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Event-triggered control of a class of cascade switched nonlinear systems

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Abstract In this paper, the event-triggered control of a class of cascaded switched nonlinear systems is studied. By applying a designed event-triggered sampling strategy, the minimum inter-event interval is obtained and the Zeno behavior is successfully avoided during the sampling process. Besides, the globally uniform boundedness of the closed-loop system is achieved by virtue of the average dwell-time method and the stability of the switched nonlinear system is guaranteed. A numerical example is provided in the end to support the main results.

Keywords Switched nonlinear system · Globally uniform boundedness · Event-triggered control

1 Introduction

Switched systems, which are a class of special hybrid systems, include a group of continuous or discrete subsystems and a switching rule governing the activated subsystem [1]. Switched control has much more superiorities than the traditional single control in modeling complex systems. Thus, switched systems have an extensive range of practical applications in many fields of control, such as mechanical systems, chemical procedure systems, and power systems [2,3]. Moreover, switched control provides more effective techniques than the traditional single control, which even can improve transient performance and control precision of systems better [4]. Generally speaking, the switched control has become one of the hottest topics in the control field.

With the increasing integration and complexity of the industry, switched systems play a more critical role than ever before; more and more results of switched systems have been obtained in recent years. Stability is a fundamental and essential issue when analyzing the performance of systems [5]. Compared with the non-switched systems, the switched systems have hybrid dynamic features, so it is more of complexity and challenge to analyze the stability of the switched systems. For a class of switched nonlinear systems whose subsystems are not supposed to be stable when the delay is present, Wang et al. provided sufficient conditions to ensure that the system is globally exponentially stable [6]. Wu et al. investigated the stability of switched systems with stochastic switching signals by using the probability analysis method [7]. By utilizing the nonlinear related techniques, the stability criteria for switched nonlinear sampled data systems are derived in [8]. Zhang et al. studied the stability of cascaded switched systems by using an input-to-state

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Lyapunov function method [9]. In addition to stability, there are many issues worthy of studying for switched nonlinear systems. Su et al. studied H_∞ control problem for cascaded switched nonlinear systems in [10]. By the virtue of multiple Lyapunov functional and average dwell-time approach, Park et al. constructed a fault estimator and the H_∞ fault estimation problem for a class of discrete-time switched nonlinear systems was solved [11]. Moreover, the dissipative control problem for wireless networked control systems was concerned in [12]. Zheng et al. proposed a performance index for a class of switched nonlinear systems in [13], which can be viewed as the mixed H_∞ performance and passive performance.

Many results show that the system may be unstable if the system switches so fast, even if every subsystem is stable. The simplest way to control the switching frequency is the activating time of each subsystem should not be too short; therefore, the concept of dwell time is proposed [5]. However, it is too restrictive to specify a dwell time. Thus, it is of interest to relax the condition of the dwell time concept, allowing the possibility of switching fast when necessary and then compensating for it by switching sufficiently slowly later. The concept of average dwell time serves this purpose. As a proven effective design method, the average dwell time technology has been adopted in [11–14]; what's more, this technology was also used to study asynchronous control of switched systems [9].

The feedback controllers of switched systems are mostly continuous in the existing results. However, controllers are implemented on a digital platform in the actual operation. Therefore, an effective research method, event-triggered control, plays an important role in this situation. Event-triggered control means that the system performs control tasks after an external event occurs [15]. In detail, its signal samplings and controller operations are triggered by a specific event, rather than being executed regularly over time [16]. There is a lot of interaction between sensors, controllers, and actuators, which leads to information load problems and a waste of computational resources [17]. These problems can be effectively solved by applying the event-triggered control method. Owing to the excellent capability, the event-triggered strategy is used in the research of many control problems. Xiao et al. studied the event-triggered control of discrete-time and continuous-time switched linear systems in [18] and [19], respectively. Su et al. adopted the event-triggered mechanism to the study of sliding mode control of hybrid switched systems [20]. Li et al. applied the event-triggered method to study the H_∞ control problem for switched systems [21]. Qi and Cao introduced an event-triggered mechanism with a fixed threshold and the finite-time boundedness in [22], and the stabilization of switched linear systems is concerned.

Generally, if the event-triggered scheme is introduced into the system, then in the process of control, the actuator performs the operations in time-varying periods, besides, the intervals between events are also variable. That is, infinite events may appear in a limited time. Thence, Zeno behavior was concerned in [23]. It is a challenging task to exclude Zeno behavior. In the existing research results, Ni et al. proposed the positive lower bound for inter-event intervals for a class of linear systems [24]; Fei and Zhao extended this method to switched linear systems [25]. Li and Lian designed a Zeno-free event-triggered scheme for switched linear systems [26]. However, due to the complexity of switched nonlinear systems, there still are few results referring to this class of systems by virtue of the event-triggered scheme, not to mention the results of excluding the Zeno behavior. In [27], Li et al. introduced the event-triggered scheme to the output tracking control problem of switched nonlinear strict-feedback systems, while the Zeno behavior was not considered. Fortunately, this problem was concerned by Huo et al. where they studied the adaptive fuzzy output feedback control of MIMO switched nonlinear systems [28].

Most of the current-based event-triggered control theories refer to linear systems and are not fully applicable to nonlinear systems. In addition, switched systems are different from non-switched systems; even though each subsystem is stable, the switched system may be unstable if the switching signal is not selected properly. The study of event-triggered control for switched nonlinear systems is not the simple superposition of the properties of each subsystems. At the same time, cascade systems are an important class of systems [29]. This class of systems has special structure and facilitates the wide range of applications. Therefore, it is necessary and meaningful to study the event-triggered control of cascade switched nonlinear systems.

Motivated by the above discussion, we mainly studied the event-triggered control problem for switched nonlinear systems by an average dwell-time method. Our main contributions are in the following aspects:

(1) For the cascaded switched nonlinear system, an event-triggered control scheme is first presented, under which the communication resources are effectively saved.
(2) For the resulting closed-loop system, the sufficient condition for globally uniform boundedness is obtained by the virtue of an average dwell time method, and the exponential stability is guaranteed by the conditions we provided.
(3) The case that infinite events may happen in a limited time interval is considered. The minimum interval of inter-event lower bound is calculated by applying the event-triggered sampling mechanism and the Zeno behavior is successfully excluded during the sampling procedure.

Notations: Unless otherwise stated, the notations used in this paper are standard. R^n denotes the Euclidean space with n dimensions. N represents the natural number. The superscript T denotes matrix transposition. $P > 0$ means that the matrix P is real symmetric and positive definite. I represents the identity matrix with appropriate dimension. $\|\cdot\|$ denotes the Euclidean vector norm.

2 Preliminaries

Consider the following cascade switched nonlinear system

$$\begin{cases} \dot{x}_1(t) = A_{1\sigma(t)}x_1(t) + A_{2\sigma(t)}x_2(t) + B_{\sigma(t)}u_{\sigma(t)}(t) \\ \dot{x}_2(t) = f_{2\sigma(t)}(x_2(t)) \\ y(t) = C_{\sigma(t)}x_1(t) \end{cases} \tag{1}$$

where $x = [x_1, x_2]^T$ is the state vector and $x_1(t) \in R^{n-d}, x_2(t) \in R^d$; $y(t) \in R^q$ means the output of the system; $\sigma(t) : [0, \infty) \to \bar{M} = \{1, 2, \ldots, M\}$ is a piecewise constant value function with respect to time, called switching signal, where $\bar{M}$ is a finite index set, the ith subsystem is active when $\sigma(t) = i$. $u_i(t) \in R^m$ represents the control input of subsystem i. A_{1i}, A_{2i}, B_i, C_i are constant matrices with appropriate dimensions, $i \in \bar{M}$. $f_{2i}(\cdot)$ are known smooth vector fields with appropriate dimensions and $f_{2i}(0) = 0$

According to the switching signal $\sigma(t)$, we can get a group switching sequence presenting as follows:

$$\{x_{t_0}; (l_0, t_0), (l_1, t_1), \ldots, (l_i, t_i), \ldots | l_i \in \bar{M}, i \in N\} \tag{2}$$

where l_i means the serial number of the subsystem and t_i is the switching instant. That is, the l_i subsystem is active when $t \in [t_i, t_{i+1})$. Supposing that there is no system state jump at the moment of switching.

Denoting instant with an event happens by $\{\hat{t}_k\}_{k=0}^{\infty}$, $\hat{t}_k < \hat{t}_{k+1}$, and the error $e(t) = x_1(t) - x_1(\hat{t}_k)$. The condition of event-triggered can be described as

$$\|e(t)\|^2 \geq \eta\|x_1(t)\|^2 + \varepsilon \tag{3}$$

where η and ε are positive parameters. Sampling is performed immediately once the event is triggered. The value $e(t)$ will be reduced to 0 if an event happens; besides, it begins to increase until the next event happens. Assume that when $\hat{t}_0 = t_0$, the first event occurs. With the state $x_1(\hat{t}_k)$ sampled at the time $\hat{t}_k$, we can describe the next sampling instant $\hat{t}_{k+1}$ by

$$\hat{t}_{k+1} = \inf\left\{t > \hat{t}_k \mid \|e(t)\|^2 = \eta\|x_1(t)\|^2 + \varepsilon\right\} \tag{4}$$

A controller is obtained under the above condition (3). Assume that there are n samplings occurred during $t \in [t_i, t_{i+1})$, and denote the first sampling instant by $\hat{t}_{k+1}$. Under the switching sequence (2), each interval corresponds to an active subsystem; then, the controller can be designed as follows:

$$u_{\sigma(t)} = u_i = \begin{cases} K_i x_1(\hat{t}_k), t \in [t_i, \hat{t}_{k+1}) \\ K_i x_1(\hat{t}_{k+1}), t \in [\hat{t}_{k+1}, \hat{t}_{k+2}) \\ \quad \vdots \\ K_i x_1(\hat{t}_{k+n}), t \in [\hat{t}_{k+n}, t_{i+1}) \end{cases} \tag{5}$$

where K_i is the controller gain.

Definition 1 [13] For any σ and $\forall t \geq s \geq 0$, $N_\sigma(s, t)$ means the number of switchings on the interval (s, t). If $N_\sigma \leq 1$ and $t - s \leq \tau_d$, then τ_d is called a dwell time; for $\tau_a > \tau_d$ and $N_0 \geq 1$, if there exists $N_\sigma(s, t) \leq N_0 + ((t - s)/\tau_a)$, then τ_a is called the average dwell time.

Definition 2 [14] For any constant $\gamma > 0$, if there is a switching signal σ and a constant $\widetilde{\beta}$, satisfying

$$\|x(t_0)\| < \gamma \Rightarrow \|x(t)\| < \widetilde{\beta}, \forall t > t_0 \tag{6}$$

where $\widetilde{\beta} = \widetilde{\beta}(\gamma) < \infty$ is positive and independent from t_0, then the considered switched nonlinear system is globally uniformly bounded.

Lemma 1 *[14] For any u, v and positive definite matrix G,*

$$u^T v + v^T u \leq u^T G u + v^T G^{-1} v \tag{7}$$

holds, where u, v are real vectors and G has the appropriate dimension.

3 Main results

We will study the stability of the switched nonlinear system (1) in this section.

3.1 Event-triggered control

Substitute the controller (5) into the system (1) with the error $e(t) = x_1(t) - x_1(\hat{t}_{k+j}), (j = 0, 1, \ldots, n)$ for $\forall t \in [t_i, \hat{t}_{i+1}), [\hat{t}_{i+1}, \hat{t}_{i+2}), \ldots, [\hat{t}_{i+n}, t_{i+1})$, and the closed-loop system is presented as

$$\begin{cases} \dot{x}_1(t) = (A_{1i} + B_i K_i) x_1(t) + A_{2i} x_2(t) - B_i K_i e(t) \\ \dot{x}_2(t) = f_{2i}(x_2(t)) \\ y(t) = C_i x_1(t) \end{cases} \tag{8}$$

Theorem 1 *Consider the closed-loop system (8). If the system (8) satisfies the following conditions:*

(1) For the given scalars $\eta > 0, \mu > 1, \varepsilon > 0$ and $N_0 \geq 1$, there exist matrices $P_i > 0, P_j > 0$ and K_i for $\forall i, j \in \bar{M}$ satisfying

$$\begin{bmatrix} A_{1i}^T P_i + P_i B_i K_i + P_i A_{1i} + K_i^T B_i^T P_i + (1+\eta) I & P_i B_i K_i \\ * & -I \end{bmatrix} < 0 \tag{9}$$

and

$$P_i \leq \mu P_j, \; P_j \leq \mu P_i \tag{10}$$

(2) For the given constants $\beta > 0, \alpha_1 > 0, \alpha_2 > 0$, there exist functions $W_i(x_2)$ satisfying

$$\frac{\partial W(x_2)}{\partial x_2} f_{2i}(x_2) \leq -\beta \|x_2\|^2 \tag{11}$$

and

$$\alpha_1 \|x_2\|^2 \leq W_i(x_2) \leq \alpha_2 \|x_2\|^2, \tag{12}$$

then for any switching signal σ satisfying $\tau_a > (\ln \mu)/\gamma$, the system (1) is globally uniform bounded, where $\gamma > 0$ can be calculated from valid matrix solutions (9) and (10); the state exponentially converges to the bounded region

$$B(\varepsilon) = \{x(t) \mid \|x(t)\| \leq \left[\frac{\varepsilon e^{\gamma \tau_a}((\widetilde{\mu} - 1) e^{\gamma \tau_a N_0} + 1) - \varepsilon \widetilde{\mu}}{\varsigma \gamma (e^{\gamma \tau_a} - \widetilde{\mu})}\right]^{\frac{1}{2}}\} \tag{13}$$

where $\widetilde{\mu} = \max\left\{\mu, \frac{\alpha_2}{\alpha_1}\right\}$, ς is a scalar which will be determined later.

Proof Construct a Lyapunov function as follows:

$$V(t) = V_{\sigma(t)}(x_1, x_2) = x_1^T(t) P_{\sigma(t)} x_1(t) + l W_{\sigma(t)}(x_2) \tag{14}$$

when $\sigma(t) = i$, by virtue of Lemma 1, the derivative of V_i is

$$\begin{aligned} \dot{V}_i(t) = {} & x_1^T \left[(A_{1i} + B_i K_i)^T P_i + P_i (A_{1i} + B_i K_i)\right] x_1 \\ & + x_2^T A_{2i}^T P_i x_1 \\ & + x_1^T P_i A_{2i} x_2 - e^T K_i^T B_i^T P_i x_1 - x_1^T P_i B_i K_i e \\ & + \frac{\partial W_i(x_2)}{\partial x_2} f_{2i} \\ \leq {} & x_1^T \left[(A_{1i} + B_i K_i)^T P_i + P_i (A_{1i} + B_i K_i)\right] x_1 \\ & + 2 x_1^T P_i A_{2i} x_2 \\ & + e^T e + x_1^T P_i B_i K_i K_i^T B_i^T P_i x_1 - l\beta \|x_2\|^2 \end{aligned} \tag{15}$$

Notice that there exist two positive constants n_i and m_i satisfying

$$\|A_{2i} x_2\| \leq n_i \|x_2\|, \; \|x_1^T P_i\| \leq m_i \|x_1\|$$

Let $p = \max_{i \in M} \{n_i m_i\}$, we have

$$\begin{aligned} \dot{V}_i(t) \leq {} & x_1^T \left[(A_{1i} + B_i K_i)^T P_i + P_i (A_{1i} + B_i K_i)\right] \\ & x_1 + e^T e + x_1^T P_i B_i K_i K_i^T B_i^T P_i x_1 \\ & + 2p \|x_1\| \|x_2\| - l\beta \|x_2\|^2 \\ \leq {} & x_1^T \left[(A_{1i} + B_i K_i)^T P_i + P_i (A_{1i} \right. \\ & \left. + B_i K_i) + P_i B_i K_i K_i^T B_i^T P_i + I\right] x_1 \end{aligned}$$

$$
\begin{aligned}
&+e^{T}e+2P\|x_1\|\|x_2\|-l\beta\|x_2\|^2-\|x_1\|^2\\
&\le x_1^T\Big[(A_{1i}+B_iK_i)^TP_i+P_i(A_{1i}+B_iK_i)\\
&\quad+P_iB_iK_iK_i^TB_i^TP_i+I\Big]x_1\\
&\quad+e^Te-\left(\|x_1\|^2+2P\|x_1\|\|x_2\|+p^2\|x_2\|^2\right)\\
&\quad+p^2\|x_2\|^2-l\beta\|x_2\|^2\\
&\quad\le x_1^TQ_ix_1+\varepsilon-\frac{\beta l}{2\alpha_2}W_i(x_2)+\frac{\beta l}{2\alpha_2}W_i(x_2)\\
&\quad-l\beta\|x_2\|^2+p^2\|x_2\|^2\\
&\le-\frac{\lambda_{min}(-Q_i)}{\lambda_{max}(P_i)}x_1^TP_ix_1+\varepsilon-\frac{\beta l}{2\alpha_2}W_i(x_2)\\
&\quad-\left(\frac{1}{2}l\beta-p^2\right)\|x_2\|^2 \qquad (16)
\end{aligned}
$$

where $Q_i=(A_{1i}+B_iK_i)^T\,P_i+P_i\,(A_{1i}+B_iK_i)+P_iB_iK_iK_i^TB_i^TP_i+(1+\eta)\,I$.

Let $\gamma_i=\min\left\{\frac{\lambda_{\min}(-Q_i)}{\lambda_{\max}(P_i)},\frac{\beta l}{2\alpha_2}\right\}$, $l\ge\frac{2p^2}{\beta}$. According to the linear matrix inequality (9), we can get $Q_i<0$; therefore,

$$
\dot{V}_i\le-\gamma_iV_i+\varepsilon \qquad (17)
$$

Integrating (17) from t_i to t yields

$$
V_i(t)\le e^{-\gamma_i(t-t_i)}V_i(t_i)+\varepsilon\int_{t_i}^{t}e^{-\gamma_i(t-s)}ds \qquad (18)
$$

According to the inequalities (10), (12), and (14), we have

$$
V_{l_i}(t_i)\le\tilde{\mu}V_{l_{i-1}}\left(t_i^-\right),\forall l_i,l_{i-1}\in\bar{M} \qquad (19)
$$

where $\tilde{\mu}=\max\left\{\mu,\frac{\alpha_2}{\alpha_1}\right\}$. Let $\gamma=\min\limits_{\forall l_i\in\bar{M}}\gamma_{l_i}>0$. Combining with the inequality (18) and $N_\sigma\,((t-t_i)/\tau_a)+N_0$, we have

$$
\begin{aligned}
V_{\sigma(t)}(t)&=V_{l_i}(t)\\
&\le e^{-\gamma(t-t_i)}\tilde{\mu}V_{l_{i-1}}(t_i^-)\\
&\quad+\frac{\varepsilon}{\gamma}\left(1-e^{-\gamma(t-t_i)}\right)\\
&\le e^{-\gamma(t-t_i)}\tilde{\mu}\Big(e^{-\gamma(t-t_i)}V_{l_{i-1}}(t_{i-1})\\
&\quad+\frac{\varepsilon}{\gamma}\left(1-e^{-\gamma(t-t_i)}\right)\Big)+\frac{\varepsilon}{\gamma}\left(1-e^{-\gamma(t-t_i)}\right)\\
&\le e^{-\gamma(t-t_i)}\tilde{\mu}V_{l_{i-1}}(t_{i-1})\\
&\quad+\frac{\varepsilon\tilde{\mu}}{\gamma}\left(e^{-\gamma(t-t_i)}-e^{-\gamma(t-t_{i-1})}\right)\\
&\quad+\frac{\varepsilon}{\gamma}\left(1-e^{-\gamma(t-t_i)}\right)\\
&\le e^{-\gamma(t-t_i)}\tilde{\mu}^2V_{l_{i-2}}(t_{i-1}^-)\\
&\quad+\frac{\varepsilon\tilde{\mu}}{\gamma}\left(e^{-\gamma(t-t_i)}-e^{-\gamma(t-t_{i-1})}\right)\\
&\quad+\frac{\varepsilon}{\gamma}\left(1-e^{-\gamma(t-t_i)}\right)\\
&\quad\vdots\\
&\le e^{-\gamma(t-t_0)}\tilde{\mu}^{N_\sigma(t_0,t)}V_{l_0}(t_0)\\
&\quad+\frac{\varepsilon\tilde{\mu}^{N_\sigma(t_1,t)}}{\gamma}\left(e^{-\gamma(t-t_2)}-e^{-\gamma(t-t_1)}\right)\\
&\quad+\frac{\varepsilon\tilde{\mu}^{N_\sigma(t_2,t)}}{\gamma}\left(e^{-\gamma(t-t_3)}-e^{-\gamma(t-t_2)}\right)\\
&\quad+\cdots+\frac{\varepsilon\tilde{\mu}^2}{\gamma}\left(e^{-\gamma(t-t_{i-1})}-e^{-\gamma(t-t_{i-2})}\right)\\
&\quad+\frac{\varepsilon\tilde{\mu}}{\gamma}\left(e^{-\gamma(t-t_i)}-e^{-\gamma(t-t_{i-1})}\right)\\
&\quad+\frac{\varepsilon}{\gamma}\left(1-e^{-\gamma(t-t_i)}\right)\\
&\le e^{-\gamma(t-t_0)}\tilde{\mu}^{N_\sigma(t_0,t)}\Big(V_{l_0}(t_0)\\
&\quad-\frac{\varepsilon}{\gamma\tilde{\mu}}\Big)+\frac{\varepsilon(\tilde{\mu}-1)}{\gamma}\sum_{k=0}^{N_\sigma(t_2,t)}\tilde{\mu}^ke^{-\gamma(t-t_{i-k})}+\frac{\varepsilon}{\gamma}\\
&\le e^{-(\gamma-(\ln\tilde{\mu}/\tau_a))(t-t_0)}\tilde{\mu}^{N_0}\Big(V_{l_0}(t_0)\\
&\quad-\frac{\varepsilon\tilde{\mu}}{\gamma}\Big)+\frac{\varepsilon(\tilde{\mu}-1)}{\gamma}e^{\gamma\tau_aN_0}\sum_{k=0}^{N_\sigma(t_2,t)}e^{k(\ln\tilde{\mu}-\gamma\tau_a)}\\
&\quad+\frac{\varepsilon}{\gamma} \qquad (20)
\end{aligned}
$$

From (14), we know that

$$
\begin{aligned}
V_{l_i}(t)&=x_1^T(t)P_{l_i}x_1(t)+lW_{l_i}(x_2)\\
&\ge\min_{\forall l_i\in M}\left(\lambda\left(P_{l_i}\right)\right)\|x_1(t)\|^2\\
&\quad+l\alpha_1\|x_2(t)\|^2\ge\varsigma\|x(t)\|^2
\end{aligned}
$$

and

$$
\begin{aligned}
V_{t_0}&\le\max_{\forall l_i\in M}\left(\lambda\left(P_{l_i}\right)\right)\|x_1(t_0)\|^2\\
&\quad+l\alpha_2\|x_2(t_0)\|^2\le\xi\|x(t_0)\|^2
\end{aligned}
$$

where $\varsigma=\min\left\{\min\limits_{\forall k_i\in\bar{M}}\left(\lambda\left(P_{k_i}\right)\right),l\alpha_1\right\}$, $\xi=\max\left\{\max\limits_{\forall k_i\in\bar{M}}\left(\lambda\left(P_{k_i}\right)\right),l\alpha_2\right\}$.

Thus, according to the above inequalities, we can obtain that

$$\begin{aligned}\|x(t)\|^2 &\le \frac{1}{\varsigma}V_{l_i}(t)\\ &\le \frac{\xi}{\varsigma}e^{-(\gamma-(\ln\tilde{\mu}/\tau_a))(t-t_0)}\tilde{\mu}^{N_0}\left\{\|x(t_0)\|^2-\frac{\varepsilon}{\gamma\tilde{\mu}\xi}\right\}\\ &\quad+\frac{\varepsilon(\tilde{\mu}-1)}{\varsigma\gamma}e^{\gamma\tau_a N_0}\sum_{k=0}^{N_\sigma(t_2,t)}e^{k(ln\tilde{\mu}-\gamma\tau_a)}+\frac{\varepsilon}{\varsigma\gamma} \quad (21)\end{aligned}$$

The condition $\tau_a > (\ln\tilde{\mu})/\gamma$ means that $\gamma-((\ln\tilde{\mu})/\tau_a) > 0$ and $\ln\tilde{\mu}-\gamma\tau_a < 0$. Then, according to inequality (21), we have

$$\begin{aligned}\|x(t)\|^2 \le &\frac{\xi}{\varsigma}e^{-(\gamma-(\ln\tilde{\mu}/\tau_a))(t-t_0)}\tilde{\mu}^{N_0}\left\{\|x(t_0)\|^2-\frac{\varepsilon}{\gamma\tilde{\mu}\xi}\right\}\\ &+\frac{\varepsilon(\tilde{\mu}-1)}{\varsigma\gamma}e^{\gamma\tau_a N_0}\sum_{k=0}^{N_\sigma(t_2,t)}e^{k(ln\tilde{\mu}-\gamma\tau_a)}+\frac{\varepsilon}{\varsigma\gamma} \quad (22)\end{aligned}$$

which can guarantee the uniform boundedness of the system (1).

Remark 1 Let $\varepsilon = 0$, the inequality (22) can be rewritten as

$$\|x(t)|^2 \le \frac{\xi}{\varsigma}e^{-(\gamma-(\ln\tilde{\mu}/\tau_a))(t-t_0)}\tilde{\mu}^{N_0}\|x(t_0)\|^2$$

Thus, when $\varepsilon = 0$, the switched nonlinear system (8) is exponentially stable.

Remark 2 According to Theorem 1, we assume the controller gain is $K_i = B_i^T P_i$, and the inequality (9) can be transformed the following LMIs:

$$\begin{bmatrix} X_iA_{1i}^T+2B_iB_i^T+A_{1i}X_i & B_iB_i^T & X_i\\ B_i^TB_i & -X_i & 0\\ X_i & 0 & -(1+\eta)I\end{bmatrix} < 0,\ X_i > I \quad (23)$$

where $X_i = P_i^{-1}$, $i \in \bar{M}$.

Remark 3 There is a lot of interaction between sensors, controllers, and actuators, which leads to information load problems and a waste of computational resources. Different from periodic sampling, under the event-triggered mechanism, data will only be transmitted when the system meets the set requirements, so that the amount of calculation can be saved.

3.2 Minimum inter-event interval

The Zeno behavior can be prevented in time-triggered sampling control by setting a positive time interval between two consecutive samples. However, this method cannot successfully be used in event-triggered sampling control. Therefore, a new method to exclude Zeno behavior is proposed in this section.

Theorem 2 *Under the condition in inequality (3), the lower bound of the minimum inter-event interval is a positive scalar.*

Proof Firstly, we assume that there exist n samplings on $[t_i, t_{i+1})$, and the corresponding moments are $\hat{t}_{k+1}, \hat{t}_{k+2}, \ldots, \hat{t}_{k+n}$. On any interval of $[t_i, \hat{t}_{k+1})$, $[\hat{t}_{k+1}, \hat{t}_{k+2}), \ldots, [\hat{t}_{k+n}, \hat{t}_{i+1})$, the state $x_1(\hat{t}_{k+j})$, $j = 0, 1, \ldots, n$ are always constants and $e(t) = x_1(t) - x_1(\hat{t}_{k+j})$. Thus, we can obtain that

$$\begin{aligned}\dot{e}(t) = \dot{t}_1(t) &= A_{1i}x_1(t)+A_{2i}x_2(t)+B_iK_ix_1(\hat{t}_{k+j})\\ &= A_{1i}\left(e(t)+x_1(\hat{t}_{k+l})\right)+A_{2i}x_2(t)+B_iK_ix_1(\hat{t}_{k+j})\\ &= A_{1i}e(t)+(A_{1i}+B_iK_i)x_1(\hat{t}_{k+j})+A_{2i}x_2(t)\end{aligned} \quad (24)$$

Therefore,

$$\begin{aligned}e(t) = e^{A_{1i}(t-\hat{t}_{k+j})}e(\hat{t}_{k+j}) &+ \int_{\hat{t}_{k+j}}^{t} e^{A_{1i}(t-s)}\left((A_{1i}\right.\\ &\left.+B_iK_i)x_1(\hat{t}_{k+j})+A_{2i}x_2(s)\right)ds\end{aligned}$$

Due to $e(\hat{t}_{k+j}) = x_1(\hat{t}_{k+j}) - x_1(\hat{t}_{k+j}) = 0$, we have

$$\begin{aligned}e(t) = \int_{\hat{t}_{k+j}}^{t} &e^{A_{1i}(t-s)}\left((A_{1i}+B_iK_i)x_1(\hat{t}_{k+j})\right.\\ &\left.+A_{2i}x_2(s)\right)ds\end{aligned}$$

Thence,

$$\begin{aligned}\|e(t)\| &\le \int_{\hat{t}_{k+j}}^{t} e^{\|A_{1i}\|(t-s)}\left(\|(A_{1i}+B_iK_i)x_1(\hat{t}_{k+j})\|\right.\\ &\quad\left.+\|A_{2i}x_2(s)\|\right)ds\\ &\le \int_{\hat{t}_{k+j}}^{t} e^{\|A_{1i}\|(t-s)}\|(A_{1i}+B_iK_i)\|\|x_1(\hat{t}_{k+j})\|ds\\ &\quad+\int_{\hat{t}_{k+j}}^{t} e^{\|A_{1i}\|(t-s)}\|A_{2i}\|\|x_2(s)\|ds\end{aligned}$$

According to (22), we can find a positive constant $\tilde{\beta}$ such that

$$\|e(t)\| \le \int_{\hat{t}_{k+j}}^{t} e^{\|A_{1i}\|(t-s)}\|(A_{1i}+B_iK_i)\|\|x_1(\hat{t}_{k+j})\|ds$$

$$+\int_{\hat{t}_{k+j}}^{t} e^{\|A_{1i}\|(t-s)}\|A_{2i}\|\sqrt{\beta}ds$$
$$\leq \chi(\hat{t}_{k+j})\int_{\hat{t}_{k+j}}^{t} e^{\|A_{1i}\|(t-s)}ds$$

where $\chi(\hat{t}_{k+j}) = \|((A_{1i} + B_i K_i))\| + \|A_{2i}\|\sqrt{\beta}$. If $\|A_{1i}\| \neq 0$, then

$$\|e(t)\| \leq \frac{\chi(\hat{t}_{k+j})}{\|A_{1i}\|}\left(e^{\|A_{1i}\|(t-s)} - 1\right)$$

According to (4), when $\|e(t)\|^2 = \eta\|x_1(t)\|_2+\varepsilon$, the next event will occur. Let $T = t-\hat{t}_{k+j}$ denote the lower bound of inter-event interval, and T is determined by

$$\frac{\chi(\hat{t}_{k+j})}{\|A_{1i}\|}\left(e^{\|A_{1i}\|(t-s)} - 1\right) = \sqrt{\eta\|x_1\|^2+\varepsilon}$$

Thus

$$T = \frac{1}{\|A_{1i}\|}\ln\left(\frac{\|A_{1i}\sqrt{\eta\|x_1\|^2+\varepsilon}}{\chi(\hat{t}_{k+j})} + 1\right)$$

that means $T > 0$ for any given sampling instant.

4 Numerical example

In this section, we will prove the feasibility of the proposed methods by a numerical example.

Example Consider system (1) with the subsystem, where

$$A_{11} = \begin{bmatrix} -4.8 & 1 \\ 0 & -2.9 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.18 \\ 1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad f_{21} = -x_2 - x_2\sin^2 x_2;$$
$$A_{12} = \begin{bmatrix} -5 & 1 \\ 0 & -3 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 \\ 1 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad f_{22} = -x_2 - x_2$$

$cos^2 x_2$.

Set $\eta = 1, \alpha_1 = 0.4, \alpha_2 = 0.5, \beta = 1, \mu = 1.1, \varepsilon = 0, \kappa_1 = 0.9426, \kappa_2 = 1.3515$. Choose $W_1 = \frac{1}{2}x_2^2$, $W_2 = \frac{2}{5}x_2^2$, and $l = 1$. The controller gains are chosen as

$$K_1 = \begin{bmatrix} -0.2 & -0.5 \end{bmatrix}, K_2 = \begin{bmatrix} -0.4 & -0.6 \end{bmatrix}.$$

By solving inequalities (9) and (10) in Theorem 1, we get

$$P_1 = \begin{bmatrix} 0.3952 & 0.0285 \\ 0.0285 & 0.5649 \end{bmatrix}, P_2 = \begin{bmatrix} 0.3687 & 0.0115 \\ 0.0115 & 0.5150 \end{bmatrix}.$$

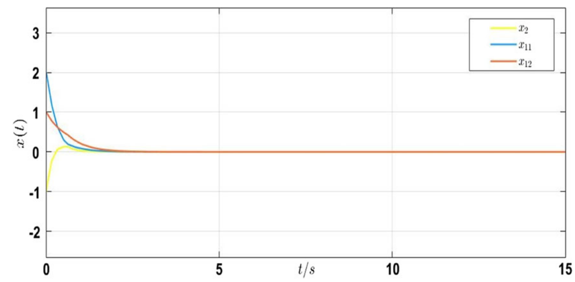

Fig. 1 State responses

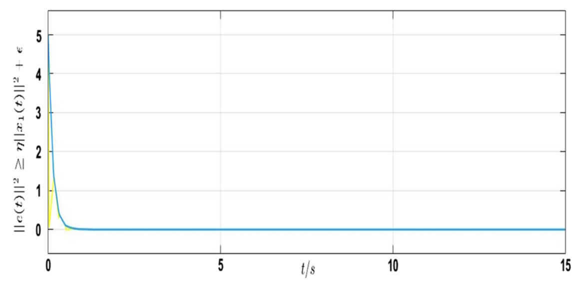

Fig. 2 Event-triggered condition

$\gamma = 1.76, \tilde{\mu} = 1.25$ and $\tau_a^* = (\ln\tilde{\mu})/\gamma = 0.127$. Obviously, the conditions of Theorem 1 are satisfied; therefore, the considered system is exponentially stable under the switching signal σ. Figures. 1, 2, 3, 4, 5, 6 show the results of the simulation, respectively. Figure 1 illustrates the state response of the system. From Fig. 1, we can see that the state of the system converges to zero, which indicates that the system is stable; Figure 2 gives the event-triggered condition; Figure 3 demonstrates the switching signal; Figure 4 displays the control input of the system. Figures 5 and 6 depict the time-triggered instants and event-triggered instants, respectively. One can obtain that, under the time-triggered scheme, there are 60 sampling instants in $[0, 15s]$, by using the event-triggered scheme, only 45 sampling instants. In other words, 75% data information is used to stabilize the controlled system, which proves that the designed event-triggered scheme in (3) can save the communication resource effectively.

5 Conclusions

In this paper, the problem of event-triggered control has been studied for the cascade switched nonlinear system. With the application of the average dwell time

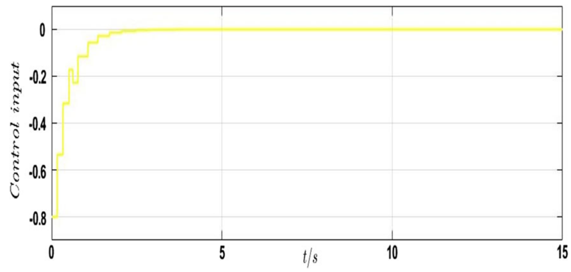

Fig. 3 Control input

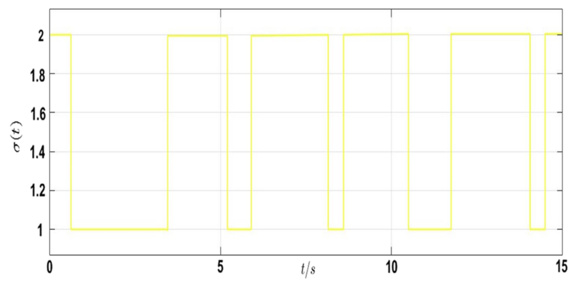

Fig. 4 Switching signal

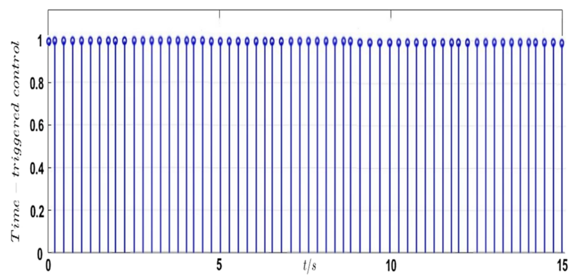

Fig. 5 Time-triggered instants

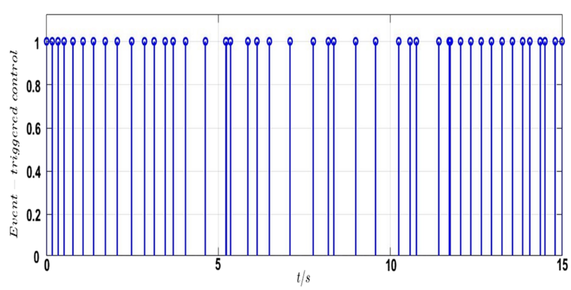

Fig. 6 Event-triggered instants

technique and the Lyapunov function method, we have obtained sufficient conditions to guarantee the system to be globally uniformly bounded and exponentially stable. What's more, we have obtained a lower bound of the minimum inter-event interval to preclude the Zeno behavior in the procedure of event-triggered sampling. At present, the event-triggered control problems of the cascaded switched nonlinear systems still need further research. For example, noise signals are widely existing in the world [30] and Zeno behavior influencing the performance of systems. Therefore, considering the effect of noises in the framework of event-triggered systems and excluding the Zeno behavior in real time are our further research goals.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Xiang, W., Johnson, T.T.: Event-triggered control for continuous-time switched linear systems[J]. IET Control Theory Appl. **11**(11), 1694–1703 (2017)
2. Lian, J., Li, C., Xia, B.: Sampled-data control of switched linear systems with application to an F-18 aircraft. IEEE Trans. Ind. Electron. **64**(2), 1332–1340 (2016)
3. Xiang, W., Lam, J., Li, P.: On stability and H_∞ control of switched systems with random switching signals[J]. Automatica **95**, 419–425 (2018)
4. Wang, J., Zhao, J.: On improving transient performance in tracking control for switched systems with input saturation via composite nonlinear feedback[J]. Int. J. Robust Nonlinear Control **26**(3), 509–518 (2016)
5. Xiang, W.: Necessary and sufficient condition for stability of switched uncertain linear systems under dwell-time constraint[J]. IEEE Trans. Autom. Control **61**(11), 3619–3624 (2016)
6. Wang, Y.E., Sun, X.M., Mazenc, F.: Stability of switched nonlinear systems with delay and disturbance[J]. Automatica **69**, 78–86 (2016)
7. Wu, X., Tang, Y., Cao, J., et al.: Stability analysis for continuous-time switched systems with stochastic switching signals[J]. IEEE Trans. Autom. Control **63**(9), 3083–3090 (2017)
8. Wang, Z., Chen, G., Ba, H.: Stability analysis of nonlinear switched systems with sampled-data controllers[J]. Appl. Math. Comput. **357**, 297–309 (2019)
9. Fei, Z., Shi, S., Zhao, C., et al.: Asynchronous control for 2-D switched systems with mode-dependent average dwell time[J]. Automatica **79**, 198–206 (2017)
10. Su, Q., Jia, X.: Finite-time H_∞ control of cascade nonlinear switched systems under state-dependent switching[J]. Int. J. Control Autom. Syst. **16**(1), 120–128 (2018)
11. Mathiyalagan, K., Park, J.H., Sakthivel, R.: Observer-based dissipative control for networked control systems: A

switched system approach[J]. Complexity **21**(2), 297–308 (2015)
12. Park, J.H., Mathiyalagan, K., Sakthivel, R.: Fault estimation for discrete-time switched nonlinear systems with discrete and distributed delays[J]. Int. J. Robust Nonlinear Control **26**(17), 3755–3771 (2016)
13. Zheng, Q., Zhang, H., Ling, Y., et al.: Mixed H_∞ and passive control for a class of nonlinear switched systems with average dwell time via hybrid control approach[J]. J. Frankl. Inst. **355**(3), 1156–1175 (2018)
14. Niu, B., Wang, D., Alotaibi, N.D., et al.: Adaptive neural state-feedback tracking control of stochastic nonlinear switched systems: an average dwell-time method[J]. IEEE Trans. Neural Netw. Learn. Syst. **30**(4), 1076–1087 (2018)
15. Linsenmayer, S., Hertneck, M., Allgower, F.: Linear weakly hard real-time control systems: time- and event-triggered stabilization. IEEE Trans. Autom. Control (2020). https://doi.org/10.1109/TAC.2020.3000981
16. Wang, Y., Jia, Z., Zhao, R., et al.: Event-triggered control for switched systems with both continuous-time and discrete-time subsystems[J]. Int. J. Syst. Sci. **51**(1), 180–190 (2020)
17. Wang, Y., Zheng, W.X., Zhang, H.: Dynamic event-based control of nonlinear stochastic systems[J]. IEEE Trans. Autom. Control **62**(12), 6544–6551 (2017)
18. Xiao, X., Park, J.H., Zhou, L., et al.: Event-triggered control of discrete-time switched linear systems with network transmission delays[J]. Automatica **111**, 108585 (2020)
19. Xiao, X., Zhou, L., Ho, D.W.C., et al.: Event-triggered control of continuous-time switched linear systems[J]. IEEE Trans. Autom. Control **64**(4), 1710–1717 (2018)
20. Su, X., Liu, X., Shi, P., et al.: Sliding mode control of hybrid switched systems via an event-triggered mechanism[J]. Automatica **90**, 294–303 (2018)
21. Li, H., Chen, Z., Wu, L., et al.: Event-triggered control for nonlinear systems under unreliable communication links[J]. IEEE Trans. Fuzzy Syst. **25**(4), 813–824 (2016)
22. Qi, Y., Cao, M.: Finite-time boundedness and stabilisation of switched linear systems using event-triggered controllers[J]. IET Control Theory Appl. **11**(18), 3240–3248 (2017)
23. Lamperski, A., Ames, A.D.: Lyapunov theory for Zeno stability. IEEE Trans. Autom. Control **58**(100), 112 (2013)
24. Ni, W., Zhao, P., Wang, X., et al.: Event-triggered control of linear systems with saturated inputs[J]. Asian J. Control **17**(4), 1196–1208 (2015)
25. Fei, Z., Guan, C., Zhao, X.: Event-triggered dynamic output feedback control for switched systems with frequent asynchronism. IEEE Trans. Autom. Control **65**(7), 3120–3127 (2020)
26. Li, C., Lian, J.: Event-triggered feedback stabilization of switched linear systems using dynamic quantized input[J]. Nonlinear Anal. Hybrid Syst. **31**, 292–301 (2019)
27. Li, H., Zhang, X., Feng, G.: Event-triggered output feedback control of switched nonlinear systems with input saturation. IEEE Trans. Cybern. (2020). https://doi.org/10.1109/TCYB.2020.2965142
28. Huo, X., Ma, L., Zhao, X., et al.: Event-triggered adaptive fuzzy output feedback control of MIMO switched nonlinear systems with average dwell time. Appl. Math. Comput. **365**, 124665 (2020)
29. Mathiyalagan, K., Park, J.H., Sakthivel, R.: Finite-time boundedness and dissipativity analysis of networked cascade control systems[J]. Nonlinear Dyn. **84**(4), 2149–2160 (2016)
30. Jiao, T., Zheng, W.X., Xu, S.: Unified stability criteria of random nonlinear time-varying impulsive switched systems[J]. IEEE Trans. Circuits Syst. I Regul. Pap. **67**(9), 3099–3112 (2020)