Nonlinear Dyn (2021) 105:2539–2548
https://doi.org/10.1007/s11071-021-06642-6

ORIGINAL PAPER

Dark wave, rogue wave and perturbation solutions of Ivancevic option pricing model

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Received: 4 May 2021 / Accepted: 16 June 2021 / Published online: 19 July 2021

Abstract Under investigation in this paper is the Ivancevic option pricing model. Based on trial function method, rogue wave and dark wave solutions are constructed. By means of symbolic computation, these analytical solutions are obtained with the Maple. Perturbation solutions are obtained through direct perturbation method. These results will enrich the existing literature of the Ivancevic option pricing model. Dynamical characteristics for rogue waves and dark waves are exhibited by using three-dimensional plots, curve plots, density plots and contour plots.

Keywords Trial function method · Tanh expansion method · Direct perturbation method · Ivancevic option pricing equation

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1 Introduction

Most of nonlinear phenomena can be easily studied by nonlinear partial differential equations (NPDEs) [1–8]. Researchers have studied various kinds of waves through this powerful tool, such as mixed lump wave [9], multi-waves [10], three-wave [11], breather [12], rogue waves [13–15], multiple complex soliton [16], bright and dark soliton [17], complex wave [18], soliton solution [19–22], traveling wave solutions [23], lump solution [24–28], dark waves [29], double-wave solutions [30], interaction solution [31–38]. At the same time, various methods have been developed to study these NPDEs, such as, Hirota bilinear method [39], the general bilinear techniques [40], bilinear neural network method [41–44], the tanh method [45], extended tanh method [46], improved (G'/G)-expansion [47], sine-cosine method [48], tanh-coth method [49], Lie group method [50], modified transformed rational function method [51]. Most of these methods listed above can be regarded as trial function method. Considering following general form of NPDEs,

$$P\left(\psi, \psi_t, \psi_s, \psi_{ss}, \ldots\right) = 0, \tag{1}$$

where ψ is a complex function. To find the analytical solutions of Eq. (1), the trial function is constructed as follows:

$$\psi = [\Psi_0 + \Psi(\xi)]\, \mathrm{e}^{i(c_2 t + k_2 s)}, \tag{2}$$

where $\xi = c_1 t + k_1 s$, $\Psi(\xi)$ can be any function with the independent variable ξ, such as $\tanh(\xi)$, $\cos(\xi)$, $\tanh(\xi) + \cos(\xi)$ and so on. $\Psi(\xi)$ can even be an arbitrary function $F(\xi)$ or $F(\xi) + F^2(\xi)$ and the like. Next, substituting Eq. (2) into Eq. (1), extracting the coefficients of $\mathrm{e}^{i(c_2 t + k_2 s)}$ and then collecting the coefficients of $\Psi(\xi)$ in both real part and imaginary part, the system of equations can be obtained. Solving these equations, the constraint solutions of the coefficients in the original equation Eq. (2) and the trial function Eq. (1) will be obtained. By introducing these coefficient solutions into the trial function Eq. (1), the explicit solution ψ of Eq. (1) will be obtained. However, it is rare to study nonlinear option equations by using these powerful tools.

In this paper, we investigate the following Ivancevic option pricing model:

$$\mathrm{i}\partial_t \psi = -\frac{1}{2}\sigma \partial_{ss}\psi - \beta|\psi|^2\psi, \quad (\mathrm{i} = \sqrt{-1}). \tag{3}$$

This is a wave-form, nonlinear, stochastic and adaptive option pricing model. This model was first proposed by Ivancevic in Ref. [52] to satisfy both behavioral and efficient markets, where σ means the volatility, which represents either stochastic process itself or just a constant. Landau coefficient $\beta = \beta(r, w)$ means the adaptive market potential. In simplest nonadaptive scenario, β is equal to the r, which represents interest rate, while in the adaptive case, $\beta(r, w)$ can be related to the market temperature and it depends on the set of adjustable parameters $\{W_i\}$. The independent variable t represents time, and s represents asset price. Response variable $\psi(s, t)$ represents the option price wave function, and it is the probability density function $|\psi(s, t)|^2$ that represents the potential field. A novel analytical technique for the solution of time-fractional Ivancevic option pricing model has been studied by Jena et. al. [53].

The organization of this paper is as follows. In Sect. 2, dark wave solutions of Eq. (3) will be obtained through the tanh expansion method. In Sect. 3, rogue wave solutions of Ivancevic option pricing model will be obtained via trial function method. The dynamical characteristics of corresponding rogue waves will be exhibited through curve plots, 3D plots, density plots and contour plots. In Sect. 4, perturbation solutions are obtained through direct perturbation method. Section 5 will conclude this paper.

2 Dark wave of Ivancevic option pricing model

To get the financial dark wave solutions of Eq. (3), a transformation is given as follows:

$$\psi = e^{i(c_2 t + k_2 s)}\Psi(\xi), \tag{4}$$

where $\xi = c_1 t + k_1 s$. Substituting transformation (4) into Eq. (3), we get a complex equation,

$$e^{i(tc_2 + sk_2)}\left(\frac{\left(\frac{d^2}{d\xi^2}\Psi(\xi)\right)\sigma k_1^2}{2} + (i\sigma k_1 k_2 + ic_1)\left(\frac{d}{d\xi}\Psi(\xi)\right) + \Psi(\xi)\left(\beta|\Psi(\xi)|^2 - \frac{\sigma k_2^2}{2} - c_2\right)\right) = 0. \tag{5}$$

The real and imaginary parts of Eq. (5) are extracted as follows:

$$(\sigma k_1 k_2 + c_1)\left(\frac{d}{d\xi}\Psi(\xi)\right), \tag{6}$$

$$\frac{\left(\frac{d^2}{d\xi^2}\Psi(\xi)\right)\sigma k_1^2}{2} + \beta\Psi(\xi)|\Psi(\xi)|^2 - \frac{\Psi(\xi)\sigma k_2^2}{2} - \Psi(\xi)c_2. \tag{7}$$

Making the following transformation to Eq. (7),

$$\Psi(\xi) = a_0 + a_1 \tanh(\xi), \tag{8}$$

The real part of Eq. (5) is transformed as

$$\tanh^3(\xi)\sigma a_1 k_1^2 - a_1\tanh(\xi)\sigma k_1^2 + \tanh(\xi)\,|a_0 + a_1\tanh(\xi)|^2\,\beta a_1 + |a_0 + a_1\tanh(\xi)|^2\,\beta a_0 - \frac{\tanh(\xi)\sigma a_1 k_2^2}{2} - \frac{\sigma a_0 k_2^2}{2} - \tanh(\xi)a_1 c_2 - a_0 c_2. \tag{9}$$

The term of $\frac{d}{d\xi}\Psi(\xi)$ in Eq. (8) and the terms in Eq. (9) having same order of $\tanh(\xi)$ are collected. Then, equating these equations to 0, a system of equations for concerned parameters is obtained as follows:

$$\begin{aligned} &\sigma k_1 k_2 + c_1 = 0,\\ &\left(\beta a_0^2 - \frac{\sigma k_2^2}{2} - c_2\right)a_0 = 0,\\ &\beta a_1^3 + \sigma a_1 k_1^2 = 0,\\ &3\beta a_0 a_1^2 = 0,\\ &3a_1\left(\beta a_0^2 + \left(-\frac{k_1^2}{3} - \frac{k_2^2}{6}\right)\sigma - \frac{c_2}{3}\right) = 0. \end{aligned} \tag{10}$$

The following three sets of solutions of Eq. (10) are obtained

$$\text{Case1: } \Big\{ a_0 = a_0, a_1 = 0, c_1 = -\sigma k_1 k_2, c_2 = \beta a_0^2 - \frac{\sigma k_2^2}{2}, k_1 = k_1, k_2 = k_2 \Big\},$$

$$\text{Case2: } \Big\{ a_0 = 0, a_1 = \sqrt{-\frac{\sigma}{\beta}} k_1, c_1 = -\sigma k_1 k_2, c_2 = -\sigma k_1^2 - \frac{1}{2}\sigma k_2^2, k_1 = k_1, k_2 = k_2 \Big\},$$

$$\text{Case3: } \Big\{ a_0 = 0, a_1 = -\sqrt{-\frac{\sigma}{\beta}} k_1, c_1 = -\sigma k_1 k_2, c_2 = -\sigma k_1^2 - \frac{1}{2}\sigma k_2^2, k_1 = k_1, k_2 = k_2 \Big\}. \tag{11}$$

Substituting case 1 of Eq. (11) into (8), the explicit solution ψ_1 of Eq. (3) via transformation (4) is obtained,

$$\psi_1 = \mathrm{a}_0 e^{\frac{i\left(2t\beta a_0^2 - t\sigma k_2^2 + 2sk_2\right)}{2}}. \tag{12}$$

Substituting case 2 of Eq. (11) into (8), the explicit solution ψ_2 of Eq. (3) via transformation (4) is obtained,

$$\psi_2 = e^{-\frac{i}{2}\left(2t\sigma k_1^2 + t\sigma k_2^2 - 2k_2 s\right)} \sqrt{-\frac{\sigma}{\beta}} k_1 \tanh\left(-\sigma t k_1 k_2 + k_1 s\right). \tag{13}$$

Substituting case3 of Eq. (11) into (8), the explicit solution ψ_3 of Eq. (3) via transformation (4) is obtained,

$$\psi_3 = -e^{-\frac{i}{2}\left(2t\sigma k_1^2 + t\sigma k_2^2 - 2k_2 s\right)} \sqrt{-\frac{\sigma}{\beta}} k_1 \tanh\left(c_1 t + k_1 s\right). \tag{14}$$

Some appropriate values in Eq. (14) are given as: $\beta = 5, \sigma = 3, k_2 = 2, k_1 = 4$, to analyze the dynamics properties briefly. The wave function ψ_3 with only two independent variables of time t and asset price s is obtained as follows:

$$\psi_3 = \frac{4\sqrt{15}}{5} \mathrm{i} \tanh(4s - 24t) \mathrm{e}^{\mathrm{i}(2(s-27t))}. \tag{15}$$

Figure 1 shows the three-dimensional plots, density plot, curve plots and contour plot for the strength $|\psi_3|$ of dark wave solutions for Eq. (15).

3 Rogue wave of Ivancevic option pricing model

To obtain the analytical solutions of Eq. (3), a transformation is given as follows:

$$\psi = \mathrm{e}^{\mathrm{i}(\mathrm{ps}+\mathrm{qt})} \left(\psi_0 + v_y\right), \tag{16}$$

where

$$v = a \ln\left(b + y^2\right), \quad y = \mathrm{e}^{\mathrm{ks}-\omega \mathrm{t}},$$

from Eq. (16), the terms in Eq. (3) are obtained as follows:

$$\begin{aligned}
\psi &= \frac{\exp\left(\mathrm{i}(\mathrm{ps}+\mathrm{qt})\right)\left(\psi_0 y^2 + 2ay + b\psi_0\right)}{y^2+b},\\
|\psi| &= \frac{\left(y^2+b\right)\psi_0 + 2ay}{y^2+b},\\
\frac{\partial}{\partial t}\psi &= \left(\frac{iq\left(\psi_0 y^2 + 2ay + b\psi_0\right)}{y^2+b} + \left(\frac{-2\psi_0 \omega y^2 - 2a\omega y}{y^2+b} + \frac{2\omega y^2\left(\psi_0 y^2 + 2ay + b\psi_0\right)}{\left(y^2+b\right)^2}\right)\right)\exp\left(\mathrm{i}(\mathrm{ps}+\mathrm{qt})\right),\\
\frac{\partial}{\partial s}\psi &= \left(\frac{\mathrm{ip}\left(\psi_0 y^2 + 2ay + b\psi_0\right)}{y^2+b} + \frac{2kya\left(-y^2+b\right)}{\left(y^2+b\right)^2}\right)\exp\left(i(ps+qt)\right),\\
\frac{\partial^2}{\partial s^2}\psi &= \left(-\frac{\left(\psi_0 y^2 + 2ay + b\psi_0\right)p^2}{y^2+b} + \frac{4Ikya\left(-y^2+b\right)p}{\left(y^2+b\right)^2} + \frac{2ak^2 y\left(y^4 - 6by^2 + b^2\right)}{\left(y^2+b\right)^3}\right)\exp\left(\mathrm{i}(\mathrm{ps}+\mathrm{qt})\right).
\end{aligned} \tag{17}$$

Substituting Eq. (17) into Eq. (3), we get the following algebraic equation,

$$\frac{-q\left(\psi_0 y^2 + 2ay + b\psi_0\right)}{y^2+b} + \left(\frac{-2\omega\psi_0 y^2 - 2a\omega y}{y^2+b} + \frac{2\omega y^2\left(\psi_0 y^2 + 2ay + b\psi_0\right)}{\left(y^2+b\right)^2}\right)\mathrm{i} + \frac{\sigma\left(-\frac{\left(\psi_0 y^2+2ay+b\psi_0\right)p^2}{y^2+b} + \frac{4\,\mathrm{i}kya\left(-y^2+b\right)p}{\left(y^2+b\right)^2} + \frac{2ak^2y\left(y^4-6by^2+b^2\right)}{\left(y^2+b\right)^3}\right)}{2} + \frac{\beta\left(\left(y^2+b\right)\psi_0 + 2ay\right)^2\left(\psi_0 y^2 + 2ay + b\psi_0\right)}{\left(y^2+b\right)^3} = 0. \tag{18}$$

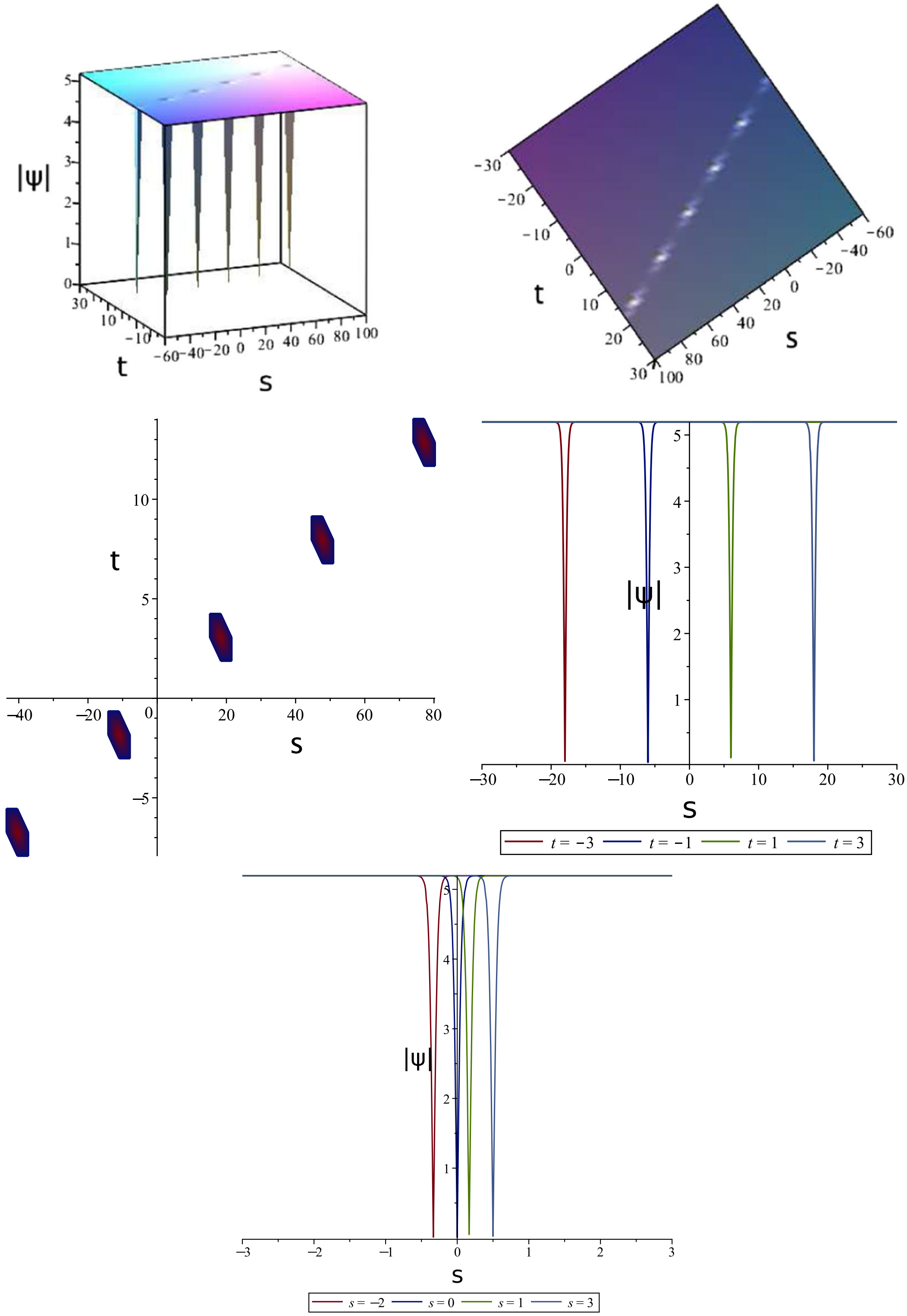

Fig. 1 (Color online) The three-dimensional plots, density plot, curve plots and contour plot for the strength $|\psi_3|$ of dark wave solutions

We collect the terms in Eq. (18) having same order of y and make them zero; the system of equations are obtained as follows:

$$\begin{aligned}
&2\beta\,{\psi_0}^3 - p^2\sigma\psi_0 - 2q\psi_0 = 0,\\
&2b^3\beta\,{\psi_0}^3 - b^3p^2\sigma\psi_0 - 2b^3q\psi_0 = 0,\\
&6b\beta\,{\psi_0}^3 - 3b\,p^2\sigma\psi_0 + 24a^2\beta\psi_0 - 6bq\psi_0 = 0,\\
&6b^2\beta\,{\psi_0}^3 - 3b^2p^2\sigma\psi_0 + 24a^2b\beta\psi_0 - 6b^2q\psi_0 = 0,\\
&24ab\beta\,{\psi_0}^2 - 12ab\,k^2\sigma - 4ab\,p^2\sigma\\
&\quad + 16a^3\beta - 8abq = 0,\\
&\quad - 4\,\mathrm{i}akp\sigma + 12a\beta\,{\psi_0}^2 + 2a\,k^2\sigma - 2a\,p^2\sigma\\
&\quad + 4\,\mathrm{i}a\omega - 4aq = 0,\\
&4\,\mathrm{i}a\,b^2kp\sigma + 12a\,b^2\beta\,{\psi_0}^2 + 2a\,b^2k^2\sigma - 2a\,b^2p^2\sigma\\
&\quad - 4\,\mathrm{i}a\,b^2\omega - 4a\,b^2q = 0.
\end{aligned} \tag{19}$$

Solving Eq. (19), we get the constraint relationship between the coefficients as follows:

$$\begin{aligned}
&\text{case1: } \left\{a = \sqrt{\tfrac{b\sigma}{\beta}}k,\ p = \tfrac{\omega}{k\sigma},\ q = \tfrac{k^4\sigma^2-\omega^2}{2k^2\sigma},\ \psi_0 = 0\right\},\\
&\text{case2: } \left\{a = -\sqrt{\tfrac{b\sigma}{\beta}}k,\ p = \tfrac{\omega}{k\sigma},\ q = \tfrac{k^4\sigma^2-\omega^2}{2k^2\sigma},\ \psi_0 = 0\right\}.
\end{aligned} \tag{20}$$

Substituting the case 1 in Eq. (20) into Eq. (17), the explicit solution ψ of Eq. (3) via transformation (16) is obtained,

$$\psi = \frac{2e^{i\left(\frac{\omega s}{k\sigma}+\frac{(k^4\sigma^2-\omega^2)t}{2k^2\sigma}\right)}\sqrt{\frac{b\sigma}{\beta}}k\mathrm{e}^{\mathrm{ks}-\omega\mathrm{t}}}{\left(e^{ks-\omega t}\right)^2 + b}. \tag{21}$$

In order to analyze the dynamics of the solution, some parameters in Eq. (21) are given as follows:

$$\omega = 2, k = 3, b = 3, \beta = 2, \sigma = 4. \tag{22}$$

Figure 2 shows the three-dimensional plots, density plot, curve plots and contour plot of the strength $|\psi|$ of rogue wave solutions for Eq. (21).

4 Perturbation solutions of Ivancevic option pricing model

As we all know, there is white noise in option model. In order to restore the real situation of the option model, we add a perturbation term to the Ivancevic option pricing model Eq. (3) and the Ivancevic option pricing model with loss is obtained as follows:

$$\mathrm{i}\partial_t\psi + \frac{1}{2}\sigma\partial_{ss}\psi + \beta|\psi|^2\psi = -i\epsilon\psi, \quad (\mathrm{i} = \sqrt{-1}). \tag{23}$$

To obtain the perturbation solutions, ψ is expanded as follows:

$$\begin{aligned}
\psi &= \mathrm{e}^{\epsilon(a+\mathrm{i}b)}\psi'(\xi,\tau,\epsilon)\\
&= \mathrm{e}^{\epsilon(a+\mathrm{i}b)}\left[\psi_0(\xi,\tau) + \epsilon\psi_1(\xi,\tau) + O\left(\epsilon^2\right)\right],
\end{aligned} \tag{24}$$

where $a = a(t,s), b = b(t,s), \xi = \xi(t,s,\Gamma), \tau = \tau(t,s,\Gamma)$ and $\{\xi,\tau\}$ satisfy the following relationship,

$$\{\xi,\tau\} \stackrel{\epsilon\to 0}{\longrightarrow} \{t,s\}. \tag{25}$$

Substituting Eq. (24) into Eq. (23),

$$\begin{aligned}
&\mathrm{i}\left(\frac{\sigma}{2}\psi_{0\tau\tau}\tau_s^2 + \mathrm{e}^{2\epsilon a}\beta\,|\psi_0|^2\,\psi_0\right) - \psi_{0\xi}\xi_t\\
&\quad+\epsilon\left\{\left[\mathrm{i}\left(\frac{\sigma}{2}\psi_{1\tau\tau}\tau_s^2 + e^{2\epsilon a}\beta\left(2\,|\psi_0|^2\,\psi_1 + \psi_0^2\psi_1^*\right)\right.\right.\right.\\
&\quad-\psi_{1\xi}\xi_t\Big]\\
&\quad+\left[\frac{\sigma}{2}\left(\mathrm{i}a_{tt} - b_{tt}\right) - a_t - \mathrm{i}b_t - 1\right]\psi_0\\
&\quad\left[\frac{1}{\epsilon}\left(\mathrm{i}\frac{\sigma}{2}\tau_{ss} - \tau_t\right) + \sigma\tau_s\left(\mathrm{i}a_s - b_s\right)\right]\psi_{0\tau}\\
&\quad\left.+\mathrm{i}\sigma\frac{\xi_s}{\epsilon}\tau_s\psi_{0\xi\tau}\right\} = O\left(\epsilon^2\right).
\end{aligned} \tag{26}$$

Let the coefficient of the same power of ϵ be zero, and the following approximate equations are obtained,

$$-\psi_{0\xi\xi}\xi + \mathrm{i}\left(\frac{\sigma}{2}\psi_{0\tau\tau}\tau_s^2 + \mathrm{e}^{2\epsilon a}\beta\,|\psi_0|^2\,\psi_0\right) = 0, \tag{27}$$

$$\begin{aligned}
&-\psi_{1\xi}\xi_t + \mathrm{i}\left[\frac{\sigma}{2}\psi_{1\tau\tau}\tau_s^2 + \mathrm{e}^{2\epsilon a}\beta\left(2\,|\psi_0|^2\,\psi_1 + \psi_0^2\psi_1^*\right)\right]\\
&\quad+\left[\frac{\sigma}{2}\left(\mathrm{i}a_{ss} - b_{ss}\right) - a_t - \mathrm{i}b_t - 1\right]\psi_0\\
&\quad+\left[\frac{1}{\epsilon}\left(\mathrm{i}\frac{\sigma}{2}\tau_{ss} - \tau_t\right) + \sigma\tau_s\left(\mathrm{i}a_s - b_s\right)\right]\psi_{0\tau}\\
&\quad+i\sigma\frac{\xi_s}{\epsilon}\tau_s\psi_{0\xi\tau} = 0.
\end{aligned} \tag{28}$$

Because ψ_0 in Eq. (27) is not explicitly related to ϵ, the following relationship can be obtained,

$$\xi_t = \mathrm{e}^{2\epsilon a}, \tau_s = \mathrm{e}^{\epsilon a}, \tag{29}$$

so ψ_0 is the exact solution of Eq. (3). From Eq. (24), ψ_1 is the solution of following equation,

$$\begin{aligned}
&-\psi_{1\xi} + \mathrm{i}\left[\frac{sigma}{2}\psi_{1\tau\tau} + \beta_2\left(2\,|\psi_0|^2\,\psi_1 + \psi_0^2\psi_1^*\right)\right]\\
&\quad= 0.
\end{aligned} \tag{30}$$

From Ref [54], the solution of Eq. (30) can be $\psi_1 = \psi_{0\xi}$ or $\psi_1 = \psi_{0\tau}$. For a given nontrivial solution ψ_0, Eqs. (28-30) are consistent in any t and s, so the

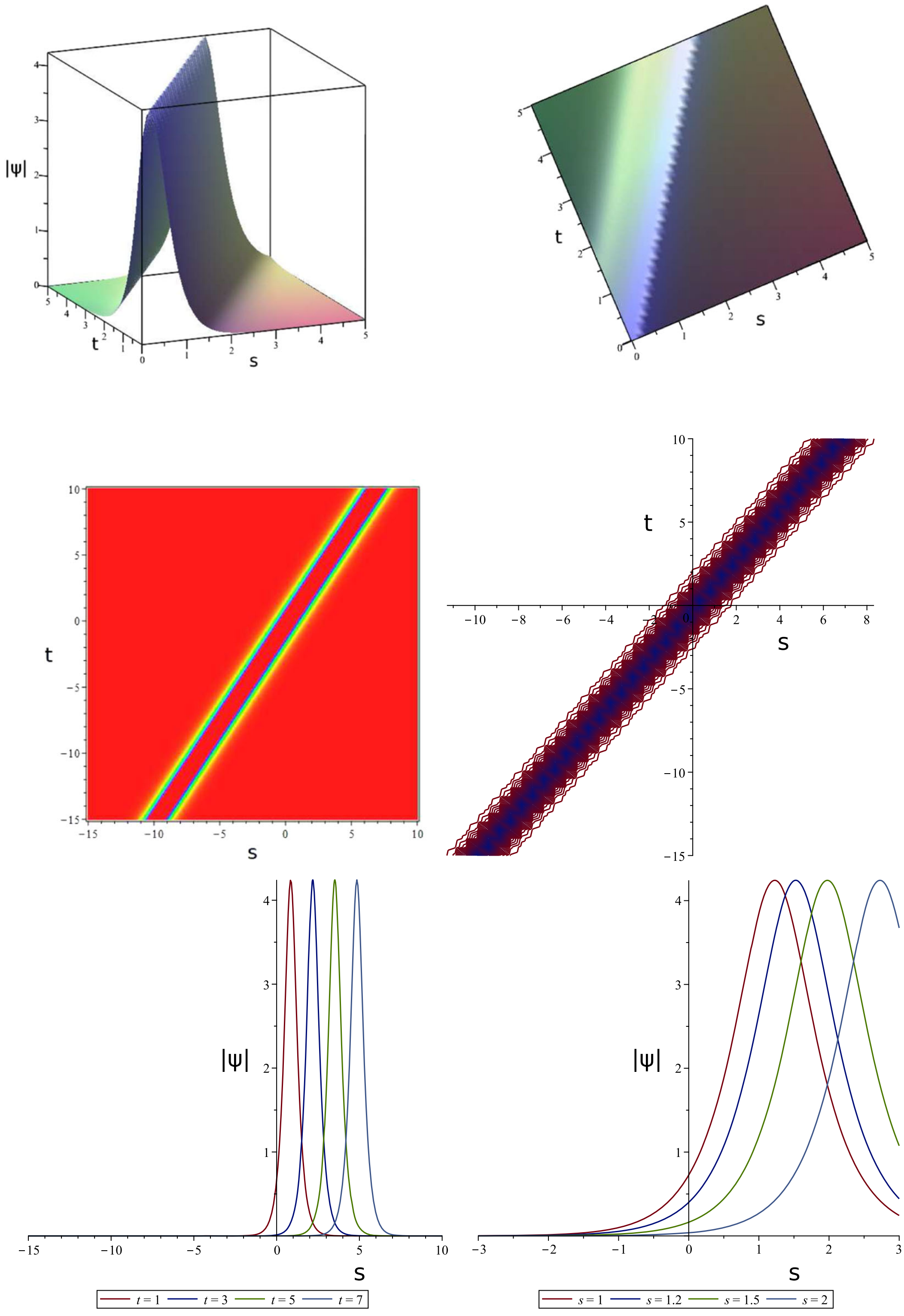

Fig. 2 (Color online) The three-dimensional plots, density plot, curve plots and contour plot of the strength $|\psi|$ of rogue wave solutions for Eq. (21) by choosing $\omega = 2, k = 3, b = 3, \beta = 2, \sigma = 4$

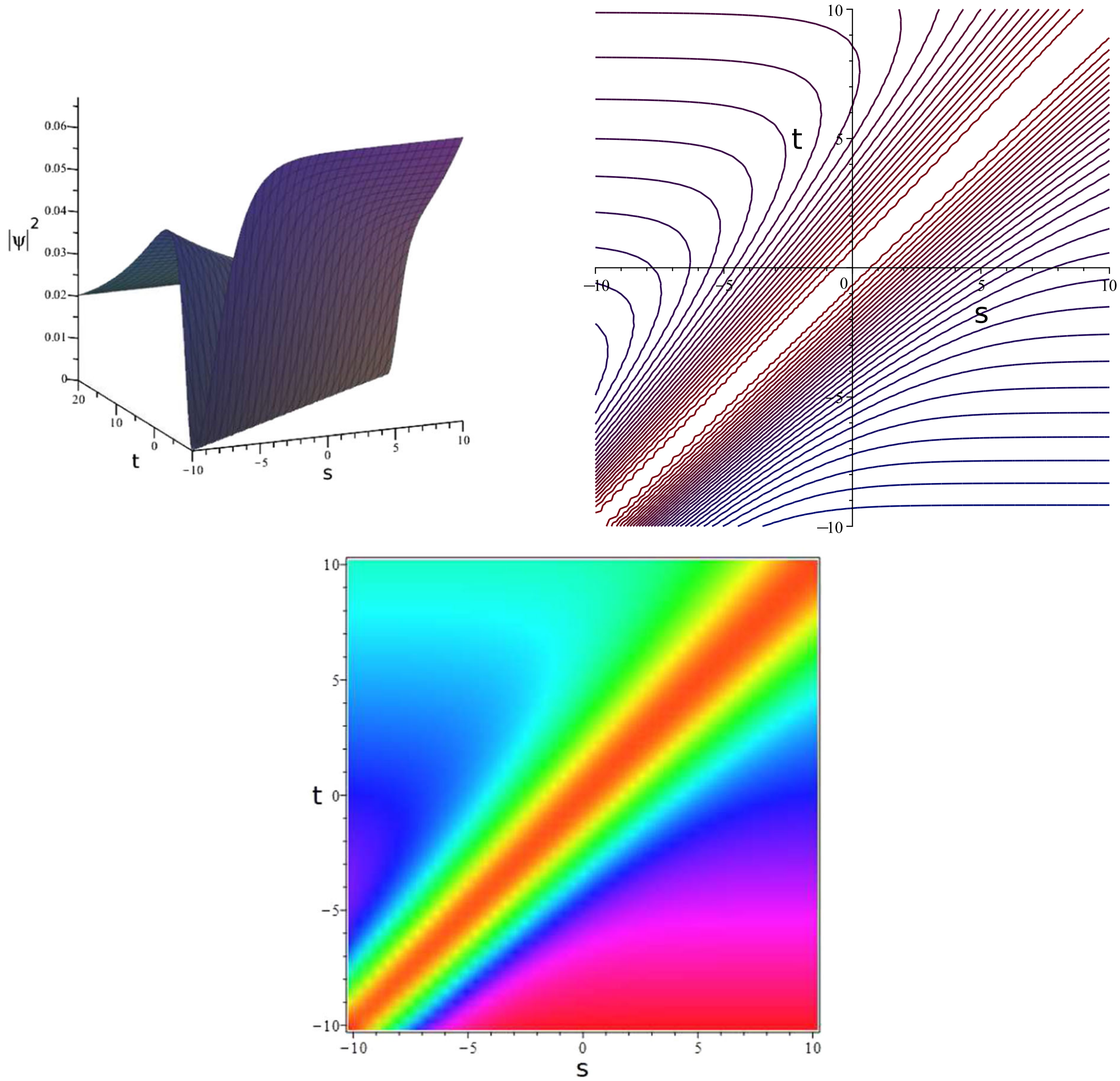

Fig. 3 (Color online) The three-dimensional plot, density plot and contour plot of the intensity $|\psi|^2$ of perturbation solutions for Eq. (33) by choosing $\beta = -4$, $\sigma = 2$, $\epsilon = 0.01$, $k_1 = 0.3$, $k_2 = 0.5$

last three terms of Eq. (28) are all equal to zero and we can get,

$$\begin{aligned} &\xi_s = 0, \\ &\frac{sigma}{2}\left(\mathrm{i}a_{ss} - b_{ss}\right) - a_t - \mathrm{i}b_t - 1 = 0, \\ &\frac{1}{\epsilon}\left(\mathrm{i}\frac{sigma}{2}\tau_{ss} - \tau_t\right) + \sigma_1\tau_s\left(\mathrm{i}a_s - b_s\right) = 0. \end{aligned} \tag{31}$$

The solutions of Eq. (29) and Eq. (31) are obtained,

$$a = -2t, b = \frac{s^2}{\sigma_1}, \tau = \mathrm{e}^{-2\epsilon t}s, \xi = \frac{1}{4\epsilon}\left(1 - \mathrm{e}^{-4\epsilon t}\right). \tag{32}$$

Substituting the exact solution Eq. (32) into Eq. (14), through transformation (24) and $\psi_1 = \psi_{0\xi}$, the perturbation solutions of Ivancevic option pricing model Eq. (14) can be obtained as follows:

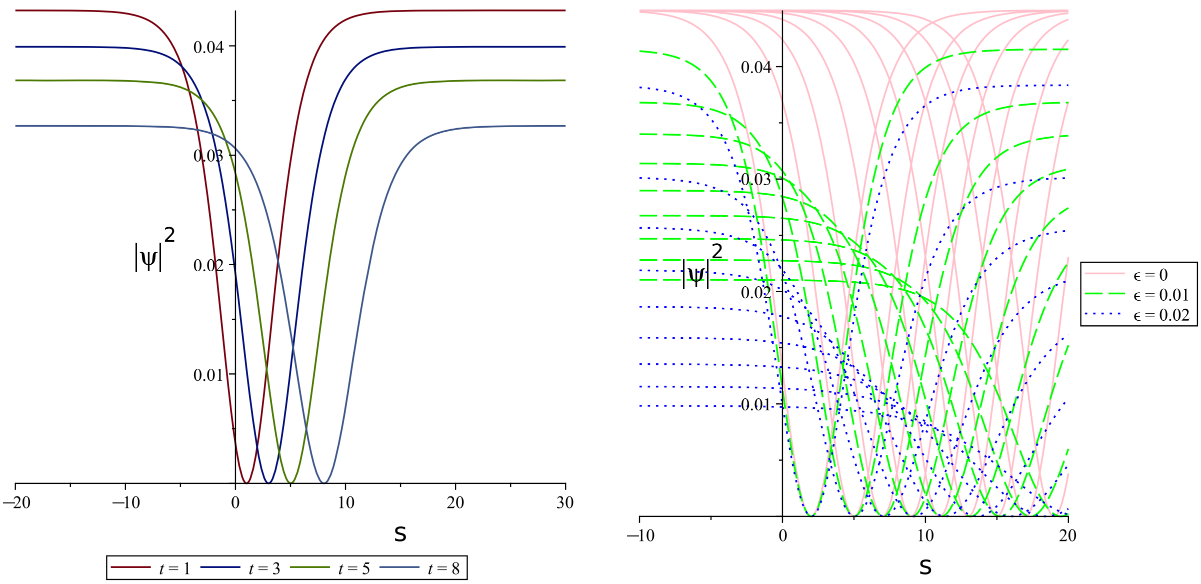

Fig. 4 (Color online) The curve plots of the intensity $|\psi|^2$ of perturbation solutions for Eq. (33) by choosing $\beta = -4$, $\sigma = 2$, $\epsilon = 0.01$(left), $k_1 = 0.3$, $k_2 = 0.5$

$$\begin{aligned}\psi &= -\frac{1}{2}\mathrm{e}^{\frac{\epsilon\left(\mathrm{I}s^2-2t\sigma\right)}{\sigma}}\mathrm{e}^{\frac{\frac{\mathrm{I}}{8}\left(8\,\mathrm{e}^{-2\epsilon t}sk_2\epsilon+2\sigma k_1^2\mathrm{e}^{-4\epsilon t}+\sigma k_2^2\mathrm{e}^{-4\epsilon t}-2\sigma k_1^2-\sigma k_2^2\right)}{\epsilon}}\\ &\quad\sqrt{-\frac{\sigma}{\beta}}\,k_1\\ &\quad\left(2\,\mathrm{I}\epsilon k_1^2\tanh\left(\frac{k_1\left(4\,\mathrm{e}^{-2\epsilon t}s\epsilon+\sigma k_2\mathrm{e}^{-4\epsilon t}-\sigma k_2\right)}{4\epsilon}\right)\sigma\right.\\ &\quad+\mathrm{I}\epsilon\tanh\left(\frac{k_1\left(4\,\mathrm{e}^{-2\epsilon t}s\epsilon+\sigma k_2\mathrm{e}^{-4\epsilon t}-\sigma k_2\right)}{4\epsilon}\right)\sigma k_2^2\\ &\quad-2\epsilon\left(\tanh^2\left(\frac{k_1\left(4\,\mathrm{e}^{-2\epsilon t}s\epsilon+\sigma k_2\mathrm{e}^{-4\epsilon t}-\sigma k_2\right)}{4\epsilon}\right)\right)\sigma k_1k_2\\ &\quad+2\epsilon k_1\sigma k_2-2\tanh\\ &\quad\left.\left(\frac{k_1\left(4\,\mathrm{e}^{-2\epsilon t}s\epsilon+\sigma k_2\mathrm{e}^{-4\epsilon t}-\sigma k_2\right)}{4\epsilon}\right)\right).\end{aligned}\tag{33}$$

By choosing $\beta = -4$, $\sigma = 2$, $\epsilon = 0.01$, $k_1 = 0.3$, $k_2 = 0.5$ in Eq. (32), the three-dimensional plot, density plot and contour plot of the intensity $|\psi|^2$ of perturbation solutions for Eq. (33) are shown well in Fig. 3. Figure 4 shows the curve plots of Eq. (33), from which we can find that perturbation solutions decays rapidly with the increase in ϵ.

5 Conclusions

In this work, we have constructed the rogue wave solutions and the dark wave solutions of Ivancevic option pricing model by choosing some different trial functions. With the help of symbolic computing technology, the rogue wave solutions of Ivancevic option pricing model are obtained via trial function method and the dark wave solutions of Ivancevic option pricing model are obtained via tanh method. Perturbation solutions are obtained through direct perturbation method. Various curve plots, density plot, three-dimensional plots and contour plots, and dynamical characteristics of these waves are shown well using Maple.

Data availability statements Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study

Declaration

Conflict of interest The authors declare that they have no conflict of interest concerning the publication of this manuscript.

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