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ORIGINAL PAPER

Adaptive neural control of state-constrained MIMO nonlinear systems with unmodeled dynamics

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Abstract In this paper, a robust adaptive neural control scheme is proposed for a class of multi-input multi-output nonlinear systems in pure-feedback form with unmodeled dynamics and full state constraints. Radial basis function neural networks are employed to approximate and compensate for the unknown nonlinear continuous functions. By introducing nonlinear symmetric mapping, the full state-constrained tracking control problem of the multi-input multi-output pure-feedback system is transformed into a novel equivalent unconstrained one. For the transformed systems, a dynamic surface control method is applied to remove the difficulties for the multiple explosion of complexity problem. The use of Nussbaum-type function removes the need for any assumption on the function of control gain. By combining variable separation technique and the function's monotonously increasing property, the restrictive assumption of the dynamic disturbances caused by unmodeled dynamics is relaxed. One advantage is that only one adjustable parameter is used in the controller design. It is proved that all the closed-loop signals remain semi-globally uniformly ultimately bounded with good tracking performance, while the system states never violate the constraints.

Keywords Nonlinear MIMO systems · Adaptive neural control · Full state constraints · Dynamic surface control · Unmodeled dynamics

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1 Introduction

In recent decades, adaptive control of nonlinear systems has been a rapidly developed research area, and many significant works have been established [1]. However, the systems to which adaptive control schemes can be applied may contain the so-called matching condition, which, unfortunately, are quite restrictive. To remove this condition, adaptive backstepping, as a recursive and systematic technique, was first proposed in [2] to obtain asymptotically tracking and globally stable controller, in which the control input was not matched with dynamic nonlinearities. Furthermore, intensive developments have been achieved on adaptive control of nonlinear systems in strict-feedback or lower-triangular form via backstepping, for instance, [3] for output-feedback control, [4] for sampled-data control and [5] for quantized control. Despite the efforts, the applicability of adaptive backstepping control schemes is subject to the repeated differentiations on virtual controllers at each recursive step, which may cause explosion of complexity problem. The work of D. Swaroop [6] initiated a surge of interest in providing the ease in nonlinear control design and implementation. By introducing a first-order filter at each step of recursive procedure, dynamic surface control (DSC) allows a design to prevent the problem of "explosion of terms" without model differentiation. For matters mentioned above, adaptive neural tracking control problem of multi-input multi-output (MIMO) nonlinear systems subject to full state constraints and unmodeled dynamics is considered in this paper.

In most applications, due to the practical limitations of physical devices or the requirements on control performance and safety operation, many practical systems are always subject to output/state constraints or input constraints [7,8]. Violation of such constraints may lead to instability, performance degradation, or danger. To address this issue, some classical algorithms are employed to handle the problem of constraints, such as model predictive control, extremum-seeking control and nonlinear reference governor. Recently, barrier Lyapunov function (BLF) method has become a common tool to deal with output or state constraints since it was first proposed to design asymptotically stable controllers for a feedback linearizable system with constraints [9]. The main idea of this methodology is that the control Lyapunov function will turn to infinity when the signals approach the constraints, imposing hard bound on the associated signals. Based on BLF and adaptive backstepping, a great deal of results have been obtained to control uncertain nonlinear systems with constraints([10–15], to just name a few). Taking the asymmetric state constraints into account in [16], the authors constructed a piecewise BLF, which made the stability analysis complicated in order to avoid discontinuity. In [17], a tan-type BLF was developed to cope with the full state constraints for high-order uncertain nonlinear systems, and this BLF can be extended to address free-constrained systems. However, such a form is limited to the symmetric constraints. Note that the abovementioned schemes require the so-called feasibility condition on virtual controllers with BLF, which poses complexity and difficulty for the implementation of stable control. To relax such restriction, a nonlinear state-dependent function was first introduced in [18], with which the feasibility condition was completely circumvented. This method has also been extended to practical tracking control [19], adaptive DSC [20]. Recently, based on nonlinear mapping (NM), an output-constrained control was studied in [21] for a class of nonlinear strict-feedback systems, and the constraint was guaranteed as long as the boundedness of the transformed unconstrained system was ensured. However, the uncertain terms in the related literatures [10–16,18–21] were compelled to be parametrically decomposable, namely the nonlinear functions in system dynamics were either linearly parameterized or assumed to extract known functions multiplying unknown constants.

On the other hand, Neural networks (NNs) [22] or Fuzzy Logic Systems (FLSs) [23] with the inherent capabilities in function approximation were claimed to model unknown functions merged in uncertain nonlinear systems. With the aid of NNs/FLSs and BLFs, considerable efforts on output tracking, event-triggered control, finite-time control and proportional-integral control have been given to prevent the violation of full state constraints for various uncertain nonlinear systems with completely unknown functions. In [24], the adaptive fuzzy control scheme was developed, which confirmed the asymptotic tracking of the nonlinear nonstrict-feedback systems. However, those studies are focused on single-input single-output (SISO) nonlinear systems. An adaptive fuzzy control was investigated in [25] for MIMO systems in the nested multiple coupling structure by constructing BLFs in the backstepping design, while in [26], the adaptive control of uncertain MIMO nonlinear block-triangular systems was concerned by fusing neural function approximation with novel integral BLFs. Despite the efforts, there still remains a significant issue needs to be addressed in most NNs/FLSs-based control schemes, where the number of estimated parameters to be tuned normally gets larger as the neural nodes becomes higher [25,26], dramatically leading to a computational cost for learning. Approximation-based backstepping control schemes were proposed by updating norms of the unknown fuzzy or neural weight vector [27–30], in which only one adaptive parameter was involved in each step of the recursive design, substantially reducing the online computational burden. One parameter-based adaptive control strategies have been investigated in [20,31,32], which derive from the consistency structure of the adaptation law in backstepping/DSC analysis. It is noted that, compared with SISO nonlinear systems, an effective solution for eliminating adaptive parameters for MIMO nonlinear systems with completely unknown functions is much more demanding and complicated. Furthermore, when the purpose of preventing full state constraints violation is incorporated, the control design becomes more challenging.

The problem of unmodeled dynamics resides in almost all real systems because of the external disturbance, measurement noise and modeling errors. Their existences frequently make the system performances degradation, in addition to the instability of controlled system. Hence, the study on the unmodeled dynamics problem is of great significance and a series of valuable results were reported in [33–41]. In early days, by adaptive backstepping algorithms, several classic control frameworks were established in [33,34] for parametric strict-feedback nonlinear systems with unmodeled dynamics. Further, to handle nonlinear systems containing unknown functions, in [35,36], adaptive backstepping design approach was incorporated with fuzzy or neural networks compensators for SISO strict-feedback nonlinear systems with unmodeled dynamics. In [37,38], two robust adaptive control schemes formulated to achieve semiglobal tracking stability were proposed for nonaffine pure-feedback nonlinear systems in presence of unmodeled dynamics. In an attempt to relax the system structure, the works in [39] and [40] developed two adaptive approximation-based strategies for SISO nonstrict-feedback systems with unmodeled dynamics, and their extension to interconnected systems with unmodeled dynamics in nonstrict-feedback form was also discussed in [41]. Even though tremendous works have been presented for versatile nonlinear systems with unmodeled dynamics, the limited control algorithm results in a common assumption that the upper bound function of the dynamic disturbance with respect to unmodeled dynamics should be of lower-triangular structure, i.e., $\Delta_i(z, x, t) \leq \varphi_i(|\bar{x}_i|) + \phi_i(\|z\|)$, $\varphi_i(|\bar{x}_i|)(\bar{x}_i = [x_1, \ldots, x_i]^T)$ only contains previous i state variables. This presupposition is crucial in practice. In [42], adaptive fuzzy tracking control was addressed for a class of SISO nonlinear systems with unmodeled dynamics, in which the upper bound function may be a function of whole state variables, but it has to be a known smooth function. When the upper bound function appears as an unknown function containing the whole state variables of the full state-constrained MIMO systems, the existing approximation-based recursive control strategies may be invalidated.

Motivated by the above statements, this paper investigates a single-adaptive-parameter-based DSC scheme for a class of uncertain MIMO nonlinear systems with full state constraints and unmodeled dynamics. Compared with output constraints, full state constraints are more difficult to conduct. Based on the transformed function [18–20] or nonlinear mapping [21,38,43] method, only one state needs to be transformed into the new variable for output constraints, while full state constraints problem is considered to transform the entire states into the new free-constrained ones, which made the controller design more complicated. On the basis of the monotonously increasing characteristics of the upper bound function of dynamic disturbances, a variable separation approach is applied to overcome the difficulties in strong coupled terms. Nussbaum functions are used to handle the unknown control directions and the problem of unmodeled dynamics is tackled by introducing a dynamic signal. Through nonlinear symmetric mapping and Lyapunov stability analysis, it is proved that the full state constraints can never be violated. The main contributions of this paper lie in:

(1) Compared with the existing works [9–16] on the control of state-constrained nonlinear systems which mainly utilize BLF technique, while in this note, by applying nonlinear symmetric mapping, the full state-constrained MIMO nonlinear system is transformed into a novel pure-feedback MIMO system without any constraints. Thus, the traditional Lyapunov functions can directly be employed and the controllers need not be redesigned.
(2) By developing the novel algorithm, the assumption can be classified as the least restrictive imposed condition available for dynamic disturbance caused by unmodeled dynamics, of which the upper bound function can cover the whole states of nonlinear MIMO systems.
(3) Based on the maximal norm of the weight vector estimation technique, the proposed design only needs one adaptive parameter to construct the neural controller for the entire MIMO systems, substantially alleviating the computation burden.

The plan for this paper is as follows. Section 2 gives the problem statement and the key nonlinear symmetric mapping techniques. Following that, the adaptive neural dynamic surface controller and stability analysis for uncertain state-constrained MIMO nonlinear systems subject to unmodeled dynamics are presented in Sect. 3. A numerical simulation example is studied in Sect. 4. Lastly, we draw the conclusion of this paper.

2 Problem statement and preliminaries

Consider the following uncertain MIMO nonlinear systems with unmodeled dynamics:

$$\begin{cases} \dot{z} = q(z, x_{1,1}, t) \\ \dot{x}_{i,j} = f_{i,j}(\bar{x}_{i-1}, \bar{x}_{i,j+1}) + x_{i,j+1} + \Delta_{i,j}(z, X, t) \\ \dot{x}_{i,n_i} = f_{i,n_i}(\bar{x}_i) + g_{i,n_i}(\bar{x}_i)u_i + \Delta_{i,n_i}(z, X, t) \\ y_i = x_{i,1} \end{cases} \tag{1}$$

where $i = 1, \ldots, N$, $j = 1, \ldots, n_i - 1$, $X = [x_1^{\mathrm{T}}, \ldots, x_N^{\mathrm{T}}]^{\mathrm{T}}$ with $x_i = [x_{i,1}, \ldots, x_{i,n_i}]^{\mathrm{T}} \in R^{n_i}$, $\bar{x}_i = [x_1^{\mathrm{T}}, \ldots, x_i^{\mathrm{T}}]^{\mathrm{T}} \in R^i$, and $\bar{x}_{i,j} = [x_{i,1}, \ldots, x_{i,j}]^{\mathrm{T}} \in R^j$ are the system state vectors. u_i and $y_i = x_{i,1}$ denote the system control inputs and outputs, respectively. $\Delta_{i,1}(z, X, t)$, $\Delta_{i,2}(z, X, t), \ldots, \Delta_{i,n_i}(z, X, t)$ are the unknown dynamic disturbances with $z \in R^{n_0}$ being the unmodeled dynamics. $f_{i,1}(\cdot)$, $f_{i,2}(\cdot) \ldots, f_{i,n_i}(\cdot)$ and $g_{1,n_1}(\cdot), g_{2,n_2}(\cdot), \ldots, g_{N,n_N}(\cdot)$ are totally unknown smooth functions and control coefficients. All the states $x_{i,j}$ are subject to the open set $\Omega_{x_{i,j}} := \{x_{i,j} : |x_{i,j}| < k_{b_{i,j}}\}$, where $k_{b_{i,j}}$ are known positive scalars due to performance requirements and physical constraints.

Control objective. Given the reference signal y_{di} for system (1) under the aforementioned symmetric state constraints, design adaptive neural controllers such that the tracking errors $y_i - y_{di}$ converge to some small regions, and all the closed-loop signals are semi-globally uniformly ultimately bounded (SGUUB) with full state constraints being strictly obeyed.

Definition 1 The unmodeled dynamics z is exponentially input-to-state practically stable (exp-ISpS) provided that there exist a Lyapunov function $V(z)$ and functions $\bar{\alpha}_1, \bar{\alpha}_2$ of class $\mathcal{K}_\infty$ such that

$$\bar{\alpha}_1(\|z\|) \le V(z) \le \bar{\alpha}_2(\|z\|) \tag{2}$$

and there exist two positive constants c, d and a class $\mathcal{K}_\infty$ function γ such that

$$\frac{\partial V(z)}{\partial z} q(z, x_{1,1}, t) \le -cV(z) + \gamma(|x_{1,1}|) + d \tag{3}$$

where c and d are known positive constants, γ is a known $\mathcal{K}_\infty$ function.

Assumption 1 The unmodeled dynamics z is exponentially input-to-state practically stable (exp-ISpS).

Assumption 2 The uncertain dynamic disturbances $\Delta_{i,j}(z, X, t)$ are upper bounded such that $|\Delta_{i,j}(z, X, t)| \le \varphi_{i,j}(\|X\|) + \phi_{i,j}(\|z\|)$, where $\varphi_{i,j}(\cdot)$ are unknown strictly increasing smooth functions and $\phi_{i,j}(\cdot)$ are nondecreasing continuous functions.

Remark 1 Let $\bar{a} = \max\{a_{m,l}, b, c, l = 1, \ldots, n_m, m = 1, \ldots, N\}$. From the increasing property of $\varphi_{i,j}(\cdot)$ in Assumption 2, we have $\varphi_{i,j}\left(\sum_{m=1}^{N}\sum_{l=1}^{n_m} a_{m,l} + b + c\right) \le \varphi_{i,j}(p_0\bar{a}) \le \sum_{m=1}^{N}\sum_{l=1}^{n_m} \varphi_{i,j}(p_0 a_{m,l}) + \varphi_{i,j}(p_0 b) + \varphi_{i,j}(p_0 c)$ with $p_0 = \sum_{m=1}^{N} n_m + 2$. Note that $\varphi_{i,j}(\cdot)$ is a smooth function, and $\varphi_{i,j}(0) = 0$; therefore, there exists a smooth function $q_{i,j}(s)$ such that $\varphi_{i,j}(s) = s q_{i,j}(s)$, which leads to

$$\begin{aligned} &\varphi_{i,j}\left(\sum_{m=1}^{N}\sum_{l=1}^{n_m} a_{m,l} + b + c\right) \\ &\quad \le \sum_{m=1}^{N}\sum_{l=1}^{n_m} p_0 a_{m,l} q_{i,j}\left(p_0 a_{m,l}\right) + \varphi_{i,j}\left(p_0 b\right) \\ &\quad +\varphi_{i,j}\left(p_0 c\right) \end{aligned}$$

Assumption 3 It holds that $|y_{di}| \le d_i^* < k_{b_{i,1}}$ with d_i^* being a known positive constant. The desired trajectory vector $[y_{di}, \dot{y}_{di}, \ddot{y}_{di}]^{\mathrm{T}}$ is available, continuous and satisfies $\Omega_{di} = \{[y_{di}, \dot{y}_{di}, \ddot{y}_{di}]^{\mathrm{T}} : y_{di}^2 + \dot{y}_{di}^2 + \ddot{y}_{di}^2 \le B_{i0}\} \in R^3$, where $B_{i0} > 0, i = 1, \ldots, N$ are known constants.

To implement the stable controller design, the following lemmas and concepts are needed.

The continuous function $N(\xi)$ is defined as a Nussbaum-type function when it carries the properties $\lim_{s\to+\infty} \sup \frac{1}{s}\int_0^s N(\xi)\, d\xi = +\infty$ and $\lim_{s\to-\infty} \sup \frac{1}{s}\int_0^s N(\xi)\, d\xi = -\infty$. Commonly used Nussbaum-type functions satisfying the above conditions are $\xi^2 \sin(\xi)$, $e^{\xi^2}\cos((\pi/2)\xi)$ and $\xi^2\cos(\xi)$ as stated in [44]. In this paper, we chose $e^{\xi^2}\cos((\pi/2)\xi)$ to handle the unknown control gains.

Lemma 1 *[43] Let $V(t) \ge 0$ and $\xi_i(t)$ be smooth functions defined on $[0, t_f)$, and consider an even Nussbaum-type function $N(\xi_i)$. If the following inequality holds*

$$V(t) \le a + e^{-bt}\sum_{i=1}^{N}\int_0^t \left(g_i\left(\bar{x}_i(\tau)\right)N(\xi_i)+1\right)\dot{\xi}_i e^{b\tau}d\tau \quad (4)$$

where $a > 0$ and $b > 0$ are constants, and $g_i(\bar{x}_i(t))$ is a time-varying parameter which takes values in the unknown closed intervals $D := [d^-, d^+]$, with $0 \notin D$, then $V(t)$, $\sum_{i=1}^{N}\int_0^t \left(g_i\left(\bar{x}_i(\tau)\right)N(\xi_i)+1\right)\dot{\xi}_i e^{b\tau}d\tau$, $\xi_i(t)$ must be bounded on $[0, t_f)$.

Lemma 2 *[45] The following inequality holds for any vectors $a, b \in R^n$:*

$$a^T b \le \frac{\varepsilon^m \|a\|^m}{m} + \frac{\|b\|^n}{n\varepsilon^n}$$

where $\varepsilon > 0$, $m > 1$, $n > 1$ and $(m-1)(n-1) = 1$.

Lemma 3 *[33] Suppose there exists an exp-ISpS Lyapunov function V for a system $\dot{z} = q(z, x_{1,1}, t)$, i.e., (2) and (3) hold, then, for any initial conditions $z_0 = z(t_0)$, $v_0 = v(t_0)$ with any initial instant $t_0 > 0$, any constant $\bar{c} \in (0, c)$, for any continuous function $\bar{\gamma}\left(|x_{1,1}|\right) \ge \gamma\left(|x_{1,1}|\right)$, there exist a finite time $T_0 = \max\{0, \log\left(V(z_0)/v_0\right)/(c-\bar{c})\} \ge 0$, a nonnegative function $D(t_0, t)$, and a signal described as follows:*

$$\dot{v}(t) = -\bar{c}v(t) + \bar{\gamma}(|x_{1,1}|) + d$$

such that $V(z) \le v(t) + D(t_0, t)$ with $D(t_0, t) = \max\{0, e^{-c(t-t_0)}V(z_0) - e^{-\bar{c}(t-t_0)}v_0\}$, and $D(t_0, t) = 0$ for all $t \ge t_0 + T_0$.

Remark 2 Common to the existing literatures [33–41] on dynamic disturbance caused by unmodeled dynamics is the restrictive triangularity condition, where the upper bound function $\varphi_{i,j}(\cdot)$ is required to be a function of the current states in order to make the corresponding backstepping-based control schemes feasible, which is obviously impractical. Note that in this paper, the upper bound function term $\varphi_{i,j}(\cdot)$ contains the whole states of MIMO nonlinear systems. In this sense, we relax the assumption broadly used in other works. However, the more general model of dynamic disturbance makes the controller design more challenging and difficult.

2.1 Neural network approximation

We use radial basis function neural networks (RBFNNs) to approximate the unknown continuous function in the uncertain MIMO system due to its good capabilities in function approximation. Suppose that $h(Z) : R^n \to R$ be an unknown nonlinear function. Then, the function approximator of RBFNN $W^T S(Z)$ can precisely estimate any continuous function $h(Z)$ with sufficiently large number of neural nodes, over the compact set Ω_Z as follows

$$h(Z) = W^{*T} S(Z) + \delta(Z) \quad (5)$$

where $Z \in \Omega_Z \subset R^q$ is the input vector, $S(Z) = [s_1(Z), \ldots, s_l(Z)]^T \in R^l$ donates the basis function vector with $s_j(Z)$ determined by the Gaussian function, which can be expressed as

$$s_j(Z) = \exp\left[-\frac{(Z-c_j)^T(Z-c_j)}{\phi_j^2}\right] \quad (6)$$

where $j = 1, \ldots, l$, $c_j = [c_{j1}, \ldots, c_{jq}]^T$ and ϕ_j are the center of receptive field and the width of Gaussian function, respectively. Theoretically, the approximation accuracy can be high enough via increasing the number of neural nodes, which implies $\delta(Z) \to 0$ when $l \to \infty$. However, we cannot choose infinite number of neural nodes in reality. Thus, approximate error $\delta(Z)$ would be nonzero. Actually, we can increase l to obtain a less approximate error.The ideal constant weighting vector W^* is defined as $W^* = \arg\min_{W\in R^l}\left[\sup_{Z\in\Omega_Z}\left|h(Z) - W^T S(Z)\right|\right]$. Note that W^* is only used for the purpose of stability analysis, its actual value is not in need.

2.2 Nonlinear symmetric mapping

In this paper, nonlinear symmetric mapping technique is used to handle full state constraints.

Define the following one-to-one nonlinear mapping $M : x_{i,j} \to s_{i,j}$:

$$s_{i,j} = M_{i,j}(x_{i,j}) := \log\frac{k_{b_{i,j}} + x_{i,j}}{k_{b_{i,j}} - x_{i,j}}, \ |x_{i,j}(0)| < k_{b_{i,j}} \quad (7)$$

where log(•) denotes the natural logarithm of •.

From (7), we obtain that its inverse mapping is

$$x_{i,j} = \frac{e^{s_{i,j}} - 1}{e^{s_{i,j}} + 1}k_{b_{i,j}} \quad (8)$$

The derivative of $s_{i,j}$ with respect to t leads to

$$\dot{s}_{i,j} = \frac{e^{s_{i,j}} + e^{-s_{i,j}} + 2}{2k_{b_{i,j}}}\dot{x}_{i,j} \quad (9)$$

Then, system (1) can be described as follows:

$$\begin{cases} \dot{z} = q(z, x_{1,1}, t) \\ \dot{s}_{i,j} = F_{i,j}(\bar{s}_{i-1}, \bar{s}_{i,j+1}) \\ \qquad +s_{i,j+1} + C_{i,j}(s_{i,j})D_{i,j}(z, S, t) \\ \dot{s}_{i,n_i} = F_{i,n_i}(\bar{s}_i) + C_{i,n_i}(s_{i,n_i})g_{i,n_i}(\bar{s}_i)u_i \\ \qquad +C_{i,n_i}(s_{i,n_i})D_{i,n_i}(z, S, t) \end{cases} \tag{10}$$

where $\bar{s}_i = [s_1^T, \ldots, s_i^T]^T$, $s_i = [s_{i,1}, \ldots, s_{i,n_i}]^T$, $\bar{s}_{i,j} = [s_{i,1}, \ldots, s_{i,j}]^T$, $i = 1, \ldots, N$. $S = [s_1^T, \ldots, s_N^T]^T$.

$$C_{i,j}(s_{i,j}) = \frac{e^{s_{i,j}} + e^{-s_{i,j}} + 2}{2k_{b_{i,j}}}$$
$$F_{i,j}(\bar{s}_{i-1}, \bar{s}_{i,j+1}) = C_{i,j}(s_{i,j}) \left(f_{i,j}\left(\bar{x}_{i-1}, x_{i,j+1}\right) + x_{i,j+1}\right) - s_{i,j+1}$$
$$j = 1, \ldots, n_i - 1, i = 1, \ldots, N$$
$$F_{i,n_i}(\bar{s}_i) = C_{i,n_i}(s_{i,n_i}) f_{i,n_i}(\bar{x}_i)$$
$$g_{i,n_i}(\bar{s}_i) = g_{i,n_i}(\bar{x}_i), i = 1, \ldots, N$$
$$D_{i,j}(z, S, t) = \Delta_{i,j}(z, X, t), j = 1, \ldots, n_i, i = 1, \ldots, N$$

The reference signal is rewritten as follows:

$$\hat{y}_{di} = \log \frac{k_{b_{i,1}} + y_{di}}{k_{b_{i,1}} - y_{di}} \tag{11}$$

Therefore, it holds that as long as $s_{i,j}$ can be made bounded, then, for the initial condition $|x_{i,j}(0)| < k_{b_{i,j}}$, $x_{i,j}$ will not exceed the predefined region $\Omega_{x_{i,j}}$ by (7), which implies the full state constraints are never violated.

Remark 3 By employing nonlinear symmetric mapping, we transform a class of MIMO pure-feedback state-constrained system to the unconstrained one (10) and (11), which also has a pure-feedback form, but without full state constraints. Thus, the adaptive backstepping-based DSC can be directly extended to guarantee tracking properties and full state constraints, and the adaptive neural controller need not be redesigned.

Remark 4 Different from the BLF-based algorithm dealing with state/output constraints, it is shown that the nonlinear symmetric mapping technique allows a design where selected Lyapunov functions are all smooth functions, thus avoiding extra efforts due to the adoption of piecewise BLFs that are made to ensure the continuity and differentiability of the virtual control functions.

3 Main results

In this section, we will propose an adaptive DSC-based neural control scheme for MIMO nonlinear systems with full state constraints. To facilitate controller design, we introduce the following coordinate transformation:

$$z_{i,1} = s_{i,1} - \hat{y}_{di} \tag{12}$$
$$z_{i,j+1} = s_{i,j+1} - \beta_{i,j+1} \tag{13}$$
$$y_{i,j+1} = \beta_{i,j+1} - \alpha_{i,j} \tag{14}$$

where $i = 1, \ldots, N$, $j = 1, \ldots, n_i - 1$. For the jth subsystem, the intermediate control (virtual control) $\alpha_{i,j}$ is required to be in the following form:

$$\alpha_{i,j} = -k_{i,j}z_{i,j} - \frac{1}{2a_{i,j}^2} z_{i,j}\hat{\lambda} S_{i,j}^{\mathrm{T}}\left(Z_{i,j}\right) S_{i,j}\left(Z_{i,j}\right) \tag{15}$$

where $k_{i,j} > 0$ and $a_{i,j} > 0$ are design parameters. $S_{i,j}(Z_{i,j})$ is the basis function vector of RBFNNs with $Z_{i,j}$ serving as the neural input to be specified shortly, and $\hat{\lambda}$ is the estimation of an unknown constant λ with

$$\lambda = \max\left\{\left\|W_{i,j}^*\right\|^2, i = 1, \ldots, N, j = 1, \ldots, n_i\right\} \tag{16}$$

The neural adaptation law is given by

$$\dot{\hat{\lambda}} = \sum_{i=1}^{N}\sum_{j=1}^{n_i} \frac{\gamma}{2a_{i,j}^2} z_{i,j}^2 S_{i,j}^{\mathrm{T}}\left(Z_{i,j}\right) S_{i,j}\left(Z_{i,j}\right) - \sigma\hat{\lambda} \tag{17}$$

where γ and σ are positive design parameters. For ease of description, the following notations are defined.

$$Y = [y_1^T, \ldots, y_N^T]^T, y_i = [y_{i,2}, \ldots, y_{i,n_i}]^T$$
$$Z = [z_1^T, \ldots, z_N^T]^T, z_i = [z_{i,1}, \ldots, z_{i,n_i}]^T \tag{18}$$

Lemma 4 *For the whole states vector $X = [x_1^T, \ldots, x_N^T]^T$ of the MIMO nonlinear systems (1), the following inequality holds:*

$$\|X\| \leq \sum_{i=1}^{N}\sum_{j=1}^{n_i} \rho_{i,j}\left|z_{i,j}\right| + D^* + \varphi(Y) \tag{19}$$

with $\rho_{i,j}(\hat{\lambda}) = k_{b_{i,j}}\left(1 + k_{i,j} + \frac{1}{2a_{i,j}^2}\hat{\lambda}\right)$, $\rho_{i,n_i} = k_{b_{i,n_i}}$, and $D^* = \sum_{i=1}^{N} d_i k_{b_{i,1}}$ with $d_i = \log\frac{k_{b_{i,1}}+d_i^*}{k_{b_{i,1}}-d_i^*}$, for $i = 1, \ldots, N$, $j = 1, \ldots, n_i - 1$.

Proof From Assumption 3, (8), (12)-(15) and using $\sum_{i=1}^{N}\sum_{j=2}^{n_i}\left|y_{i,j}\right|k_{b_{i,j}} \leq \varphi(Y)$ with $\varphi(Y)$ being a non-negative continuous function, one has

$$\begin{aligned}
\|X\| &= \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left|x_{i,j}\right| \\
&= \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left|\frac{e^{s_{i,j}}-1}{e^{s_{i,j}}+1}\right|k_{b_{i,j}} \leq \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left|s_{i,j}\right|k_{b_{i,j}} \\
&= \sum_{i=1}^{N}\left|z_{i,1}+\hat{y}_{di}\right|k_{b_{i,1}} \\
&\quad + \sum_{i=1}^{N}\sum_{j=2}^{n_i}\left|z_{i,j}+y_{i,j}+\alpha_{i,j-1}\right|k_{b_{i,j}} \\
&\leq \sum_{i=1}^{N}\left|z_{i,1}\right|k_{b_{i,1}} + \sum_{i=1}^{N}d_i k_{b_{i,1}} \\
&\quad + \sum_{i=1}^{N}\sum_{j=2}^{n_i}\left(\left|z_{i,j}\right|+\left|y_{i,j}\right|\right)k_{b_{i,j}} \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\left|\alpha_{i,j}\right|k_{b_{i,j}} \\
&\leq \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left|z_{i,j}\right|k_{b_{i,j}} \\
&\quad + \sum_{i=1}^{N}\sum_{j=2}^{n_i-1}k_{b_{i,j}}\left(k_{i,j}+\frac{1}{2a_{i,j}^2}\hat{\lambda}\right)\left|z_{i,j}\right| \\
&\quad + D^* + \varphi(Y) \\
&= \sum_{i=1}^{N}\sum_{j=1}^{n_i}\rho_{i,j}\left|z_{i,j}\right| + D^* + \varphi(Y) \qquad (20)
\end{aligned}$$

Remark 5 It should be stressed that Lemma 4 plays a crucial role in removing the restrictive triangularity conditions on dynamic disturbances $\Delta_{i,j}(\cdot)$ as imposed in most existing works [33–41], because it sets up a relation between the norm of the whole states vector X and the norm of $z_{i,j}$. As shown in the sequel, the upper bound function $\varphi_{i,j}(\cdot)$ can be decomposed into a sum of the continuous functions with respect to $z_{i,j}$ by combining variable separation approach. Thus, there is no need for the triangularity conditions indispensable for developing a backstepping-based recursive design.

At the present stage, we carry out the neural control design step by step.

Step 1: Considering (10) and the derivative of $z_{i,1}$ in (12) yields

$$\begin{aligned}
\dot{z}_{i,1} &= \dot{s}_{i,1} - \dot{\hat{y}}_{di} \\
&= F_{i,1}\left(\bar{s}_{i-1},\bar{s}_{i,2}\right) + s_{i,2} \\
&\quad + C_{i,1}\left(s_{i,1}\right)D_{i,1}(z,S,t) - \dot{\hat{y}}_{di} \qquad (21)
\end{aligned}$$

Choose a Lyapunov candidate as $V_{z_{i,1}} = \frac{1}{2}z_{i,1}^2$. By taking the derivative of $V_{z_{i,1}}$ and using (21), (13) and (14), one has

$$\begin{aligned}
\dot{V}_{z_{i,1}} &= z_{i,1}\left[F_{i,1}\left(\bar{s}_{i-1},\bar{s}_{i,2}\right)\right. \\
&\quad \left. + s_{i,2} + C_{i,1}\left(s_{i,1}\right)D_{i,1}(z,S,t) - \dot{\hat{y}}_{di}\right] \\
&= z_{i,1}\left(z_{i,2}+y_{i,2}+\alpha_{i,1}\right) \\
&\quad + z_{i,1}\left[F_{i,1}\left(\bar{s}_{i-1},\bar{s}_{i,2}\right) - \dot{\hat{y}}_{di}\right] \\
&\quad + z_{i,1}C_{i,1}\left(s_{i,1}\right)D_{i,1}(z,S,t) \qquad (22)
\end{aligned}$$

According to the Definition of z in (2), we have $||z|| \leq \bar{\alpha}_1^{-1}(V(z))$. In view of Lemma 3, we have

$$||z|| \leq \bar{\alpha}_1^{-1}\left(v(t)+D_0\right) \qquad (23)$$

where D_0 is a positive constant. Upon using Assumption 2, (23) and employing Lemma 2, Lemma 4, we obtain

$$\begin{aligned}
& z_{i,1}C_{i,1}\left(s_{i,1}\right)\Delta_{i,1}(z,X,t) \leq \left|z_{i,1}\right|C_{i,1}\left(s_{i,1}\right)\left|\Delta_{i,1}(z,X,t)\right| \\
&\leq \left|z_{i,1}\right|C_{i,1}\left(s_{i,1}\right) \\
&\quad \left[\varphi_{i,1}\left(\sum_{i=1}^{N}\sum_{j=1}^{n_i}\rho_{i,j}\left|z_{i,j}\right| + D^* + \varphi(Y)\right) + \phi_{i,1}(\|z\|)\right] \\
&\leq \left|z_{i,1}\right|C_{i,1}\left(s_{i,1}\right)\sum_{m=1}^{N}\sum_{l=1}^{n_m}\varphi_{i,1}\left(p_0\rho_{m,l}\left|z_{m,l}\right|\right) \\
&\quad + \left|z_{i,1}\right|C_{i,1}\left(s_{i,1}\right)\varphi_{i,1}\left(p_0 D^*\right) \\
&\quad + \left|z_{i,1}\right|C_{i,1}\left(s_{i,1}\right)\varphi_{i,1}\left(p_0\varphi(Y)\right) \\
&\quad + \left|z_{i,1}\right|C_{i,1}\left(s_{i,1}\right)\phi_{i,1}\left(\bar{\alpha}_1^{-1}\left(v(t)+D_0\right)\right) \\
&\leq \frac{1}{2}C_{i,1}^2\left(s_{i,1}\right)z_{i,1}^2 + \sum_{m=1}^{N}\sum_{l=1}^{n_m}z_{m,l}^2\bar{\varphi}_{i,1}^2 \\
&\quad + z_{i,1}^2C_{i,1}^2\left(s_{i,1}\right)\varphi_{i,1}^2\left(p_0 D^*\right) \\
&\quad + z_{i,1}^2C_{i,1}^2\left(s_{i,1}\right)\varphi_{i,1}^2\left(p_0\varphi(Y)\right) \\
&\quad + z_{i,1}^2C_{i,1}^2\left(s_{i,1}\right)\phi_{i,1}^2\left(\bar{\alpha}_1^{-1}\left(v(t)+D_0\right)\right) + \frac{3}{4} \qquad (24)
\end{aligned}$$

where $p_0=\sum_{m=1}^{N}n_m+2, \bar{\varphi}_{i,1}^2=\frac{1}{2}p_0^2\rho_{m,l}^2q_{i,1}^2\left(p_0\rho_{m,l}\left|z_{m,l}\right|\right)$. Substituting (24) into (22) yields

$$\begin{aligned}\dot{V}_{z_{i,1}} \leq\ & z_{i,1}\alpha_{i,1}+\frac{1}{4}z_{i,2}^2+\frac{1}{4}y_{i,2}^2+z_{i,1}\\ &\times\left[F_{i,1}\left(\bar{s}_{i-1},\bar{s}_{i,2}\right)+2z_{i,1}-\dot{\hat{y}}_{di}\right]\\ &+z_{i,1}^2C_{i,1}^2\left(s_{i,1}\right)\\ &\times\left[\frac{1}{2}+\varphi_{i,1}^2\left(p_0D^*\right)+\varphi_{i,1}^2\left(p_0\varphi(Y)\right)\right.\\ &\left.+\phi_{i,1}^2\left(\bar{\alpha}_1^{-1}\left(v(t)+D_0\right)\right)\right]\\ &+\sum_{m=1}^{N}\sum_{l=1}^{n_m}z_{m,l}^2\bar{\varphi}_{i,1}^2+\frac{3}{4}\end{aligned}\tag{25}$$

By introducing a first-order low-pass filter, that is, $\tau_{i,2}\dot{\beta}_{i,2}+\beta_{i,2}=\alpha_{i,1},\beta_{i,2}(0)=\alpha_{i,1}(0)$, where $\tau_{i,2}>0$, and using (14), (15), we arrive at

$$\begin{aligned}\dot{y}_{i,2}=&-\frac{y_{i,2}}{\tau_{i,2}}-\dot{\alpha}_{i,1}\\ =&-\frac{y_{i,2}}{\tau_{i,2}}+\left[k_{i,1}\dot{z}_{i,1}+\frac{1}{2a_{i,1}^2}\dot{z}_{i,1}\hat{\lambda}\left|\left|S_{i,1}\left(Z_{i,1}\right)\right|\right|^2\right.\\ &+\frac{1}{2a_{i,1}^2}z_{i,1}\dot{\hat{\lambda}}\left|\left|S_{i,1}\left(Z_{i,1}\right)\right|\right|^2\\ &\left.+\frac{1}{2a_{i,1}^2}z_{i,1}\hat{\lambda}\frac{d\left|\left|S_{i,1}\left(Z_{i,1}\right)\right|\right|^2}{dt}\right]\end{aligned}\tag{26}$$

$$\left|\dot{y}_{i,2}+\frac{y_{i,2}}{\tau_{i,2}}\right|\leq\eta_{i,2}\left(Z,Y,\hat{\lambda},y_{di},\dot{y}_{di},\ddot{y}_{di},v(t)\right)\tag{27}$$

where $\eta_{i,2}\left(Z,Y,\hat{\lambda},y_{di},\dot{y}_{di},\ddot{y}_{di},v(t)\right)$ is a continuous function. Consequently, we have

$$\begin{aligned}y_{i,2}\dot{y}_{i,2}\leq&-\frac{y_{i,2}^2}{\tau_{i,2}}+\left|y_{i,2}\right|\eta_{i,2}\\ &\left(Z,Y,\hat{\lambda},y_{di},\dot{y}_{di},\ddot{y}_{di},v(t)\right)\\ \leq&-\frac{y_{i,2}^2}{\tau_{i,2}}+y_{i,2}^2+\frac{1}{4}\eta_{i,2}^2\end{aligned}\tag{28}$$

Step $j(2\leq j\leq n_i-1)$: Taking derivative of $z_{i,j}$ in (13) yields

$$\begin{aligned}\dot{z}_{i,j}=\ &\dot{s}_{i,j}-\dot{\beta}_{i,j}=F_{i,j}(\bar{s}_{i-1},\bar{s}_{i,j+1})+s_{i,j+1}\\ &+C_{i,j}(s_{i,j})D_{i,j}(z,S,t)-\dot{\beta}_{i,j}\end{aligned}\tag{29}$$

Consider a Lyapunov function candidate as $V_{z_{i,j}}=\frac{1}{2}z_{i,j}^2$. Then, the time derivative of $V_{z_{i,j}}$ along (13), (14) and (29) is given by

$$\begin{aligned}\dot{V}_{z_{i,j}}=\ &z_{i,j}(z_{i,j+1}+y_{i,j+1}+\alpha_{i,j})\\ &+z_{i,j}\left[F_{i,j}(\bar{s}_{i-1},\bar{s}_{i,j+1})-\dot{\beta}_{i,j}\right]\\ &+z_{i,j}C_{i,j}(s_{i,j})D_{i,j}(z,S,t)\end{aligned}\tag{30}$$

By Assumption 2, Remark 1 and Lemma 2, one has

$$\begin{aligned}&z_{i,j}C_{i,j}\left(s_{i,j}\right)\Delta_{i,j}(z,X,t)\\ &\leq|z_{i,j}|C_{i,j}\left(s_{i,j}\right)\left(\varphi_{i,j}(\|X\|)+\phi_{i,j}(\|z\|)\right)\\ &\leq|z_{i,j}|C_{i,j}\left(s_{i,j}\right)\varphi_{i,j}\\ &\quad\times\left(\sum_{m=1}^{N}\sum_{l=1}^{n_m}\rho_{m,l}\left|z_{m,l}\right|+D^*+\varphi(Y)\right)\\ &\quad+|z_{i,j}|C_{i,j}\left(s_{i,j}\right)\phi_{i,j}(\|z\|)\\ &\leq\frac{1}{2}C_{i,j}^2\left(s_{i,j}\right)z_{i,j}^2+\sum_{m=1}^{N}\sum_{l=1}^{n_m}z_{m,l}^2\bar{\varphi}_{i,j}^2\\ &\quad+z_{i,j}^2C_{i,j}^2\left(s_{i,j}\right)\varphi_{i,j}^2\left(p_0D^*\right)+\frac{3}{4}\\ &\quad+z_{i,j}^2C_{i,j}^2\left(s_{i,j}\right)\varphi_{i,j}^2\left(p_0\varphi(Y)\right)\\ &\quad+z_{i,j}^2C_{i,j}^2\left(s_{i,j}\right)\phi_{i,j}^2\left(\bar{\alpha}_1^{-1}\left(v(t)+D_0\right)\right)\end{aligned}\tag{31}$$

where $\bar{\varphi}_{i,j}^2=\frac{1}{2}p_0^2\rho_{m,l}^2q_{i,j}^2\left(p_0\rho_{m,l}\left|z_{m,l}\right|\right)$. Substituting (31) into (30) gives

$$\begin{aligned}\dot{V}_{z_{i,j}}&\leq z_{i,j}\left(z_{i,j+1}+y_{i,j+1}+\alpha_{i,j}\right)\\ &+z_{i,j}\left[F_{i,j}\left(\bar{s}_{i-1},\bar{s}_{i,j+1}\right)\right.\\ &\quad+\frac{1}{2}C_{i,j}^2\left(s_{i,j}\right)z_{i,j}+z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\varphi_{i,j}^2\left(p_0D^*\right)\\ &\quad+z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\varphi_{i,j}^2\left(p_0\varphi(Y)\right)\\ &\quad\left.+z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\phi_{i,j}^2\left(\bar{\alpha}_1^{-1}\left(v(t)+D_0\right)\right)\right]\\ &\quad+\sum_{m=1}^{N}\sum_{l=1}^{n_m}z_{m,l}^2\bar{\varphi}_{i,j}^2-z_{i,j}\dot{\beta}_{i,j}+\frac{3}{4}\\ &\leq z_{i,j}\alpha_{i,j}+\frac{1}{4}z_{i,j+1}^2\\ &\quad+\frac{1}{4}y_{i,j+1}^2+z_{i,j}\left[2z_{i,j}-\dot{\beta}_{i,j}+F_{i,j}\left(\bar{s}_{i-1},\bar{s}_{i,j+1}\right)\right.\\ &\quad+\frac{1}{2}C_{i,j}^2\left(s_{i,j}\right)z_{i,j}+z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\varphi_{i,j}^2\left(p_0D^*\right)\\ &\quad+z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\varphi_{i,j}^2\left(p_0\varphi(Y)\right)\\ &\quad\left.+z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\phi_{i,j}^2\left(\bar{\alpha}_1^{-1}\left(v(t)+D_0\right)\right)\right]\\ &\quad+\sum_{m=1}^{N}\sum_{l=1}^{n_m}z_{m,l}^2\bar{\varphi}_{i,j}^2+\frac{3}{4}\end{aligned}\tag{32}$$

Define $\beta_{i,j+1}$ as follows:

$$\tau_{i,j+1}\dot{\beta}_{i,j+1}+\beta_{i,j+1}=\alpha_{i,j},\ \beta_{i,j+1}(0)=\alpha_{i,j}(0)\tag{33}$$

where $\tau_{i,j+1} > 0$ is a small time constant given by designer.

From (33) and (14), we have $|\dot{y}_{i,j+1} + \frac{y_{i,j+1}}{\tau_{i,j+1}}| = |-\dot{\alpha}_{i,j}| \leq \eta_{i,j+1}(Z, Y, \hat{\lambda}, y_{di}, \dot{y}_{di}, \ddot{y}_{di}, v(t))$ with $-\dot{\alpha}_{i,j} = \frac{1}{2a_{i,j}^2}\dot{z}_{i,j}\hat{\lambda}\| S_{i,j}\left(Z_{i,j}\right)\|^2 + \frac{1}{2a_{i,j}^2}z_{i,j}\dot{\hat{\lambda}}\|S_{i,j}\left(Z_{i,j}\right)\|^2 + \frac{1}{2a_{i,j}^2}z_{i,j}\hat{\lambda}\frac{d\| S_{i,j}(Z_{i,j})\|^2}{dt} + k_{i,j}\dot{z}_{i,j}$, where $\eta_{i,j+1}$ is a continuous function. Therefore, one can obtain

$$y_{i,j+1}\dot{y}_{i,j+1} \leq -\frac{y_{i,j+1}^2}{\tau_{i,j+1}} + y_{i,j+1}^2 + \frac{1}{4}\eta_{i,j+1}^2 \tag{34}$$

Step n_i: The actual control law u_i will be proposed in this step. Taking derivative of z_{i,n_i} in (13) along the last equation in (10) yields

$$\begin{aligned}\dot{z}_{i,n_i} &= \dot{s}_{i,n_i} - \dot{\beta}_{i,n_i}\\ &= F_{i,n_i}\left(\bar{s}_i\right) + C_{i,n_i}\left(s_{i,n_i}\right) g_{i,n_i}\left(\bar{s}_i\right) u_i\\ &\quad + C_{i,n_i}\left(s_{i,n_i}\right) D_{i,n_i}(z, S, t) - \dot{\beta}_{i,n_i}\end{aligned} \tag{35}$$

The control law is given as

$$\begin{aligned}u_i &= \frac{N\left(\xi_i\right)}{C_{i,n_i}\left(s_{i,n_i}\right)}\\ &\quad \left[k_{i,n_i}z_{i,n_i} + \frac{1}{2a_{i,n_i}^2}z_{i,n_i}\hat{\lambda}S_{i,n_i}^{\mathrm{T}}\left(Z_{i,n_i}\right)S_{i,n_i}\left(Z_{i,n_i}\right)\right]\end{aligned} \tag{36}$$

$$\dot{\xi}_i = k_{i,n_i}z_{i,n_i}^2 + \frac{1}{2a_{i,n_i}^2}z_{i,n_i}^2\hat{\lambda}S_{i,n_i}^{\mathrm{T}}\left(Z_{i,n_i}\right)S_{i,n_i}\left(Z_{i,n_i}\right) \tag{37}$$

Choosing the Lyapunov function candidate $V_{z_{i,n_i}} = \frac{1}{2}z_{i,n_i}^2$, the derivative of $V_{z_{i,n_i}}$ is $\dot{V}_{z_{i,n_i}} = z_{i,n_i}C_{i,n_i}\left(s_{i,n_i}\right) g_{i,n_i}\left(\bar{x}_i\right)u_i + z_{i,n_i}C_{i,n_i}\left(s_{i,n_i}\right)\Delta_{i,n_i}(z, X, t) + z_{i,n_i}\left[F_{i,n_i}\left(\bar{s}_i\right) - \dot{\beta}_{i,n_i}\right]$. For the uncertain disturbance with respect to unmodeled dynamics $z_{i,n_i}C_{i,n_i}\left(s_{i,n_i}\right)\Delta_{i,n_i}(z, X, t)$ similar to (31), Let $j = n_i$ in (31), and the specific process is omitted for simplicity. Thus, applying (36) and (37) into $\dot{V}_{z_{i,n_i}}$ yields

$$\begin{aligned}\dot{V}_{z_{i,n_i}} &\leq \left(g_{i,n_i}\left(\bar{x}_i\right)N\left(\xi_i\right) + 1\right)\dot{\xi}_i - \dot{\xi}_i\\ &\quad + z_{i,n_i}\Big[z_{i,n_i}C_{i,n_i}^2\left(s_{i,n_i}\right)\varphi_{i,n_i}^2\left(p_0D^*\right)\\ &\quad + z_{i,n_i}C_{i,n_i}^2\left(s_{i,n_i}\right)\varphi_{i,n_i}^2\left(p_0\varphi(Y)\right)\\ &\quad + \frac{1}{2}C_{i,n_i}^2\left(s_{i,n_i}\right)z_{i,n_i} - \dot{\beta}_{i,n_i}\\ &\quad + z_{i,n_i}C_{i,n_i}^2\left(s_{i,n_i}\right)\phi_{i,n_i}^2\left(\bar{\alpha}_1^{-1}\left(v(t) + D_0\right)\right)\\ &\quad + F_{i,n_i}\left(\bar{s}_i\right)\Big]\\ &\quad + \sum_{m=1}^{N}\sum_{l=1}^{n_m} z_{m,l}^2\bar{\varphi}_{i,n_i}^2 + \frac{3}{4}\end{aligned} \tag{38}$$

where $\bar{\varphi}_{i,n_i}^2 = \frac{1}{2}p_0^2\rho_{m,l}^2q_{i,n_i}^2\left(p_0\rho_{m,l}|z_{m,l}|\right)$.

To present the stability analysis, we choose the total Lyapunov function candidate as follows:

$$V_n = \sum_{i=1}^{N}\sum_{j=1}^{n_i} V_{z_{i,j}} + \sum_{i=1}^{N}\sum_{j=1}^{n_i-1} y_{i,j+1}^2 + \frac{\tilde{\lambda}^2}{2\gamma} \tag{39}$$

where $\gamma > 0$ is a design parameter, and $\tilde{\lambda} = \hat{\lambda} - \lambda$.

Since for $p > 0$ and $B_{i0} > 0$, the set $\Omega_n := \{[Z, Y, \lambda]^T, V_n \leq p\}$ and $\Omega_{di} = \{[y_{di}, \dot{y}_{di}, \ddot{y}_{di}]^T : y_{di}^2 + \dot{y}_{di}^2 + \ddot{y}_{di}^2 \leq B_{i0}\} \subset R^3$ are compact in R^{p_n} with $p_n = 2\sum_{i=1}^{N} n_i - N + 1$ and R^3, respectively. Therefore, we obtain $z_{i,j}, y_{i,l}, \hat{\lambda} \in L_\infty, i = 1, \ldots, N, j = 1, \ldots, n_i, l = 2, \ldots, n_i$. Note that $y_{di} \in L_\infty$ and $z_{i,1} = s_{i,1} - \hat{y}_{di}, s_{i,1} = z_{i,1} + \hat{y}_{di} \in L_\infty$, then it is ensured that $v(t) \in L_\infty$. Thus, over the compact set $\Omega_n \times \Omega_{di}$, there exists a positive constant $M_{i,l}$ such that $|\eta_{i,l}| \leq M_{i,l}\,(i = 1, \ldots, N, l = 2, \ldots, n_i)$, and $|\varphi(Y)| \leq Y^*$ on Ω_n.

Theorem 1 *Consider the uncertain MIMO nonlinear systems (1) subject to symmetric full state constraints controlled by the adaptive neural controllers (15), (36) with adaptation law (17) under nonlinear mapping (7), (8) and Assumptions 1-3. For bounded initial conditions, satisfying $V_n(0) < p$, and $x_{i,j}(0) \in \Omega_{x_{i,j}}$, there exist constants $k_{i,j} > 0$, $\tau_{i,j} > 0$, $\sigma > 0$ such that the full state constraints never be violated, i.e., $x_{i,j} \in \Omega_{x_{i,j}}$, $\forall t \geq 0$, and all the closed-loop signals are SGUUB. In addition, $k_{i,j}$ and $\tau_{i,j}$ satisfy*

$$\begin{cases} k_{i,1} \geq \frac{\alpha_0}{2} + 1\\ k_{i,j} \geq \frac{\alpha_0}{2} + \frac{5}{4}, \; j = 2, \ldots, n_i, i = 1, \ldots, N\\ \frac{1}{\tau_{i,j+1}} \geq \frac{5}{4} + \frac{\alpha_0}{2}, \; j = 1, \ldots, n_i - 1\\ \alpha_0 \leq \sigma \end{cases} \tag{40}$$

Proof Consider the overall Lyapunov function candidate $V = V_n$, differentiating $V(t)$ along (25), (32) and (38) leads to

$$\dot{V} \leq \sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\left(z_{i,j}\alpha_{i,j} + \frac{1}{4}z_{i,j+1}^2 + \frac{1}{4}y_{i,j+1}^2\right) + \sum_{i=1}^{N} z_{i,n_i}\left[F_{i,n_i}\left(\bar{s}_i\right) - \dot{\beta}_{i,n_i}\right] + \sum_{i=1}^{N} z_{i,1}\left[F_{i,1}\left(\bar{s}_{i-1}, \bar{s}_{i,2}\right) + 2z_{i,1} - \dot{\hat{y}}_{di}\right] + \sum_{i=1}^{N}\left[\left(g_{i,n_i}\left(\bar{x}_i\right)N\left(\xi_i\right) + 1\right)\dot{\xi}_i - \dot{\xi}_i\right] + \sum_{i=1}^{N}\sum_{j=2}^{n_i-1} z_{i,j}\left[F_{i,j}\left(\bar{s}_{i-1}, \bar{s}_{i,j+1}\right) + 2z_{i,j} - \dot{\beta}_{i,j}\right] + \sum_{i=1}^{N}\sum_{j=1}^{n_i} z_{i,j}\left[\frac{1}{2}C_{i,j}^2\left(s_{i,j}\right)z_{i,j} + z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\varphi_{i,j}^2\left(p_0 D^*\right) + z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\varphi_{i,j}^2 \left(p_0 Y^*\right) + z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\phi_{i,j}^2\left(\bar{\alpha}_1^{-1}\left(v(t) + D_0\right)\right)\right] + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\sum_{m=1}^{N}\sum_{l=1}^{n_m} z_{m,l}^2\bar{\varphi}_{i,j}^2 + \sum_{i=1}^{N}\frac{3}{4}n_i - \frac{\tilde{\lambda}}{\gamma}\dot{\hat{\lambda}} + \sum_{i=1}^{N}\sum_{j=1}^{n_i-1} y_{i,j+1}\dot{y}_{i,j+1} \tag{41}$$

Notice that by rearranging the sequence, we obtain

$$\sum_{i=1}^{N}\sum_{j=1}^{n_i}\sum_{m=1}^{N}\sum_{l=1}^{n_m} z_{m,l}^2\bar{\varphi}_{i,j}^2 = \sum_{i=1}^{N}\sum_{j=1}^{n_i} z_{i,j}^2 \sum_{m=1}^{N}\sum_{l=1}^{n_m}\frac{1}{2}p_0^2\rho_{i,j}^2 q_{m,l}^2\left(p_0\rho_{i,j}\left|z_{i,j}\right|\right) \tag{42}$$

Replacing (42) into (41), it follows that

$$\dot{V} \leq \sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\left(z_{i,j}\alpha_{i,j} + \frac{1}{4}z_{i,j+1}^2 + \frac{1}{4}y_{i,j+1}^2\right) + \sum_{i=1}^{N}\left[\left(g_{i,n_i}\left(\bar{x}_i\right)N\left(\xi_i\right) + 1\right)\dot{\xi}_i - \dot{\xi}_i\right] + \sum_{i=1}^{N}\sum_{j=1}^{n_i} z_{i,j}h_{i,j}\left(Z_{i,j}\right) + \sum_{i=1}^{N}\frac{3}{4}n_i - \frac{\tilde{\lambda}}{\gamma}\dot{\hat{\lambda}} + \sum_{i=1}^{N}\sum_{j=1}^{n_i-1} y_{i,j+1}\dot{y}_{i,j+1} \tag{43}$$

where

$$h_{i,1}\left(Z_{i,1}\right) = F_{i,1}\left(\bar{s}_{i-1}, \bar{s}_{i,2}\right) + 2z_{i,1} - \dot{\hat{y}}_{di} + z_{i,1}\sum_{m=1}^{N}\sum_{l=1}^{n_m}\frac{1}{2}p_0^2\rho_{i,1}^2 q_{m,l}^2\left(p_0\rho_{i,1}\left|z_{i,1}\right|\right) + \frac{1}{2}z_{i,1}C_{i,1}^2(s_{i,1}) + z_{i,1}C_{i,1}^2\left(s_{i,1}\right)\varphi_{i,1}^2\left(p_0 D^*\right) + z_{i,1}C_{i,1}^2\left(s_{i,1}\right)\varphi_{i,1}^2\left(p_0 Y^*\right) + z_{i,1}C_{i,1}^2\left(s_{i,1}\right)\phi_{i,1}^2 \left(\bar{\alpha}_1^{-1}\left(v(t) + D_0\right)\right)$$

$$Z_{i,1} = \left[\bar{s}_{i-1}, \bar{s}_{i,2}, z_{i,1}, v(t), \dot{\hat{y}}_{di}\right]^T \in R^{\sum_{l=1}^{i-1} n_l + 5} \tag{44}$$

$$h_{i,j}\left(Z_{i,j}\right) = F_{i,j}\left(\bar{s}_{i-1}, \bar{s}_{i,j+1}\right) + 2z_{i,j} - \dot{\beta}_{i,j} + z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\varphi_{i,j}^2\left(p_0 Y^*\right) + \frac{1}{2}z_{i,j}C_{i,j}^2\left(s_{i,j}\right) + z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\varphi_{i,j}^2\left(p_0 D^*\right) + z_{i,j}C_{i,j}^2\left(s_{i,j}\right)\phi_{i,j}^2\left(\bar{\alpha}_1^{-1}\left(v(t) + D_0\right)\right) + z_{i,j}\sum_{m=1}^{N}\sum_{l=1}^{n_m}\frac{1}{2}p_0^2\rho_{i,j}^2 q_{m,l}^2\left(p_0\rho_{i,j}|z_{i,j}|\right)$$

$$Z_{i,j} = \left[\bar{s}_{i-1}, \bar{s}_{i,j+1}, z_{i,j}, v(t), \dot{\beta}_{i,j}\right]^T \in R^{\sum_{l=1}^{i-1} n_l + j + 4},\ j = 2, \ldots, n_i - 1 \tag{45}$$

$$h_{i,n_i}\left(Z_{i,n_i}\right) = F_{i,n_i}\left(\bar{s}_i\right) - \dot{\beta}_{i,n_i} + z_{i,n_i}\sum_{m=1}^{N}\sum_{l=1}^{n_m}\frac{1}{2}p_0^2\rho_{i,n_i}^2 q_{m,l}^2\left(p_0\rho_{i,n_i}|z_{i,n_i}|\right) + \frac{1}{2}z_{i,n_i}C_{i,n_i}^2\left(s_{i,n_i}\right) + z_{i,n_i}C_{i,n_i}^2\left(s_{i,n_i}\right)\varphi_{i,n_i}^2\left(p_0 D^*\right) + z_{i,n_i}C_{i,n_i}^2\left(s_{i,n_i}\right)\varphi_{i,n_i}^2\left(p_0 Y^*\right) + z_{i,n_i}C_{i,n_i}^2\left(s_{i,n_i}\right)\phi_{i,n_i}^2\left(\bar{\alpha}_1^{-1}\left(v(t) + D_0\right)\right)$$

$$Z_{i,n_i} = \left[\bar{s}_i, z_{i,n_i}, v(t), \dot{\beta}_{i,n_i}\right]^T \in R^{\sum_{l=1}^{i} n_l + 3} \tag{46}$$

Then, we employ RBFNN to approximate the unknown function, i.e., $h_{i,j}(Z_{i,j}) = W_{i,j}^{*\,T} S_{i,j}(Z_{i,j}) + \delta_{i,j}(Z_{i,j})$ with $\left|\delta_{i,j}\left(Z_{i,j}\right)\right| \leq \varepsilon_{i,j}\left(Z, Y, \hat{\lambda}, y_{di}, \dot{y}_{di}, v(t)\right)$, where $\varepsilon_{i,j}$ is a continuous function. Thus, for $i = 1, \ldots, N$,

$j = 1, \ldots, n_i$

$$\begin{aligned} & z_{i,j} h_{i,j}\left(Z_{i,j}\right) \\ & = z_{i,j} W_{i,j}^{*\,T} S_{i,j}(Z_{i,j}) + z_{i,j}\delta_{i,j}(Z_{i,j}) \\ & \le \frac{1}{2a_{i,j}^2} z_{i,j}^2 \lambda S_{i,j}^T\left(Z_{i,j}\right) S_{i,j}\left(Z_{i,j}\right) \\ & \quad + \frac{1}{2} a_{i,j}^2 + \left|z_{i,j}\right| \varepsilon_{i,j} \\ & \le \frac{1}{2a_{i,j}^2} z_{i,j}^2 \lambda S_{i,j}^T\left(Z_{i,j}\right) S_{i,j}\left(Z_{i,j}\right) \\ & \quad + \frac{1}{2} a_{i,j}^2 + z_{i,j}^2 + \frac{1}{4}\varepsilon_{i,j}^2 \end{aligned} \tag{47}$$

In view of $\Omega_n \times \Omega_{di}$ is compact in R^{p_n+3}, there exists a constant $N_{i,j} > 0$ such that $|\varepsilon_{i,j}| \le N_{i,j}$ $(i = 1, \ldots, N, j = 1, \ldots, n_i)$ on $\Omega_n \times \Omega_{di}$. Substituting (28), (34), (37) and (47) into (43), and using (15), (17), we arrive at

$$\begin{aligned} \dot{V} \le & \sum_{i=1}^{N}\left(-k_{i,1} + 1\right) z_{i,1}^2 + \sum_{i=1}^{N}\sum_{j=2}^{n_i}\left(-k_{i,j} + \frac{5}{4}\right) z_{i,j}^2 \\ & + \sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\left(-\frac{1}{\tau_{i,j+1}} + \frac{5}{4}\right) y_{i,j+1}^2 \\ & - \frac{\sigma}{\gamma}\tilde{\lambda}\hat{\lambda} + \sum_{i=1}^{N}\left[g_{i,n_i}\left(\bar{x}_i\right) N\left(\xi_i\right) + 1\right]\dot{\xi}_i \\ & + \sum_{i=1}^{N}\frac{3}{4} n_i + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(\frac{1}{2} a_{i,j}^2 + \frac{1}{4} N_{i,j}^2\right) \\ & + \sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\frac{1}{4} M_{i,j+1}^2 \end{aligned} \tag{48}$$

Applying the inequality $-\tilde{\lambda}\hat{\lambda} = -\tilde{\lambda}(\tilde{\lambda} + \lambda) \le -\tilde{\lambda}^2 + \frac{\tilde{\lambda}^2}{2} + \frac{\lambda^2}{2} = -\frac{\tilde{\lambda}^2}{2} + \frac{\lambda^2}{2}$ and (40) yields

$$\dot{V}(t) \le -\alpha_0 V + \sum_{i=1}^{N}\left[g_{i,n_i}\left(\bar{x}_i\right) N\left(\xi_i\right) + 1\right]\dot{\xi}_i + \mu_0 \tag{49}$$

where $\mu_0 = \sum_{i=1}^{N}\frac{3}{4} n_i + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(\frac{1}{2} a_{i,j}^2 + \frac{1}{4} N_{i,j}^2\right) + \sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\frac{1}{4} M_{i,j+1}^2 + \frac{\sigma}{2\gamma}\lambda^2$. Multiplying both sides by $e^{\alpha_0 t}$ and integrating it over $[0, t]$, (49) can be further expressed as

$$\begin{aligned} V(t) \le & \frac{\mu_0}{\alpha_0} + \left[V(0) - \frac{\mu_0}{\alpha_0}\right] e^{-\alpha_0 t} \\ & + e^{-\alpha_0 t}\sum_{i=1}^{N}\int_0^t\left[g_i\left(\bar{x}_i\right) N\left(\xi_i\right) + 1\right]\dot{\xi}_i e^{\alpha_0 \tau} d\tau \end{aligned} \tag{50}$$

As $\frac{\mu_0}{\alpha_0} + \left[V(0) - \frac{\mu_0}{\alpha_0}\right] e^{-\alpha_0 t} \le V(0) + \frac{\mu_0}{\alpha_0}$, then, in view of Lemma 1, it is established that $V(t)$, ξ_i and $\sum_{i=1}^{N}\int_0^t\left[g_i\left(\bar{x}_i\right) N\left(\xi_i\right) + 1\right]\dot{\xi}_i e^{\alpha_0\tau} d\tau$ remain bounded on $[0, t_f)$. From the discussion in [46], we can obtain that the above conclusion is true for $t_f \to \infty$. Let $\sum_{i=1}^{N}\int_0^t\left[g_i\left(\bar{x}_i\right) N\left(\xi_i\right) + 1\right]\dot{\xi}_i e^{\alpha_0\tau} d\tau \le \mu_1$, (50) becomes

$$0 \le V(t) \le \frac{\mu_0}{\alpha_0} + \left[V(0) - \frac{\mu_0}{\alpha_0} + \mu_1\right] e^{-\alpha_0 t} \tag{51}$$

Therefore, the signals $z_{i,j}$, $y_{i,j}$ and $\hat{\lambda}$ are SGUUB. Furthermore, it is ensured that $\alpha_{i,j}$ and $\beta_{i,j+1}$ also remain SGUUB. Then, it follows from (13) and (14) that $s_{i,j} = z_{i,j} + y_{i,j} + \alpha_{i,j-1} \in L_\infty$, which implies that $x_{i,j} \in \Omega_{x_{i,j}}$ from (7), namely the full state constraints are not violated. Since $x_{1,1} \in \Omega_{x_{i,j}}$, we obtain $v(t) \in L_\infty$. According to definition 1, we known that $\|z\| \le \bar{\alpha}_1^{-1}(V(t))$, then based on Lemma 3, we have $\|z\| \le \bar{\alpha}_1^{-1}(v(t) + D(t_0, t))$. Because of $v(t) \in L_\infty$ and $D(t_0, t) \in L_\infty$, $\|z\| \in L_\infty$. From (51), one has

$$|z_{i,1}| \le \sqrt{2\frac{\mu_0}{\alpha_0} + 2\left[V(0) - \frac{\mu_0}{\alpha_0} + \mu_1\right] e^{-\alpha_0 t}} \tag{52}$$

Remark 6 It is worth noting that the size of $z_{i,1}$ depends on the design parameters γ, σ, $k_{i,j}$, $a_{i,j}$ as well as the initial conditions $z_{i,j}(0)$, $\hat{\lambda}(0)$. It is clear that decreasing σ helps to reduce μ_0, and increasing $k_{i,j}$ might result in small α_0; thus, it will help to reduce $\frac{\mu_0}{\alpha_0}$. In addition, decreasing initial values $z_{i,j}(0)$ will help to reduce $V(0)$. Therefore, $z_{i,1}$ as $t \to \infty$ can be made arbitrarily small.

Recalling (8) and (11), we obtain

$$\begin{aligned} y_i - y_{di} & = \frac{2k_{b_{i,1}}\left(e^{s_{i,1}} - e^{\hat{y}_{di}}\right)}{\left(e^{s_{i,1}} + 1\right)\left(e^{\hat{y}_{di}} + 1\right)} \\ & = \frac{2k_{b_{i,1}} e^{s_{i,1}}\left(1 - e^{-z_{i,1}}\right)}{\left(e^{s_{i,1}} + 1\right)\left(e^{s_{i,1} - z_{i,1}} + 1\right)} \end{aligned} \tag{53}$$

Upon using mean value theorem, it is obviously that $1 - e^{-z_{i,1}} = z_{i,1} e^{-\lambda_{z_{i,1}} z_{i,1}}$ is established when $\lambda_{z_{i,1}}$ varies in the interval (0, 1). Thus, we have

$$|y_i - y_{di}| \leq 2k_{b_{i,1}} e^{s_{i,1}} e^{-\lambda_{z_{i,1}} z_{i,1}} |z_{i,1}| \tag{54}$$

From the above analysis, we conclude that $z_{i,1}$ can be made small enough by selecting the design parameters appropriately, in view of $s_{i,1} \in L_\infty$, as a consequence, the tracking error $y_i - y_{di}$ as $t \to \infty$ can achieve arbitrarily small value. The proof of Theorem 1 is completed.

Remark 7 It is noted that $V_n(0) < p$ can be ensured by setting the bounded initial value. Moreover, the actual value of p is not necessary, which is only required for the purpose of analysis of closed-loop stability and boundedness.

Remark 8 Although several full state-constrained control schemes based on DSC/command filter and transformed function/nonlinear mapping have been developed in [18,47,48], these existing works did not consider that the system appeared as MIMO and the effect of unmodeled dynamics. Among the few exceptions are [20,38,49]. In [20], via employing a novel transformed function, the adaptive controller design was developed for uncertain nonlinear MIMO state-constrained systems. However, the uncertainties considered in [20] were bounded by known nonlinear functions, while in [49], a one-to-one nonlinear mapping was applied to deal with the symmetric full state constraints for a class of MIMO strict-feedback nonlinear systems, and the finite-time command filter control scheme can compensate the filtering error with better control performance compared with DSC. In our study, a more general uncertain MIMO pure-feedback nonlinear system with unknown functions is considered, in which RBFNNs are used to model unknown functions. In [38], an adaptive control strategy in combination of DSC and nonlinear mapping was proposed for a class of SISO pure-feedback nonlinear systems with unmodeled dynamics. However, the uncertain disturbance with respect to unmodeled dynamics has to satisfy a triangular assumption, which is replaced by a less restrictive condition in this study. Comparing with [18,20,38,47–49], the proposed control scheme is the first attempt for uncertain MIMO pure-feedback nonlinear systems subject to unmodeled dynamics and full state constraints without involving the so-called feasibility condition.

4 Simulation verification

In this section, the effectiveness of the developed control scheme is illustrated from a simulation. The following MIMO nonlinear system is written as

$$\begin{cases} \dot{z} = -z + 0.5x_{1,1}^2 \sin(x_{1,1}) \\ \dot{x}_{1,1} = x_{1,1} + \frac{1}{5}x_{1,2}^3 + x_{1,2} + \Delta_{1,1} \\ \dot{x}_{1,2} = x_{1,1}x_{1,2} + (1 + 0.1\sin(x_{1,1}x_{1,2}))u_1 + \Delta_{1,2} \\ \dot{x}_{2,1} = x_{2,1}e^{-0.5x_{2,1}x_{1,1}} + x_{2,1}^2 x_{2,2} + x_{2,2} + \Delta_{2,1} \\ \dot{x}_{2,2} = x_{2,1}x_{2,2}^2 + (3 - \cos(x_{1,1}x_{1,2}x_{2,1}x_{2,2}))u_2 + \Delta_{2,2} \end{cases} \tag{55}$$

where $\Delta_{1,1} = 2z\sin(t)$, $\Delta_{1,2} = 0.2z\cos(0.5x_{1,2}) - 0.5x_{1,1}$, $\Delta_{2,1} = 0.2zx_{1,1}x_{1,2}x_{2,1}\sin(x_{2,2})$, $\Delta_{2,2} = 0.1z\cos(0.5x_{2,2})$. The reference signals are given as $y_{d1} = 0.5\sin(t) + \sin(0.5t)$ and $y_{d2} = \sin(0.5t)$. In order to cope with the unmodeled dynamics, we choose $\dot{v}(t) = -v(t) + 2.5x_{1,1}^4 + 0.625$ as the dynamic signal. The state-constrained parameter values are set to $k_{b_{1,1}} = 1$, $k_{b_{1,2}} = 2$, $k_{b_{2,1}} = 1.5$, $k_{b_{2,2}} = 2$. The initial conditions are selected as $x_1(0) = x_2(0) = [0.1, 0]^T$, $\hat{\lambda}(0) = 0.5$, $z(0) = 0.1$, $v(0) = 0.1$, $\xi_1(0) = \xi_2(0) = 0.2$, $\beta_{1,2}(0) = 0.1$,$\beta_{2,2}(0) = 0.5$.

The adaptive neural controllers are designed as follows:

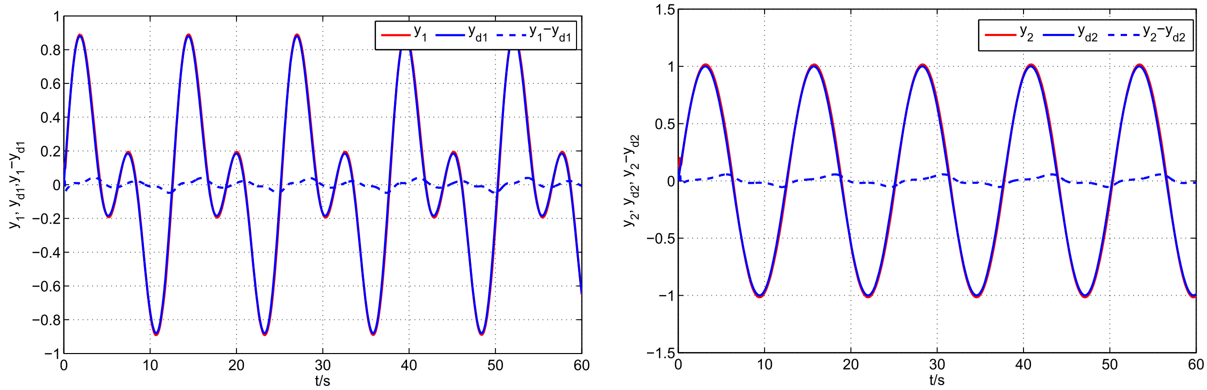
$$\alpha_{i,1} = -k_{i,1}z_{i,1} - \frac{1}{2a_{i,1}^2} z_{i,1}\hat{\lambda}S_{i,1}^{\mathrm{T}}\left(Z_{i,1}\right)S_{i,1}\left(Z_{i,1}\right) \tag{56}$$

$$\tau_{i,2}\dot{\beta}_{i,2} + \beta_{i,2} = \alpha_{i,1} \tag{57}$$

$$u_i = \frac{N\left(\xi_i\right)}{C_{i,2}\left(s_{i,2}\right)}\left[k_{i,2}z_{i,2} + \frac{1}{2a_{i,2}^2} z_{i,2}\hat{\lambda}S_{i,2}^{\mathrm{T}}\left(Z_{i,2}\right)S_{i,2}\left(Z_{i,2}\right)\right] \tag{58}$$

$$\dot{\hat{\lambda}} = \sum_{i=1}^{2}\sum_{j=1}^{2} \frac{\gamma}{2a_{i,j}^2} z_{i,j}^2 S_{i,j}^{\mathrm{T}}\left(Z_{i,j}\right)S_{i,j}\left(Z_{i,j}\right) - \sigma\hat{\lambda} \tag{59}$$

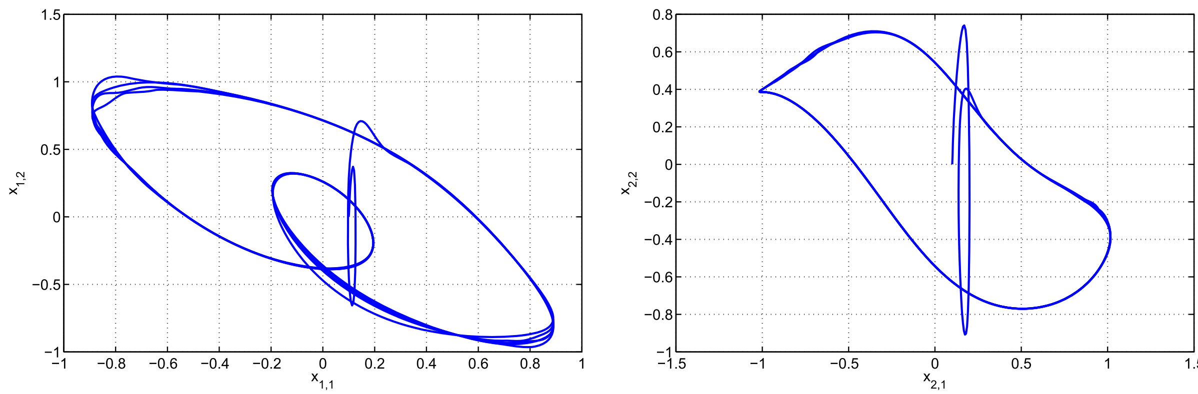
where $i = 1, 2$. $z_{i,1} = s_{i,1} - \dot{\hat{y}}_{di}$, $z_{i,2} = s_{i,2} - \beta_{i,2}$. From (7) and (11), $s_{i,1}$, $s_{i,2}$, $\hat{y}_{di}$ $C_{i,2}\left(s_{i,2}\right)$ are readily calculated. The design parameters are chosen as $k_{1,1} = 9$, $k_{1,2} = 6$, $k_{2,1} = 10$, $k_{2,2} = 2.5$, $a_{1,1} = a_{1,2} = a_{2,1} = a_{2,2} = 1$, $\tau_{1,2} = \tau_{2,2} = 0.01$, $\sigma = 0.01$, and $\gamma = 100$. We employ 4 RBFNNs having nine neural nodes to estimate unknown continuous functions. The input vectors are $Z_{1,1} = [s_{1,1}, s_{1,2}, z_{1,1}, \dot{\hat{y}}_{d1}, v(t)]^T$, $Z_{1,2} = [s_{1,1}, s_{1,2}, z_{1,1}, \dot{\beta}_{1,2}, v(t)]^T$, $Z_{2,1} = [s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}, z_{2,1}, \dot{\hat{y}}_{d2}, v(t)]^T$, and $Z_{2,2} = [s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2},$



(a) Output y_1, desired trajectory y_{d1}, and tracking error $y_1 - y_{d1}$

(b) Output y_2, desired trajectory y_{d2}, and tracking error $y_2 - y_{d2}$

Fig. 1 System outputs follow desired trajectories



(a) Phase portrait of states $x_{1,1}$ and $x_{1,2}$

(b) Phase portrait of states $x_{2,1}$ and $x_{2,2}$

Fig. 2 System full state constraints

$z_{2,2}, \dot{\beta}_{2,2}, v(t)]^T$. $S_{i,j}(Z_{i,j}) = [s_{i,j1}(Z_{i,j}), \ldots, s_{i,jl}(Z_{i,j})]^T \in R^l$ denotes the basis function vector with $s_{i,jq}$ being the Gaussian function, which can be expressed as

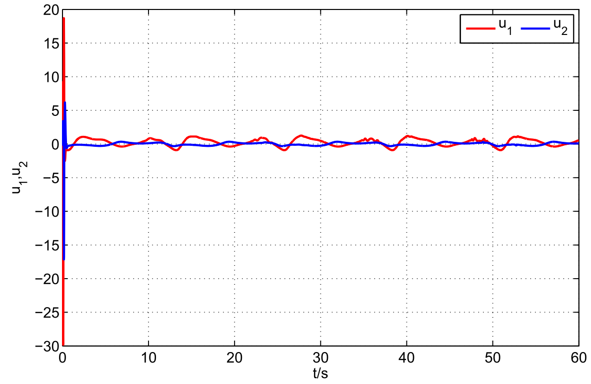
$$s_{i,jq}(Z_{i,j}) = e^{-\frac{(Z_{i,j}-c_{i,jq})^T((Z_{i,j}-c_{i,jq})}{\phi_{i,jq}^2}} \tag{60}$$

where $i = 1, 2$, $j = 1, 2$, $q = 1, \ldots, l$, and $l = 9$, $c_{i,jq} = [c_{i,j1}, \ldots, c_{i,jq_{i,j}}]^T$, and $q_{1,1} = q_{1,2} = 5$, $q_{2,1} = q_{2,2} = 7$. $c_{1,1q} = (q - 5)[1, 1, 1, 1, 1]^T$, $c_{1,2q} = 0.5(q - 5)[1, 1, 1, 1, 1]^T$, $c_{2,1q} = (q - 5)[1, 1, 1, 1, 1, 1, 1]^T$, $c_{2,2q} = 0.5(q - 5)[1, 1, 1, 1, 1, 1, 1]^T$.

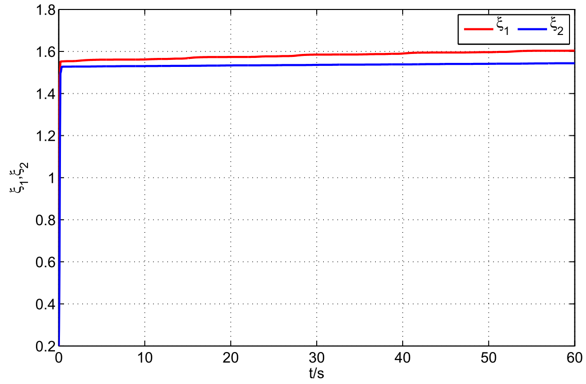
The simulation results are listed in Figures. 1-3. Figure. 1(a) and Figure. 1(b) show that the good tracking performance is guaranteed, while the state trajectories of MIMO systems are displayed in Figure. 2 which demonstrate that all the states remain in their respective constraint regions $|x_{1,1}| \leq 1$, $|x_{1,2}| \leq 2$, $|x_{2,1}| \leq 1.5$, and $|x_{2,2}| \leq 2$ for all $t \geq 0$. In addition, Figure. 3 displays that all the other closed-loop variables are bounded.

Remark 9 Let $V(z) = z^2$, there exist class K_∞ functions $\bar{\alpha}_1(|z|) = 0.5z^2$ and $\bar{\alpha}_2(|z|) = 2z^2$, such that $\bar{\alpha}_1(|z|) \leq V(z) \leq \bar{\alpha}_2(|z|)$. The time derivative of $V(z)$ is

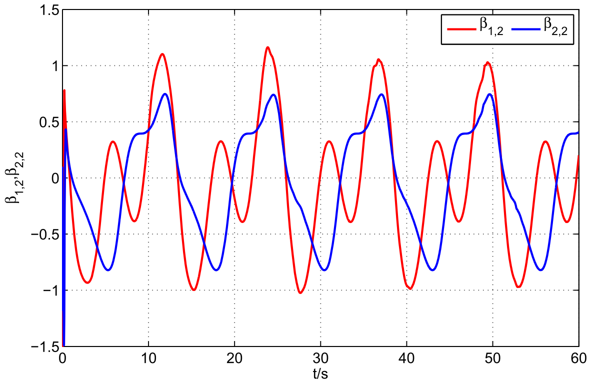
$$\begin{aligned}\dot{V}(z) &\leq -2z^2 + |z|x_{1,1}^2 \\ &\leq -z^2 + x_{1,1}^4 \\ &\leq -z^2 + 2.5x_{1,1}^4 + 0.625\end{aligned}$$



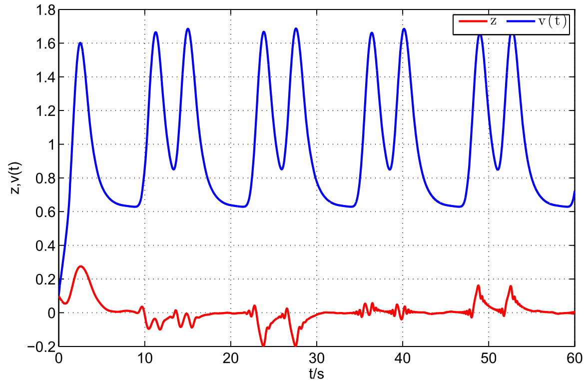
(a) Control inputs u_1 and u_2



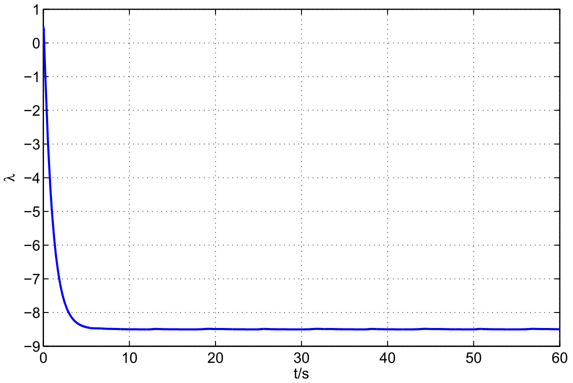
(b) Nussbaum parameters ξ_1 and ξ_2



(c) Outputs of the first-order filters $\beta_{1,2}$ and $\beta_{2,2}$



(d) unmodeled dynamics z and dynamic signal $v(t)$



(e) Adaptive parameter $\hat{\lambda}$

Fig. 3 Closed-loop signals

Therefore, it is easy to verify that the designed dynamic signal $v(t)$ in simulation 1 is reasonable.

5 Conclusions

In this research, the adaptive neural dynamic surface control problem was addressed for a class of uncertain

MIMO nonlinear systems with full state constraints and unmodeled dynamics. RBFNN compensator has played an important role to approximate the unknown continuous package functions. The novelties of our results are as follows: (i) for the first time, the nonlinear symmetric mapping technique combining with DSC is extended to full state-constrained MIMO nonlinear systems, with which the piecewise BLF is prevented ; (ii) by using variable splitting approach and the structure consistency of virtual controllers, the triangularity conditions on the uncertain disturbances caused by unmodeled dynamics are removed; and (iii) only one online parameter is adjusted for the entire MIMO nonlinear systems, the efficiency of computation can be largely improved. We have shown that the SGUUB tracking is achieved without violation of full state constraints, and the boundedness of all closed-loop signals is guaranteed by both the theoretical analysis and simulation verification. Further developments will concentrate on: (i) how to solve the problem of output feedback control of full state-constrained MIMO nonlinear systems with unmodeled dynamics; (ii) how to extend the adaptive neural dynamic surface control scheme to confirm the good tracking performance of the uncertain MIMO systems with both state and input constraints.

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Data availability Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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