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Input-output finite-time IT2 fuzzy dynamic sliding mode control for fractional-order nonlinear systems

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Abstract In this work, the issue of input-output finite-time stabilization of fractional-order nonlinear systems represented by interval type-2 fuzzy models is discussed. Specifically, the addressed system takes into account more realistic factors such as uncertainties, nonlinearities, disturbances, and state delays. A new dynamic sliding-mode control (SMC) scheme for interval type-2 fuzzy models is developed in order to eliminate the commonly held assumption that all subsystems share the same input matrix (i.e. $B^i \neq B$), which is considered in the majority of fuzzy SMC scheme results. Based on input-output finite-time stabilization properties and the proposed control scheme, the goal of this work is to reduce the impact of uncertainties, nonlinearities, disturbances, and state delays while ensuring that the signal variables arrive at a domain within the designed fixed-time level. Furthermore, the required criteria are expressed as linear matrix inequalities, which can be solved by using MATLAB linear matrix inequality toolbox. Following that, three numerical examples, including the permanent magnet synchronous motor model and the single-link robot arm model, are provided to validate the proposed control scheme.

Keywords Fractional-order systems · Interval type-2 fuzzy models · Dynamic sliding-mode control · Input-output finite-time stabilization

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1 Introduction

Fractional calculus is a powerful tool for describing memory and hereditary qualities in different kinds of real-world fields, such as mathematics, biology, engineering and so on. Many irregular engineering models can be represented precisely and concisely with the help of fractional-order (FO) derivatives, making the research of fractional calculus more meaningful. It is also mentioned that the fractional calculus can be considered as a superset of integer calculus, which is another significant benefit. As a result, FO systems are more accurate and reliable than integral-order systems in representing mathematical models in a variety of areas of researches, including dynamical systems and control theory [1–6]. For instance, the control algorithm for Lithium-ion battery applications based on FO electrical systems can be presented in [6].

The concept of the type-1 fuzzy set together with their fuzzy logic model has been proven to be an efficient tool for representing various situations in fuzzy set theory [7–11]. However, the results are not available for type-1 fuzzy models when the grades of membership contain uncertain information. It is essential to find out an approach to solve the problem that the grades of membership contain uncertain information. Mendel et al. [12] proposed an interval type-2 (IT2) fuzzy logic model which can be used to handle uncertain grades of membership. Due to the merits of IT2 fuzzy sets over type-1 fuzzy sets, the authors in [13] introduced a lower and upper membership functions approach to deal with the parameter uncertainties in the nonlinear plants and designed an IT2 fuzzy state feedback controller for IT2 fuzzy models. In particular, the introduction of the IT2 fuzzy set is to compensate for the inefficiency of the type-1 fuzzy set in modeling uncertainties. In recent years, the IT2 fuzzy models have been used effectively in a wide range of applications, including control classifications [14–18]. The authors in [14] investigated the relation between type-1 and IT2 fuzzy models via using type-reduction and defuzzification methods. In [17], the issue of IT2 sampled data stability results have been investigated for nonlinear systems against parameter uncertainties. By using IT2 fuzzy models, the authors in [18] investigated the stabilization problem for nonlinear networked control systems under cyber attacks. With the rapid growth of fuzzy logic theory, several works have been reported by combining fractional calculus with fuzzy logic models [19–23].

Uncertainties and disturbances are frequently enco untered in practical control systems, because it is very difficult to obtain exact mathematical models because of many reasons such as environmental noises, model errors, and aging of systems. The presence of uncertainties and disturbances may cause instability or bad performance on controlled systems. As a well-known robust control method in the control fields, the sliding-mode control (SMC) scheme has a lot of attractive merits, such as good transient performance, system order reduction, high convergence speed and strong robustness against system uncertainties, external disturbances and model errors. The basic procedure of SMC is that, in finite time, a suitable discontinuous control law is used to drive the system states to the sliding surface and then drive the states to move along the surface to the origin. Several interesting results on the SMC-based stabilization problem for various fuzzy models have been addressed during the past few decades [24–27].

It is noted that all those SMC schemes in above results rely on an assumption that all linear local models of the fuzzy system share a common input matrix, that is, the input matrices B^i for all the subsystems is the same (i.e. $B^i = B$), which is very restrictive. It is noted that many real plants, such as the well-known inverted pendulum on a cart, permanent magnet synchronous motor, single-link robot arm model, do not satisfy this assumption. In order to overcome this constraint assumption, the dynamic SMC scheme is developed for fuzzy systems under robust H_∞ control designs [28]. Recently, there are only a few effective results in the literature that remove the restrictive assumption that all local models share the same control input matrix [29–32]. Furthermore, the fuzzy FOSMC has emerged as a powerful control design [33]. The application of FO calculus provides an extra degree of freedom that facilitates the design of more flexible and powerful control methods that satisfy system specifications. The FOSMC designs were shown to exhibit minimal chattering, robust performance against variations in gain, and the ability to reject noise and output disturbances [22,23]. In order to get more accurate model, the FOSMC design is developed for a class of IT2 fuzzy models.

Furthermore, we generally require system outputs to stay within a specified bound during a short operation time period. However, most of the stabilization results cannot meet our requirements under these conditions. As a result, the concept of finite-time stabilization for linear time-varying systems was established in [34]. Following that, Amato et al. [35] introduced a special case of finite-time stabilization known as input-output finite-time stabilization for linear systems. The input-output finite-time stabilization focuses exclusively on the input-output features or state fluctuations of the dynamical system within a finite predefined time interval. In particular, if the system's input is bounded in a specified time interval, the system's output must be also bounded during that time interval. Out of that finite time interval, the system may become unstable. The specified finite time interval is employed in the system states, which is proved to be efficient in improving the system performance [36–40].

Motivated by the above facts, this paper studies the dynamic SMC scheme for a class of IT2FO fuzzy systems against uncertainties, nonlinearities, disturbances and state delays. The main contributions of this paper are listed as follows:

(1) An input-output finite-time stabilization problem for delayed FO nonlinear systems with IT2 fuzzy models is presented for the first time.
(2) In contrast to the existing fuzzy SMC scheme in [24–27,37], this work employs the FO dynamic SMC scheme, in which the control input coefficient matrices of all linear subsystems do not have to be the same (i.e. $B^i \neq B$).
(3) The desired stabilization criteria of IT2FO fuzzy augmented system is investigated using input-output finite-time stabilization properties by constructing an appropriate FO Lyapunov functional in conjunction with the Schur complement.

Finally, the derived theoretical results are validated through three numerical examples, including the permanent magnet synchronous motor and single-link robot arm models. The validation section demonstrates that the results developed in this paper are effective for the system under consideration and reduce the convergence time when compared to recent result in [37].

2 Problem formulation and preliminaries

2.1 IT2FO fuzzy system description

There are several definitions that exist for the FO derivative of x_t. In particular, the most commonly used FO derivatives are Riemann-Liouville, Grunwald-Letnikov and Caputo FO derivatives. The Caputo FO derivative is used in this paper because it is more appropriate in many engineering applications.

Definition 1 [1,2] The Caputo fractional derivative of FO α of function $c(t)$ is defined as

$$\mathcal{D}^{\alpha} c(t) = \frac{1}{\Gamma(\kappa-\alpha)} \frac{d^{\kappa}}{dt^{\kappa}} \int_{t_0}^{t} (t-i)^{\kappa-\alpha-1} c(i) di,$$

where α is the order of the derivative, such that $\kappa - 1 \leq \alpha < \kappa$, $\kappa \in \mathbb{Z}^{+}$ and $\Gamma(r) = \int_0^{\infty} t^{r-1} e^{-t} dt$.

Consider the FO system with uncertainties, time delay, nonlinearities and disturbances that is modeled by using IT2 fuzzy models with the ith rule as follows: IF $\mathscr{S}_1(x_t)$ is s_1^i, $\mathscr{S}_2(x_t)$ is $s_2^i, \dots,$ and $\mathscr{S}_r(x_t)$ is s_r^i, THEN

$$\begin{aligned} D^{\alpha} x_t =& (A^i + \Delta A_t^i) x_t + (A^{\tau i} + \Delta A_t^{\tau i}) x_{(t-\tau)} \\ &+ (B^i + \Delta B_t^i) u_t + E^i f(t, x_t) + B^{wi} w_t, \\ z_t =& C^i x_t, \\ x_t =& \phi_t, t \in [-\tau, 0], i \in \mathbb{S} := \{1, 2, \dots, n\} \end{aligned} \quad (1)$$

where $\mathscr{S}_{\varsigma}(x_t)$ is the premise variable; s_{ς}^i ($i \in \mathbb{S}$ and $\varsigma = \{1, 2, \dots, r\}$) indicates the IT2 fuzzy set; $x_t \in \mathfrak{R}^n$, $z_t \in \mathfrak{R}^z$, $u_t \in \mathfrak{R}^m$, $w_t \in \mathfrak{R}^w$, denote the system state, controlled output, control input and external disturbances, respectively; A^i, $A^{\tau i}$, B^i, B^{wi}, E^i and C^i ($i \in \mathbb{S}$) are known real matrices; τ denotes the constant time delay and ϕ_t indicates an initial state value defined on $[-\tau, 0]$. The unknown nonlinear vector function $f(t, x_t)$ satisfies the local Lipschitz condition on $\Sigma \subset \mathfrak{R}^n$, that is, there exists a known real constant matrix $\mathscr{F}$ so that $||f(t, x_t^1) - f(t, x_t^2)|| \leq ||\mathscr{F}(x_t^1 - x_t^2)||$, $\forall \{x_t^1, x_t^2\} \in \Sigma$. This model uncertainties ΔA_t^i, ΔB_t^i and $\Delta A_t^{\tau i}$ are unknown matrices following $[\Delta A_t^i \ \Delta B_t^i \ \Delta A_t^{\tau i}] = \mathscr{N}^{1i} \nu_t [\mathscr{N}^{2i} \ \mathscr{N}^{3i} \ \mathscr{N}^{4i}]$, where $\mathscr{N}^{\bar{g} i}$ ($\bar{g} = 1, 2, 3, 4,$ & $i \in \mathbb{S}$) are known constant matrices and ν_t indicates an unknown matrix in which satisfies $\nu_t^T \nu_t < I$.

Here, the scalars r and n denote the number of premise variables and IF-THEN rules of the system, respectively. Then, the firing strength of the ith rule can be expressed in the following form:

$$V^i(x_t) = [\underline{\upsilon}^i(x_t), \overline{\upsilon}^i(x_t)], \ i \in \mathbb{S},$$

where $\overline{\upsilon}^i(x_t) = \prod_{\varsigma=1}^{r} \bar{\mathfrak{l}}_{s_{\varsigma}^i}(\varrho_{\varsigma}(x_t)) \geq 0$, $\underline{\upsilon}^i(x_t) = \prod_{\varsigma=1}^{r} \underline{\mathfrak{l}}_{s_{\varsigma}^i}(\varrho_{\varsigma}(x_t)) \geq 0$. The upper and the lower grade of membership, the upper and the lower membership functions are depicted by $\bar{\mathfrak{l}}_{s_{\varsigma}^i}(\varrho_{\varsigma}(x_t)) \in [0, 1]$, $\underline{\mathfrak{l}}_{s_{\varsigma}^i}(\varrho_{\varsigma}(x_t)) \in [0, 1]$, $\overline{\upsilon}^i(x_t)$ and $\underline{\upsilon}^i(x_t)$, respectively. According to the property $\underline{\mathfrak{l}}_{s_{\varsigma}^i}(\varrho_{\varsigma}(x_t)) \leq \bar{\mathfrak{l}}_{s_{\varsigma}^i}(\varrho_{\varsigma}(x_t))$ that $\underline{\upsilon}^i(x_t) \leq \overline{\upsilon}^i(x_t)$ is always satisfied for all i. Then, the defuzzified systems are written as

$$\begin{aligned} D^{\alpha} x_t =& \sum_{i=1}^{n} \upsilon^i(x_t) \Big[(A^i + \Delta A_t^i) x_t + (A^{\tau i} + \Delta A_t^{\tau i}) x_{(t-\tau)} \\ &+ (B^i + \Delta B_t^i) u_t + E^i f(t, x_t) + B^{wi} w_t \Big], \\ z_t =& \sum_{i=1}^{n} \upsilon^i(x_t) C^i x_t, \end{aligned} \quad (2)$$

with $v^i(x_t) = \underline{m}^i(x_t)\underline{v}^i(x_t) + \overline{m}^i(x_t)\overline{v}^i(x_t)$, where $\underline{m}^i(x_t)$ and $\overline{m}^i(x_t)$ are nonlinear functions such that the following conditions are satisfied: $\underline{m}^i(x_t)+\overline{m}^i(x_t) = 1$, $0 \le \underline{m}^i(x_t) \le \overline{m}^i(x_t) \le 1$ and $\sum_{p=1}^{m} v^i(x_t) = 1$ in which $v^i(x_t)$ is regarded as the grade of fuzzy membership functions.

The primary objective of this paper is to design the dynamic SMC scheme, such that the IT2FO argument system is input-output finite-time stabilization. In order to achieve the desired objective, the following definition and lemma are presented.

Definition 2 According to [40], for given scalars $c_1 > 0$, $c_2 > 0$, $T > 0$ and weight matrix $\mathcal{F} > 0$, under the zero initial condition, $\phi_t = 0$, the system (2) is input-output finite-time stabilization with the parameters $(c_1, c_2, \mathcal{F}, T)$, if system (2) satisfies

$$\forall w_t \in \mathcal{R} \implies z_t^T \mathcal{F} z_t < c_2^2, \ \forall t \in [0, T],$$

where $\mathcal{R} = \{w_t \in \mathcal{L}_2[0, T] : \int_0^T w_u^T w_u du \le c_1^2\}$.

Lemma 1 *[1] Given matrices $\xi_1 = \xi_1^T$, ξ_2 and ξ_3 of appropriate dimensions, the relationship*

$$\xi_1 + \xi_2\delta_t\xi_3 + \xi_3^T\delta_t^T\xi_2^T < 0,$$

is verified for all δ_t satisfying $\delta_t^T\delta_t \le I$, if and only if there exists some $\lambda > 0$ such that

$$\xi_1 + \lambda\xi_2\xi_2^T + \lambda^{-1}\xi_3^T\xi_3 < 0.$$

Remark 1 This paper focuses on designing a dynamic SMC scheme to deal with delayed IT2FO fuzzy systems, in which, the sliding surface function is combined linearly with states and inputs. The results in [24–27,37] require that all the control coefficient matrices of the addressed systems are same. However, this requirement is not realistic for many real-world systems. To be precise, in this paper, the restriction on the input matrices of the considered systems is relaxed.

3 Dynamic SMC analysis and synthesis

In this section, we mainly study the input-output finite-time stabilization of IT2FO fuzzy systems (2). To effectively handle the stabilization criteria, a fuzzy dynamic SMC is developed. Based on the designed IT2FO fuzzy augmented system, three theorems on input-output finite-time stabilization for IT2FO fuzzy systems (2) are proposed via dynamic SMC scheme with and without uncertainties.

3.1 Design of dynamic SMC scheme

Although fuzzy SMC scheme have been widely discussed to study the stabilization of fuzzy systems in [24–27,37], these controls are applicable only when a very restrictive assumption is satisfied, that is, all local linear models of fuzzy systems share the same input matrix. Interestingly, newly developed dynamic SMC schemes do not need such a restrictive assumption [28]. Then, the design of the sliding surface is provided in the following form:

$$s_t = \mathcal{O}_x x_t + \mathcal{O}_u u_t = \bar{\mathcal{O}}\bar{x}_t = 0, \tag{3}$$

where $\bar{x}_t = [x_{1t}, \ldots, x_{nt}, u_{1t}, \ldots, u_{mt}]^T$, $\mathcal{O}_x \in \mathfrak{R}^{m\times n}$, $\mathcal{O}_u \in \mathfrak{R}^{m\times m}$, $\bar{\mathcal{O}} = [\mathcal{O}_x, \mathcal{O}_u]$, and $\mathcal{O}_u$ is assumed to be nonsingular.

Then, to stabilize the IT2FO fuzzy systems (2), the FO dynamic SMC scheme is constructed in the following form:

$$D^\alpha u_t = \sum_{i=1}^{n} v^i(x_t)\big[-\mathcal{O}_{ux}\big(A^i x_t + A^{\tau i}x_{(t-\tau)} + B^i u_t\big)\big] - (p + l_t)\mathcal{O}_u^{-1}\mathrm{sgn}(s_t), \tag{4}$$

where $\mathcal{O}_{ux} = \mathcal{O}_u^{-1}\mathcal{O}_x$, $l_t = \sum_{i=1}^n v^i(x_t)||\mathcal{O}_x|| \times \Big[||\mathcal{N}^{1i}||||\mathcal{N}^{2i}||||x_t|| + ||\mathcal{N}^{1i}||||\mathcal{N}^{4i}||||x_{(t-\tau)}|| + ||\mathcal{N}^{1i}||||\mathcal{N}^{3i}||||u_t|| + ||E^i||||f(t, x_t)|| + ||B^{wi}||c_t\Big]$, where $p > 0$, and c_t is the known uniform upper bound of the disturbance w_t.

Let $\mathcal{L}_1 = [I_n, 0_{n\times m}]^T$, $\mathcal{L}_2 = [0_{m\times n}, I_m]^T$, $\bar{A}^i = [A^i, B^i]$, $\bar{A}^{\tau i} = [A^{\tau i}, 0_{n\times m}]$, $\Delta\bar{A}_t^i = [\Delta A_t^i, \Delta B_t^i]$, $\Delta\bar{A}_t^{\tau i} = [\Delta A_t^{\tau i}, 0_{n\times m}]$ and $\bar{C}^i = [C_i, 0_{n\times m}]$. On the basis of above, the IT2FO fuzzy systems (2) with the designed controller (4) can be rewritten in a compact form as

$$D^\alpha \bar{x}_t = \sum_{i=1}^{n} v^i(x_t)\big[\big((\mathcal{L}_1 - \mathcal{L}_2\mathcal{O}_{ux})\bar{A}^i + \mathcal{L}_1\Delta\bar{A}_t^i\big)\bar{x}_t + \big((\mathcal{L}_1 - \mathcal{L}_2\mathcal{O}_{ux})\bar{A}^{\tau i} + \mathcal{L}_1\Delta\bar{A}_t^{\tau i}\big)\bar{x}_{(t-\tau)}$$

$$
\begin{aligned}
&+\mathscr{L}_1 E^i f(t,x_t)+\mathscr{L}_1 B^{wi} w_t\big]\\
&-\mathscr{L}_2(p+l_t)\mathcal{O}_u^{-1}\mathrm{sgn}(s_t),\\
z_t=&\sum_{i=1}^{n} v^i(x_t)\bar{C}^i\bar{x}_t. \qquad (5)
\end{aligned}
$$

Subsequently, we will prove that the states of the augmented system will be drifted and kept on the sliding-mode surface in a finite time. In other words, the reachability of the sliding surface is guaranteed by the following theorem.

Theorem 1 *The states of the IT2FO fuzzy augmented system (5) will be reached onto the sliding-mode surface (3) in a finite time* $t \le \sqrt[\alpha]{\frac{||s_0||^2\Gamma(\alpha+1)}{2p}}$.

Proof Choose the function $\mathcal{S}_t = s_t^T s_t$ for all $t > 0$. Then, along the states of the IT2FO augmented system (5), one has

$$
\begin{aligned}
D^\alpha \mathcal{S}_t =& 2s_t^T D^\alpha s_t\\
=& 2s_t^T\bar{\mathcal{O}}D^\alpha\bar{x}_t\\
=& 2\sum_{i=1}^{n} v^i(x_t)s_t^T\bar{\mathcal{O}}\big[\big((\mathscr{L}_1-\mathscr{L}_2\mathcal{O}_{ux})\bar{A}^i\\
&+\mathscr{L}_1\Delta\bar{A}_t^i\big)\bar{x}_t+\big((\mathscr{L}_1-\mathscr{L}_2\mathcal{O}_{ux})\bar{A}^{\tau i}\\
&+\mathscr{L}_1\Delta\bar{A}_t^{\tau i}\big)\bar{x}_{(t-\tau)}+\mathscr{L}_1E^i f(t,x_t)\\
&+\mathscr{L}_1 B^{wi}w_t-\mathscr{L}_2(p+l_t)\mathcal{O}_u^{-1}\mathrm{sgn}(s_t)\big]. \qquad (6)
\end{aligned}
$$

With the fact that $\bar{\mathcal{O}}(\mathscr{L}_1-\mathscr{L}_2\mathcal{O}_{ux})=0$, one has

$$
\begin{aligned}
D^\alpha \mathcal{S}_t =& 2\sum_{i=1}^{n} v^i(x_t)s_t^T\bar{\mathcal{O}}\big[\mathscr{L}_1\Delta\bar{A}_t^i\bar{x}_t+\mathscr{L}_1\Delta\bar{A}_t^{\tau i}\bar{x}_{(t-\tau)}\\
&+\mathscr{L}_1E^i f(t,x_t)+\mathscr{L}_1B^{wi}w_t\\
&-\mathscr{L}_2(p+l_t)\mathcal{O}_u^{-1}\mathrm{sgn}(s_t)\big]\\
=& 2\sum_{i=1}^{n} v^i(x_t)s_t^T\mathcal{O}_x\big[\Delta\bar{A}_t^i\bar{x}_t+\Delta\bar{A}_t^{\tau i}\bar{x}_{(t-\tau)}\\
&+E^i f(t,x_t)+B^{wi}w_t\big]-2l_t||s_t||-2p||s_t||\\
\le& -2p||s_t|| = -2p\sqrt{\mathcal{S}_t}\\
\le& -2p<0,\ \text{for all } s_t\neq 0. \qquad (7)
\end{aligned}
$$

To demonstrate that the sliding motion occurs in a finite time, we utilize Lemma 1 in [3] and (7) that

$$
D^\alpha||s_t||^2 \le 2s_t^T D^\alpha s_t \le -2p, \qquad (8)
$$

which implies that there exists $M_t \ge 0$ such that

$$
D^\alpha||s_t||^2 \le -2p-M_t. \qquad (9)
$$

Taking Laplace transform on both sides of (9), one has

$$
\begin{aligned}
&s^\alpha N_s - s^{\alpha-1}n_0 = -\frac{2p}{s}-M_s,\\
&\Longrightarrow\ N_s = \frac{n_0}{s}-\frac{2p}{s^{\alpha+1}}-\frac{M_s}{s^\alpha}. \qquad (10)
\end{aligned}
$$

where $N_s=\int_0^\infty e^{-st}n_t dt$, $M_s=\int_0^\infty e^{-st}M_t dt$, and $n_t=||s_t||^2$. Taking inverse Laplace transform on (10), it yields

$$
||s_t||^2 = ||s_0||^2-\frac{2pt^\alpha}{\Gamma(\alpha+1)}-\int_0^t(t-\tau)^{(\alpha-1)}M_\tau d\tau. \qquad (11)
$$

In this way, $||s_t||^2=0$ implies that the system states converge to the sliding surface $s(t)=0$, that is

$$
||s_0||^2-\frac{2pt^\alpha}{\Gamma(\alpha+1)}-\int_0^t(t-\tau)^{(\alpha-1)}M_\tau d\tau = 0. \qquad (12)
$$

Since $\int_0^t(t-\tau)^{(\alpha-1)}M_\tau d\tau \ge 0$, from (12) one has

$$
\begin{aligned}
&||s_0||^2-\frac{2pt^\alpha}{\Gamma(\alpha+1)} \ge 0.\\
&\Longrightarrow\ t \le \sqrt[\alpha]{\frac{||s_0||^2\Gamma(\alpha+1)}{2p}}.
\end{aligned}
$$

Hence, the system states can arrive at the sliding surface (3) within a finite time $t \le \sqrt[\alpha]{\frac{||s_0||^2\Gamma(\alpha+1)}{2p}}$. Hence, the proof of this theorem is completed. □

3.2 Analysis of the sliding motion

It is noted that in Theorem 1, the reachability of the system states (2) in a finite-time is established with the aid of dynamic SMC scheme (4). In this subsection, we will analyze the input-output finite-time stabilization of the sliding motion.

Theorem 2 *Consider the IT2FO fuzzy systems (2) with* $\Delta A_t^i=0$, $\Delta B_t^i=0$, $\Delta A_t^{\tau i}=0$ *and let* $\tau, c_1>0, c_2>0, T>0, a>0, \alpha\in(0,1]$ *be given scalars. Under* *a given matrix $\mathscr{F}$, the IT2FO fuzzy augmented system (5) is input-output finite-time stabilization with respect to $(c_1, c_2, \mathcal{F}, T)$, if there exist positive definite matrix $\mathscr{P} \in \Re^{(m+n)\times(m+n)}$, the real matrices $\mathscr{Z}_g^T = \mathscr{Z}_g \in \Re^{(m+n)\times(m+n)}$, $\mathscr{K}^{gi} \in \Re^{m\times(m+n)}$, $(g = 1, 2,\ \&\ i \in \mathbb{S})$ such that the following inequalities are satisfied:*

$$[\bar{\phi}]_{6\times 6} = \begin{bmatrix} [\phi]_{4\times 4} & \vartheta_1 & \tau^{\alpha}\alpha^{-1}\mathscr{Z}_2\vartheta_2 \\ * & -I & 0 \\ * & * & -\tau^{\alpha}\alpha^{-1}\mathscr{Z}_2 \end{bmatrix} < 0, \quad (13)$$

$$(\bar{C}^i)^T \mathcal{F} \bar{C}^i - \mathscr{P} < 0, \quad (14)$$

$$\mathcal{R} < \frac{c_2^2}{e^{aT} c_1^2} I, \quad (15)$$

$$\begin{bmatrix} \mathscr{Z}_1 & 0 \\ 0 & \mathscr{Z}_2 \end{bmatrix} \geq 0, \quad (16)$$

where

$$\begin{aligned} \phi_{11} =& \mathscr{P}\big(\mathscr{L}_1\bar{A}^i - \mathscr{L}_2\mathscr{K}^{1i}\big) + \big(\mathscr{L}_1\bar{A}^i - \mathscr{L}_2\mathscr{K}^{1i}\big)^T\mathscr{P} \\ &+ \tau^{\alpha}\alpha^{-1}\mathscr{Z}_1 + (1-a)\mathscr{P}, \quad \phi_{13} = \mathscr{P}\mathscr{L}_1 B^{wi}, \\ \phi_{12} =& \mathscr{P}\big(\mathscr{L}_1\bar{A}^{\tau i} - \mathscr{L}_2\mathscr{K}^{2i}\big), \quad \phi_{14} = \mathscr{P}E^i\mathscr{L}_1, \\ \phi_{22} =& -\mathscr{P}, \quad \phi_{33} = -\mathcal{R}, \quad \vartheta_1 = [\mathscr{F}\ \ 0\ \ 0\ \ 0]^T, \\ \phi_{44} =& -I, \quad \vartheta_2 = [\mathscr{L}_1\bar{A}^i\ \ \mathscr{L}_1\bar{A}^{\tau i}\ \ \mathscr{L}_1 B^{wi}\ \ \mathscr{L}_1 E^i]^T \end{aligned}$$

and the remaining terms of $[\bar{\phi}]_{6\times 6}$ are zero.

Proof We choose $\mathscr{P} = \mathscr{X}^{-1}$ and a Lyapunov function for the IT2FO fuzzy augmented system (5) as

$$\mathscr{W}_t = \bar{x}_t^T \mathscr{P} \bar{x}_t. \quad (17)$$

The constructed sliding surface matrix is provided as $s_t = \mathscr{L}_2^T \mathscr{P}\bar{x}_t$, then we determine that $\mathscr{L}_2^T\mathscr{P}\bar{x}_t = 0$ on the sliding surface. In light of [1,2], one can compute the derivative of the Lyapunov functional as

$$\begin{aligned} D^{\alpha}\mathscr{W}_t \leq& \mathrm{Sym}\Big\{\bar{x}_t^T\mathscr{P}\sum_{i=1}^{n} v^i(x_t)\Big[\mathscr{L}_1\bar{A}^i\bar{x}_t + \mathscr{L}_1\bar{A}^{\tau i}\bar{x}_{(t-\tau)} \\ &+ \mathscr{L}_1 E^i f(t,x_t) + \mathscr{L}_1 B^{wi} w_t\Big]\Big\}. \end{aligned} \quad (18)$$

Furthermore, for any real matrices $\mathscr{Z}_g = \mathscr{Z}_g^T$, $g = 1, 2$, satisfying $\mathrm{diag}\{\mathscr{Z}_1, \mathscr{Z}_2\} \geq 0$, the following inequality holds:

$$\begin{aligned} &\tau^{\alpha}\alpha^{-1}\begin{bmatrix} \bar{x}_t \\ D^{\alpha}(\bar{x}_t) \end{bmatrix}^T \begin{bmatrix} \mathscr{Z}_1 & 0 \\ 0 & \mathscr{Z}_2 \end{bmatrix}\begin{bmatrix} \bar{x}_t \\ D^{\alpha}(\bar{x}_t) \end{bmatrix} - \\ &\int_{t-\tau}^{t}(t-v)^{\alpha-1}\begin{bmatrix} \bar{x}_t \\ D^{\alpha}(\bar{x}_t) \end{bmatrix}^T \begin{bmatrix} \mathscr{Z}_1 & 0 \\ 0 & \mathscr{Z}_2 \end{bmatrix}\begin{bmatrix} \bar{x}_t \\ D^{\alpha}(\bar{x}_t) \end{bmatrix} dv \geq 0. \end{aligned} \quad (19)$$

From (18) and (19), we can obtain that

$$\begin{aligned} D^{\alpha}\mathscr{W}_t \leq& \mathrm{Sym}\Big\{\bar{x}_t^T\mathscr{P}\sum_{i=1}^{n} v^i(x_t)\big[\mathscr{L}_1\bar{A}^i\bar{x}_t + \mathscr{L}_1\bar{A}^{\tau i}\bar{x}_{(t-\tau)} \\ &+ \mathscr{L}_1 E^i f(t,x_t) + \mathscr{L}_1 B^{wi} w_t\big]\Big\} \\ &+ \tau^{\alpha}\alpha^{-1}\bar{x}_t^T\mathscr{Z}_1\bar{x}_t + \tau^{\alpha}\alpha^{-1}(D^{\alpha}(\bar{x}_t))^T\mathscr{Z}_2 D^{\alpha}(\bar{x}_t) \\ &- \int_{t-\tau}^{t}(t-v)^{\alpha-1}\begin{bmatrix} \bar{x}_t \\ D^{\alpha}(\bar{x}_t) \end{bmatrix}^T \begin{bmatrix} \mathscr{Z}_1 & 0 \\ 0 & \mathscr{Z}_2 \end{bmatrix} \\ &\times \begin{bmatrix} \bar{x}_t \\ D^{\alpha}(\bar{x}_t) \end{bmatrix} dv. \end{aligned} \quad (20)$$

Moreover, it can be shown that

$$\bar{x}_t^T\mathscr{F}^T\mathscr{F}\bar{x}_t - f^T(t,x_t)f(t,x_t) \geq 0. \quad (21)$$

Since $\bar{x}_t$ satisfies for $-\tau \leq \theta \leq 0$, $q > 1$ and $\mathscr{W}(t+\theta, \bar{x}(t+\theta)) \leq q\mathscr{W}(t, \bar{x}_t)$, one can conclude that

$$q\bar{x}_t^T\mathscr{P}\bar{x}_t - \bar{x}_{(t-\tau)}^T\mathscr{P}\bar{x}_{(t-\tau)} \geq 0. \quad (22)$$

On the basis of $\mathscr{L}_2^T\mathscr{P}\bar{x}_t = 0$, the derivative of $\mathscr{W}_t$ can be obtained as follows

$$\begin{aligned} D^{\alpha}\mathscr{W}_t \leq& \sum_{i=1}^{n} v^i(x_t)\Big[\mathrm{Sym}\Big\{\bar{x}_t^T\mathscr{P}\big[\mathscr{L}_1\bar{A}^i\bar{x}_t + \mathscr{L}_1\bar{A}^{\tau i}\bar{x}_{(t-\tau)} \\ &+ \mathscr{L}_1 E^i f(t,x_t) + \mathscr{L}_1 B^{wi} w_t\big]\Big\} + q\bar{x}_t^T\mathscr{P}\bar{x}_t \\ &+ \tau^{\alpha}\alpha^{-1}\bar{x}_t^T\mathscr{Z}_1\bar{x}_t + \tau^{\alpha}\alpha^{-1}(D^{\alpha}(\bar{x}_t))^T\mathscr{Z}_2 D^{\alpha}(\bar{x}_t) \\ &- \bar{x}_{(t-\tau)}^T\mathscr{P}\bar{x}_{(t-\tau)} - \mathrm{Sym}\{\underbrace{\bar{x}_t^T\mathscr{P}\mathscr{L}_2}_{0}[\mathcal{O}_{ux}\bar{A}^i\bar{x}_t \\ &+ \mathcal{O}_{ux}\bar{A}^i\bar{x}_{(t-\tau)}]\}\Big] - \int_{t-\tau}^{t}(t-v)^{\alpha-1}\begin{bmatrix} \bar{x}_t \\ D^{\alpha}(\bar{x}_t) \end{bmatrix}^T \\ &\times \begin{bmatrix} \mathscr{Z}_1 & 0 \\ 0 & \mathscr{Z}_2 \end{bmatrix}\begin{bmatrix} \bar{x}_t \\ D^{\alpha}(\bar{x}_t) \end{bmatrix} dv, \\ =& \sum_{i=1}^{n} v^i(x_t)\Big[\mathrm{Sym}\Big\{\bar{x}_t^T\mathscr{P}\big[\mathscr{L}_1\bar{A}^i\bar{x}_t + \mathscr{L}_1\bar{A}^{\tau i}\bar{x}_{(t-\tau)} \\ &+ \mathscr{L}_1 E^i f(t,x_t) + \mathscr{L}_1 B^{wi} w_t\big]\Big\} \end{aligned}$$

$$+\tau^{\alpha}\alpha^{-1}\bar{x}_t^T\mathscr{Z}_1\bar{x}_t+\tau^{\alpha}\alpha^{-1}\Big(\mathscr{L}_1\bar{A}^i\bar{x}_t+\mathscr{L}_1\bar{A}^{\tau i}\bar{x}_{(t-\tau)}+\mathscr{L}_1E^if(t,x_t)+\mathscr{L}_1B^{wi}w_t\Big)^T\times\mathscr{Z}_2\Big(\mathscr{L}_1\bar{A}^i\bar{x}_t+\mathscr{L}_1\bar{A}^{\tau i}\bar{x}_{(t-\tau)}+\mathscr{L}_1E^if(t,x_t)+\mathscr{L}_1B^{wi}w_t\Big)+q\bar{x}_t^T\mathscr{P}\bar{x}_t-\bar{x}_{(t-\tau)}^T\mathscr{P}\bar{x}_{(t-\tau)}-\mathrm{Sym}\{\underbrace{\bar{x}_t^T\mathscr{P}\mathscr{L}_2}_{0}[\mathscr{K}^{1i}\bar{x}_t+\mathscr{K}^{2i}\bar{x}_{(t-\tau)}]\}\Big]-\int_{t-\tau}^{t}(t-v)^{\alpha-1}\begin{bmatrix}\bar{x}_t\\D^{\alpha}(\bar{x}_t)\end{bmatrix}^T\begin{bmatrix}\mathscr{Z}_1&0\\0&\mathscr{Z}_2\end{bmatrix}\times\begin{bmatrix}\bar{x}_t\\D^{\alpha}(\bar{x}_t)\end{bmatrix}dv, \tag{23}$$

where $\mathscr{K}_{gi}\in\mathfrak{R}^{m\times(m+n)}$, $g=1,2$, are matrices to be determined.

Let us denote $\Omega_t^T=\Big[\bar{x}_t^T \quad \bar{x}_{(t-\tau)}^T \quad w_t^T \quad f(t,x_t)\Big]$. The following bound relationship can be easily derived on the basis of the derivation so far:

$$D^{\alpha}\mathscr{W}_t-a\mathscr{W}_t-w_t^T\mathcal{R}w_t\leq\sum_{i=1}^{n}v^i(x_t)\Big[\Omega_t^T[\phi]_{4\times4}\Omega_t+\bar{x}_t^T\mathscr{F}^T\mathscr{F}\bar{x}_t+\tau^{\alpha}\alpha^{-1}\Big(\mathscr{L}_1\bar{A}^i\bar{x}_t+\mathscr{L}_1\bar{A}^{\tau i}\bar{x}_{(t-\tau)}+\mathscr{L}_1E^if(t,x_t)+\mathscr{L}_1B^{wi}w_t\Big)^T\mathscr{Z}_2\Big(\mathscr{L}_1\bar{A}^i\bar{x}_t+\mathscr{L}_1\bar{A}^{\tau i}\bar{x}_{(t-\tau)}+\mathscr{L}_1E^if(t,x_t)+\mathscr{L}_1B^{wi}w_t\Big)\Big]-\int_{t-\tau}^{t}(t-v)^{\alpha-1}\begin{bmatrix}\bar{x}_t\\D^{\alpha}(\bar{x}_t)\end{bmatrix}^T\begin{bmatrix}\mathscr{Z}_1&0\\0&\mathscr{Z}_2\end{bmatrix}\begin{bmatrix}\bar{x}_t\\D^{\alpha}(\bar{x}_t)\end{bmatrix}dv, \tag{24}$$

where the elements of $[\phi]_{4\times4}$ are displayed in theorem statement.

Now talking $q\to1^+$ in (24), the following inequality holds for some small $h>0$,

$$D^{\alpha}\mathscr{W}_t\leq-h||\bar{x}_t||^2-\int_{t-\tau}^{t}(t-s)^{\alpha-1}\begin{bmatrix}\bar{x}_t\\D^{\alpha}(\bar{x}_t)\end{bmatrix}^T\times\begin{bmatrix}\mathscr{Z}_1&0\\0&\mathscr{Z}_2\end{bmatrix}\begin{bmatrix}\bar{x}_t\\D^{\alpha}(\bar{x}_t)\end{bmatrix}ds\leq-h||\bar{x}_t||^2. \tag{25}$$

In the view of Schur complement, the following inequality is true if the condition in (13) is met:

$$D^{\alpha}\mathscr{W}_t-a\mathscr{W}_t-w_t^T\mathcal{R}w_t<0. \tag{26}$$

Multiplying e^{-at} on both sides of the above inequality and integrating it from 0 to t, one can obtain

$$e^{-at}\mathscr{W}_t-\mathscr{W}_0<\int_0^t e^{-au}w_u^T\mathcal{R}w_u du, \tag{27}$$

which can be further written, under the zero initial condition $x_0=0$, as follows:

$$\mathscr{W}_t<e^{at}\int_0^t e^{-au}w_u^T\mathcal{R}w_u du. \tag{28}$$

In addition, for any weight matrix $\mathcal{F}$, the controlled output vector z_t and subject to the conditions (15) and (28), we can obtain that

$$z_t^T\mathcal{F}z_t=\sum_{i=1}^{n}v^i(x_t)\bar{x}_t^T(\bar{C}^i)^T\mathcal{F}\bar{C}^i\bar{x}_t\leq\bar{x}_te^{-at}\mathscr{P}\bar{x}_t\leq\mathscr{W}_t\leq e^{at}\lambda_{\max}(\mathcal{R})\int_0^t w_u^T\mathcal{R}w_u du<e^{aT}\frac{c_2^2}{e^{at}c_1^2}c_1^2=c_2^2,\ \forall t\in[0,T]. \tag{29}$$

In the view of Definition 1, it can be seen that the IT2FO fuzzy augmented system (5) is input-output finite-time stabilization with respect to $(c_1,c_2,\mathcal{F},T)$. Hence, the proof is completed. □

The established sufficient conditions in Theorem 2 are not in the form of linear matrix inequalities (LMIs) due to the presence of the time-varying structural uncertainties. In this regard, one can determine the gain matrices of the dynamic SMC scheme to stabilize the system (2) by using the following theorem.

Theorem 3 *For given scalars τ, $c_1>0$, $c_2>0$, $T>0$, $a>0$, $\alpha\in(0,1]$ and a matrix $\mathscr{F}$, the IT2FO fuzzy augmented system (5) is input-output finite-time stabilization with respect to $(c_1,c_2,\mathcal{F},T)$, if there exist the positive definite matrix $\mathscr{X}\in\mathfrak{R}^{(m+n)\times(m+n)}$, real matrices $\mathscr{M}^{gi}\in\mathfrak{R}^{m\times(m+n)}$, $(g=1,2,\ \&\ i\in\mathbb{S})$ and scalars ϵ^i, $(i\in\mathbb{S})$ ensuring the following constraints:*

$$\begin{bmatrix}[\hat{\phi}]_{6\times6}&\tilde{W}_1&\epsilon^i\tilde{W}_2\\ *&-\epsilon^iI&0\\ *&*&-\epsilon^iI\end{bmatrix}<0, \tag{30}$$

$$\begin{bmatrix}-\mathscr{X}&(\bar{C}^i\mathscr{X})^T\\ *&-\mathcal{F}I\end{bmatrix}<0, \tag{31}$$

$$\mathcal{R} < \frac{c_2^2}{e^{aT} c_1^2} I, \tag{32}$$

where $[\hat{\phi}]_{6\times 6} = \begin{bmatrix} [\tilde{\phi}]_{4\times 4} & \tilde{\vartheta}_1 & \tau^\alpha \alpha^{-1} \tilde{\vartheta}_2 \\ * & -I & 0 \\ * & * & -\tau^\alpha \alpha^{-1} I \end{bmatrix}$

$$\begin{aligned}
\tilde{\phi}_{11} =& \big(\mathcal{L}_1 \bar{A}^i \mathcal{X} - \mathcal{L}_2 \mathcal{M}^{1i}\big) + \big(\mathcal{L}_1 \bar{A}^i \mathcal{X} - \mathcal{L}_2 \mathcal{M}^{1i}\big)^T \\
& + \mathcal{X}(\tau^\alpha \alpha^{-1} + 1 - a), \\
\tilde{\phi}_{12} =& \big(\mathcal{L}_1 \bar{A}^{\tau i} \mathcal{X} - \mathcal{L}_2 \mathcal{M}^{2i}\big), \quad \tilde{\phi}_{13} = \mathcal{L}_1 B^{wi}, \\
\tilde{\phi}_{14} =& E^i \mathcal{L}_1, \quad \tilde{\phi}_{22} = -\mathcal{X}, \quad \tilde{\phi}_{33} = -\mathcal{R}, \quad \tilde{\phi}_{44} = -I, \\
\tilde{\vartheta}_1 =& [\mathcal{F}\mathcal{X} \;\; 0 \;\; 0 \;\; 0]^T, \\
\tilde{\vartheta}_2 =& [\mathcal{L}_1 \bar{A}^i \mathcal{X} \;\; \mathcal{L}_1 \bar{A}^{\tau i} \mathcal{X} \;\; \mathcal{L}_1 B^{wi} \;\; \mathcal{L}_1 E^i]^T, \\
\tilde{W}_1 =& [\bar{\mathcal{N}}^{2i} \mathcal{X} \;\; \bar{\mathcal{N}}^{4i} \mathcal{X} \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T, \\
\tilde{W}_2 =& [(\mathcal{L}_1 \bar{\mathcal{N}}^{1i})^T \;\; 0 \;\; 0 \;\; 0 \;\; (\tau^\alpha \alpha^{-1} \mathcal{L}_1 \bar{\mathcal{N}}^{1i})^T \;\; 0]^T
\end{aligned}$$

and the remaining terms of $[\tilde{\phi}]_{8\times 8}$ *are zero. In addition, we can obtain the sliding surface matrix* $\bar{\mathcal{O}} = \mathcal{L}_2^T \mathcal{X}^{-1}$.

Proof According to Lemma 1 and the inequality (24), by replacing $\mathcal{L}_1 \bar{A}^i = \big(\mathcal{L}_1 \bar{A}^i + \mathcal{L}_1 \Delta \bar{A}_t^i\big)$ and $\mathcal{L}_1 \bar{A}^{\tau i} = \big(\mathcal{L}_1 \bar{A}^{\tau i} + \mathcal{L}_1 \Delta \bar{A}_t^{\tau i}\big)$, one can observe that $[\bar{\phi}]_{6\times 6}$ is equivalent to the following condition:

$$\begin{aligned}
[\hat{\bar{\phi}}]_{6\times 6} =& [\bar{\phi}]_{6\times 6} + W_1 \nu_t W_2 + W_2^T \nu_t^T W_1^T \\
\le& [\bar{\phi}]_{6\times 6} + \epsilon^i W_1 W_1^T + (\epsilon^i)^{-1} W_2^T W_2,
\end{aligned} \tag{33}$$

where $W_1 = \big[(\mathcal{P}\bar{\mathcal{N}}^{1i})^T \; 0\;0\;0\;0 \; (\tau^\alpha \alpha^{-1} I \bar{\mathcal{N}}^{1i})^T\big]^T$, $W_2 = \big[\mathcal{L}_1 \bar{\mathcal{N}}^{2i} \; \mathcal{L}_1 \bar{\mathcal{N}}^{4i} \; 0\;0\;0\;0\big]$ and the elements of $[\bar{\phi}]_{6\times 6}$ are displayed in Theorem 1. Based on Schur complement, the term in right-hand side of (33) is true if and only if there exist scalars ϵ^i, $(i \in \mathbb{S})$ such that

$$[\vartheta]_{8\times 8} = \begin{bmatrix} [\bar{\phi}]_{6\times 6} & \epsilon^i W_1 & \epsilon^i W_2^T \\ * & -\epsilon^i I & 0 \\ * & * & -\epsilon^i I \end{bmatrix}. \tag{34}$$

Pre- and post-multiplying the matrix $[\vartheta]_{8\times 8}$ by $\mathrm{diag}\{\mathcal{X}, \mathcal{X}, I, I, I, I, I, I\}$, and letting $\mathcal{M}^{1i} = \mathcal{K}^{1i}\mathcal{X}$, $\mathcal{M}^{2i} = \mathcal{K}^{2i}\mathcal{X}$, $\mathcal{P} = \mathcal{Z}_1$, $I = \mathcal{Z}_2$, the inequality (34) can be rewritten as (30). Thus, the proof is completed. □

Remark 2 Theorems 2 and 3 present the input-output finite-time stabilization conditions for IT2FO fuzzy systems with and without uncertainties and time delays by using dynamic SMC scheme. Next, if we consider $\Delta B_t^i = 0$, $A^{\tau i} = 0$, $\Delta A_t^{\tau i} = 0$ in (2) then the IT2FO fuzzy augmented system (5) can be rewritten as follows

$$\begin{aligned}
D^\alpha \bar{x}_t =& \sum_{i=1}^n \upsilon^i(x_t)\big[\big((\mathcal{L}_1 - \mathcal{L}_2 \mathcal{O}_{ux})\bar{A}^i + \mathcal{L}_1 \Delta \bar{A}_t^i\big)\bar{x}_t \\
& + \mathcal{L}_1 E^i f(t, x_t) + \mathcal{L}_1 B^{wi} w_t\big] \\
& - \mathcal{L}_2 (p + l_t) \mathcal{O}_u^{-1} \mathrm{sgn}(s_t),
\end{aligned} \tag{35}$$

where $\mathcal{L}_1 = [I_n, 0_{n\times m}]^T$, $\mathcal{L}_2 = [0_{m\times n}, I_m]^T$, $\bar{A}^i = [A^i, B^i]$, $\Delta \bar{A}_t^i = [\Delta A_t^i, 0_{n\times m}]$. In order to better illustrate the benefits of the proposed control design, Corollary 1 is given. Subsequently, using the same lines as in the proof of Theorems 1 and 2, we can obtain the following results.

Corollary 1 *Let* $c_1 > 0$, $c_2 > 0$, $T > 0$, $a > 0$, $\alpha \in (0, 1]$ *be given scalars. Assume that there exist positive definite matrix* $\mathcal{X} \in \Re^{(m+n)\times(m+n)}$*, any real matrices* $\mathcal{M}^i \in \Re^{m\times(m+n)}$*,* $(i \in \mathbb{S})$ *and scalars* ϵ^i*,* $(i \in \mathbb{S})$*, such that the following constraints hold:*

$$[\check{\phi}]_{6\times 6} < 0, \tag{36}$$

$$\begin{bmatrix} -\mathcal{X} & (\bar{C}^i \mathcal{X})^T \\ * & -\mathcal{F} I \end{bmatrix} < 0, \tag{37}$$

$$\mathcal{R} < \frac{c_2^2}{e^{aT} c_1^2} I, \tag{38}$$

where

$$\begin{aligned}
\check{\phi}_{11} =& \big(\mathcal{L}_1 \bar{A}^i \mathcal{X} - \mathcal{L}_2 \mathcal{M}^i\big) + \big(\mathcal{L}_1 \bar{A}^i \mathcal{X} - \mathcal{L}_2 \mathcal{M}^i\big)^T - a\mathcal{X} \\
\check{\phi}_{12} =& \mathcal{L}_1 E^i, \quad \check{\phi}_{13} = \mathcal{L}_1 B^{wi}, \quad \check{\phi}_{22} = -I, \\
\check{\phi}_{33} =& -\mathcal{R}, \quad \check{\phi}_{14} = \mathcal{X}\mathcal{F}^T, \quad \check{\phi}_{15} = (\bar{\mathcal{N}}^{2i}\mathcal{X})^T, \\
\check{\phi}_{16} =& \epsilon^i \bar{\mathcal{N}}^{1i}, \quad \check{\phi}_{44} = -I, \quad \check{\phi}_{44} = -\epsilon^i I, \quad \check{\phi}_{66} = -\epsilon^i I.
\end{aligned}$$

Then, the IT2FO fuzzy augmented system (5) is input-output finite-time stabilization with respect to $(c_1, c_2, \mathcal{F}, T)$*. Furthermore, the control gain can be calculated by* $\bar{\mathcal{O}} = \mathcal{L}_2^T \mathcal{X}^{-1}$.

Remark 3 In recent years, the investigation of the dynamic SMC scheme for fuzzy systems has achieved an enormous growth [29–32]. Unfortunately, majority of the investigated results are based on Lyapunov stability over an infinite-time interval with no consideration for prescribed performance. However, in many real-world systems, the researchers need the prescribed system performance within a finite-time interval. For this reason, the input-output finite-time stabilization problem for IT2FO fuzzy systems via the dynamic SMC scheme is addressed in this paper.

Remark 4 Note that the designed dynamic SMC scheme in (4) contains a switching term $\mathrm{sgn}(s(t))$, which would create a chattering phenomenon when executed. In order to reduce that phenomenon, a smooth continuous term $s_t/(||s_t|| + 0.01)$ can be utilized in the place of $\mathrm{sgn}(s_t)$. Thus, the above-mentioned factors are used during the simulations in Sect. 4.

4 Illustrative examples

This section provides three examples to show the usefulness and superiority of the constructed control method. In Example 1, we utilized artificial parameters to demonstrate the effectiveness of the proposed dynamic SMC scheme. In Example 2, the efficiency of the designed dynamic SMC scheme is validated via a permanent magnet synchronous motor (PMSM) model [23]. At last, Example 3 compares the designed dynamic SMC scheme with fuzzy SMC scheme in [37].

Example 1 Consider the IT2FO fuzzy systems with two fuzzy rules and the following parameters as

$$A^1 = \begin{bmatrix} -2 & 4 & 1 \\ 7 & 2.8 & 2 \\ 9 & 4 & -8.5 \end{bmatrix}, A^{\tau 1} = \begin{bmatrix} 0.5 & 0.4 & 0.7 \\ 1 & 0.5 & 0.1 \\ 0.3 & 0.1 & 0.2 \end{bmatrix},$$
$$E^1 = 0.1I,$$
$$A^2 = \begin{bmatrix} -2 & 5 & 1 \\ 7 & 1.8 & 2 \\ 9 & 4 & -8 \end{bmatrix}, A^{\tau 2} = \begin{bmatrix} 0.2 & 0.2 & 0.4 \\ 1 & 0.3 & 0.1 \\ 0.5 & 0.2 & 0.1 \end{bmatrix},$$
$$E^2 = 0.2I,$$
$$B^1 = \begin{bmatrix} 2 \\ 0 \\ -0.7 \end{bmatrix}, B^{w1} = \begin{bmatrix} 0.3 \\ -0.4 \\ 0.1 \end{bmatrix}, \mathscr{N}^{11} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix},$$
$$B^2 = \begin{bmatrix} 3 \\ 0 \\ -0.6 \end{bmatrix}, B^{w2} = \begin{bmatrix} 0.4 \\ -0.1 \\ 0.2 \end{bmatrix}, \mathscr{N}^{12} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix},$$
$$\mathscr{N}^{21} = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix}, \mathscr{N}^{31} = 0.1, \mathscr{N}^{41} = \begin{bmatrix} 0.5 & 0.2 & 0.3 \end{bmatrix},$$
$$\mathscr{N}^{22} = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}, \mathscr{N}^{32} = 0.2, \mathscr{N}^{42} = \begin{bmatrix} 0.2 & 0.1 & 0.2 \end{bmatrix},$$

Table 1 Sliding surface gain matrices $\mathcal{H}_x$ and $\mathcal{H}_u$

Cases	$\mathcal{H}_x$			$\mathcal{H}_u$
Case (i)	[23.6478	24.5597	5.4172]	4.1141
Case (ii)	[42.9429	43.5328	9.1784]	5.9163
Case (iii)	[67.0106	73.9049	19.0162]	7.0639

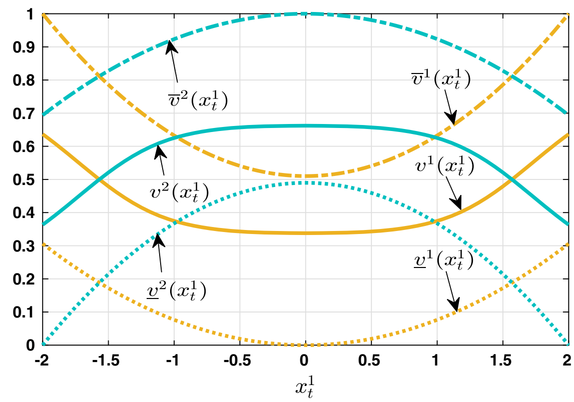

Fig. 1 Membership function

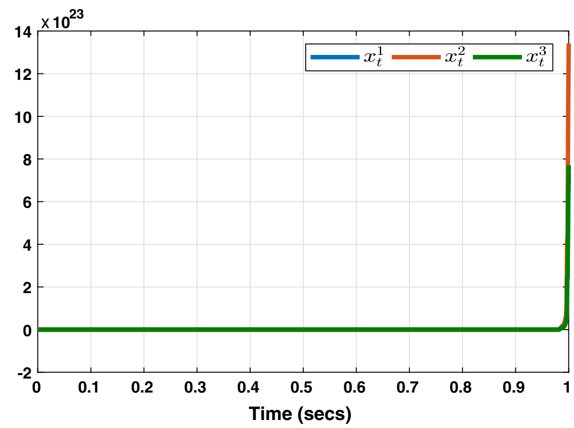

Fig. 2 State responses of the open-loop system

$$C^1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, C^2 = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix}, \mathscr{F} = 0.9I.$$

Figure 1 is an image of the curves about the upper and lower membership functions which is selected as

$$\underline{v}_1(x_t^1) = \frac{-\mathfrak{r} + \mathfrak{r}_{max}}{\mathfrak{r}_{max} - \mathfrak{r}_{min}}, \overline{v}_2(x_t^1) = \frac{\mathfrak{r} - \mathfrak{r}_{min}}{\mathfrak{r}_{max} - \mathfrak{r}_{min}}$$
with $\mathfrak{h} = 5$,
$$\overline{v}_1(x_t^1) = \frac{-\mathfrak{r} + \mathfrak{r}_{max}}{\mathfrak{r}_{max} - \mathfrak{r}_{min}}, \underline{v}_2(x_t^1) = \frac{\mathfrak{r} - \mathfrak{r}_{min}}{\mathfrak{r}_{max} - \mathfrak{r}_{min}}$$
with $\mathfrak{h} = 8$.

Table 2 Minimum c_2 for different c_1 and T in Example 1

T/c_1	$c_1 = 0.1$	$c_1 = 0.2$	$c_1 = 0.3$	$c_1 = 0.4$	$c_1 = 0.5$
$T = 5$	0.1816	0.3632	0.5448	0.7264	0.9079
$T = 7$	0.2007	0.4014	0.6021	0.8027	1.0034
$T = 10$	0.2332	0.4663	0.6995	0.9327	1.1658

where $\mathfrak{r} = -\mathfrak{h} - 0.09\mathfrak{h}(x_t^1)^2$ in which $\mathfrak{r}_{max} = -5$, $\mathfrak{r}_{min} = -10.88$ and choose the uncertain parameter $\mathfrak{h} \in [5, 8]$. Correspondingly, we choose $\underline{m}_i(x_t^1) = 0.5(\sin(x_t^1))^2$, $\overline{m}_i(x_t^1) = 1 - \underline{m}_i(x_t^1)$, for all $i = 1, 2$, which satisfy $\sum_{i=1}^{2} v^i(x_t) = 1$ to indicate the nonlinear functions of the proposed IT2FO fuzzy systems (2).

Meanwhile, the parameters concerned with proposed approach are given as $\alpha = 0.96$, $\tau = 0.5$, $a = 0.1$, $c_1 = 0.2$, $c_2 = 0.3632$, $p = 1.2$, $T = 5$, and $\mathcal{F} = 2$. To distinguish the efficiency between the addressed system and its deduced systems, three cases are considered, which are Case (i) Without uncertainties and nonlinearities; Case (ii) Without uncertainties and with nonlinearities; and Case (iii) With uncertainties and nonlinearities. Utilizing Theorem 3 with the above-mentioned parameter values, a feasible solution of the LMIs in (30)-(32) are obtained with the dynamic SMC gains and unknown matrices, which are displayed in Table 1.

Based on input-output finite-time stabilization property, the dynamic SMC scheme is as follows:

$$D^{\alpha}u_t = \sum_{i=1}^{2} v^i(x_t)\big[- \mathcal{O}_u^{-1}\mathcal{O}_x\big(A^i x_t + A^{\tau i}x_{(t-\tau)} + B^i u_t\big)\big] - (1.2 + l_t)\mathcal{O}_u^{-1}\text{sgn}(s_t),$$

where $l_t = \sum_{i=1}^{2} v^i(x_t)||\mathcal{O}_x||\Big[||\mathcal{N}^{1i}||||\mathcal{N}^{2i}||||x_t|| + ||\mathcal{N}^{1i}||||\mathcal{N}^{4i}||||x_{(t-\tau)}|| + ||\mathcal{N}^{1i}||||\mathcal{N}^{3i}||||u_t|| + ||E^i||||f(t, x_t)|| + ||B^{wi}||c_t\Big]$.

Table 2 demonstrates the optimal minimum values of c_2 for ensuring the IT2FO fuzzy augmented system finite-time bounded for various c_1 and T. At the same time, we choose the disturbance vectors and initial conditions as $w_t = \frac{0.01\sin(\pi t)}{e^{0.02t}}$, $\phi_t = [0, 0]^T$, and $u_0 = 0$. The unknown nonlinear functions as $f(t, x_t) = [\iota_t(x_t^1)^3 \;\; \iota_t(x_t^2)^3 \;\; \iota_t(x_t^1)^3]^T$, where ι_t stands for random input vector with an upper bound of 0.4. With the obtained dynamic SMC gains, the simulation graphs of the IT2FO fuzzy systems (2) are displayed in Figs. 2-6. When there are no dynamic SMC gains, the unstable state trajectories of the uncontrolled IT2FO fuzzy augmented system can be easily inspected as shown in Fig. 2. Based on the obtained dynamic SMC gains $\mathcal{O}_x$ and $\mathcal{O}_u$, we can find the IT2FO fuzzy augmented system is input-output finite-time stabilization in Fig. 3. Meantime, the evaluation of the dynamic SMC input and sliding-surface trajectories are displayed in Figs. 4 and 5. Eventually, the simulation of $z_t^T\mathcal{F}z_t$ under the dynamic SMC gains $\mathcal{O}_x$ and $\mathcal{O}_u$ is presented in Fig. 6, where the objective constraint of this paper is satisfied. The necessity and importance of the designed dynamic SMC scheme are therefore observed in Fig. 6. In other words, depending on the behavior of the IT2FO fuzzy augmented system, the simulation curve does not exceed the value of $c_2 = 0.3632$. As a result, clearly reveals that a satisfactory performance is achieved in all three cases, however the system performance under the three case is more effective. This illustrates the adaptability and superiority of the proposed design strategy.

Example 2 For the sake of verifying the proposed results, the PMSM [23] model is considered. It is expected that there happens to be a time delay in this system, which is described as:

$$D^{0.97}x_t = \sum_{i=1}^{2} v^i(x_t)\Big[(1-\mathfrak{c})(A^i + \Delta A_t^i)x_t + \mathfrak{c}(A^i + \Delta A_t^i)x_{(t-\tau)} + B^i u_t + E^i f(t, x_t) + B^{wi}w_t\Big],$$

$$z_t = \sum_{i=1}^{2} v^i(x_t)C^i x_t,$$

where

$$A^1 = \begin{bmatrix} -l_1 & 0 & l_1 \\ 0 & -1 & l_3 \\ l_2 & -l_3 & -1 \end{bmatrix}, E^1 = \begin{bmatrix} 0.16 & 0.25 & 0.14 \\ 0.02 & 0.03 & 0.01 \\ 0.12 & 0.12 & 0.13 \end{bmatrix},$$

Table 3 Minimum c_2 for different c_1 and T in Example 2

T/c_1	$c_1 = 0.1$	$c_1 = 0.2$	$c_1 = 0.3$	$c_1 = 0.4$	$c_1 = 0.5$
$T = 10$	0.2332	0.4663	0.6995	0.9327	1.1658
$T = 50$	1.7229	3.4457	5.1686	6.8915	8.6143
$T = 100$	20.9888	41.9776	62.9664	83.9552	104.1044

$$A^2 = \begin{bmatrix} -l_1 & 0 & l_1 \\ 0 & -1 & l_4 \\ l_2 & -l_4 & -1 \end{bmatrix}, E^2 = \begin{bmatrix} 0.25 & 0.13 & 0.12 \\ 0.03 & 0.02 & 0.02 \\ 0.13 & 0.17 & 0.15 \end{bmatrix},$$
$$B^i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, B^{wi} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathscr{N}^{11} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix},$$
$$\mathscr{N}^{12} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix},$$
$$\mathscr{N}^{21} = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix}, \mathscr{N}^{22} = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}, C^i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$
$$\mathscr{F} = 0.5I,$$

with $\mathfrak{c} = 0.1, l_1 = -1.9, l_2 = 5, l_3 = -1.5, l_4 = 3$ and $i = 1, 2$. The upper and lower membership functions of the IT2FO fuzzy systems are borrowed from [16]. Furthermore, we choose $\alpha = 0.97$, $\tau = 0.6$, $a = 0.1$, $c_1 = 0.1$, $c_2 = 0.2332$, $p = 1.2$, $T = 10$ and $\mathcal{F} = 2$. Based on the LMIs (30)-(32), by utilizing the toolbox of MATLAB 2017b, we can obtain the dynamic SMC gains as follows:

$$\mathcal{O}_x = \begin{bmatrix} 31.3305 & 3.2072 & 21.6726 \end{bmatrix}, \quad \text{and} \quad \mathcal{O}_u = 0.0902.$$

The unknown nonlinearities, disturbances and initial conditions are taken the same as in Example 1. The validation of the simulations are displayed in Figs. 7-10. It is clear from Fig. 7 that the designed dynamic SMC scheme can effectively eliminate the effects of uncertainties, nonlinearities and disturbances, and guarantee the input-output finite-time stabilization of the IT2FO fuzzy augmented systems. The control input and sliding-surface trajectories are plotted in Figs. 8 and 9, respectively. The simulation $z_t^T \mathcal{F} z_t$ curve imply that the aimed object is well achieved (i.e. $z_t^T \mathcal{F} z_t < c_2^2$) in Fig. 10. On the basis of the LMIs (30)-(32), the optimal minimum value c_2 which ensures the input-output finite-time stabilization of the considered system addressed is calculated for different values of c_1 and T. Detailed statistics is provided in Table 3.

Remark 5 If we consider the $\alpha = 1$ in system (2), then the fractional-order IT2 fuzzy systems with uncertainties and disturbances will degenerate to an integer-order one. Correspondingly, the input-output stability results in Theorem 3 and Corollary 1 are still valid for the integer order type-1 fuzzy model, which were discussed in [37].

Example 3 For comparison purpose, we consider the single-link robot arm model whose dynamics are specified in [37] and are not provided here for brevity. The corresponding system matrices are chosen as follows:

$$D^\alpha x_t = \sum_{i=1}^{2} v^i(x_t)\Big[(A^i + \Delta A_t^i)x_t + B^i u_t + E^i f(t, x_t) + B^{wi} w_t\Big],$$
$$z_t = \sum_{i=1}^{2} v^i(x_t) C^i x_t,$$

where

$$A^1 = \begin{bmatrix} 0 & 1 \\ -\mathscr{G}_1\mathscr{G}_2 & -\mathscr{G}_3 \end{bmatrix}, A^2 = \begin{bmatrix} 0 & 1 \\ -\mathfrak{c}\mathscr{G}_1\mathscr{G}_2 & -\mathscr{G}_3 \end{bmatrix},$$
$$B^i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B^{wi} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \mathscr{N}^{1i} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix},$$
$$E^i = I, \mathscr{N}^{2i} = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}, C^i = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ i = 1, 2,$$

with $\mathfrak{c} = 0.01/\pi$, $\mathscr{G}_1 = 9.81$, $\mathscr{G}_2 = 0.5$ and $\mathscr{G}_3 = 2$. The parameters are chosen $w_t = 0.1e^{-0.001t}$, $p = 1.2$, $c_1 = 0.1$, $c_2 = 0.7$, $\mathcal{F} = I$ and $T = 1$. Solving the LMIs (36)-(38), by utilizing the toolbox of MATLAB 2017b, we can obtain that

$$\mathcal{O}_x = \begin{bmatrix} 0.0205 & 0.0039 \end{bmatrix}, \quad \text{and} \quad \mathcal{O}_u = 0.0001.$$

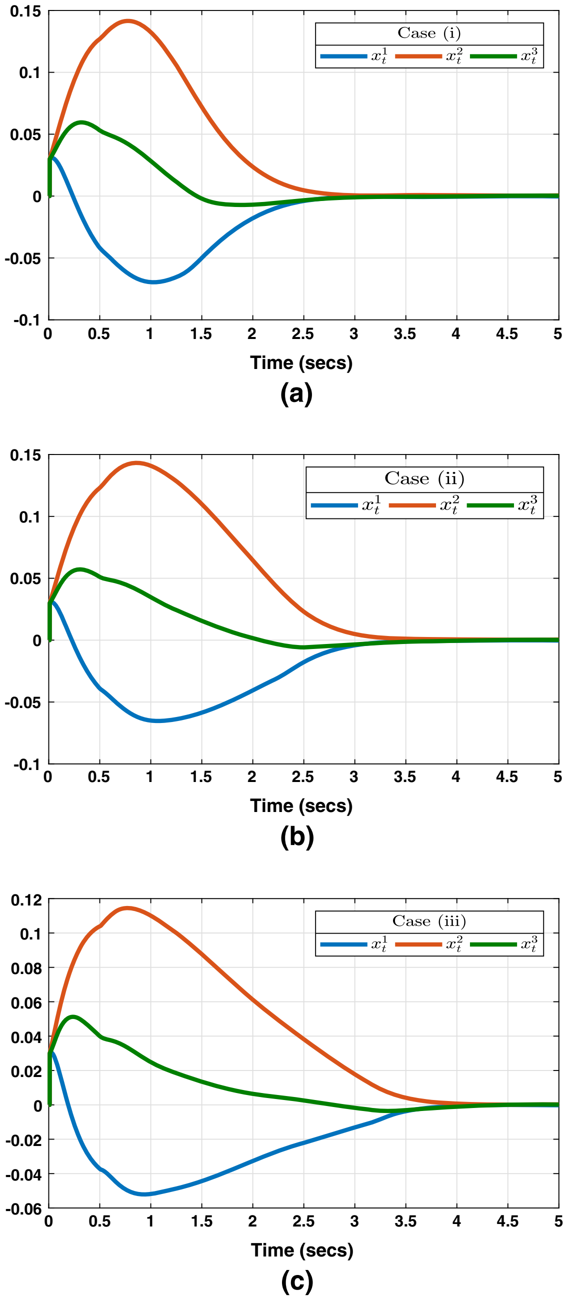

Fig. 3 State responses of the closed-loop system

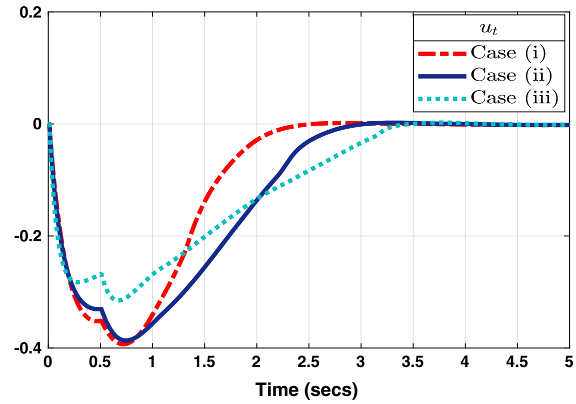

Fig. 4 Control input signal u_t

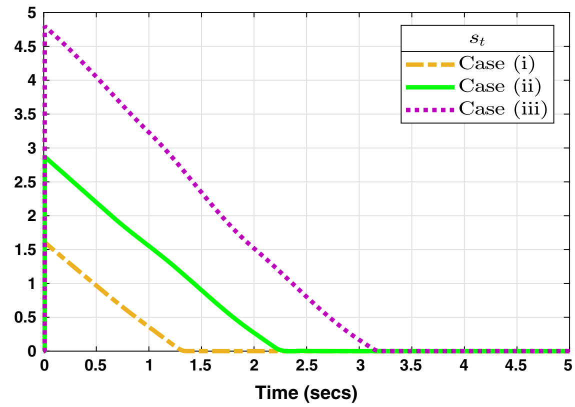

Fig. 5 Sliding-surface trajectory s_t

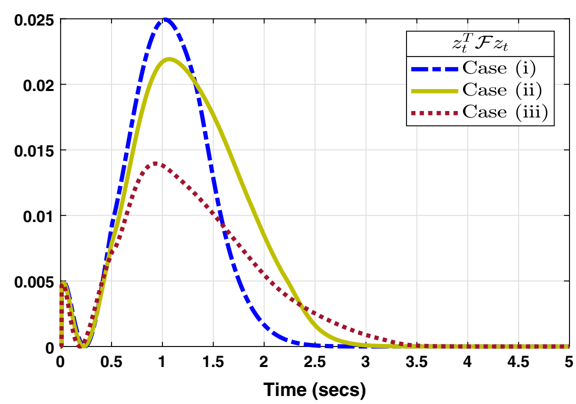

Fig. 6 The trajectory of the weighted output $z_t^T \mathcal{F} z_t$

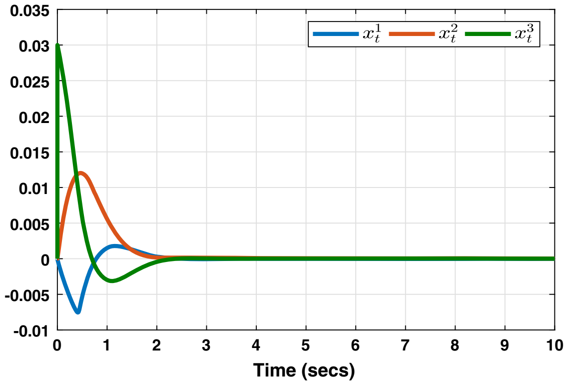

Fig. 7 State responses of the closed-loop system

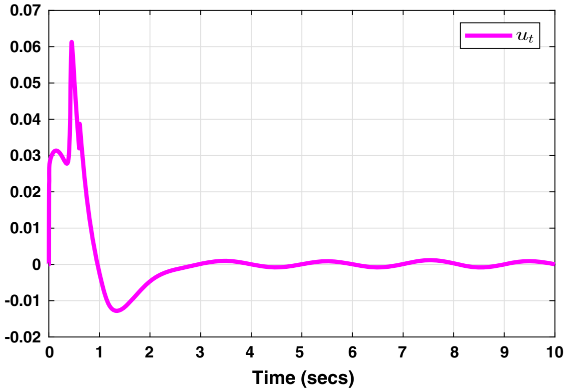

Fig. 8 Control input signal u_t

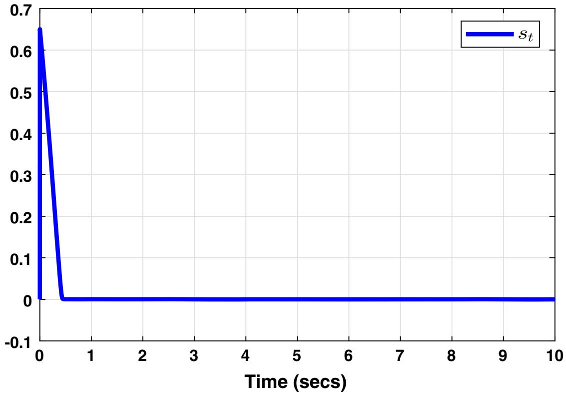

Fig. 9 Sliding-surface trajectory s_t

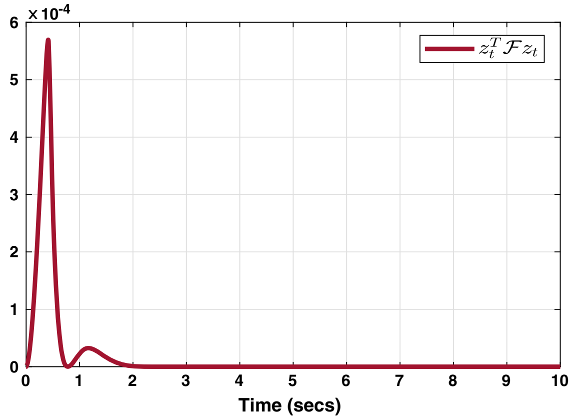

Fig. 10 The trajectory of the weighted output $z_t^T \mathcal{F} z_t$

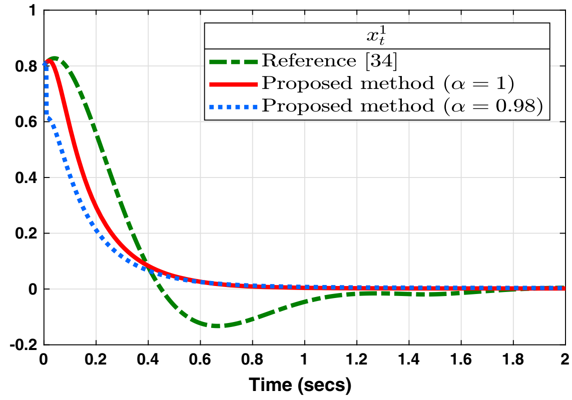

Fig. 11 Comparison of the system state x_t^1

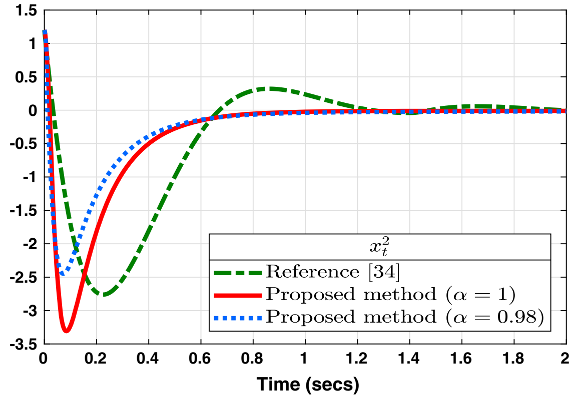

Fig. 12 Comparison of the system state x_t^2

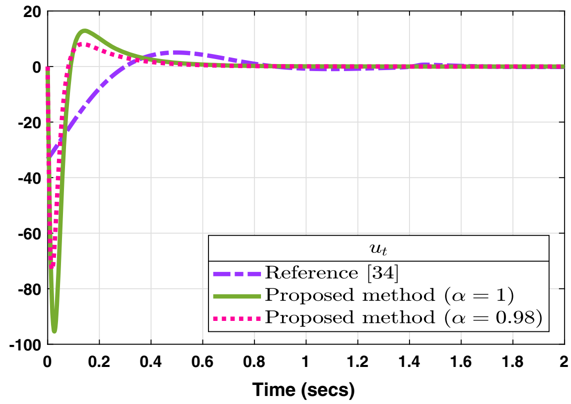

Fig. 13 Comparison of the control input u_t

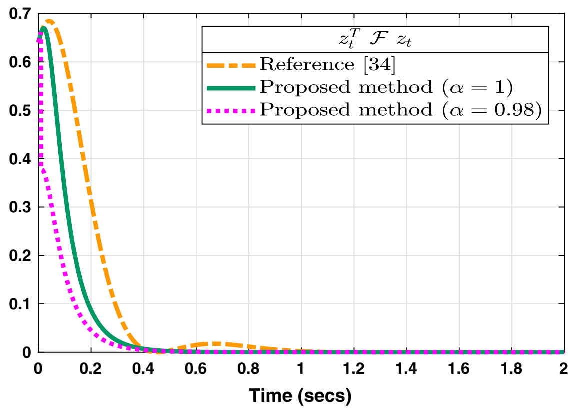

Fig. 14 Comparison of the weighted output $z_t^T \mathcal{F} z_t$

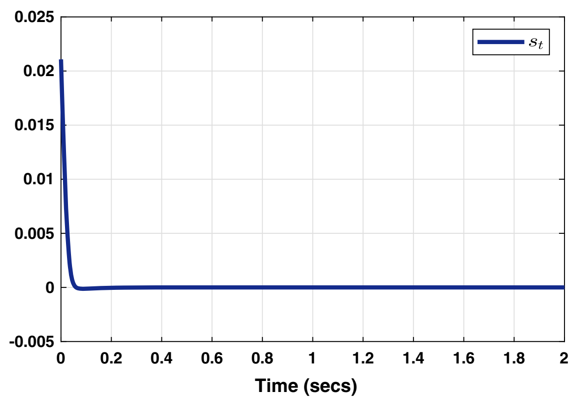

Fig. 15 Sliding-surface trajectory s_t

For comparison, the parameter in dynamic SMC scheme is chosen as

$$D^{\alpha} u_t = \sum_{i=1}^{2} v^i(x_t)\big[-\mathcal{O}_u^{-1}\mathcal{O}_x\big(A^i x_t + B^i u_t\big)\big] - (1.2 + l_t)\mathcal{O}_u^{-1}\mathrm{sgn}(s_t),$$

where $l_t = \sum_{i=1}^{2} v^i(x_t)||\mathcal{O}_x||\Big[||\mathcal{N}^{1i}||||\mathcal{N}^{2i}||||x_t|| + ||\mathcal{N}^{1i}||||\mathcal{N}^{3i}||||u_t|| + ||E^i||||f(t,x_t)|| + ||B^{wi}||c_t\Big]$. With the same initial condition $\phi_t = [0.8, 1.2]^T$, Figs. 10-13 show the same system states, control input, and weighted output under the methods in [37] and this paper with different α. From Figs. 11 and 14, it can be seen that the designed dynamic SMC scheme in this paper shows faster sates responses than those of the paper [37]. Then, the corresponding control inputs are plotted in Fig. 12. In particular, Fig. 13 shows that the weighted output $z_t^T \mathcal{F} z_t$ of SMC scheme [37] based input-output finite-time stabilization is significantly larger than dynamic SMC scheme proposed in this paper, which means that the input-output finite-time stabilization of the system addressed is achieved within the suggested time interval and very quick convergence, that is $z_t^T \mathcal{F} z_t \leq 0.7$ for $t \in [0, 2]$. Fig. 15 plots the sliding-surface of the designed control scheme. All those figures strongly demonstrate the superiority of the designed dynamic SMC scheme.

Remark 6 It is worth pointing out that the design algorithm proposed in Theorems 2 and 3 depends on several positive scalars, namely τ, c_1, c_2, a and ϵ^i $(i \in \mathbb{S})$. In general, these parameters are chosen in a random manner within the margin level specified in their definitions and without violating the feasibility of the obtained LMI-based criterion during the investigation of system stability. Among these scalars, c_2 plays a vital role in obtaining the desired finite-time stability criterion. Since it ensures the settling time of achieving stability over a finite domain, the choice of values of c_2 highly affects the feasibility of the proposed stability condition in Theorem 3. Moreover, it is worth noting that in practical situations, it is more reasonable to choose sufficiently small values for c_2. Furthermore, the matrices $\mathscr{X}$, $\mathscr{M}^{gi}$, ($g = 1, 2,\ \& \ i \in \mathbb{S}$), denote the positive definite matrix and real matrices, respectively. According to the system matrices and above-mentioned scalars values, the matrices $\mathscr{X}$, $\mathscr{M}^{gi}$, ($g = 1, 2,\ \& \ i \in \mathbb{S}$) can be determined by solving Theorem 3 using a standard convex optimization toolbox.

5 Conclusion

The finite-time prescribed performance control design for delayed FO nonlinear systems against uncertainties, nonlinearities and disturbances was investigated in this paper. In particular, IT2 fuzzy models were utilized to identify the local nonlinear dynamics and a new set of parameters were used to achieve the desired system performance. The dynamic SMC was then designed in such a way that the IT2FO fuzzy systems achieves input-output finite-time stabilization with a specified time interval. The stabilization criteria were established in terms of linear matrix inequalities via Lyapunov functional methods. At last, the developed results were verified through three numerical examples.

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Availability of data and material Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Conflict of interest The authors declare that there is no conflict of interest.

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