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ORIGINAL PAPER

How to wake up the electric synapse coupling between neurons?

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Abstract Both electric and chemical synapses play an important role in receiving and propagating signals, and thus an isolated neuron and neurons in networks can be activated to trigger appropriate firing modes. The synaptic plasticity enables adaptive regulation in the channel current along the synapse connection, and the intrinsic field energy in the media is pumped to keep possible balance between neurons. The Hamilton energy function in biophysical neurons addressed its dependence of energy on firing modes, and the same energy in neurons seldom indicates complete synchronization because the energy function is often composed of more than two variables in the neuron models. In this paper, we claim that the creation and waking up of synapses connection result from the diversity in field energy of neurons. From physical viewpoint, each neuron can be considered as a complex charged body and any external stimulus will change the distribution of field energy. For two and more neurons, the external stimulus-induced fluctuation in field energy will activate the synapses of neurons, and more synaptic connections will be enhanced for keeping energy balance. That is, the coupling intensity via synapse connection to neurons will be regulated in a possible way. In this work, two simple neural circuits are mapped into feasible neuron models for investigating the energy pumping and propagation by adjusting the intensity along the coupling channel until energy balance is approached. Furthermore, a similar case is explored in a chain network, and it is found that continuous energy pumping to the adjacent neurons will build up a network connected by synapses. These results clarify that synapses connection is activated between neurons because of gradient diversity in field energy in neurons and networks, and then synapse connections are waken up effectively when field energy is propagated to and received by adjacent neurons. That is, synapse connections are formed and waken when gradient field energy is shared by more neurons. The main contribution of this work is that a reliable criterion is suggested to explain how energy diversity controls the creation of synapse and the enhancement of synapse connection to neurons. That is, the biophysical mechanism of synaptic connection is controlled by the energy diversity between neurons, and the synapse coupling will terminate its increase in the coupling intensity until reaching energy balance between neurons even in neural network.

Keywords Hamilton energy · Synapse function · Neuron · Synchronization

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1 Introduction

An isolated neuron can be stimulated to present a variety of firing modes [1–5]; in particular, the involvement of periodic stimulus and taming in noisy disturbance can induce stochastic resonance [6–8] and distinct regularity in firing patterns can be developed. In generic neuron models, most of the previous works considered that all external stimuli on the neuron can be approached by using equivalent transmembrane current and the excitability can be changed effectively. In fact, neurons have been developed and tamed to show specific sensing functions and thus different functional regions are formed in the brain. Auditory neurons [9–12] are sensitive to capture acoustic wave within specific frequency band. Visual neurons [13–16] are capable of discerning optical signal and illumination emitted from objects. Some neurons are able to percept the temperature effect on mode transition in neural activities, e.g., temperature has important impact on the conductance of ion channels [17–20] and enzyme activity. These deterministic neuron models can reproduce and predict the main dynamical properties of biological neurons, while the external applied external stimuli are described by simple functions, for example, periodic, chaotic and even accompanied by noise. In fact, the external optical signal across eyes and acoustic wave across ears are often composite of signals with certain frequency band [21, 22], and the nonlinear response and mode selection in electric activities are dependent on all the stimuli synchronously.

On the other hand, specific electric components are incorporated into some nonlinear circuits and they can be controlled to percept external physical signals. For example, a phototube [23–26] can be connected to neural circuits and an artificial eye can be implemented to detect the external illumination, and special films can be coated on the phototube to realize wave filtering. A piezoelectric ceramics [21, 27–31] connection to the neural circuit can enhance its sensitivity to external mechanical vibration and acoustic wave; similarly, special films can also be coated for possible selection in frequency band and noise reduction. When thermistor [32–35] is incorporated into one branch circuit of the neural circuits, a slight change in temperature can be perceived because the channel current across the thermistor can change the firing modes in electrical activities greatly. The involvement of memristor [36–40] connecting to neural circuit can enrich its dynamics dependence on initial values, and physical field effect can be estimated effectively.

In the nervous system, 80% neurons are excitatory, while the rest 20% neurons are inhibitory [41, 42]. That is, each neuron is surrounded by others when the same neurons are assembled in certain functional regions. From physical viewpoint [43], the intracellular ions and also the extracellular ions contributed to the field energy for each cell, and any external stimuli on neuron will break the balance and field energy distribution is changed by releasing/pumping ions (calcium, potassium and sodium) rapidly. Therefore, action potential is triggered to affect adjacent neurons, and firing modes can be induced continuously. In fact, the release of field energy is effective to affect the most adjacent neurons and a partial of energy can be captured and encoded by the adjacent neurons. As a result, the connection bridge is built, and synapse function is activated. In this paper, two identical generic neural circuits and neurons are waken up, and the energy pumping and propagation are activated to keep a possible balance in field energy between neurons. During the energy release and capture, the coupling channels are tamed and waken up, and synapse connections are formed. These results indicate that the formation and creation of synapse coupling result from the energy propagation and pumping between neurons than for reaching synchronization.

2 Scheme and results

Most of the nonlinear circuits can be tamed and controlled to reproduce similar firing modes observed from electrical activities in biological neurons, and further incorporation of some special electric components can enhance their additive perception functions and abilities. These electric components can estimate the effect of external magnetic field, temperature changes, acoustic wave and illumination by generating equivalent channel currents, and then the neural activities can be regulated synchronously and effectively. In Ref.[44], a two-variable neuron model is developed from a simple neural circuit composed of one capacitor, induction coil, nonlinear resistor, constant voltage source and two linear resistors, and this neural circuit can simulate all the firing modes by applying a time-varying voltage source. In fact, the forcing source can be derived from the photocurrent across the phototube, piezoelectric voltage across the piezoelectric ceramics, silicon photocell and even the additive branch circuit composed of capacitor, memristor, Josephson junction [45–48] and induction coil for capturing external electromagnetic radiation energy effectively. In Fig. 1, a simple neural circuit composed of RLC (resistors, capacitor and induction coil) is presented, and the time-varying voltage source can be replaced by a phototube or piezoelectric ceramics, which can produce continuous stimulus to excite this neural circuit.

According to the physical Kirchhoff's law, circuit equations can be obtained to bridge a connection between these physical variables, and the output voltage across the capacitor and channel current across the induction coil can be, respectively, estimated by

$$\begin{cases} C\dfrac{\mathrm{d}V_C}{\mathrm{d}t} = \dfrac{V_S - V_C}{R_S} - i_L - i_{\mathrm{NR}}; \\ L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} = V_C + E - Ri_L; \end{cases} \tag{1}$$

where V_c, i_L and i_{NR} represent the output voltage from capacitor, channel current across induction coil and channel current across nonlinear resistor, respectively. For the nonlinear resistor NR, the channel current is often approached by [44]

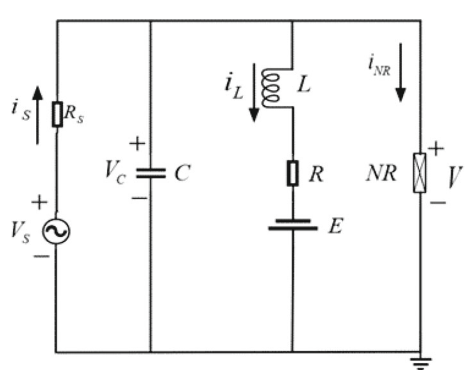

Fig. 1 Schematic diagram for a neural circuit driven by variant voltage source. *NR* is a nonlinear resistor, *C* is the capacitor, *L* represents the induction coil, *R* and R_S are the linear resistors, *E* is a constant voltage source, and V_S is the external voltage source

$$i_{\mathrm{NR}} = -\frac{1}{\rho}\left(V - \frac{1}{3}\frac{V^3}{V_0^2}\right); \tag{2}$$

where ρ, V_0 and V denote the resistance in the linear region, cutoff voltage and output voltage across the nonlinear resistor, respectively. For further nonlinear analysis, standard scale transformation [49] is applied for the variables and parameters defined in Eq. (2) and Eq. (3) as follows:

$$\begin{cases} x = \dfrac{V_C}{V_0}, y = \dfrac{\rho i_L}{V_0}, \tau = \dfrac{t}{\rho C}, a = \dfrac{E}{V_0}, b = \dfrac{R}{\rho}; \\ c = \dfrac{\rho^2 C}{L}, \xi = \dfrac{\rho}{R_S}, u_S = \dfrac{V_S}{V_0}; \end{cases} \tag{3}$$

As a consequence, a generic neuron model driven by external stimulus can be obtained by

$$\begin{cases} \dot{x} = \dfrac{\mathrm{d}x}{\mathrm{d}\tau} = x(1-\xi) - \dfrac{1}{3}x^3 - y + \xi u_S; \\ \dot{y} = \dfrac{\mathrm{d}y}{\mathrm{d}\tau} = c(x - by + a); \end{cases} \tag{4}$$

where the variables x and y describe the membrane potential and recovery variable for slow current, respectively. The excitability of the neuron will be changed by external stimulus u_s; as a result, the generic neuron can be induced to present a variety of firing modes. As is well known, the capacitor and induction coil will be injected and pumped field energy when this neural circuit is awaken by adjusting the external forcing current continuously. The physical field energy is mainly kept in the two electric components, and it is estimated by

$$W_E = \frac{1}{2}\mathrm{C}V_c^2 + \frac{1}{2}\mathrm{L}i_L^2. \tag{5}$$

By the same way, the physical energy is mapped into dimensionless Hamilton energy [50] as follows:

$$H = \frac{W_E}{\mathrm{C}V_0^2} = \frac{1}{2}x^2 + \frac{1}{2c}y^2; \tag{6}$$

Indeed, the Hamilton energy for each neuron is dependent on the firing mode, intrinsic parameters and variables completely. Therefore, neurons will contain different energy values when neurons are applied with different stimuli and/or controlled in some intrinsic parameters. That is, any slight parameter mismatch and diversity in excitability in neurons will induce distinct gradient distribution of field energy. As a result, field energy will be pumped from neurons with high energy to those neurons in lower energy; in this way, the coupling channel is awaked and built via synapse connections. During the energy pumping, the coupling intensity is regulated carefully until energy balance is activated well. For two neurons, the processing of coupling channel is controlled by

$$\begin{cases} \dot{x} = x(1-\xi) - \frac{1}{3}x^3 - y + \xi u_S + k(x' - x); \\ \dot{y} = c(x - by + a); \\ \dot{x}' = x'(1-\xi) - \frac{1}{3}x'^3 - y' + \xi u'_S + k(x - x'); \\ \dot{y}' = c(x' - by' + a); \quad \dot{k} = \mathrm{d}k/\mathrm{d}\tau = r\Theta(\Delta H - \varepsilon); \end{cases} \tag{7}$$

where the parameter ε is a tiny number, and the coupling intensity k for the synapse connection is controlled by the Heaviside function $\Theta(\Delta H)$ as follows:

$$\begin{aligned} \Delta H &= |H(x,y) - H(x',y')| \\ &= \left|\frac{1}{2}x^2 + \frac{1}{2c}y^2 - \frac{1}{2}x'^2 - \frac{1}{2c}y'^2\right|; \end{aligned} \tag{8}$$

The gain r is considered as a constant, and the coupling intensity k is increased linearly with time as $k \sim rt$ before reaching energy balance between two neurons. In fact, the coupling intensity can also be approached by $k \sim \exp(rt)$ when the gain r is very small. In fact, a fast increase in the coupling intensity continuously can enhance the energy pumping and synchronization approach. For two identical neurons, energy balance is critical for reaching complete synchronization; otherwise, energy pumping and propagation will be continued along the coupling channels.

For more neurons with energy diversity, for simplicity, chain network is discussed and the dynamics evolution can be regulated as follows:

$$\begin{cases} \dot{x}_i = x_i(1-\xi) - \frac{1}{3}x_i^3 - y_i + \xi u_S^i + k_i(x_{i+1} + x_{i-1} - 2x_i); \\ \dot{y}_i = c(x_i - by_i + a); \ \dot{k}_i = r\Theta(\Delta H_i - \varepsilon). \end{cases} \tag{9}$$

In case of the adjacent neighbor coupling and propagation, the energy difference and exchange are mainly considered between the two nearest neighbor neurons; it is estimated by

$$\begin{aligned} \Delta H_i &= |H_i(x_i,y_i) - H_{i-1}(x_{i-1},y_{i-1})| + |H_i(x_i,y_i) \\ &\quad - H_{i+1}(x_{i+1},y_{i+1})| \\ &= \left|\frac{1}{2}x_i^2 + \frac{1}{2c}y_i^2 - \frac{1}{2}x_{i-1}^2 - \frac{1}{2c}y_{i-1}^2\right| \\ &\quad + \left|\frac{1}{2}x_i^2 + \frac{1}{2c}y_i^2 - \frac{1}{2}x_{i+1}^2 - \frac{1}{2c}y_{i+1}^2\right|. \end{aligned} \tag{10}$$

For the coupling intensity k in Eq. (7) and k_i in Eq. (9), the threshold constant ε can also be selected with zero, which means that the error for energy function should be close to zero completely before terminating a further increase in the coupling intensity. In fact, the constant ε can be selected with tiny value as 0.01 and 0.1 because the synchronization approaches often need transient period when the coupling intensity is beyond the threshold. In the network, more coupling channels are built when energy is continuously pumped to adjacent neurons forwardly, and more synapse connections are created to propagate the energy to distant neurons with lower energy. That is, synapse connections result from the energy propagation among neurons and thus the neural network is completely awaken. To discern the synchronization stability between two neurons, the error function is often defined by

$$\Delta\theta = \sqrt{(x - x')^2 + (y - y')^2}. \tag{11}$$

Complete synchronization can be stabilized within certain transient period for two identical neurons controlled in Eq. (7) because the coupling intensity is further increased; as a result, the error function will be decreased to zero soon. For more neurons in the network, the statistical synchronization factor [51] is defined by using mean field theory, and it is estimated by

$$F = \frac{1}{N}\sum_{i=1}^{N} x_i\ ; \ R = \frac{<F^2> - <F>^2}{\frac{1}{N}\sum_{i=1}^{N}\left(<x_i^2> - <x_i>^2\right)}; \tag{12}$$

where the symbol $< * >$ indicates average calculation over time and N is the node number in the network. It indicates that perfect synchronization is realized when the synchronization factor R is much close to 1 and the network tends to become homogeneous, while lower value for synchronization factor R accounts for non-perfect synchronization and distinct spatial patterns can be developed in the network. According to Eq. (6), any diversity in the initial values and parameter c will induce different Hamilton energy. In fact, slight diversity in the parameter c will generate parameter mismatch, and these non-identical neurons can be controlled to reach phase synchronization than complete synchronization. To discern the difference diversity in Hamilton energy for neurons, the initial values for the variables are selected with large difference. On the other hand, diversity in excitability occurs when neurons are applied with different stimuli synchronously. In biological neural network and functional regions in the nervous system, each neuron is kept energy balance and it is affected by the electromagnetic field emitted by other neurons. Therefore, it is not necessary to activate all the synapse connections until distinct gradient distribution in energy is induced by applying external stimuli on some neurons in this region or community network.

When the parameters a, b, c, ξ for neurons and gain r for coupling channel are fixed, the evolution of field energy (and Hamilton energy) of neurons and synchronization stability can be investigated. For identical neurons, the same parameters a, b, c, ξ and external stimuli can be selected; the coupling intensity k (and k_i) is increased from zero and controlled by the gain r completely. Within certain transient period, complete synchronization can be stabilized between identical neurons. On the other hand, neurons with the same parameters ($a = 0.7$, $b = 0.8$, c = 0.1, $\xi = 0.15$) and different external stimuli show some diversities. Therefore, these neurons become non-identical and phase lock/synchronization can be induced when the coupling channels are built and adjusted for activating the synapses connection. The energy pumping is terminated when neurons are stabilized in complete synchronization. In fact, it is also interesting to investigate whether energy balance can be realized under phase synchronization and phase lock when the coupling intensity is regulated for two neurons driven by external stimuli with diversity. For each isolated neuron, the firing mode can be controlled by the angular frequency in the external stimulus; for example, the neuron will present bursting, spiking, chaotic and periodic firing patterns at $\omega = 0.002, 0.012, 0.16, 0.5$ when the parameters are fixed at $a = 0.7$, $b = 0.8$,c = 0.1, $\xi = 0.15$, $A = 6.66$ in the absence of noise. For simplicity, we consider the case for two identical neurons, and then it takes different transient periods for energy balance and pumping to activate the synapse connection completely. The increase in coupling intensity is dependent on the firing modes of the neurons until reaching complete energy balance between neurons. In Fig. 2, the self-regulation of synaptic coupling is estimated for neurons within different firing modes.

In case of bursting, spiking and periodic firing in neuron, the energy balance between neurons can be realized due to continuous diffusion in the electromagnetic field energy, and the coupling intensity for synapse connection can reach a certain saturation value, which is relative to the firing mode in the neurons. And it indicates that these neurons can be stabilized in complete synchronization in finite transient period. The coupling intensity is further increased, and synchronization becomes difficult when two neurons are excited in chaotic states, and the energy pumping is continued all the time. In Fig. 3, the error evolution of Hamilton energy for the coupled neurons is calculated during the creation of synapse connections.

It is confirmed that a slight higher step in the coupling intensity can realize energy balance between two neurons quickly when both of them are excited in bursting, spiking and even distinct periodic firing modes. However, the energy pumping is continued between two chaotic neurons and energy balance becomes difficult when the coupling intensity is further increased in the synaptic connection. For further illumination, the error function for two coupled neurons in different firing modes is calculated in Fig. 4, respectively.

The results in Fig. 4 confirmed that two neurons (bursting, spiking or periodic firing) can reach complete synchronization when the coupling intensity is increased slightly during the energy propagation, and synapse connections are waken effectively. However, complete synchronization becomes unstable, while phase lock and phase synchronization become available between two chaotic neurons under single channel coupling (Fig. 5).

It is interesting to discuss the similar case in the network, as described in Eq. (9), the collective behaviors in chain network are investigated by creating more coupling channels when energy diversity between neurons is controlled. In our study, no-flux boundary condition is applied for the network and the

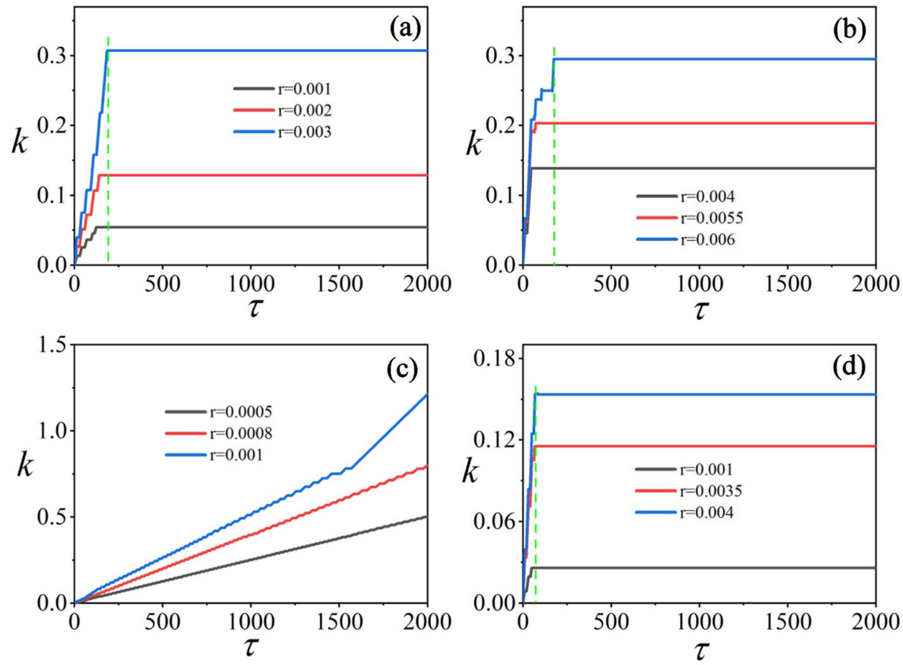

Fig. 2 Creation of synaptic connection with the increase in coupling intensity in synapse. For **a** $\omega = 0.002$, bursting neurons; **b** $\omega = 0.012$, spiking neurons; **c** $\omega = 0.16$, chaotic neurons; **d** $\omega = 0.5$, periodic firing neurons. The parameters are fixed at $a = 0.7$, $b = 0.8$, $c = 0.1$, $\xi = 0.15$, $A = 6.66$, $\varepsilon = 0.00001$, and initial values are selected as (0.2, 0.1, 0.1, 0.1, 0.0)

transient period is about 2000 time units for estimating the synchronization factors. The collective behaviors are dependent on the local kinetics and the properties of coupling channels, and the energy propagation along the synapses is critical for realizing synchronization and energy balance. Firstly, the neuron in the network is activated with bursting state, and then synchronization approach is investigated by calculating the distribution for synchronization factors when the coupling channels are activated to reach different saturation values in the coupling intensity.

With a further increase in the coupling intensity, the synchronization factor is also increased for stabilizing the collective behaviors of neural network, and these bursting neurons reach complete synchronization within finite transient period. Furthermore, the evolution of energy error between adjacent neurons defined in Eq. (10) and evolution of membrane potential of neurons are calculated in Fig. 6 by activating the synapses intensity with different steps.

Within finite transient period, these bursting neurons tend to reach complete synchronization and adjacent neurons are coupled to keep energy balance when the synapses are waken completely due to continuous energy pumping and propagation along the network. It is interesting to discuss the case when neurons in the network are excited in presenting spiking modes, and the synchronization factors are calculated in Fig. 7.

When comparing the curve in Fig. 7 with the case for bursting neurons shown in Fig. 5, the network begins to obtain a higher synchronization factor with increasing the coupling intensity when synapse connections are activated because of continuous energy propagation between spiking neurons in the network. By the same way, the evolution of spatial patterns with the membrane potentials and energy diversity between adjacent neurons is, respectively, presented in Fig. 8.

Indeed, the transient period for synchronization approach is shorten greatly when the coupling intensity is increased with higher step, and adjacent neurons reach energy balance well because continuous energy propagation tames the synapse channels well. As mentioned above, two chaotic neurons encounter some difficulty by applying bidirectional coupling with one variable (single coupling channel). It is important to discuss the similar case in the network composed of chaotic neurons, and synchronization factors in the network composed of chaotic neurons are presented in Fig. 9.

When comparing the results in Fig. 9 with the two previous cases in the network composed of bursting neurons and spiking neurons, the curve for synchronization factors shows more irregular fluctuation than

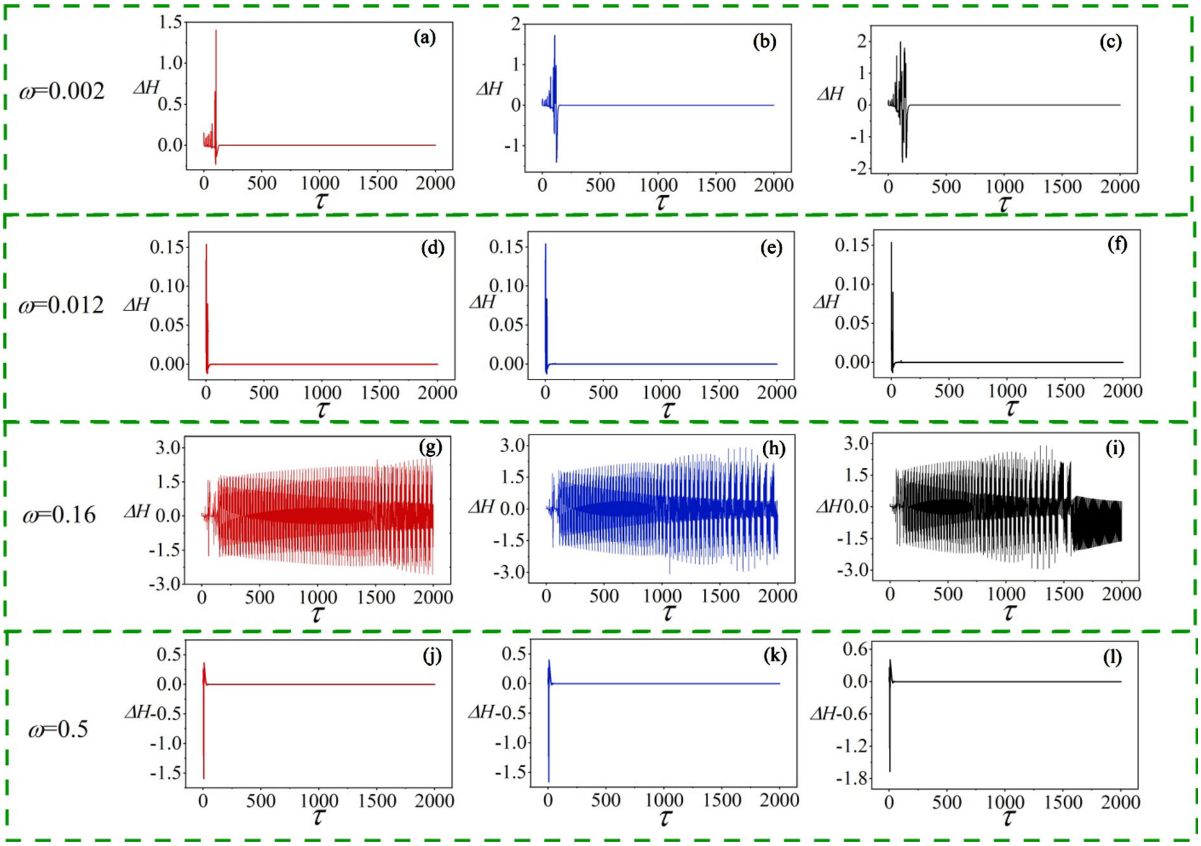

Fig. 3 Energy balance between neurons when synapse connection is created. For **a** $r = 0.001$; **b** $r = 0.002$; **c** $r = 0.003$; **d** $r = 0.004$; **e** $r = 0.0055$; **f** $r = 0.006$; **g** $r = 0.0005$; **h** $r = 0.0008$; **i** $r = 0.001$; **j** $r = 0.001$; **k** $r = 0.0035$; **l** $r = 0.004$. The parameters are fixed at $a = 0.7$, $b = 0.8$, $c = 0.1$, $\xi = 0.15$, $A = 6.66$, $\varepsilon = 0.00001$

monotonous changes with the increase in coupling intensity when synapse connections are enhanced. However, the values for synchronization factors tend to increase by applying higher gains in the coupling intensity for synaptic connections to neurons in the network. In Fig. 10, the evolution of membrane potentials and diversity in energy between adjacent neurons in chaotic firing modes are estimated as well.

For chaotic neurons, a further increase in the coupling intensity is helpful to enhance the spatial regularity, and energy balance between adjacent neurons in the network is controlled effectively when the synapse connections are waken by propagating the energy between neurons in the network when complete synchronization is not reached. Finally, the synchronization stability in the neural network is discussed when each neuron is excited to present distinct periodic firing modes, and the synchronization factors are estimated by increasing the coupling intensity carefully.

Similar to the case for bursting neurons and spiking neurons, the synchronization factors show regular but monotonous increase, and then it reaches a saturation value when the coupling intensity between neurons is further increased. It indicates that these neurons can reach complete synchronization and the neural network tends to become homogeneous and uniform greatly. Furthermore, the evolution of the network is presented by showing the spatial patterns for membrane potential and energy diversity between adjacent neurons in Fig. 12.

The network develops its collective behaviors under the creation of synapse connections when energy is propagated between adjacent neurons, and synchronization stability is controlled with a further increase in the coupling intensity for these periodic neurons. Due to fast and effective energy propagation,

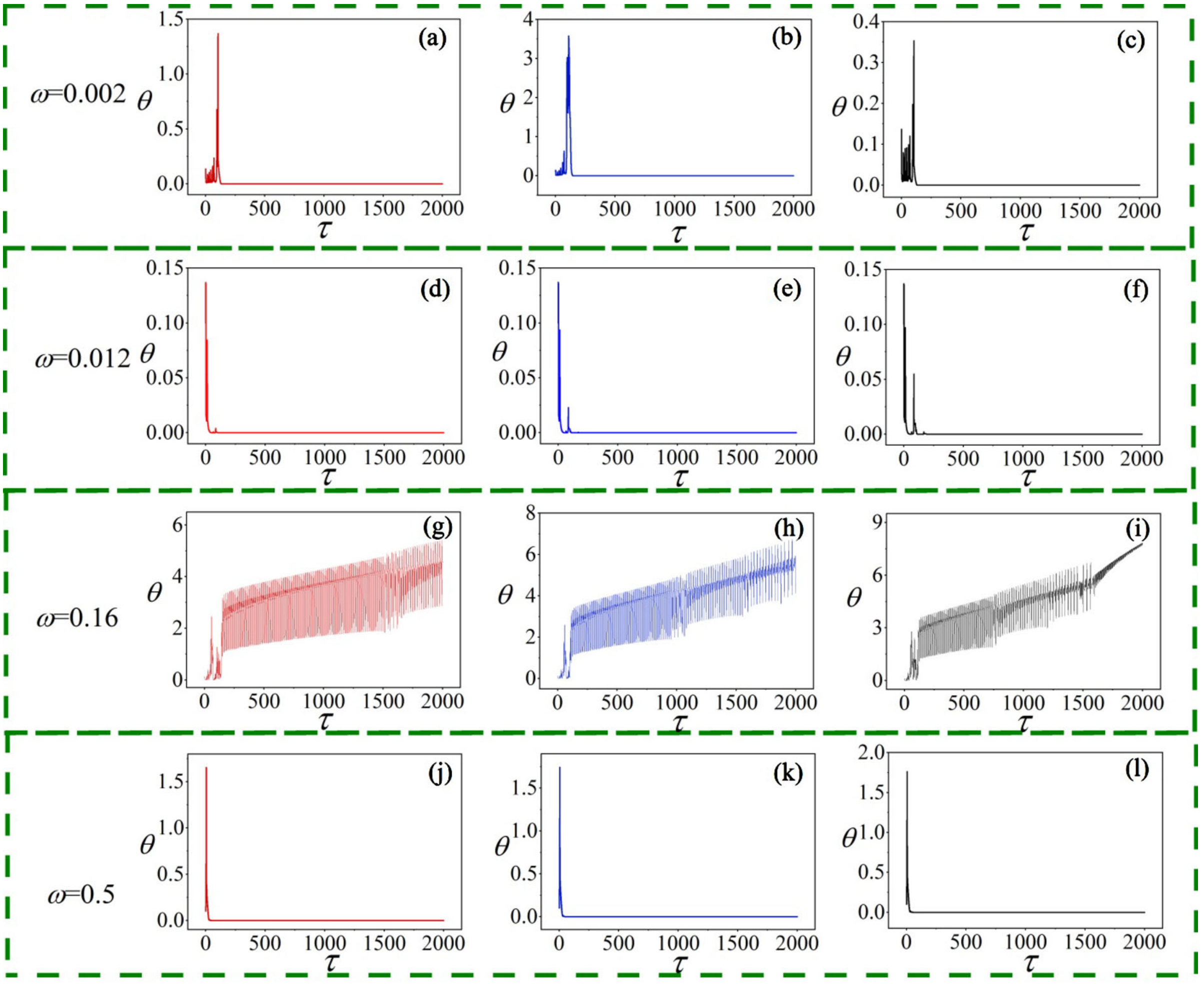

Fig. 4 Evolution of error functions for two coupled neurons with different gains in the coupling intensity. For **a** r= 0.001; **b**r = 0.002; **c** r = 0.003; **d** r = 0.004; **e** r = 0.0055; **f** r = 0.006; **g** r = 0.0005; **h** r = 0.0008; **i** r = 0.001; **j** r = 0.001; **k** r = 0.0035; **l** r = 0.004. The parameters are fixed at a = 0.7, b = 0.8, c = 0.1, ξ = 0.15, A = 6.66, ε = 0.00001

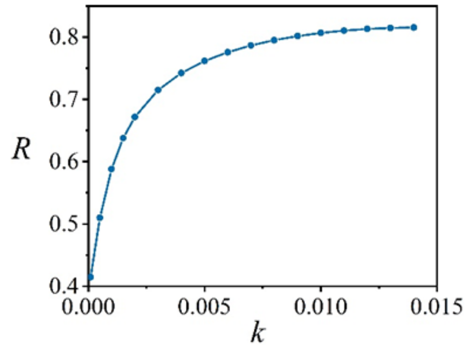

Fig. 5 Synchronization factors in the network composed of bursting neurons. The parameters are fixed at a = 0.7, b = 0.8, c = 0.4, ξ = 0.15, A = 6.66, ω = 0.002, ε = 0.00001, and all the neurons are selected with the same initial value (0.2, 0.1)

the coupling channels are built and synapse connections are created for reaching energy balance between neurons; as a result, these neurons in periodic firing modes also reach complete synchronization effectively.

From physical viewpoint, neurons in each cluster network and community used to keep energy balance and synapse connections are created to decrease energy diversity, and any external stimuli on a few neurons will break the energy balance because of external energy injection. As a result, the absorbed energy will be shared and propagated to other neurons by creating more synapse connections and some coupling channels are tamed with higher coupling intensity as well. As claimed in Ref. [52], the

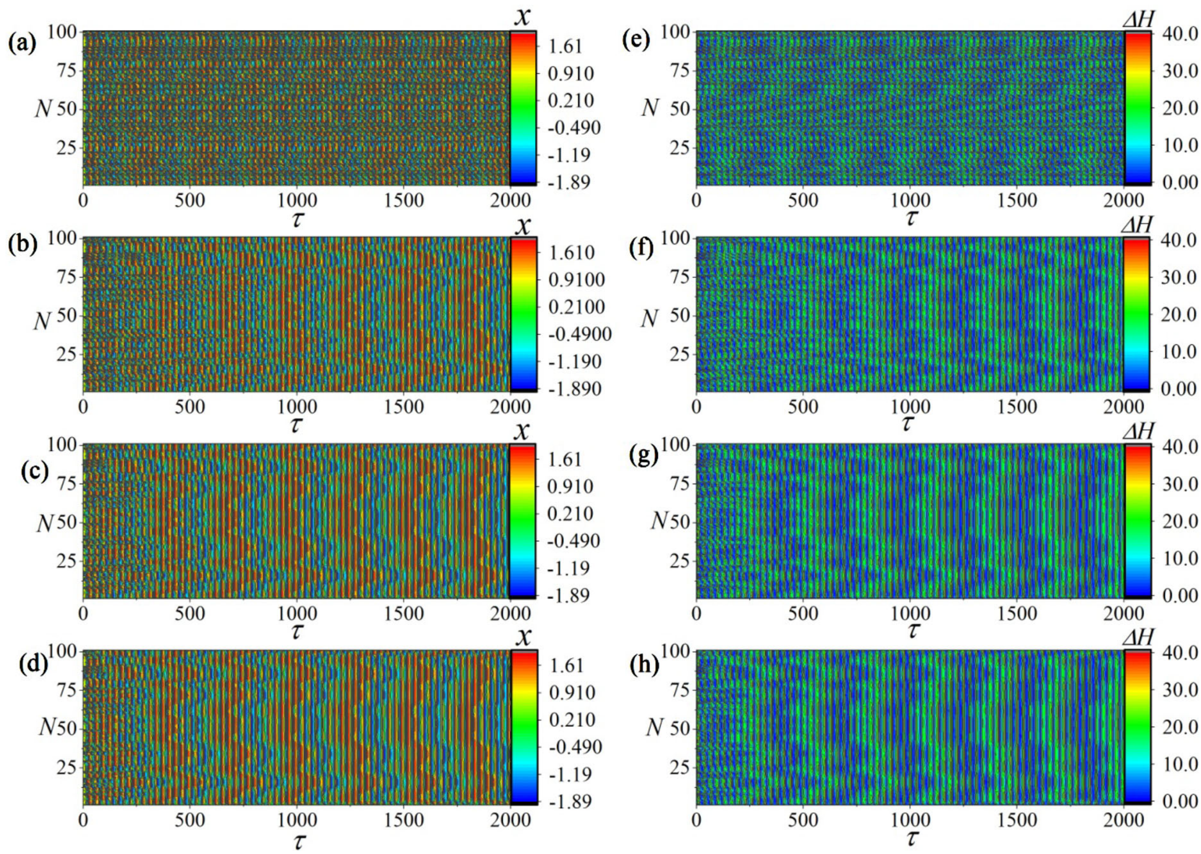

Fig. 6 Evolution of membrane potential and energy diversity between adjacent neurons of the chain network. For **a**, **e** $r = 0.0001$; **b**, **f** $r = 0.004$; **c**, **g**$r = 0.01$; **d**, **h**$r = 0.014$, and the parameters are fixed at $a = 0.7$, $b = 0.8$, c = 0.4, $\xi = 0.15$, $A = 6.66$, $\omega = 0.002$, $\varepsilon = 0.00001$, initial values (0.2, 0.1, 0.0) for each neuron

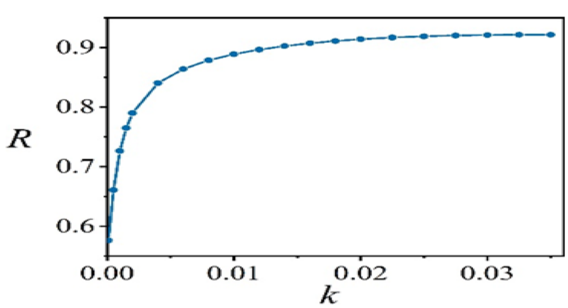

Fig. 7 Synchronization factors in the network composed of spiking neurons. The parameters are fixed at $a = 0.7$, $b = 0.8$, c = 0.4, $\xi = 0.15$, $A = 6.66$, $\omega = 0.012$, $\varepsilon = 0.00001$, and all the neurons are selected with the same initial value (0.2, 0.1)

formation and creation of autapse result from the injury of axon in the neuron, and then auxiliary loop via synapse is guided to propagate and correct the blocked signal propagation in some interneurons. For an isolated neuron driven by autapse, adaptive selection and regulation of synaptic intensity and time delay will induce energy release or pumping effectively by adjusting the firing modes because the intrinsic Hamilton energy is much dependent on the firing modes of neuron. When more autapses are created in neurons in the network, local distribution of autapses [53–60] will regulate the collective behaviors of neural networks by developing continuous pulses or wave fronts. As a result, energy distribution is controlled completely. That is, the creation of autapse and electric synapses confirms the self-adaption of biological neurons, and thus they can behave the most suitable firing modes in electric activities. Each neuron holds certain field energy, and it is affected by other neurons via field coupling uniformly. When energy is pumped and propagated directly to any neurons, the connection channel is open and synapse connections are enhanced during continuous pumping in energy between neurons in the network. Where the

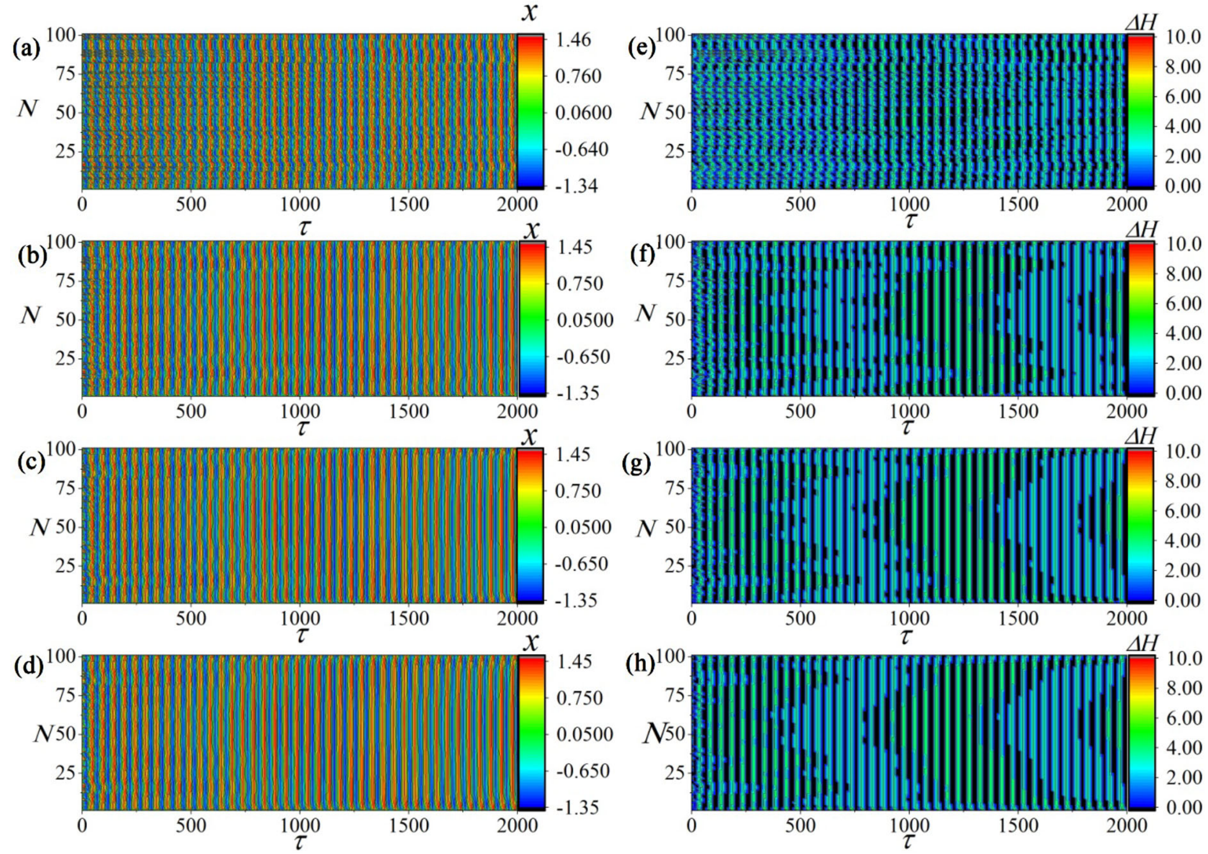

Fig. 8 Evolution of membrane potential and energy diversity between adjacent neurons of the chain network. For **a**, **e** $r = 0.0001$; **b**, **f** $r = 0.01$; **c**, **g** $r = 0.02$; **d**, **h** $r = 0.03$, and the parameters are fixed at $a = 0.7$, $b = 0.8$, c = 0.4, $\xi = 0.15$, $A = 6.66$, $\omega = 0.012$, $\varepsilon = 0.00001$, initial values (0.2, 0.1, 0.0) for each neuron

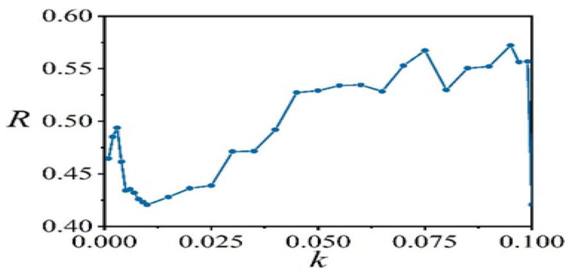

Fig. 9 Synchronization factors in the network composed of chaotic neurons. The parameters are fixed at $a = 0.7$, $b = 0.8$, c = 0.4, $\xi = 0.15$, $A = 6.66$, $\omega = 0.16$, $\varepsilon = 0.00001$, and all the neurons are selected with the same initial value (0.2, 0.1)

sun shines, there is life. The world has no roads, but only man walk more and the roads appear. It is the energy sharing that we cooperate and compete with each other; therefore, we all bridge to the world with different ways. For neurons, these synaptic connections play as roads when signals and energy are propagated. As a result, continuous exchanges of energy between neurons are effective to tame and develop synaptic connections for signal propagation and these connections or links behave like roads. Synapses are activated for keeping energy balance among neurons when any of them are stimulated by external stimuli, which induces instability and balance in the field energy of the neural network.

3 Open problems

In realistic nervous systems, long-range connections with certain probability can also be awaken and created between neurons besides the nearest neighbor connection, and some neurons in the same functional region can be connected in cluster network as well.

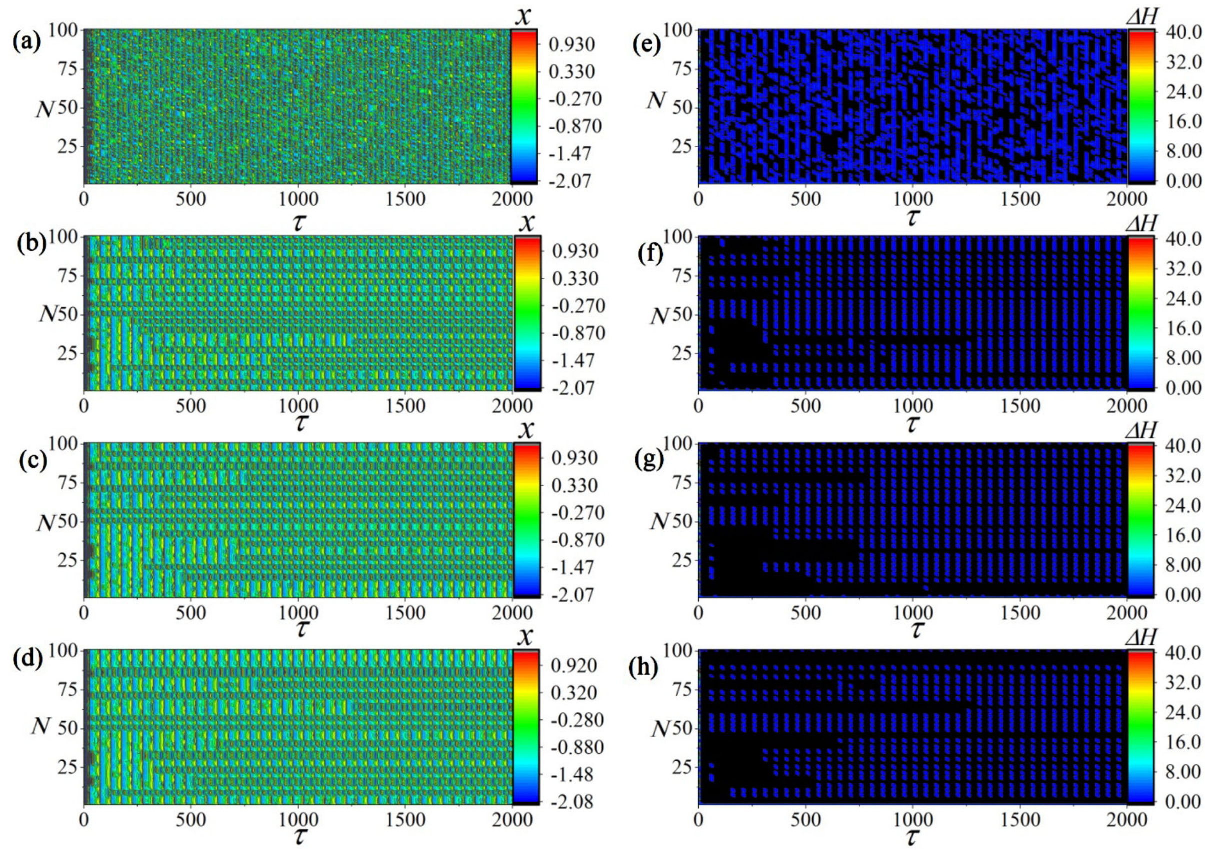

Fig. 10 Evolution of membrane potential and energy diversity between adjacent neurons of the chain network. For **a**, **e** $r = 0.003$; **b**, **f** $r = 0.055$; **c**, **g** $r = 0.075$; **d**, **h** $r = 0.095$, and the parameters are fixed at $a = 0.7$, $b = 0.8$, $c = 0.4$, $\xi = 0.15$, $A = 6.66$, $\omega = 0.16$, $\varepsilon = 0.00001$, initial values (0.2, 0.1, 0.0) for each neuron

That is, small-world connection is more effective to propagate and share the energy among neurons in the network, and some synapses are activated to connect other distant neurons when energy is propagated along links in long range. Similar to the above discussion, small-world network can also be tamed and developed by creating synapse connections in long range under certain probability along the links for energy pumping and propagation, and the coupling intensity along each link (bridge) can be controlled and increased with certain step. When energy is dispersed between neurons, all the neurons tend to keep energy balance and then reach possible synchronization stability by keeping chemical or electric synapses connection with appropriate intensity and channel current. On the contrary, when energy is assembled and gathered to certain neurons, the energy is pumped to certain communities in the network and more synapse connections will be inhibited, and desynchronization

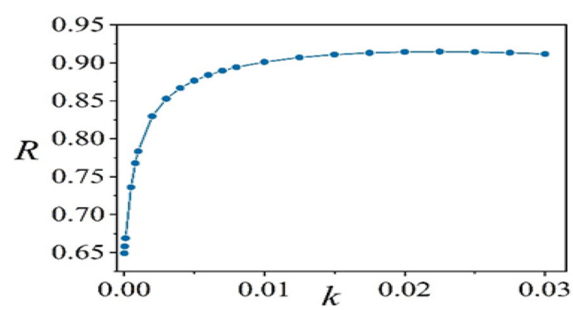

Fig. 11 Synchronization factors in the network composed of neurons within periodic firing. The parameters are fixed at $a = 0.7$, $b = 0.8$, $c = 0.4$, $\xi = 0.15$, $A = 6.66$, $\omega = 0.5$, $\varepsilon = 0.00001$, and all the neurons are selected with the same initial value (0.2, 0.1)

occurs in the network accompanied with distinct spatial patterns. For example, local poisoning in some ion channels will block the energy propagation and balance between neurons, and synchronization

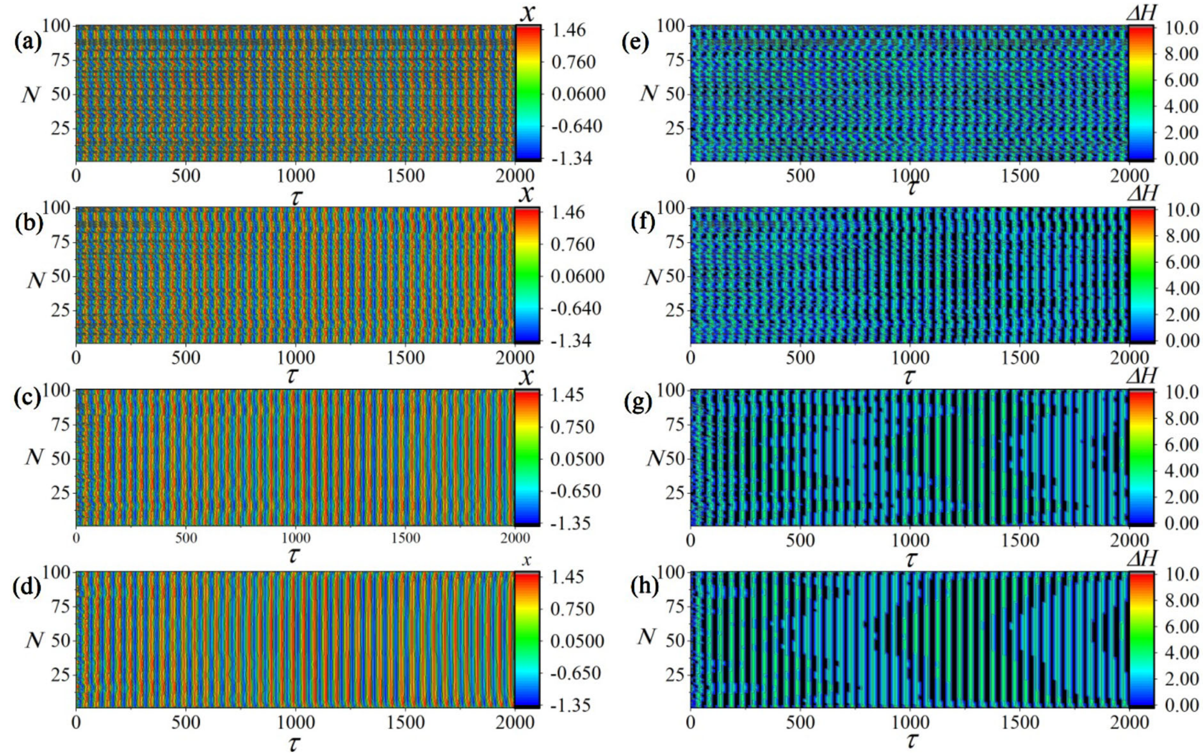

Fig. 12 Evolution of membrane potential and energy diversity between adjacent neurons of the chain network. For **a**, **e** $r = 0.0001$; **b**, **f** $r = 0.001$; **c**, **g** $r = 0.01$; **d**, **h** $r = 0.03$, and the parameters are fixed at $a = 0.7$, $b = 0.8$, $c = 0.4$, $\xi = 0.15$, $A = 6.66$, $\omega = 0.5$, $\varepsilon = 0.00001$, initial values (0.2, 0.1, 0.0) for each neuron

realization becomes difficult in the neural network. From dynamical viewpoint, the coupling intensity along the synapse connections/links will be decreased adaptively until synchronization stability is corrupted completely. The neural networks disable their synapse connections, and energy is collected and piled to certain neurons as follows:

$$\begin{cases} \dot{x}_i = x_i(1-\xi) - \frac{1}{3}x_i^3 - y_i + \xi u_S^i + k_i(x_{i+1} + x_{i-1} - 2x_i); \\ \dot{y}_i = c(x_i - by_i + a);\ \dot{k}_i = -r\Theta(\varepsilon - \Delta H_i). \end{cases} \tag{13}$$

That is, the coupling intensity is continuously decreased and complete synchronization is corrupted; as a result, some synapse connections are suppressed. The similar discussion can also be used for the instability in small-world network by pumping energy to certain neurons, and more links in the network are cut off to suppress the synapse connections as follows:

$$\begin{cases} \dot{x}_i = x_i(1-\xi) - \frac{1}{3}x_i^3 - y_i + \xi u_S^i + k_i \sum_{j=1, j\neq i}^{N} \sigma_{ij}(x_j - x_i); \\ \dot{y}_i = c(x_i - by_i + a);\ \dot{k}_i = -r\Theta(\varepsilon - \Delta H_i); \end{cases} \tag{14}$$

where the connection matrix σ_{ij} can be carefully adjusted to describe the scale for long-range connection probability, the subscript *ij* denotes the node position in the network, $\sigma_{ij} = 1$ when the node *i* connects to the node *j*, otherwise, $\sigma_{ij} = 0$. The gain k_i in the coupling intensity for the *ith* link can be selected certain vales beyond the threshold for reaching synchronization; as a result, continuous energy pumping means the breaking off along this link and thus synapse connection is suppressed. That is, when the energy for all the neurons is pumped to a few of neurons in the network, the synapse connections will be terminated and bidirectional coupling along the synapses is switched off. In a noisy condition and in the presence of electromagnetic radiation, the formation and creation of synapses can also be confirmed when energy is pumped from neurons stimulated by external current or field to other neurons in the network. Another thing is that we just discussed the case for creating electric synapses connection, and neurons are coupled with gap junction. In fact, similar study can be applied for creating chemical synapses connection between neurons, and neurons in networks with different topological structure, boundary conditions and noisy disturbance can also be considered. The increase in coupling intensity can also be selected with other different functions, e.g., exponential increase or intermittent increase as well.

4 Conclusions

From dynamical viewpoint, neurons and nonlinear oscillators in the network can be connected via biophysical (electric and chemical) synapses and artificial synapse, and adaptive adjustment in the coupling intensity along these links will stabilize synchronization, and also energy balance between neurons/oscillators can be realized effectively. It is ever believed that coupling channels and bridge connections should be built before energy exchange and pumping between chaotic oscillators. The synapses and dendrites of neurons are flexible, and continuous energy propagation and exchange can drive them to build possible links and thus the synapse connections are activated. Furthermore, channel current is induced and it becomes more effective to realize synchronization and energy balance between neurons. That is, when ions and charges are propagated from presynaptic terminal of neuron to postsynaptic terminal of another neuron, the distribution of electromagnetic field for the neurons is changed, and then field energy is exchanged; as a result, synapse connection is formed and switched on. In this paper, a simple generic neural circuit is used to discuss the release of synapse function when field energy is pumped between neural circuits. The coupling intensity is increased with time during the energy propagation and the coupling channel is switched on. When neurons keep energy balance, the coupling intensity stops its increase and the synapse connection is activated completely. Complete synchronization become available for two identical neurons, and neural network composed of identical neurons with bursting, bursting and even periodic firing modes can also be controlled to become synchronous and homogeneous when synapse connection is further enhanced by increasing the coupling intensity. However, complete synchronization becomes difficult for chaotic neurons when the synapse connections and coupling are further enhanced via a single channel between neurons. These results inform that the creation of synapse coupling results from the diversity in field energy in neurons, and continuous energy pumping will activate the synapse function by building appropriate connections, which is more effective to regulate the energy pumping and propagation. When all neurons are coupled with higher intensity, they will be controlled to reach balance in energy and complete synchronization, and the same firing modes are controlled effectively. That is, the energy flow controls the creation and connection of synapses between neurons. In addition, similar criterion can be considered for exploring the creation and enhancement of chemical synapses connected to neurons and networks as well.

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Data availability All data generated or analyzed during this study are included in this article.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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