



Consensus of nonlinear multiagent systems with intermittent dynamic event-triggered protocols

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Abstract This paper studies the consensus of leader-following multiagent systems with nonlinear dynamics. Consider that control protocols for the consensus work under an intermittent framework due to inevitable factors; meanwhile, the event-triggered mechanism is introduced, so as to reduce the update frequency of the control protocols. In particular, threshold parameters in the event-triggered conditions are supposed to be dynamically changed, and the minimum event-triggered intervals are set to be positive in advance. Furthermore, according to whether event-triggered conditions depend on combined measurements or a single measurement, two forms of event-triggered schemes are designed; then, the corresponding distributed control protocols are demonstrated. Based on the graph theory and Lyapunov function method, sufficient conditions and a concrete algorithm for the consensus of multiagent systems are presented. It is shown that the intermittent dynamic event-triggered protocols given in this paper can effectively reduce the update frequency of control and exclude Zeno behavior. Finally, detailed

numerical examples are supplied to illustrate the proposed results.

Keywords Consensus · Intermittent framework · Event-triggered protocol · Nonlinear dynamics · Zeno-free mechanism

1 Introduction

A multiagent system is composed of multiple interacting agents, which can complete specific tasks through information exchange and cooperation among the agents. There are many real systems that can be modeled as multiagent systems, such as clustered animals, sensor networks, smart power grids, and so on. During the past decades, the cooperative control problem of multiagent systems has been widely addressed [1–4], wherein the consensus is regarded as an important issue [5, 6]. In general, the consensus of multiagent systems implies that all the agents can reach a common value by adopting appropriate control protocols. Multiagent systems in the study of consensus can be divided into two main types, that is, leader-following systems and leaderless systems [7–10]. Particularly, a leader-following system includes multiple followers and one leader, then all the followers converge to the trajectory of the leader if the consensus is realized, in other words, the state of the leader can be used as the target of control. Therefore, in the leader-following consensus, all the agents

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can achieve a prescribed behavior, which can be applied to the tracking problem.

In order to achieve the consensus of multiagent systems, a number of cooperative control strategies have been reported in the existing results, such as adaptive control [11], output feedback control [12], optimal control [13], and so on. It is worth mentioning that the controller applied to an agent mainly depends on the information from its neighboring agents via communication links. However, because of the existence of inevitable factors including cyber-attacks and the transmission distance greater than the effective sensing range, it is unrealistic for an agent to sense the information all the time. On the other hand, control protocols may be disabled due to the sensor or actuator failures. Accordingly, an intermittent framework is more practical compared with a continuous one. In the intermittent scheme, protocols are required to work for a period of time, then rest or recover from failures for the next interval, which can also extend the service life of the control equipment. So far, the intermittent control protocols have been adopted for the consensus. For example, by utilizing the intermittent pinning control, synchronization of nonlinear coupled networks was discussed [14]. The consensus of multiagent systems in a heterogeneous coupling network was realized through intermittent scheme [15]. In [16], an intermittent pinning strategy on time scales was put forward, and in [17], the consensus of second-order multiagent systems with intermittent approach was investigated.

In addition to the intermittent framework described above, many current works are concerned with economic and effective ways for designing control laws, such as impulsive control, sampled-data control, and so on [18, 19]. In recent years, it has been shown that the update frequency of control can be reduced by adopting event-triggered mechanism [20], since the instants for updating are discrete rather than continuous. Compared with the sampled-data control employing the time-triggered approach, in the event-triggered strategy, those instants are decided by the given specific conditions, then update intervals are not fixed and unnecessary updates can be avoided. To be specific, in the time-triggered mechanism, updates of control depend on the time device, therefore, at each new sampling instant, even if the received information changes a little, the control still needs to be updated. Until now, event-triggered schemes have been used for the consensus of multiagent systems. The consensus in switching

networks via event-triggered control was investigated [21], the consensus of stochastic multiagent systems with event-triggered strategy was proposed [22], and a fully distributed event-triggered method for the consensus was developed [23].

It should be noted that the event-triggered mechanism may cause Zeno phenomenon, which implies that the controlled system will be instable owing to the existence of infinite switches in finite time. Therefore, Zeno behavior should be excluded in practical control protocols with the triggering mechanism, that is, event-triggered intervals need to be strictly positive. Some related results demonstrated the design of event-triggered conditions for avoiding Zeno behavior [24–27]. In particular, in the literature [25–27], the minimum triggering interval is set to be positive in advance. In addition, from the perspective of reducing the update frequency of control, the event-triggered conditions that can bring about larger triggering intervals are more effective. Recently, dynamic event-triggered schemes have been put forward and discussed [28–33]. It is found that if the threshold parameters in triggering conditions are supposed to be dynamic, the update frequency can be further reduced. By introducing a dynamic variable, a new class of dynamic event-triggered mechanism was presented [28]. In [29, 30], the consensus of linear multiagent systems under dynamic strategies was investigated, and the control problem of switched systems under dynamic event-triggered method was addressed [31, 32].

Based on the above discussion, the design of practical and effective control protocols for the consensus deserves further investigation. In this work, we put forward intermittent dynamic event-triggered control protocols, and the main contributions can be summarized as follows.

1. This paper proposes intermittent dynamic event-triggered schemes for the consensus of multiagent systems. To the authors' knowledge, there exist few results of the event-triggered mechanism that is both dynamic and works in the intermittent framework. On the one hand, the cost of updating the control can be saved in the dynamic strategies. On the other hand, compared with the available results on the dynamic event-triggered mechanism, such as [29, 30], the protocols in this paper can work well for multiagent systems subject to cyber-attacks or actuator failures. Moreover, under the intermittent

framework, the life of the control devices can be prolonged.

2. As is well known, Zeno behavior should be excluded in the event-triggered control protocols, therefore, the minimum triggering interval is set to be positive in advance for excluding Zeno behavior, which is partially motivated by the current results involving Zeno-free method [26,27]. However, it should be noted that this paper focuses on the dynamic triggered mechanism and multiagent systems with nonlinear dynamics [34,35], then, how to design the minimum interval is challenging and innovative.
3. Two forms of event-triggered conditions are given in the paper, where combined measurements and a single measurement are utilized, respectively, furthermore, the characteristics of these two schemes are compared and illustrated. In addition, in comparison with our previous work [36], the event-triggered protocols in this paper are more advanced, since the protocols are dynamic and Zeno-free, and the leader in the multiagent system is not required to be static.

Organization In Sect. 2, multiagent systems are modeled, and intermittent dynamic event-triggered control protocols for the consensus are formulated. In Sect. 3, two types of dynamic event-triggered schemes are put forward. Section 4 shows detailed numerical examples. Finally, Sect. 5 gives concluding remarks.

Notations Let \mathbb{R}^m be the m -dimensional Euclidean space. $\|\cdot\|$ denotes the Euclidean norm of a vector or the spectral norm of a matrix, and $|\cdot|$ indicates the length of an interval. \otimes denotes the Kronecker product. \cup , \cap , and \setminus represent the union, intersection, and relative complement in set operations, respectively. $\text{diag}\{\cdot\cdot\}$ indicates a diagonal matrix. $P > 0$ means that the symmetric matrix P is positive definite. I_m is supposed to be m -dimensional identity matrix.

2 Problem formulation and preliminaries

2.1 Multiagent systems and graph theory

Consider that a leader-following multiagent system is composed of N followers and one leader labeled by 0, and each agent has nonlinear dynamics, which is described as

$$\begin{cases} \dot{x}_i(t) = f(t, x_i(t)) + u_i(t), & i = 1, 2, \dots, N, \\ \dot{x}_0(t) = f(t, x_0(t)), \end{cases} \quad (1)$$

where $x_i(t) \in \mathbb{R}^m$ is the state of the i th follower, and $x_0(t) \in \mathbb{R}^m$ is the state of the leader. $f(t, \cdot) \in \mathbb{R}^m$ denotes the nonlinear dynamics. $u_i(t) \in \mathbb{R}^m$ indicates the control protocol applied to the i th follower, which mainly utilizes the information from neighboring followers and the leader. Obviously, the leader is an isolated agent. It should be noted that the nonlinear dynamics is supposed to be the same for all the agents, which is similar to the existing related results, such as in the literature [37,38].

Generally, the protocol $u_i(t)$ is given as (2) if it works under a continuous framework:

$$u_i(t) = c \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t) - x_i(t)) + cb_i(x_0(t) - x_i(t)), \quad (2)$$

where $c > 0$ represents the coupling strength, \mathcal{N}_i denotes the set of the neighbors for the i th follower. $a_{ij} \geq 0$ and $b_i \geq 0$ represent the weights of the information from the j th follower and the leader, respectively. Moreover, it is usually supposed that $b_i = 0$ for most of the followers, which means that just a fraction of the followers are pinned by the leader.

The communication topology of a multiagent system can be represented as a graph. Take the N followers in (1) for illustration, the graph is described as $\mathcal{G} = (\mathcal{W}, \mathcal{E}, A)$, where \mathcal{W} is the set of the N followers or nodes, and \mathcal{E} is the set of communication channels or edges. $A = (a_{ij})_{N \times N}$ is called the adjacency matrix. If there is an edge from the j th agent to the i th agent, then $a_{ij} > 0$, otherwise, $a_{ij} = 0$. Furthermore, $a_{ij} > 0$ implies that the information from the j th agent affects the state $x_i(t)$ through the protocol $u_i(t)$, therefore, the j th agent can be regarded as a neighbor for the i th agent. Besides the matrix A , the Laplacian matrix L is an important element for the graph, which is defined as $L = (l_{ij})_{N \times N}$ with $l_{ii} = \sum_{j=1}^N a_{ij}$ and $l_{ij} = -a_{ij}$ ($i \neq j$). In addition, a directed path for two nodes in a graph refers to a sequence of ordered edges between them. A graph is called to have a spanning tree if there is a node owning directed paths to any other node, and this node can be taken as the root of the graph.

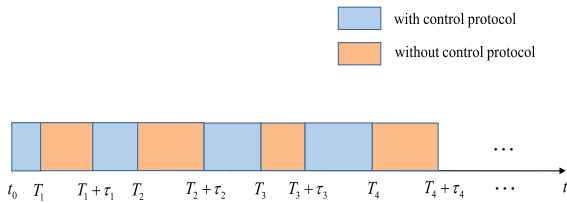


Fig. 1 Illustration for intermittent control protocols

2.2 Consensus and event-triggered protocols

First, the definition of consensus for the multiagent system (1) is presented as follows.

Definition 1 The multiagent system (1) is called to realize consensus if, for any given initial conditions $x_i(0)$ ($i = 1, 2, \dots, N$), $\lim_{t \rightarrow \infty} \|x_i(t) - x_0(t)\| = 0$ holds.

Remark 1 It is obvious that the states of the followers converge to that of the leader if the consensus is realized, then the leader-following consensus can be regarded as the tracking problem [15].

The main objective of this work is to design appropriate and effective protocols $u_i(t)$ for the consensus of the system (1). However, it is difficult to require the protocol to be continuous as (2) due to cyber-attacks, actuator failures, and so on. Accordingly, intermittent strategies may be more practical. Suppose that the initial time is t_0 , and the protocol fails during the intervals $[T_k, T_k + \tau_k)$ ($k = 1, 2, \dots$). Without loss of generality, assume $\tau_k \geq \tau_0 > 0$ and $T_{k+1} > T_k + \tau_k$. Figure 1 presents the time intervals for the intermittent protocol. Moreover, let $\Delta^f(t_0, t_1)$ be the set of all intervals that the protocol $u_i(t)$ fails in the period $[t_0, t_1]$, then $\Delta^f(t_0, t_1)$ is described as $\Delta^f(t_0, t_1) = \bigcup_{k=1,2,\dots} [T_k, T_k + \tau_k) \cap [t_0, t_1]$. Instead, $\Delta^w(t_0, t_1) = [t_0, t_1] \setminus \Delta^f(t_0, t_1)$ denotes the set of all intervals that the protocol works.

On the other hand, as described in Sect. 1, so as to reduce the update frequency of control, the event-triggered scheme is utilized for the protocol, then the control protocol $u_i(t)$ is given as

$$u_i(t) = \begin{cases} c \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) \\ + cb_i(\hat{x}_0(t) - \hat{x}_i(t)), & t \in \Delta^w(t_0, t_1), \\ 0, & t \in \Delta^f(t_0, t_1), \end{cases} \quad (3)$$

where $i = 1, 2, \dots, N$. $\hat{x}_i(t)$ and $\hat{x}_j(t)$ denote the states of the i th and j th agents at certain event-triggered

instants, respectively, which will be discussed later. Obviously, the event-triggered instants will decide the updating of the control.

Suppose that event-triggered instants of the i th agent are t_1^i, t_2^i, \dots with $t_1^i = t_0$, furthermore, since we focus on the intermittent protocols, the event-triggered mechanism is assumed to only work during $\Delta^w(t_0, t_1)$. Additionally, in order to avoid Zeno behavior, which can induce the instability of the controlled system, Zeno-free mechanism is utilized, that is, assume that the current triggering instant is t_q^i , then the next triggering instant t_{q+1}^i satisfies

$$t_{q+1}^i = t_q^i + \max\{\delta_q^i, \varepsilon_i\}, \quad (4)$$

where $\varepsilon_i > 0$, and δ_q^i is obtained through the following dynamic event-triggered condition:

$$\delta_q^i = \inf_{t > t_q^i, t \in \Delta^w(t_0, t_1)} \{t - t_q^i | g_i(t) \geq \eta_i(t)\} \quad (5)$$

for $i = 1, 2, \dots, N$, $q = 1, 2, \dots$, $g_i(t)$ is a specific function relying on the information from the neighboring agents, and the form of $g_i(t)$ will be presented in Sect. 3. Moreover, the threshold parameter $\eta_i(t)$ is dynamic with a law as

$$\dot{\eta}_i(t) = \begin{cases} -\alpha_i \eta_i(t), & t \in \Delta^w(t_0, t_1), \\ 0, & t \in \Delta^f(t_0, t_1), \end{cases} \quad (6)$$

where α_i is a positive constant, and $\eta_i(t_0) > 0$.

Remark 2 Zeno behavior means that there exist infinite switches in finite time, accordingly, strictly positive event-triggered intervals $t_{q+1}^i - t_q^i$ can guarantee the exclusion of Zeno behavior. Therefore, motivated by the results in [26,27], this paper adopts Zeno-free mechanism, that is, $t_{q+1}^i - t_q^i \geq \varepsilon_i > 0$, and the selection of parameters ε_i will be given later. Recently, a different method for designing the parameter $\eta_i(t)$ was presented [30], which may play a better role in adjusting $\eta_i(t)$. However, since the intermittent framework is introduced in this work, the condition (6) is more suitable.

It should be noted that t_{q+1}^i may locate in $\Delta^f(t_0, t_1)$ when $t_{q+1}^i = t_q^i + \varepsilon_i$ and $t_q^i + \varepsilon_i \in \Delta^f(t_0, t_1)$. However, the information from the neighboring agents at t_{q+1}^i may be unavailable if $t_{q+1}^i \in \Delta^f(t_0, t_1)$ due to the

intermittent scheme. Accordingly, $t_{q+1}^i = t_q^i + \varepsilon_i$ is replaced with $t_{q+1}^i = \min_{T_k + \tau_k \geq t_q^i + \varepsilon_i} \{T_k + \tau_k\}$ if $t_q^i + \varepsilon_i \in \Delta^f(t_0, t_1)$.

Remark 3 If $\eta_i(t)$ in the condition (5) equals zero, then the condition changes into $g_i(t) \geq 0$, which has been employed in the existing literature such as our previous work [36], and this case will be illustrated in Sect. 4. On the other hand, in order to further increase event-triggered intervals, dynamic event-triggered control with time regularization was proposed [39]. However, it is difficult to apply the time regularization technique to this work, since the event-triggered time sequences $\{t_1^i, t_2^i, \dots\}$ can be non-identical for different agents.

Obviously, $\eta_i(t) > 0$ can be guaranteed by the law (6) and $\eta_i(t_0) > 0$. Specifically, as described in Fig. 1, if $t \in [t_0, T_1)$, $\eta_i(t) = \eta_i(t_0) \exp\{-\alpha_i(t - t_0)\} > 0$, and if $t \in [T_1, T_1 + \tau_1)$, $\eta_i(t) = \eta_i(T_1) = \eta_i(t_0) \exp\{-\alpha_i(T_1 - t_0)\} > 0$, therefore, $\eta_i(t) = \eta_i(t_0) \exp\{-\alpha_i(t - \tau_k - \tau_{k-1} - \dots - \tau_1 - t_0)\} > 0$ when $t \in [T_k + \tau_k, T_{k+1})$, and $\eta_i(t) = \eta_i(T_{k+1}) > 0$ for $t \in [T_{k+1}, T_{k+1} + \tau_{k+1})$. In conclusion, $\eta_i(t) > 0$ is verified. Moreover, the threshold parameter $\eta_i(t) > 0$ in the condition $g_i(t) \geq \eta_i(t)$ brings about larger event-triggered intervals δ_q^i compared with the case of $\eta_i(t) \equiv 0$ (see Sect. 4 for more details). Consequently, the dynamic event-triggered mechanism can further reduce the number of control updates. It should be noted that the condition $g_i(t) \geq \eta_i(t)$ is detected continuously when $t \in \Delta^w(t_0, t_1)$, while in some existing results, the corresponding condition can be detected discretely [20]. However, since the intermittent framework, dynamic parameter, and nonlinear dynamics are introduced in the model of this paper, it will be difficult to deal with the discrete event-triggered detector.

In short, as described in Fig. 2, the event-triggered detector and the controller work under the same intermittent framework, in other words, the failures of sensor and actuator are synchronous. Of course, the failures can be asynchronous, however, this problem is more complicated, which will be focused on in our future work. The control input $u_i(t)$ is updated according to $\hat{x}_i(t)$, and kept a constant through the zero-order hold (ZOH) until the new event is triggered. It should be noted that ZOH is usually employed in the event-triggered scheme [39], in addition, the first-order hold

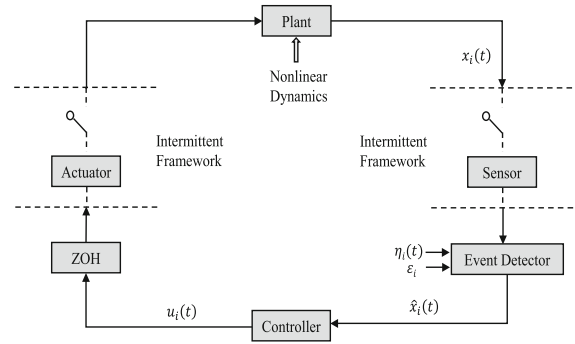


Fig. 2 Block diagram of the controlled multiagent system

(FOH) is also utilized for the networked control systems [40].

2.3 Preliminaries

This subsection provides some essential definition, lemmas, and assumptions.

Definition 2 ([41]) A nonsingular matrix H is called to be an M -matrix if each off-diagonal element is non-positive, and all of its eigenvalues have positive real parts.

Lemma 1 ([42]) For a graph $\mathcal{G} = (\mathcal{W}, \mathcal{E}, A)$ containing N nodes, if there exists a leader node having a directed path to every other node in \mathcal{G} , then the matrix $L + B$ is an M matrix, where L is the Laplacian matrix of the graph \mathcal{G} , and $B = \text{diag}\{b_1, b_2, \dots, b_N\}$, b_i denotes the weight of the edge from the leader node to the i th node ($i = 1, 2, \dots, N$).

Lemma 2 ([43]) If $H \in \mathbb{R}^{N \times N}$ is an M -matrix, then there is a vector $(p_1, p_2, \dots, p_N)^T \in \mathbb{R}^N$ such that $PH + H^T P > 0$, where $P = \text{diag}\{p_1, p_2, \dots, p_N\}$ and $p_i > 0$. In addition, H^{-1} exists.

Lemma 3 ([41]) For matrices $\mathcal{U}_1, \mathcal{U}_2, \mathcal{V}_1$ and \mathcal{V}_2 with appropriate dimensions, the Kronecker product \otimes satisfies:

- (i) $(\rho \mathcal{U}_1) \otimes \mathcal{U}_2 = \mathcal{U}_1 \otimes (\rho \mathcal{U}_2) = \rho(\mathcal{U}_1 \otimes \mathcal{U}_2)$, where ρ is a constant.
- (ii) $(\mathcal{U}_1 + \mathcal{U}_2) \otimes \mathcal{V}_1 = \mathcal{U}_1 \otimes \mathcal{V}_1 + \mathcal{U}_2 \otimes \mathcal{V}_1$.
- (iii) $(\mathcal{U}_1 \otimes \mathcal{U}_2)(\mathcal{V}_1 \otimes \mathcal{V}_2) = (\mathcal{U}_1 \mathcal{V}_1) \otimes (\mathcal{U}_2 \mathcal{V}_2)$.
- (iv) $(\mathcal{U}_1 \otimes \mathcal{U}_2)^T = \mathcal{U}_1^T \otimes \mathcal{U}_2^T$.

Assumption 1 There exists $\gamma > 0$ such that $\|f(t, x_i) - f(t, x_j)\| \leq \gamma \|x_i - x_j\|$ for all $x_i, x_j \in \mathbb{R}^m$.

Assumption 2 The leader node 0 in the graph \mathcal{G} has a directed path to every follower, where \mathcal{G} represents the topology of the multiagent system (1).

Assumption 3 There exist $n_0 \geq 0$ and $T^f > 1$ satisfying

$$|\Delta^f(t_0, t_1)| \leq n_0 + \frac{t_1 - t_0}{T^f} \tag{7}$$

for all $t_0 \geq 0$, and $t_1 > t_0$, where $|\Delta^f(t_0, t_1)|$ represents the length of time when the control protocol $u_i(t)$ fails within $[t_0, t_1]$.

Remark 4 The parameter $T^f > 1$ in Assumption 3 guarantees that the time without control protocols is less than $t_1 - t_0$. In addition, Assumption 3 has been utilized in current results on cyber-attacks [33,44].

3 Main results

3.1 Protocol with combined measurements

This subsection presents the consensus under the intermittent dynamic event-triggered protocol with combined measurements. First, let $\hat{x}_i(t)$, $\hat{x}_j(t)$, and $\hat{x}_0(t)$ in (3) be $x_i(t_q^i)$, $x_j(t_q^j)$, and $x_0(t_q^0)$, respectively, where $t_q^i \leq t$ indicates the latest triggering instant of the i th agent. Accordingly, the protocol (3) for $t \in \Delta^w(t_0, t_1)$ is rewritten as

$$u_i(t) = c \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t_q^j) - x_i(t_q^i)) + cb_i(x_0(t_q^0) - x_i(t_q^i)), \tag{8}$$

then the protocol $u_i(t)$ is updated only at each discrete event-triggered instant t_q^i ($q = 1, 2, \dots$) of the i th agent, which reduces the number of updates. According to the definition of the Laplacian matrix L , $u_i(t)$ as (8) can be described as

$$u_i(t) = \begin{cases} -c \sum_{j=1}^N l_{ij}x_j(t_q^j) \\ +cb_i(x_0(t_q^0) - x_i(t_q^i)), & t \in \Delta^w(t_0, t_1), \\ 0, & t \in \Delta^f(t_0, t_1). \end{cases} \tag{9}$$

Furthermore, t_q^i is decided by the event-triggered mechanism introduced in Sect. 2, where $g_i(t)$ in (5) is defined as

$$g_i(t) = \|\beta_i(t_q^i) - \beta_i(t)\|^2 - \sigma_i \|\beta_i(t)\|^2, \tag{10}$$

where $i = 1, 2, \dots, N$, $\beta_i(t) = \sum_{j=1}^N l_{ij}x_j(t) - b_i(x_0(t) - x_i(t))$, and $\beta_i(t_q^i) = \sum_{j=1}^N l_{ij}x_j(t_q^j) - b_i(x_0(t_q^0) - x_i(t_q^i))$. The parameter $\sigma_i > 0$. $\beta_i(t_q^i) - \beta_i(t) \triangleq \vartheta_i(t)$ denotes combined measurement relying on the information from neighbors at the triggering instant t_q^i and the current time t .

Remark 5 Obviously, the condition $g_i(t) \geq \eta_i(t)$ in (5) changes into $\|\vartheta_i(t)\|^2 \geq \sigma_i \|\beta_i(t)\|^2 + \eta_i(t)$. Then, if $\|\vartheta_i(t)\|^2 \geq \sigma_i \|\beta_i(t)\|^2 + \eta_i(t)$, an event is triggered.

Denote the consensus error to be $e_i(t) = x_i(t) - x_0(t)$ ($i = 1, 2, \dots, N$), based on the multiagent system (1) and the control protocol (9), one has

$$\dot{e}_i(t) = \begin{cases} f(t, x_i(t)) - f(t, x_0(t)) \\ -c(\vartheta_i(t) + \beta_i(t)), & t \in \Delta^w(t_0, t_1), \\ f(t, x_i(t)) - f(t, x_0(t)), & t \in \Delta^f(t_0, t_1). \end{cases} \tag{11}$$

Next, conditions guaranteeing the consensus are given in Theorem 1.

Theorem 1 *The leader-following multiagent system (1) under the intermittent control protocol (9) with the dynamic event-triggered mechanism, i.e., described as (4)–(6) and (10), will realize the consensus if Assumptions 1–3 and the following conditions hold.*

- (i) *The coupling strength $c > \frac{2\gamma\bar{p}}{2\kappa_1 - \theta\bar{p}^2}$, where $0 < \theta < \frac{2\kappa_1}{\bar{p}^2}$, $\bar{p} = \max\{p_1, p_2, \dots, p_N\}$, $\kappa_1 = \min_{i=1,2,\dots,N} \{\lambda_i(\frac{PH+H^T P}{2})\}$.*
- (ii) *The parameter σ_i in (10) satisfies $0 < \sigma_i \leq \bar{\sigma}_1 < \bar{\sigma}$ for $i = 1, 2, \dots, N$, where $\bar{\sigma} < \frac{2\theta c\kappa_1 - 2\theta\gamma\bar{p} - c\theta^2\bar{p}^2}{c\kappa_2}$, and $\kappa_2 = \max_{i=1,2,\dots,N} \{\lambda_i(H^T H)\}$.*
- (iii) *The parameter α_i in (6) satisfies $\alpha_i > \frac{c}{2\bar{\theta}}$ for $i = 1, 2, \dots, N$.*
- (iv) *The parameter T^f in Assumption 3 satisfies $T^f > \frac{\rho_1 + \rho_2}{\rho_1}$, where $\rho_1 = -\max_{i=1,2,\dots,N} \{\frac{c}{2\bar{\theta}} - \alpha_i, \frac{2\gamma\bar{p} + c\theta\bar{p}^2 - 2c\kappa_1 + \frac{c}{\bar{\theta}}\bar{\sigma}\kappa_2}{\bar{p}}\}$, and $\rho_2 = 2\gamma$.*

(v) The parameter ε_i in (4) satisfies $0 < \varepsilon_i \leq \frac{1}{\varpi} \times \ln \left(\sqrt{\frac{\bar{\sigma}_2}{N}} + 1 \right)$, where $\varpi = \gamma \|H\| \|H^{-1}\| + c \|H\| \times \sqrt{\bar{\sigma}_2 N} + c \|H\|$, and $\bar{\sigma}_2 = \bar{\sigma} - \bar{\sigma}_1$.

Proof According to Assumption 2 and Lemma 1, the matrix $H = L + B$ is an M -matrix, where L is the Laplacian matrix and $B = \text{diag}\{b_1, b_2, \dots, b_N\}$, then there is a matrix $P = \text{diag}\{p_1, p_2, \dots, p_N\}$ satisfying $PH + H^T P > 0$ due to Lemma 2. Therefore, select the Lyapunov function as

$$V(t) = \frac{1}{2} \sum_{i=1}^N p_i e_i^T(t) e_i(t) + \sum_{i=1}^N \eta_i(t). \tag{12}$$

When $t \in \Delta^w(t_0, t_1)$, based on the error system (11), the law (6), and Assumption 1, one can get

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N p_i e_i^T(t) (f(t, x_i(t)) - f(t, x_0(t)) \\ &\quad - c(\vartheta_i(t) + \beta_i(t))) + \sum_{i=1}^N (-\alpha_i \eta_i(t)) \\ &\leq \gamma \sum_{i=1}^N p_i e_i^T(t) e_i(t) - c \sum_{i=1}^N p_i e_i^T(t) \vartheta_i(t) \\ &\quad - c \sum_{i=1}^N p_i e_i^T(t) \beta_i(t) - \sum_{i=1}^N \alpha_i \eta_i(t), \end{aligned} \tag{13}$$

moreover, for any $\theta > 0$,

$$\begin{aligned} &-c \sum_{i=1}^N p_i e_i^T(t) \vartheta_i(t) \\ &\leq \frac{c}{2} [\theta \sum_{i=1}^N p_i^2 e_i^T(t) e_i(t) + \frac{1}{\theta} \sum_{i=1}^N \vartheta_i^T(t) \vartheta_i(t)], \end{aligned} \tag{14}$$

and according to the event-triggered conditions (4), (5) and (10), we assume that

$$\sum_{i=1}^N \vartheta_i^T(t) \vartheta_i(t) \leq \sum_{i=1}^N (\bar{\sigma} \beta_i^T(t) \beta_i(t) + \eta_i(t)), \tag{15}$$

where σ_i in (10) satisfies $\sigma_i \leq \bar{\sigma}_1 < \bar{\sigma}$. It is worth pointing out that $\sum_{i=1}^N \vartheta_i^T(t) \vartheta_i(t) \leq \sum_{i=1}^N (\bar{\sigma} \beta_i^T(t) \beta_i(t) + \eta_i(t))$ is employed, instead of $\sum_{i=1}^N \vartheta_i^T(t) \vartheta_i(t) \leq$

$\sum_{i=1}^N (\sigma_i \beta_i^T(t) \beta_i(t) + \eta_i(t))$, since the triggering mechanism is given as $t_{q+1}^i = t_q^i + \max\{\delta_q^i, \varepsilon_i\}$ rather than $t_{q+1}^i = t_q^i + \delta_q^i$ for avoiding Zeno behavior.

In addition, it can be verified that $\beta_i(t) = \sum_{j=1}^N h_{ij} \times e_j(t)$ with h_{ij} being the element of matrix H , therefore,

$$-c \sum_{i=1}^N p_i e_i^T(t) \beta_i(t) = -ce^T(t) (PH \otimes I_m) e(t), \tag{16}$$

where $e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T \in \mathbb{R}^{Nm}$.

Based on (13)–(16), one can get

$$\begin{aligned} \dot{V}(t) &\leq (\gamma \bar{p} + \frac{c}{2} \theta \bar{p}^2 - c\kappa_1) e^T(t) e(t) \\ &\quad + \frac{c}{2\theta} \sum_{i=1}^N (\bar{\sigma} \beta_i^T(t) \beta_i(t) + \eta_i(t)) - \sum_{i=1}^N \alpha_i \eta_i(t), \end{aligned} \tag{17}$$

where $\bar{p} = \max\{p_1, p_2, \dots, p_N\}$, and the parameter $\kappa_1 = \min_{i=1,2,\dots,N} \{\lambda_i(\frac{PH+H^T P}{2})\}$, which denotes the minimum eigenvalue of $\frac{PH+H^T P}{2}$. Obviously, if the coupling strength $c > \frac{2\gamma \bar{p}}{2\kappa_1 - \theta \bar{p}^2}$, where $0 < \theta < \frac{2\kappa_1}{\bar{p}^2}$, i.e., the condition (i) in Theorem 1 is satisfied, $\gamma \bar{p} + \frac{c}{2} \theta \bar{p}^2 - c\kappa_1 < 0$ holds.

Furthermore, due to $\sum_{i=1}^N \beta_i^T(t) \beta_i(t) = e^T(t) (H^T H \otimes I_m) e(t)$, then

$$\begin{aligned} \dot{V}(t) &\leq (\gamma \bar{p} + \frac{c}{2} \theta \bar{p}^2 - c\kappa_1 + \frac{c}{2\theta} \bar{\sigma} \kappa_2) e^T(t) e(t) \\ &\quad + \sum_{i=1}^N (\frac{c}{2\theta} - \alpha_i) \eta_i(t), \end{aligned} \tag{18}$$

where $\kappa_2 = \max_{i=1,2,\dots,N} \{\lambda_i(H^T H)\}$. Therefore, if the conditions (ii) and (iii) in Theorem 1 are guaranteed, $\gamma \bar{p} + \frac{c}{2} \theta \bar{p}^2 - c\kappa_1 + \frac{c}{2\theta} \bar{\sigma} \kappa_2 < 0$, and $\frac{c}{2\theta} - \alpha_i < 0$. Obviously, one has

$$\dot{V}(t) \leq -\rho_1 V(t), \tag{19}$$

where $-\rho_1 = \max_{i=1,2,\dots,N} \left\{ \frac{2\gamma \bar{p} + c\theta \bar{p}^2 - 2c\kappa_1 + \frac{c}{\theta} \bar{\sigma} \kappa_2}{\bar{p}}, \frac{c}{2\theta} - \alpha_i \right\} < 0$.

When $t \in \Delta^f(t_0, t_1)$, according to (11) and (6), one has

$$\dot{V}(t) = \sum_{i=1}^N p_i e_i^T(t) (f(t, x_i(t)) - f(t, x_0(t))) \leq \rho_2 V(t), \tag{20}$$

where $\rho_2 = 2\gamma > 0$.

Therefore, $\dot{V}(t) \leq -\rho_1 V(t)$ holds when $t \in \Delta^w(t_0, t_1)$, while $\dot{V}(t) \leq \rho_2 V(t)$ holds for $t \in \Delta^f(t_0, t_1)$, moreover, $V(t)$ is a continuous function, then for any $t \leq t_1$,

$$V(t) \leq V(t_0) \exp\{-\rho_1 |\Delta^w(t_0, t_1)|\} \exp\{\rho_2 |\Delta^f(t_0, t_1)|\}, \tag{21}$$

where $|\Delta^w(t_0, t_1)|$ and $|\Delta^f(t_0, t_1)|$ indicate the length of time with or without control protocols, respectively. It is obvious that if $-\rho_1 |\Delta^w(t_0, t_1)| + \rho_2 |\Delta^f(t_0, t_1)| < -\epsilon(t_1 - t_0)$ with ϵ being a small positive constant, then $\lim_{t \rightarrow \infty} V(t) = 0$. On the other hand, $-\rho_1 |\Delta^w(t_0, t_1)| + \rho_2 |\Delta^f(t_0, t_1)| < -\epsilon(t_1 - t_0)$ is ensured by $|\Delta^f(t_0, t_1)| < \frac{\rho_1}{\rho_1 + \rho_2} (t_1 - t_0)$ due to $|\Delta^w(t_0, t_1)| = t_1 - t_0 - |\Delta^f(t_0, t_1)|$. Furthermore, if Assumption 3 is satisfied, then $T^f > \frac{\rho_1 + \rho_2}{\rho_1}$, i.e., the condition (iv) in Theorem 1, can ensure $|\Delta^f(t_0, t_1)| < \frac{\rho_1}{\rho_1 + \rho_2} (t_1 - t_0)$.

On the other hand, $\lim_{t \rightarrow \infty} V(t) = 0$ implies that $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ and $\lim_{t \rightarrow \infty} \eta_i(t) = 0$, moreover, $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ means that $\lim_{t \rightarrow \infty} \|x_i(t) - x_0(t)\| = 0$, in other words, the consensus of the multi-agent system (1) is realized.

It should be noted that $\sum_{i=1}^N \vartheta_i^T(t) \vartheta_i(t) \leq \sum_{i=1}^N (\bar{\sigma} \beta_i^T(t) \beta_i(t) + \eta_i(t))$ as (15) plays an essential role. As described above, the event-triggered instant satisfies $t_{q+1}^i = t_q^i + \max\{\delta_q^i, \varepsilon_i\}$, where δ_q^i is defined as $\delta_q^i = \inf_{t > t_q^i, t \in \Delta^w(t_0, t_1)} \{t - t_q^i \mid \|\vartheta_i(t)\|^2 \geq \sigma_i \|\beta_i(t)\|^2 + \eta_i(t)\}$ based on (5) and (10). Accordingly, divide the N followers into two non-overlapping parts $\Omega_1(t)$ and $\Omega_2(t)$ at time t , which satisfy that $t_{q+1}^i = t_q^i + \delta_q^i$ for $i \in \Omega_1(t)$, and $t_{q+1}^i = t_q^i + \varepsilon_i$ for $i \in \Omega_2(t)$.

Moreover, for $i \in \Omega_1(t)$, as soon as $\|\vartheta_i(t)\|^2 \geq \sigma_i \|\beta_i(t)\|^2 + \eta_i(t)$ holds, a new event is triggered, then $\vartheta_i(t)$ is set to be a zero vector, which implies that

$$\begin{aligned} \sum_{i \in \Omega_1(t)} \|\vartheta_i(t)\|^2 &\leq \sum_{i \in \Omega_1(t)} (\sigma_i \|\beta_i(t)\|^2 + \eta_i(t)) \\ &\leq \sum_{i=1}^N (\bar{\sigma}_1 \|\beta_i(t)\|^2 + \eta_i(t)) \end{aligned} \tag{22}$$

due to $\sigma_i \leq \bar{\sigma}_1$. Then, if the following (23) holds for $i \in \Omega_2(t)$, (15) can be guaranteed.

$$\sum_{i \in \Omega_2(t)} \|\vartheta_i(t)\|^2 \leq \sum_{i=1}^N \bar{\sigma}_2 \|\beta_i(t)\|^2, \tag{23}$$

where $\bar{\sigma}_2 = \bar{\sigma} - \bar{\sigma}_1$. For each $i \in \Omega_2(t)$, $\|\vartheta_i(t)\| \leq \sqrt{\frac{\bar{\sigma}_2}{N}} \|\beta(t)\|$ is a sufficient condition for (23), where $\beta(t) = (\beta_1^T(t), \beta_2^T(t), \dots, \beta_N^T(t))^T$. Therefore, the parameter ε_i corresponds to the time interval from 0 to $\sqrt{\frac{\bar{\sigma}_2}{N}}$ for $\|\vartheta_i(t)\|/\|\beta(t)\|$. Analysis of the time interval is given as follows.

Due to $\beta(t) = (H \otimes I_m) e(t)$, based on Lemma 2, one gets $e(t) = (H^{-1} \otimes I_m) \beta(t)$, then from (11),

$$\begin{aligned} \|\dot{\beta}(t)\| &\leq \|H \otimes I_m\| \|\dot{e}(t)\| \\ &\leq \|H\| (\gamma \|H^{-1}\| \|\beta(t)\| + cN \|\vartheta_i(t)\| + c \|\beta(t)\|). \end{aligned} \tag{24}$$

On the other hand,

$$\begin{aligned} \frac{d}{dt} \left(\frac{\|\vartheta_i(t)\|}{\|\beta(t)\|} \right) &= \frac{d}{dt} \left[\frac{(\vartheta_i^T(t) \vartheta_i(t))^{1/2}}{(\beta^T(t) \beta(t))^{1/2}} \right] \\ &= \frac{(\vartheta_i^T \dot{\vartheta}_i) - 1/2 \vartheta_i^T \dot{\vartheta}_i (\beta^T \dot{\beta})^{1/2}}{\|\beta(t)\|^2} \\ &\quad - \frac{(\vartheta_i^T \vartheta_i)^{1/2} (\beta^T \dot{\beta})^{-1/2} \beta^T \dot{\beta}}{\|\beta(t)\|^2} \\ &\leq \frac{\|\dot{\vartheta}_i(t)\|}{\|\beta(t)\|} + \frac{\|\vartheta_i(t)\| \|\dot{\beta}(t)\|}{\|\beta(t)\|^2}, \end{aligned} \tag{25}$$

where $\|\dot{\vartheta}_i(t)\| = \|\dot{\beta}_i(t)\| \leq \|\dot{\beta}(t)\|$. Owing to $\|\vartheta_i(t)\| \leq \sqrt{\frac{\bar{\sigma}_2}{N}} \|\beta(t)\|$ and (24), one can obtain

$$\frac{d}{dt} \left(\frac{\|\vartheta_i(t)\|}{\|\beta(t)\|} \right) \leq \varpi \left(1 + \frac{\|\vartheta_i(t)\|}{\|\beta(t)\|} \right), \tag{26}$$

where $\varpi = \gamma \|H\| \|H^{-1}\| + c \|H\| \sqrt{\bar{\sigma}_2 N} + c \|H\|$. Consequently, $\frac{\|\vartheta_i(t)\|}{\|\beta(t)\|}$ satisfies the bound of $\frac{\|\vartheta_i(t)\|}{\|\beta(t)\|} \leq \varphi(t)$, where $\varphi(t)$ denotes the solution of $\dot{\varphi} = \varpi(1 +$

φ) with $\varphi(0) = 0$, then $t = \frac{1}{\varpi} \ln \left(\sqrt{\frac{\varpi_2}{N}} + 1 \right)$ is the value for $\varphi(t) = \sqrt{\frac{\varpi_2}{N}}$. Accordingly, if the parameter ε_i satisfies the condition (v) in Theorem 1, i.e., $0 < \varepsilon_i \leq \frac{1}{\varpi} \ln \left(\sqrt{\frac{\varpi_2}{N}} + 1 \right)$, then (23) holds, which means that (15) can be ensured. This completes the proof. \square

Remark 6 In the proof of Theorem 1, there are two main steps, one is stability analysis of the error system, and the other is selection of the time interval for excluding Zeno behavior. The conditions proposed in Theorem 1 include the coupling strength c in control protocol, the parameters σ_i, α_i , and ε_i in dynamic event-triggered mechanism, and the requirement of T^f for intermittent framework.

In order to illustrate the protocols clearly, Algorithm 1 is provided.

3.2 Protocol with a single measurement

It should be noted that in the previous subsection, $\vartheta_i(t)$ in the event-triggered condition is defined as $\beta_i(t_q^i) - \beta_i(t)$, which is taken as combined measurements, and the control protocol $u_i(t)$ is given as (9), consequently, at the triggering instant t_q^i of the i th agent, the states of all the agents should be available. This subsection presents another scheme, where the protocol is described as

$$u_i(t) = \begin{cases} -c \sum_{j=1}^N l_{ij} x_j(t_{q'}^j) \\ +cb_i(x_0 - x_i(t_q^i)), & t \in \Delta^w(t_0, t_1), \\ 0, & t \in \Delta^f(t_0, t_1), \end{cases} \quad (27)$$

where $t_{q'}^j \leq t$ and $t_q^i \leq t$ denote the latest event-triggered instant of the j th and i th agents, respectively. Moreover, the leader is supposed to satisfy $f(t, x_0(t)) = 0$, that is, the state x_0 is an equilibrium point. If the consensus is realized, then $\lim_{t \rightarrow \infty} \|x_i(t) - x_0\| = 0$ holds, which was also investigated in [45] by utilizing continuous feedback control.

Remark 7 Compared with (9), the control protocol $u_i(t)$ as (27) requires the information $x_j(t_{q'}^j)$ of the j th agent at the latest event-triggered instant $t_{q'}^j$, rather than t_q^i , where $t_{q'}^j$ and t_q^i can be non-identical. Consequently, for the i th agent, $u_i(t)$ will be updated at $t_{q'}^j$ besides t_q^i .

Algorithm 1 Intermittent dynamic event-triggered control protocol for the consensus

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1: Initialize:  $t_0, t_1, c, \sigma_i, \alpha_i, \varepsilon_i, T^f, \eta_i(t_0), x_i(t_0), x_0(t_0), t_1^i = t_0,$ 
    $q = 1, \Delta^w(t_0, t_1).$ 
2: if  $t \in \Delta^w(t_0, t_1)$  then
3:   for each  $i \in \{1, 2, \dots, N\}$  do
4:     Update  $u_i(t): u_i(t) = c \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) +$ 
        $cb_i(\hat{x}_0(t) - \hat{x}_i(t));$ 
5:     Calculate  $x_i(t): \dot{x}_i(t) = f(t, x_i(t)) + u_i(t);$ 
6:     Calculate  $\eta_i(t): \dot{\eta}_i(t) = -\alpha_i \eta_i(t);$ 
7:   end for
8:   Calculate  $x_0(t): \dot{x}_0(t) = f(t, x_0(t));$ 
9:   for each  $i \in \{1, 2, \dots, N\}$  do
10:    Calculate  $\beta_i(t): \beta_i(t) = \sum_{j=1}^N l_{ij} x_j(t) - b_i(x_0(t) -$ 
       $x_i(t));$ 
11:    Calculate  $\vartheta_i(t): \vartheta_i(t) = \beta_i(t_q^i) - \beta_i(t);$ 
12:    if  $t > t_q^i$  and  $\|\vartheta_i(t)\|^2 - \sigma_i \|\beta_i(t)\|^2 \geq \eta_i(t)$  then
13:       $\delta_q^i = t - t_q^i;$ 
14:      if  $\delta_q^i \geq \varepsilon_i$  then
15:         $t_{q+1}^i = t_q^i + \delta_q^i;$ 
16:      else
17:         $t_{q+1}^i = t_q^i + \varepsilon^i;$ 
18:      if  $t_q^i + \varepsilon^i \notin \Delta^w(t_0, t_1)$  then
19:         $t_{q+1}^i = \min_{T_k + \tau_k \geq t_q^i + \varepsilon^i} \{T_k + \tau_k\};$ 
20:      end if
21:    end if
22:     $q = q + 1;$ 
23:  end if
24:  for each  $i \in \{0, 1, 2, \dots, N\}$  do
25:    Update  $\hat{x}_i(t)$  based on  $t_q^i;$ 
26:  end for
27: end for
28: else
29:   for each  $i \in \{0, 1, 2, \dots, N\}$  do
30:    Calculate  $x_i(t): \dot{x}_i(t) = f(t, x_i(t));$ 
31:   end for
32: end if

```

Furthermore, let $\vartheta_i(t) = x_i(t_q^i) - x_i(t)$, which is a measurement depending on its own information for the i th agent, and the function $g_i(t)$ is given as

$$g_i(t) = \|x_i(t_q^i) - x_i(t)\|^2 - \sigma_i \|\beta_i(t)\|^2, \quad (28)$$

where $i = 1, 2, \dots, N$, $\beta_i(t)$ is defined the same as in (10).

Similarly, denote the consensus error to be $e_i(t) = x_i(t) - x_0$ ($i = 1, 2, \dots, N$), based on the multi-agent system (1), the control protocol (27), and $\vartheta_i(t) = x_i(t_q^i) - x_i(t)$, one has

$$\dot{e}_i(t) = \begin{cases} f(t, x_i(t)) - f(t, x_0(t)) \\ -c \sum_{j=1}^N h_{ij}(\vartheta_j(t) + e_j(t)), \quad t \in \Delta^w(t_0, t_1), \\ f(t, x_i(t)) - f(t, x_0(t)), \quad t \in \Delta^f(t_0, t_1). \end{cases} \tag{29}$$

It should be noted that if the leader is not supposed to be static, then the item $cb_i(x_0 - x_i(t_q^i))$ in Eq. (27) should be changed into $cb_i(x_0(t_q^i) - x_i(t_q^i))$. Correspondingly, we cannot get the consensus error system (29). Therefore, the leader is supposed to be static. Conditions guaranteeing the consensus are given in Theorem 2, which is similar to Theorem 1.

Theorem 2 *The leader-following multiagent system (1) under the intermittent control protocol (27) with the dynamic event-triggered mechanism, i.e., described as (4)–(6) and (28), will realize the consensus if Assumptions 1–3 and the following conditions hold.*

- (i) *The coupling strength $c > \frac{2\gamma\bar{p}}{2\kappa_1 - \theta\kappa_3}$, where $0 < \theta < \frac{2\kappa_1}{\kappa_3}$, $\bar{p} = \max\{p_1, p_2, \dots, p_N\}$, $\kappa_1 = \min_{i=1,2,\dots,N} \{\lambda_i(\frac{PH+H^T P}{2})\}$, $\kappa_3 = \max_{i=1,2,\dots,N} \{\lambda_i(PH H^T P)\}$.*
- (ii) *The parameter σ_i in (28) satisfies $0 < \sigma_i \leq \bar{\sigma}_1 < \bar{\sigma}$ for $i = 1, 2, \dots, N$, where $\bar{\sigma} < \frac{2\theta c\kappa_1 - 2\theta\gamma\bar{p} - c\theta^2\kappa_3}{c\kappa_2}$, and $\kappa_2 = \max_{i=1,2,\dots,N} \{\lambda_i(H^T H)\}$.*
- (iii) *The parameter α_i in (6) satisfies $\alpha_i > \frac{c}{2\bar{\theta}}$ for $i = 1, 2, \dots, N$.*
- (iv) *The parameter T^f in Assumption 3 satisfies $T^f > \frac{\rho_1 + \rho_2}{\rho_1}$, where $\rho_1 = -\max_{i=1,2,\dots,N} \{\frac{c}{2\bar{\theta}} - \alpha_i, \frac{2\gamma\bar{p} + c\theta\kappa_3 - 2c\kappa_1 + \frac{c}{\bar{\theta}}\bar{\sigma}\kappa_2}{\bar{p}}\}$, and $\rho_2 = 2\gamma$.*
- (v) *The parameter ε_i in (4) satisfies $0 < \varepsilon_i \leq \frac{1}{\bar{\omega}} \times \ln\left(\frac{1}{\|H^{-1}\|} \sqrt{\frac{\bar{\sigma}_2}{N}} + 1\right)$, where $\bar{\omega} = \gamma\|H\|\|H^{-1}\| + c\|H\|^2\sqrt{\bar{\sigma}_2 N} + c\|H\|$, and $\bar{\sigma}_2 = \bar{\sigma} - \bar{\sigma}_1$.*

Proof Choose the Lyapunov function the same as (12), when $t \in \Delta^w(t_0, t_1)$, based on the error system (29), one can get

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N p_i e_i^T(t)(f(t, x_i(t)) - f(t, x_0(t))) \\ &\quad - c \sum_{j=1}^N h_{ij}(\vartheta_j(t) + e_j(t)) + \sum_{i=1}^N (-\alpha_i \eta_i(t)) \\ &\leq \gamma \sum_{i=1}^N p_i e_i^T(t) e_i(t) - c \sum_{i=1}^N p_i e_i^T(t) \sum_{j=1}^N h_{ij} \vartheta_j(t) \\ &\quad - c \sum_{i=1}^N p_i e_i^T(t) \beta_i(t) - \sum_{i=1}^N \alpha_i \eta_i(t), \end{aligned} \tag{30}$$

moreover, for any $\theta > 0$,

$$\begin{aligned} &-c \sum_{i=1}^N p_i e_i^T(t) \sum_{j=1}^N h_{ij} \vartheta_j(t) \\ &= -ce^T(t)(PH \otimes I_m)\vartheta(t) \\ &\leq \frac{c}{2} [\theta\kappa_3 \sum_{i=1}^N e_i^T(t) e_i(t) + \frac{1}{\theta} \sum_{i=1}^N \vartheta_i^T(t) \vartheta_i(t)], \end{aligned} \tag{31}$$

where $\vartheta(t) = (\vartheta_1^T(t), \vartheta_2^T(t), \dots, \vartheta_N^T(t))^T$, and the parameter $\kappa_3 = \max_{i=1,2,\dots,N} \{\lambda_i(PH H^T P)\}$. The following proof is similar to that of Theorem 1, then the conditions (i)–(iv) of Theorem 2 can be obtained.

Next, since $\vartheta_i(t)$ is changed in the event-triggered mechanism, the parameter ε_i in (4) needs to be reselected. Based on (25) in Theorem 1, one can get that

$$\frac{d}{dt} \left(\frac{\|\vartheta_i(t)\|}{\|\beta(t)\|} \right) \leq \frac{\|\dot{\vartheta}_i(t)\|}{\|\beta(t)\|} + \frac{\|\vartheta_i(t)\| \|\dot{\beta}(t)\|}{\|\beta(t)\|^2}$$

still holds. Meanwhile, owing to $\vartheta_i(t) = x_i(t_q^i) - x_i(t)$, one has $\|\dot{\vartheta}_i(t)\| = \|\dot{x}_i(t)\| = \|\dot{e}_i(t)\| \leq \|H^{-1}\| \|\beta(t)\|$, and

$$\begin{aligned} \|\dot{\beta}(t)\| &\leq \|H \otimes I_m\| \|\dot{e}(t)\| \\ &\leq \|H\|(\gamma\|H^{-1}\| \|\beta(t)\| + cN\|H\| \|\vartheta_i(t)\| \\ &\quad + c\|\beta(t)\|). \end{aligned} \tag{32}$$

Then, we have

$$\frac{d}{dt} \left(\frac{\|\vartheta_i(t)\|}{\|\beta(t)\|} \right) \leq \varpi \left(\|H^{-1}\| + \frac{\|\vartheta_i(t)\|}{\|\beta(t)\|} \right), \tag{33}$$

where $\varpi = \gamma\|H\|\|H^{-1}\| + c\|H\|^2\sqrt{\bar{\sigma}_2 N} + c\|H\|$. Consequently, $\frac{\|\vartheta_i(t)\|}{\|\beta(t)\|}$ satisfies the bound of $\frac{\|\vartheta_i(t)\|}{\|\beta(t)\|} \leq$

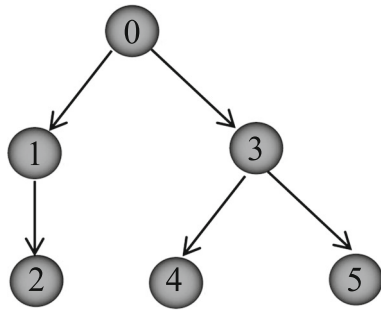


Fig. 3 The graph for communication topology of the multiagent system

$\varphi(t)$, where $\varphi(t)$ denotes the solution of $\dot{\varphi} = \varpi(\|H^{-1}\| + \varphi)$ with $\varphi(0) = 0$, then the value for $\varphi(t) = \sqrt{\frac{\sigma_2}{N}}$ is $t = \frac{1}{\varpi} \ln\left(\frac{1}{\|H^{-1}\|} \sqrt{\frac{\sigma_2}{N}} + 1\right)$. Therefore, the parameter ε_i should satisfy the condition (v) in Theorem 2, i.e., $0 < \varepsilon_i \leq \frac{1}{\varpi} \ln\left(\frac{1}{\|H^{-1}\|} \sqrt{\frac{\sigma_2}{N}} + 1\right)$. This completes the proof. \square

Remark 8 $\vartheta_i(t)$ depends on the single measurement as $x_i(t_q^i) - x_i(t)$, then it is not required to save the information $x_j(t_q^i)$. However, the control protocol $u_i(t)$ as (27) may be updated more frequently than that as (9). The corresponding algorithm can be obtained according to Algorithm 1, thus the detail is omitted here.

4 Numerical simulations

This section demonstrates specific numerical examples for illustrating the consensus of multiagent systems via intermittent dynamic event-triggered control protocols.

Consider that a multiagent system consists of five followers ($N = 5$) and one leader, where the communication topology is shown in Fig. 3, obviously, the leader 0 has a directed path to every follower. Moreover, suppose that $f(t, x_i(t)) = 0.05 \sin(x_i(t))$, the weights of edges equal one and $x_i(t) \in \mathbb{R}$ for simplicity, where $i = 0, 1, \dots, 5$.

According to Fig. 3, the matrix H is represented

as $H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}$, then we have the matrix $P = \text{diag}\{0.5, 0.25, 0.75, 0.25, 0.25\}$ based on Lemma 2.

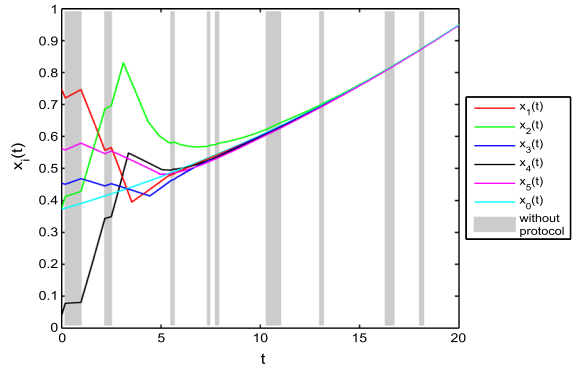


Fig. 4 States $x_i(t)$ of the agents in Example 1

Example 1 First, the event-triggered mechanism is designed as (4)–(6) and (10), where $\vartheta_i(t) = \beta_i(t_q^i) - \beta_i(t)$ denotes combined measurements. The control protocol $u_i(t)$ is given as (9). Correspondingly, the parameters can be obtained through calculation, where $\bar{p} = 0.75$, $\kappa_1 = 0.1938$, $\kappa_2 = 3.7321$. Therefore, in order to satisfy the conditions (i)–(v) of Theorem 1, for $i = 1, 2, \dots, 5$, choose $\theta = 0.1$, the coupling strength $c = 0.5$ in the control protocol, $\bar{\sigma} = 0.004$ and $\sigma_i = 0.0035$ in the event-triggered condition, $\alpha_i = 2.6$ for $\eta_i(t)$, $T^f = 5.7$ ($\rho_1 = 0.0214$ and $\rho_2 = 0.1$) in Assumption 3 for intermittent framework, and $\varepsilon_i = 0.008$ ($\varpi = 1.2009$) for Zeno-free scheme. Additionally, assume that $t_0 = 0$, $t_1 = 20s$, the initial conditions $x_i(0)$ are randomly generated, and $\eta_i(0) = 100$. By employing Algorithm 1, the numerical results are presented in Figs. 4, 5, and 6 and Table 1.

In Fig. 4, the states of the followers and leader are provided. Obviously, the followers track the trajectory of the leader, which shows that the consensus is achieved based on Definition 1. Additionally, the time intervals without control protocol are also given, where the length of the time intervals $|\Delta^f(0, 20)| = 3.0191$, then Assumption 3 is satisfied.

Figure 5 demonstrates the intermittent control protocols $u_i(t)$, where $u_i(t) = 0$ during $\Delta^f(0, 20)$. Furthermore, as the consensus realizes, all the protocols converge to zero. Particularly, the protocols are updated discretely owing to the event-triggered mechanism.

In order to display the discrete instants when the protocols are updated, Fig. 6 supplies the event-triggered instants, i.e., t_q^i ($q = 1, 2, \dots; i = 1, 2, \dots, 5$), which implies that the mechanism greatly reduces the number of the control updates. On the other hand, the

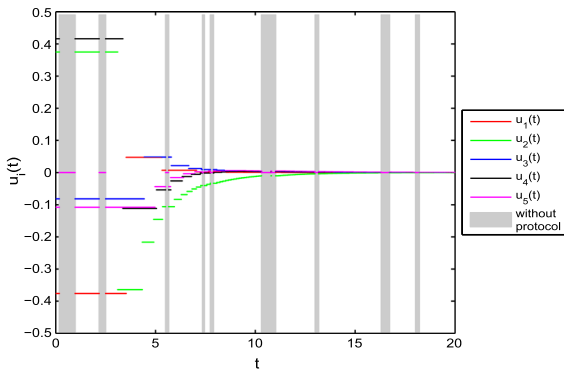


Fig. 5 Intermittent control protocols $u_i(t)$ in Example 1

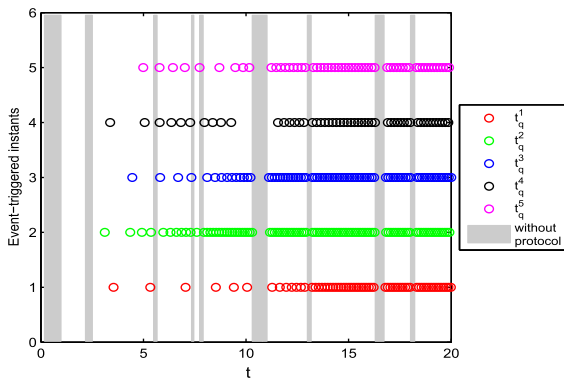


Fig. 6 Event-triggered instants t_q^i in Example 1

Table 1 Event-triggered times in the static and dynamic strategies

Agent i	1	2	3	4	5
Static strategy	139	466	138	300	103
Dynamic strategy	64	100	81	54	58

event-triggered instants of different agents can be non-identical, and Zeno behavior is excluded due to the minimum time interval $\varepsilon_i > 0$.

Moreover, for the event-triggered times during $[0, 20s)$, Table 1 provides a comparison between two cases, i.e., in the condition (5), $\eta_i(t) \equiv 0$ and $\eta_i(t)$ is updated with the law (6), which correspond to the static and dynamic event-triggered strategies, respectively. Clearly, the times in the dynamic strategy are much less than those in the static situation.

Example 2: Second, the event-triggered mechanism is designed as (4)–(6) and (28), where $\vartheta_i(t) = x_i(t_q^i) - x_i(t)$ denotes a single measurement. The control pro-

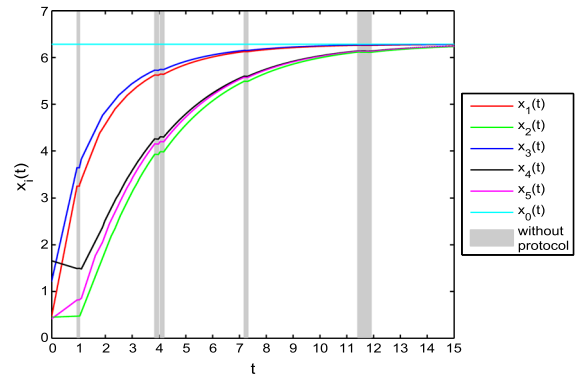


Fig. 7 States $x_i(t)$ of the agents in Example 2

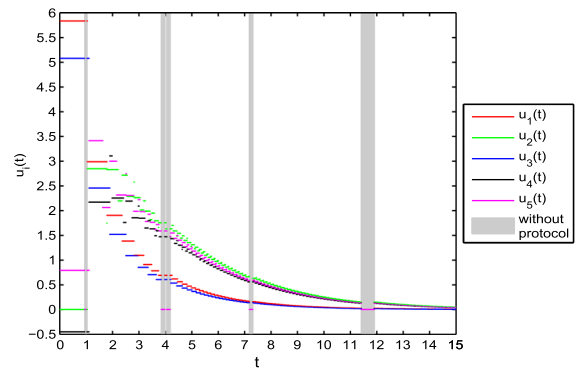


Fig. 8 Intermittent control protocols $u_i(t)$ in Example 2

ocol $u_i(t)$ is given as (27). The parameters $\bar{p} = 0.75$, $\kappa_1 = 0.1938$, $\kappa_2 = 3.7321$, and $\kappa_3 = 0.6998$, then if select $c = 0.5$, $\bar{\sigma} = 0.004$, $\sigma_i = 0.0035$, and $\alpha_i = 2.6$, which are the same as in Example 1, the conditions (i)–(iii) of Theorem 2 are also satisfied. Besides, the conditions (iv) and (v) can be guaranteed by $T^f = 9.2$ ($\rho_1 = 0.0122$, $\rho_2 = 0.1$) and $\varepsilon_i = 0.004$ ($\varpi = 1.2459$). The initial conditions $x_i(0)$ except $x_0(0)$ are randomly generated, while $x_0(0) = 2\pi$, and $\eta_i(0) = 100$. $t_0 = 0$, and $t_1 = 15s$. The corresponding results for illustrating Theorem 2 are given in Figs. 7, 8, and 9 and Table 2. On the other hand, since the parameter T^f in this example is larger than that in Example 1, the length without control $|\Delta^f(0, 15)| = 0.9712$.

Figure 7 presents the states of the followers and leader, which implies that the consensus is realized. Compared with Fig. 4 in Example 1, the state of the leader in Fig. 7 is a constant for ensuring $f(t, x_0(t)) = 0$. In Fig. 8, the control protocols as (27) are given, and it can be seen that the protocols are updated more frequently than those in Fig. 5, since for the i th agent,

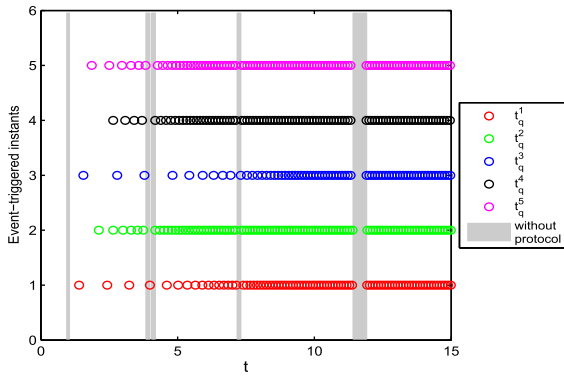


Fig. 9 Event-triggered instants t_q^i in Example 2

Table 2 Event-triggered times in the static and dynamic strategies

Agent i	1	2	3	4	5
Static strategy	138	136	138	146	138
Dynamic strategy	86	105	74	100	102

besides its own triggering instants t_q^i , $u_i(t)$ is updated at the triggering instants of its neighbors t_q^j , which is different from the protocols in Example 1.

Figure 9 shows the event-triggered instants produced by the mechanism (4)–(6) and (28). Furthermore, as presented in Table 2, during [0, 15s), the event-triggered times of the agents with dynamic strategy are 86, 105, 74, 100, and 102, respectively, which implies that the dynamic event-triggered approach reduces the update frequency more effectively compared with the static scheme.

In the two examples above, for simplicity, the agents are assumed to move on the one-dimensional space, i.e., $x_i(t) \in \mathbb{R}$. It should be noted that for the physical systems moving on higher-dimensional space, if the conditions in the theorems can be guaranteed, the designed control algorithm should also be applicable.

5 Conclusion and discussion

In order to realize the consensus of nonlinear multiagent systems, this paper has investigated how to improve control protocols. On the one hand, suppose that the protocols are intermittent. On the other hand, event-triggered scheme has been utilized and developed for reducing the update frequency of control. In

particular, the event-triggered strategies under intermittent framework are not only dynamic but also Zeno-free. Two forms of the event-triggered protocols have been provided and discussed, where combined measurements and a single measurement have been adopted in the event-triggered conditions. Correspondingly, we put forward the sufficient conditions for the consensus via the proposed schemes, and designed the concrete algorithm. It can be found that the conditions are related to the coupling strength, parameters in the event-triggered mechanism, and the length of time without control. However, we focused on the fixed communication topology rather than switching topologies, since both the event-triggered mechanism and intermittent framework have been considered, then if the topologies are supposed to be switching, the exclusion of Zeno behavior needs further analysis. Furthermore, the consensus conditions depend on the communication topology, i.e., the matrix H . Therefore, only the fixed communication topology has been studied, and the case under switching topologies is a problem to be solved.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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