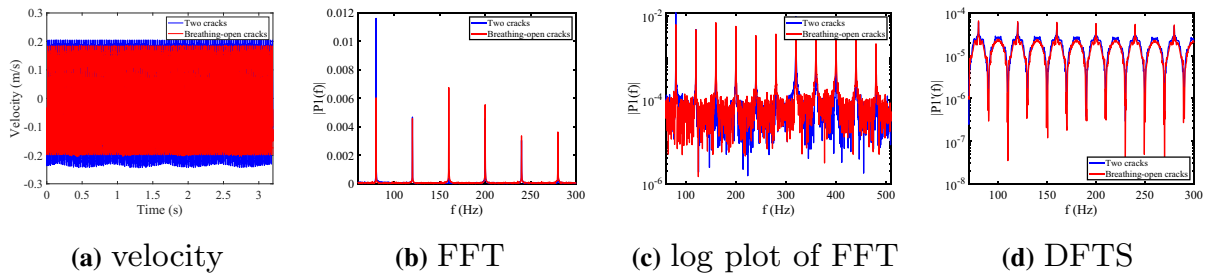


Fig. 24 Vibration responses at $q = 0.040$ and 0.207 of the cantilever beam for breathing and open cracks, respectively

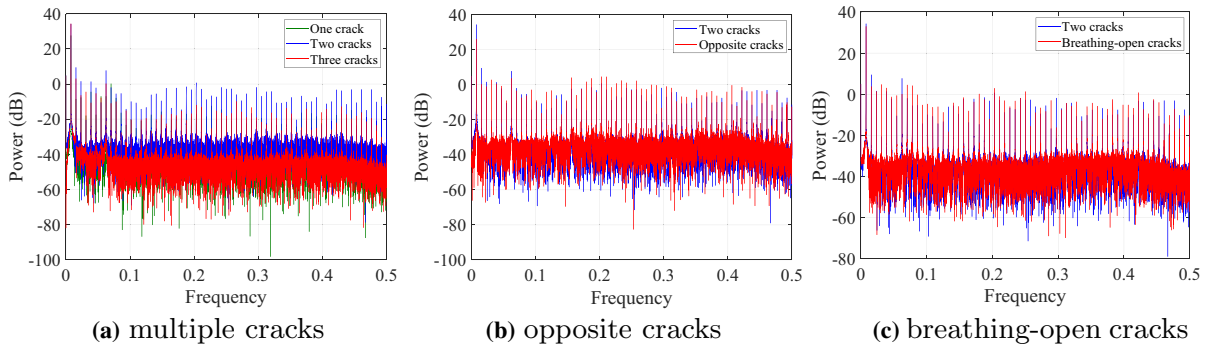


Fig. 25 Power spectral density (PSD) of vibration responses of the cantilever beam: **a** multiple cracks, **b** opposite cracks, **c** breathing-open cracks

ity of the crack. The proposed method is developed to overcome the usual limitations and difficulties found in existing typical methods for identifying single and multiple cracks in beam-like structures. The strength of the proposed method has been proven numerically and experimentally. Several beam models and damage scenarios are investigated, and noise immunity is tested by contaminating the signals with Gaussian noise. Moreover, the ability to identify and quantify single and multiple breathing cracks under different noisy conditions is one of the major advantages of the proposed method. Furthermore, the effects of crack and excitation parameters on the nonlinear dynamic characteristics have been investigated. On the other hand, identifying multiple breathing cracks is an issue of broader importance but a more significant challenge than identifying single breathing cracks. As an exploration to identify multiple damages in beam-like structures, different beam models containing multiple, opposite, and mixed (breathing-open) cracks have been investigated. In all the above-mentioned damage scenarios, the proposed DFTS method has shown outstanding results for the identification of breathing cracks in beam structures and can be highly recommended and promoted for structural damage detection and quantification to handle complex issues found in several practical applications. Hence, future work intends to extend the concept of DFTS and include other types of applications such as composite structures and real bridges.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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