Nonlinear Dyn (2022) 110:1455–1471
https://doi.org/10.1007/s11071-022-07674-2

ORIGINAL PAPER

Finite-time H_∞ predictive control for stochastic networked control systems with delays and packet dropouts

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Received: 20 November 2021 / Accepted: 18 June 2022 / Published online: 12 July 2022

Abstract The work investigates the finite-time H_∞ predictive control problem for stochastic networked control systems (NCSs) with communication constraints. Firstly, by absorbing the phenomena of unmeasurable state and missing measurements, the non-fragile observer (NFO)-based networked predictive control (NPC) strategy is obtained to dispose the time delays and packet dropouts (TD-PDs). To replace previous prediction strategy, a disturbance-based prediction mechanism is adopted. Moreover, based on it, a novel NPC system model is constructed, where the missing measurement is first considered in studying NPC system with TD-PDs, sufficient conditions are derived to ensure the closed-loop systems (CLSs) stochastic finite-time boundedness (SFTB) with a prescribed H_∞ performance. Subsequently, criteria for co-designing both the uniform NFO-based predictive controller and the NFO are calculated based on the bounded TD-PDs. Finally, an illustrative example clearly verifies the usefulness of the main result.

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Keywords Finite-time H_∞ predictive control · Stochastic networked control systems (NCSs) · Time delays and packet dropouts (TD-PDs)

1 Introduction

NCSs have been employed in the past few years for their wide application in some areas, such as intelligent transportation systems, telesurgery, telerobotics, and other scientific areas. Compared with the traditional control point-to-point hard connection, NCSs connect the controller with sensors and actuators through the network. Hence, NCSs own a lot of advantages, which are recalled in [1–5]. However, NCSs also have shortcomings caused by the network, such as intermittent packet dropouts (PDs), time delays (TDs), which can degrade system performance and even cause instability [6,7]. These two issues have been studied, and several methods have been employed in recent years. To deal with them, several interesting methods have been developed, which could be divided into two classifications: one is model-based methods and the other is data-driven methods [8]. Model-based approaches include time delay system approach [9,10], switched system approach [11,12], stochastic system approach [13,14], NPC approach [15–17], etc. The main steps of time delay system method involve incorporating the TD-PDs in the networked control loop into the system input delay, permitting the closed-loop networked system can be formed as a system with a time-varying delay. More specially, we can construct an appropriate Lyapunov–Krasovskii function and derive less conservative stability conditions for system stability when using a time delay system approach. In [9], a novel discrete orthogonal polynomials is proposed and general summation inequalities are derived, then based on it, the hierarchical stability criteria is developed. For the linear discrete systems subject to an interval time-varying delay, a new inequality is applied [10]. Another important approach is the NPC scheme, which has been proposed to actively control the system when suffering from these two communication constraints. It is a compensation method that can be used to deal with TD-PDs [18–22]. NPC as a new approach in adaptation framework, can predict the future system dynamics in a finite horizon and takes the specialties of NCSs into a full account. In recent years, it becomes more and more popular because of its less conservative analysis and design. In fact, this control is efficient for guaranteeing the infinite time stability of the NPC systems, which is a closed-loop system (CLS) that depends on predictive control.

However, it is well-known that finite-time stability (FTS) is a more representative of stability than infinite time stability in engineering systems [23–25], and some research on finite time control for NPC systems have also been reported [26,27]. In [26], a novel predictive networked control methods is derived, then novel finite-time state-feedback and output-feedback stabilization controllers are proposed to compensate the time delays and data packet dropouts. In [27], by considering modeling uncertainties, a sufficient condition is given to ensure the FTS of the augmented CLS, where both bilinear matrix inequality and linear matrix inequality conditions are derived. But it must be noted that these systems are both linear systems without considering missing measurements.

We all know that actual systems always have some stochastic elements. The mistakes of model parameter identification could be represented by these stochastic factors. As a result, the research on network control of stochastic systems is both required and important in practice. In the actual network, the measurement signal may be lost during network transmission because of some complex situation, such as sensor aging, sensor intermittent failure, bandwidth limitation, network congestion or accidental loss of part of the collected data [28–30]. In order to measure the internal state completely and continuously, the observer-based control schemes are employed. After that, more and more researchers have paid attention to the constructs of state estimation and observer-based controller (OBC) [31–33]. Although it is significant to complete state estimation and OBC design, these methods cannot be used to handle systems that are subject to small disturbances, which is caused by those special factors we mentioned above. Fortunately, when the NFO method was proposed, the disturbance of the observer coefficients could be dealt with [34–36]. Through introducing Bernoulli distributions to describe missing measurement, Wu et al. investigate the non-fragile guaranteed cost control for discrete-time Takagi-Sugeno fuzzy Markov jump systems with time-varying delays [34]. With the help of a simplified NFO, Liu et al. construct a novel linear switching surface scheme for the uncertain stochastic Markovian jump system [35]. Wang et al. design the quantized H_∞ controllers for a class of nonlinear stochastic time-delay network-based systems with probabilistic data missing [36].

However, the NFO method has not been applied to the NPC systems, especially when designing NPC. How to perfectly combine NFO with predictive control to obtain the CLSs and analyze of the system stability is an open issue, which encourages us to shorten such a gap. Additionally, it can be concluded that the SFTB and H_∞ performance problems have not been discussed for stochastic NPC systems with external disturbances, missing measurements [37,38], TD-PDs [26,39,40], which deserves further investigation. Consequently, there is an urgent need to address the SFTB with H_∞ performance problem for stochastic NPC systems and develop a predictive disposition method accordingly.

Summarizing the above discussion, we devote ourselves to studying the SFTB with H_∞ performance for NCSs by utilizing a NFO-based NPC scheme. Then, we can dispose the TD-PDs well when the system is suffering from exogenous disturbances and the available probability information of the missing measurements. The major contributions can be highlighted by the following three aspects:

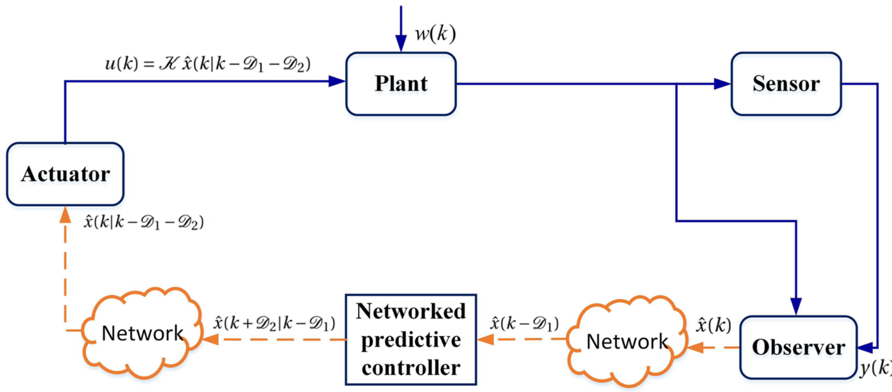

Fig. 1 Architecture of stochastic NPC systems with TD-PDs

(1) Considering the missing measurement phenomena, the NFO is designed in one step prediction, and the NFO-based NPC approach is proposed to actively dispose of the TD-PDs, which can simplify the CLSs obtained.
(2) Through the proposed NPC approach, a new model of stochastic NPC system is established for the first time by taking missing measurements, exogenous disturbances, TD-PDs into consideration, which can reflect more realistic dynamical behaviors.
(3) Sufficient conditions for SFTB with a desired H_∞ performance are presented with the uniform controllers and observers developed by Lyapunov theory, which based on the bounded TD-PDs.

The remaining of this paper have been arranged. In Sect. 2, a new NFO-based NPC scheme is proposed to handle the predictive control problem, and within the unavoidable communication constraints, a new model of stochastic NPC system is established. Section 3 derives some sufficient conditions for ensuring SFTB with a prescribed H_∞ performance. Then, the desired controller and observer are obtained. Section 4 provides a numerical example, and Sect. 5 concludes this paper.

Notation: $\mathbb{R}^n$ stands for the n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ matrices. $\mathbb{E}\{\cdot\}$ denotes that mathematical expectation of "$.$". $\hat{x}(\varpi|\chi)$ represents the prediction of current moment χ to the moment ϖ. $*$ indicates symmetric term. $\mathcal{Q}$, $\mathcal{Q}^T$, $\mathcal{Q}^{-1}$, and $\mathcal{Q} > 0$ denote the transpose of $\mathcal{Q}$, the inverse of $\mathcal{Q}$, and $\mathcal{Q}$ being a symmetric positive-definite matrix, respectively. The vector $x = col(x_1, \cdots, x_n)$ is given by $[x_1^T, x_2^T, \cdots, x_n^T]^T$. diag$\{A, B\}$ stands for the block-diagonal matrix. $\lambda_{max}(P)(\lambda_{min}(P))$, respectively, denote the largest (smallest) eigenvalue of matrix P. I means the identity matrix. $\otimes$ represents the Kronecker product.

2 Problem formulation

As shown in Fig. 1, communication networks exist in both the feedback channel and the forward channel. Considering that the following discrete-time NCSs are subject to missing measurements and exogenous disturbances

$$\begin{cases} x(k+1) = \mathcal{A}x(k) + \mathcal{B}u(k) + \mathfrak{D}w(k), \\ y(k) = \alpha(k)Cx(k), \\ z(k) = \mathfrak{E}x(k) + \mathfrak{F}w(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ means the system state, $u(k) \in \mathbb{R}^m$ represents the control input, $y(k) \in \mathbb{R}^l$ denotes the measured output, $z(k) \in \mathbb{R}^z$ means control output, $w(k)$ denotes the disturbance input vector and satisfies $\sum_{i=0}^N w^T(k)w(k) \leq \mathfrak{d}$. Appropriately, dimensional matrices $\mathcal{A} \in \mathbb{R}^{n\times n}$, $\mathcal{B} \in \mathbb{R}^{n\times m}$, $\mathfrak{D} \in \mathbb{R}^{n\times s}$, $C \in \mathbb{R}^{l\times n}$, $\mathfrak{E} \in \mathbb{R}^{z\times n}$ and $\mathfrak{F} \in \mathbb{R}^{z\times s}$ denote system matrices, respectively. The random variable $\alpha(k)$ characterizes the probabilistic missing phenomena. Also, $\alpha(k)$ is Bernoulli-distributed white sequences, and its probabilities are written by

$$\text{Prob}\{\alpha(k) = 1\} = \alpha, \ \text{Prob}\{\alpha(k) = 0\} = 1 - \alpha.$$

Remark 1 We should take the unmeasurable state and missing measurements into account simultaneously. In the actual network, the measurement signal may be lost during network transmission because of some complex situation [28]. The sensor output often suffers from probabilistic signal losses such as channel congestion, multi-path fading, transmission rejection, faulty network hardware or drivers, and so on. A general model is the Bernoulli distribution model, where 0 means an entire missing of signals and 1 represents the intactness. NCSs (1) are more general than the systems in [26].

Before proceeding further, it is assumed that:

Assumption 1 n_b denotes the TDs in feedback channel and n_w denotes the TDs in forward channel. Besides, the PDs are represented as n_l in two channels. For generally speaking, PDs might be considered TDs due to its consistency, and this treatment approach is also specified in [21,26]. Additionally, the communication delay in the feedback channel and forward channel are $\mathcal{D}_1 = n_b + n_l$ and $\mathcal{D}_2 = n_w + n_l$, respectively.

Assumption 2 The data transmission mode is single packet transmission. It is assumed that the TD-PDs are bounded. $\bar{\mathcal{D}}_1$ denotes the maximum value of $\mathcal{D}_1$, and $\bar{\mathcal{D}}_2$ denotes the maximum value of $\mathcal{D}_2$.

Actually, we desire to design the NFO to deal with the missing measurement, and the NFO is given for system (1) as follows:

$$\begin{cases} \hat{x}(k-\mathcal{D}_1+1|k-\mathcal{D}_1) = \mathcal{A}\hat{x}(k-\mathcal{D}_1|k-\mathcal{D}_1-1) \\ \qquad + \mathcal{B}u(k-\mathcal{D}_1) \\ \qquad + \mathfrak{D}w(k-\mathcal{D}_1) \\ \qquad + (\mathcal{L} + \triangle\mathcal{L}(k-\mathcal{D}_1)) \\ \qquad \times \big(y(k-\mathcal{D}_1) - \hat{y}(k-\mathcal{D}_1)\big), \\ \hat{y}(k-\mathcal{D}_1) = \alpha(k-\mathcal{D}_1)C\hat{x}(k-\mathcal{D}_1|k-\mathcal{D}_1-1), \end{cases} \tag{2}$$

where $\hat{x}(k-\mathcal{D}_1+1|k-\mathcal{D}_1) \in \mathbb{R}^n$ is the one-step prediction of state, $\hat{y}(k-\mathcal{D}_1)$ stands for the observer output. $\mathcal{L}$ denotes the observer gain, it can de derived later in this paper. $\triangle\mathcal{L}$ is the uncertain observer gain and $\triangle\mathcal{L} = M_{\mathcal{L}}Q_{\mathcal{L}}(k)N_{\mathcal{L}}$. $Q_{\mathcal{L}}(k)$ is a real uncertain matrix with Lebesgue measurable element which satisfies $Q_{\mathcal{L}}(k)^T Q_{\mathcal{L}}(k) \leq I$.

Then, the state predictions from time $k-\mathcal{D}_1+1$ to $k+\mathcal{D}_2-1$ on the controller side is constructed by

$$\begin{aligned} \hat{x}(k-\mathcal{D}_1+\iota|k-\mathcal{D}_1) = &\mathcal{A}\hat{x}(k-\mathcal{D}_1+\iota-1|k-\mathcal{D}_1) \\ &+ \mathcal{B}u(k-\mathcal{D}_1+\iota-1) \\ &+ \mathfrak{D}w(k-\mathcal{D}_1+\iota-1), \\ &\iota = 2,3,\ldots,\mathcal{D}_1+\mathcal{D}_2, \end{aligned} \tag{3}$$

where $\hat{x}(k-\mathcal{D}_1+\iota|k-\mathcal{D}_1) \in \mathbb{R}^n$ represents the state prediction of ι step forward for the current time $k-\mathcal{D}_1$.

From (3), the predictive values are given as

$$\begin{aligned} \hat{x}(k-\mathcal{D}_1+2|k-\mathcal{D}_1) = &\mathcal{A}\hat{x}(k-\mathcal{D}_1+1|k-\mathcal{D}_1) \\ &+ \mathcal{B}u(k-\mathcal{D}_1+1) \\ &+ \mathfrak{D}w(k-\mathcal{D}_1+1), \\ \hat{x}(k-\mathcal{D}_1+3|k-\mathcal{D}_1) = &\mathcal{A}^2\hat{x}(k-\mathcal{D}_1+1|k-\mathcal{D}_1) \\ &+ \mathcal{A}\mathcal{B}u(k-\mathcal{D}_1+1) \\ &+ \mathcal{A}\mathfrak{D}w(k-\mathcal{D}_1+1) \\ &+ \mathcal{B}u(k-\mathcal{D}_1+2) \\ &+ \mathfrak{D}w(k-\mathcal{D}_1+2). \end{aligned}$$

Thus, the state prediction of $k+\mathcal{D}_1$ to time k on the controller side is calculated as

$$\begin{aligned} \hat{x}(k|k-\mathcal{D}_1) = &\mathcal{A}^{\mathcal{D}_1-1}\hat{x}(k-\mathcal{D}_1+1|k-\mathcal{D}_1) \\ &+ \sum_{\iota=2}^{\mathcal{D}_1} \mathcal{A}^{\mathcal{D}_1-\iota}\mathcal{B}u(k+\iota-\mathcal{D}_1-1) \\ &+ \sum_{\iota=2}^{\mathcal{D}_1} \mathcal{A}^{\mathcal{D}_1-\iota}\mathfrak{D}w(k+\iota-\mathcal{D}_1-1). \end{aligned} \tag{4}$$

According to (4), $\hat{x}(k+1|k-\mathcal{D}_1)$ can be represented as

$$\begin{aligned} \hat{x}(k+1|k-\mathcal{D}_1) = &\mathcal{A}\hat{x}(k|k-\mathcal{D}_1) + \mathcal{B}u(k) + \mathfrak{D}w(k) \\ = &\mathcal{A}^{\mathcal{D}_1}\hat{x}(k-\mathcal{D}_1+1|k-\mathcal{D}_1) \\ &+ \sum_{\iota=2}^{\mathcal{D}_1} \mathcal{A}^{\mathcal{D}_1-\iota+1}\mathcal{B}u(k+\iota-\mathcal{D}_1-1) \\ &+ \sum_{\iota=2}^{\mathcal{D}_1} \mathcal{A}^{\mathcal{D}_1-\iota+1}\mathfrak{D}w(k+\iota-\mathcal{D}_1-1) \\ &+ \mathcal{B}u(k) + \mathfrak{D}w(k). \end{aligned}$$

Then, $\hat{x}(k+\mathcal{D}_2|k-\mathcal{D}_1)$ can be obtained as

$$\begin{aligned} \hat{x}(k+\mathcal{D}_2|k-\mathcal{D}_1) = &\mathcal{A}^{\mathcal{D}_1+\mathcal{D}_2-1}\hat{x}(k-\mathcal{D}_1+1|k-\mathcal{D}_1) \\ &+ \sum_{\iota=2}^{\mathcal{D}_1+\mathcal{D}_2} \mathcal{A}^{\mathcal{D}_1-\iota+\mathcal{D}_2}\mathcal{B} \\ &\times u(k+\iota-\mathcal{D}_1-1) \\ &+ \sum_{\iota=2}^{\mathcal{D}_1+\mathcal{D}_2} \mathcal{A}^{\mathcal{D}_1-\iota+\mathcal{D}_2}\mathfrak{D} \\ &\times w(k+\iota-\mathcal{D}_1-1). \end{aligned}$$

Furthermore, $\hat{x}(k+\mathcal{D}_2|k-\mathcal{D}_1+1)$ is derived as follows:

$$\begin{aligned}\hat{x}(k+\mathcal{D}_2|k-\mathcal{D}_1+1) =&\mathcal{A}^{\mathcal{D}_1+\mathcal{D}_2-2}\\ &\times \hat{x}(k-\mathcal{D}_1+2|k-\mathcal{D}_1+1)\\ &+\sum_{\iota=3}^{\mathcal{D}_1+\mathcal{D}_2}\mathcal{A}^{\mathcal{D}_1-\iota+\mathcal{D}_2}\mathcal{B}\\ &\times u(k+\iota-\mathcal{D}_1-1)\\ &+\sum_{\iota=3}^{\mathcal{D}_1+\mathcal{D}_2}\mathcal{A}^{\mathcal{D}_1-\iota+\mathcal{D}_2}\mathfrak{D}\\ &\times w(k+\iota-\mathcal{D}_1-1).\end{aligned} \quad (5)$$

Denote $e(k-\mathcal{D}_1+1)=x(k-\mathcal{D}_1+1)-\hat{x}(k-\mathcal{D}_1+1|k-\mathcal{D}_1)$, then substitute (2) into (5), we have

$$\begin{aligned}\hat{x}(k+\mathcal{D}_2|k-\mathcal{D}_1+1) =&\hat{x}(k+\mathcal{D}_2|k-\mathcal{D}_1)\\ &+\mathcal{A}^{\mathcal{D}_1+\mathcal{D}_2-2}\\ &\times(\mathcal{L}+\triangle\mathcal{L}(k-\mathcal{D}_1+1))\\ &\times\alpha(k-\mathcal{D}_1+1)Ce(k-\mathcal{D}_1+1).\end{aligned} \quad (6)$$

Then, the state prediction of $k-\mathcal{D}_1$ to $k+\mathcal{D}_2$ can be derived as follows:

$$\begin{aligned}\hat{x}(k+\mathcal{D}_2|k-\mathcal{D}_1) =&\hat{x}(k+\mathcal{D}_2|k-\mathcal{D}_1+2)\\ &-\mathcal{A}^{\mathcal{D}_1+\mathcal{D}_2-3}(\mathcal{L}+\triangle\mathcal{L}(k-\mathcal{D}_1+2))\\ &\times\alpha(k-\mathcal{D}_1+2)Ce(k-\mathcal{D}_1+2)\\ &-\mathcal{A}^{\mathcal{D}_1+\mathcal{D}_2-2}(\mathcal{L}+\triangle\mathcal{L}(k-\mathcal{D}_1+1))\\ &\times\alpha(k-\mathcal{D}_1+1)Ce(k-\mathcal{D}_1+1)\\ &\cdots\\ =&\hat{x}(k+\mathcal{D}_2|k+\mathcal{D}_2-1)\\ &-\sum_{i=0}^{\mathcal{D}_1+\mathcal{D}_2-2}\mathcal{A}^i\\ &\times(\mathcal{L}+\triangle\mathcal{L}(k+\mathcal{D}_2-i-1))\\ &\times\alpha(k+\mathcal{D}_2-i-1)\\ &\times Ce(k+\mathcal{D}_2-i-1).\end{aligned}$$

By employing the designed NPC scheme, the state prediction $\hat{x}(k|k-\mathcal{D}_1-\mathcal{D}_2)$ on the actuator side at time k, the following is obtained:

$$\begin{aligned}\hat{x}(k|k-\mathcal{D}_1-\mathcal{D}_2) =&\hat{x}(k|k-1)-\sum_{i=0}^{\mathcal{D}_1+\mathcal{D}_2-2}\mathcal{A}^i\\ &\times(\mathcal{L}+\triangle\mathcal{L}(k-i-1))\\ &\times\alpha(k-i-1)Ce(k-i-1).\end{aligned}$$

The control input has not been updated since the state prediction of the current time is delivered to the actuator. In this note, we are interested in designing the following appropriate controller:

$$u(k)=u(k|k-\mathcal{D}_1-\mathcal{D}_2)=\mathcal{K}\hat{x}(k|k-\mathcal{D}_1-\mathcal{D}_2), \quad (7)$$

where the controller gain $\mathcal{K}$ can be generated by the finite-time control theory. Based on the above analysis, the CLSs can be obtained as

$$\begin{cases}x(k+1)=\Gamma x(k)+\Theta(\mathcal{D}_1,\mathcal{D}_2)E(k)+\mathfrak{D}w(k),\\ E(k+1)=\Lambda E(k),\end{cases} \quad (8)$$

where

$$\begin{aligned}&\Gamma=\mathcal{A}+\mathcal{B}\mathcal{K},\quad E(k)=[e(k)^T,e(k-1)^T,\\ &\qquad\qquad\qquad\qquad \ldots,e(k-\mathcal{D}_1-\mathcal{D}_2+1)^T]^T,\\ &e(k)=x(k)-\hat{x}(k|k-1),\\ &\Theta(\mathcal{D}_1,\mathcal{D}_2)=-\mathcal{B}\mathcal{K}\,[\,I_n\ (\mathcal{L}+\triangle\mathcal{L}(k-1))\alpha(k-1)C\\ &\qquad \mathcal{A}(\mathcal{L}+\triangle\mathcal{L}(k-2))\alpha(k-2)C\ \cdots\\ &\qquad \mathcal{A}^{\mathcal{D}_1+\mathcal{D}_2-2}(\mathcal{L}+\triangle\mathcal{L}(k-\mathcal{D}_1-\mathcal{D}_2+1))\\ &\qquad \times\alpha(k-\mathcal{D}_1-\mathcal{D}_2+1)C\,],\\ &\Lambda=\mathrm{diag}\{\mathcal{A}-(\mathcal{L}+\triangle\mathcal{L}(k))\alpha(k)C,\\ &\qquad \mathcal{A}-(\mathcal{L}+\triangle\mathcal{L}(k-1))\alpha(k-1)C,\cdots,\\ &\qquad \mathcal{A}-(\mathcal{L}+\triangle\mathcal{L}(k-\mathcal{D}_1-\mathcal{D}_2+1))\\ &\qquad \times\alpha(k-\mathcal{D}_1-\mathcal{D}_2+1)C\}.\end{aligned}$$

Remark 2 This new prediction strategy makes a contribution to the model of CLSs simpler. If we use the previous NPC scheme [16,17], then one-step prediction is obtained as follows:

$$\begin{aligned}\hat{x}(k-\mathcal{D}_1+1|k-\mathcal{D}_1) =&\mathcal{A}\hat{x}(k-\mathcal{D}_1|k-\mathcal{D}_1-1)\\ &+\mathcal{B}u(k-\mathcal{D}_1)\\ &+(\mathcal{L}+\triangle\mathcal{L}(k-\mathcal{D}_1))\\ &\times\big(y(k-\mathcal{D}_1)\\ &-\alpha(k-\mathcal{D}_1)\\ &\times C\hat{x}(k-\mathcal{D}_1|k-\mathcal{D}_1-1)\big).\end{aligned}$$

Similar to the previous derivation, we derive

$$\begin{aligned}\hat{x}(k|k-\mathcal{D}_1-\mathcal{D}_2) =&\hat{x}(k|k-1)-\sum_{\iota=0}^{\mathcal{D}_1+\mathcal{D}_2-2}\\ &\times\mathcal{A}^\iota(\mathcal{L}+\triangle\mathcal{L}(k-\iota-1))\\ &\times\alpha(k-\iota-1)Ce(k-\iota-1)\\ &+\sum_{\iota=0}^{\mathcal{D}_1+\mathcal{D}_2-2}\mathcal{A}^\iota\mathfrak{D}w(k-\iota-1).\end{aligned}$$

Hence, the CLSs can be described as

$$\begin{cases} x(k+1) =(\mathcal{A}+\mathcal{B}\mathcal{K})x(k) - \mathcal{B}\mathcal{K} \\ \quad \times \big(e(k) + \sum_{\iota=0}^{\mathcal{D}_1+\mathcal{D}_2-2} \mathcal{A}^{\iota} \\ \quad \times (\mathcal{L}+\triangle\mathcal{L}(k-\iota-1)) \\ \quad \times \alpha(k-\iota-1)Ce(k-\iota-1) \\ \quad - \sum_{\iota=0}^{\mathcal{D}_1+\mathcal{D}_2-2} \mathcal{A}^{\iota}\mathfrak{D}w(k-\iota-1)\big) + \mathfrak{D}w(k), \\ e(k-\jmath-1) =\big(\mathcal{A}-(\mathcal{L}+\triangle\mathcal{L}(k-\jmath))\alpha(k-\jmath)C\big) \\ \quad \times e(k-\jmath),\ \jmath=0,1,\ldots,\mathcal{D}_1+\mathcal{D}_2-1. \end{cases} \tag{9}$$

Compared with (8), $\sum_{\iota=0}^{\mathcal{D}_1+\mathcal{D}_2-2}\mathcal{A}^{\iota}\mathfrak{D}w(k-\iota-1)$ as an extra item exists in CLSs (9), which will increases the complexity of (9). Even if (9) can be integrated, additional conditions $w(k-\zeta+1) = \aleph w(k-\zeta)$, $\zeta = 0, 1\ldots, \mathcal{D}_1+\mathcal{D}_2-1$ are required. Due to the matrix $\aleph$, it will increase the conservativeness of the system. In addition, if we cannot find such a matrix, it may be impossible to get the CLS like (8), not to mention the corresponding controller and observer.

We added $\mathfrak{D}w(k-\mathcal{D}_1)$ to the above analysis to make the one-step prediction more general and the resulting CLSs more simple. The new NPC scheme is inspired by [15–17], but it is different from these ones.

Denoting variable $\bar{x}(k) = col\big(x(k), E(k)\big)$, (8) is reformulated as follows:

$$\begin{cases} \bar{x}(k+1) = \bar{\mathcal{A}}\bar{x}(k) + \bar{\mathfrak{D}}w(k), \\ z(k) = \bar{\mathfrak{E}}\bar{x}(k) + \bar{\mathfrak{F}}w(k), \end{cases} \tag{10}$$

where

$$\bar{\mathcal{A}} = \begin{bmatrix} \Gamma & \Theta(\mathcal{D}_1,\mathcal{D}_2) \\ 0 & \Lambda \end{bmatrix},\ \bar{\mathfrak{D}} = \begin{bmatrix}\mathfrak{D}^T & 0 \cdots 0\end{bmatrix}^T, \\ \bar{\mathfrak{E}} = \begin{bmatrix}\mathfrak{E} & 0 \cdots 0\end{bmatrix},\ \bar{\mathfrak{F}} = \mathfrak{F}.$$

In order to study system (10) over a finite time interval, we must recall some definitions and lemmas as follows:

Definition 1 [41] Given positive scalars $0 < c_1 < c_2$, $\mathfrak{d} > 0$, $\gamma > 0$, and $N \in \mathbb{Z}^+$, a positive define matrix $\bar{\Re}$, if

$$\tau_1 : \mathbb{E}\left\{\bar{x}^T(0)\bar{\Re}\bar{x}(0) \le c_1\right\} \Rightarrow \tau_2 : \mathbb{E}\left\{\bar{x}^T(k)\bar{\Re}\bar{x}(k)\right\} \le c_2,$$

holds, (10) is said to be SFTB.

Remark 3 In fact, it should be indicated that if the augmented system (10) is SFTB, then NCSs (1) is also SFTB. The reason for this conclusion is as follows. Due to $\bar{\Re} = \text{diag}\underbrace{\{\Re, \Re, \ldots, \Re\}}_{\mathcal{D}_1+\mathcal{D}_2+1} = I_{(\mathcal{D}_1+\mathcal{D}_2+1)} \otimes \Re, \Re > 0$, we can use simple derivation to get $\mathbb{E}\left\{x^T(k)\Re x(k)\right\} < \mathbb{E}\left\{\bar{x}^T(k)\bar{\Re}\bar{x}(k)\right\} \le c_2$. That is to say, we can have the conclusion that the (1) is SFTB with respect to $(c_1, c_2, \Re, N, \mathfrak{d})$.

Definition 2 [42] For given scalars $0 < c_1 < c_2, \mathfrak{d} > 0$, $\gamma > 0$, and $N \in \mathbb{Z}^+$, a positive define matrix $\bar{\Re}$, if

$$\mathbb{E}\left\{\sum_{k=0}^{N} z^T(k)z(k)\right\} < \gamma^2\mathbb{E}\left\{\sum_{k=0}^{N} w^T(k)w(k)\right\},$$

holds, under the zero-initial condition and $w^T(k)w(k) < \mathfrak{d}$, the CLSs (10) can be said to be SFTB with a prescribed H_∞ performance index γ.

Lemma 1 *[43] Given matrices $W = W^T$, H and U of appropriate dimensions, if the condition of $\triangle^T(k)\triangle(k) \le I$ satisfies all $\triangle(k)$, the following inequality is true*

$$W + H\triangle(k)U + U^T\triangle^T(k)H^T < 0,$$

if and only if for some $\varepsilon > 0$, $W + \varepsilon^{-1}HH^T + \varepsilon U^TU < 0$.

On the basis of the previous statement and analysis, we design a NFO-based predictive controller for (1) such that these requirements are met. First, the concerned system is SFTB and have a H_∞ performance with respect to $(c_1, c_2, I_{(\mathcal{D}_1+\mathcal{D}_2+1)} \otimes \bar{\Re}, N, \mathfrak{d})$. Second, communication delays are disposed well.

3 Main results

The SFTB and SFTB with H_∞ performance problems for (10) are given. Further, sufficient conditions are derived under communication constraints. Due to the different TD-PDs, we first choose $\mathcal{D}_1 + \mathcal{D}_2 = 2$ which is the minimum value of the sum of TD-PDs.

3.1 Stability Analysis

Theorem 1 *For given the TDs $\mathcal{D}_1$, PDs $\mathcal{D}_2$, $c_1 < c_2$, N, $\mathfrak{d}$, matrices $\mathcal{K}$ and $\mathcal{L}$, and positive definite matrices $\Re$. The concerned system (10) is SFTB with respect to $(c_1, c_2, I_3 \otimes \Re, N, \mathfrak{d})$ if there exist a constant $\beta \ge 1$, matrices $\mathcal{P} > 0$, $Q > 0$ satisfying*

$$\begin{bmatrix} \tilde{\mathcal{A}}^T\mathcal{P}\tilde{\mathcal{A}} - \beta\mathcal{P} & \tilde{\mathcal{A}}^T\mathcal{P}\bar{\mathfrak{D}} \\ * & \bar{\mathfrak{D}}^T\mathcal{P}\bar{\mathfrak{D}} - \beta Q \end{bmatrix} < 0, \tag{11}$$

$$\bar{\sigma}_{\mathcal{P}}c_1 + \sigma_Q\mathfrak{d} < \beta^{-N}c_2\underline{\sigma}_{\mathcal{P}}. \tag{12}$$

Proof To begin with, we construct Lyapunov functional candidate

$$V(x(k)) = \bar{x}^T(k)\mathcal{P}\bar{x}(k), \tag{13}$$

Along the trajectory of (10), we calculate $V(x(k+1))$ and its mathematical expectation. It yields that

$$\begin{aligned}\mathbb{E}\{V(x(k+1))\} =& \bar{x}^T(k)\tilde{\mathcal{A}}^T\mathcal{P}\tilde{\mathcal{A}}\bar{x}(k)\\ &+ 2\bar{x}^T(k)\tilde{\mathcal{A}}^T\mathcal{P}\bar{\mathfrak{D}}w(k)\\ &+ w^T(k)\bar{\mathfrak{D}}^T\mathcal{P}\bar{\mathfrak{D}}w(k),\end{aligned}$$

where

$$\tilde{\mathcal{A}} = \begin{bmatrix}\Gamma & \tilde{\Theta}(\mathcal{D}_1,\mathcal{D}_2)\\ 0 & \tilde{\Lambda}\end{bmatrix},$$

$$\tilde{\Lambda} = \begin{bmatrix}\mathcal{A}-\alpha(\mathcal{L}+\triangle\mathcal{L}(k))C & 0\\ 0 & \mathcal{A}-\alpha(\mathcal{L}+\triangle\mathcal{L}(k-1))C\end{bmatrix},$$

$$\tilde{\Theta}(\mathcal{D}_1,\mathcal{D}_2) = -\mathcal{B}\mathcal{K}\left[I_n \;\; \alpha(\mathcal{L}+\triangle\mathcal{L}(k-1))C\right].$$

Then,

$$\begin{aligned}&\mathbb{E}\{V(x(k+1))\} - \beta V(x(k)) - \beta w^T(k)Qw(k)\\ &= \bar{x}^T(k)(\tilde{\mathcal{A}}^T\mathcal{P}\tilde{\mathcal{A}} - \beta\mathcal{P})\bar{x}(k) + 2\bar{x}^T(k)\tilde{\mathcal{A}}^T\mathcal{P}\bar{\mathfrak{D}}w(k)\\ &\quad + w^T(k)\big(\bar{\mathfrak{D}}^T\mathcal{P}\bar{\mathfrak{D}} - \beta Q\big)w(k)\\ &= \eta^T(k)\Upsilon\eta(k),\end{aligned}$$

where

$$\eta^T(k) = [\bar{x}^T(k) \;\; w^T(k)],$$
$$\Upsilon = \begin{bmatrix}\tilde{\mathcal{A}}^T\mathcal{P}\tilde{\mathcal{A}} - \beta\mathcal{P} & \tilde{\mathcal{A}}^T\mathcal{P}\bar{\mathfrak{D}}\\ * & \bar{\mathfrak{D}}^T\mathcal{P}\bar{\mathfrak{D}} - \beta Q\end{bmatrix}.$$

Thus, it is noticed from (11) that

$$\mathbb{E}\{V(x(k+1))\} - \beta V(x(k)) + \beta w^T(k)Qw(k) < 0.$$

Let $\sigma_Q = \lambda_{max}(Q)$ and according to the above relation, we get

$$\mathbb{E}\{V(x(k+1))\} < \beta\mathbb{E}\{V(x(k))\} + \beta\sigma_Q\mathbb{E}\left\{w^T(k)w(k)\right\}.$$

Note that $\beta \geq 1$, we readily infer that

$$\begin{aligned}\mathbb{E}\{V(x(k))\} <& \beta^k\Bigg(\mathbb{E}\{V(x(0))\}\\ &+\sigma_Q\mathbb{E}\left\{\sum_{l=0}^{k-1}\beta^{l-k+1}w_{k-l-1}^T w_{k-l-1}\right\}\Bigg)\\ \leq& \beta^k\big(\mathbb{E}\{V(x(0))\} + \sigma_Q\mathfrak{d}\big)\\ \leq& \beta^N\big(\mathbb{E}\{V(x(0))\} + \sigma_Q\mathfrak{d}\big).\end{aligned}$$

Denoting $\tilde{\mathcal{P}} = \bar{\Re}^{-1/2}\mathcal{P}\bar{\Re}^{-1/2}$, $\bar{\sigma}_{\mathcal{P}} = \lambda_{max}(\tilde{\mathcal{P}})$, $\underline{\sigma}_{\mathcal{P}} = \lambda_{min}(\tilde{\mathcal{P}})$, it indicates that

$$\begin{aligned}\mathbb{E}\{V(x(0))\} &= \mathbb{E}\left\{\bar{x}^T(0)\mathcal{P}\bar{x}(0)\right\}\\ &\leq \lambda_{max}(\tilde{\mathcal{P}})\mathbb{E}\left\{\bar{x}^T(0)\bar{\Re}\bar{x}(0)\right\}\\ &= \lambda_{max}(\tilde{\mathcal{P}})\mathbb{E}\left\{\bar{x}^T(0)(I_3\otimes\Re)\bar{x}(0)\right\} \leq \bar{\sigma}_{\mathcal{P}}c_1.\end{aligned} \tag{14}$$

Notice that

$$\begin{aligned}\mathbb{E}\{V(x(k))\} &= \mathbb{E}\left\{\bar{x}^T(k)\mathcal{P}\bar{x}(k)\right\}\\ &\geq \lambda_{min}(\tilde{\mathcal{P}})\mathbb{E}\left\{\bar{x}^T(k)\bar{\Re}\bar{x}(k)\right\}\\ &\geq \underline{\sigma}_{\mathcal{P}}\mathbb{E}\left\{\bar{x}^T(k)\bar{\Re}\bar{x}(k)\right\}.\end{aligned} \tag{15}$$

Hence, taking (12)–(15) into consideration, it follows that

$$\begin{aligned}\mathbb{E}\left\{\bar{x}^T(k)\bar{\Re}\bar{x}(k)\right\} &= \mathbb{E}\left\{\bar{x}^T(k)(I_3\otimes\Re)\bar{x}(k)\right\}\\ &\leq \frac{\beta^N(\bar{\sigma}_{\mathcal{P}}c_1 + \sigma_Q\mathfrak{d})}{\underline{\sigma}_{\mathcal{P}}} < c_2.\end{aligned} \tag{16}$$

It is clear from (16) that $\mathbb{E}\left\{\bar{x}^T(k)\bar{\Re}\bar{x}(k)\right\} \leq c_2$. According to Definition 1, one can conclude that SFTB of the CLSs (10) have been achieved. That proof is now complete. □

3.2 Guaranteed Performance Analysis

The analysis of the H_∞ performance are conducted, and sufficient conditions which ensure (10) is SFTB are also provided.

Theorem 2 *For given the TDs $\mathcal{D}_1$, PDs $\mathcal{D}_2$, $c_1 < c_2$, N, $\mathfrak{d}$, matrices $\mathcal{K}$ and $\mathcal{L}$, and positive definite matrices $\Re$. The concerned system (10) is SFTB and possesses a prescribed H_∞ performance with respect to $(c_1, c_2, I_3\otimes\Re, N, \mathfrak{d})$ if there exist a scalar $\beta \geq 1$, matrices $\Im$ and $\mathcal{P} > 0$ satisfying*

$$\begin{bmatrix}\Pi^{(1)} & \Pi^{(2)}\\ * & \Pi^{(4)}\end{bmatrix} < 0, \tag{17}$$

$$\bar{\sigma}_{\mathcal{P}}c_1 + \beta\mathfrak{d} < \beta^{-N}c_2\underline{\sigma}_{\mathcal{P}}, \tag{18}$$

where

$$\Pi^{(1)} = \begin{bmatrix}\mathcal{P} - \Im - \Im^T & 0\\ * & -I\end{bmatrix},\; \Pi^{(2)} = \begin{bmatrix}\Im\tilde{\mathcal{A}} & \Im\bar{\mathfrak{D}}\\ \bar{\mathfrak{E}} & \bar{\mathfrak{F}}\end{bmatrix},$$
$$\Pi^{(4)} = \mathrm{diag}\{-\beta\mathcal{P}, -\beta I\}.$$

Proof Based on Theorem 1, construct Lyapunov functional candidate and obtain

$$\begin{aligned} &\mathbb{E}\{V(x(k+1))\} \\ &\quad -\beta V(x(k)) - \beta w^T(k)w(k) + z^T(k)z(k) \\ &\qquad = \zeta^T(k)\aleph\zeta(k), \end{aligned} \tag{19}$$

where

$$\zeta^T(k) = [\bar{x}^T(k) \;\; w^T(k)],$$
$$\aleph = \begin{bmatrix} \tilde{\mathcal{A}}^T\mathcal{P}\tilde{\mathcal{A}} + \bar{\mathfrak{E}}^T\bar{\mathfrak{E}} - \beta\mathcal{P} & \tilde{\mathcal{A}}^T\mathcal{P}\bar{\mathfrak{D}} + \bar{\mathfrak{E}}^T\bar{\mathfrak{F}} \\ * & \bar{\mathfrak{D}}^T\mathcal{P}\bar{\mathfrak{D}} + \bar{\mathfrak{F}}^T\bar{\mathfrak{F}} - \beta I \end{bmatrix}.$$

By using the basic inequality $\mu^T\nu + \nu^T\mu \leq \mu^T\mathfrak{Q}\mu + \nu^T\mathfrak{Q}^{-1}\nu$ (where μ and ν are vectors of compatible dimensions), we can achieve $P - \Im - \Im^T \geq -\Im P^{-1}\Im^T$. By substituting it into (17), we have

$$\begin{bmatrix} -\Im\mathcal{P}^{-1}\Im^T & 0 & \Im\tilde{\mathcal{A}} & \Im\bar{\mathfrak{D}} \\ * & -I & \bar{\mathfrak{E}} & \bar{\mathfrak{F}} \\ * & * & -\beta\mathcal{P} & 0 \\ * & * & * & -\beta I \end{bmatrix} < 0. \tag{20}$$

If we pre- and post-multiply the inequality (20) with $\mathrm{diag}\{\Im^{-T}\mathcal{P}, I, I, I\}$, then we get

$$\begin{bmatrix} -\mathcal{P} & 0 & \mathcal{P}\tilde{\mathcal{A}} & \mathcal{P}\bar{\mathfrak{D}} \\ * & -I & \bar{\mathfrak{E}} & \bar{\mathfrak{F}} \\ * & * & -\beta\mathcal{P} & 0 \\ * & * & * & -\beta I \end{bmatrix} < 0. \tag{21}$$

From (17) and Schur complement, we derive

$$\begin{aligned} \mathbb{E}\{V(x(k+1))\} <& \beta\mathbb{E}\{V(x(k))\} \\ &+ \beta\mathbb{E}\left\{w^T(k)w(k)\right\} \\ &- \mathbb{E}\left\{z^T(k)z(k)\right\}. \end{aligned} \tag{22}$$

It follows from (22) that

$$\begin{aligned} \mathbb{E}\{V(x(k))\} <& \beta\mathbb{E}\{V(x(k-1))\} \\ &+ \beta\mathbb{E}\left\{w^T(k-1)w(k-1)\right\} \\ &- \mathbb{E}\left\{z^T(k-1)z(k-1)\right\} \\ <& \cdots \\ <& \beta^k\mathbb{E}\{V(x(0))\} + \beta \\ &\times \mathbb{E}\left\{\sum_{\xi=0}^{k-1}\beta^\xi w(k-\xi-1)^T w(k-\xi-1)\right\} \\ &- \mathbb{E}\left\{\sum_{\xi=0}^{k-1}\beta^\xi z(k-\xi-1)^T z(k-\xi-1)\right\}. \end{aligned} \tag{23}$$

From the zero initial condition and $V(x(k)) \geq 0$, we can obtain the following inequality:

$$\begin{aligned} &\beta\mathbb{E}\left\{\sum_{\xi=0}^{k-1}\beta^\xi w(k-\xi-1)^T w(k-\xi-1)\right\} \\ &\quad > \mathbb{E}\left\{\sum_{\xi=0}^{k-1}\beta^\xi z(k-\xi-1)^T z(k-\xi-1)\right\}. \end{aligned} \tag{24}$$

Using $\beta \geq 1$, it is deduced that

$$\begin{aligned} &\mathbb{E}\left\{\sum_{\xi=0}^{k-1} z(k-\xi-1)^T z(k-\xi-1)\right\} \\ &\quad < \mathbb{E}\left\{\sum_{\xi=0}^{k-1}\beta^\xi z(k-\xi-1)^T z(k-\xi-1)\right\} \\ &\quad < \beta\mathbb{E}\left\{\sum_{\xi=0}^{k-1}\beta^\xi w(k-\xi-1)^T w(k-\xi-1)\right\} \\ &\quad < \beta^k\mathbb{E}\left\{\sum_{\xi=0}^{k-1} w(k-\xi-1)^T w(k-\xi-1)\right\}. \end{aligned} \tag{25}$$

According to (25), we can easily get that

$$\mathbb{E}\left\{\sum_{k=0}^{N} z^T(k)z(k)\right\} < \gamma^2\mathbb{E}\left\{\sum_{\xi=0}^{N} w^T(k)w(k)\right\}, \tag{26}$$

with $\gamma = \sqrt{\beta^{N+1}}$. This completes the proof. □

3.3 Observer-Based NPC and Observer Design

Note that Theorem 2 is not suitable for designing the NPC controller and observer. It needs to be linearized. By Lemma 1, we present an equivalent statement of Theorem 2 as follows:

Theorem 3 *For given the TDs $\mathcal{D}_1$, PDs $\mathcal{D}_2$, scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\alpha > 0$, $c_1 < c_2$, N, and $\mathfrak{d}$, and positive definite matrices $\Re$. Consider the concerned system (10) with respect to $(c_1, c_2, I_3 \otimes \Re, N, \mathfrak{d})$ is SFTB and possesses a prescribed H_∞ performance, if there exist scalars $\beta \geq 1$ and $\sigma_1 \leq 1$, proper symmetric matrices $\mathcal{P}$, $\Im_{11}$, $\Im_{22}$, $\Im_{33}$, matrices Z_1, M_1, Y_1 and Y_2 satisfy the following LMIs:*

$$\begin{bmatrix} \tilde{\Phi}^{(1)} & \tilde{\Phi}^{(2)} \\ * & \tilde{\Phi}^{(4)} \end{bmatrix} < 0, \tag{27}$$

$$\Im_{11}\mathcal{B} = \mathcal{B}Z_1,\ \sigma_1\Re < \mathcal{P} < \Re,\ c_1 + \beta\mathfrak{d} < \beta^{-N}\sigma_1 c_2, \quad (28)$$

where

$$\tilde{\Phi}^{(1)} = \begin{bmatrix} \Phi_{11} & \mathcal{P}_{12} & \mathcal{P}_{13} & 0 & \Phi_{15} & \Phi_{16} & \Phi_{17} & \Im_{11}\mathfrak{D} \\ * & \Phi_{22} & \mathcal{P}_{23} & 0 & 0 & \Phi_{26} & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 & \Phi_{37} & 0 \\ * & * & * & -I & \mathfrak{E} & 0 & 0 & \mathfrak{F} \\ * & * & * & * & \Phi_{55} & \Phi_{56} & \Phi_{57} & 0 \\ * & * & * & * & * & \Phi_{66} & \Phi_{67} & 0 \\ * & * & * & * & * & * & \Phi_{77} & 0 \\ * & * & * & * & * & * & * & -\beta I \end{bmatrix},$$

$$\tilde{\Phi}^{(2)} = \begin{bmatrix} BM_1M_{\mathcal{L}} & 0 \\ 0 & \Im_{22}M_{\mathcal{L}} \\ \Im_{33}M_{\mathcal{L}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Phi}^{(4)} = \mathrm{diag}\{-\varepsilon_2 I, -\varepsilon_1 I\}, \quad \Phi_{11} = \mathcal{P}_{11} - \Im_{11} - \Im_{11}^T,$$

$$\Phi_{15} = \Im_{11}\mathcal{A} + \mathcal{B}M_1, \quad \Phi_{16} = -\mathcal{B}M_1,$$

$$\Phi_{17} = \begin{bmatrix} \mathcal{B}^T\Im_{11}^T & \alpha C^T M_2^T \\ -I & 0 \\ 0 & -I \end{bmatrix},$$

$$\Phi_{22} = \mathcal{P}_{22} - \Im_{22} - \Im_{22}^T, \quad \Phi_{26} = \Im_{22}\mathcal{A} - \alpha Y_1 C,$$

$$\Phi_{33} = \mathcal{P}_{33} - \Im_{33} - \Im_{33}^T, \quad \Phi_{37} = \Im_{33}\mathcal{A} - \alpha Y_2 C,$$

$$\Phi_{55} = -\beta\mathcal{P}_{11}, \quad \Phi_{56} = -\beta\mathcal{P}_{12}, \quad \Phi_{57} = -\beta\mathcal{P}_{13},$$

$$\Phi_{66} = -\beta\mathcal{P}_{22} + \varepsilon_1\alpha^2 C^T N_{\mathcal{L}}^T N_{\mathcal{L}} C, \quad \Phi_{67} = -\beta\mathcal{P}_{23},$$

$$\Phi_{77} = -\beta\mathcal{P}_{33} + \varepsilon_2\alpha^2 C^T N_{\mathcal{L}}^T N_{\mathcal{L}} C.$$

Meanwhile, the controller gain matrix and the NFO gain matrix can be achieved as

$$\mathcal{K} = Z_1^{-1}M_1,\ \mathcal{L} = \Im_{22}^{-1}Y_1. \quad (29)$$

Proof First, (17) can be reorganized as

$$\Xi + \Gamma_1 Q_{\mathcal{L}}(k)\Gamma_2 + \Gamma_2^T Q_{\mathcal{L}}^T(k)\Gamma_1^T + \Gamma_3 Q_{\mathcal{L}}(k)\Gamma_4 + \Gamma_4^T Q_{\mathcal{L}}^T(k)\Gamma_3^T < 0,$$

where

$$\Xi = \begin{bmatrix} \mathcal{P} - \Im - \Im^T & 0 & \Im\check{\mathcal{A}} & \Im\bar{\mathfrak{D}} \\ * & -I & \mathfrak{E} & \mathfrak{F} \\ * & * & -\beta\mathcal{P} & 0 \\ * & * & * & -\beta I \end{bmatrix},$$

$$\check{\mathcal{A}} = \begin{bmatrix} \mathcal{A} + \mathcal{B}\mathcal{K} & -\mathcal{B}\mathcal{K} & -\alpha\mathcal{B}\mathcal{K}\mathcal{L}C \\ 0 & \mathcal{A} - \alpha\mathcal{L}C & 0 \\ 0 & 0 & \mathcal{A} - \alpha\mathcal{L}C \end{bmatrix},$$

$$\tilde{\mathcal{A}} = \check{\mathcal{A}} + \begin{bmatrix} \Im_{11}\mathcal{B}\mathcal{K}M_{\mathcal{L}} \\ 0 \\ \Im_{33}M_{\mathcal{L}} \end{bmatrix} Q_{\mathcal{L}} \left[0\ 0\ -\alpha N_{\mathcal{L}}C\right] + \begin{bmatrix} 0 \\ \Im_{22}M_{\mathcal{L}} \\ 0 \end{bmatrix} Q_{\mathcal{L}} \left[0\ -\alpha N_{\mathcal{L}}C\ 0\right],$$

$$\Gamma_1 = \left[M_{\mathcal{L}}^T\mathcal{K}^T\mathcal{B}^T\Im_{11}^T\ 0\ M_{\mathcal{L}}^T\Im_{33}^T\ 0\ 0\ 0\ 0\ 0\right]^T,$$

$$\Gamma_2 = \left[0\ 0\ 0\ 0\ 0\ 0\ -\alpha N_{\mathcal{L}}C\ 0\right],$$

$$\Gamma_3 = \left[0\ M_{\mathcal{L}}^T\Im_{22}^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right]^T,$$

$$\Gamma_4 = \left[0\ 0\ 0\ 0\ 0\ -\alpha N_{\mathcal{L}}C\ 0\ 0\right].$$

According to Lemma 1, there exist $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that $\Xi + \varepsilon_2^{-1}\Gamma_1\Gamma_1^T + \varepsilon_2\Gamma_2^T\Gamma_2 + \varepsilon_1^{-1}\Gamma_3\Gamma_3^T + \varepsilon_1\Gamma_4^T\Gamma_4 < 0$. Note that

$$\mathcal{P} = \begin{bmatrix} \mathcal{P}_{11} & \mathcal{P}_{12} & \mathcal{P}_{13} \\ * & \mathcal{P}_{22} & \mathcal{P}_{23} \\ * & * & \mathcal{P}_{33} \end{bmatrix}, \quad \Im = \begin{bmatrix} \Im_{11} & 0 & 0 \\ 0 & \Im_{22} & 0 \\ 0 & 0 & \Im_{33} \end{bmatrix}. \quad (30)$$

In what follows, by using Schur complement, we can get

$$\begin{bmatrix} \Xi_{11} & 0 & \Xi_{13} & \Im_{11}\bar{\mathfrak{D}} & \Im_{11}\mathcal{B}\mathcal{K}M_{\mathcal{L}} & 0 \\ * & -I & \mathfrak{E} & \mathfrak{F} & 0 & \Im_{22}M_{\mathcal{L}} \\ * & * & \Xi_{33} & 0 & \Im_{33}M_{\mathcal{L}} & 0 \\ * & * & * & -\beta I & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & -\varepsilon_1 I \end{bmatrix} < 0,$$

where

$$\Xi_{11} = \mathrm{diag}\{\mathcal{P}_{11} - \Im_{11} - \Im_{11}^T, \mathcal{P}_{22} - \Im_{22} - \Im^T, \mathcal{P}_{33} - \Im_{33} - \Im_{33}^T\},$$

$$\Xi_{13} = \begin{bmatrix} \Im_{11}(\mathcal{A} + \mathcal{B}\mathcal{K}) & -\Im_{11}\mathcal{B}\mathcal{K} & -\alpha\Im_{11}\mathcal{B}\mathcal{K}\mathcal{L}C \\ 0 & \Im_{22}(\mathcal{A} - \mathcal{L}C) & 0 \\ 0 & 0 & \Im_{33}(\mathcal{A} - \mathcal{L}C) \end{bmatrix},$$

$$\Xi_{33} = \begin{bmatrix} -\beta\mathcal{P}_{11} & -\beta\mathcal{P}_{12} & -\beta\mathcal{P}_{13} \\ * & -\beta\mathcal{P}_{22} + \varepsilon_1\alpha^2 C^T N_{\mathcal{L}}^T N_{\mathcal{L}} C & -\beta\mathcal{P}_{23} \\ * & * & -\beta\mathcal{P}_{33} + \varepsilon_2\alpha^2 C^T N_{\mathcal{L}}^T N_{\mathcal{L}} C \end{bmatrix}.$$

Note that $\Im$ is non-singular. One can see that from (27)–(28) which contains an equation constraint. In order to treat $\Im_{11}\mathcal{B} = \mathcal{B}Z_1$, we should use the inequality to approximate the equation, which is proposed in [32]. As a result, the (17)–(18) feasibility problem has been transformed into the (27)–(28) feasibility problem. Then, we denote $Z_1\mathcal{K} = M_1$ and $\Im_{22}\mathcal{L} = Y_1$. It is not difficult to obtain $\mathcal{K}$ and $\mathcal{L}$. Furthermore, by taking $\mathcal{P} = \Re^{1/2}\tilde{\mathcal{P}}\Re^{1/2}$ into account, we can have $\sigma_1\Re < \mathcal{P} < \Re$ if $\lambda_{min}(\tilde{\mathcal{P}}) > \sigma_1$, $\lambda_{max}(\tilde{\mathcal{P}}) < 1$. From the above analysis, we can get (18). Now, the proof is completed. □

Remark 4 The existence problem given in the condition can be interpreted as the indefinite solution (or infinitely many solutions) of Y_1 and Y_2 in the simulation. When the condition is met, $Y_1 = Y_2$ is selected to obtain the unique solution of $\mathcal{L}$. In Theorem 3, conditions $\Im_{22}\mathcal{L} = Y_1$ and $\Im_{33}\mathcal{L} = Y_2$ can exist independently. We can get the observer $\mathcal{L}$ of (29) if the two conditions can be combined. We hypothesis that $\Im_{33} = \Im_{22}$ and $Y_2 = Y_1$, then the uniform observer can be obtained successfully. This assumption played a considerable part in obtaining reasonable observer.

Remark 5 Some sufficient conditions are provided to guarantee the closed-loop system is SFTB in Theorem 3, and the desired controller and observer exist. It should be noticed that the condition in Theorem 3 is dependent on the scalars ε_1, ε_2, α, β, c_1, c_2, N, $\mathfrak{d}$ and σ_1. If we fix some positive values for $\varepsilon_1, \varepsilon_2, \alpha$, then the conditions of Theorem 3 can be formally expressed as an LMI feasibility problem with $\mathcal{P}$, $\Im_{11}$, $\Im_{22}$, $\Im_{33}$, Z_1, M_1, Y_1, Y_2, σ_1 as optimization parameters. As previously stated, this enables the finite-time control problem to be incorporated into the general framework of multi-objective synthesis.

For fixed $\varepsilon_1, \varepsilon_2, \alpha, c_2, N$ and $\mathfrak{d}$, the maximum allowable value of β decreases as the value of σ_1 decreases or c_1 increases. When ε_1, ε_2, α, c_1, N, $\mathfrak{d}$ are given, the minimum allowable value of c_2 for different β and σ_1 can be calculated. As a result, when the condition in Theorem 3 is used, it is reasonable to study these parameters (ε_1, ε_2, α, β, c_1, c_2, N, $\mathfrak{d}$ and σ_1) together in some specifics.

For different TD-PDs conditions, we should obtain the sufficient conditions of SFTB with H_∞ performance. Similar to the result in Theorem 3, a NFO-based controller and the corresponding observer are designed to guarantee that stochastic NPC systems are SFTB and have H_∞ performance.

Corollary 1 *For given the TDs $\mathcal{D}_1$, PDs $\mathcal{D}_2$, scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\alpha > 0$, $c_1 < c_2$, N, $\mathfrak{d}$, and positive define matrices $\Re$. Consider the concerned system (10) with respect to $(c_1, c_2, I_{(\mathcal{D}_1+\mathcal{D}_2+1)} \otimes \Re, N, \mathfrak{d})$ is SFTB and possesses a prescribed H_∞ performance, if there exist constants $\beta \geq 1$ and $\sigma_1 \leq 1$, proper symmetric matrices $\mathcal{P}$, $\Im_{ii}$, $i \in \{1, 2, \ldots, \mathcal{D}_1 + \mathcal{D}_2 + 1\}$, matrices Z_1, M_1, M_2, Y_j, $j \in \{1, 2, \ldots, \mathcal{D}_1 + \mathcal{D}_2\}$ satisfy the following LMIs:*

$$\begin{bmatrix} \Phi^{(1)} & \Phi^{(2)} \\ * & \Phi^{(4)} \end{bmatrix} < 0, \tag{31}$$

$$\Im_{11}\mathcal{B} = \mathcal{B}Z_1, \ \sigma_1\Re < \mathcal{P} < \Re,$$
$$c_1 + \beta\mathfrak{d} < \beta^{-N}\sigma_1 c_2, \tag{32}$$

where

$$\Phi^{(2)} = \left[\begin{array}{cc} \mathcal{B}M_1\mathcal{A}^{\mathcal{D}_1+\mathcal{D}_2-2}M_{\mathcal{L}} & \mathcal{B}M_1\mathcal{A}^{\mathcal{D}_1+\mathcal{D}_2-3}M_{\mathcal{L}} \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & \Im_{(\mathcal{D}_1+\mathcal{D}_2)(\mathcal{D}_1+\mathcal{D}_2)}M_{\mathcal{L}} \\ \Im_{(\mathcal{D}_1+\mathcal{D}_2+1)(\mathcal{D}_1+\mathcal{D}_2+1)}M_{\mathcal{L}} & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{array}\right.$$
$$\left.\begin{array}{ccc} \cdots & \mathcal{B}M_1M_{\mathcal{L}} & 0 \\ \cdots & 0 & \Im_{22}M_{\mathcal{L}} \\ \cdots & \Im_{33}M_{\mathcal{L}} & 0 \\ \ddots & \vdots & \vdots \\ 0 & 0 & 0 \\ \cdots & 0 & 0 \\ \cdots & 0 & 0 \\ \ddots & \vdots & \vdots \\ \cdots & 0 & 0 \end{array}\right],$$

$$\Phi^{(1)} = \left[\begin{array}{ccccc} \Phi_{11} & \mathcal{P}_{12} & \cdots & \mathcal{P}_{1(\mathcal{D}_1+\mathcal{D}_2+1)} & 0 \\ * & \Phi_{22} & \cdots & \mathcal{P}_{2(\mathcal{D}_1+\mathcal{D}_2+1)} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & \Phi_{(\mathcal{D}_1+\mathcal{D}_2+1)(\mathcal{D}_1+\mathcal{D}_2+1)} & 0 \\ * & * & \cdots & * & -I \\ * & * & \cdots & * & * \\ * & * & \cdots & * & * \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & * & * \\ * & * & \cdots & * & * \end{array}\right.$$
$$\begin{array}{cc} \Phi_{1(\mathcal{D}_1+\mathcal{D}_2+3)} & \Phi_{1(\mathcal{D}_1+\mathcal{D}_2+4)} \\ 0 & \Psi_{2(\mathcal{D}_1+\mathcal{D}_2+4)} \\ \vdots & \vdots \\ 0 & 0 \\ \mathfrak{E} & 0 \\ \Phi_{(\mathcal{D}_1+\mathcal{D}_2+3)(\mathcal{D}_1+\mathcal{D}_2+3)} & \Phi_{(\mathcal{D}_1+\mathcal{D}_2+3)(\mathcal{D}_1+\mathcal{D}_2+4)} \\ \vdots & \vdots \\ * & * \\ * & * \end{array}$$
$$\left.\begin{array}{ccc} \cdots & \Phi_{1(2\mathcal{D}_1+2\mathcal{D}_2+3)} & \Im_{11}\mathfrak{D} \\ \cdots & 0 & 0 \\ \ddots & \vdots & \vdots \\ \cdots & \Phi_{(\mathcal{D}_1+\mathcal{D}_2+1)(2\mathcal{D}_1+2\mathcal{D}_2+3)} & 0 \\ \cdots & 0 & \mathfrak{F} \\ \cdots & \Phi_{(\mathcal{D}_1+\mathcal{D}_2+3)(2\mathcal{D}_1+2\mathcal{D}_2+3)} & 0 \\ \ddots & \vdots & \vdots \\ \cdots & \Phi_{(2\mathcal{D}_1+2\mathcal{D}_2+3)(2\mathcal{D}_1+2\mathcal{D}_2+3)} & 0 \\ \cdots & * & -\beta I \end{array}\right],$$

$$\Phi^{(4)} = \mathrm{diag}\{-\varepsilon_{(\mathcal{D}_1+\mathcal{D}_2)}I, \ldots, -\varepsilon_2 I, -\varepsilon_1 I\},$$
$$\Phi_{1(l+\mathcal{D}_1+\mathcal{D}_2+2)} = \Im_{11}\mathcal{A} + \mathcal{B}M_1, l = 1,$$
$$\Phi_{1(l+\mathcal{D}_1+\mathcal{D}_2+3)} = -\mathcal{B}M_1, l = 1,$$
$$\Phi_{1(l+\mathcal{D}_1+\mathcal{D}_2+4)} = \begin{bmatrix} (\mathcal{A}^{l-1})^T\mathcal{B}^T\Im_{11}^T & \alpha C^T M_2^T \\ -I & 0 \\ 0 & -I \end{bmatrix},$$

$$l = 1, \ldots, \mathcal{D}_1 + \mathcal{D}_2 - 1,$$
$$\Phi_{ll} = \mathcal{P}_{ll} - \Im_{ll} - \Im_{ll}^T,$$
$$l = 1, \ldots, 2\mathcal{D}_1 + 2\mathcal{D}_2 + 3,$$
$$\Phi_{(l+1)(l+5)} = \Im_{(l+1)(l+1)}\mathcal{A} - \alpha Y_l C,$$
$$l = 1, \ldots, \mathcal{D}_1 + \mathcal{D}_2 + 1,$$
$$\Phi_{(l+\mathcal{D}_1+\mathcal{D}_2+2)(l+\mathcal{D}_1+\mathcal{D}_2+2)} = -\beta\mathcal{P}_{ll}, l = 1,$$
$$\Phi_{(l+\mathcal{D}_1+\mathcal{D}_2+2)(l+\mathcal{D}_1+\mathcal{D}_2+2)} = -\beta\mathcal{P}_{ll} + \varepsilon_{l-1}\alpha^2 C^T N_{\mathcal{L}}^T N_{\mathcal{L}} C,\ l = 2, \ldots, \mathcal{D}_1 + \mathcal{D}_2 + 1,$$
$$\Phi_{(l+\mathcal{D}_1+\mathcal{D}_2+2)(l+\mathcal{D}_1+\mathcal{D}_2+3)} = -\beta\mathcal{P}_{l(l+1)},$$
$$l = 1, \ldots, \mathcal{D}_1 + \mathcal{D}_2,$$
$$\Phi_{(l+\mathcal{D}_1+\mathcal{D}_2+2)(l+\mathcal{D}_1+\mathcal{D}_2+4)} = -\beta\mathcal{P}_{l(l+2)},$$
$$l = 1, \ldots, \mathcal{D}_1 + \mathcal{D}_2 - 1, \cdots,$$
$$\Phi_{(l+\mathcal{D}_1+\mathcal{D}_2+2)(l+2\mathcal{D}_1+2\mathcal{D}_2+2)} = -\beta\mathcal{P}_{l(l+\mathcal{D}_1+\mathcal{D}_2)}, l = 1.$$

Furthermore, the controller gain matrix and observer gain matrix are given by

$$\mathcal{K} = Z_1^{-1} M_1,\ \mathcal{L} = \Im_{22}^{-1} Y_1. \tag{33}$$

The matrix $\bar{\mathcal{A}}$, $\mathcal{P}$, and $\Im$ is as follows:

$$\bar{\mathcal{A}} = \begin{bmatrix} \mathcal{A}+\mathcal{B}\mathcal{K} & -\mathcal{B}\mathcal{K} & -\mathcal{B}\mathcal{K}\mathcal{L}\mathcal{C} & \cdots & -\mathcal{B}\mathcal{K}\mathcal{A}^{\mathcal{D}_1+\mathcal{D}_2-2}\mathcal{L}\mathcal{C} \\ 0 & \mathcal{A}-\mathcal{L}\mathcal{C} & 0 & \cdots & 0 \\ 0 & 0 & \mathcal{A}-\mathcal{L}\mathcal{C} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \mathcal{A}-\mathcal{L}\mathcal{C} \end{bmatrix},$$

$$\mathcal{P} = \begin{bmatrix} \mathcal{P}_{11} & \mathcal{P}_{12} & \cdots & \mathcal{P}_{1(\mathcal{D}_1+\mathcal{D}_2+1)} \\ * & \mathcal{P}_{22} & \cdots & \mathcal{P}_{2(\mathcal{D}_1+\mathcal{D}_2+1)} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \mathcal{P}_{(\mathcal{D}_1+\mathcal{D}_2+1)(\mathcal{D}_1+\mathcal{D}_2+1)} \end{bmatrix},$$
$$\Im = \begin{bmatrix} \Im_{11} & 0 & \cdots & 0 \\ 0 & \Im_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Im_{(\mathcal{D}_1+\mathcal{D}_2+1)(\mathcal{D}_1+\mathcal{D}_2+1)} \end{bmatrix}. \tag{34}$$

Proof The $\mathcal{P}$ and $\Im$ are shown above. This corollary is the extension of Theorem 3, and there is still an existential problem with obtaining the corresponding unique observer when $\mathcal{D}_1 + \mathcal{D}_2$ is fixed. Similar to the process of Theorem 3 and from (31–32), we let $Z_1\mathcal{K} = M_1, \Im_{ii}\mathcal{L} = Y_j,\ i \in \{2, \ldots, \mathcal{D}_1+\mathcal{D}_2+1\},\ j \in \{1, 2, \ldots, \mathcal{D}_1 + \mathcal{D}_2\}$. If we find matrices $\Im_{22}$ and Y_1 to meet all constraints of $\Im_{ii}\mathcal{L} = Y_j,\ i \in \{2, \ldots, \mathcal{D}_1 + \mathcal{D}_2 + 1\},\ j \in \{1, 2, \ldots, \mathcal{D}_1 + \mathcal{D}_2\}$, we can get the unique $\mathcal{L}$. In addition, different matrix gain $\mathcal{L}$ and $\mathcal{K}$ can be obtained because of different $\mathcal{D}_1 + \mathcal{D}_2$. □

Remark 6 It is obvious that we have solved the problems of SFTB and H_∞ performance of stochastic NPC systems with unavoidable constraints. From Theorem 3 and its corollary, we use the slacking variable $\Im$ to achieve the controller and observer co-designed. Since there are a few results in the existing literature concerned with unavoidable constraints of stochastic NPC systems simultaneously [19,20], the result of Theorem 3 and Corollary 1 fills this gap and is also the main contribution of this paper.

Remark 7 In our work, the feasible solution is obtained by utilizing $\mathbb{E}\left\{\bar{x}^T(k)\bar{\Re}\bar{x}(k)\right\} \le (\bar{\mathcal{D}}_1 + \bar{\mathcal{D}}_2 + 1)c_4$. As we know, $\bar{\mathcal{D}}_1 + \bar{\mathcal{D}}_2$ is the upper bound of $\mathcal{D}_1 + \mathcal{D}_2$. Meanwhile, a feasible solution based on $\mathcal{D}_1 + \mathcal{D}_2$ has also been developed for further study. Furthermore, for each value of $\mathcal{D}_1 + \mathcal{D}_2$, we can have the corresponding $\mathcal{K}$ and $\mathcal{L}$. That means that different TD-PDs have different effects on the transient performance of the system. Due to Corollary 1, a uniform $\bar{\mathcal{K}}$ and $\bar{\mathcal{L}}$ has been provided to achieve SFTB with a desired H_∞ performance for each NPC system.

Remark 8 In this paper, controllers and observers are designed for the fixed upper bounds of TD-PDs. If the upper bound is given, the corresponding controller and observer are calculated. It is observed that our method can be used to deal with the random delays and time-varying delays. The reason is that both these two kinds of communication delay have upper bounds in real-world application systems. Most of the options for dealing with the random delays and time-varying delays assume that delay is bounded [17,18]. Thus, the proposed method is valid in practice. However, the upper bound cannot be exhausted in different occasions and applications. Hence, our simulation results provide some cases.

Remark 9 The total number of variables in Theorem 1 and Theorem 1 in [21] are calculated as

$$\hat{N} = (n(n+1)/2)(\mathcal{D}_1 + \mathcal{D}_2 + 1) + s(s+1)/2$$

and

$$\check{N} = (n(n+1)/2)(i+1) + s(s+1)/2, \imath \in 0, 1, \ldots, \mathcal{D}_1 + \mathcal{D}_2,$$

respectively, where i stands for the subsystem mode. The number of subsystem is determined by $\mathcal{D}_1 + \mathcal{D}_2$.

Due to $\bar{\mathcal{D}}_1 + \bar{\mathcal{D}}_2$, there is an upper bound on the computation complexity of these two theorems. The value is $(n(n+1)/2)(\bar{\mathcal{D}}_1+\bar{\mathcal{D}}_2+1)+s(s+1)/2$. Furthermore, we can see the solution of $\hat{N}-\check{N}=(n(n+1)/2)(\mathcal{D}_1+\mathcal{D}_2-i),\ i\in 0,1,\ldots,\mathcal{D}_1+\mathcal{D}_2$. Three cases should be discussed. Case 1: when $i>\mathcal{D}_1+\mathcal{D}_2$, we can get $\hat{N}<\check{N}$, our result is more efficient than the result in [21]. Case 2: when $i=\mathcal{D}_1+\mathcal{D}_2$, we can get $\hat{N}=\check{N}$. Case 3: when $i<\mathcal{D}_1+\mathcal{D}_2$, the result is $\hat{N}>\check{N}$. For Cases 2–3, even though our results are the same or have more computation complexity than the results in [21], the cone complementary algorithm used in [21] has some disadvantages. It requires more computing time and is memory consuming, which is mentioned in Remark 4 in [21]. In conclusion, our approach is obviously more effective than the approach in [21].

4 Illustrative examples

To verify the result proposed in the work, a numerical example is exploited to demonstrate the effectiveness of the proposed design. A servo motor control system is chosen [17], and discrete-time model of this system can be presented by

$$\mathcal{A}=\begin{bmatrix}1.12 & 0.213 & -0.335\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix},\ \mathcal{B}=\begin{bmatrix}1\\0\\0\end{bmatrix},$$

$$C=\begin{bmatrix}0.0541\ 0.1150\ 0.0001\end{bmatrix},\ \mathfrak{D}=\begin{bmatrix}0.1\\0\\0\end{bmatrix},$$

$$\mathfrak{E}=\begin{bmatrix}0.04\ 0.01\ 0.01\end{bmatrix},\ \mathfrak{F}=\begin{bmatrix}0.1\ 0\ 0\end{bmatrix}^T.$$

In this section, the success probabilities of the stochastic variable α is measured to be 0.6. Thus, in the following description, we use $\sqrt{\bar{x}(0)^T\bar{\mathfrak{R}}\bar{x}(0)}$ to represent $\mathbb{E}\left\{\sqrt{\bar{x}(0)^T\bar{\mathfrak{R}}\bar{x}(0)}\right\}$, and use $\sqrt{\bar{x}^T(k)\bar{\mathfrak{R}}\bar{x}(k)}$ to indicate $\mathbb{E}\left\{\sqrt{\bar{x}^T(k)\bar{\mathfrak{R}}\bar{x}(k)}\right\}$. First, a case where there are no PDs in two channels that is considered. To put it more simply, $n_l=0$. Then, we investigate the second situation of (1) with PDs, which will have an impact on the system stability. As a result, the next two cases will be examined.
Case A Without PDs.

We first show the co-design method of $\mathcal{K}$ and $\mathcal{L}$. We consider $n_b=n_w=1,\ n_l=0,\ \mathcal{D}_1+\mathcal{D}_2=n_b+n_l+n_w+n_l=1+1=2$. Then, it is selected that $\bar{\mathfrak{R}}=I,\ N=10,\ \mathfrak{d}=0.6,\ \sigma_1=0.01,\ \beta=1.01,\ c_1=2.5$. The external disturbance is

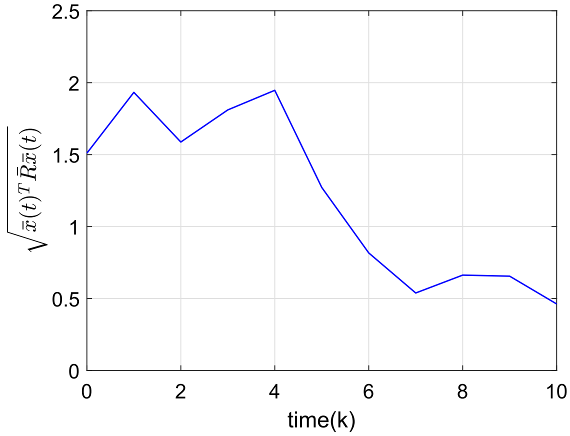

Fig. 2 $\sqrt{\bar{x}(k)^T\bar{\mathfrak{R}}\bar{x}(k)}$ trajectory ($\mathcal{D}_1+\mathcal{D}_2=2$)

defined as $w(k)=e^{-0.4k}\sin(k)$, and it is checked that $\sum_{k=0}^{10}w^T(k)w(k)=0.5312<\mathfrak{d}=0.6$. In this simulation, we take $\bar{\mathcal{D}}_1+\bar{\mathcal{D}}_2$ is 7 [17]. Applying Theorem 3, the matrices $\mathcal{K}$ and $\mathcal{L}$ are obtained by

$$\mathcal{K}=\begin{bmatrix}-0.3594\ -0.1468\ -0.0407\end{bmatrix},$$
$$\mathcal{L}=\begin{bmatrix}13.0726\ 8.6996\ 10.4610\end{bmatrix}^T,$$

with a minimum H_∞ performance index $\gamma=1.0563$. Besides, considering that the initial states are given as $x^T(0)=[0.5\ -0.8\ 0.5]$ and $e^T(0)=[0.5\ -0.8\ 0.5]$, then we have $\sqrt{\bar{x}(0)^T\bar{\mathfrak{R}}\bar{x}(0)}<\sqrt{c_1}=1.5811$. Furthermore, the minimum values of $c_2=68.6194$ is obtained.

The simulation results are figured in Figs. 2, 3, 4, 5, 6 and 7. From Fig. 2, we can claim that $\sqrt{\bar{x}(k)^T\bar{\mathfrak{R}}\bar{x}(k)}$ is smaller than $\sqrt{c_2}=8.2837$ over the time period [0, 10], which indicates that the augmented state of (10) could be SFTB with a prescribed H_∞ performance level γ by the NPC strategy proposed.

Figure 3 includes the system state and errors. The maximum value of $\sqrt{x(k)^T\mathfrak{R}x(k)}$ is 1.9519 due to $x_1(k)=0.7811,\ x_2(k)=1.0069,\ x_3(k)=1.4785$ at $k=4$. And the maximum value of $\sqrt{\bar{x}(k)^T\bar{\mathfrak{R}}\bar{x}(k)}$ is 1.9525. We can conclude that the maximum value of $\sqrt{x(k)^T\mathfrak{R}x(k)}$ is no bigger than the maximum value of $\sqrt{\bar{x}(k)^T\bar{\mathfrak{R}}\bar{x}(k)}$, which verify Remark 3. The state trajectory is shown in Figs. 4, 5, 6 and 7. Figures 4, 5, 6 and 7 show the phase portrait and projective portrait, respectively. Based on Figs. 5, 6 and 7, the response of the state is within a circle with a radius of 8.2837, and finally reaches the center of the circle. It is clear that the overshoot of $\sqrt{x(k)^T\mathfrak{R}x(k)}$ is smaller than $\sqrt{c_2}$. From

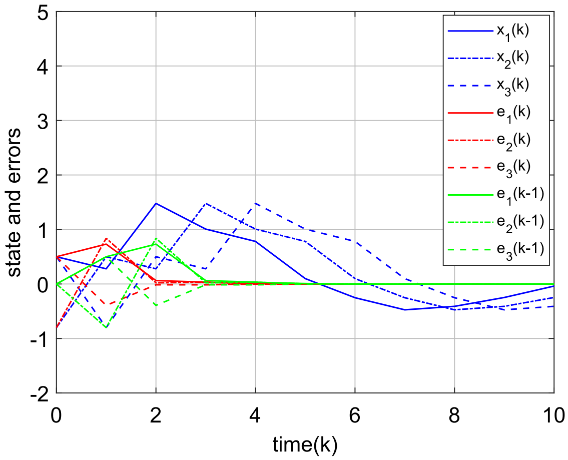

Fig. 3 State and errors ($\mathcal{D}_1 + \mathcal{D}_2 = 2$)

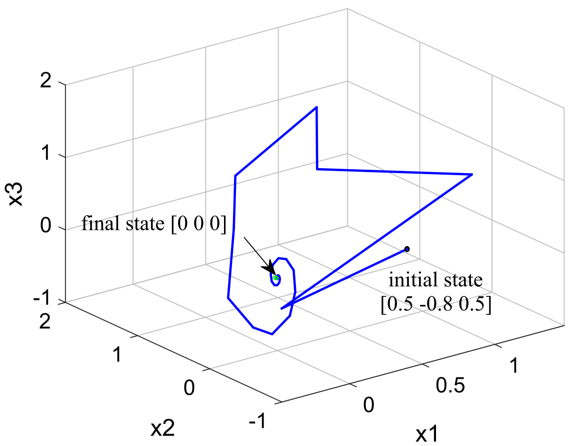

Fig. 4 Phase portrait in (x_1, x_2, x_3) space ($\mathcal{D}_1 + \mathcal{D}_2 = 2$)

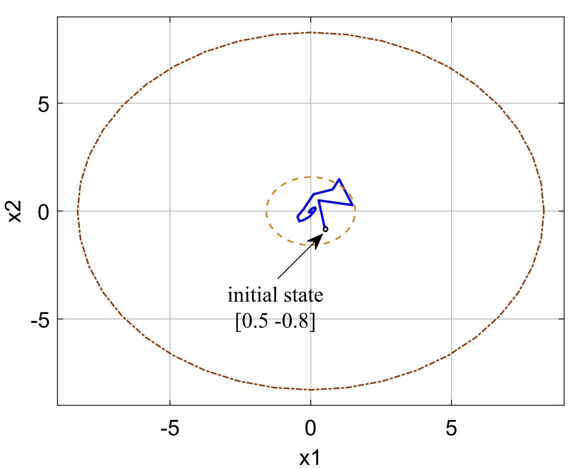

Fig. 5 Projective portrait in $(x1, x2)$ plane ($\mathcal{D}_1 + \mathcal{D}_2 = 2$)

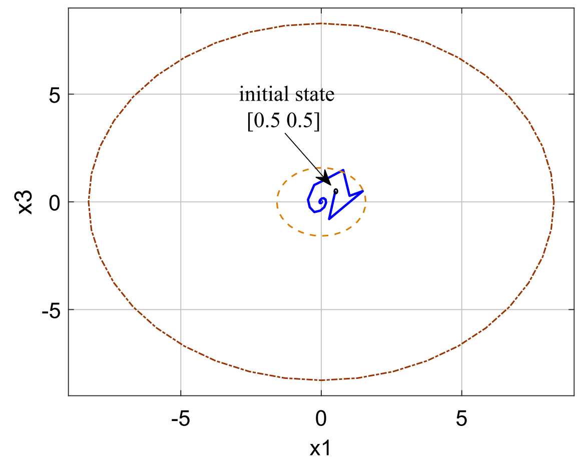

Fig. 6 Projective portrait in $(x1, x3)$ plane ($\mathcal{D}_1 + \mathcal{D}_2 = 2$)

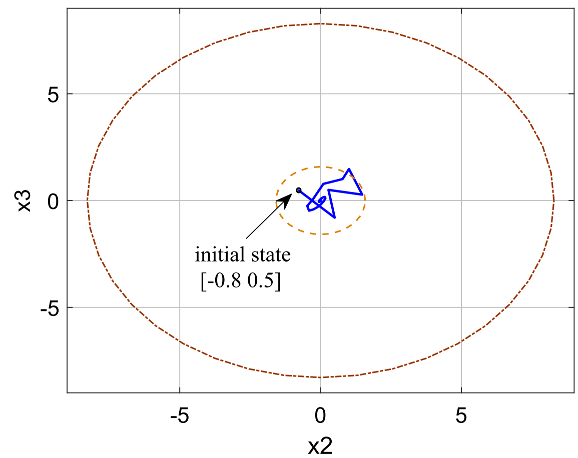

Fig. 7 Projective portrait in $(x2, x3)$ plane ($\mathcal{D}_1 + \mathcal{D}_2 = 2$)

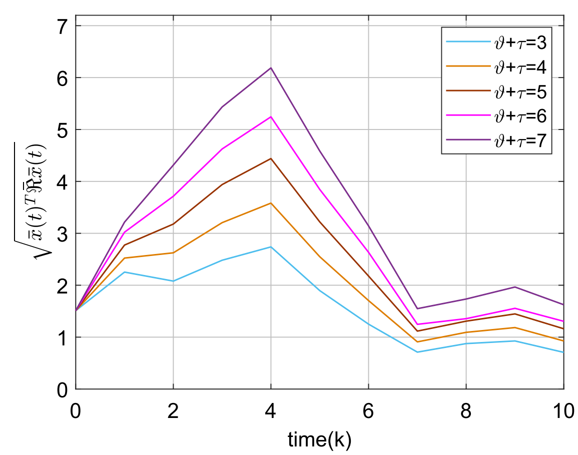

Fig. 8 $\sqrt{\bar{x}(k)^T\bar{\Re}\bar{x}(k)}$ trajectories for different $\mathcal{D}_1 + \mathcal{D}_2$

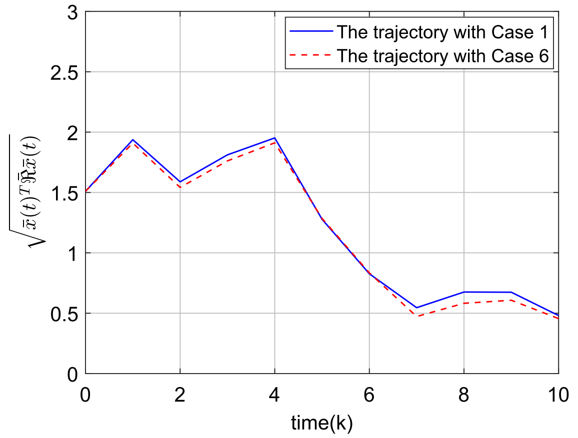

Fig. 9 $\sqrt{\bar{x}(k)^T\bar{\Re}\bar{x}(k)}$ trajectories of Case 1 & Case 6 ($\mathcal{D}_1+\mathcal{D}_2=2$)

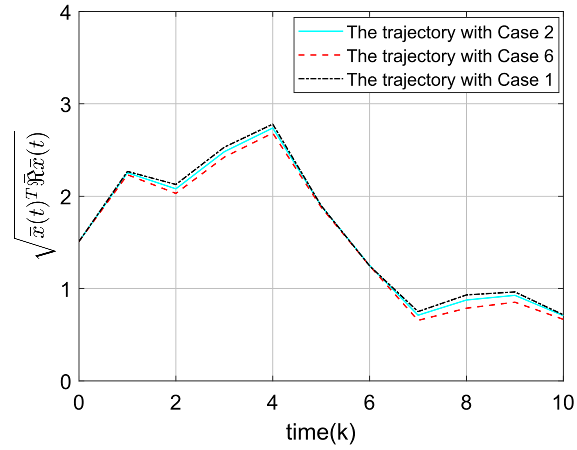

Fig. 10 $\sqrt{\bar{x}(k)^T\bar{\Re}\bar{x}(k)}$ trajectories of Case 2 & Case 6, 1 ($\mathcal{D}_1+\mathcal{D}_2=3$)

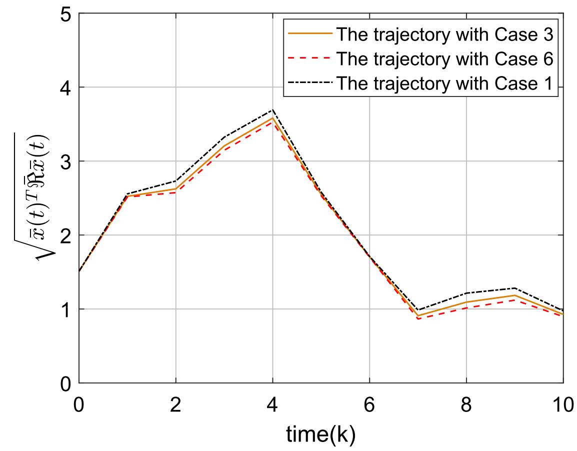

Fig. 11 $\sqrt{\bar{x}(k)^T\bar{\Re}\bar{x}(k)}$ trajectories of Case 3 & Case 6, 1 ($\mathcal{D}_1+\mathcal{D}_2=4$)

Figs. 3, 4, 5, 6 and 7, we can see that the state $x(k)$ of (10) could converge to the steady state. Therefore, it is found that the stochastic NPC systems are stabilized by the proposed NPC scheme.

Case B With PDs.

Next, we show how TD-PDs affect the performance of control. We consider different values of TD-PDs in this proposal, $\sqrt{\bar{x}(k)^T\bar{\Re}\bar{x}(k)}$ trajectories are derived using the same basic parameters. With these conditions, the second simulation results are drawn as solid lines in many colors, which shown in Fig. 8. This diagram shows that the bigger sum of TD-PDs, the bigger overshoot magnitude of $\sqrt{\bar{x}(k)^T\bar{\Re}\bar{x}(k)}$. Moreover, by comparing Figs. 8 and 2, it is observed that in the second situation, the transient performance has been degraded in comparison with that achieved in the first situation.

According to Figs. 2, 8, and Table 1 for various $\mathcal{D}_1+\mathcal{D}_2$, the system states begin within $\sqrt{c_1}$ and are limited within $\sqrt{c_2}$. As shown in Fig. 2 and 8, the simulation results verify that the system is SFTB with respect to the chosen parameters. Table 1 shows the NFO gain and predictive controller gain, which have been calculated by the condition of terminal bound c_2. Cases 1–5 show the obtained $\mathcal{K}$ and $\mathcal{L}$ according to $\bar{x}^T(k)\bar{\Re}\bar{x}(k) \leq (\mathcal{D}_1+\mathcal{D}_2+1)c_4$. Case 6 shows the obtained $\bar{\mathcal{K}}$ and $\bar{\mathcal{L}}$ based on $\bar{x}^T(k)\bar{\Re}\bar{x}(k) \leq (\bar{\mathcal{D}}_1+\bar{\mathcal{D}}_2+1)c_4$. Figures 9, 10, 11, 12, 13 and 14 depict the comparison of Case 6 with Cases 1–5, respectively. As we know, poor transient performance means that the overshoot magnitude is bigger. From the pictures, the overshoot magnitude of $\bar{x}^T(k)\bar{\Re}\bar{x}(k)$ with $\bar{\mathcal{K}}$ and $\bar{\mathcal{L}}$ is just equal to the overshoots for which the corresponding controller and observer of Cases 1–5 are utilized. That is to say, along with the $\bar{\mathcal{K}}$ and $\bar{\mathcal{L}}$, the system transient also possesses good performance compared with the ones we mentioned before. Indeed, it has been mentioned in Remark 7. When $\mathcal{K}$ and $\mathcal{L}$ are calculated by the fixed $\mathcal{D}_1+\mathcal{D}_2=2$, the system transient performance is worse than that of Cases 2–6 systems with the controller and observer. Actually, we can reach the same conclusion for any fixed $\mathcal{D}_1+\mathcal{D}_2$, such as 3, 4, and so on. Therefore, it is of great significance for the uniform controller and observer.

Figures 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 and 14 demonstrate that transient performance and the needed index γ of the closed-loop NPC systems is guaranteed,

Table 1 Predictive controller gain and NFO gain for different sums of TD-PDs

Different sum of TD-PDs	Predictive controller gain	NFO gain
Case 1: $\mathcal{D}_1 + \mathcal{D}_2 = 2$	$\mathcal{K} = \begin{bmatrix} -0.3594 & -0.1468 & -0.0407 \end{bmatrix}$	$\mathcal{L} = \begin{bmatrix} 13.0726 & 8.6996 & 10.4610 \end{bmatrix}^T$
Case 2: $\mathcal{D}_1 + \mathcal{D}_2 = 3$	$\mathcal{K} = \begin{bmatrix} -0.3575 & -0.1433 & -0.0374 \end{bmatrix}$	$\mathcal{L} = \begin{bmatrix} 12.8393 & 8.5739 & 10.4960 \end{bmatrix}^T$
Case 3: $\mathcal{D}_1 + \mathcal{D}_2 = 4$	$\mathcal{K} = \begin{bmatrix} -0.3648 & -0.1402 & -0.0349 \end{bmatrix}$	$\mathcal{L} = \begin{bmatrix} 12.6488 & 8.4499 & 10.5444 \end{bmatrix}^T$
Case 4: $\mathcal{D}_1 + \mathcal{D}_2 = 5$	$\mathcal{K} = \begin{bmatrix} -0.3645 & -0.1382 & -0.0331 \end{bmatrix}$	$\mathcal{L} = \begin{bmatrix} 12.5962 & 8.4277 & 10.5501 \end{bmatrix}^T$
Case 5: $\mathcal{D}_1 + \mathcal{D}_2 = 6$	$\mathcal{K} = \begin{bmatrix} -0.3661 & -0.1281 & -0.0199 \end{bmatrix}$	$\mathcal{L} = \begin{bmatrix} 12.7055 & 8.5446 & 10.4902 \end{bmatrix}^T$
Case 6: $\mathcal{D}_1 + \mathcal{D}_2 = 7$	$\bar{\mathcal{K}} = \begin{bmatrix} -0.3651 & -0.1360 & -0.0294 \end{bmatrix}$	$\bar{\mathcal{L}} = \begin{bmatrix} 12.5255 & 8.3916 & 10.5608 \end{bmatrix}^T$

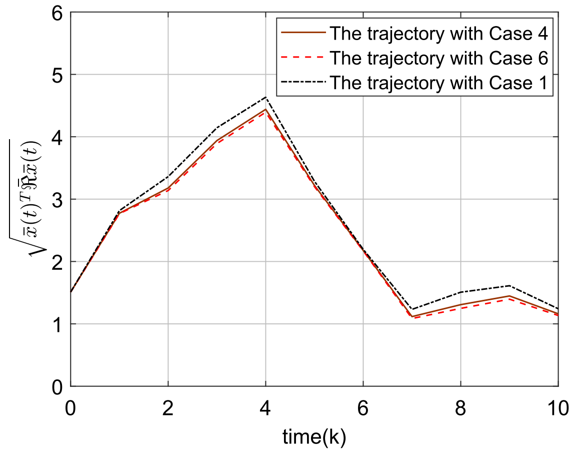

Fig. 12 $\sqrt{\bar{x}(k)^T\bar{\Re}\bar{x}(k)}$ trajectories of Case 4 & Case 6, 1 ($\mathcal{D}_1 + \mathcal{D}_2 = 5$)

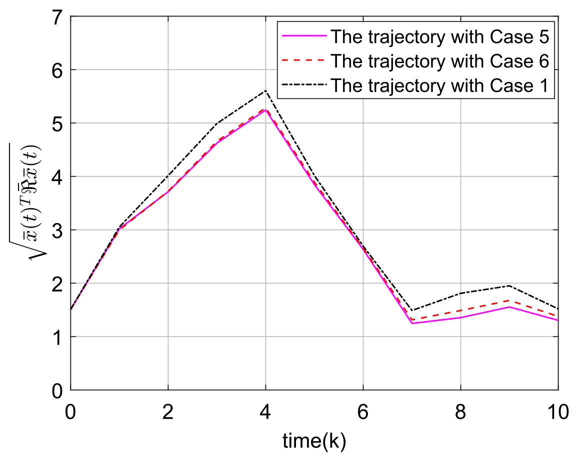

Fig. 13 $\sqrt{\bar{x}(k)^T\bar{\Re}\bar{x}(k)}$ trajectories of Case 5 & Case 6, 1 ($\mathcal{D}_1 + \mathcal{D}_2 = 6$)

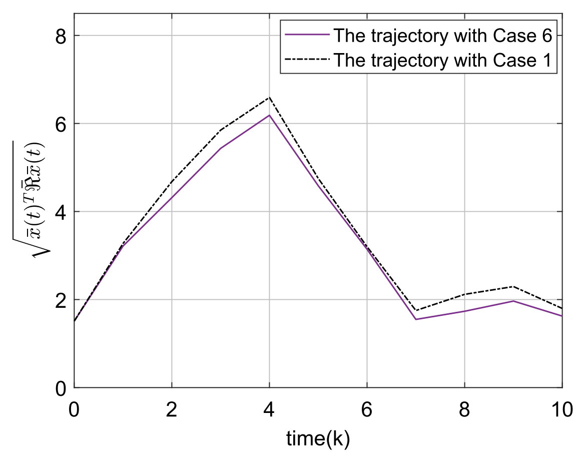

Fig. 14 $\sqrt{\bar{x}(k)^T\bar{\Re}\bar{x}(k)}$ trajectories of Case 6 & Case 1 ($\mathcal{D}_1 + \mathcal{D}_2 = 7$)

and the TD-PDs are actively disposed by a uniform controller and observer when the system is suffering from communication constraints.

5 Conclusion

We made one of the first attempts in this paper to investigate finite-time H_∞ predictive control problems for stochastic NCSs with unavoidable communication constraints. First, due to the unmeasurable state and missing measurements, the NFO has been constructed. Then, a NFO-based NPC scheme has been found to be efficient in terms of achieved the proposed prediction strategy. Moreover, a novel model of NPC systems has been built, reflecting the effects of the proposed NPC and communication delay. The SFTB of the resulting system with TD-PDs with a desirable H_∞ performance criterion is derived. Moreover, the uniform controller gain and observer gain are obtained with the aid of the bounded TD-PDs, which can be well-handed.

There are several possible avenues for future research. On the one hand, only a few cases are given in the simulation, other cases can be extended by the method in the paper. On the other hand, it will be interesting to see if the existing solution to the NPC can be extended to NCSs that are subject to cyber-attacks and whose network security is threatened. Another consideration is extending the proposed method to switched systems and Markovian jump systems.

Acknowledgements This paper is supported by the central Guidance on Local Science and Technology Development Fund of Tibet Autonomous Region (No.XZ202201YD0002C).

Data availability The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflicts of interest The authors declare that they have no conflict of interest.

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