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Distributed output feedback leader-following consensus for nonlinear multiagent systems with time delay

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Abstract This paper concentrates on the output feedback control problem for a class of nonlinear multiagent systems governed by the high-order strict-feedback model with time delay. Within the dynamic gain technique and the Lyapunov-like method, the dynamic gain state observer for each agent is put forward with the hope to compensate the impact induced by the immeasurable state variables, and then the distributed leader-following consensus protocols which are independent of the time delay on the agent state are designed such that the output of each follower can asymptotically track that of the leader. Besides, the problem considered is extended into the general case where the Lipschitz growth rates of the nonlinear function are unknown time-varying functions. Finally, simulation examples are performed to illustrate the validity and effectiveness of the proposed approach.

Keywords Leader-following consensus · Dynamic gain · Output feedback · Time delay

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1 Introduction

During the past couple of decades, the distributed consensus problems for multiagent systems have been drawing an ever increasing concern due to the description of various physical systems such as mobile robots, flocks or swarms, and unmanned aerial vehicles (see [1–11] and the references therein). Recently, the nonlinear multiagent systems which are described by the strict-feedback form and satisfy Lipschitz conditions have been widely regarded as a control target owing to their vast applications [12–17]. In [12], the general case where the nonlinear characteristic of the agents are described by feedforward nonlinearities with the growth rate being unknown priori was considered. Based on this, Chang *et al* [12] both proposed the state feedback regulation protocol and the output feedback regulation protocol such that the tracking performance was well-guaranteed. The distributed prescribed finite-time observer was first designed for a strict-feedback nonlinear system with external disturbance in [13]. Zhang *et al* [14] designed the distributed control protocols such that the leader-following consensus was achieved for the nonlinear multiagent systems which were supposed to satisfy Lipschitz conditions with time-varying gains. The event-triggered output feedback controllers were developed for a class of switched nonlinear strict-feedback systems, where the nonlinear functions are supposed to be bounded by a continuous function of the output multiplied by unmeasured states [15,16]. In [17], the consensus problem was investigated for the multiagent systems with unknown smooth nonlinear term. Although these consensus control mechanisms express better tracking performance for the nonlinear multiagent systems, they are difficult to be generalized to the delayed nonlinear counterparts.

At present, the outcomes about the consensus problem investigation for multiagent systems with time delay only have focused on the its linear parts, and that of the nonlinear counterparts still has not obtained enough attention. In practical engineering applications, however, the time delay is inevitable. For instance, state delay, input delay as well as communication delay are often encountered in the multiagent systems (see [19–23] and the references therein). The delay effect will degrade system performance at certain degree, and even cause the instability of the system. Therefore, researchers gradually show solicitude for the consensus analysis for nonlinear multiagent systems with time delay [24–27]. Hua *et al* [24] explored the leader-following output consensus problem for a class of nonlinear multiagent system with delay measurements under the directed communication graph. Li *et al* [25] further investigated this problem by establishing the dynamic gain compensator. Chen *et al* [26] proposed a novel control strategy for asymptotically stabilizing chained nonholonomic systems with input delay by utilizing the input-state-scaling technique and the static gain control method.

It is worthy that most of the consensus protocols of nonlinear multiagent systems were formulated under the assumption that the state variables of each agent were available [28–31]. This assumption, however, was much serious for some practical systems and also limited the practicality of control framework derived from the state feedback viewpoint. Generally speaking, not all state information is available in practical applications, instead only a few part of information in terms of output can be measured. Hence, the state observer shall be developed with the potential ability to tackle the impact induced by the immeasurable state variables. With the help of the dynamic gain method, You *et al* [29] made a profound investigation for the leader-following consensus of the higher-order stochastic nonlinear multiagent systems, and the distributed observer-type controller was formulated using the relative output measurements of neighboring agents. In [28], the lower triangular system was considered, and the output feedback controller was established. Zhang *et al* [30] further extended output feedback control method to the case of unmodeled dynamics. Another limitation of the theoretical results are obtained based on an assumption that the Lipschitz growth rates are known constants.

Regarding statements presented above, the current research primarily focuses on the leader-following consensus problem for a class of nonlinear multiagent systems with time delay. The agent dynamics are assumed to be in the strict-feedback form and satisfy Lipschitz conditions both with fixed gains and time-varying gains. The investigation seems to be distinguished mainly resulted from the evident challenging summarized as follows:

1. How to analyze the influence induced by the time delay and immeasurable state variables as well as nonlinear terms?
2. How to formulate the distributed controller for each follower by only using the agent's output and the relative output of its neighbor agents?
3. How to design the dynamic gain such that the leader-following consensus problem of nonlinear multiagent systems with time delay on the state can be addressed?

Thereby, in this paper, the output-feedback consensus control problem for nonlinear multiagent systems with time delay has been explored. The main novelties include the following three folds:

1. The practical case that the multiagent systems suffered from intrinsic nonlinear characteristic and time delay as well as the immeasurable state variables is considered.
2. The state observer of each agent is established by applying dynamic gain method to compensate the effect of the immeasurable state variables.
3. The distributed output feedback consensus protocols are proposed such that the output of each agent can track that of the leader both for cases that the Lipschitz growth rates of the nonlinear function are known constants and unknown time-varying function.

2 Preliminaries and problem formulation

This section covers some terminologies on the graph, and the model as well as the problem will be addressed.

2.1 Graph theory

The weighted undirected graph is described by $\mathcal{G} \triangleq (V, E, \mathcal{A})$, where $V = \{v_1, v_2, \ldots, v_N\}$ is the vertex set, $E \subseteq V \times V$ denotes the edge set, and the weighted adjacency matrix is represented by $\mathcal{A} = [a_{ij}]_{N\times N}$. The edge (v_j, v_i) is included in the edge set E if and only if the agent i and agent j can obtain information from each other. The adjacency matrix $\mathcal{A}$ is defined such that $a_{ij} > 0$ yields $(v_j, v_i) \in E$, otherwise $a_{ij} = 0$. Without loss of generality, the self-loop is excluded in this paper, *i.e*, $a_{ii} = 0$. The path of the undirected graph between vertex v_i and v_j is a sequence of edges (v_i, v_{k_1}),(v_{k_1}, v_{k_2}),…,(v_{k_n}, v_j). The undirected graph is said to be connected if there exists a path between any two vertex. The Laplacian matrix of the graph $\mathcal{G}$ is defined as $\mathcal{L} = [\ell_{ij}]_{N\times N}$, where $\ell_{ij} = -a_{ij}$, if $i \neq j$ and $\ell_{ii} = d_i = \sum_{i=1}^{j} a_{ij}$. A subgraph $\mathcal{H}$ of $\mathcal{G}$ is said to be an induced subgraph if two vertices are adjacent in $\mathcal{H}$ only if they are adjacent in $\mathcal{G}$. The component of $\mathcal{G}$ is an induced subgraph which is maximal, subjected to be connected. The composition graph $\bar{\mathcal{G}}$ is associated with the system containing N agents and a leader. $\bar{\mathcal{G}}$ consists of $\mathcal{G}$ and a leader with some edges describing the relationships between some agents and the leader. The connection weighted matrix $\mathcal{B}$ is determined as $\mathcal{B} = \text{diag}\{b_1, b_2, \ldots, b_N\}$ where $b_i > 0$ implies the agent i can obtain information from the leader; otherwise, $b_i = 0$. $\bar{\mathcal{G}}$ is said to be connected if at least one agent in each component can obtain information from the leader. Denote $\hat{\mathcal{L}} = \mathcal{L} + \mathcal{B}$.

2.2 Model and problem formulation

Consider a flock of nonlinear multiagent systems with $N + 1$ agents containing N followers consecutively labeled from 1 to N and a leader indexed by 0. The dynamics of ith agent can be explicitly described by the following uncertain delayed nonlinear strict-feedback form:

$$\begin{aligned} \dot{x}_{i,m}(t) &= x_{i,m+1}(t) + h_m(t, \underline{x}_{i,m}(t), \underline{x}_{i,m}(t-\tau(t))),\\ \dot{x}_{i,n}(t) &= u_i(t) + h_n(t, \underline{x}_{i,n}(t), \underline{x}_{i,n}(t-\tau(t))), \qquad (1)\\ y_i(t) &= x_{i,1}(t), \end{aligned}$$

where $i = 0, 1, ..., N$, $m = 1, 2, \ldots, n-1$, n stands for the dimension of the dynamics of each

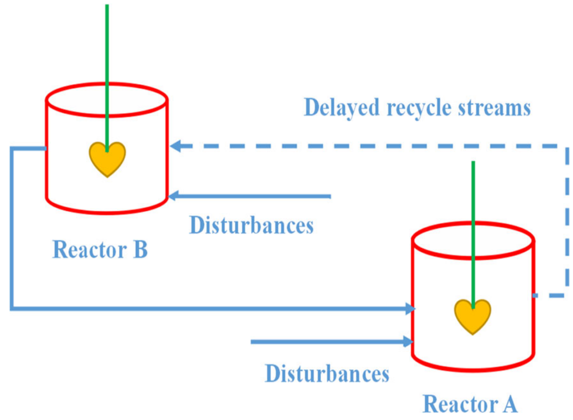

Fig. 1 Two-stage chemical reactors

agent. $\underline{x}_{i,m}(t) = \text{col}\{x_{i,1}(t), x_{i,2}(t), ..., x_{i,m}(t)\} \in \mathbb{R}^m$ and $\underline{x}_{i,m}(t-\tau(t)) = \text{col}\{x_{i,1}(t-\tau(t)), x_{i,2}(t-\tau(t)), ..., x_{i,m}(t-\tau(t))\} \in \mathbb{R}^m$ denote the state vector without or with the time delay on the state, respectively. $u_i(t) \in \mathbb{R}$ and $y_i(t) \in \mathbb{R}$ refer to as the control input and the output of the ith agent. Without any particular statement, the control input of the leader $u_0(t)$ is supposed to satisfying $u_0(t) = 0$. The nonlinear term $h_m(t, \underline{x}_{i,m}(t), \underline{x}_{i,m}(t-\tau(t))) : \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$ represents the intrinsic uncertain nonlinear features.

Remark 1 Recently, there has been a trend in regarding the nonlinear multiagent systems as a control target owing to their vast applications [12–17]. Most of the related results, however, have been implicitly paid attention to the case where the nonlinear multiagent systems were not subjected to the delay effect, which inevitably results in the limitation of the applications. It is worthy that many real physical systems only can be modeled as the system with time delay [24–27], such as the chemical reactors shown in Fig. 1. Hence, it is necessary to investigate the consensus problem for multiagent systems subjected to the delay on state.

Just as the statements in the introduction, the state variables except the output signals of some practical systems may not be available as a result of complicated and volatile environment. Hence, regarding the nonlinear multiagent systems with time delay on the state, how to put forward an effective distributed consensus protocol only based on their relative output information such that the output of each follower can asymptotically track that of the leader is one of the challenging topics. Hence, the primary objective of this paper is to address this problem. To be specific, we concentrate upon determining the dynamic gain of the state observer and formulating the distributed consensus protocol such that the output of each follower can asymptotically track that of the leader. Before proceeding, some necessary assumptions and lemmas shall be imposed.

Assumption 1 Suppose that the time delay $\tau(t)$ and its derivative satisfies

$$0 \leq \tau(t) \leq \tau^*, \quad \dot{\tau}(t) \leq \bar{\tau} < 1, \quad \forall t, \tag{2}$$

where τ^* and $\bar{\tau}$ are positive scalars.

Assumption 2 Suppose that there exist positive constants c_{k1} and c_{k2} such the following inequality holds for each $m = 1, 2, \ldots, n$

$$\begin{aligned} &|h_m(t, \underline{x}_{i,m}(t), \underline{x}_{i,m}(t - \tau(t))) \\ &\quad - h_m(t, \underline{x}_{0,m}(t), \underline{x}_{0,m}(t - \tau(t)))| \\ &\quad \leq \sum_{k=1}^{m} c_{k1}|x_{ik} - x_{0k}| + \sum_{k=1}^{m} c_{k2}|x_{ik\tau} - x_{0k\tau}| \ \ \forall t, \end{aligned} \tag{3}$$

where $x_{ik\tau} = x_{i,k}(t - \tau(t))$.

Assumption 3 The augmented graph $\bar{\mathcal{G}}$ associated with the communication topology of the multiagent systems is fixed and connected.

Lemma 1 [14] *Let* $A = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}$, $b = \begin{bmatrix} 1, \underbrace{0, 0, \ldots, 0}_{n-1} \end{bmatrix}^{\mathrm{T}}$, $c = \begin{bmatrix} \underbrace{0, 0, \cdots,}_{n-1} 1 \end{bmatrix}^{\mathrm{T}}$, *where* I_{n-1} *is an identity matrix of order* $n-1$. *There exist matrices* $Q = [q_1, q_2, \ldots, q_n]^{\mathrm{T}}$, $L = [l_1, l_2, \ldots, l_n]^{\mathrm{T}}$, *such that* $M_i, i = 1, 2$ *are hurwitz stable, where* $M_1 = I_N \otimes A - (\hat{\mathcal{L}} \otimes (cQ^{\mathrm{T}}))$ *and* $M_2 = I_N \otimes A - I_N \otimes (Lb^{\mathrm{T}})$. *In other words, there are positive definite matrices* $P_1 \in \mathbb{R}^{nN \times nN}$, $P_2 \in \mathbb{R}^{nN \times nN}$, *and positive constants* η_1, η_2 *such that*

$$P_1 M_1 + M_1^{\mathrm{T}} P_1 \leq -\eta_1 P_1, \tag{4}$$

$$P_2 M_2 + M_2^{\mathrm{T}} P_2 \leq -\eta_2 P_2. \tag{5}$$

Remark 2 The conditions in assumptions 1 and 2 are reasonable in many physical systems (*e. g.*, inverted pendulums and chemical reactors). Without loss of generality, let

$$\begin{aligned} &h_m(t, \underline{x}_{i,m}(t), \underline{x}_{i,m}(t - \tau(t))) \\ &\quad = \sum_{k=1}^{m} f(x_{ik}(t)) + \sum_{k=1}^{m} g(x_{ik}(t - \tau(t))) \end{aligned}$$

Then,

$$\begin{aligned} &\Big| h_m(t, \underline{x}_{i,m}(t), \underline{x}_{i,m}(t - \tau(t))) \\ &\quad - h_m(t, \underline{x}_{0,m}(t), \underline{x}_{0,m}(t - \tau(t))) \Big| \\ &\quad = \Big| \sum_{k=1}^{m} f(x_{ik}(t)) - \sum_{k=1}^{m} f(x_{0k}(t)) \\ &\qquad + \sum_{k=1}^{m} g(x_{ik}(t - \tau(t))) - \sum_{k=1}^{m} g(x_{0k}(t - \tau(t))) \Big| \\ &\quad \leq \sum_{k=1}^{m} \Big| f(x_{ik}(t)) - f(x_{0k}(t)) \Big| \\ &\qquad + \sum_{k=1}^{m} \Big| g(x_{ik}(t - \tau(t))) + g(x_{0k}(t - \tau(t))) \Big| \end{aligned}$$

If the functions $f(\cdot)$ and $g(\cdot)$ both satisfy the Lipschitz condition, then, there exist positive constants c_{k1} and c_{k2} such that the assumption is hold. For instance, the function $f(\cdot)$ and $h(\cdot)$ are defined as $f(\cdot) = \sin(\cdot)$ and $h(\cdot) = \cos(\cdot)$. Both of them satisfy the Lipschitz condition because of their bounded derivative. One notes that if the time delay is not considered, *i. e.*, $\tau(t) = 0$, the assumption 2 is transformed into a widespread one, which has been widely applied in the analysis of the output feedback control for nonlinear multiagent systems without time delay (Please refer to [20,29]). In addition, assumption 3 is a general and necessary assumption, which is often used to describe the communication topology among the agents (see [14,19]).

3 Main results

In this section, the distributed leader-following consensus protocols are established by applying the dynamic gain method. In order to facilitate later analysis for the ith agent with immeasurable state variables, the state observer for each agent is formulated as:

$$\dot{\hat{x}}_{i,m}(t) = \hat{x}_{i,m+1}(t) + l_m K^m (x_{i,1}(t) - \hat{x}_{i,1}(t)),$$
$$\dot{\hat{x}}_{i,n}(t) = u_i(t) + l_n K^n (x_{i,1}(t) - \hat{x}_{i,1}(t)), \tag{6}$$

where $\hat{x}_{i,m}(t)$ $(i = 0, 1, 2, \ldots, N, m = 1, 2, \ldots, n)$ is the estimate of the state variables $x_{i,m}(t)$, l_m is the coefficient selected such that the condition (5) in lemma 1 is satisfied, and $K(t) \geq 1$ denotes the dynamic gain to be specified subsequently. K^m refers to as the mth power of K.

Define

$$e_{i,m}(t) = x_{i,m}(t) - x_{0,m}(t),$$
$$z_{i,m} = \hat{x}_{i,m}(t) - \hat{x}_{0,m}(t),$$
$$m = 1, 2, \ldots, n, i = 1, 2, \ldots, N,$$
$$e_i = \left[e_{i,1}, \ldots, e_{i,n}\right]^{\mathrm{T}}, z_i = \left[z_{i,1}, \ldots, z_{i,n}\right]^{\mathrm{T}},$$
$$e = \left[e_1^{\mathrm{T}}, \ldots, e_N^{\mathrm{T}}\right]^{\mathrm{T}}, z = \left[z_1^{\mathrm{T}}, \ldots, z_N^{\mathrm{T}}\right]^{\mathrm{T}}.$$

According to (1), the tracking error system for ith agent can be described as

$$\begin{aligned}\dot{e}_{i,m}(t) &= e_{i,m+1}(t) + h_m(t, \underline{x}_{i,m}(t), \underline{x}_{i,m}(t - \tau(t))) \\ &\quad - h_m(t, \underline{x}_{0,m}(t), \underline{x}_{0,m}(t - \tau(t))), \\ \dot{e}_{i,n}(t) &= u_i(t) + h_n(t, \underline{x}_{i,n}(t), \underline{x}_{i,n}(t - \tau(t))) \\ &\quad - h_n(t, \underline{x}_{0,n}(t), \underline{x}_{0,n}(t - \tau(t))). \end{aligned} \tag{7}$$

From (6), one has

$$\dot{z}_{i,m}(t) = z_{i,m+1}(t) + l_m K^m (e_{i,1}(t) - z_{i,1}(t)),$$
$$\dot{z}_{i,n}(t) = u_i(t) + l_n K^n (e_{i,1}(t) - z_{i,1}(t)). \tag{8}$$

Therefore, the leader-following consensus problem of the multiagent system (1) is transformed into the stabilization problems of (7) and (8). Let $s_{i,m} = e_{i,m} - z_{i,m}$, $m = 1, 2, \ldots, n$, $i = 1, 2, \ldots, N$, one has

$$\begin{aligned}\dot{s}_{i,m}(t) &= s_{i,m+1}(t) - l_m K^m s_{i,1}(t) \\ &\quad + h_m(t, \underline{x}_{i,m}(t), \underline{x}_{i,m}(t - \tau(t))) \\ &\quad - h_m(t, \underline{x}_{0,m}(t), \underline{x}_{0,m}(t - \tau(t))), \\ \dot{s}_{i,n}(t) &= -l_n K^n s_{i,1}(t) + h_n(t, \underline{x}_{i,n}(t), \underline{x}_{i,n}(t - \tau(t))) \\ &\quad - h_n(t, \underline{x}_{0,n}(t), \underline{x}_{0,n}(t - \tau(t))). \end{aligned} \tag{9}$$

For ease of derivation, the following transformations are conducted:

$$\tilde{z}_{i,m}(t) = K^{1-m-\iota} z_{i,m}(t), \tilde{s}_{i,m}(t) = K^{1-m-\iota} s_{i,m}(t), \tag{10}$$

where ι is a positive scalar.

Thus, the systems (8) and (9) can be respectively reorganized as

$$\begin{aligned}\dot{\tilde{z}}_{i,m}(t) &= K \tilde{z}_{i,m+1}(t) + (1 - m - \iota) \frac{\dot{K}}{K} \tilde{z}_{i,m}(t) \\ &\quad + l_m K \tilde{s}_{i,1}(t), \\ \dot{\tilde{z}}_{i,n}(t) &= K u_i(t) + (1 - n - \iota) \frac{\dot{K}}{K} \tilde{z}_{i,n}(t) \\ &\quad + l_n K \tilde{s}_{i,1}(t), \end{aligned} \tag{11}$$

and

$$\begin{aligned}\dot{\tilde{s}}_{i,m}(t) &= K \tilde{s}_{i,m+1}(t) + (1 - m - \iota) \frac{\dot{K}}{K} \tilde{s}_{i,m}(t) \\ &\quad - l_m K \tilde{s}_{i,1}(t) \\ &\quad + K^{1-m-\iota} \left[h_m(\underline{x}_{i,m}(t), \underline{x}_{i,m}(t - \tau(t))) \right. \\ &\quad \left. - h_m(t, \underline{x}_{0,m}(t), \underline{x}_{0,m}(t - \tau(t)))\right], \\ \dot{\tilde{s}}_{i,n}(t) &= (1 - n - \iota) \frac{\dot{K}}{K} \tilde{s}_{i,n}(t) - l_n K \tilde{s}_{i,1}(t) \\ &\quad + K^{1-n-\iota} \left[h_n(t, \underline{x}_{i,n}(t), \underline{x}_{i,n}(t - \tau(t))) \right. \\ &\quad \left. - h_n(t, \underline{x}_{0,n}(t), \underline{x}_{0,n}(t - \tau(t)))\right]. \end{aligned} \tag{12}$$

With all the analysis above taking into account, we are now in a position to formulate our main result, which describes distributed consensus protocol for the nonlinear multiagent systems in the presence of state delay and immeasurable state variables simultaneously. The main result of this paper is proposed in the manner of theorem 1.

Theorem 1 *Suppose that assumptions 1-3 hold for the multiagent systems (1). Then, the output of each follower can ultimately asymptotically track that of the leader under the following distributed output feedback consensus protocol*

$$\begin{aligned}u_i(t) &= -Q^{\mathrm{T}} \Gamma \left(\sum_{j=1}^{N} a_{ij} (\hat{x}_i(t) \right. \\ &\quad \left. - \hat{x}_j(t)) + b_i (\hat{x}_i(t) - \hat{x}_0(t)) \right), \end{aligned} \tag{13}$$

$$\dot{K}(t) = \max\left\{-\phi K^2(t) + \varrho K(t), 0\right\}, \tag{14}$$

where $\hat{x}_i(t)$, $i = 0, 1, 2, \ldots, N$ *refers to as the estimate of the state variables* $x_i(t)$*, which is defined in (6),* $\Gamma = \text{diag}\{K^n, K^{n-1}, \ldots, K\}$, $Q = [q_1, q_2, \ldots, q_n]^{\text{T}}$ *denotes the control variables,* ϕ *and* ϱ *are positive coefficients of the dynamic gain* $K(t)$ *determined such that* $K(t) \geq 1$.

Proof According to (13), the control protocol of ith agent $u_i(t)$ can be reorganized as

$$u_i = -(\Lambda_i \otimes Q^{\text{T}})\tilde{z}, \tag{15}$$

where Λ_i is the ith row of the matrix $\hat{\mathcal{L}}$. $\tilde{z}_i(t) = \text{col}\{\tilde{z}_{i,1}(t), \tilde{z}_{i,2}(t), \ldots, \tilde{z}_{i,n}(t)\}$, $\tilde{z}(t) = [\tilde{z}_1^{\text{T}}(t), \ldots, \tilde{z}_N^{\text{T}}(t)]^{\text{T}}$. Substituting (15) into (11), one obtains

$$\begin{aligned}\dot{\tilde{z}}_{i,m} &= K\tilde{z}_{i,m+1} + (1 - m - \iota)\frac{\dot{K}}{K}\tilde{z}_{i,m} + l_m K\tilde{s}_{i,1},\\ \dot{\tilde{z}}_{i,n} &= -K(\Lambda_i \otimes Q^{\text{T}})\tilde{z} + (1 - n - \iota)\frac{\dot{K}}{K}\tilde{z}_{i,n}\\ &\quad + l_n K\tilde{s}_{i,1}.\end{aligned} \tag{16}$$

Define $\tilde{s}_i(t) = \text{col}\{\tilde{s}_{i,1}(t), \tilde{s}_{i,2}(t), \ldots, \tilde{s}_{i,n}(t)\}$. Then, the systems (12) and (16) can be rewritten as

$$\begin{aligned}\dot{\tilde{z}}_i(t) &= KA\tilde{z}_i(t) - K(\Lambda_i \otimes cQ^{\text{T}})\tilde{z}(t)\\ &\quad - \frac{\dot{K}}{K}(\iota I_n + G)\tilde{z}_i(t) + KL\tilde{s}_{i,1}(t),\end{aligned} \tag{17}$$

$$\begin{aligned}\dot{\tilde{s}}_i(t) &= K(A - Lb^{\text{T}})\tilde{s}_i(t)\\ &\quad - \frac{\dot{K}}{K}(\iota I_n + G)\tilde{s}_i(t) + f_i(t),\end{aligned} \tag{18}$$

where $f_i(t) = [f_{i,1}(t), \ldots, f_{i,n}(t)]^{\text{T}}$, $f_{i,m}(t) = K^{1-m-\iota}\left[h_m(t, \underline{x}_{i,m}, \underline{x}_{i,m\tau}) - h_m(t, \underline{x}_{0,m}, \underline{x}_{0,m\tau})\right]$, with $m = 1, 2, \ldots, n$ and ι is defined in (10), $\underline{x}_{i,m\tau} = \underline{x}_{i,m}(t - \tau(t))$, $G = \text{diag}\{0, 1, \ldots, n-1\}$. Let $\tilde{s}(t) = [\tilde{s}_1^{\text{T}}(t), \ldots, \tilde{s}_N^{\text{T}}(t)]^{\text{T}}$, $f(t) = [f_1^{\text{T}}(t), \ldots, f_N^{\text{T}}(t)]^{\text{T}}$, thus, one has

$$\begin{aligned}\dot{\tilde{z}}(t) &= K(I_N \otimes A - \hat{\mathcal{L}} \otimes cQ^{\text{T}})\tilde{z}(t)\\ &\quad - \frac{\dot{K}}{K}(I_N \otimes D)\tilde{z}(t) + K(\hat{s}(t) \otimes L),\end{aligned} \tag{19}$$

$$\begin{aligned}\dot{\tilde{s}}(t) &= K(I_N \otimes (A - Lb^{\text{T}}))\tilde{s}(t)\\ &\quad - \frac{\dot{K}}{K}(I_N \otimes D)\tilde{s}(t) + f(t),\end{aligned} \tag{20}$$

with $D = \iota I_n + G$, $\hat{s}(t) = \left[\tilde{s}_{1,1}(t), \tilde{s}_{2,1}(t), \ldots, \tilde{s}_{N,1}(t)\right]^{\text{T}}$.

In what follows, by utilizing the Lyapunov stability theory, the stability problems of systems (19) and (20) will be resolved. So as to achieve this objective, the following Lyapunov function candidate shall be established

$$V(t) = \sum_{k=1}^{3} V_k(t) \tag{21}$$

with

$$\begin{aligned}V_1(t) &= \tilde{z}^{\text{T}}(t)P_1\tilde{z}(t),\\ V_2(t) &= \tilde{s}^{\text{T}}(t)P_2\tilde{s}(t),\\ V_3(t) &= \frac{e^{\rho\tau^*}}{1-\bar{\tau}}\int_{t-\tau(t)}^{t} e^{-\rho(t-\mu)}(\lambda_2\|P_2\|\|\tilde{z}(\mu)\|^2\\ &\quad + \lambda_3\|P_2\|\|\tilde{s}(\mu)\|^2)d\mu,\end{aligned}$$

where ρ, λ_2, and λ_3 are positive scalars.

The derivative of $V(t)$ along (19) and (21) is expressed as

$$\begin{aligned}\dot{V}_1(t) &= 2\tilde{z}^{\text{T}}P_1\dot{\tilde{z}}\\ &= 2\tilde{z}^{\text{T}}P_1\left[K(I_N \otimes A - \hat{\mathcal{L}} \otimes cQ^{\text{T}})\tilde{z}\right]\\ &\quad + 2\tilde{z}^{\text{T}}P_1\left[-\frac{\dot{K}}{K}(I_N \otimes D)\tilde{z} + K(\hat{s} \otimes L)\right],\end{aligned} \tag{22}$$

$$\begin{aligned}\dot{V}_2(t) &= 2\tilde{s}^{\text{T}}P_2\dot{\tilde{s}}\\ &= 2\tilde{s}^{\text{T}}P_2\left[K(I_N \otimes (A - Lb^{\text{T}}))\tilde{s}\right.\\ &\quad \left.-\frac{\dot{K}}{K}(I_N \otimes D)\tilde{s} + f\right],\end{aligned} \tag{23}$$

$$\begin{aligned}\dot{V}_3(t) &\leq -\rho V_3 + \frac{\lambda_2 e^{\rho\tau^*}}{1-\bar{\tau}}\|P_1\|\|\tilde{z}\|^2 + \frac{\lambda_3 e^{\rho\tau^*}}{1-\bar{\tau}}\|P_2\|\|\tilde{s}\|^2\\ &\quad - \lambda_2\|P_2\|\|\tilde{z}_\tau\|^2 - \lambda_3\|P_2\|\|\tilde{s}_\tau\|^2.\end{aligned} \tag{24}$$

Based on lemma 1, one has

$$K\tilde{z}^{\text{T}}(P_1M_1 + M_1^{\text{T}}P_1)\tilde{z} \leq -\eta_1 K\tilde{z}^{\text{T}}P_1\tilde{z}, \tag{25}$$

According to the contributions in [25], there exists strictly positive constant ι_i such that $-\iota_i P_i \leq P_i(I_N \otimes$

$D)+(I_N\otimes D)P_i \leq \iota_i P_i$ for $i=1,2$. Hence, one has

$$-2\frac{\dot{K}}{K}\tilde{z}^{\mathrm{T}}P_1(I_N\otimes D)\tilde{z} \leq -\iota_1\frac{\dot{K}}{K}\tilde{z}^{\mathrm{T}}P_1\tilde{z},$$
$$-2\frac{\dot{K}}{K}\tilde{s}^{\mathrm{T}}P_2(I_N\otimes G)\tilde{s} \leq -\iota_2\frac{\dot{K}}{K}\tilde{s}^{\mathrm{T}}P_2\tilde{s}. \tag{26}$$

On the other hand, one obtains

$$\begin{aligned}2K\tilde{z}^{\mathrm{T}}P_1(\hat{s}\otimes L) &\leq 2K\|P_1\|\sqrt{\sum_{j=1}^{n} l_j^2}\|\tilde{z}\|\|\tilde{s}\| \\ &\leq \alpha_1 K\tilde{z}^{\mathrm{T}}P_1\tilde{z}+\alpha_2 K\tilde{s}^{\mathrm{T}}P_2\tilde{s},\end{aligned} \tag{27}$$

where $\alpha_1=\frac{\|P_1\|\sqrt{\sum_{j=1}^{n} l_j^2}}{\lambda_{\min}(P_1)}$, $\alpha_2=\frac{\|P_1\|\sqrt{\sum_{j=1}^{n} l_j^2}}{\lambda_{\min}(P_2)}$.

According to lemma 1, one gets

$$2\tilde{s}^{\mathrm{T}}P_2K(I_N\otimes(A-Lb^{\mathrm{T}}))\tilde{s} \leq -\eta_2 K\tilde{s}^{\mathrm{T}}P_2\tilde{s}. \tag{28}$$

In addition, combining with assumption 2, one obtains

$$\begin{aligned}|f_{i,m}| &= K^{1-m-\iota}\left|h_m(t,\underline{x}_{i,m},\underline{x}_{i,m\tau})-h_m(t,\underline{x}_{0,m},\underline{x}_{0,m\tau})\right| \\ &\leq K^{1-m-\iota}\left[\sum_{k=1}^{m}c_{k1}|x_{ik}-x_{0k}|+\sum_{k=1}^{m}c_{k2}|x_{ik\tau}-x_{0k\tau}|\right] \\ &\leq K^{1-m-\iota}\left[\sum_{k=1}^{m}c_{k1}|s_{ik}+z_{ik}|+\sum_{k=1}^{m}c_{k2}|s_{ik\tau}+z_{ik\tau}|\right] \\ &\leq K^{1-m-\iota}\left[\sum_{k=1}^{m}K^{m-1+\iota}c_{k1}\,|\tilde{s}_{ik}+\tilde{z}_{ik}| \right.\\ &\quad\left.+\sum_{k=1}^{m}K_{\tau}^{m-1+\iota}c_{k2}\,|\tilde{s}_{ik\tau}+\tilde{z}_{ik\tau}|\right]\end{aligned} \tag{29}$$

where $K_\tau=K(t-\tau(t))$, $m=1,2,\ldots,n$ and ι is defined in (10).

One notes that $m-1+\iota>0$. Based on the condition (14) and $\tau(t)\geq 0$, one has $K(t)\geq K(t-\tau(t))\geq 1$. That is, $K_\tau^{m-1+\iota}\leq K^{m-1+\iota}$, which implies

$$|f_{i,m}| \leq \sum_{k=1}^{m}c_{k1}\,|\tilde{s}_{ik}+\tilde{z}_{ik}|+\sum_{k=1}^{m}c_{k2}\,|\tilde{s}_{ik\tau}+\tilde{z}_{ik\tau}|\,. \tag{30}$$

Therefore, one has

$$\begin{aligned}2\tilde{s}^{\mathrm{T}}P_2 f &\leq 2\|\tilde{s}\|\|P_2\|(c_1(\|\tilde{s}\|+\|\tilde{z}\|) \\ &\quad+c_2(\|\tilde{s}_\tau\|+\|\tilde{z}_\tau\|),\end{aligned} \tag{31}$$

where $c_1=\max\left\{\sqrt{\frac{n(n+1)}{2}}c_{k1}\right\}$, $c_2=\max\left\{\sqrt{\frac{n(n+1)}{2}}c_{k2}\right\}$, $k=1,2,\ldots,n$.

By applying the Young's inequality, it yields

$$2c_1\|P_2\|\|\tilde{s}\|\|\tilde{z}\| \leq \frac{c_1^2}{\lambda_1}\|P_2\|\|\tilde{s}\|^2+\lambda_1\|P_2\|\|\tilde{z}\|^2, \tag{32}$$

$$2c_2\|P_2\|\|\tilde{s}\|\|\tilde{z}_\tau\| \leq \frac{c_2^2}{\lambda_2}\|P_2\|\|\tilde{s}\|^2+\lambda_2\|P_2\|\|\tilde{z}_\tau\|^2, \tag{33}$$

$$2c_2\|P_2\|\|\tilde{s}\|\|\tilde{s}_\tau\| \leq \frac{c_2^2}{\lambda_3}\|P_2\|\|\tilde{s}\|^2+\lambda_3\|P_2\|\|\tilde{s}_\tau\|^2, \tag{34}$$

where λ_1, λ_2 and λ_3 are positive constants.

Substituting equations (32)–(34) into (31), one has

$$\begin{aligned}2\tilde{s}^{\mathrm{T}}P_2 f &\leq \alpha_3\tilde{z}^{\mathrm{T}}P_1\tilde{z}+\alpha_4\tilde{s}^{\mathrm{T}}P_2\tilde{s} \\ &\quad+\lambda_2\|P_2\|\|\tilde{z}_\tau\|^2+\lambda_3\|P_2\|\|\tilde{s}_\tau\|^2,\end{aligned} \tag{35}$$

where $\alpha_3=\frac{\lambda_1\|P_2\|}{\lambda_{\min}(P_1)}$, $\alpha_4=\left(2c_1\|P_2\|+\frac{c_1^2\|P_2\|}{\lambda_1}break+\frac{c_2^2\|P_2\|}{\lambda_2}+\frac{c_2^2\|P_2\|}{\lambda_3}\right)/\lambda_{\min}(P_2)$.

According to system (6) and equations (22)–(25), the derivative of $V(t)$ can be reorganized as

$$\begin{aligned}\dot{V}(t) &\leq -\rho V_3-\left[(\eta_1-\alpha_1-\iota_1\phi)\,K+(\iota_1\varrho-\bar{\alpha}_3)\right]V_1(t) \\ &\quad-\left[(\eta_2-\alpha_2-\iota_2\phi)\,K+(\iota_2\varrho-\bar{\alpha}_4)\right]V_2(t).\end{aligned} \tag{36}$$

where $\bar{\alpha}_3=\alpha_3+\frac{\lambda_2 e^{\rho\tau^*}\|P_1\|}{(1-\bar{\tau})\lambda_{\min}(P_1)}$, $\bar{\alpha}_4=\alpha_4+\frac{\lambda_3 e^{\rho\tau^*}\|P_2\|}{(1-\bar{\tau})\lambda_{\min}(P_2)}$.

One can select the parameters ϕ and ϱ such that

$$\eta_1-\alpha_1-\iota_1\phi>0,\quad \eta_2-\alpha_2-\iota_2\phi \tag{37}$$
$$\iota_1\varrho-\bar{\alpha}_3>0,\quad \iota_2\varrho-\bar{\alpha}_4>0 \tag{38}$$

Hence, there exist positive constants β_1 and β_2 such that

$$\dot{V}(t) \leq -\beta V_1(t), \tag{39}$$

where $\beta = \min\{\rho, \beta_1, \beta_2\}$. Further, one gets

$$V(t) \leq V(0)\exp(-\beta t). \tag{40}$$

Hence, one has

$$\|\tilde{z}\|^2 \leq \frac{V(0)}{\lambda_{\min}(P_1)}\exp(-\beta t),$$
$$\|\tilde{s}\|^2 \leq \frac{V(0)}{\lambda_{\min}(P_2)}\exp(-\beta t). \tag{41}$$

From the definitions of $\tilde{z}(t)$ and $\tilde{s}(t)$, one further obtains

$$z_{im}^2 \leq \frac{V(0)K_m^{2(\iota+m-1)}}{\lambda_{\min}(P_1)}\exp(-\beta t), \tag{42}$$
$$s_{im}^2 \leq \frac{V(0)K_m^{2(\iota+m-1)}}{\lambda_{\min}(P_2)}\exp(-\beta t). \tag{43}$$

Combining with the fact that $K(0) \leq K(t) \leq K_m$ with $K_m = \max\{K(0), \frac{\varrho}{\phi}\}$, which yields

$$x \lim_{t\to\infty} |x_{im} - x_{0m}| \leq \lim_{t\to\infty} |z_{im} + s_{im}| \leq \lim_{t\to\infty} |z_{im}| + \lim_{t\to\infty} |s_{im}| \to 0, \tag{44}$$

which completes the proof of theorem 1. □

Remark 3 The leader-following consensus issue of the nonlinear multiagent system with time delay is well-resolved if the coefficients ϕ and ϱ of the dynamic gain $K(t)$ are determined. The coefficients ϕ and ϱ play the key role of the consensus problem, which can be summarized as follows: whether or not the conditions (37) and (38) are satisfied is depend on selections of the ϕ and ϱ, which further determines the stability analysis of systems (1). Besides, by selecting positive constants ϕ and ϱ, one can derive that the time-varying function $K(t)$ is a nondecreasing one and satisfies $K(0) \leq K(t) \leq K_m$. On the other hand, since the time-delay $\tau(t)$ is nonnegative and bounded, thus, one can further conclude that $K(t) \geq K(t - \tau(t))$.

Remark 4 Regarding aforementioned stability analysis and controller design procedures, the distributed consensus controller of ith agent only required its own relative output and the output information of its neighbors is designed, in which there are some control parameters shall be specified. The corresponding design procedures are summarized as follows:

1. By solving the LMIs presented in lemma 1, the positive matrices P_1 and P_2 can be obtained.
2. Based on the assumption (3), the parameters c_1 and c_2 can be specified.
3. Choosing appropriate parameters $\lambda_i (i = 1, 2, 3)$, the coefficients $\alpha_1, \alpha_2, \bar{\alpha}_3, \bar{\alpha}_4$ can be directly computed.
4. Based on (37) and (38), specify ϕ and ϱ.

Remark 5 In [14], the output feedback consensus control problem was investigated by establishing the dynamic gain observer for each agent. Besides, the distributed leader-following consensus protocol was proposed by using the relative outputs and the estimation of the state variables of its neighbors. Different from this pioneering works, the practical case that the nonlinear multiagent systems subjected to the time delay is considered. Thus, the works in [14] can be regard as a special case of this work with $\tau(t) = 0$. Compared with the works in [24,25], the dynamic gain compensator was established constituted the relative outputs of all followers, which is replaced by the state observer just consisted of its own output in our works. Therefore, the number of the communication variable can be reduced at certain degree.

It is worthy that the Lipschitz growth rates are known constants in assumption 2, which will limit the applications of the theoretical results. Hence, in this part, the condition of the Lipschitz growth rates will be relaxed and the output feedback control theoretical based on the general case will be followed.

Assumption 4 Consider the function $h_m(t, \underline{x}_{i,m}, \underline{x}_{i,m}(t - \tau(t)))$. Suppose that there exist nonnegative constants ϵ_i and υ_i, $i = 1, 2$ satisfying $2\upsilon_2 < \upsilon_1$ such that the following inequality holds for each $m = 1, 2, \ldots, n$

$$\begin{aligned} &|h_m(t, \underline{x}_{i,m}, \underline{x}_{i,m}(t - \tau(t))) \\ &\quad - h_m(t, \underline{x}_{0,m}(t), \underline{x}_{0,m}(t - \tau(t)))| \\ &\quad \leq \sum_{k=1}^{m} \epsilon_1 e^{\upsilon_1 t} |x_{ik} - x_{0k}| \\ &\qquad + \sum_{k=1}^{m} \epsilon_2 e^{\upsilon_2 t} |x_{ik\tau} - x_{0k\tau}| \quad \forall t, \end{aligned} \tag{45}$$

where $x_{ik\tau} = x_{ik}(t - \tau(t))$.

As stated in [14], the growth nonlinearities with respect to unmeasured state components play the key role in the output feedback control problem. Hence, the growth condition is necessary in addressing nonlinearities depending on unmeasured states. Compared the known constants Lipschitz growth rates, the distributed consensus controller design for the unknown time-varying Lipschitz growth rates seems to be difficult mainly resulted from the evident challenging summarized as follows: the upper bound of the derivative of the Lyapunov function is dependent of the unknown time-varying Lipschitz growth rates, hence, how to design the dynamic gain such that the effect induced by the time-varying Lipschitz growth rates contains certain challenging.

In what follows, the distributed leader-following consensus protocol for the case that Lipschitz growth rates of the nonlinear functions are unknown time-varying functions will be proposed in the manner of following theorem.

Theorem 2 *If the assumptions (1), (3) and (4) are satisfied, then the output of each follower can ultimately asymptotically track that of the leader by designing the observer (6)-based distributed linear-like controller (13) with*

$$\dot{K}(t) = \max\left\{-\phi K^2 + \varrho_1 e^{\upsilon_1 t} K, 0\right\}, \tag{46}$$

where ϕ, ϱ_1 are positive constants specified subsequently such that $K(t) \geq 1$.

Proof Combining with assumption 4, one obtains

$$\begin{aligned}
|f_{i,m}| &= K^{1-m-\iota}\left|h_m(t, \underline{x}_{i,m}, \underline{x}_{i,m\tau}) - h_m(t, \underline{x}_{0,m}, \underline{x}_{0,m\tau})\right| \\
&\leq K^{1-m-\iota}\left[\sum_{k=1}^{m} \epsilon_1 e^{\upsilon_1 t}|x_{ik} - x_{0k}| + \sum_{k=1}^{m} \epsilon_2 e^{\upsilon_2 t}|x_{ik\tau} - x_{0k\tau}|\right] \\
&\leq K^{1-m-\iota}\left[\sum_{k=1}^{m} \epsilon_1 e^{\upsilon_1 t}|s_{ik} + z_{ik}| + \sum_{k=1}^{m} \epsilon_2 e^{\upsilon_2 t}|s_{ik\tau} + z_{ik\tau}|\right] \\
&\leq K^{1-m-\iota}\left[\sum_{k=1}^{m} K^{m-1+\iota}\epsilon_1 e^{\upsilon_1 t}\,|\tilde{s}_{ik} + \tilde{z}_{ik}| + \sum_{k=1}^{m} K_\tau^{m-1+\iota}\epsilon_2 e^{\upsilon_2 t}\,|\tilde{s}_{ik\tau} + \tilde{z}_{ik\tau}|\right] \\
&\leq \sum_{k=1}^{m} \epsilon_1 e^{\upsilon_1 t}\left(|\tilde{s}_{ik}| + |\tilde{z}_{ik}|\right) + \sum_{k=1}^{j} \epsilon_2 e^{\upsilon_2 t}\left(|\tilde{s}_{ik\tau}| + |\tilde{z}_{ik\tau}|\right),
\end{aligned} \tag{47}$$

which yields

$$\begin{aligned}
2\tilde{s}^{\mathrm{T}} P_2 f &\leq 2\|\tilde{s}\|\|P_2\|(\omega_1 e^{\upsilon_1 t}(\|\tilde{s}\| + \|\tilde{z}\|) \\
&\quad + \omega_2 e^{\upsilon_2 t}(\|\tilde{s}_\tau\| + \|\tilde{z}_\tau\|),
\end{aligned} \tag{48}$$

where $\omega_1 = \sqrt{\frac{n(n+1)}{2}}\epsilon_1$, $\omega_2 = \sqrt{\frac{n(n+1)}{2}}\epsilon_2$.

Following the similar procedures in (35), eq. (48) can be reorganized as

$$\begin{aligned}
2\tilde{s}^{\mathrm{T}} P_2 f &\leq \zeta_1 e^{\upsilon_1 t} V_1(t) + (\zeta_2 + \zeta_3 e^{(2\upsilon_2 - \upsilon_1)t}) e^{\upsilon_1 t} V_2(t) \\
&\quad + \lambda_2\|P_2\|\|\tilde{z}_\tau\|^2 + \lambda_3\|P_2\|\|\tilde{s}_\tau\|^2
\end{aligned} \tag{49}$$

where $\zeta_1 = \frac{\lambda_1\|P_2\|\omega_1}{\lambda_{\min}(P_1)}$, $\zeta_2 = \frac{2\|P_2\|\omega_1}{\lambda_{\min}(P_2)} + \frac{\|P_2\|\omega_1}{\lambda_1\lambda_{\min}(P_2)}$, $\zeta_3 = \frac{\|P_2\|\omega_2^2}{\lambda_2\lambda_{\min}(P_2)} + \frac{\|P_2\|\omega_2^2}{\lambda_3\lambda_{\min}(P_2)}$.

By utilizing the Lyapunov function (21) and following the similar procedures in theorem 1, one has

$$\begin{aligned}
\dot{V}(t) &\leq -\rho V_3(t) - ((\eta_1 - \alpha_1 - \iota_1\phi) K \\
&\quad + \left((\iota_1\varrho_1 - \zeta_1) e^{\upsilon_1 t} - \zeta_4)\right) V_1(t) \\
&\quad - ((\eta_2 - \alpha_2 - \iota_2\phi) K \\
&\quad + \left((\iota_2\varrho_1 - (\zeta_2 + \zeta_3 e^{(2\upsilon_2 - \upsilon_1)t}))e^{\upsilon_1 t} - \zeta_5\right)\Big) V_2(t).
\end{aligned} \tag{50}$$

where $\zeta_4 = \frac{\lambda_2 e^{\rho\tau^*}\|P_1\|}{(1-\bar{\tau})\lambda_{\min}(P_1)}$, $\zeta_5 = \frac{\lambda_3 e^{\rho\tau^*}\|P_2\|}{(1-\bar{\tau})\lambda_{\min}(P_2)}$.

Choose parameter ϕ such that

$$\eta_1 - \alpha_1 - \iota_1\phi > 0,\ \eta_2 - \alpha_2 - \iota_2\phi > 0 \tag{51}$$

$$\iota_1\varrho_1 - \zeta_1 > 0,\ \iota_2\varrho_1 - \zeta_2 - \zeta_3 > 0 \tag{52}$$

. In addition, one knows that there must be instant t_1^* and t_2^* satisfying $e^{\upsilon_1 t} > \frac{\zeta_4}{\iota_1\varrho_1 - \zeta_1}$ and $e^{\upsilon_1 t} > \frac{\zeta_5}{\iota_2\varrho_1 - \zeta_2 - \zeta_3}$ for $t \in [t^*, \infty)$ with $t^* = \max\{t_1^*, t_2^*\}$.

Hence, let $\bar{\mu} = \max\{\eta_1 - \alpha_1 - \iota_1\phi, \eta_2 - \alpha_2 - \iota_2\phi\}$, for $t \in [t^*, \infty)$, there exist a positive constant $\mu = \min\{\bar{\mu}K(0), \rho\}$ such that

$$\dot{V}(t) \le -\mu V(t). \tag{53}$$

Further, one gets

$$\dot{V}(t) \le V(t^*)\exp(-\mu(t-t^*)), \quad t \in [t^*, \infty), \tag{54}$$

which implies that

$$\begin{aligned} \|\tilde{z}\|^2 &\le \frac{V(t^*)}{\lambda_{\min}(P_1)}\exp\left(-\mu(t-t^*)\right), \\ \|\tilde{s}\|^2 &\le \frac{V(t^*)}{\lambda_{\min}(P_2)}\exp\left(-\mu(t-t^*)\right). \end{aligned} \tag{55}$$

Similar as the previous procedures, one has

$$z_{im}^2 \le \gamma_1 K^{2(\iota+m-1)}\exp(-\mu t), \tag{56}$$
$$s_{im}^2 \le \gamma_2 K^{2(\iota+m-1)}\exp(-\mu t), \tag{57}$$

where $\gamma_1 = \frac{V(t^*)e^{\mu t^*}}{\lambda_{\min}(P_1)}$, $\gamma_2 = \frac{V(t^*)e^{\mu t^*}}{\lambda_{\min}(P_2)}$.

According to (46), one notes that $K(t) \le \max\{K(0), \frac{\varrho_1 e^{\upsilon_1 t}}{\phi}\}$, which yields

$$z_{im}^2 \le \gamma_1 \max\{K_0^{\delta_m}, \xi_m e^{\delta_m \upsilon_1 t}\}\exp(-\mu t), \tag{58}$$
$$s_{im}^2 \le \gamma_2 \max\{K_0^{\delta_m}, \xi_m e^{\delta_m \upsilon_1 t}\}\exp(-\mu t), \tag{59}$$

where $\delta_m = 2(\iota+m-1)$, $\xi_m = (\frac{\varrho_1}{\phi})^{\delta_m}$, $K_0^{\delta_m} = (K_0)^{\delta_m}$.

By choosing

$$\rho > \delta_n \upsilon_1 + \kappa, \quad K(0) \ge \max\left\{1, \frac{\delta_n \upsilon_1 + \kappa}{\bar{\mu}}\right\} \tag{60}$$

with κ being any positive scalar, one can ensure that $\lim_{t\to\infty} z_{im}^2 \to 0$ and $\lim_{t\to\infty} s_{im}^2 \to 0$. The proof is completed for theorem 2. □

4 Simulation example

In this part, the simulation examples are presented to verify the effectiveness of the proposed protocols.

Example 1 Consider the following chemical reactor with delayed recycle streams, whose dynamic models can be described as follows:

$$\begin{aligned} \dot{x}_{i1} &= -\frac{1}{\alpha_{i1}}x_{i1} - \beta_{i1}x_{i1} \\ &\quad + \frac{1-\theta_{i2}}{\varpi_{i1}}x_{i2} + F_1(\cdot) \\ \dot{x}_{i2} &= -\frac{1}{\alpha_{i2}}x_{i2} - \beta_{i2}x_{i2} + \frac{\theta_{i1}}{\varpi_{i2}}x_{i1}(t-\tau(t)) \\ &\quad + \frac{h_{i2}}{\varpi_{i2}}u_i + F_2(\cdot) \\ y_i &= x_{i1}(t) \end{aligned} \tag{61}$$

where x_{i1}, x_{i2} represent the compositions; α_{i1}, α_{i2} denote the reactor residence times; β_{i1}, β_{i2} stand for the reaction variables; θ_{i2} refers to as the recycle flow rate; ω_{i2} denotes the feed rate; ϖ_{i1}, ϖ_{i1} denotes the reactor volumes; F_{i1} and F_{i2} stand for the intrinsic nonlinear features of the following chemical reactor; $\tau(t)$ is the unknown time-varying delay. Motivated by the work in [24,25], the corresponding simulation parameters are defined as: $\alpha_{i1} = \alpha_{i2} = 10$; $\beta_{i1} = 0.02$; $\beta_{i2} = 0.05$, $\theta_{i1} = 0.2$, $\theta_{i2} = 0.2$, $\varpi_{i1} = \varpi_{i2} = 0.8$; $h_{i2} = 0.8$; $F_1(\cdot) = 0.03x_{i1}$, $F_2 = -0.25x_{i2}(t-\tau(t))$, $\tau(t) = 0.6 + 0.2\sin(t)$. Substituting these parameters into (61) leads to

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} + \delta_1(x_{i1}(t)) \\ \dot{x}_{i2} &= u_i + \delta_2(\underline{x}_{i2}(t), \underline{x}_{i2}(t-\tau(t))) \end{aligned} \tag{62}$$

where $\delta_1(x_{i1}(t)) = -0.09x_{i1}$, $\delta_2(\underline{x}_{i2}(t), \underline{x}_{i2}(t-\tau(t))) = -0.15x_{i2} - 0.25x_{i1}(t-\tau(t)) - 0.25x_{i2}(t-\tau(t))$.

Further, one has

$$|\delta_1| \le 0.09|x_{i1}|$$

$$|\delta_2| \le 0.15\sum_{j=1}^{2}|x_{ij}(t)| + 0.25\sum_{j=1}^{2}|x_{ij}(t-\tau(t))|$$

Obviously, the assumption (2) about the nonlinear function is satisfied. From (6), the distributed dynamic observer for system (62) is formulated as

$$\begin{aligned} \dot{\hat{x}}_{i,1} &= \hat{x}_{i,2} + l_1 K(x_{i,1} - \hat{x}_{i,1}) \\ \dot{\hat{x}}_{i,2} &= u_i + l_2 K^2(x_{i,1} - \hat{x}_{i,1}) \end{aligned} \tag{63}$$

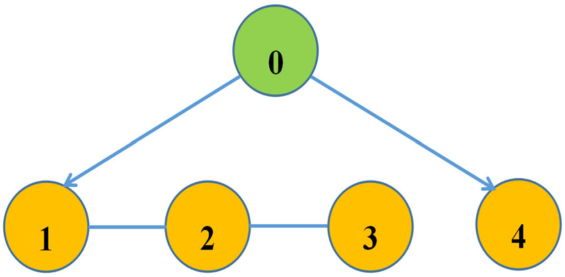

Fig. 2 Communication topology of multiagent systems (61)

The communication topology graph is shown in Fig. 2, where the leader is indexed by 0 and followers are indexed from 1 to 4. The weighted adjacent matrix of graph $\mathcal{G}$ and connection weight matrix are given as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \text{diag}\{1, 0, 0, 1\}.$$

Let $l_1 = 0.6$, $l_2 = 0.8$, $q_1 = q_2 = 6$, $\eta_1 = 0.3$, $\rho = 0.1$, $\eta_2 = 0.2$, $\iota_1 = 0.1$, $\iota_2 = 0.1$. By solving the conditions in Lemma 1, the positive definite matrix P_1 and P_2 can be obtained. Hence, one has $\lambda_{\min}(P_1) = 0.0768$, $\lambda_{\min}(P_2) = 0.1654$, $\|P_1\| = 0.9742$, $\|P_2\| = 0.4098$. By directly computation, one has $c_1 = 0.2598$, $c_2 = 0.4330$, $\tau^* = 0.8$, $\bar{\tau} = 0.2$. Based on conditions (37) and (38), the coefficients of the dynamical gain $K(t)$ can be specified as $\phi = 7.1849$ and $\varrho = 67.2705$. The simulation results are shown in Figs. 3, 4, 5, 6, 7, 8, 9, 10, which further verify the effectiveness of the theoretical results. Figures 3 and 4 show the signal response curves of the system (61) with initial condition $x_{0,1}(t) = 1.6$, $x_{0,2}(t) = 0.1$, $x_{1,1}(t) = 0.1$, $x_{1,2}(t) = 0.4$, $x_{2,1}(t) = 0.7$, $x_{2,2}(t) = 0.3$, $x_{3,1}(t) = 0.9$, $x_{3,2}(t) = 0.6$, $x_{4,1}(t) = 1.3$, $x_{4,2}(t) = 0.9$. The control input response curves and the dynamic gain response curve are shown in Figs. 5, 6 with the initial condition $u_i(t) = 0 (i = 0, 1, \ldots, 4)$ and $K(t) = 5$, respectively. The state variables of the dynamic gain observer are shown in Figs. 9, 10 with the initial condition $\hat{x}_{0,1}(t) = 0.1$, $\hat{x}_{0,2}(t) = 0.1$, $\hat{x}_{1,1}(t) = 0.5$, $\hat{x}_{1,2}(t) = 0.4$, $\hat{x}_{2,1}(t) = 0.3$, $\hat{x}_{2,2}(t) = 0.3$, $\hat{x}_{3,1}(t) = 0.1$, $\hat{x}_{3,2}(t) = 0.6$, $\hat{x}_{4,1}(t) = 0.7$, $\hat{x}_{4,2}(t) = 0.9$. Based on these, one can conclude that the validity of theorem 1 is well-illustrated by the simulation example 1.

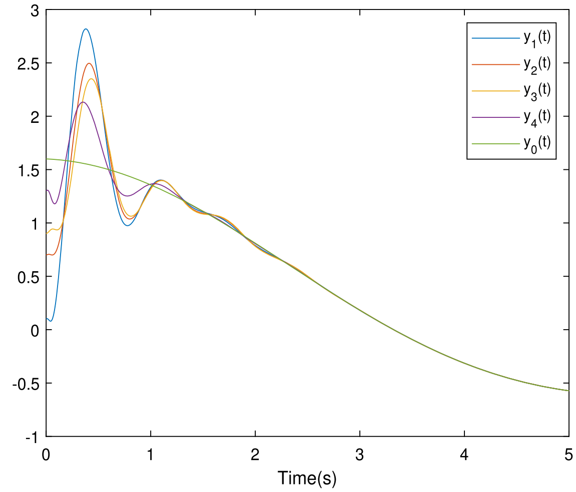

Fig. 3 Response curves of the output y_i of system (61), $i = 0, 1, 2, 3, 4$

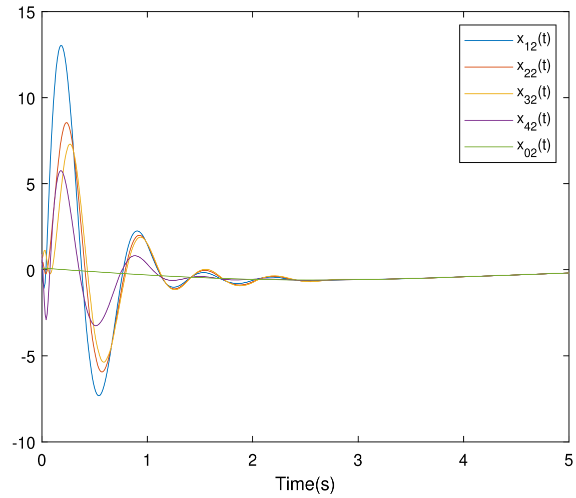

Fig. 4 Response curves of the state variables x_{i2} of system (61), $i = 0, 1, 2, 3, 4$

Example 2 To show the effectiveness and the validity of theorem 2, the following multiagent systems are considered as:

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1} + h_m(t, \underline{x}_{i,m}(t), \underline{x}_{i,m}(t - \tau(t))), \\ \dot{x}_{i,n} &= u_i(t) + h_n(t, \underline{x}_{i,n}(t), \underline{x}_{i,n}(t - \tau(t))), \\ y_i(t) &= x_{i,1}(t), \end{aligned} \tag{64}$$

where $x_i = [x_{i,1}, x_{i,2}]^{\mathrm{T}} \in \mathbb{R}^2$, $y_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ represent the output measurement and the control signal of agent i, respectively. $h_1(t, x_{i,1}) =$

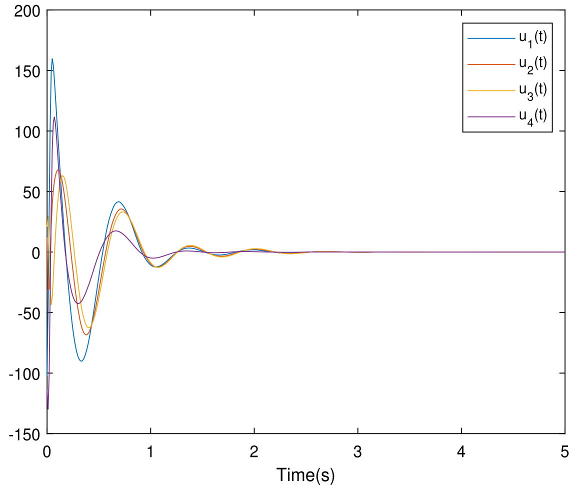

Fig. 5 Response curves of the control input u_i of system (61) $i = 1, 2, 3, 4$

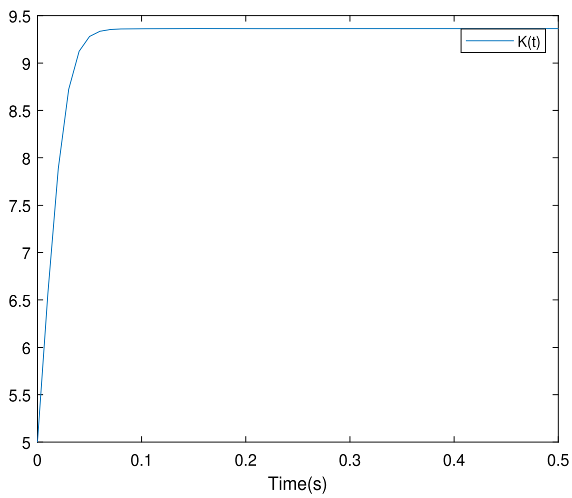

Fig. 6 Response curve of the dynamic gain parameter $K(t)$ of system (61)

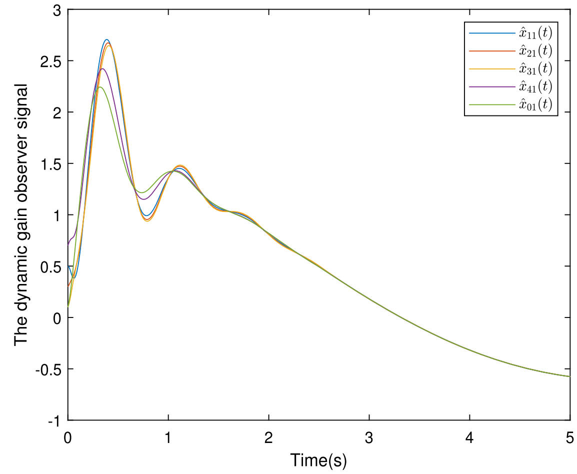

Fig. 7 Response curves of the output of the observer $\hat{x}_{i1}$ of system (61)

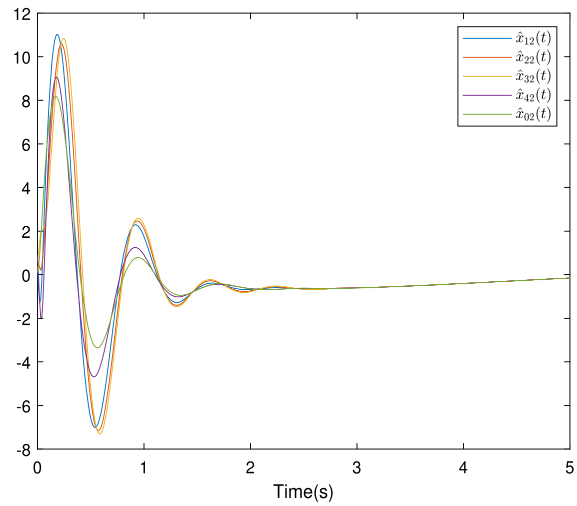

Fig. 8 Response curves of the state variables of observer $\hat{x}_{i2}$ of system (61)

$0.2e^{0.2t}x_{i,1}+0.25e^{0.05t}x_{i,1}(t-\tau(t)), h_2(t, x_{i,1}, x_{i,2}) = 0.3e^{0.2t}x_{i,2}+0.25e^{0.05t}x_{i,1}(t-\tau(t))-0.25e^{0.05t}x_{i,2}(t-\tau(t))$. The communication graph is shown in Fig. 9, where the leader is indexed by 0 and followers are indexed from 1 to 4. The weighted adjacent matrix of graph $\mathcal{G}$ and connection weight matrix are given as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \text{diag}\{1, 0, 0, 1\}.$$

From the definition of nonlinear functions, one knows that the nonlinear terms satisfy assumption (4) with $\epsilon_1 = 0.2$, $\epsilon_2 = 0.3$, $\upsilon_1 = 2$, $\upsilon_2 = 0.5$, and other simulation parameters are given in example 1. The coefficients ϕ and ϱ_1 of the dynamical gain $k(t)$ can be specified as $\phi = 6.0656$ and $\varrho_1 = 26.0715$. The simulation results are shown in Figs. 10, 10, 12, which further verify the effectiveness of the theoretical results. Figure 10 shows the response curves of the closed-loop systems (64) with initial condition $x_0(t) = [0, 0]^{\mathrm{T}}$, $x_i(t) = [-i, 2i]^{\mathrm{T}}(i = 1, 2, 3, 4)$. The control input response curves and the dynamic gain

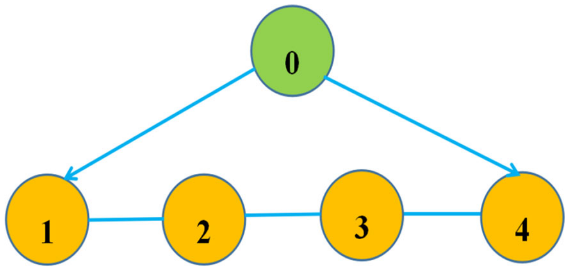

Fig. 9 Communication topology of multiagent systems (64)

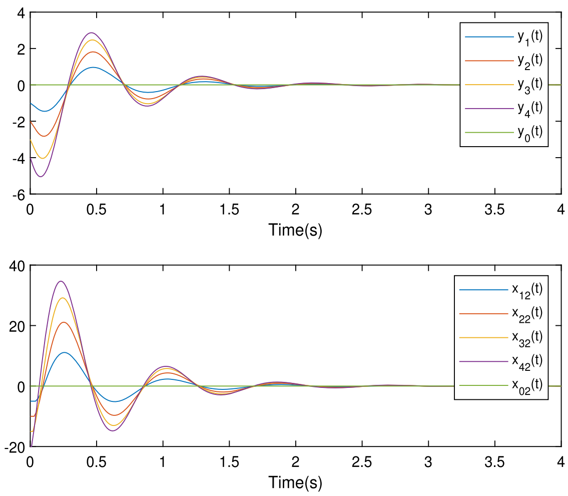

Fig. 10 Response trajectories of the closed-loop systems(64)

response curve are shown in Fig. 11 with the initial condition $u_i(t) = 0(i = 0, 1, \ldots, 4)$ and $K(t) = 9$, respectively. The state variables of the dynamic gain observer are shown in Fig. 12 with the initial condition $\hat{x}_i(t) = [0, 0]^{\mathrm{T}}(i = 0, 1, 2, \ldots, 5)$. Hence, one can conclude that the validity and the effectiveness of theorem 2 are well-illustrated by the simulation example 2.

5 Conclusion

In this paper, the output feedback consensus issue has been addressed for a class of nonlinear multiagent systems with time delay. In order to compensate the effect induce by the immeasurable state variables, the dynamic gain observer for each agent was formulated. With the help of the coordinate transformation, the consensus problems were transformed into the stabilization problem, which overcomes the explosion of complexity problem of the back-stepping method. By virtual of the Lyapunov-like approach, the distributed consensus protocols were established both for the case that the Lipschitz growth rates were known constants and unknown time-varying functions. The simulation examples have been performed to verify the effectiveness of the consensus agreements presented in this paper. Other interesting research directions would be the further extension of the current outcomes to the output feedback leader-following consensus problems for multiagent systems under weighted directed graph (see [12,13])

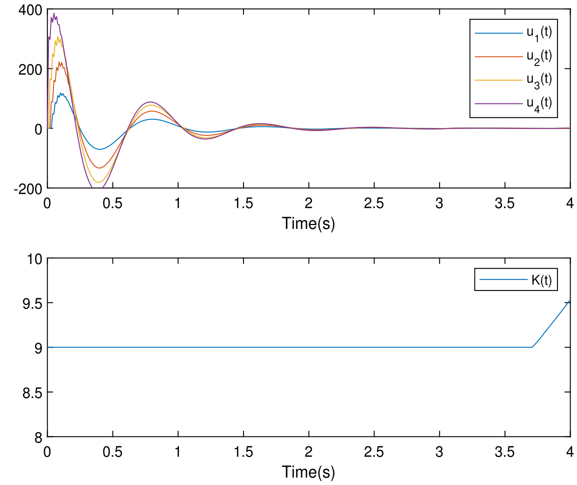

Fig. 11 Response curves of the control input u_i and dynamical gain $K(t)$ of the closed-loop systems(64)

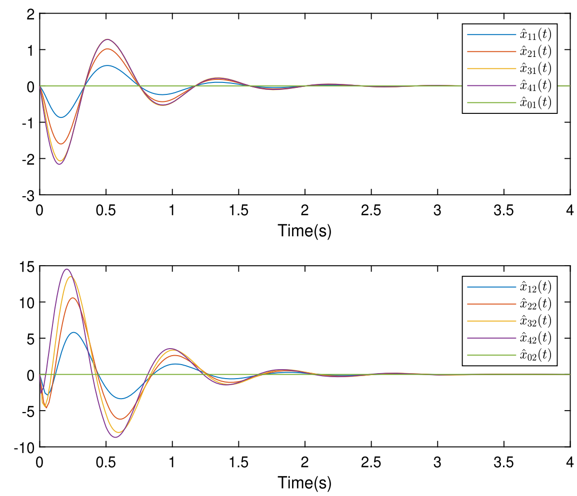

Fig. 12 Response trajectories of the dynamical observer of the closed-loop systems(64)

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any experiments with human or animal participants performed by any of the authors.

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