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ORIGINAL PAPER

On active disturbance rejection control for lower-triangular systems with mismatched nonlinear uncertainties and unknown time-varying control coefficients

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Abstract It is a challenging issue to achieve the normal operation of control systems despite mismatched nonlinear uncertainties and unknown time-varying control coefficients. Based on the signs of control coefficients rather than nominal values or approximative mathematical expressions, the paper proposes a new active disturbance rejection control to tackle mismatched nonlinear uncertainties and unknown values of time-varying control coefficients. The design procedure can be concluded by three steps: determining the equivalent integrators chain form, constructing the extended state observer to estimate total disturbance and designing a dynamical system to generate the input approaching the desired input signal. Then, under a mild assumption for mismatched nonlinear uncertainties and unknown time-varying control coefficients, the paper rigorously analyzes the bounds of tracking error, estimating error and the error between the actual and desired inputs. Based on the presented error bounds, the tracking error with respect to the desired trajectory can be close to zero during the whole time period by suitably enlarging the observer parameter. The theoretical results reveal the strong robustness of the proposed method to mismatched nonlinear uncertainties and unknown time-varying control coefficients. Finally, by constructing the relationship between the observer parameter and the parameter in dynamical input design, the adjustable controller parameters remain observer parameter and feedback gain, which is friendly to practitioners.

Keywords Nonlinear uncertain system · Active disturbance rejection control · Mismatched uncertainty · Unknown time-varying control coefficient

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1 Introduction

Uncertainties, including unmodeled nonlinear dynamics, external disturbances and parametric perturbations, are ubiquitous in practice. In control science and technology, it is a central issue to ensure the normal operation of control systems despite various uncertainties [1].

Motivated by this important objective, numerous control strategies have been substantially developed, such as proportional-integral-derivative (PID) control [2], adaptive control [3], fuzzy control [4,5], neural network-based control [6–8] and disturbance rejection methods [9–12], just to name a few. In [2], the capability of PID control to handle nonlinear uncertainties was proved. The reference [3] comprehensively introduced the design and application of adaptive control, which can tackle a large scope of parametric uncertainties. The design of fuzzy control for nonlinear uncertainties was reviewed in [4]. More importantly, for nonlinear systems with unknown control coefficients, a novel fuzzy controller was developed in [5]. Besides, for nonlinear lower-triangular systems, neural network-based designs were proposed in [6,7]. In addition, various disturbance rejection designs have been proposed in the past decades [9–12], which can online estimate and compensate for uncertainties.

In recent years, disturbance rejection methods have drawn lots of attention from researches due to the simplicity in practical implementation and the superior performance to handle nonlinear uncertainties. Active disturbance rejection control (ADRC) is one of the most popular designs among various disturbance rejection methods, which has been successfully applied to flight systems [13–15], motion control systems [16,17] and process control systems [18–20], just to name a few. In the framework of ADRC, the extended state observer (ESO) is novelly constructed to estimate the total effect of various uncertainties, named as "total disturbance" [21], and then the control input is composed of the compensation for total disturbance and the feedback of integrators chain states.

In the last two decades, the theoretical foundation of ADRC has been substantially established. For the ESO which is the vital component in ADRC, the literature [22] investigated the convergence of ESO and further showed the bound of estimating error. The closed-loop stability of ADRC was firstly presented in [23], where the considered uncertainties and their derivatives with respect to the time were assumed to be bounded. For the linear time-invariant uncertain systems, the literature [24] theoretically illustrated the satisfied tracking performance of ADRC despite a large scope of parametric variations. In [25], by assuming the existence of the Lyapunov functions related with uncertainties, the capability of ADRC to deal with nonlinear uncertainties was proved. By assuming that the uncertainties and their partial derivatives are bounded if the states are in a bounded set, the references [26,27] rigorously studied the convergence and the transient performance of ADRC-based closed-loop system. It is remarkable that the conditions in [26,27] can depict a wide class of nonlinear uncertainties in practice. By utilizing the concept of total disturbance, the literatures [28,29] illuminated the capability of ADRC to handle the mismatched nonlinearities in both dynamical systems and measurement models. Besides, several successful modifications of ADRC have been made for nonlinear uncertain systems with other complicated practical factors, such as time delay [30,31] and stochastic uncertainties [32,33].

Up to now, the theoretical results have demonstrated the effectiveness of ADRC to tackle the time-varying disturbances and the nonlinear internal uncertainties dependent on system states. In the conventional ADRC design, it is remarkable that the information of the control coefficient is required to design the ESO and the compensation term for total disturbance [12,34,35]. In practical systems, the true values of control coefficients are usually unknown [24], which promotes the development of ADRC based on the nominal values or the approximative mathematical expressions of control coefficients. For the ADRC based on the nominal information of control coefficients, the literatures [24,26,28] quantitatively analyzed the stability region of uncertain control coefficients. Unfortunately, compared with the capability to handle the nonlinear uncertainties dependent on system states, the capability of the conventional ADRC to deal with the uncertainties of control coefficients is limited [36]. Besides, in some practical systems, it is difficult to obtain the nominal values or the approximative mathematical expressions of control coefficients [18,19,30]. Hence, it is significant to design a new ADRC which is featured with the strong robustness to uncertainties and does not rely on the nominal values or the approximative mathematical expressions of control coefficients.

The paper studies the control problem for a class of lower-triangular nonlinear uncertain systems. Based on the signs of control coefficients rather than the nominal values or the approximative mathematical expressions, a new ADRC design is proposed to handle the mismatched nonlinear uncertainties and the unknown values of control coefficients. The design procedure of the new ADRC is separated into three parts: (1) determining the transformation from the original states to the states of an equivalent integrators chain system, (2) designing the ESO to estimate the total disturbance and the states of the integrators chain system, and (3) forcing the actual input to track the desired input signal by designing a dynamical system. By rigorously analyzing the closed-loop properties, the bounds of tracking error, estimating error and the error between the actual and desired inputs are explicitly shown as the functions of control parameters. Based on the detailed expressions of error bounds, it is demonstrated that the satisfied closed-loop performance can be obtained despite a wide class of uncertainties by suitably enlarging the ESO's parameter. Moreover, by meticulously studying the relationship between the original states and the integrators chain states, the paper removes the bounded hypothesis for integrators chain states, which is required in [21]. The main contributions of the paper are as follows.

(1) In the conventional ADRC designs [12,26,28,34], the nominal values or the approximative mathematical expressions of control coefficients are required. In the paper, a new ADRC based on the signs of control coefficients is proposed, which can deal with a large scope of uncertain control coefficients.
(2) Compared with the mismatched bounded disturbances [5] and Lipschitz continuous uncertainties [7], the paper considers the mismatched uncertainties with nonlinear growth with respect to system states. Moreover, despite a wide class of nonlinear uncertainties, the satisfied tracking performance in the whole time period is proved.
(3) The tuning principle of the control parameters is provided. Especially, the relationship between the parameter of input dynamical system and the observer parameter is explicitly shown.

The rest of the paper has the following organization. In Sect. 2, the problem formulation is given. In Sect. 3, the new ADRC based on the signs of control coefficients is proposed. In Sect. 4, the theoretical analysis of the closed-loop system is presented. The simulation studies are shown in Sect. 5. The conclusion is presented in Sect. 6.

1.1 Notations

The following notations are used throughout the paper. $y^{(k)}(t)$ represents the k-th order derivative of y with respect to the variable t for $k \geq 1$ and $y^{(0)}(t) \triangleq y(t)$. The notations $|\cdot|$ and $\|\cdot\|$ are the absolute value of a scalar and the 2-norm of a vector or a matrix, respectively. The notation $\mathrm{diag}(a_1, a_2, \cdots, a_m)$ represents a diagonal matrix with the dimension $m \times m$, whose i-th diagonal element is a_i. For a given real symmetric matrix M, the maximal eigenvalue of M is denoted as $\lambda_{\max}(M)$ and the minimal eigenvalue of M is denoted as $\lambda_{\min}(M)$. The following useful matrices are introduced.

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{n\times n}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{n\times 1},$$
$$B_f = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{(n+1)\times 1}, \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{n\times 1}, \tag{1}$$
$$A_e = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}_{(n+1)\times(n+1)}, \quad C_e = \begin{bmatrix} C \\ 0 \end{bmatrix}_{(n+1)\times 1}.$$

The function $\mathrm{sgn}(\cdot)$ represents the sign function, which satisfies

$$\mathrm{sgn}(a) = \begin{cases} 1, & \text{if } a > 0, \\ 0, & \text{if } a = 0, \\ -1, & \text{if } a < 0. \end{cases} \tag{2}$$

2 Problem formulation

Consider the following class of lower-triangular nonlinear uncertain systems with unknown control coefficients.

$$\begin{cases} \dot{x}_i(t) = \theta_i(t)x_{i+1}(t) + \phi_i(x_1(t), \cdots, x_i(t), t), & 1 \leq i \leq n-1, \\ \dot{x}_n(t) = \theta_n(t)u(t) + \phi_n(x_1(t), \cdots, x_n(t), t), \\ \dot{z}(t) = g(z, x, t), \\ y(t) = x_1(t), \quad t \geq t_0, \end{cases} \tag{3}$$

where $x_i(t) \in R$ $(1 \leq i \leq n)$ are the system states, $x(t) \triangleq [x_1(t) \ \cdots \ x_n(t)]^T \in R^n$ is the system state vector, $z(t) \in R^m$ is the state vector of zero dynamics, $y(t) \in R$ is the measured output to be controlled, $u(t) \in R$ is the control input, $g(\cdot)$ represents the dynamics of $z(t)$, $\phi_i(\cdot)$ $(1 \leq i \leq n)$ represent the uncertainties in various channels which might be mismatched, and $\theta_i(t)$ $(1 \leq i \leq n)$ are the unknown time-varying control coefficients. As shown in [37], the signs of the control coefficients $\theta_i(t)$, i.e., $\mathrm{sgn}(\theta_i(t))$ $(1 \leq i \leq n)$, represent the control directions of the system (3). The paper considers the situation that the control directions $\mathrm{sgn}(\theta_i(t))$ $(1 \leq i \leq n)$ are known. However, both the approximative mathematical expressions and the nominal values of the control coefficients $\theta_i(t)$ are unknown.

Based on the control directions $\mathrm{sgn}(\theta_i(t))$ rather than the nominal values or the approximative mathematical expressions of control coefficients, the control objective of the system (3) is to design the control input $u(t)$ such that the output $y(t)$ can track the reference signal $r(t)$ despite the mismatched nonlinear uncertainties $\phi_i(\cdot)$ $(1 \le i \le n)$.

Remark 1 The system (3) can model plenty of practical processes, such as flight systems [15], motion control systems [16] and process control systems [20]. More importantly, under the strong nonlinearity of the unknown functions $\phi_i(\cdot)$, it is challenging to achieve the output regulation task only based on the control directions $\text{sgn}(\theta_i(t))$.

Before the detailed control design, the following assumptions for the reference signal $r(t)$, the mismatched nonlinear uncertainties $\phi_i(\cdot)$ $(1 \le i \le n)$ and the dynamics of $z(t)$ are introduced.

Assumption 1 There exists a positive constant M_r such that $\sup_{t\ge t_0} |r^{(i)}(t)| \le M_r$ for $0 \le i \le n+1$.

Remark 2 Assumption 1 implies that the reference signal and its derivatives are bounded, which is rational in practice [34].

Assumption 2 The functions $\phi_i(\cdot)$ and $\theta_i(\cdot)$ are $(n+1-i)$-th order differentiable with respect to their variables for $1 \le i \le n$. There exist positive constants $\bar{M}_{\theta,i}$ and $\underline{M}_{\theta,i}$ and continuous functions $\psi_{\phi,i}(x_1, \cdots, x_i)$ such that

$$\sup_{t\ge t_0, 0\le j\le n+1-i} |\theta_i^{(j)}(t)| \le \bar{M}_{\theta,i}, \quad \inf_{t\ge t_0} |\theta_i(t)| \ge \underline{M}_{\theta,i} > 0, \tag{4}$$

$$\sup_{t\ge t_0} \left| \frac{\partial^{\Sigma_{k=1}^{i+1} j_k} \phi_i(x_1, \cdots, x_i, t)}{\partial x_1^{j_1} \cdots \partial x_i^{j_i} \partial t^{j_{i+1}}} \right| \le \psi_{\phi,i}(x_1, \cdots, x_i), \tag{5}$$

for $\Sigma_{k=1}^{i+1} j_k \le n+i-1$, $1 \le i \le n$ and $j_p \ge 0$ $(1 \le p \le i+1)$.

Remark 3 Assumption 2 describes a large scope of internal nonlinear uncertainties and external disturbances in practice, which is more general than the assumptions in [5,7]. From (4), the control coefficients and their derivatives are assumed to be bounded. Since the practical systems are controllable, the control coefficients $\theta_i(t)$ are assumed to be nonzero for $t \ge t_0$, as shown in (4) [38]. Hence, (4) further implies that the control directions $\text{sgn}(\theta_i(t))$ will not change. We remark that the assumption for $\theta_i(t)$ is a common one [5,37]. As for the nonlinear uncertainties $\phi_i(\cdot)$, (5) implies that the uncertainties and their partial derivatives are bounded by some continuous functions $\psi_{\phi,i}(\cdot)$ dependent on system states. Due to (5), the uncertainties $\phi_i(\cdot)$ and their partial derivatives are assumed to be bounded if the system states are bounded, which allows the uncertainties to grow nonlinearly with respect to system states.

Assumption 3 There exists a radially unbounded positive-definite function $V_z(z)$ such that

$$\dot{V}_z(z(t)) = \frac{\mathrm{d}V_z}{\mathrm{d}z} g(z, x, t) \le 0, \quad \forall V_z(z) \ge r_z(\|x\|), \quad t \ge t_0, \tag{6}$$

where $r_z(\cdot)$ is a nonnegative continuous increasing function.

Remark 4 By regarding $x(t)$ as the input of the z-subsystem, Assumption 3 implies that the z-subsystem is uniformly input-state-stable [26]. For the system (3) being a linear time-invariant system, Assumption 3 is equivalent to that the system (3) is a minimum phase plant.

Remark 5 As shown in the conventional ADRC design [12,26,28,34], the nominal values or the approximative mathematical expressions of control coefficients are required, which play important roles in the design of ESO and compensation term. However, it is sometimes hard to acquire the nominal values or the approximative mathematical expressions in practical systems [18,19], which leads to the ruleless tuning of nominal control coefficients. In the other hand, due to physical mechanism, the control directions, i.e., the signs of the control coefficients, can be easily determined. Hence, the paper aims to develop a new ADRC based on the control directions rather than the nominal values or the approximative mathematical expressions of control coefficients.

In the next section, an ADRC design based on the control directions $\text{sgn}(\theta_i(t))$ is proposed to achieve the output regulation task despite multiple uncertainties.

3 ADRC based on control directions

In this section, a new ADRC based on control directions is proposed. The corresponding control diagram is shown in Fig. 1. The rest of this section consists of the following three parts.

1. The equivalent integrators chain form for the system (3) is investigated, which shows the essential relationship between the input and the output. Moreover, the output regulation task is transmitted to the state tracking task.
2. Based on the equivalent integrators chain system, an ESO is presented to estimate the derivatives of output and the unknown term named as "total disturbance."
3. Via the estimations from ESO, the control input is generated by a dynamical system, which forces the input to track the desired input signal.

3.1 Analysis for the equivalent integrators chain form

In this subsection, the equivalent integrators chain form for the system (3) is investigated.

Denote the new state vector $\tilde{x}(t) = [\tilde{x}_1(t) \cdots \tilde{x}_n(t)]^T \in R^n$ as follows.

$$\begin{cases} \tilde{x}_1(t) = x_1(t), \\ \tilde{x}_i(t) = \dot{\tilde{x}}_{i-1}(t) \\ \quad = \left(\prod_{j=1}^{i-1} \theta_j(t)\right) x_i(t) + \sum_{j=1}^{i-2} \frac{\mathrm{d}^{j-1}}{\mathrm{d}t^{j-1}} \left(\frac{\mathrm{d}\left(\prod_{k=1}^{i-j-1} \theta_k(t)\right)}{\mathrm{d}t} x_{i-j}(t) \right) \\ \quad + \sum_{j=1}^{i-1} \frac{\mathrm{d}^{j-1}}{\mathrm{d}t^{j-1}} \left(\left(\prod_{k=0}^{i-j-1} \theta_k(t) \right) \phi_{i-j}(x_1, \cdots, x_{i-j}, t) \right), \quad 2 \le i \le n, \end{cases} \tag{7}$$

where $\theta_0(t) = 1$ for $t \ge t_0$.

The following proposition illustrates the relationship between the state vectors x and $\tilde{x}$.

Proposition 1 *Consider the transformation (7) under Assumption 2. Then, there exist two continuous mappings satisfying*

$$\gamma(x,t) \triangleq \begin{bmatrix} \gamma_1(x,t) \\ \vdots \\ \gamma_{n+1}(x,t) \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_n \\ t \end{bmatrix},$$

$$\varphi(\tilde{x},t) \triangleq \begin{bmatrix} \varphi_1(\tilde{x},t) \\ \vdots \\ \varphi_{n+1}(\tilde{x},t) \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \\ t \end{bmatrix}, \tag{8}$$

and

$$\sup_{t \ge t_0, \|x\| \le \varrho_x} \left\{ |\gamma_i(x,t)|, \left\| \frac{\partial \gamma_i(x,t)}{\partial x} \right\|, \left| \frac{\partial \gamma_i(x,t)}{\partial t} \right| \right\} \le \psi_\gamma(\varrho_x), \quad \forall \varrho_x \ge 0, \ \forall 1 \le i \le n, \tag{9}$$

$$\sup_{t \ge t_0, \|\tilde{x}\| \le \tilde{\varrho}_x} \left\{ |\varphi_i(\tilde{x},t)|, \left\| \frac{\partial \varphi_i(\tilde{x},t)}{\partial \tilde{x}} \right\|, \left| \frac{\partial \varphi_i(\tilde{x},t)}{\partial t} \right| \right\} \le \psi_\varphi(\tilde{\varrho}_x), \quad \forall \tilde{\varrho}_x \ge 0, \ \forall 1 \le i \le n, \tag{10}$$

where $\psi_\gamma(\cdot)$ and $\psi_\varphi(\cdot)$ are nonnegative continuous increasing functions dependent on $\bar{M}_{\theta,i}$, $\underline{M}_{\theta,i}$ and $\psi_{\phi,i}$ for $1 \le i \le n$.

The proof of Proposition 1 is given in Appendix.

Remark 6 In the existing studies [21,28], the bounds of the mappings $\gamma_i(\cdot)$ and $\varphi_i(\cdot)$ are provided in additional assumptions. In this paper, by rigorously analyzing the detailed expressions of the mappings $\gamma_i(\cdot)$ and $\varphi_i(\cdot)$, we prove that $\gamma_i(\cdot)$ and $\varphi_i(\cdot)$ and their partial derivatives satisfy (9)–(10). Hence, the assumptions for the bounds of $\gamma_i(\cdot)$ and $\varphi_i(\cdot)$ are removed in the paper.

Based on the transformation (7) and Proposition 1, the system (3) can be rewritten as the following integrators chain system.

$$\begin{cases} \dot{\tilde{x}}_i(t) = \tilde{x}_{i+1}(t), \quad 1 \le i \le n-1, \\ \dot{\tilde{x}}_n(t) = b(t)u(t) + f(\tilde{x}(t), t), \\ \dot{z}(t) = g(z, \varphi_1(\tilde{x},t), \cdots, \varphi_n(\tilde{x},t), t), \\ y(t) = \tilde{x}_1(t), \quad t \ge t_0, \end{cases} \tag{11}$$

with the initial condition $\tilde{x}(t_0) = [\gamma_1(x(t_0), t_0) \cdots \gamma_n(x(t_0), t_0)]^T$. The control coefficient $b(t)$ and the uncertainty $f(\tilde{x}(t), t)$ have the following form.

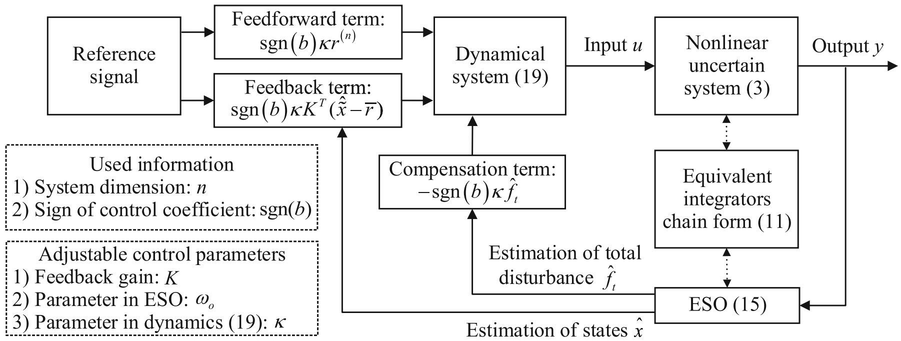


Fig. 1 Control diagram for the proposed ADRC

$$\begin{cases} b(t) = \Pi_{i=1}^{n}\theta_i(t), \\ f(\tilde{x},t) = \Sigma_{j=1}^{n-1}\dfrac{\mathrm{d}^{j-1}}{\mathrm{d}t^{j-1}}\left(\dfrac{\mathrm{d}\left(\Pi_{k=1}^{n-j}\theta_k(t)\right)}{\mathrm{d}t}\varphi_{n+1-j}(\tilde{x},t)\right) \\ \qquad + \Sigma_{j=1}^{n}\dfrac{\mathrm{d}^{j-1}}{\mathrm{d}t^{j-1}}\left(\left(\Pi_{k=0}^{n-j}\theta_k(t)\right)\tilde{\phi}_{n+1-j}(\tilde{x},t)\right), \end{cases} \tag{12}$$

where $\tilde{\phi}_i(\tilde{x},t) = \phi_i(\varphi_1(\tilde{x},t),\cdots,\varphi_i(\tilde{x},t),t)$ for $1 \le i \le n$.

If the nominal value of the control coefficient $b(t)$, denoted as $\bar{b}(t)$, can be obtained, the conventional ADRC can be applied to the system (11) by regarding $f(\tilde{x},t)+(b(t)-\bar{b}(t))u(t)$ as the total disturbance [28,34]. However, only the sign of the control coefficient $b(t)$ is known in the paper, rather than the nominal value or the approximative mathematical expression.

$$\mathrm{sgn}(b(t)) = \Pi_{i=1}^{n}\mathrm{sgn}(\theta_i(t)). \tag{13}$$

Next, based on the integrators chain form (11) and the sign of control coefficient $b(t)$, an ADRC design will be proposed.

3.2 ESO design

Since the nominal value of the control coefficient $b(t)$ is unknown, the total disturbance of the system (11) is denoted as

$$f_t(\tilde{x},u,t) \triangleq b(t)u(t) + f(\tilde{x},t). \tag{14}$$

Then, the following ESO is presented to estimate $\tilde{x}$ and f_t.

$$\begin{bmatrix} \dot{\hat{\tilde{x}}}(t) \\ \dot{\hat{f}}_t(t) \end{bmatrix} = A_e\begin{bmatrix} \hat{\tilde{x}}(t) \\ \hat{f}_t(t) \end{bmatrix} + L_e\left(y(t) - C_e^T\begin{bmatrix} \hat{\tilde{x}}(t) \\ \hat{f}_t(t) \end{bmatrix}\right), \tag{15}$$

where $\hat{\tilde{x}}(t) = [\hat{\tilde{x}}_1(t)\ \cdots\ \hat{\tilde{x}}_n(t)]^T \in R^n$ is the estimation for the state vector $\tilde{x}(t)$ and $\hat{f}_t(t) \in R$ is the estimation for the total disturbance $f_t(\tilde{x},u,t)$. In addition, the constant vector $L_e \in R^{(n+1)\times 1}$ is the tunable parameter vector of ESO such that the matrix $A_L \triangleq A_e - L_eC_e^T$ is Hurwitz. Owing to [39], the following concise tuning method of L_e is presented.

$$L_e = \left[\varsigma_1\omega_o\ \varsigma_2\omega_o^2\ \cdots\ \varsigma_{n+1}\omega_o^{n+1}\right]^T, \\ \varsigma_i = \frac{(n+1)!}{(n+1-i)!i!},\quad \omega_o > 0, \tag{16}$$

which ensures that all the eigenvalues of A_L are set at $-\omega_o$.

Next, a dynamical design of ADRC input based on the estimations from ESO will be presented.

3.3 Dynamical input design

Firstly, the desired control input is introduced. For the system (11), the desired closed-loop system satisfies the following form.

$$\begin{cases} \dot{\tilde{x}}_i^*(t) = \tilde{x}_{i+1}^*(t), \quad 1 \le i \le n-1, \\ \dot{\tilde{x}}_n^*(t) = -K^T(\tilde{x}^*(t) - \bar{r}(t)) + r^{(n)}(t), \\ \dot{z}^*(t) = g(z^*, \varphi_1(\tilde{x}^*,t),\cdots,\varphi_n(\tilde{x}^*,t),t), \\ y^*(t) = \tilde{x}_1^*(t), \quad t \ge t_0, \quad \tilde{x}^*(t_0) = \tilde{x}(t_0), \end{cases} \tag{17}$$

where $\tilde{x}^*(t) \triangleq [\tilde{x}_1^*(t)\ \cdots\ \tilde{x}_n^*(t)] \in R^n$ is the state vector of the desired system, $z^*(t) \in R^m$ is the state vector of the desired zero dynamics, $y^*(t) \in R$ is the desired output, $\bar{r}(t) \triangleq [r(t)\ \cdots\ r^{(n-1)}(t)]^T \in R^n$ and the constant vector $K \in R^{n\times 1}$ is feedback gain vector satisfying that the matrix $A_K \triangleq A - BK^T$ is Hurwitz.

Remark 7 For the desired system (17), the output $y^*(t)$ can exponentially converge to the reference signal $r(t)$ with the desired convergence rate by tuning the feedback gain vector K. Moreover, the system states $(\tilde{x}^*(t), z^*(t))$ are bounded for $t \ge t_0$.

By comparing the integrators chain system (11) with the desired system (17), the desired control input can be obtained as follows.

$$u^*(t) = \frac{-f(\tilde{x},t) - K^T(\tilde{x}(t) - \bar{r}(t)) + r^{(n)}(t)}{b(t)}. \quad (18)$$

Inspired by the approximative dynamic inversion method in [40], we design the following dynamical system to generate the actual input $u(t)$ which can approach the desired input signal $u^*(t)$.

$$\begin{aligned}\dot{u}(t) = &-\mathrm{sgn}(b(t))\kappa(\omega_o)(\hat{f}_t(t)\\ &+K^T(\hat{\tilde{x}}(t) - \bar{r}(t)) - r^{(n)}(t)),\end{aligned} \quad (19)$$

where $\kappa(\omega_o) > 0$ is a function to be designed.

The parameter $\kappa(\omega_o)$ should be carefully selected to ensure that the dynamics (19) is "slower" than the dynamics of the ESO (15) [41]. However, the explicit tuning law of $\kappa(\omega_o)$ has not been provided in the existing studies. To make the proposed method more friendly to practitioners, the tuning law of $\kappa(\omega_o)$ is depicted in the following assumption.

Assumption 4 The increasing function $\kappa(\omega_o) > 0$ for $\omega_o > 0$ and

$$\begin{aligned}&\lim_{\omega_o\to\infty} \frac{\ln \omega_o}{\sqrt{\kappa(\omega_o)}} = 0,\\ &\lim_{\omega_o\to\infty} \frac{\kappa(\omega_o)}{\omega_o} = 0.\end{aligned} \quad (20)$$

Assumption 4 describes the growth rate of the increasing function $\kappa(\omega_o)$ which grows faster than the log function $f_1(\omega_o) = (\ln \omega_o)^2$ and slower than the linear function $f_2(\omega_o) = \omega_o$. It is remarkable that the function $\kappa(\omega_o) = \omega_o^k$ with $0 < k < 1$ can satisfy Assumption 4.

Remark 8 This remark provides the design ideology of the dynamical system (19). Firstly, the dynamics of $u(t)$ is supposed to satisfy the following equation, which can force $u(t)$ to track $u^*(t)$.

$$\dot{u}(t) = -\varpi(t)(u(t) - u^*(t)), \quad (21)$$

where $\varpi(t) > 0$ is a function to be designed. By substituting (18) into (21), we have

$$\begin{aligned}\dot{u}(t) &= -\frac{\varpi(t)}{b(t)}(b(t)u(t) + f(\tilde{x},t)\\ &\quad + K^T(\tilde{x}(t) - \bar{r}(t)) - r^{(n)}(t))\\ &= -\frac{\varpi(t)}{b(t)}(f_t(\tilde{x},u,t)\\ &\quad + K^T(\tilde{x}(t) - \bar{r}(t)) - r^{(n)}(t)).\end{aligned} \quad (22)$$

Then, by substituting the estimations $\hat{f}_t$ and $\hat{\tilde{x}}$ into (22), the control input can be designed as follows.

$$\begin{aligned}\dot{u}(t) = &-\frac{\varpi(t)}{b(t)}(\hat{f}_t(t)\\ &+K^T(\hat{\tilde{x}}(t) - \bar{r}(t)) - r^{(n)}(t)).\end{aligned} \quad (23)$$

Finally, the dynamical system of input (19) is obtained by designing $\varpi(t) = \kappa(\omega_o)|b(t)|$.

4 Performance analysis of closed-loop system

The performance of the closed-loop system based on the ADRC design (15) and (19) is investigated in this section.

The following theorem shows the satisfactory transient performance of the closed-loop system based on the proposed ADRC despite a wide class of uncertainties.

Theorem 1 *Consider the system (3) with Assumptions 1–4. Let $u(t) = 0$ for $t \in [t_0, t_u)$ where*

$$\begin{cases} t_u = t_0 + 2nc_{\varsigma 2}\dfrac{\max\{\ln(\omega_o\varrho_0),\ 0\}}{\sqrt{\kappa(\omega_o)}}, \quad \varrho_0 = \max\limits_{2\le i\le n} |\tilde{x}_i(t_0) - \hat{\tilde{x}}_i(t_0)|^{\frac{1}{n}},\\ c_{\varsigma 2} = \lambda_{\max}(P_\varsigma), \quad A_\varsigma^T P_\varsigma + P_\varsigma A_\varsigma = -I, \quad A_\varsigma = \begin{bmatrix} -\varsigma_1 & 1 & 0 & \cdots & 0\\ \vdots & 0 & \ddots & \ddots & \vdots\\ \vdots & \vdots & \ddots & \ddots & 0\\ -\varsigma_n & 0 & \cdots & 0 & 1\\ -\varsigma_{n+1} & 0 & \cdots & \cdots & 0 \end{bmatrix}. \end{cases} \quad (24)$$

For $t \ge t_u$, $u(t)$ is designed according to (15) and (19). Then, there exist positive constants η_i^* $(1 \le i \le 5)$ and ω^* dependent on $(x(t_0), \hat{\tilde{x}}(t_0), \hat{f}_t(t_0), M_r,$

$\underline{M}_{\theta,i}$, $\bar{M}_{\theta,i}$, $\psi_{\phi i}$, K) *such that*

$$\sup_{t\geq t_0}|y(t)-y^*(t)| \leq \eta_1^* \max\left\{\frac{\ln\omega_o}{\sqrt{\kappa(\omega_o)}}, \frac{\kappa(\omega_o)}{\omega_o}, \frac{1}{\kappa(\omega_o)}\right\}, \tag{25}$$

$$\left\|\begin{bmatrix}\tilde{x}(t)-\hat{\tilde{x}}(t)\\ f_t(\tilde{x},u,t)-\hat{f}_t(t)\end{bmatrix}\right\| \leq \eta_2^*\left(\frac{\kappa(\omega_o)}{\omega_o}+e^{-\eta_3^*\omega_o(t-t_u)}\right), \quad \forall t\geq t_u, \tag{26}$$

$$|u(t)-u^*(t)| \leq \eta_4^*\left(\frac{\kappa(\omega_o)}{\omega_o}+\frac{1}{\kappa(\omega_o)}+e^{-\eta_5^*\kappa(\omega_o)(t-t_u)}\right), \quad \forall t\geq t_u, \tag{27}$$

for any $\omega_o \geq \omega^*$.

Theorem 1 demonstrates that the tracking error between the actual and desired outputs, the estimating error of the ESO (15) and the error between the actual and desired inputs are bounded. Furthermore, (25)–(27) explicitly show the bounds of the tracking error, estimating error and the error between the actual and desired inputs. More importantly, as shown in (25), the tracking error between the actual and ideal outputs can be sufficiently small for $t \geq t_0$ by tuning ω_o to be suitably large, which ensures the satisfied transient performance despite various uncertainties. In addition, (26)–(27) imply that both the estimating error and the error between the actual and desired inputs can converge into a neighborhood with a small boundary by suitably enlarging ω_o.

Remark 9 In the proposed design (15) and (19), the adjustable parameters are κ, K and ω_o. According to Assumption 4, the parameter κ can be designed as a function with respect to ω_o, such as $\kappa = \omega_o^k$ for a constant k satisfying $0 < k < 1$. The feedback gain K should satisfy that the matrix A_K is Hurwitz, which determines the convergence rate of the desired output y^* generated by the system (17). Based on Theorem 1, the observer parameter ω_o should be suitably large to ensure the small tracking error and estimating error.

Remark 10 The design (24) is a common way to avoid the peaking phenomenon of ESO [26,29]. Moreover, since plenty of practical systems satisfy the initial condition that $\tilde{x}(t_0) = [0 \ \cdots \ 0]^T$, by designing the initial values of ESO as $\hat{\tilde{x}}(t_0) = [0 \ \cdots \ 0]^T$, then it can be obtained that $t_u = t_0$. In addition, for the initial values satisfying $\varrho_0 \geq \frac{1}{\omega_o}$, it can be deduced from (24) and Assumption 4 that t_u can be close to t_0 by suitably enlarging ω_o.

To simplify the proof of Theorem 1, Propositions 2–3 are introduced. Proposition 2 describes the bounds of the uncertainty $f(\tilde{x},t)$, the control coefficient $b(t)$ and their derivatives. Proposition 3 provides the closed-loop form and the bounds of the uncertain terms in closed-loop system. The proofs of Propositions 2–3 are given in Appendix.

Proposition 2 *Let Assumption 2 holds. For any given positive constant* $\tilde{\varrho}_x$*, the functions* $b(t)$ *and* $f(\tilde{x},t)$ *in (12) satisfy the following equations:*

$$\sup_{t\geq t_0}\left\{|(b(t))^{-1}|, |b(t)|, |\dot{b}(t)|\right\} \leq \psi_b, \tag{28}$$

$$\sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x}\left\{|f(\tilde{x},t)|, \left\|\frac{\partial f(\tilde{x},t)}{\partial\tilde{x}}\right\|, \left|\frac{\partial f(\tilde{x},t)}{\partial t}\right|\right\} \leq \psi_f(\tilde{\varrho}_x), \tag{29}$$

where the positive constant ψ_b *and the non-decreasing function* $\psi_f(\cdot)$ *are dependent on* $\bar{M}_{\theta,i}$, $\underline{M}_{\theta,i}$ *and* $\psi_{\phi,i}$ *for* $1 \leq i \leq n$.

Denote the tracking error vector, estimating error vector and the error between the actual and desired inputs as follows.

$$e(t) = \tilde{x}(t) - \tilde{x}^*(t),$$
$$\zeta(t) = T_1^{-1}\begin{bmatrix}\tilde{x}(t)-\hat{\tilde{x}}(t)\\ f_t(\tilde{x},u,t)-\hat{f}_t(t)\end{bmatrix}, \quad \delta_u(t) = u(t)-u^*(t), \tag{30}$$

where $T_1 = \mathrm{diag}(\omega_o^{-n}, \cdots, \omega_o^{-1}, 1)$. Then, the following proposition presents the closed-loop system and further analyzes the properties of the uncertain terms in closed-loop system.

Proposition 3 *Let Assumptions 1–3 hold. Design* $u(t) = 0$ *for* $t \in [t_0, t_u)$*, and design* $u(t)$ *by (15) and (19) for* $t \in [t_u, \infty)$*. Then, the closed-loop system is shown as follow.*

$$\begin{cases}\dot{e}(t) = Ae(t) + B\Delta_{e0}(e,t),\\ \dot{z}(t) = g(z, \varphi_1(\tilde{x}^*+e,t), \cdots, \varphi_n(\tilde{x}^*+e,t), t),\\ \dot{\zeta}(t) = \omega_o A_\zeta \zeta(t) + B_f\Delta_{\zeta 0}(e,t),\\ \dot{\delta}_u(t) = -|b(t)|\kappa\delta_u(t) + \Delta_{\delta_u 0}(e,\zeta,\omega_o,\kappa,t),\end{cases} \quad t\in[t_0,t_u), \tag{31}$$

$$\begin{cases} \dot{e}(t) = A_K e(t) + B\Delta_{e1}(\delta_u, t), \\ \dot{z}(t) = g(z, \varphi_1(\tilde{x}^* + e, t), \cdots, \varphi_n(\tilde{x}^* + e, t), t), \\ \dot{\zeta}(t) = \omega_o A_\varsigma \zeta(t) + B_f \Delta_{\zeta 1}(e, \zeta, \delta_u, \omega_o, \kappa, t), \\ \dot{\delta}_u(t) = -|b(t)|\kappa\delta_u(t) + \Delta_{\delta_u 1}(e, \zeta, \delta_u, \omega_o, \kappa, t), \end{cases} \quad t \in [t_u, \infty). \tag{32}$$

Moreover, the uncertain terms Δ_{e0}, $\Delta_{\zeta 0}$, $\Delta_{\delta_u 0}$, Δ_{e1}, $\Delta_{\zeta 1}$ and $\Delta_{\delta_u 1}$ have the following bounds.

$$\begin{cases} |\Delta_{e0}| \le \pi_{e0}(\varrho_e), \quad |\Delta_{e1}| \le \psi_b|\delta_u|, \\ |\Delta_{\zeta 0}| \le \pi_{\zeta 0}(\varrho_e), \quad |\Delta_{\zeta 1}| \le \pi_{\zeta 1}(\varrho_e) + \pi_\omega(\omega_o^*)\kappa\|\zeta\| + (\kappa+1)\pi_{\delta_u}(\varrho_e, \varrho_u), \\ |\Delta_{\delta_u 0}| \le \pi_{\delta_u 0}(\varrho_e) + \pi_\omega(\omega_o^*)\kappa\|\zeta\|, \quad |\Delta_{\delta_u 1}| \le \pi_{\delta_u 1}(\varrho_e, \varrho_u) + \pi_\omega(\omega_o^*)\kappa\|\zeta\|, \end{cases} \tag{33}$$

for $e \in \{e|\ \|e\| \le \varrho_e\}$, $\zeta \in \{\zeta|\ \|\zeta\| \le \varrho_\zeta\}$, $\delta_u \in \{\delta_u|\ |\delta_u| \le \varrho_u\}$ and $\omega_o \in \{\omega_o|\ \omega_o \ge \omega_o^*\}$ with any given positives ϱ_e, ϱ_ζ, ϱ_u and ω_o^*. The functions $\pi_{e0}(\cdot)$, $\pi_{\zeta 0}(\cdot)$, $\pi_{\zeta 1}(\cdot)$, $\pi_{\delta_u}(\cdot)$, $\pi_{\delta_u 0}(\cdot)$ and $\pi_{\delta_u 1}(\cdot)$ are non-decreasing and are dependent on K, M_r, $\bar{M}_{\theta,i}$, $\underline{M}_{\theta,i}$ and $\psi_{\phi,i}$ for $1 \le i \le n$. The function $\pi_\omega(\cdot)$ is non-increasing and is dependent on $\bar{M}_{\theta,i}$, $\underline{M}_{\theta,i}$ and K.

Based on Propositions 2–3, the proof of Theorem 1 is presented as follows.

Proof of Theorem 1 With the mapping from (x, t) to $(\tilde{x}, t)$ presented in Proposition 1 and the discussions in Sect. 3.1, the plant (3) can be rewritten as the integrators chain system (11). Owing to Propositions 2–3, the closed-loop system is formulated as (31)–(32).

Since the matrices A_K and A_ς are Hurwitz, there exist positive definite matrices P_K and P_ς such that $A_K^T P_K + P_K A_K = -I$ and $A_\varsigma^T P_\varsigma + P_\varsigma A_\varsigma = -I$. Then, the following Lyapunov functions are introduced.

$$V_K(t) = e^T(t)P_K e(t), \quad V_\varsigma(t) = \zeta^T(t)P_\varsigma\zeta(t),$$
$$V_u(t) = \frac{\delta_u^2(t)}{2}. \tag{34}$$

Let c_{k1} and c_{k2} be the minimal and maximal eigenvalues of P_K and denote $c_{\varsigma 1}$ and $c_{\varsigma 2}$ as the minimal and maximal eigenvalues of P_ς. Then, the following inequalities hold.

$$c_{k1}\|e(t)\|^2 \le V_K(t) \le c_{k2}\|e(t)\|^2,$$
$$c_{\varsigma 1}\|\zeta(t)\|^2 \le V_\varsigma(t) \le c_{\varsigma 2}\|\zeta(t)\|^2. \tag{35}$$

Next, we analyze the properties of the closed-loop system for $t \in [t_0, t_u)$ and $t \in [t_u, \infty)$.

Part 1: The analysis for the closed-loop system for $t \in [t_0, t_u)$.

Owing to (11) and (17), the initial condition satisfies that $e(t_0) = 0$. The dynamical systems (31)–(32) imply the continuity of $e(t)$. Hence, for a sufficiently small positive constant Δt, there exists a positive constant η_{e1} such that

$$\|e(t)\| \le \eta_{e1}, \quad \forall t \in [t_0, t_0 + \Delta t]. \tag{36}$$

According to Assumption 4, it can be verified that $\lim_{\omega_o\to\infty} \frac{\ln \omega_o}{\sqrt{\kappa(\omega_o)}} = 0$. In addition, it can be verified from (24) that $\lim_{\omega_o\to\infty} t_u = t_0$. Hence, there exists a positive constant ω_1 such that $t_u - t_0 \le \Delta t$ for $\omega_o \ge \omega_1$. Combined with the statement (36), it can be concluded that $\|e(t)\| \le \eta_{e1}$ for $t \in [t_0, t_u)$ and $\omega_o \ge \omega_1$.

Due to Proposition 1, the bound of $x(t)$ for $t \in [t_0, t_u)$ satisfies that $\sup_{t_0 \le t \le t_u} \|x(t)\| \le n\psi_\varphi(\eta_{e1} + M_{x^*})$. Combined with Assumption 3, the following equation is satisfied.

$$\dot{V}_z(z(t)) \le 0, \quad \forall V_z(z) \ge r_z(n\psi_\varphi(\eta_{e1} + M_{x^*})),$$
$$t_0 \le t \le t_u, \tag{37}$$

which further implies that

$$\sup_{t_0 \le t \le t_u} V_z(z(t)) \le \eta_{V_z 1}$$
$$\triangleq r_z(n\psi_\varphi(\eta_{e1} + M_{x^*})). \tag{38}$$

Based on the bound of $e(t)$ for $t \in [t_0, t_u)$, the dynamics (31) implies that

$$\begin{aligned} \sup_{t_0 \le t \le t_u} \|e(t)\| &\le (t_u - t_0) \cdot \sup_{t_0 \le t \le t_u} \{\|A\|\|e(t)\| \\ &\quad + \|B\||\Delta_{e0}(e, t)|\} \\ &\le 2nc_{\varsigma 2}(\|A\|\eta_{e1} \\ &\quad + \pi_{e0}(\eta_{e1}))\frac{\ln \omega_o + |\ln \rho_0|}{\sqrt{\kappa}}. \end{aligned} \tag{39}$$

Due to (31) and (33), the following dynamics of $\sqrt{V_\varsigma(t)}$ and $\sqrt{V_u(t)}$ hold for $t \in [t_0, t_u]$.

$$\begin{cases} \frac{d\sqrt{V_\varsigma(t)}}{dt} = \frac{-\omega_o\|\zeta\|^2 + 2\zeta^T P_\varsigma B_f \Delta_{\zeta 0}}{2\sqrt{V_\varsigma}} \\ \qquad \leq -\frac{\omega_o}{2c_{\varsigma 2}}\sqrt{V_\varsigma(t)} + \frac{\|P_\varsigma\|\pi_{\zeta 0}(\eta_{e1})}{\sqrt{c_{\varsigma 1}}}, \\ \frac{d\sqrt{V_u(t)}}{dt} = \frac{-|b|\kappa|\delta_u|^2 + \delta_u \Delta_{\delta_u 0}}{2\sqrt{V_u}} \\ \qquad \leq -\psi_b^{-1}\kappa\sqrt{V_u(t)} + \frac{\sqrt{2}\pi_{\delta_u 0}(\eta_{e1}) + \sqrt{2}\pi_\omega(\omega_1)\kappa\|\zeta(t)\|}{2}, \end{cases} \tag{40}$$

for $\omega_o \geq \omega_1$. With the help of Gronwall lemma, it can be deduced from (40) that $\sqrt{V_\varsigma(t)}$ has the following bound.

$$\sqrt{V_\varsigma(t)} \leq \frac{2c_{\varsigma 2}\|P_\varsigma\|\pi_{\zeta 0}(\eta_{e1})}{\sqrt{c_{\varsigma 1}}\omega_o} + \sqrt{V_\varsigma(t_0)}e^{-\frac{\omega_o(t-t_0)}{2c_{\varsigma 2}}}, \quad t \in [t_0, t_u]. \tag{41}$$

Next, the bounds of $\sqrt{V_\varsigma(t_u)}$ and $\sqrt{V_u(t_u)}$ will be analyzed.

Based on the initial condition $\varrho_0 > \frac{1}{\omega_o}$, the initial value of $\sqrt{V_\varsigma}$ satisfies that $\sqrt{V_\varsigma(t_0)} \leq n\sqrt{c_{\varsigma 2}}(\omega_o^{n-1}\varrho_0^n + \eta_{f0})$ where $\eta_{f0} = |\hat{f}_t(t_0) - f_t(\tilde{x}(t_0), u(t_0), t_0)|$. Owing to the definition of t_u (24), the following inequalities hold for $\frac{\omega_o}{\kappa} \geq 1$ and $\kappa \geq \max\{1, \frac{1}{4\psi_b^{-2}c_{\varsigma 2}^2}\}$.

$$\begin{cases} \sqrt{V_\varsigma(t_0)}e^{-\frac{\omega_o(t_u-t_0)}{2c_{\varsigma 2}}} = \sqrt{V_\varsigma(t_0)}e^{-\frac{n\omega_o}{\sqrt{\kappa}}\ln(\omega_o\varrho_0)} \\ \qquad \leq \frac{n\sqrt{c_{\varsigma 2}}}{\omega_o} + \frac{n\sqrt{c_{\varsigma 2}}\eta_{f0}}{\omega_o^n\varrho_0^n}, \\ e^{-\psi_b^{-1}\kappa(t_u-t_0)} = e^{-2nc_{\varsigma 2}\psi_b^{-1}\sqrt{\kappa}\ln(\omega_o\varrho_0)} \leq \frac{1}{\varrho_0^n\omega_o^n}. \end{cases} \tag{42}$$

Owing to (41)–(42), the following inequalities are satisfied for any $\omega_o \geq \omega_2$, $\kappa \geq \kappa_2$ and $\frac{\omega_o}{\kappa} \geq \tau_2$, where $\omega_2 = \max\{\kappa_2\tau_2, \omega_1\}$, $\kappa_2 = \max\{1, \frac{1}{4\psi_b^{-2}c_{\varsigma 2}^2}\}$ and $\tau_2 = \max\{1, 4\psi_b^{-1}c_{\varsigma 2}\}$.

$$\begin{cases} \sqrt{V_\varsigma(t_u)} \leq \eta_{V_\varsigma 1}(\omega_2) \triangleq \frac{2c_{\varsigma 2}\|P_\varsigma\|\pi_{\zeta 0}(\eta_{e1})}{\sqrt{c_{\varsigma 1}}\omega_2} + \frac{n\sqrt{c_{\varsigma 2}}}{\omega_2} + \frac{n\sqrt{c_{\varsigma 2}}\eta_{f0}}{\omega_2^n\varrho_0^n}, \\ \sqrt{V_u(t_u)} \leq \eta_{V_u 1}(\omega_2, \kappa_2) \triangleq \frac{\sqrt{2}\pi_{\delta_u 0}(\eta_{e1})}{2\psi_b^{-1}\kappa_2} + \frac{2\sqrt{2}\pi_\omega(\omega_2)c_{\varsigma 2}\|P_\varsigma\|\pi_{\zeta 0}(\eta_{e1})\tau_2}{c_{\varsigma 1}\psi_b^{-1}} \\ \qquad + \frac{\sqrt{V_u(t_0)}}{\omega_2^n\varrho_0^n} + \frac{\sqrt{2}\pi_\omega(\omega_2)n\sqrt{c_{\varsigma 2}}}{\sqrt{c_{\varsigma 1}}\psi_b^{-1}\omega_2} + \frac{\sqrt{2}\pi_\omega(\omega_2)n\sqrt{c_{\varsigma 2}}\eta_{f0}}{\sqrt{c_{\varsigma 1}}\psi_b^{-1}\omega_2^n\varrho_0^n}. \end{cases} \tag{43}$$

Based on Gronwall lemma and (40)–(41), the bound of $\sqrt{V_u(t)}$ is obtained as follows.

$$\begin{aligned} \sqrt{V_u(t)} &\leq \frac{\sqrt{2}\pi_{\delta_u 0}(\eta_{e1})}{2\psi_b^{-1}\kappa} + \sqrt{V_u(t_0)}e^{-\psi_b^{-1}\kappa(t-t_0)} \\ &\quad + \int_{t_0}^t e^{-\psi_b^{-1}\kappa(t-s)}\sqrt{2}\pi_\omega\kappa\|\zeta(s)\|ds \\ &\leq \frac{\sqrt{2}\pi_{\delta_u 0}(\eta_{e1})}{2\psi_b^{-1}\kappa} \\ &\quad + \frac{2\sqrt{2}\pi_\omega\kappa c_{\varsigma 2}\|P_\varsigma\|\pi_{\zeta 0}(\eta_{e1})}{c_{\varsigma 1}\psi_b^{-1}\omega_o} \\ &\quad + \sqrt{V_u(t_0)}e^{-\psi_b^{-1}\kappa(t-t_0)} \\ &\quad + \frac{\sqrt{2}\pi_\omega\kappa\sqrt{V_\varsigma(t_0)}}{\sqrt{c_{\varsigma 1}}\left(\frac{\omega_o}{2c_{\varsigma 2}} - \psi_b^{-1}\kappa\right)} \\ &\quad e^{-\psi_b^{-1}\kappa(t-t_0)}(1 - e^{-(\frac{\omega_o}{2c_{\varsigma 2}} - \psi_b^{-1}\kappa)(t-t_0)}). \end{aligned}$$

Considering the initial condition satisfying $\varrho_0 \leq \frac{1}{\omega_o}$, (24) implies that $t_u = t_0$. It can be directly proved that $\|e(t_u)\|$, $V_z(z(t_u))$, $\sqrt{V_\varsigma(t_u)}$ and $\sqrt{V_u(t_u)}$ are bounded for $\omega_o \geq \omega_2$. Without loss of generality, the bounds of $\|e(t_u)\|$, $V_z(z(t_u))$, $\sqrt{V_\varsigma(t_u)}$ and $\sqrt{V_u(t_u)}$ can be also denoted as η_{e1}, $\eta_{V_z 1}$, $\eta_{V_\varsigma 1}$ and $\eta_{V_u 1}$, respectively.

Part 2: The analysis for the closed-loop system for $t \in [t_u, \infty)$.

According to the control design (19) and the closed-loop system (31)–(32), the variables e,ζ,z and u are continuous at t_u. To simplify the mathematical expressions in this part, we introduce the following notations.

$$\begin{cases} \eta_{V_K 2} \triangleq \max\{\sqrt{c_{k2}}\eta_{e1}, 4\|P_K\|\psi_b\eta_{\delta_u 2}\}, \quad \eta_{V_\varsigma 2} \triangleq \eta_{V_\varsigma 1}, \\ \eta_{V_u 2} \triangleq \max\{\eta_{V_u 1}, 2\pi_\omega(\omega_2)\eta_{\zeta 2}\psi_b\}, \\ \eta_{V_z 2} \triangleq r_z(n\psi_\varphi(\eta_{e2} + M_{x^*})), \\ \eta_{e2} \triangleq \frac{\eta_{V_K 1}}{\sqrt{c_{k1}}}, \quad \eta_{\zeta 2} \triangleq \frac{\eta_{V_\zeta 2}}{\sqrt{c_{\zeta 1}}}, \quad \eta_{\delta_u 2} \triangleq \sqrt{2}\eta_{V_u 2}. \end{cases} \tag{44}$$

Then, we proceed to prove that there exist positive constants $(\omega_3, \kappa_3, \tau_3)$ such that $(e(t), z(t), \zeta(t), \delta_u(t))$ stay in a bounded set $\Omega_1 \triangleq \{(e, z, \zeta, \delta_u) \mid \sqrt{V_K} \le \eta_{V_K 2}, \sqrt{V_z} \le \eta_{V_z 2}, \sqrt{V_\varsigma} \le \eta_{V_\varsigma 2}, \sqrt{V_u} \le \eta_{V_u 2}\}$ for $\omega_o \ge \omega_3$, $\kappa \ge \kappa_3$, $\frac{\omega_o}{\kappa} \ge \tau_3$ and $t \ge t_u$. The proof consists of the following four steps:

Step 1 We assume that there exists $t^* \in [t_u, \infty)$ such that $\sqrt{V_\varsigma(t^*)} = \eta_{V_\varsigma 2}$. Besides, $\sqrt{V_\varsigma(t)} \le \eta_{V_\varsigma 2}$, $\sqrt{V_K(t)} \le \eta_{V_K 2}$, $V_z(t) \le \eta_{V_z 2}$ and $\sqrt{V_{\delta_u}(t)} \le \eta_{V_u 2}$ for $t \in [t_u, t^*]$. Then, it can be deduced that $\|e(t)\| \le \eta_{e2}$, $\|\zeta(t)\| \le \eta_{\zeta 2}$ and $|\delta_u(t)| \le \eta_{\delta_u 2}$ for $t \in [t_u, t^*]$. Owing to the dynamics (32) and the bound of $\Delta_{\zeta 1}$ (33), the derivative of $\sqrt{V_\varsigma(t^*)}$ satisfies

$$\frac{d\sqrt{V_\varsigma(t^*)}}{dt} \le -\frac{\omega_o \eta_{V_\varsigma 2}}{2c_{\varsigma 2}} + \frac{\|P_\varsigma\|(\pi_{\zeta 1}(\eta_{e2}) + (\kappa+1)\pi_{\delta_u}(\eta_{e2}, \eta_{\delta_u 2}) + \pi_\omega(\omega_o)\kappa\eta_{\zeta 2})}{\sqrt{c_{\varsigma 1}}}. \quad (45)$$

By selecting $\omega_3 = \max\{\omega_2, 6c_{\varsigma 2}\|P_\varsigma\|(\pi_{\zeta 1}(\eta_{e2}) + \pi_{\delta_u}(\eta_{e2}, \eta_{\delta_u 2}))/(\eta_{V_\varsigma 2}\sqrt{c_{\varsigma 1}})\}$ and $\tau_3 = 6c_{\varsigma 2}\|P_\varsigma\| (\pi_\omega(\omega_2)\eta_{\zeta 2} + \pi_{\delta_u}(\eta_{e2}, \eta_{\delta_u 2}))/(\eta_{V_\varsigma 2}\sqrt{c_{\varsigma 1}})$, (45) directly implies that $\frac{d\sqrt{V_\varsigma(t^*)}}{dt} < 0$ for the (ω_o, κ) satisfying $\omega_o \ge \omega_3$ and $\frac{\omega_o}{\kappa} \ge \tau_3$.

Step 2 We assume that there exists $t^* \in [t_u, \infty)$ such that $V_z(t^*) = \eta_{V_z 2}$. Besides, $\sqrt{V_u(t)} \le \eta_{V_u 2}$, $\sqrt{V_K(t)} \le \eta_{V_K 2}$, $V_z(t) \le \eta_{V_z 2}$ and $\sqrt{V_\varsigma(t)} \le \eta_{V_\varsigma 2}$ for $t \in [t_u, t^*]$. Proposition 1, the bound of $x(t)$ satisfies that

$$\sup_{t_u \le t \le t^*} \|x(t)\| \le n\psi_\varphi \left(\sup_{t_u \le t \le t^*} \|\tilde{x}(t)\| \right) \le n\psi_\varphi(\eta_{e2} + M_{x^*}), \quad \forall t \in [t_u, t^*]. \quad (46)$$

The definition of $\eta_{V_z 2}$ (44) implies that $\eta_{V_z 2} \ge r_z(\sup_{t_u \le t \le t^*} \|x(t)\|) \ge r_z(\|x(t^*)\|)$. Due to Assumption 3, it can be verified that $\dot{V}_z(z(t^*)) \le 0$.

Step 3 We assume that there exists $t^* \in [t_u, \infty)$ such that $\sqrt{V_u(t^*)} = \eta_{V_u 2}$. Besides, $\sqrt{V_u(t)} \le \eta_{V_u 2}$, $\sqrt{V_K(t)} \le \eta_{V_K 2}$, $V_z(t) \le \eta_{V_z 2}$ and $\sqrt{V_\varsigma(t)} \le \eta_{V_\varsigma 2}$ for $t \in [t_u, t^*]$. Hence, $\|e(t)\| \le \eta_{e2}$, $\|\zeta(t)\| \le \eta_{\zeta 2}$, $|\delta_u(t)| \le \eta_{\delta_u 2}$ and $|(b(t))^{-1}| \ge \psi_b^{-1}$ for $t \in [t_u, t^*]$. Based on (32)–(33), the derivative of $\sqrt{V_u(t^*)}$ satisfies

$$\frac{d\sqrt{V_u(t^*)}}{dt} \le -\psi_b^{-1}\eta_{V_u 2}\kappa + \frac{\sqrt{2}}{2}(\pi_{\delta_u 1}(\eta_{e2}, \eta_{\delta_u 2}) + \pi_\omega \kappa \eta_{\zeta 2}). \quad (47)$$

Owing to the definition of $\eta_{V_u 2}$ (44), it can be verified that $-\frac{\psi_b^{-1}\eta_{V_u 2}}{2} + \pi_\omega(\omega_o)\eta_{\zeta 2} = -\pi_\omega(\omega_2)\eta_{\zeta 2} \le 0$ for any $\omega_o \ge \omega_3$. By choosing $\kappa_3 = \frac{2\sqrt{2}\pi_{\delta_u 1}(\eta_{e2}, \eta_{\delta_u 2})}{\psi_b^{-1}\eta_{V_u 2}}$, the following inequality holds for $\kappa \ge \kappa_3$.

$$-\frac{\psi_b^{-1}\eta_{V_u 2}}{2}\kappa + \frac{\sqrt{2}\pi_{\delta_u 1}(\eta_{e2}, \eta_{\delta_u 2})}{2} < 0. \quad (48)$$

Hence, $\frac{d\sqrt{V_u(t^*)}}{dt} < 0$ for $\omega_o \ge \omega_3$ and $\kappa \ge \kappa_3$.

Step 4 We assume that there exists $t^* \in [t_u, \infty)$ such that $\sqrt{V_K(t^*)} = \eta_{V_K 2}$. Besides, $\sqrt{V_K(t)} \le \eta_{V_K 2}$, $\sqrt{V_\varsigma(t)} \le \eta_{V_\varsigma 2}$, $V_z(t) \le \eta_{V_z 2}$ and $\sqrt{V_u(t)} \le \eta_{V_u 2}$ for $t \in [t_u, t^*]$. Hence, $\|e(t)\| \le \eta_{e2}$, $\|\zeta(t)\| \le \eta_{\zeta 2}$ and $|\delta_u(t)| \le \eta_{\delta_u 2}$ for $t \in [t_u, t^*]$. According to (32), (33) and (44), the derivative of $\sqrt{V_K(t^*)}$ satisfies

$$\frac{d\sqrt{V_K(t^*)}}{dt} \le -\frac{\|e(t^*)\|}{2\sqrt{V_K(t^*)}}(\|e(t^*)\| - 2\|P_K\|\psi_b\eta_{\delta_u 2}) < 0. \quad (49)$$

Due to Steps 1–4, it can be concluded that the variables $e(t)$, $z(t)$, $\zeta(t)$ and $\delta_u(t)$ stay in Ω_1 for $t \in [t_u, \infty)$ if the parameters satisfy $\omega_o \ge \omega_3$, $\kappa \ge \kappa_3$ and $\frac{\omega_o}{\kappa} \ge \tau_3$. Next, we will analyze the bounds of $\sqrt{V_K(t)}$, $\sqrt{V_\varsigma(t)}$ and $\sqrt{V_u(t)}$ for $t \ge t_u$.

The analysis of the bound of $\sqrt{V_\varsigma(t)}$. Due to (32)–(33), for $t \ge t_u$, we have

$$\frac{d\sqrt{V_\varsigma(t)}}{dt} \le -\frac{\omega_o}{2c_{\varsigma 2}}\sqrt{V_\varsigma(t)} + \frac{\|P_\varsigma\|(\pi_{\zeta 1}(\eta_{e2}) + (\kappa+1)\pi_{\delta_u}(\eta_{e2}, \eta_{\delta_u 2}) + \pi_\omega(\omega_o)\kappa\eta_{\zeta 2})}{\sqrt{c_{\varsigma 1}}}. \quad (50)$$

Combined with Gronwall lemma, $\sqrt{V_\varsigma(t)}$ has the following bound for $\omega_o \ge \omega_3$, $\kappa \ge \kappa_3$ and $\frac{\omega_o}{\kappa} \ge \tau_3$.

$$\sqrt{V_\varsigma(t)} \le \eta_{V_\varsigma 2} e^{-\frac{\omega_o(t-t_u)}{2c_{\varsigma 2}}} + \theta_{\varsigma 1}\frac{1}{\omega_o} + \theta_{\varsigma 2}\frac{\kappa}{\omega_o}, \quad \forall t \in [t_u, \infty), \quad (51)$$

where $\theta_{\varsigma 1} = (\|P_\varsigma\|(\pi_{\zeta 1}(\eta_{e2}) + \pi_{\delta_u}(\eta_{e2}, \eta_{\delta_u 2}))/\sqrt{c_{\varsigma 1}}$ and $\theta_{\varsigma 2} = (\|P_\varsigma\|(\pi_{\delta_u}(\eta_{e2}, \eta_{\delta_u 2}) + \pi_\omega(\omega_3)\eta_{\zeta 2}))/\sqrt{c_{\varsigma 1}}$.

The analysis of the bound of $\sqrt{V_u(t)}$. Owing to (32), (33) and (51), for $t \geq t_u$, there is

$$\begin{aligned}\frac{\mathrm{d}\sqrt{V_u(t)}}{\mathrm{d}t} &\leq -\psi_b^{-1}\kappa\sqrt{V_u(t)} \\ &\quad + \frac{\sqrt{2}}{2}(\pi_{\delta_u 1}(\eta_{e2}, \eta_{\delta_u 2}) + \pi_\omega \kappa\|\zeta(t)\|) \\ &\leq -\psi_b^{-1}\kappa\sqrt{V_u(t)} \\ &\quad + \frac{\sqrt{2}}{2}\left(\pi_{\delta_u 1}(\eta_{e2}, \eta_{\delta_u 2}) + \kappa\frac{\pi_\omega\left(\eta_{V_\varsigma 2}e^{-\frac{\omega_o}{2c_{\varsigma 2}}(t-t_u)} + \theta_{\varsigma 1}\frac{1}{\omega_o} + \theta_{\varsigma 2}\frac{\kappa}{\omega_o}\right)}{\sqrt{c_{\varsigma 1}}}\right).\end{aligned}$$

Combined with Gronwall lemma, the bound of $\sqrt{V_u(t)}$ for $\omega_o \geq \omega_3, \kappa \geq \kappa_3$ and $\frac{\omega_o}{\kappa} \geq \tau_4 \triangleq \max\{\tau_3, 4c_{\varsigma 2}\psi_b^{-1}\}$ is shown as follows.

$$\begin{aligned}\sqrt{V_u(t)} &\leq \eta_{V_u 2}e^{-\psi_b^{-1}\kappa(t-t_u)} + \frac{\theta_{u1}}{\omega_o} + \frac{\theta_{u2}\kappa}{\omega_o} \\ &\quad + \frac{\theta_{u3}}{\kappa} + \theta_{u4}\int_{t_u}^{t} e^{-\psi_b^{-1}\kappa(t-s)}e^{-\frac{\omega_o}{2c_{\varsigma 2}}(s-t_u)}\mathrm{d}s \\ &\leq (\eta_{V_u 2} + \frac{\theta_{u4}}{\psi_b^{-1}\kappa_3})e^{-\psi_b^{-1}\kappa(t-t_u)} + \theta_{u1}\frac{1}{\omega_o} \\ &\quad + \theta_{u2}\frac{\kappa}{\omega_o} + \theta_{u3}\frac{1}{\kappa}, \quad \forall t \in [t_u, \infty),\end{aligned} \tag{52}$$

where $\theta_{u1} = \frac{\sqrt{2}\pi_\omega(\omega_3)\theta_{\varsigma 1}}{2\psi_b^{-1}\sqrt{c_{\varsigma 1}}}$, $\theta_{u2} = \frac{\sqrt{2}\pi_\omega(\omega_3)\theta_{\varsigma 2}}{2\psi_b^{-1}\sqrt{c_{\varsigma 1}}}$, $\theta_{u3} = \frac{\sqrt{2}\pi_{\delta_u 1}(\eta_{e2}, \eta_{\delta_u 2})}{2\psi_b^{-1}}$ and $\theta_{u4} = \frac{\sqrt{2}\pi_\omega(\omega_3)\eta_{V_\varsigma 2}}{2\psi_b^{-1}\sqrt{c_{\varsigma 1}}}$.

The analysis of the bound of $\sqrt{V_K(t)}$. By denoting $\tilde{t}_u = t_u + \frac{\ln\omega_o}{\psi_b^{-1}\sqrt{\kappa}}$, it can be deduced from (32) and (39) that

$$\begin{cases}\sup_{t_u \leq t \leq \tilde{t}_u}\sqrt{V_K(t)} \leq \sqrt{c_{k2}}\left(\|e(t_u)\| + (\|A_K\|\eta_{e2} + \psi_b\eta_{\delta_u 2})\frac{\ln\omega_o}{\psi_b^{-1}\sqrt{\kappa}}\right) \\ \qquad \leq \theta_e\frac{\ln\omega_o}{\sqrt{\kappa}}, \\ e^{-\psi_b^{-1}\kappa(\tilde{t}_u - t_u)} = \frac{1}{\omega_o^{\sqrt{\kappa}}} \leq \frac{1}{\omega_o},\end{cases} \tag{53}$$

for $\omega_o \geq \omega_3$, $\kappa \geq \kappa_4 \triangleq \max\{\kappa_3, 1\}$ and $\frac{\omega_o}{\kappa} \geq \tau_4$, where $\theta_e = 2\sqrt{c_{k2}}nc_{\varsigma 2}(\|A\|\eta_{e1} + \pi_{e0}(\eta_{e1})) + \frac{\sqrt{c_{k2}}\|A_K\|\eta_{e2} + \sqrt{c_{k2}}\psi_b\eta_{\delta_u 2}}{\psi_b^{-1}}$. Then, the bound of $\sqrt{V_K(t)}$ for $t \geq \tilde{t}_u$ is analyzed. According to (32), (33) and (52)–(53), the dynamics of $\sqrt{V_K(t)}$ satisfies the following equation for $t \geq \tilde{t}_u$.

$$\begin{aligned}\frac{d\sqrt{V_K(t)}}{dt} &\leq -\frac{\sqrt{V_K(t)}}{2c_{k2}} + \frac{\|P_K\|\psi_b|\delta_u(t)|}{\sqrt{c_{k1}}} \\ &\leq -\frac{\sqrt{V_K(t)}}{2c_{k2}} + \frac{\sqrt{2}\|P_K\|\psi_b}{\sqrt{c_{k1}}} \\ &\quad \left((\eta_{V_u 2} + \frac{\theta_{u4}}{\psi_b^{-1}\kappa})e^{-\psi_b^{-1}\kappa(t-t_u)}\right. \\ &\quad \left. + \frac{\theta_{u1}}{\omega_o} + \frac{\theta_{u2}\kappa}{\omega_o} + \frac{\theta_{u3}}{\kappa}\right) \\ &\leq -\frac{\sqrt{V_K(t)}}{2c_{k2}} + \frac{\sqrt{2}\|P_K\|\psi_b}{\sqrt{c_{k1}}}\left((\eta_{V_u 2}\right. \\ &\quad \left. + \frac{\theta_{u4}}{\psi_b^{-1}\kappa} + \theta_{u1})\frac{1}{\omega_o} + \frac{\theta_{u2}\kappa}{\omega_o} + \frac{\theta_{u3}}{\kappa}\right)\end{aligned}$$

for $\omega_o \geq \omega_3$, $\kappa \geq \kappa_4$ and $\frac{\omega_o}{\kappa} \geq \tau_4$. With the help of Gronwall lemma, we get the following bound of $\sqrt{V_K}$ for $\omega_o \geq \omega_3$, $\kappa \geq \kappa_4$, and $\frac{\omega_o}{\kappa} \geq \tau_4$.

$$\begin{aligned}\sup_{t \geq \tilde{t}_u}\sqrt{V_K(t)} &\leq \sqrt{V_K(\tilde{t}_u)} + \frac{\theta_{e1}}{\omega_o} + \frac{\theta_{e2}\kappa}{\omega_o} + \frac{\theta_{e3}}{\kappa} \\ &\leq \theta_e\frac{\ln\omega_o}{\sqrt{\kappa}} + \frac{\theta_{e1}}{\omega_o} + \frac{\theta_{e2}\kappa}{\omega_o} + \frac{\theta_{e3}}{\kappa},\end{aligned} \tag{54}$$

where $\theta_{e1} = \frac{4c_{k2}\|P_K\|\psi_b(\eta_{V_u 2} + \frac{\theta_{u4}}{\psi_b^{-1}\kappa_3} + \theta_{u1})}{\sqrt{c_{k1}}}$, $\theta_{e2} = \frac{4c_{k2}\|P_K\|\psi_b\theta_{u2}}{\sqrt{c_{k1}}}$ and $\theta_{e3} = \frac{4c_{k2}\|P_K\|\psi_b\theta_{u3}}{\sqrt{c_{k1}}}$.

According to Assumption 4, there exists a positive constant $\omega_4 \geq \omega_3$ such that

$$\kappa(\omega_o) \geq \kappa_4, \quad \frac{\omega_o}{\kappa(\omega_o)} \geq \tau_4, \tag{55}$$

for any $\omega_o \geq \omega_4$. Notice that $\frac{1}{\omega_o} \leq \frac{\kappa}{\omega_o}$ for $\kappa \geq \kappa_4 \geq 1$. With the combination of the bounds of $\sup_{t_0 \leq t \leq t_u}\|e(t)\|$, $\sqrt{V_K(t)}$, $\sqrt{V_\varsigma(t)}$ and $\sqrt{V_u(t)}$, i.e., (39), (51), (52), (53) and (54), the equations (25)–(27) hold for $\omega_o \geq \omega_4$. □

5 Simulation

In this section, the simulation for an application example, Chua's circuit, is presented.

Figure 2 describes Chua's circuit, which is featured with strong nonlinearity and can generate chaotic response [42]. The mathematical model of Chua's circuit is presented as follows [37,42].

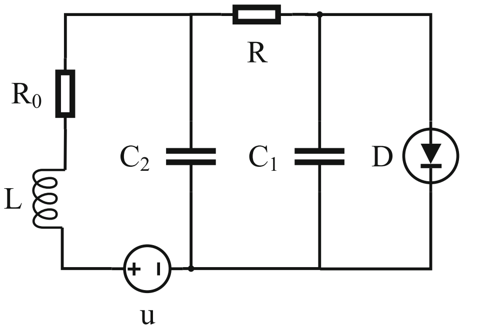


Fig. 2 Chua's circuit

$$\begin{cases} \dot{x}_1(t) = -\dfrac{1}{C_1 R}x_1(t) + \dfrac{1}{C_1 R}x_2(t) - \dfrac{1}{C_1} f_D(x_1), \\ \dot{x}_2(t) = -\dfrac{1}{C_2 R}x_1(t) + \dfrac{1}{C_2 R}x_2(t) + \dfrac{1}{C_2}x_3(t), \\ \dot{x}_3(t) = \dfrac{1}{L}u(t) - \dfrac{1}{L}x_2(t) - \dfrac{R_0}{L}x_3(t), \end{cases} \tag{56}$$

where $x_1(t)$ and $x_2(t)$ represent the voltages across the capacitor C_1 and C_2, $x_3(t)$ is the current through the inductor L, $u(t)$ is the input voltage, $f_D(x_1)$ represents the nonlinear current caused by the nonlinear resistor D, and R and R_0 are resistances. The units of voltage, current, capacitance, inductance and resistance are volt (V), ampere (A), farad (F), henry (H) and ohm (Ω), respectively.

The control objective is to design the input voltage $u(t)$ such that the system states $(x_1(t), x_2(t), x_3(t))$ can track the reference signal (0, 0, 0).

the detailed values of the system parameters, i.e., C_1, C_2, L, R and R_0, are unknown for control design, whereas the signs of system parameters can be directly verified by physical mechanism:

$$\begin{aligned} &C_1 > 0\,(F), \quad C_2 > 0\,(F), \quad L > 0\,(H), \\ &R > 0\,(\Omega), \quad R_0 \geq 0\,(\Omega). \end{aligned} \tag{57}$$

According to [37,42], the unknown nonlinear function $f_D(x_1)$ satisfies that $f_D(0) = 0$.

Remark 11 The presented control objective is a classical stabilization problem [37,42]. However, the methods proposed in [37,42] require the measurements for all states, i.e., x_1, x_2 and x_3. In this paper, only the measurement of x_1 is utilized for the stabilization problem. Moreover, the nominal values of system parameters, i.e., C_1, C_2, L, R and R_0, are unknown.

Next, the proposed ADRC is applied to the system (56). Firstly, we denote the following new states.

$$\begin{cases} \tilde{x}_1(t) = x_1(t), \\ \tilde{x}_2(t) = -\dfrac{1}{C_1 R}x_1(t) + \dfrac{1}{C_1 R}x_2(t) - \dfrac{1}{C_1} f(x_1), \\ \tilde{x}_3(t) = \left(-\dfrac{1}{C_1 R} - \dfrac{1}{C_1}\dfrac{df(x_1)}{dx_1}\right)\left(-\dfrac{x_1(t)}{C_1 R} + \dfrac{x_2(t)}{C_1 R} - \dfrac{f(x_1)}{C_1}\right) \\ \qquad + \dfrac{1}{C_1 R}\left(-\dfrac{x_1(t)}{C_2 R} + \dfrac{x_2(t)}{C_2 R} + \dfrac{x_3(t)}{C_2}\right). \end{cases} \tag{58}$$

Then, the integrators chain form is obtained as follows.

$$\begin{cases} \dot{\tilde{x}}_1(t) = \tilde{x}_2(t), \\ \dot{\tilde{x}}_2(t) = \tilde{x}_3(t), \\ \dot{\tilde{x}}_3(t) = bu(t) + f, \end{cases} \tag{59}$$

where b and f satisfy the following equation.

$$\begin{cases} b = \dfrac{1}{C_1 C_2 R L}, \\ f = -\dfrac{1}{C_1}\dfrac{d^2 f_D(x_1)}{dx_1^2}\left(-\dfrac{x_1(t)}{C_1 R} + \dfrac{x_2(t)}{C_1 R} - \dfrac{f_D(x_1)}{C_1}\right)^2 - \dfrac{1}{C_1 C_2 R^2}\left(-\dfrac{x_1(t)}{C_1 R} + \dfrac{x_2(t)}{C_1 R} - \dfrac{f_D(x_1)}{C_1}\right) \\ \quad + \dfrac{1}{C_1 C_2 R^2}\left(-\dfrac{x_1(t)}{C_2 R} + \dfrac{x_2(t)}{C_2 R} + \dfrac{x_3(t)}{C_2}\right) - \dfrac{1}{C_1 C_2 R}\left(\dfrac{x_2(t)}{L} + \dfrac{R_0 x_3(t)}{L}\right) \\ \quad + \left(-\dfrac{1}{C_1 R} - \dfrac{1}{C_1}\dfrac{df_D(x_1)}{dx_1}\right)\left(-\dfrac{1}{C_1 R} - \dfrac{1}{C_1}\dfrac{df_D(x_1)}{dx_1}\right)\left(-\dfrac{x_1(t)}{C_1 R} + \dfrac{x_2(t)}{C_1 R} - \dfrac{f_D(x_1)}{C_1}\right) \\ \quad + \dfrac{1}{C_1 R}\left(-\dfrac{1}{C_1 R} - \dfrac{1}{C_1}\dfrac{df_D(x_1)}{dx_1}\right)\left(-\dfrac{x_1(t)}{C_2 R} + \dfrac{x_2(t)}{C_2 R} + \dfrac{x_3(t)}{C_2}\right). \end{cases}$$

In the simulation, the measurement of x_1 can be obtained, while x_2 and x_3 cannot be measured. Besides,

Due to the transformation (58), it can be verified that $[x_1\ x_2\ x_3] = [0\ 0\ 0]$ is equivalent to $[\tilde{x}_1\ \tilde{x}_2\ \tilde{x}_3] =$

Fig. 3 The response curves of the state x_1 for Cases 1–3

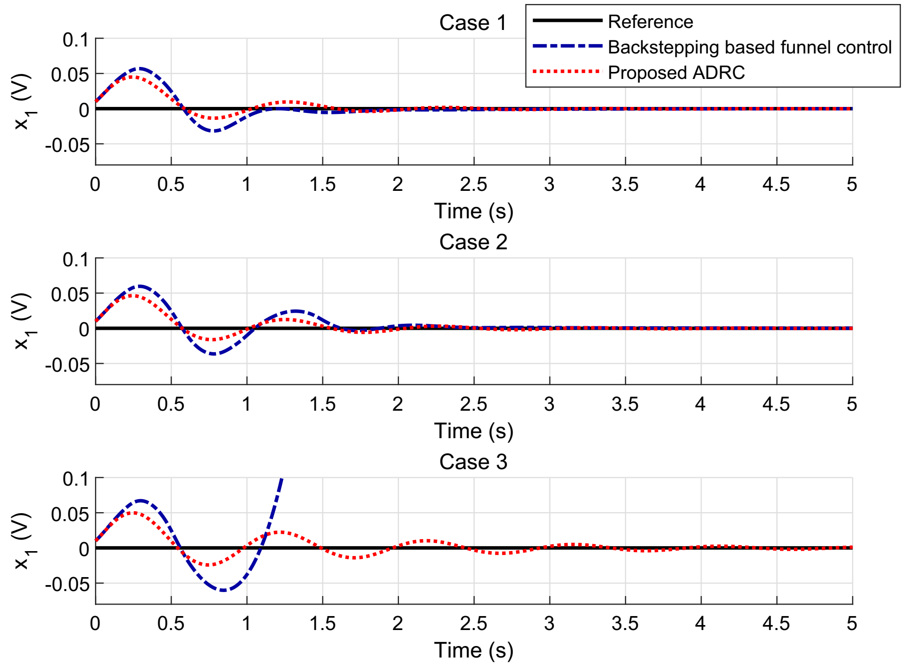
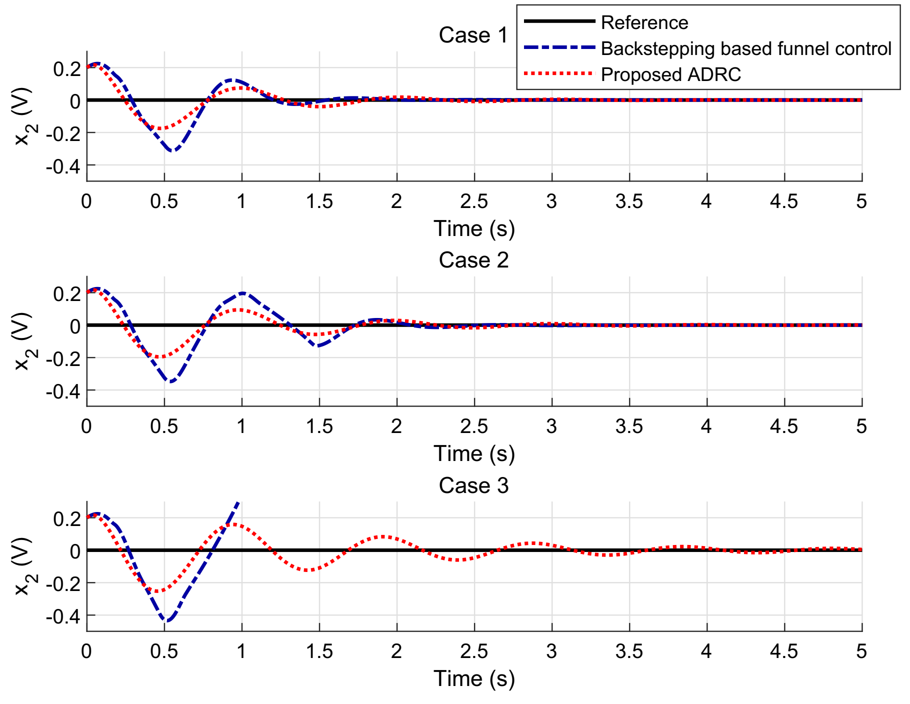


Fig. 4 The response curves of the state x_2 for Cases 1–3



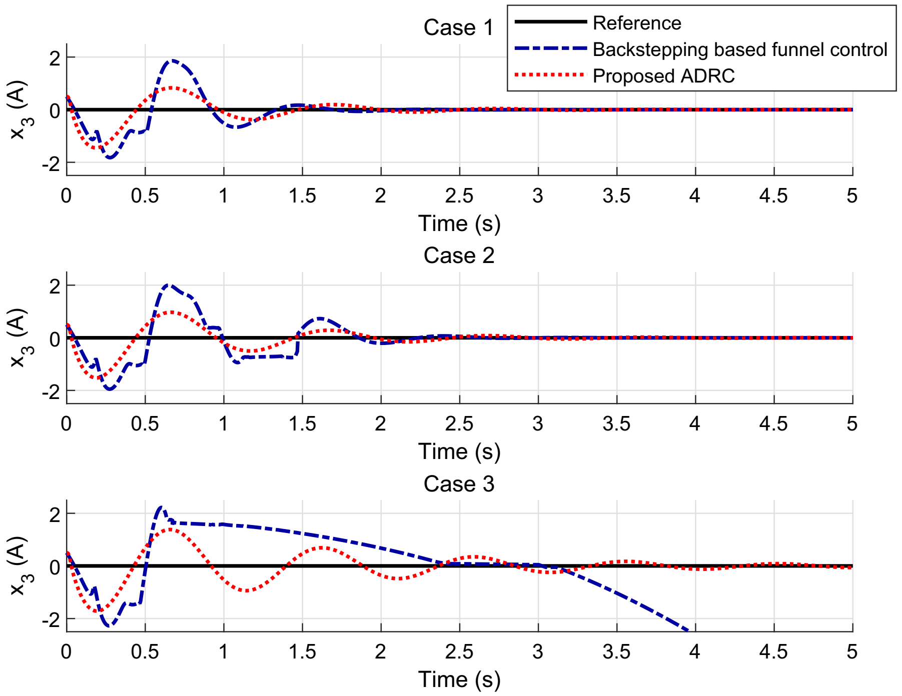


Fig. 5 The response curves of the state x_3 for Cases 1–3

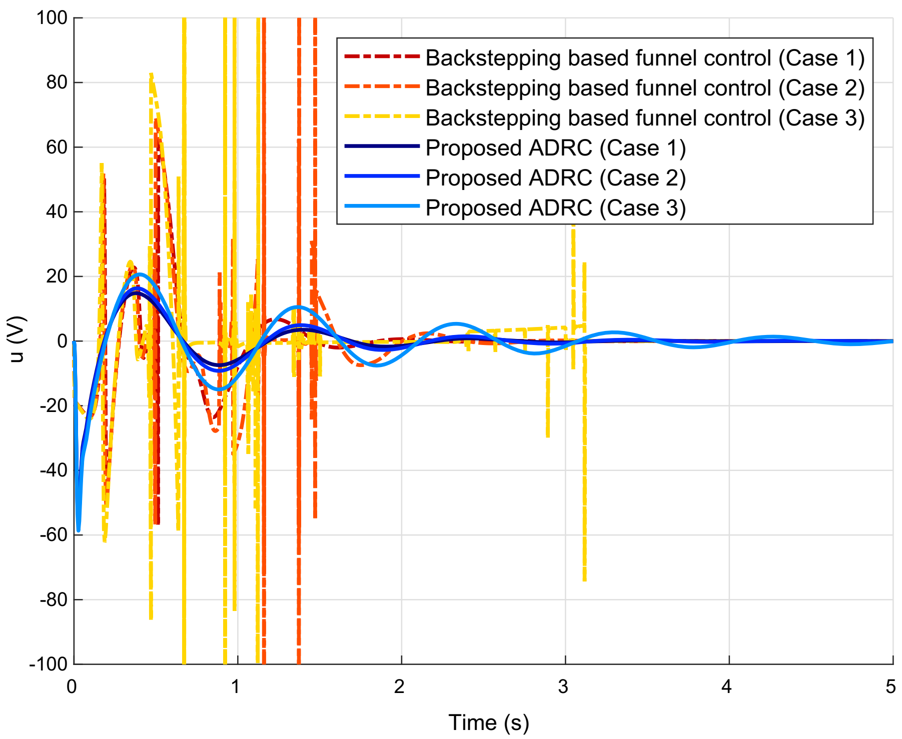


Fig. 6 The response curves of the input u for Cases 1–3

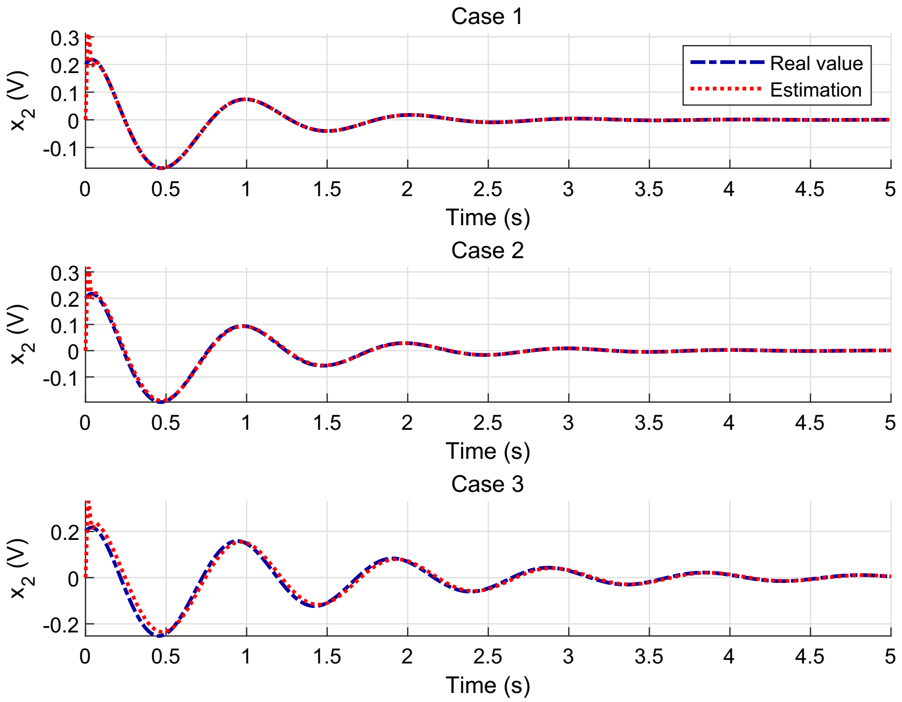


Fig. 7 The estimation of the state x_2 via ESO for Cases 1–3

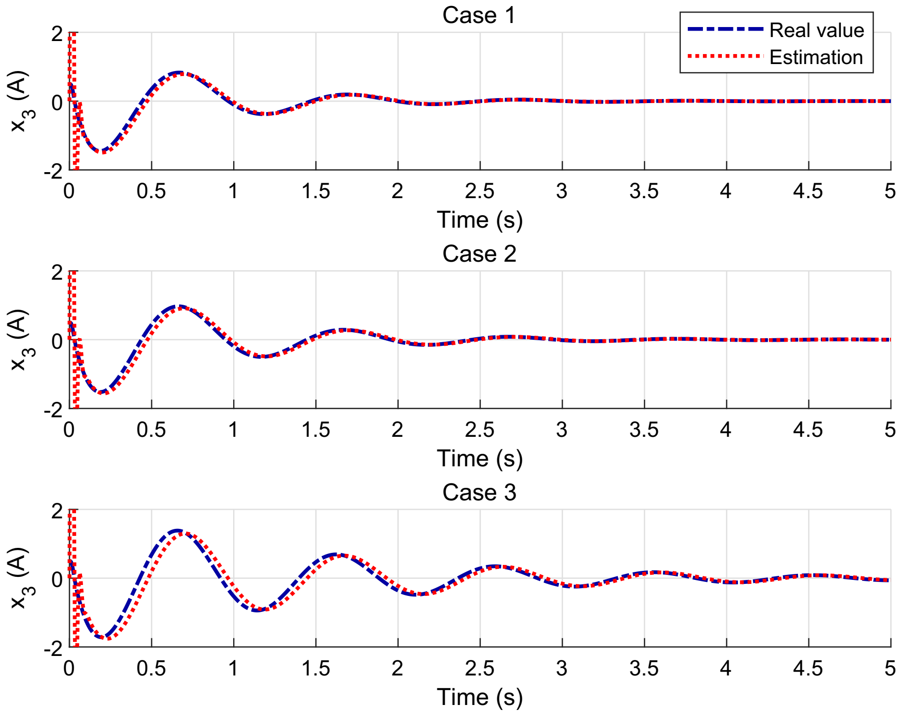


Fig. 8 The estimation of the state x_3 via ESO for Cases 1–3

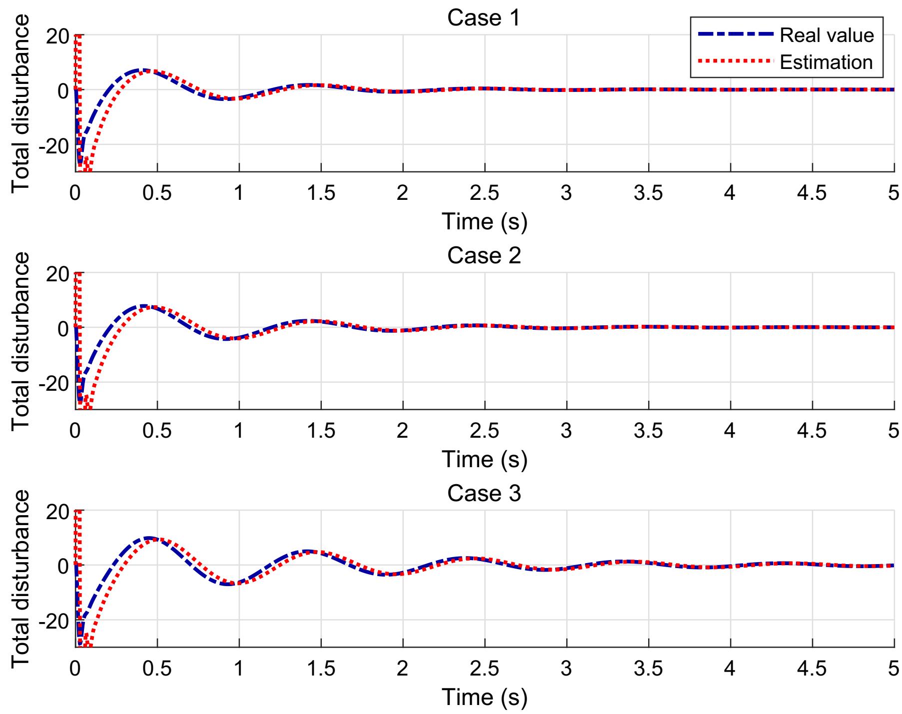


Fig. 9 The estimation of the total disturbance via ESO for Cases 1–3

[0 0 0]. Hence, the stabilization problem of the system (56) can be reformulated as the stabilization problem of the system (59).

Although the nominal value of b is unknown due to the unknown system parameters C_1, C_2, L and R, it can be verified by (57) that $\mathrm{sgn}(b) > 0$. Based on the sign of b, the proposed ADRC (15) and (19) with the following controller parameters is utilized.

$$\omega_o = 100, \quad K = [8\ 12\ 6]^T, \quad \kappa(\omega_o) = \sqrt{\omega_o}. \tag{60}$$

To investigate the capability of disturbance rejection, the following cases of uncertainties are considered, including the cubic function in [37] (Case 1).

Case 1 $f_D(x_1) = -x_1 + x_1^3$,

Case 2 $f_D(x_1) = -1.3x_1 + x_1^3$,

Case 3 $f_D(x_1) = -x_1 + x_1^3 - \sin(x_1)$.

We consider the following system parameters and initial condition, which are provided in [37].

$$\begin{cases} C_1 = C_2 = 1\ (F),\ R_0 = 0\ (\Omega),\ R = 1\ (\Omega),\ L = 2\ (H), \\ x_1(0) = 0.01\ (V),\ x_2(0) = 0.2\ (V),\ x_3(0) = 0.5\ (A). \end{cases} \tag{61}$$

The simulation results of the proposed ADRC and the following backstepping-based funnel control [37] are presented in Figs. 3–9.

$$\begin{cases} u(t) = 7r_3(t)\cos(\pi r_3(t))z_3(t), \\ z_1(t) = x_1(t), \\ z_2(t) = x_2(t) - 10r_1(t)\cos(\pi r_1(t))z_1(t), \\ z_3(t) = x_3(t) - 7r_2(t)\cos(\pi r_2(t))z_2(t), \\ r_i(t) = \dfrac{1}{1-(e^t-1)^2 z_i^2(t)}, \quad i = 1, 2, 3. \end{cases} \tag{62}$$

In addition, the integral of squared tracking error (ISE) and the integral of squared control input (ISCI) are presented in Table 1.

For Case 1, the satisfied closed-loop performance of the proposed ADRC and the backstepping-based funnel control is shown in Figs. 3–5. From Figs. 3–5 and ISE in Table 1, the closed-loop performance of the backstepping-based funnel control becomes poor for Cases 2–3. Especially for Case 3, the backstepping-based funnel control systems becomes unstable. Moreover, Figs. 7–9 depict the estimating performance of the proposed method, where the estimations for unmeasured integrators chain states and total disturbance are close to the real value. The satisfied estimating performance results in the highly consistent tracking performance of the proposed ADRC despite mismatched nonlinear uncertainty. Furthermore, according to the

Table 1 Performance indicators for proposed ADRC and backstepping-based funnel control

Methods	ISE	ISCI
Proposed ADRC (Case 1)	6.6×10^{-4}	196.4
Proposed ADRC (Case 2)	7.3×10^{-4}	225.9
Proposed ADRC (Case 3)	1.0×10^{-3}	359.9
Backstepping-based funnel control (Case 1)	1.2×10^{-3}	532.3
Backstepping-based funnel control (Case 2)	1.5×10^{-3}	17940
Backstepping-based funnel control (Case 3)	10.6	1527.9

response curves of inputs shown in Fig. 6 and ISCI in Table 1, the control energy consumption of proposed ADRC is much smaller.

6 Conclusion

For a class of lower-triangular nonlinear uncertain systems, the paper proposes a new ADRC based on the control directions rather than the nominal values or the approximative mathematical expressions of control coefficients. The design ideology can be summarized as the following three parts: (1) By transforming the original states into the states of an integrators chain system, the effects from the control input and uncertainties to the controlled output are clearly shown; (2) Based on the integrators chain form, the ESO is presented to estimate the total disturbance and the integrators chain states; (3) Inspired by the approximative dynamic inversion method, a dynamical system is designed to generate the input, which can approach the desired input signal. Moreover, by associating the parameter in dynamical input design with the ESO's parameter, the tuning method of the parameter in dynamical input design is explicitly provided. With the consideration of a large scope of mismatched nonlinear uncertainties, the transient performance of the proposed ADRC is theoretically investigated. Based on the presented theoretical results, the satisfied tracking and estimating performance can be ensured by suitably enlarging the ESO's parameter.

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Data availability The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest regarding the publication of this paper.

Appendix

Proof of Proposition 1

Part I: The analysis of the mapping φ. With the combination of the dynamics (3) and the explicit form of $\tilde{x}_i$ $(1 \le i \le n)$ (7), it can be directly verified that $\tilde{x}_i = \dot{\tilde{x}}_{i-1}$ for $2 \le i \le n$. Then, for the transformation (7), we will prove that the term

$$\tilde{\tau}_i \triangleq \Sigma_{j=1}^{i-2} \frac{d^{j-1}}{dt^{j-1}} \left(\frac{d(\Pi_{k=1}^{i-j-1} \theta_k(t))}{dt} x_{i-j} \right) + \Sigma_{j=1}^{i-1} \frac{d^{j-1}}{dt^{j-1}} \left((\Pi_{k=0}^{i-j-1} \theta_k(t)) \phi_{i-j} \right) \quad (63)$$

is a function dependent on $(x_1, \cdots, x_{i-1}, t)$ for $2 \le i \le n$ by mathematical induction in the following three steps.

Step 1 (Consider the case that $i = 2$). It can be obtained that

$$\tilde{\tau}_2 = \theta_0(t)\phi_1(x_1, t) = \phi_1(x_1, t). \quad (64)$$

Hence, $\tilde{\tau}_2$ is a function dependent on (x_1, t).

Step 2 (Consider the case that $i = k$ $(2 \le k \le n-1)$). Suppose that $\tilde{\tau}_k$ $(2 \le k \le n-1)$ is a function dependent on $(x_1, \cdots, x_{k-1}, t)$.

Step 3 (Consider the case that $i = k+1$ $(2 \le k \le n-1)$). Since $\tilde{\tau}_k$ is a function dependent on

$(x_1, \cdots, x_{k-1}, t)$, the state $\tilde{x}_k$ has the following form:

$$\tilde{x}_k(t) = (\Pi_{j=1}^{k-1}\theta_j(t))x_i(t) + \tilde{\tau}_k(x_1, \cdots, x_{k-1}, t). \tag{65}$$

By taking the derivative of $\tilde{x}_k$ and utilizing the dynamics (3), there is

$$\begin{aligned} \dot{\tilde{x}}_k(t) &= \tilde{x}_{k+1}(t) \\ &= (\Pi_{j=1}^{k-1}\theta_j(t))(\theta_k(t)x_{k+1}(t) + \phi_k(x_1, \cdots, x_k, t)) \\ &\quad + \frac{d(\Pi_{j=1}^{k-1}\theta_j(t))}{dt}x_k(t) \\ &\quad + \frac{\partial\tilde{\tau}_k(x_1, \cdots, x_{k-1}, t)}{\partial t} \\ &\quad + \Sigma_{j=1}^{k-1}\frac{\partial\tilde{\tau}_k(x_1, \cdots, x_{k-1}, t)}{\partial x_j} \\ &\quad (\theta_j x_{j+1} + \phi_j(x_1, \cdots, x_j, t)). \end{aligned} \tag{66}$$

By comparing the form (66) with (7), it can be obtained that

$$\begin{aligned} \tilde{\tau}_{k+1} &= (\Pi_{j=1}^{k-1}\theta_j(t))\phi_k(x_1, \cdots, x_k, t) \\ &\quad + \frac{d(\Pi_{j=1}^{k-1}\theta_j(t))}{dt}x_k(t) + \frac{\partial\tilde{\tau}_k}{\partial t} \\ &\quad + \Sigma_{j=1}^{k-1}\frac{\partial\tilde{\tau}_k}{\partial x_j}(\theta_j x_{j+1} + \phi_j(x_1, \cdots, x_j, t)). \end{aligned} \tag{67}$$

Due to the supposition in Step 2, (67) illustrates that $\tilde{\tau}_{k+1}$ is a function dependent on $(x_1, \cdots, x_k, t)$.

Based on the fact that $\tilde{\tau}_i$ is a function dependent on $(x_1, \cdots, x_{i-1}, t)$ for $2 \le i \le n$, (7) can be rewritten in the following form.

$$\begin{cases} \tilde{x}_1 = x_1, \\ \tilde{x}_i = (\Pi_{j=1}^{i-1}\theta_j(t))x_i + \tilde{\tau}_i(x_1, \cdots, x_{i-1}, t), \quad 2 \le i \le n. \end{cases} \tag{68}$$

According to Assumption 2, the term $\Pi_{j=1}^{i-1}\theta_j(t)$ is nonzero for $2 \le i \le n$. Then, the mapping from $(\tilde{x}, t)$ to (x, t) can be determined by the following recursive way.

From (68), the functions φ_1 and φ_2 can be obtained as follows.

$$\begin{aligned} \varphi_1(\tilde{x}, t) &= \tilde{x}_1, \\ \varphi_2(\tilde{x}, t) &= \frac{\tilde{x}_2 - \tilde{\tau}_2(x_1, t)}{\theta_1(t)} = \frac{\tilde{x}_2 - \tilde{\tau}_2(\varphi_1(\tilde{x}, t), t)}{\theta_1(t)}. \end{aligned} \tag{69}$$

Suppose that $\varphi_i(\tilde{x}, t)$ exists for $2 \le i \le k \le n-1$. Owing to (68), we can acquire the function $\varphi_{k+1}(\tilde{x}, t)$ as follows.

$$\varphi_{k+1}(\tilde{x}, t) = \frac{\tilde{x}_{k+1} - \tilde{\tau}_{k+1}(\varphi_1(\tilde{x}, t), \cdots, \varphi_k(\tilde{x}, t), t)}{\Pi_{j=1}^{k}\theta_j(t)},$$
$$2 \le k \le n-1. \tag{70}$$

Additionally, it is obvious that $\varphi_{n+1}(\tilde{x}, t) = t$. Moreover, the inverse of the mapping φ can be directly obtained by (68).

Then, the bounds of $\varphi_i(\tilde{x}, t)$ $(1 \le i \le n+1)$ are analyzed. Owing to the dynamics (3), the expression of $\tilde{\tau}_i$ (63) implies that $\tilde{\tau}_i$ is composed of the finite sums and products of x_j $(1 \le j \le i-1)$, ϕ_j $(1 \le j \le i-1)$, the partial derivatives of ϕ_j $(1 \le j \le i-1)$ up to $(i-j-1)$-th order and $\theta_j^{(p)}$ $(1 \le j \le i-1, 0 \le p \le i-j-1)$. According to Assumption 2, there exist continuous functions $\tilde{\psi}_{\tau,i}(x_1, \cdots, x_{i-1})$ $(2 \le i \le n)$ such that

$$\begin{aligned} &\sup_{t \ge t_0}|\tilde{\tau}_i(x_1, \cdots, x_{i-1}, t)| \le \tilde{\psi}_{\tau,i}(x_1, \cdots, x_{i-1}), \\ &\forall[x_1 \ \cdots \ x_{i-1}] \in R^{i-1}, \ 2 \le i \le n. \end{aligned} \tag{71}$$

By introducing the non-decreasing continuous function

$$\psi_{\tau,i}(\varrho_x) \triangleq \sup_{\|[x_1 \ \cdots \ x_{i-1}]\| \le \varrho_x} \tilde{\psi}_{\tau,i}(x_1, \cdots, x_{i-1})$$

for $\varrho_x \ge 0$, (71) implies that

$$\begin{aligned} &\sup_{t \ge t_0, \|[x_1 \ \cdots \ x_{i-1}]\| \le \varrho_x} |\tilde{\tau}_i(x_1, \cdots, x_{i-1}, t)| \\ &\le \psi_{\tau,i}(\varrho_x), \quad \forall\varrho_x \ge 0, \ 2 \le i \le n. \end{aligned} \tag{72}$$

Next, the bounds of φ_i are presented by recursive method. Denote $\tilde{\varrho}_x$ as a nonnegative constant. Based on (69), the bounds of φ_1 and φ_2 are shown as follows.

$$\begin{cases} \sup_{t \ge t_0, \|\tilde{x}\| \le \tilde{\varrho}_x} |\varphi_1(\tilde{x}, t)| \le \sup_{\|\tilde{x}\| \le \tilde{\varrho}_x} |\tilde{x}_1| \le M_{\varphi,1}(\tilde{\varrho}_x) \triangleq \tilde{\varrho}_x, \\ \sup_{t \ge t_0, \|\tilde{x}\| \le \tilde{\varrho}_x} |\varphi_2(\tilde{x}, t)| \le \sup_{t \ge t_0, \|\tilde{x}\| \le \tilde{\varrho}_x} \frac{|\tilde{x}_2| + |\tilde{\tau}_2(\varphi_1(\tilde{x}, t), t)|}{|\theta_1(t)|} \\ \qquad \le M_{\varphi,2}(\tilde{\varrho}_x) \triangleq \frac{\tilde{\varrho}_x + \psi_{\tau,2}(M_{\varphi,1}(\tilde{\varrho}_x))}{\underline{M}_{\theta,1}}. \end{cases} \tag{73}$$

Notice that $M_{\varphi,1}(\cdot)$ and $M_{\varphi,2}(\cdot)$ are increasing functions. Suppose that

$$\sup_{t \ge t_0, \|\tilde{x}\| \le \tilde{\varrho}_x} |\varphi_i(\tilde{x}, t)| \le M_{\varphi,i}(\tilde{\varrho}_x)$$

where $M_{\varphi,i}$ is a continuous increasing function for $2 \leq i \leq k$, $2 \leq k \leq n-1$ and $\tilde{\varrho}_x \geq 0$. Then, the bound of $\varphi_{k+1}(\tilde{x}, t)$ can be obtained as follows.

$$\begin{aligned} &\sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x} |\varphi_{k+1}(\tilde{x}, t)| \\ &\leq \sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x} \frac{|\tilde{x}_{k+1}| + |\tilde{\tau}_{k+1}(\varphi_1(\tilde{x}, t), \cdots, \varphi_k(\tilde{x}, t), t)|}{|\Pi_{j=1}^k \theta_j(t)|} \\ &\leq M_{\varphi,k+1}(\tilde{\varrho}_x) \\ &\triangleq \frac{\tilde{\varrho}_x + \psi_{\tau,k+1}\left(\sqrt{\Sigma_{j=1}^k (M_{\varphi,j}(\tilde{\varrho}_x))^2}\right)}{\Pi_{j=1}^k \underline{M}_{\theta,j}}. \end{aligned} \tag{74}$$

Hence, it can be concluded that there exist continuous increasing functions $M_{\varphi,i}$ $(1 \leq i \leq n)$ such that $\sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x} |\varphi_i(\tilde{x}, t)| \leq M_{\varphi,i}(\tilde{\varrho}_x)$ for $\tilde{\varrho}_x \geq 0$ and $1 \leq i \leq n$.

By taking the partial derivatives of φ_i along the dynamics (3), it can be deduced from the similar analysis (69)–(74) that there exist continuous functions $M_{\frac{\partial\varphi}{\partial\tilde{x}},i}$ and $M_{\frac{\partial\varphi}{\partial t},i}$ such that

$$\begin{aligned} &\sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x} \left\| \frac{\partial\varphi_i(\tilde{x}, t)}{\partial\tilde{x}} \right\| \\ &\leq M_{\frac{\partial\varphi}{\partial\tilde{x}},i}(\tilde{\varrho}_x), \\ &\sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x} \left| \frac{\partial\varphi_i(\tilde{x}, t)}{\partial t} \right| \\ &\leq M_{\frac{\partial\varphi}{\partial t},i}(\tilde{\varrho}_x), \quad 1 \leq i \leq n. \end{aligned} \tag{75}$$

for any $\tilde{\varrho}_x \geq 0$. Due to (73)–(75), (10) is proved by defining

$$\psi_\varphi(\tilde{\varrho}_x) \triangleq \max_{1\leq i\leq n} \left\{ M_{\varphi,i}(\tilde{\varrho}_x), M_{\frac{\partial\varphi}{\partial\tilde{x}},i}(\tilde{\varrho}_x) M_{\frac{\partial\varphi}{\partial t},i}(\tilde{\varrho}_x) \right\}.$$

Part II: The analysis of the mapping γ. Based on the analysis of $\tilde{\tau}_i$, the mapping γ can be directly determined by (68). Combined with the bounds of $\tilde{\tau}_i$ (71)–(72), (68) further implies that

$$\begin{cases} \sup_{t\geq t_0, \|x\|\leq\varrho_x} |\gamma_1(x, t)| \leq |x_1| \leq M_{\gamma,1}(\varrho_x) \triangleq \varrho_x, \\ \sup_{t\geq t_0, \|x\|\leq\varrho_x} |\gamma_i(x, t)| \leq \sup_{t\geq t_0, \|x\|\leq\varrho_x} \left(|\Pi_{j=1}^{i-1}\theta_j(t)||x_i| + |\tilde{\tau}_i| \right) \\ \qquad \leq M_{\gamma,i}(\varrho_x) \triangleq \varrho_x \Pi_{j=1} \bar{M}_{\theta,i} + \psi_{\tau,i}(\varrho_x), \quad 2 \leq i \leq n, \end{cases} \tag{76}$$

for any given $\varrho_x \geq 0$. Notice that $M_{\gamma,i}(\varrho_x)$ $(1 \leq i \leq n)$ are continuous increasing functions.

Similarly, by taking the partial derivatives of γ_i along the dynamics (3), it can be deduced from Assumption 2 that there exist continuous functions $M_{\frac{\partial\gamma}{\partial x},i}$ and $M_{\frac{\partial\gamma}{\partial t},i}$ such that

$$\begin{aligned} &\sup_{t\geq t_0, \|x\|\leq\varrho_x} \left\| \frac{\partial\gamma_i(x, t)}{\partial x} \right\| \leq M_{\frac{\partial\gamma}{\partial x},i}(\varrho_x), \\ &\sup_{t\geq t_0, \|x\|\leq\varrho_x} \left| \frac{\partial\gamma_i(x, t)}{\partial t} \right| \leq M_{\frac{\partial\gamma}{\partial t},i}(\varrho_x), \ \forall\, 1 \leq i \leq n, \end{aligned} \tag{77}$$

for any $\varrho_x \geq 0$. Based on (76)–(77) and the notation

$$\psi_\gamma(\varrho_x) \triangleq \max_{1\leq i\leq n} \left\{ M_{\gamma,i}(\varrho_x), M_{\frac{\partial\gamma}{\partial x},i}(\varrho_x) M_{\frac{\partial\gamma}{\partial t},i}(\varrho_x) \right\},$$

then (9) is proved.

Proof of Proposition 2

Firstly, the bounds of $b(t)$, $(b(t))^{-1}$ and $\dot{b}(t)$ are analyzed. According to the bounds of $\theta_i(t)$, $(\theta_i(t))^{-1}$ and $\dot{\theta}_i(t)$ in Assumption 2, the expression of $b(t)$ (12) implies that

$$\begin{cases} |b(t)| \leq \Pi_{i=1}^n \bar{M}_{\theta,i}, \\ |b^{-1}(t)| = |\Pi_{i=1}^n \theta_i^{-1}(t)| \leq \Pi_{i=1}^n \underline{M}_{\theta,i}^{-1}, \\ |\dot{b}(t)| = \Sigma_{j=1}^n |\dot{\theta}_j(t)| \Pi_{1\leq i\leq n, i\neq j} |\theta_i(t)| \leq \Pi_{i=1}^n \bar{M}_{\theta,i}. \end{cases} \tag{78}$$

By denoting $\psi_b = \max\{\Pi_{i=1}^n \bar{M}_{\theta,i}, \Pi_{i=1}^n \underline{M}_{\theta,i}^{-1}\}$, (28) is proved.

Next, the bound of $f(\tilde{x}, t)$ is analyzed. Proposition 1 illustrates that $\varphi_{n+1-j}(\tilde{x}, t) = x_{n+1-j}$ for $1 \leq j \leq n-1$. Additionally, $\tilde{\phi}_i(\tilde{x}, t) = \phi_i(x_1, \cdots, x_i, t)$ for $1 \leq i \leq n$. From the expression of $f(\tilde{x}, t)$ (12), $f(\tilde{x}, t)$ can be regarded as a function of (x, t), denoted as $\bar{f}(x, t)$.

Owing to the dynamics (3) and the notation $\bar{f}(x, t)$, it can be verified that $\bar{f}(x, t)$ is composed of the finite sums and products of x_j $(1 \leq j \leq n)$, ϕ_j $(1 \leq j \leq n)$, the partial derivatives of ϕ_j $(1 \leq j \leq n)$ up to $(n-j)$-th order and $\theta_j^{(p)}$ $(1 \leq j \leq n-1, 0 \leq p \leq n-j-1)$.

Due to Assumption 2, there exists a continuous function $\psi_{f,1}(x)$ such that

$$\sup_{t\geq t_0} |\bar{f}(x, t)| \leq \psi_{f,1}(x), \tag{79}$$

which further implies that

$$\sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x} |f(\tilde{x},t)| = \sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x} |\bar{f}(x,t)| \leq \sup_{\|\tilde{x}\|\leq\tilde{\varrho}_x} \psi_{f,1}(x), \quad \forall\tilde{\varrho}_x \geq 0. \tag{80}$$

Recalling the continuous increasing function $\psi_\gamma(\cdot)$ in Proposition 1, owing to inverse function theorem, the inverse function of $\psi_\gamma(\cdot)$ can be denoted as $\psi_\gamma^{-1}(\cdot)$ which is a continuous function in $[0,\infty)$. According to (8)–(9) in Proposition 1, the following equation holds.

$$\sup_{\|x\|\leq\psi_\gamma^{-1}(\tilde{\varrho}_x/\sqrt{n})} \|\tilde{x}\| = \sup_{\|x\|\leq\psi_\gamma^{-1}(\tilde{\varrho}_x/\sqrt{n})} \left\| \left[\gamma_1(x,t) \cdots \gamma_n(x,t)\right]\right\| \leq \sqrt{n}\psi_\gamma(\psi_\gamma^{-1}(\tilde{\varrho}_x/\sqrt{n})) = \tilde{\varrho}_x. \tag{81}$$

Based on (81), it can be verified that

$$\{\tilde{x}|\ \|\tilde{x}\| \leq \tilde{\varrho}_x\} \subset \left\{\tilde{x}|\ \|x\| \leq \psi_\gamma^{-1}(\tilde{\varrho}_x/\sqrt{n})\right\}, \quad \forall\tilde{\varrho}_x \geq 0. \tag{82}$$

With the combination of (80) and (82), it can be obtained that

$$\sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x} |f(\tilde{x},t)| \leq \sup_{\|\tilde{x}\|\leq\tilde{\varrho}_x} \psi_{f,1}(x) \leq \bar{\psi}_{f,1}(\tilde{\varrho}_x) \triangleq \sup_{\|x\|\leq\psi_\gamma^{-1}(\tilde{\varrho}_x/\sqrt{n})} \psi_{f,1}(x), \quad \forall\tilde{\varrho}_x \geq 0. \tag{83}$$

Due to the continuity of $\psi_\gamma^{-1}(\cdot)$ and $\psi_{f,1}(\cdot)$, it can be deduced that $\bar{\psi}_{f,1}(\tilde{\varrho}_x)$ is a continuous function with respect to the variable $\tilde{\varrho}_x$.

From Proposition 1, $\varphi_{n+1}(\tilde{x},t) = t$ and

$$x_i = \varphi_i(\tilde{x},t), \quad 1 \leq i \leq n. \tag{84}$$

Then, the partial derivatives of $f(\tilde{x},t)$ can be expressed as follows.

$$\begin{cases} \dfrac{\partial f(\tilde{x},t)}{\partial \tilde{x}_i} = \dfrac{\partial \bar{f}(\varphi_1(\tilde{x},t),\cdots,\varphi_n(\tilde{x},t),t)}{\partial \tilde{x}_i} \\ \qquad = \Sigma_{j=1}^n \dfrac{\partial \bar{f}(x_1,\cdots,x_n,t)}{\partial x_j}\dfrac{\partial \varphi_j(\tilde{x},t)}{\partial \tilde{x}_i}, \quad 1 \leq i \leq n, \\ \dfrac{\partial f(\tilde{x},t)}{\partial t} = \dfrac{\partial \bar{f}(x_1,\cdots,x_n,t)}{\partial t}. \end{cases} \tag{85}$$

Due to Assumption 2, with the similar derivation as (79), there exist continuous functions $\psi_{fx,j}$ and ψ_{ft} such that

$$\sup_{t\geq t_0} \left|\frac{\partial \bar{f}(x_1,\cdots,x_n,t)}{\partial t}\right| \leq \psi_{ft}(x),$$

$$\sup_{t\geq t_0} \left|\frac{\partial \bar{f}(x_1,\cdots,x_n,t)}{\partial x_j}\right| \leq \psi_{fx,j}(x), \quad 1 \leq j \leq n. \tag{86}$$

Based on the bounds of $\frac{\partial\varphi_j}{\partial x_i}$ (10) and with the similar procedure as (79)–(83), there exist continuous functions $\bar{\psi}_{f,2}(\cdot)$ and $\bar{\psi}_{f,3}(\cdot)$ such that

$$\sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x} \left\|\frac{\partial f(\tilde{x},t)}{\partial \tilde{x}}\right\| \leq \bar{\psi}_{f,2}(\tilde{\varrho}_x),$$
$$\sup_{t\geq t_0, \|\tilde{x}\|\leq\tilde{\varrho}_x} \left|\frac{\partial f(\tilde{x},t)}{\partial t}\right| \leq \bar{\psi}_{f,3}(\tilde{\varrho}_x), \quad \forall\tilde{\varrho}_x \geq 0. \tag{87}$$

By denoting $\psi_f(\tilde{\varrho}_x) \triangleq \max\{\bar{\psi}_{f,1}(\tilde{\varrho}_x), \bar{\psi}_{f,2}(\tilde{\varrho}_x), \bar{\psi}_{f,3}(\tilde{\varrho}_x)\}$, (83) and (87) imply (29).

Proof of Proposition 3

For $t \in [t_0, t_u)$, the control input satisfies that $u(t) = 0$. Hence, the dynamics of $\tilde{x}$ in (11) can be rewritten as follows.

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + Bf(\tilde{x},t). \tag{88}$$

Combined with the dynamics of $\tilde{x}^*(t)$ (17), the dynamics of $e(t)$ for $t \in [t_0, t_u)$ is shown as follows.

$$\dot{e}(t) = Ae(t) + B\Delta_{e0}(e,t), \quad t \in [t_0, t_u), \tag{89}$$

where

$$\Delta_{e0}(e,t) = f(\tilde{x}^* + e, t) + K^T(\tilde{x}^*(t) - \bar{r}(t)) - r^{(n)}(t). \tag{90}$$

Then, consider the case that $t \in [t_u, \infty)$. With the notation (1) and the desired input (18), the dynamics of $\tilde{x}$ in (11) can be reformulated as follows.

$$\begin{aligned}\dot{\tilde{x}}(t) &= A\tilde{x}(t) + B(b(t)u(t) + f(\tilde{x},t) - b(xt)u^*(t) + b(t)u^*(t)) \\ &= A\tilde{x}(t) - BK^T(\tilde{x}(t) - \bar{r}(t)) + Br^{(n)}(t) + Bb(t)\delta_u(t) \\ &= A_K e(t) + Bb(t)\delta_u(t) + \dot{\tilde{x}}^*(t).\end{aligned} \tag{91}$$

Owing to the dynamics of $\tilde{x}^*(t)$ (17), the dynamics of $e(t)$ for $t \in [t_u, \infty)$ is obtained as follows.

$$\dot{e}(t) = A_K e(t) + Bb(t)\delta_u(t), \quad t \in [t_u, \infty). \tag{92}$$

Before analyzing the closed-loop form of $\zeta(t)$ and $\delta_u(t)$, the derivative of $u^*(t)$ is calculated due to (18).

$$\begin{cases} \dot{u}^*(t) = -\dfrac{\dot{b}(t)(-f(\tilde{x},t) - K^T(\tilde{x}-\bar{r}) + r^{(n)})}{b^2(t)} \\ \quad + \dfrac{-\frac{\partial f}{\partial \tilde{x}}(A(e+\tilde{x}^*) + Bf(e+\tilde{x}^*,t)) - \frac{\partial f}{\partial t}}{b(t)} \\ \quad + \dfrac{-K^T(A(e+\tilde{x}^*) + Bf(e+\tilde{x}^*,t) - \dot{\bar{r}}) + r^{(n+1)}}{b(t)}, \\ \quad t \in (t_0, t_u), \\ \dot{u}^*(t) = -\dfrac{\dot{b}(t)(-f(\tilde{x},t) - K^T(\tilde{x}-\bar{r}) + r^{(n)})}{b^2(t)} \\ \quad + \dfrac{-\frac{\partial f}{\partial \tilde{x}}(A_K e + Bb\delta_u + \dot{\tilde{x}}^*) - \frac{\partial f}{\partial t}}{b(t)} \\ \quad + \dfrac{-K^T(A_K e + Bb\delta_u + \dot{\tilde{x}}^* - \dot{\bar{r}}) + r^{(n+1)}}{b(t)}, \\ \quad t \in (t_u, \infty). \end{cases} \tag{93}$$

With the help of Remark 8, the derivative of $u(t)$ is shown as follows.

$$\begin{aligned} \dot{u}(t) &= -sgn(b(t))\kappa(\omega_o)(\hat{f}_t(t) + K^T(\hat{\tilde{x}}(t) \\ &\quad -\bar{r}(t)) - r^{(n)}(t)) \\ &= -\kappa|b(t)|\frac{f_t(\tilde{x},u,t) + K^T(\tilde{x}(t)-\bar{r}(t)) - r^{(n)}(t) - K_e^T T_1\zeta}{b(t)} \\ &= -\kappa|b(t)|\delta_u(t) + sgn(b(t))\kappa K_e^T T_1 \zeta(t), \end{aligned} \tag{94}$$

where $K_e = [K^T\ 1]^T$. Since $T_1^{-1}A_L T_1 = \omega_o A_\varsigma$, the dynamics of ζ for $t \in [t_0, t_u)$ can be obtained from (11) and (15):

$$\dot{\zeta}(t) = \omega_o A_\varsigma \zeta(t) + B_f \Delta_{\zeta 0}(e,t), \quad t \in [t_0, t_u), \tag{95}$$

where

$$\begin{aligned} \Delta_{\zeta 0}(e,t) = \frac{\mathrm{d}f(\tilde{x},t)}{\mathrm{d}t}|_{\text{along (88)}} &= \frac{\partial f}{\partial \tilde{x}}(A(e+\tilde{x}^*) \\ &\quad + Bf(e+\tilde{x}^*,t)) + \frac{\partial f}{\partial t}. \end{aligned} \tag{96}$$

Owing to (94), the dynamics of $\zeta(t)$ for $t \in [t_u, \infty)$ is presented as follows.

$$\begin{aligned} \dot{\zeta}(t) &= \omega_o A_\varsigma \zeta(t) \\ &\quad + B_f \Delta_{\zeta 1}(e, \zeta, \delta_u, \omega_o, \kappa, t), \quad t \in [t_0, t_u), \end{aligned} \tag{97}$$

where

$$\begin{aligned} \Delta_{\zeta 1}(e,\zeta,\delta_u,\omega_o,\kappa,t) &= \frac{\mathrm{d}(b(t)u(t) + f(\tilde{x},t))}{\mathrm{d}t}|_{\text{along (91)}} \\ &= \dot{b}\delta_u + \dot{b}(-f(e+\tilde{x}^*,t) \\ &\quad - K^T(e+\tilde{x}^*-\bar{r}) + r^{(n)})/b \\ &\quad + b(-\kappa|b|\delta_u + sgn(b)\kappa K_e^T T_1 \zeta) \\ &\quad + \frac{\partial f}{\partial \tilde{x}}(A_K e + Bb\delta_u + \dot{\tilde{x}}^*) + \frac{\partial f}{\partial t}. \end{aligned} \tag{98}$$

With the combination of (93) and (94), the derivative of $\delta_u(t)$ is shown as follows.

$$\begin{cases} \dot{\delta}_u(t) = -|b(t)|\kappa\delta_u(t) + \Delta_{\delta_u 0}(e,\zeta,\omega_o,\kappa,t), \quad t \in [t_0, t_u), \\ \dot{\delta}_u(t) = -|b(t)|\kappa\delta_u(t) + \Delta_{\delta_u 1}(e,\zeta,\delta_u,\omega_o,\kappa,t), \quad t \in [t_u, \infty), \end{cases} \tag{99}$$

where

$$\begin{cases} \Delta_{\delta_u 0}(e,\zeta,\omega_o,\kappa,t) = \mathrm{sgn}(b(t))\kappa K_e^T T_1 \zeta + \dfrac{\dot{b}(t)(-f(\tilde{x},t) - K^T(e+\tilde{x}^*-\bar{r}) + r^{(n)})}{b^2(t)} \\ \qquad - \dfrac{-\frac{\partial f}{\partial \tilde{x}}(A(e+\tilde{x}^*) + Bf(e+\tilde{x}^*,t)) - \frac{\partial f}{\partial t} - K^T(A(e+\tilde{x}^*) + Bf(e+\tilde{x}^*,t) - \dot{\bar{r}}) + r^{(n+1)}}{b(t)}, \\ \Delta_{\delta_u 1}(e,\zeta,\delta_u,\omega_o,\kappa,t) = \mathrm{sgn}(b(t))\kappa K_e^T T_1 \zeta + \dfrac{\dot{b}(t)(-f(\tilde{x},t) - K^T(e+\tilde{x}^*-\bar{r}) + r^{(n)})}{b^2(t)} \\ \qquad - \dfrac{-\frac{\partial f}{\partial \tilde{x}}(A_K e + Bb\delta_u + \dot{\tilde{x}}^*) - \frac{\partial f}{\partial t} - K^T(A_K e + Bb\delta_u + \dot{\tilde{x}}^* - \dot{\bar{r}}) + r^{(n+1)}}{b(t)}. \end{cases} \tag{100}$$

As for the dynamics of $z(t)$, it can be directly obtained from (11) that

$$\dot{z}(t) = g(z, \varphi_1(\tilde{x},t), \cdots, \varphi_n(\tilde{x},t), t), \quad t \geq t_0. \tag{101}$$

Based on the dynamics of e, ζ, δ_u and z, i.e., (89), (92), (95), (97), (99) and (101), the closed-loop form can be written as (31)–(32).

Next, we will prove that the bounds of Δ_{e0}, $\Delta_{\zeta 0}$, $\Delta_{\delta_u 0}$, Δ_{e1}, $\Delta_{\zeta 1}$ and $\Delta_{\delta_u 1}$ satisfy (33) for $e \in \{e|\ \|e\| \leq \varrho_e\}$, $\zeta \in \{\zeta|\ \|\zeta\| \leq \varrho_\zeta\}$, $\delta_u \in \{\delta_u|\ |\delta_u| \leq \varrho_u\}$ and $\omega_o \in \{\omega_o|\ \omega_o \geq \omega_o^*\}$ with any given positives ϱ_e, ϱ_ζ, ϱ_u and ω_o^*. Hence, we only need to present the detailed expressions of the functions $\pi_{e0}(\cdot)$, $\pi_{\zeta 0}(\cdot)$, $\pi_{\zeta 1}(\cdot)$, $\pi_{\delta_u}(\cdot)$, $\pi_{\delta_u 0}(\cdot)$ and $\pi_{\delta_u 1}(\cdot)$.

Due to Assumption 1, there is

$$\sup_{t \geq t_0} \|\bar{r}(t)\| \leq nM_r,$$

$$\sup_{t\geq t_0}\{|r^{(n)}(t)|, |r^{(n+1)}(t)|\} \leq M_r. \tag{102}$$

Combined with the dynamics of $\tilde{x}^*$ (17), there exists a positive constant M_{x^*} such that

$$\sup_{t\geq t_0}\{\|\tilde{x}^*(t)\|, \|\dot{\tilde{x}}^*(t)\|\} \leq M_{x^*}. \tag{103}$$

According to the bounds of b, $\dot{b}$, b^{-1}, f, $\frac{\partial f}{\partial \tilde{x}}$ and $\frac{\partial f}{\partial t}$ shown in Proposition 2, (90) implies that

$$\begin{aligned}&\sup_{\|e\|\leq \varrho_e} |\Delta_{e0}(e,t)| \leq \pi_{e0}(\varrho_e)\\ &\triangleq \psi_f(\varrho_e + M_{x^*}) + \|K\|(M_{x^*} + nM_r) + M_r,\\ &\quad \forall \varrho_e \geq 0.\end{aligned} \tag{104}$$

Due to (92), it can be directly obtained that

$$|\Delta_{e1}(\delta_u, t)| = |b(t)\delta_u(t)| \leq \psi_b|\delta_u|. \tag{105}$$

Owing to (96) and (98), the bounds of $\Delta_{\zeta 0}$ and $\Delta_{\zeta 1}$ are provided as follows.

$$\begin{aligned}&\sup_{\|e\|\leq \varrho_e} |\Delta_{\zeta 0}(e,t)| \leq \pi_{\zeta 0}(\varrho_e)\\ &\triangleq \psi_f(\varrho_e + M_{x^*})\left(1 + \|A\|(\varrho_e + M_{x^*}) + \psi_f(\varrho_e + M_{x^*})\right),\\ &\sup_{\|e\|\leq \varrho_e, |\delta_u|\leq \varrho_u, \omega_o\geq \omega_o^*} |\Delta_{\zeta 1}(e,\zeta,\delta_u,\omega_o,\kappa,t)| \leq \pi_{\zeta 1}(\varrho_e)\\ &+\pi_\omega(\omega_o^*)\kappa\|\zeta\| + (\kappa+1)\pi_{\delta_u}(\varrho_e,\varrho_u),\end{aligned}$$

for any $\varrho_e \geq 0$, $\varrho_u \geq 0$ and $\omega_o^* \geq 0$, where $\pi_{\zeta 1}(\varrho_e) = \psi_b^2(\psi_f(\varrho_e + M_{x^*}) + \|K\|(\varrho_e + M_{x^*} + nM_r) + M_r) + \psi_f(\varrho_e + M_{x^*})(1 + \|A_K\|\varrho_e + M_{x^*})$, $\pi_\omega(\omega_o^*) = \max\{1, \psi_b\}\|K_e\| \cdot \|T_1(\omega_o^*)\|$ and $\pi_{\delta_u}(\varrho_e, \varrho_u) = \max\{\psi_b^2\varrho_u, \psi_b\varrho_u(1+\psi_f(\varrho_e+M_{x^*}))\}$. With the help of Proposition 2, the following bounds of $\Delta_{\delta_u 1}$ and $\Delta_{\delta_u 2}$ are obtained from (100).

$$\begin{cases}\sup_{\|e\|\leq \varrho_e} |\Delta_{\delta_u 0}| \leq \pi_{\delta_u 0}(\varrho_e) + \pi_\omega(\omega_o^*)\kappa\|\zeta\|,\\ \sup_{\|e\|\leq \varrho_e, |\delta_u|\leq \varrho_u} |\Delta_{\delta_u 1}| \leq \pi_{\delta_u 1}(\varrho_e,\varrho_u) + \pi_\omega(\omega_o^*)\kappa\|\zeta\|,\end{cases} \quad \forall \varrho_e \geq 0,\ \varrho_u \geq 0, \tag{106}$$

where $\pi_{\delta_u 0}(\varrho_e) = \max\{\psi_b, \psi_b^3\}(\psi_f(\varrho_e + M_{x^*})(2 + \|A\|(\varrho_e + M_{x^*}) + \psi_f(\varrho_e + M_{x^*})) + \|K\|(1 + \|A\|)(\varrho_e + M_{x^*} + nM_r + \psi_f(\varrho_e + M_{x^*})) + 2M_r)$ and $\pi_{\delta_u 0}(\varrho_e, \varrho_u) = \pi_{\delta_u 0}(\varrho_e) + \psi_b(\psi_f(\varrho_e + M_{x^*}) + \|K\|)(\|A_K\|\varrho_e + \psi_b\varrho_u + M_{x^*} + nM_r)$.

Based on (104)–(106), (33) is proved.

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