Nonlinear Dyn (2021) 103:1627–1644
https://doi.org/10.1007/s11071-020-06188-z

ORIGINAL PAPER

A homogeneous domination output feedback control method for active suspension of intelligent electric vehicle

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Received: 21 August 2020 / Accepted: 29 December 2020 / Published online: 12 January 2021

Abstract An active suspension of an intelligent electric vehicle driven by four in-wheel motors (IEV-DFIM) is a strong nonlinear system because of time-varying parameters in practice, which causes difficult controllability. For addressing this issue, the paper proposes a novel homogeneous output feedback control method. Firstly, an active suspension dynamic model which considers the time-varying sprung mass, stiffness coefficients and damping coefficients is built. Secondly, an active suspension control system is constructed based on the dynamic model whose uncertain and nonlinear terms do not meet the linear or high-order growing. Thirdly, the homogeneous output feedback method is developed to relax the growth condition imposed on the uncertain and nonlinear terms for the active suspension. Finally, the simulation and test are carried out to verify the effectiveness of the designed controller compared with the sliding mode control method and passive suspension.

Keywords Homogeneous output feedback control · Nonlinear control · Active suspension · Intelligent vehicle

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1 Introduction

Intelligent vehicles have attracted more and more attention due to their great advancements in enhancing vehicle safety and performance as well as traffic efficiency in the recent years [13,14,25,43,46,51]. On the other hand, the electric vehicle driven by four in-wheels motors are easy to be intelligentized because of their features [7,30,31]. Therefore, the research about the intelligent electric vehicles driven by four in-wheel motors (IEV-DFIM) will attract more and more researchers [19,20,36]. Generally speaking, control methods are the keys of the IEV-DFIM.

The suspension system is very important for the riding comfort and maneuverability of the IEV-DFIM. For the IEV-DEIM, the in-wheel motor increases the unsprung mass that influences the riding comfort and maneuverability of the vehicle, and also causes the nonlinearity and uncertainty. In order to improve the riding comfort and maneuverability of the vehicle, the active suspension is necessary. For the active suspension, the control method is crucial, which decides the performance of the suspension directly. Many researchers have studied the issue widely, and presented some control methods via modern control theory. Now the active suspension control systems mainly adopt H ∞ control method [3,49,53], explicit MPC-based RBF neural network method [6], sliding mode control method [24, 26,34], rule-based control method [44], bioinspired dynamics-based adaptive tracking control method [35], robust and linear-parameter varying methods [33], neural network control method [22,27], and adaptive fuzzy control method [21,23], et. al. New control methods such as sampled-data control method [29] and homogeneous control method [28] were also developed for the active suspension.

The proposed control methods in the aforementioned literature need at least two sensors on each actuator to obtain the kinematic parameters of the active suspension. It means that the most active suspension control methods are state feedback control. But more sensors increase the cost of the active suspension which limits the application widely in practice. In order to reduce the cost, i.e. to reduce the number of sensors in the active suspension, it is necessary to design new control methods depending on less sensors while ensuring the riding comfort and maneuverability. This kind control method can be realized via output feedback control. Researchers have proposed some effective methods in this respect such as H_∞ output feedback control methods [45], sliding mode control method [8,42,56], and other new method [1,12,47]. It should be noted that the parameters of the active suspension models used in these control methods are dealt as constants for controlling easily. In fact, the sprung mass, stiffness coefficients and damping coefficients are time-varying.

Generally speaking, the most control methods of the active suspension are developed based on the linear growth condition. In engineering, the active suspension control system can not meet the linear growth condition strictly. In the decades years, a homogeneous domination approach is presented and developed rapidly which can solve this problem. The approach has strong robustness, and can handle the linear or nonlinear terms of control systems which do not require the restrict linear growth condition [10,32,37,39,41]. Some researchers have studied the homogeneous control method in theory. For example, an output feedback controller based on homogeneous domination was designed for upper-triangular systems via adding a power integrator method [55]. Furthermore, a new design method based on adding a power integrator and the interval homogeneous domination approach for a kind of nonlinear systems with time-varying powers was proposed in [5]. And a homogeneous domination idea and recursive design were used to design a continuous adaptive state-feedback controller to ensure the time-delay systems being globally stable [48]. A Lyapunov-based homogeneous controller was designed to stabilize a perturbed chain of integrators of arbitrary order [16]. Based on the aforementioned researches, the homogeneous control method was developed further. In paper [52], a generalized homogeneous adaptive stabilization method based on a general approximated homogeneous function restraining hypothesis was developed for a class of high order nonlinear systems without controllable/observable linearization. A state feedback control strategy is presented based on homogeneous domination for a kind of high-order uncertain nonlinear systems with multiple time-delays in [54]. In [4], a homogeneous state feedback controller with an adaptive strategy was constructed. A fixed-time synchronization control method based on homogeneous system theory was proposed for master-slave coupling systems in [50]. The paper [2] studied the second order practical consensus of homogeneous sampled-data multi-agent systems, and proposed a nonlinear emulation method based on homogeneity. In [11], a homogeneous version of Gronwall-Bellman inequality is introduced into the sampled-data control systems to solve the systems' stabilization which is uncontrollable and unobservable around the origin.

But uncertain or nonlinear terms of the present methods must to be triangular and satisfy higher-order growth condition. Unfortunately, the active suspension dynamic model of our paper does not meet the aforementioned condition. Considering the aforementioned issues of the active suspension, in order to develop the homogeneous control method application for the active suspension of IEV-DFIM in engineering, we design a novel homogeneous output feedback controller to address this issues. Our contributions of this paper are following.

- An active suspension model which considers the time-varying sprung and unsprung masses, stiffness coefficient and damping coefficient is built compared with other active suspension models which consider the sprung mass, stiffness coefficient and damping coefficient as constants.

- A homogeneous observer is constructed to estimate the unmeasurable road profile that can help to improve the vehicle ride comfort.
- A homogeneous domination method is developed to design an output feedback controller to solve the problem which the uncertain and nonlinear terms are not higher-order or linear growth in the active suspension control system.

The paper is organized as follows. The active suspension control system model based on the general active suspension dynamic model is given in Sect. 2. In Sect. 3, a novel active suspension control method based on homogeneous output feedback approach is proposed for IEV-DFIM. Simulation and test are carried out to verify the effectiveness of the designed homogeneous output feedback controller compared with the sliding mode controller and passive suspension in Sect. 4. And Conclusions are given in Sect. 5.

2 Active suspension control system modeling with time-varying coefficients

When an IEV-DFIM runs on road, the sprung mass m_s will transfer between the front axle and rear axle. And the unsprung mass (tyre mass) m_u will reduce because of wear. Therefore, the sprung mass m_s and unsprung mass m_u are time-varying. On the other hand, the actuator is also nonlinear, i.e., the stiffness coefficient k_s and damping coefficient of the actuator c_s are time-varying. Of course, the stiffness coefficient k_t and damping coefficient of the tyre c_t are also time-varying with tyre pressure and load. These time-varying parameters influence the performance of the active suspension greatly. But in the existing active suspension models, these parameters are considered as constants for controlling easily. In this section, we will build an active suspension dynamic model of an IEV-DFIM considering time-varying parameters. Then the active suspension control system is presented based on the dynamic model. In fact, the active suspension used in the IEV-DFIM is similar with the traditional vehicle. Therefore, we build the active suspension dynamic model referring the paper [28]. The dynamic model is shown in Fig. 1, and described as

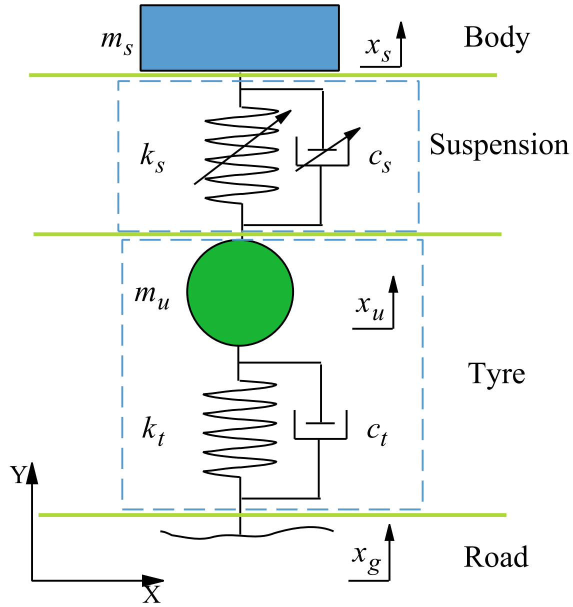

Fig. 1 The quarter-car active suspension model of an intelligent electric vehicles driven by four in-wheel motors

$$
\begin{aligned}
& m_s(t)\ddot{x}_s(t) + k_s(t)(x_s(t) - x_u(t)) \\
& \quad + c_s(t)(\dot{x}_s(t) - \dot{x}_u(t)) \\
& \quad + v(t) = 0 \\
& \quad m_u(t)\ddot{x}_u(t) - k_s(t)(x_s(t) \\
& \quad - x_u(t)) - c_s(t)(\dot{x}_s(t) - \dot{x}_u(t)) \\
& \quad + k_t(x_u(t) - x_g(t)) \\
& \quad + c_t(\dot{x}_u(t) - \dot{x}_g(t)) \\
& - v(t) = 0,
\end{aligned} \tag{1}
$$

where $m_s(t)$ and $m_u(t)$ are the sprung and unsprung mass respectively. $x_s(t)$, $x_u(t)$ and $x_g(t)$ are the vertical displacements of the sprung mass, unsprung mass and vertical road profile respectively. $k_s(t)$ and $c_s(t)$ are the stiffness and damping coefficients of the suspension between the body and tyre respectively. k_t and c_t are the stiffness and damping coefficients of a tyre respectively.

The dynamic Eq. (1) can be rewritten as

$$
\begin{aligned}
m_s(t)\ddot{x}_s(t) + m_u(t)\ddot{x}_u(t) = & - k_t(t)(x_u(t) - x_g(t)) \\
& - c_t(t)(\dot{x}_u(t) - \dot{x}_g(t)).
\end{aligned} \tag{2}
$$

Defining

$$x_1 = m_s(t)x_s(t) + m_u(t)x_u(t),$$
$$x_2 = m_s(t)\dot{x}_s(t) + m_u(t)\dot{x}_u(t),$$
$$x_3 = -k_t(t)(x_u(t) - x_g(t)), x_4 = -k_t(t)(\dot{x}_u(t) - \dot{x}_g(t)),$$
$$v = m_u(t)\ddot{x}_u(t).$$

The dynamic Eq. (2) is transferred to

$$\begin{cases} \dot{x}_1 = x_2 + \phi_1(t) \\ \dot{x}_2 = x_3 + \phi_2(t) \\ \dot{x}_3 = x_4 + \phi_3(t) \\ \dot{x}_4 = u + \phi_4(t), \end{cases} \tag{3}$$

where $\phi_1(t) = 0$, $\phi_2(t) = \frac{c_t(t)}{k_t(t)}x_4$, $\phi_3(t) = 0$, $\phi_4(t) = k_t(t)\ddot{x}_g(t)$, $u = -\frac{k_t(t)}{m_u(t)}v$.

3 Homogeneous active suspension control method base on homogeneous output feedback approach

3.1 Problem statement

Before designing the homogeneous output feedback controller of the active suspension for the IEV-DFIM, we firstly give some important definitions and lemmas which have been proposed in literature.

Definition 1 [17]: **Weighted Homogenity**: For fixed coordinates $(x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$ and real numbers $r_i > 0, i = 1, 2, \cdots, n$, the dilation $\Delta_\varepsilon(x)$ is defined as $\Delta_\varepsilon(x) = (\varepsilon^{r_1}x_1, \varepsilon^{r_2}x_2, \cdots, \varepsilon^{r_n}x_n), \forall\varepsilon > 0$, in which r_i are called as the weights of the coordination. A function $V \in C(\mathbb{R}^n, \mathbb{R})$ is homogeneous of degree τ where $\tau \in \mathbb{R}$, $\forall x \in \mathbb{R}^n \setminus 0, \varepsilon > 0$, $V(\Delta_\varepsilon(x)) = \varepsilon^\tau V(x_1, x_2, \cdots, x_n)$. A vector field $f \in C(\mathbb{R}^n, \mathbb{R}^n)$ is homogeneous of degree of τ where $\tau \in \mathbb{R}, \forall x \in \mathbb{R}^n \setminus 0, \varepsilon > 0, f_i(\Delta_\varepsilon(x)) = \varepsilon^{\tau + r_i} f_i(x)$. And a homogeneous p-norm is defined as $\|x\|_{\Delta,p} = \left(\sum_{i=1}^n |x_i|^{p/r_i}\right)^{1/p}, \forall x \in \mathbb{R}^n$ where $p \geq 1$.

Lemma 1 [17]*: V_1 and V_2 are homogeneous functions of degree τ_1 and τ_2 with respect to a same dilation weight respectively. Then $V_1 \cdot V_2$ is also homogeneous with respect to the same dilation weight and the homogeneous degree is $\tau_1 + \tau_2$.*

Lemma 2 [17]*: $V : \mathbb{R}^n \longrightarrow \mathbb{R}$ is a homogeneous function of degree τ with respect to a given dilation weight $\Delta = (r_1, \cdots, r_n)$. Then the following items hold.*

- *$\partial V/\partial x_i$ is homogeneous of degree $\tau - r_i$ with respect to the dilation weight Δ.*
- *There is a positive constant c_1 such that $V(x) \leq c_1\|x\|_\Delta^\tau$.*
- *If $V(x)$ is positive definite, there is a positive constant c_2 such that $c_2\|x\|_\Delta^\tau \leq V(x)$.*

Lemma 3 [40]*: When c and d are positive constants, the following inequality holds,*

$$|x|^c|y|^d \leq \frac{c}{c+d}\gamma|x|^{c+d} + \frac{d}{c+d}\gamma^{-c/d}|y|^{c+d}. \tag{4}$$

Lemma 4 [40]*: For a positive odd integer $p \geq 1$, the following inequality holds,*

$$|x^p - y^p| \leq p|x - y|(|x|^{p-1} + |y|^{p-1}). \tag{5}$$

Lemma 5 [15]*: For any $x_i \in \mathbb{R}, i = 1, \cdots, n$ and a real number $p \geq 1$, the following inequalities hold,*

$$\begin{aligned} &(|x_1| + \cdots + |x_n|)^p \leq n^{p-1}(|x_1|^p + \cdots + |x_n|^p), \\ &(|x_1| + \cdots + |x_n|)^{1/p} \leq |x_1|^{1/p} + \cdots + |x_n|^{1/p}. \end{aligned} \tag{6}$$

The purpose of this paper is to design a homogeneous output feedback controller of the active suspension for the IEV-DFIM. For the purpose, a homogeneous state feedback controller will be firstly designed, and based on which a homogeneous output feedback controller will be designed. Firstly, we give a definition and an assumption for the designed controller.

Definition 2 Denote $\tau = q/p$, where q is even and p is odd, then there is

$$r_i = (i-1)\tau + 1, i = 1, 2, 3, 4 \tag{7}$$

Assumption 1 *There are constants $\tau \geq 0$ and $c \geq 0$ leading to*

$$|\phi_i| \leq c(|x_1|^{i\tau+1} + \cdots + |x_i|^{p_m}), i = 1, 2, 3, 4 \tag{8}$$

where $p_m = \frac{i\tau+1}{(i-1)\tau+1}$.

3.2 Design of the homogeneous state feedback controller

Next, we present a theorem to describe the main contribution of this paper.

Theorem 1 *Based on the Assumption 1, the active suspension system (3) can be asymptotically stabilized by a homogeneous domination output feedback controller to improve the ride comfort and maneuverability of the IEV-DFIM.*

$$u(\hat{x}) = -\beta_i \xi_4^{\frac{r_i+\tau}{r_i}}, i = 1, 2, 3, 4. \tag{9}$$

where β_i is constant, ξ_4 is an expression including the homogeneous state observer $\hat{x}$ which has the following form

$$\begin{cases} \hat{x}_1 = x_1 \\ \dot{\eta}_j = f_j(x_1, \eta_2, \cdots, \eta_j) \\ \hat{x}_{j+1} = (\eta_j + k_j \hat{x}_j)^{p_j}, k_j > 0, p_j > 0, \end{cases} \tag{10}$$

where $j = 1, 2, 3$.

Proof We will firstly design an active suspension homogeneous state feedback controller according to Theorem 1.

$$u(x) = -\beta_4(x_4 + \beta_3(x_3 + \beta_2(x_2 + \beta_1 x_1^{p_1})^{p_2})^{p_3})^{p_4}, \tag{11}$$

where β_i is constant, $p_1 = r_2/r_1$, $p_2 = r_3/r_2$, $p_3 = r_4/r_3$, $p_4 = (r_4 + \tau)/r_4$. □

Next, for the following proving convenience, we construct a simple system (12) similar with the system (3), and prove that the controller (11) can stabilize the system (12).

$$\begin{cases} \dot{x}_i = x_{i+1} \\ \dot{x}_4 = u, \end{cases} \tag{12}$$

where $i = 1, 2, 3$, $u := x_5 \in \mathbb{R}$ and $y := x_1 \in \mathbb{R}$.

We select a Lyapunov function

$$V_1(x_1) = \frac{r_1}{2r_4 - \tau} x_1^{\frac{2r_4-\tau}{r_1}}, \tag{13}$$

The time derivative of the V_1 along the controller (11) is

$$\begin{aligned} \dot{V}_1 &= x_1^{\frac{2r_4-\tau}{r_1}-1} x_2 \le x_1^{\frac{2r_4-\tau}{r_1}-1}(x_2 + e x_1^{\frac{r2}{r1}}) \\ &\le x_1^{\frac{2r_4-\tau}{r_1}-1}(x_2 - x_2{}^* + x_2{}^* + e x_1^{\frac{r2}{r1}}), \end{aligned} \tag{14}$$

where x_2^* is a virtual controller, and $x_2{}^* = -\beta_1 x_1{}^{r2/r1}$, $\beta_1 = 4 + e$. Then Eq. (14) is rewritten as

$$\dot{V}_1 \le -4x_1^{\frac{2r_4}{r_1}} + x_1^{\frac{2r_4-\tau}{r_1-1}}(x_2 - x_2{}^*). \tag{15}$$

Suppose that there exists a positive definite and homogeneous C^1 Lyapunov function $V_{k-1} : \mathbb{R}^{k-1} \to \mathbb{R}$ with respect to Eq. (7) at step $k-1, k = 2, 3, 4$. And a set of C^1 virtual controllers $x_1{}^*, x_2{}^*, x_3{}^*, x_4{}^*$ are defined as

$$\begin{aligned} &x_1{}^* = 0, \xi_1 = x_1 - x_1{}^*, \\ &x_j{}^* = -\beta_{j-1}\xi_{j-1}^{\frac{r_j}{r_j-1}}, \xi_j = x_j - x_j{}^*, j = 2, 3, 4, \end{aligned} \tag{16}$$

where $\beta_1 > 0$, $\beta_j > 0$. Therefore,

$$\begin{aligned} \dot{V}_{k-1} \le &-(6-k)\sum_{j=1}^{k-1} \xi_{k-1}^{(2r_4-\tau)/r_{k-1}-1} \\ &+\xi_j^{2r_4/r_j}(x_k - x_k{}^*). \end{aligned} \tag{17}$$

We claim that Eq. (17) also holds at step k. In order to prove the claim, we construct the Lyapunov function

$$V_k(x_1, \cdots, x_k) = V_{k-1} + \frac{r_k}{2r_4 - \tau}\xi_k^{\frac{2r_4-\tau}{r_k}}. \tag{18}$$

Then the derivative of the $V_k(x_1, \cdots, x_k)$ is

$$\begin{aligned} \dot{V}_k(x_1, \cdots, x_k) \le &-(6-k)\sum_{j=1}^{k-1} \xi_j^{2r_4/r_j} + \xi_{k-1}^{\frac{2r_4-\tau}{r_k-1}-1}\xi_k \\ &+ \xi_k^{\frac{2r_4-\tau}{r_k}-1}(x_{k+1} + \phi_k(\cdot) - \sum_{l=1}^{k-1}\frac{\partial x_k{}^*}{\partial x_l}\dot{x}_l). \end{aligned} \tag{19}$$

Next, we will estimate each term in the right hand side of Eq. (19). By Yang Inequality ($p = 2r_4 - \tau - r_{k-1} = 2r_4 - r_k, q = r_k$), and denoting $c_x = 2r_4 - \tau -$

$r_{k-1}, d_x = r_k, \gamma = r_4/c_x, c_x + d_x = 2r_4$, for a onstant $c_k > 0$, there is

$$\xi_{k-1}^{\frac{2r_4-\tau}{r_{k-1}}-1}\xi_k \leq \frac{c_x}{2r_4}\gamma\xi_{k-1}^{\frac{2r_4}{r_{k-1}}} + \frac{d_x}{2r_4}\gamma^{-\frac{c_x}{d_x}}\xi_{k-1}^{\frac{2r_4}{r_k}} = \frac{1}{2}\xi_{k-1}^{\frac{2r_4}{r_{k-1}}} + c_k\xi_k^{\frac{2r_4}{r_k}}. \quad (20)$$

According to Eq. (7) and Assumption 1, one obtains

$$\frac{r_k+\tau}{r_j} = \frac{k\tau+1}{(j-1)\tau+1} = \frac{kq+p}{(j-1)q+p} \geq 1 \quad (21)$$

is an odd function.

According to Lemma (4) and Eq. (16), there exists

$$\begin{aligned} |x_j|^{\frac{k\tau+1}{(j-1)\tau+1}} &= |\xi_j - \xi_{j-1}^{\frac{r_j}{r_{j-1}}}\beta_{j-1}| \\ &\leq 2^{\frac{k\tau+1}{r_j}-1}|\xi_j^{\frac{r_k+\tau}{r_j}} - \beta_{j-1}^{\frac{r_k+\tau}{r_j}}\xi_{j-1}^{\frac{r_j}{r_{j-1}}\frac{r_k+\tau}{r_j}}| \\ &\leq 2^{\frac{k\tau+1}{r_j}-1}(|\xi_j^{\frac{r_k+\tau}{r_j}}| + \beta_{j-1}^{\frac{r_k+\tau}{r_j}}|\xi_{j-1}^{\frac{r_k+\tau}{r_{j-1}}}|). \end{aligned} \quad (22)$$

Combining Eq. (22) with Assumption (1), one obtains

$$\begin{aligned} |\phi_k| &\leq c\sum_{j=1}^{k}|\xi_j - \xi_{j-1}^{\frac{r_j}{r_{j-1}}}\beta_{j-1}| \\ &\leq c\sum_{j=1}^{k}2^{\frac{k\tau+1}{r_j}-1}(|\xi_j^{\frac{r_k+\tau}{r_j}}| + \beta_{j-1}^{\frac{r_k+\tau}{r_j}}|\xi_{j-1}^{\frac{r_k+\tau}{r_{j-1}}}|) \\ &= \bar{c}_k\sum_{j=1}^{k}|\xi_j|^{\frac{r_k+\tau}{r_j}}. \end{aligned} \quad (23)$$

The last right hand side of Eq. (19) can be estimated as the following proposition that has been proved in the paper [38].

Proposition 1 *There is a constant $\tilde{c}_k$ such that*

$$\sum_{l=1}^{k-1}\frac{\partial x_k{}^*}{\partial x_l}\dot{x}_l \leq \tilde{c}_k(|\xi_1|^{\frac{r_k+\tau}{r_1}} + \cdots + |\xi_k|^{\frac{r_k+\tau}{r_k}}). \quad (24)$$

By Lemma 3, combining Eqs. (23) and (24) yields

$$\begin{aligned} &\xi_k^{\frac{2r_4-\tau}{r_k}-1}(\phi_k(\cdot) - \sum_{l=1}^{k-1}\frac{\partial x_k{}^*}{\partial x_l}\dot{x}_l) \\ &\leq x_k^{\frac{2r_4-\tau}{r_k}-1}(\bar{c}_k + \tilde{c}_k)\sum_{l=1}^{k}|\xi_l|^{\frac{r_k+\tau}{r_l}} \\ &\leq (\bar{c}_k + \tilde{c}_k)\sum_{l=1}^{k-1}\Bigg(\frac{c}{c+d}\gamma|\xi_k^{\frac{r_l}{r_k}}|^{\frac{2r_4}{r_l}} \\ &\quad + \frac{d}{c+d}\gamma^{-\frac{c}{d}}|\xi_l|^{\frac{2r_4}{r_l}}\Bigg) \\ &\quad + (\bar{c}_k + \tilde{c}_k)\xi_l^{\frac{2r_4}{r_k}} \\ &= \frac{1}{2}\sum_{l=1}^{k-1}\xi_l^{\frac{2r_4}{r_l}} \\ &\quad + \hat{c}_k\xi_l^{\frac{2r_4}{r_k}}, \end{aligned} \quad (25)$$

where $c = \frac{2r_4-\tau-r_k}{r_l}, d = \frac{\tau+r_k}{r_l}, \frac{d}{c+d}\gamma^{-\frac{c}{d}}(\bar{c}_k+\tilde{c}_k) = \frac{1}{2}$ and $\hat{c}_k$ is a constant.

Substituting Eqs. (20) and (25) into Eq. (19) generates

$$\begin{aligned} \dot{V}_k &\leq -(6-k)\sum_{j=1}^{k-1}\xi_j^{\frac{2r_4}{r_j}} + \frac{1}{2}\xi_{k-1}^{\frac{2r_4}{r_{k-1}}} + c_k\xi_k^{\frac{2r_4}{r_k}} \\ &\quad + \xi_k^{\frac{2r_4-\tau}{r_k}-1}x_{k+1} + \frac{1}{2}\sum_{l=1}^{k-1}\xi_l^{\frac{2r_4}{r_l}} + \hat{c}_k\xi_k^{\frac{2r_4}{r_k}} \\ &\leq -(6-k)\sum_{j=1}^{k-1}\xi_j^{\frac{2r_4}{r_j}} + \sum_{l=1}^{k-1}\xi_l^{\frac{2r_4}{r_l}} + \xi_k^{\frac{2r_4-\tau}{r_k}-1}x_{k+1} \\ &\quad + (c_k + \hat{c}_k)\xi_k^{\frac{2r_4-\tau}{r_k}-1}\xi_k^{\frac{2r_k+\tau}{r_k}} \\ &= -(5-k)\sum_{j=1}^{k-1}\xi_j^{\frac{2r_4}{r_j}} \\ &\quad + \xi_k^{\frac{2r_4-\tau}{r_k}-1}(x_{k+1} + (c_k + \hat{c}_k)\xi_k^{\frac{r_k+1}{r_k}}). \end{aligned} \quad (26)$$

According to the defined virtual controller, there exists

$$x_{k+1}^* = -\beta_k\xi_k^{\frac{r_k+1}{r_k}} = -(5-k+c_k+\hat{c}_k)\xi_k^{\frac{r_k+1}{r_k}}. \quad (27)$$

Then

$$\dot{V}_k \le -(5-k)\sum_{j=1}^{k-1}\xi_j^{\frac{2r_4}{r_j}} + \xi_k^{\frac{2r_4-\tau}{r_k}-1}(x_{k+1}-x_{k+1}^* - (5-k)\xi_k^{\frac{r_k+1}{r_k}}) = -(5-k)\sum_{j=1}^{k-1}\xi_j^{\frac{2r_4}{r_j}} - (5-k)\xi_j^{\frac{2r_4}{r_k}} + \xi_k^{\frac{2r_4-\tau}{r_k}-1}(x_{k+1}-x_{k+1}^*) = -(5-k)\sum_{j=1}^{k}\xi_j^{\frac{2r_4}{r_j}} + \xi_k^{\frac{2r_4-\tau}{r_k}-1}(x_{k+1}-x_{k+1}^*). \tag{28}$$

Then the inductive proof ends. The inductive argument shows that Eq. (16) holds for $k = 5$. Therefore, the state feedback controller can be designed as

$$u = x_5^* = -\beta_4\xi_4^{\frac{r_4+\tau}{r_4}}, \tag{29}$$

which insures $\dot{V}_4 \le -(\xi_1^{2r_4/r_1} + \xi_2^{2r_4/r_2} + \xi_3^{2r_4/r_3} + \xi_4^{2r_4/r_4}) < 0$, i.e., system (12) can be globally asymptotically stabilized under the controller (11) or (29).

Next, we will prove that the state feedback controller (29) can stabilize the system (3). According to Eq. (29), one obtains

$$\dot{V}_4 \le -\sum_{j=1}^{4}\xi_j^{2r_4/r_j} + \sum_{j=1}^{4}\frac{\partial V_4(x_4)}{\partial x_j}\phi_j(\cdot). \tag{30}$$

According to Assumption 1, there exists

$$\phi_j(t, x_{i+1}, \cdots, x_4) \le \kappa(|x_1|^{p_1} + |x_2|^{p_2} + |x_3|^{p_3} + |x_4|^{p_4}) \times (|x_1|^{p_{m1}-p_1} + |x_2|^{p_{m2}-p_2} + |x_3|^{p_{m3}-p_3} + |x_4|^{p_{m4}-p_4}). \tag{31}$$

Then based on the aforementioned homogeneity definition, one can prove that $\sum_{j=1}^4 \xi_j^{2r_4/r_j}$ and $\sum_{j=1}^4 \frac{\partial V_4(x_4)}{\partial x_j}(|x_1|^{p_1}+|x_2|^{p_2}+|x_3|^{p_3}+|x_4|^{p_4})$ have the same homogeneity. According to the conclusion in the paper [9], there exists

$$\dot{V}_4 \le -\sum_{j=1}^{4}\xi_j^{2r_4/r_j} + \overline{\kappa}\sum_{j=1}^{4}\xi_j^{2r_4/r_j} \times (|x_1|^{p_{m1}-p_1} + |x_2|^{p_{m2}-p_2} + |x_3|^{p_{m3}-p_3} + |x_4|^{p_{m4}-p_4}), \tag{32}$$

when $\overline{\kappa} > 0$.

According to Assumption (1) and (11), one knows that $p_{mi} = \frac{i\tau+1}{(i-1)\tau+1}$, $p_1 = r2/r1$, $p_2 = r3/r2$, $p_3 = r4/r3$, $p_4 = (r4+\tau)/r4$, i.e., $p_{m1} = p_1$, $p_{m2} = p_2$, $p_{m3} = p_3$, $p_{m4} = p_4$. Then there is $(|x_1|^{p_{m1}-p_1} + |x_2|^{p_{m2}-p_2} + |x_3|^{p_{m3}-p_3} + |x_4|^{p_{m4}-p_4}) = 4$. Therefore,

$$\dot{V}_4 \le -\sum_{j=1}^{4}\xi_j^{2r_4/r_j} + 4\overline{\kappa}\sum_{j=1}^{4}\xi_j^{2r_4/r_j} \le \sum_{j=1}^{4}\xi_j^{2r_4/r_j}(1-4\overline{\kappa}). \tag{33}$$

That is to say, $\dot{V}_4 < 0$ if one selects suitable $\overline{\kappa}$. Therefore, The designed controller (11) can globally asymptotically stabilize the system (3).

Next, we will design the homogeneous observer and controller based on the designed state feedback controller.

3.3 Design of homogeneous state observer and controller

We construct a homogeneous observer inspired by the one used in [38] as follows

$$\begin{cases} \hat{x}_1 = x_1 \\ \dot{\eta}_i = -Ll_i\hat{x}_i \\ \hat{x}_{i+1} = (\eta_i + k_i\hat{x}_i)^{p_i}, \end{cases} \tag{34}$$

where $L \ge 1$, $l_i > 0$, $k_i > 0$, $i = 1, 2, 3$.

Based on the designed homogeneous state feedback controller, the homogeneous output feedback controller is designed as

$$u(\hat{x}) = -\beta_4(\hat{x}_4 + \beta_3(\hat{x}_3 + \beta_2(\hat{x}_2 + \beta_1\hat{x}_1^{p_1})^{p_2})^{p_3})^{p_4}, \tag{35}$$

i.e.,

$$u(\hat{x}) = -\beta_i\xi_4^{\frac{r_i+\tau}{r_i}}, i = 1, 2, 3, 4. \tag{36}$$

We construct

$$U_i = \int_{\eta_i+l_{i-1}x_{i-1}^{\frac{2r_4-r_i}{r_{i-1}}}}^{x_i^{\frac{2r_4-r_i}{r_i}}} (s^{\frac{r_{i-1}}{2r_4-r_i}} - (\eta_i + l_{i-1}x_{i-1}))\mathrm{d}s, \tag{37}$$

where $i=2,3,4$, and U_i is C^1. One obtains

$$\frac{\partial U_i}{\partial x_i}=\frac{2r_4-r_i}{r_i}x_i^{\frac{2r_4-2r_i}{r_i}}(x_i^{\frac{r_{i-1}}{r_i}}-(\eta_i+l_{i-1}x_{i-1})),$$
$$\frac{\partial U_i}{\partial \eta_i}=-(x_i^{\frac{2r_4-2r_i}{r_i}}-(\eta_i+l_{i-1}x_{i-1})^{\frac{2r_4-2r_i}{r_{i-1}}}),\quad (38)$$
$$\frac{\partial U_i}{\partial x_{i-1}}=-l_{i-1}(x_i^{\frac{2r_4-2r_i}{r_i}}-(\eta_i+l_{i-1}x_{i-1})^{\frac{2r_4-2r_i}{r_{i-1}}}).$$

Then the derivative of U_i along Eqs. (11), (12), (16), (34) is

$$\begin{aligned}\dot{U}_i=&\frac{2r_4-r_i}{r_i}x_{i+1}x_i^{\frac{2r_4-2r_i}{r_i}}(x_i^{\frac{r_{i-1}}{r_i}}-(\eta_i+l_{i-1}x_{i-1}))\\&-l_{i-1}e_i(x_i^{\frac{2r_4-r_i}{r_i}}-\hat{x}_i^{\frac{2r_4-r_i}{r_i}})\\&-l_{i-1}e_i(\hat{x}_i^{\frac{2r_4-2r_i}{r_i}}-(\eta_i+l_{i-1}x_{i-1})^{\frac{2r_4-2r_i}{r_{i-1}}}),\end{aligned}\quad (39)$$

where $e_i=x_i-\hat{x}_i$, $i=2,3,4$, and $x_5=u(\hat{x})$.

Next, we will estimate each term of the right hand side of Eq. (39). By Lemma 4, one obtains

$$-l_{i-1}e_i(x_i^{\frac{2r_4-r_i}{r_i}}-\hat{x}_i^{\frac{2r_4-r_i}{r_i}})\le -l_{i-1}e_i^{\frac{2r_4}{r_i}}2^{\frac{2r_i-2r_4}{r_i}}.\quad (40)$$

The other terms can be verified by the following propositions which have been proved in [38].

Proposition 2 *For $i=2,3$, there exists*

$$\begin{aligned}&\frac{2r_4-r_i}{r_i}x_{i+1}x_i^{\frac{2r_4-2r_i}{r_i}}(x_i^{\frac{r_{i-1}}{r_i}}-(\eta_i+l_{i-1}x_{i-1}))\\&\le\frac{1}{12}(\xi_{i-1}^{\frac{2r_4}{r_{i-1}}}+\xi_i^{\frac{2r_4}{r_i}}+\xi_{i+1}^{\frac{2r_4}{r_{i+1}}})\\&\quad+\gamma_i e_i^{\frac{2r_4}{r_i}}+g_i(l_{i-1})e_{i-1}^{\frac{2r_4}{r_{i-1}}},\end{aligned}\quad (41)$$

where α_i is a constant, g_i is a continuous function of l_{i-1} with $g_2(\cdot)=0$.

Proposition 3 *For controller (36), there exists*

$$\begin{aligned}&r_4u(\hat{x})(x_4^{\frac{2r_3}{r_4}}-(\eta_4+l_3z_3))\\&\le c(||\xi||_{\Delta_x}^{r_4+\tau}+||e||_{\Delta_x}^{r_4+\tau})(c|e_4|^{\frac{r_3}{r_4}}+l_3|e_3|)\\&\le\frac{1}{8}\sum_{i=1}^{4}\xi_i^{2r_4/r_i}+\bar{\gamma}\sum_{i=1}^{4}e_i^{2r_4/r_i}+g_4(l_3)e_3^{2r_4/r_3}.\end{aligned}\quad (42)$$

Proposition 4 *For $i=3,4$, there exists*

$$\begin{aligned}&-l_{i-1}e_i(\hat{x}_i^{\frac{2r_4-r_i}{r_i}}-(\eta_i+l_{i-1}x_{i-1})^{\frac{2r_4-r_i}{r_{i-1}}})\\&\quad\le e_i^{\frac{2r_4}{r_i}}+\frac{1}{16}(\xi_i^{\frac{2r_4}{r_i}}+\xi_{i-1}^{\frac{2r_4}{r_{i-1}}})+h_i(l_{i-1})e_{i-1}^{\frac{2r_4}{r_i}},\end{aligned}\quad (43)$$

where $h_i(l_{i-1})$ is a polynomial function.

With the help of Proposition 2–4, we can obtain the derivative of $U=\sum_{i=2}^{4}U_i$

$$\begin{aligned}\dot{U}\le&\frac{1}{2}\sum_{i=1}^{4}\xi_i^{\frac{2r_4}{r_i}}-(l_1 2^{\frac{2r_1-2r_4}{r_1}}-\gamma_2-\bar{\gamma}-g_3(l_2)\\&-h_3(l_2))e_2^{\frac{2r_4}{r_2}}-(l_2 2^{\frac{2r_3-2r_4}{r_3}}-\gamma_3-1-\bar{\gamma}-g_4(l_3)\\&-h_4(l_3))e_3^{\frac{2r_4}{r_3}}-(l_3-1-\tilde{\gamma}-\bar{\gamma})e_4^2.\end{aligned}\quad (44)$$

Now we will discuss how to determine l_i. Because the states x_2,x_3,x_4 are not measurable, system (29) generates a redundant term $\xi_4^{2r_4-\tau/r_4-1}(u-u^*)$ under controller (35). We use Proposition 5 to solve this issue.

Proposition 5 *For a constant $\bar{\gamma}\ge 0$, there exists*

$$u(\hat{x})-u^*(x)=\sum_{i=2}^{4}e_i\int_0^1\frac{\partial(\mathcal{X})}{\partial\mathcal{X}_i}\mathrm{d}\lambda,\quad (45)$$

where $\mathcal{X}=x-\lambda e$.

According to the homogeneity of $u^*(x)$ whose degree is $r_4+\tau$, the degree of $\frac{\partial(\mathcal{X})}{\partial\mathcal{X}_i}$ is $r_4+\tau-r_i$. According to the definition of homogeneous norm, there exists

$$\frac{\partial(\mathcal{X})}{\partial\mathcal{X}_i}\le c||\xi||_{\Delta_x}^{r_4+\tau-r_i}+c||e||_{\Delta_x}^{r_4+\tau-r_i}.$$

By Yong's Inequality, there exists

$$\begin{aligned}
&\xi_4^{\frac{2r_4-\tau}{r_4}-1}(u^*(\hat{x})-u(x)) \\
&\le c|\xi_4^{\frac{2r_4-\tau}{r_4}-1}|\sum_{i=2}^{4}|e_i|\times(||\xi||_{\Delta_x}^{r_4+\tau-r_i} \\
&\quad+||e||_{\Delta_x}^{r_4+\tau-r_i}) \\
&\le \sum_{i=1}^{4}\frac{\xi_i^{\frac{2r_4}{r_i}}}{4}+\tilde{\gamma}\sum_{i=2}^{4}e_i^{\frac{2r_4}{r_i}},
\end{aligned} \tag{46}$$

for a constant $\tilde{\gamma}\ge 0$.

We construct a Lyapunov function

$$W=U+V_4. \tag{47}$$

It's derivative is

$$\begin{aligned}
\dot{W}\le&\frac{1}{4}\sum_{i=1}^{4}\xi_i^{\frac{2r_4}{r_i}}-(l_1 2^{\frac{2r_1-2r_4}{r_1}} \\
&-\gamma_2-\tilde{\gamma}-\bar{\gamma}-g_3(l_2) \\
&-h_3(l_2))e_2^{\frac{2r_4}{r_2}}-(l_2 2^{\frac{2r_3-2r_4}{r_3}} \\
&-\gamma_3-1-\tilde{\gamma}-\bar{\gamma}-g_4(l_3) \\
&-h_4(l_3))e_3^{\frac{2r_4}{r_3}}-(l_3-1 \\
&-\tilde{\gamma}-\bar{\gamma})e_4^3.
\end{aligned} \tag{48}$$

Obviously, if one selects

$$\begin{aligned}
l_1&=2^{1+\frac{r_1}{2r_4-2r_1}}(\frac{1}{4}+\gamma_2 \\
&\quad+\tilde{\gamma}+\bar{\gamma}+g_3(l_2)+h_3(l_2)) \\
l_2&=2^{1+\frac{r_3}{2r_4-2r_3}}(\frac{5}{4}+\gamma_3+\tilde{\gamma} \\
&\quad+\bar{\gamma}+g_4(l_3)+h_4(l_3)) \\
l_3&=\frac{5}{4}+\tilde{\gamma}+\bar{\gamma},
\end{aligned} \tag{49}$$

Equation (48) can be rewritten as

$$\dot{W}\le-\frac{1}{4}(\xi_1^{2r_4/r_1}+\sum_{j=2}^{4}(\xi_j^{2r_4/r_j}+e_j^{2r_4/r_1})). \tag{50}$$

According to the construction of W, it can be proved that W is positive definite. And according to the constructions of ξ_i and e_i, $\dot{W}$ is negative definite. Therefore, the closed-loop system 12 is globally asymptotically stable.

We rewrite system (12), (34) and (35) as

$$\dot{X}=F(X)=(x_2,x_3,x_4,u(\hat{x}),F_5,F_6,F_7)^T. \tag{51}$$

System (51) is homogeneous, and the dilations can be selected as

$$\Delta=(\Delta_x,\Delta_\eta)=(r_1,r_2,r_3,r_4,r_5,r_6,r_7), \tag{52}$$

where $\Delta_x=(1,1+\tau,1+2\tau,1+3\tau)$, $\Delta_\eta=(1,1+\tau,1+2\tau)$. Therefore, system (51) is homogeneous of degree of τ. W is homogeneous of degree of $2r_4-\tau$, and $\dot{W}$ is homogeneous of degree of $2r_4$.

According to Lemma 2, Eq. (48) is rewritten as

$$\frac{\partial W(X)}{\partial X}\le -c_1||X||_\Delta^{2r_4}, \tag{53}$$

where

$$||X||_\Delta=\sqrt{\sum||X_i||^{2/r_i}},\ i=1,\cdots,7.$$

Next, we will prove that system (3) is stable under observer (34) and controller (35). A coordinate change is introduced firstly. Let $z_1=x_1$, $z_2=\frac{x_2}{L}$, $z_3=\frac{x_3}{L^2}$, $z_4=\frac{x_4}{L^3}$, $u(\hat{z})=\frac{u(\hat{x})}{L^4}$, system 3 is changed as

$$\begin{aligned}
\dot{z}_1&=Lz_2+\phi_1(t,z) \\
\dot{z}_2&=Lz_3+\frac{\phi_2(t,z)}{L} \\
\dot{z}_3&=Lz_4+\frac{\phi_3(t,z)}{L^2} \\
\dot{z}_4&=Lu(\hat{z})+\frac{\phi_4(t,z)}{L^3}.
\end{aligned} \tag{54}$$

Combining with Eqs. (48), (50) and (53), the derivative of the Lyapunov function for system (54) is

$$\begin{aligned}
\dot{W}_f\le&-\frac{1}{4}(\xi_1^{2r_4/r_1}+\sum_{j=2}^{4}(\xi_j^{2r_4/r_j}+e_j^{2r_4/r_j})) \\
&+\frac{\partial W_f(Z)}{\partial Z}\left[\phi_1(\cdot)\cdot\frac{\phi_2(\cdot)}{L}\cdot\frac{\phi_3(\cdot)}{L^2}\cdot\frac{\phi_4(\cdot)}{L^3}\right]^T \\
\le&-Lc_1||Z||_\Delta^{2r_4} \\
&+\frac{\partial W_f(Z)}{\partial Z}\left[\phi_1(\cdot)\cdot\frac{\phi_2(\cdot)}{L}\cdot\frac{\phi_3(\cdot)}{L^2}\cdot\frac{\phi_4(\cdot)}{L^3}\right]^T,
\end{aligned} \tag{55}$$

where $Z=[z_1\quad z_2\quad z_3\quad z_4]^T$.

Considering the characteristics of the active suspension, there exists

$$\begin{aligned}
|\phi_1(t,z)| &= 0 \le c|z_1|^{\tau+1} \\
|\phi_2(t,z)| &= \frac{c_t}{k_t}x_4 \le c(|z_1|^{\tau+1} + |Lz_2|^{\frac{2\tau+1}{\tau+1}}) \\
|\phi_3(t,z)| &= 0 \le c(|z_1|^{\tau+1} + |Lz_2|^{\frac{2\tau+1}{\tau+1}} + |L^2z_3|^{\frac{3\tau+1}{2\tau+1}}) \\
|\phi_4(t,z)| &= k_t\ddot{x}_g(t) \le c(|z_1|^{\tau+1} + |Lz_2|^{\frac{2\tau+1}{\tau+1}} \\
&\quad + |L^2z_3|^{\frac{3\tau+1}{2\tau+1}} + |L^3z_4|^{\frac{4\tau+1}{3\tau+1}}).
\end{aligned} \tag{56}$$

Because $L \ge 1$,

$$\begin{aligned}
&|\phi_1(t,z)| = 0 \\
&|\frac{\phi_2(t,z)}{L}| \le cL^{1-\frac{1}{\tau+1}}(|z_1|^{\frac{r_2+\tau}{r_1}} + |z_2|^{\frac{r_2+\tau}{r_2}}) \\
&|\frac{\phi_3(t,z)}{L^2}| = 0 \\
&|\frac{\phi_4(t,z)}{L^3}| \le cL^{1-\frac{1}{2\tau+1}}(|z_1|^{\frac{r_4+\tau}{r_1}} + |z_2|^{\frac{r_4+\tau}{r_2}} \\
&\qquad + |z_3|^{\frac{r_4+\tau}{r_3}} + z_4|^{\frac{r_4+\tau}{r_4}}).
\end{aligned} \tag{57}$$

According to the definition of homogeneous function, $\frac{\partial W(Z)}{\partial Z_i}$ is homogeneous of degree of $2r_4 - \tau - r_i$. Based on Lemma 1,

$$\begin{aligned}
&\left|\frac{\partial W_f(Z)}{\partial Z_1}\right|(|z_1|^{\frac{r_1+\tau}{r_1}}), \\
&\quad\left|\frac{\partial W_f(Z)}{\partial Z_2}\right|(|z_1|^{\frac{r_2+\tau}{r_1}} + |z_2|^{\frac{r_2+\tau}{r_2}}), \\
&\quad\left|\frac{\partial W_f(Z)}{\partial Z_3}\right|(|z_1|^{\frac{r_3+\tau}{r_1}} + |z_2|^{\frac{r_3+\tau}{r_2}} \\
&+ |z_3|^{\frac{r_3+\tau}{r_3}}), \\
&\left|\frac{\partial W_f(Z)}{\partial Z_4}\right|(|z_1|^{\frac{r_4+\tau}{r_1}} + |z_2|^{\frac{r_4+\tau}{r_2}} \\
&\quad + |z_3|^{\frac{r_4+\tau}{r_3}} + |z_4|^{\frac{r_4+\tau}{r_4}})
\end{aligned} \tag{58}$$

are homogeneous of degree of $2r_4$. Therefore, we can find a constant $\chi_i > 0$ in the Eqs. (57) and (58) to get

$$\frac{\partial W_f(Z)}{\partial Z_i}\frac{\phi_i(\cdot)}{L^{i-1}} \le \chi_i L^{1-\frac{1}{(i-1)\tau+1}}||Z||_{\Delta}^{2r_4}. \tag{59}$$

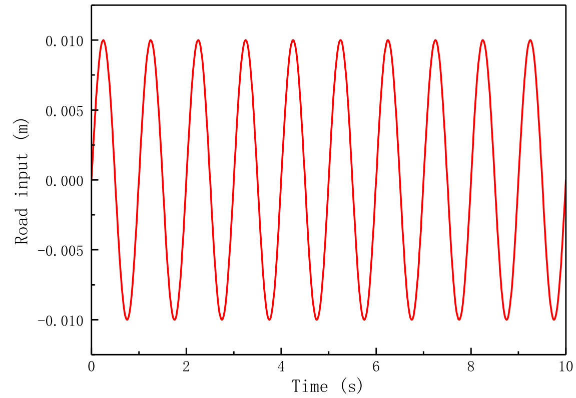

Fig. 2 The disturbance of $0.01\sin(2\pi t)$m road profile

Substituting Eq. (59) into (50) leads to

$$\dot{W}_f \le -L(c_1 - \chi_2 L^{-\frac{1}{\tau+1}} - \chi_4 L^{-\frac{1}{3\tau+1}})||Z||_{\Delta}^{2r_4}. \tag{60}$$

Therefore, as long as we select a suitable L, there exists $\dot{W}_f < 0$ which ensures $\dot{W}_f < 0$, i.e., the active suspension control system (3) is asymptotically stable under observer (34) and controller (35).

The proof of the Theorem 1 ends.

4 Simulation and test

We firstly numerically simulate the designed homogeneous output feedback controller Eq. (35) with the homogeneous observer Eq. (34) for the active suspension compared with the Sliding mode controller. The sliding mode controller is designed inspired by Reference [18]. We analyze the effectiveness of the designed homogeneous output feedback controller about the sprung mass vertical displacement and unsprung mass vertical displacement according to the suspension evaluation index. In the simulation, the parameters of the active suspension and the designed homogeneous output feedback controller are shown in Tables 1 and 2 respectively.

We select three classic road profiles: sinusoidal road profile, convergent sinusoidal road profile and step road profile, to verify the designed controller. Figures 2, 3, 4 are the selected three road profiles.

Figures 5, 6, 7 are the vertical displacement of the body under different controllers and passive suspension suffering different road profiles. The homogeneous controller can decrease the vertical displacement of the

Table 1 The parameters of an active suspension of an IEV-DFIM

Symbol	Value	Symbol	Value
m_u	49 kg	m_s	300 kg
k_s	17000 Nm	c_s	1713 Nm
k_t	200000 Nm	c_t	360 Nm

Table 2 The parameters of the designed homogeneous output feedback controller

Symbol	Value	Symbol	Value	Symbol	Value
r_1	1	r_2	1.018	r_3	1.036
r_4	1.054	p_1	1.018	p_2	1.018
p_3	1.017	p_4	1.017	k_1	1.9
k_2	2.5	k_3	2.7	τ	2/111
β_1	1.3	β_2	1.5	β_3	15
β_4	221	l_1	55	l_2	8.5
l_3	121				

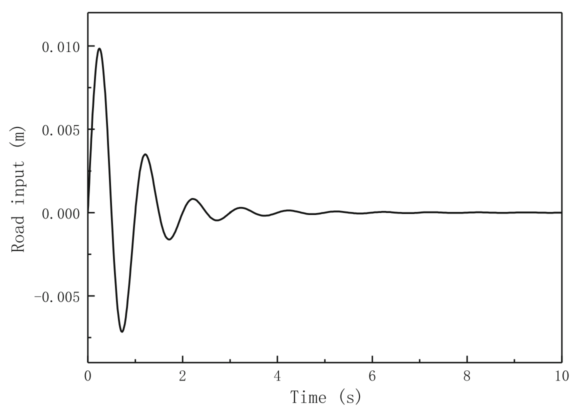

Fig. 3 The disturbance of $0.01\sin(2\pi t)/(1+t^3)$m road profile

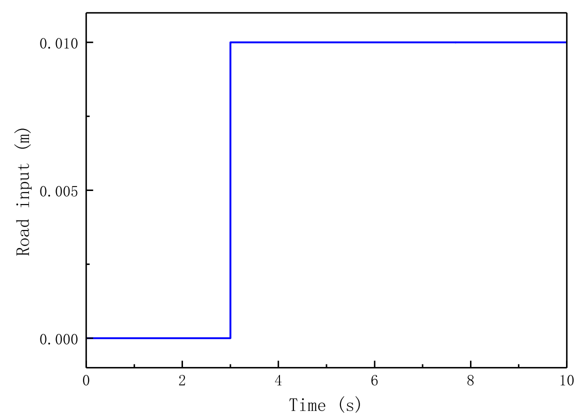

Fig. 4 The disturbance of step road profile

sprung mass under sinusoidal road profile better than the sliding mode controller as shown in Fig. 5. The vertical displacements of the sprung mass under different controllers are in Table 3. Figure 6 shows that the vertical displacement of the sprung mass is more smoother with the homogeneous controller than the sliding mode controller under $0.01\sin(2\pi t)/(1+t^3)$m road profile. Figure 7 shows that the homogeneous controller can stabilize the vertical displacement of the sprung mass smoothly under step road profile. The sliding mode controller is better than the homogeneous controller on decreasing the vertical displacement of the sprung mass, but worse on smoothness. Figures 8, 9, 10 are the outputs of the homogeneous controller and sliding mode controller under different road profiles, from which we can obtain that the outputs of the two controllers are similar under sinusoidal and $0.01\sin(2\pi t)/(1+t^3)$m road profiles. But the output of the homogeneous controller is greatly smaller than the sliding mode controller under step road profile as shown in Fig. 10, which means the homogeneous controller is more practical than the sliding mode controller.

In order to further verify the effectiveness of the designed homogeneous controller in engineering, we test the controller compared with the sliding mode con-

Table 3 The vertical displacements of the sprung mass under different controllers suffering sinusoidal road profile

Controller	Max vertical displacement (m)
Without controller	0.018
Sliding mode controller	0.008
Homogeneous controller	0.003

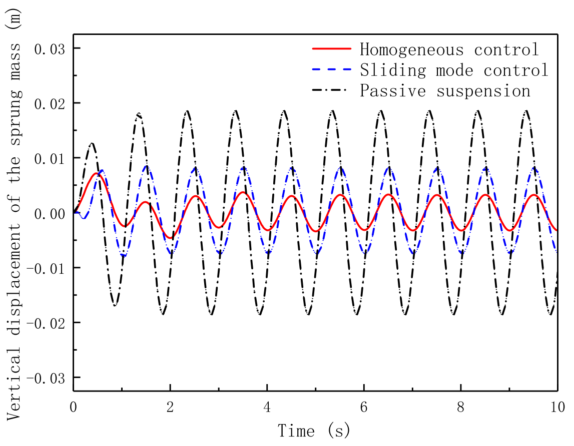

Fig. 5 The vertical displacement of the body under different controllers suffering sinusoidal road profile

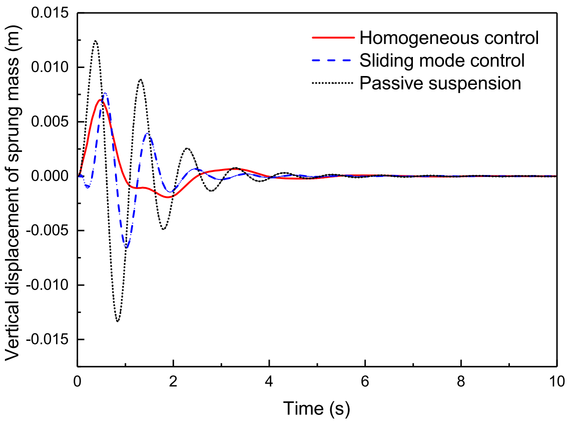

Fig. 6 The vertical displacement of the body under different controllers suffering $0.01\sin(2\pi t)/(1+t^3)$m road profile

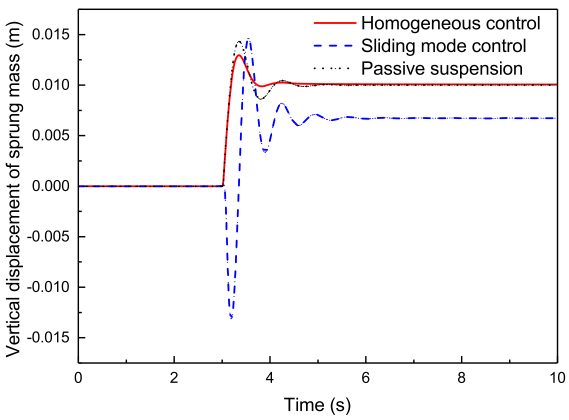

Fig. 7 The vertical displacement of the body under different controllers suffering step road profile

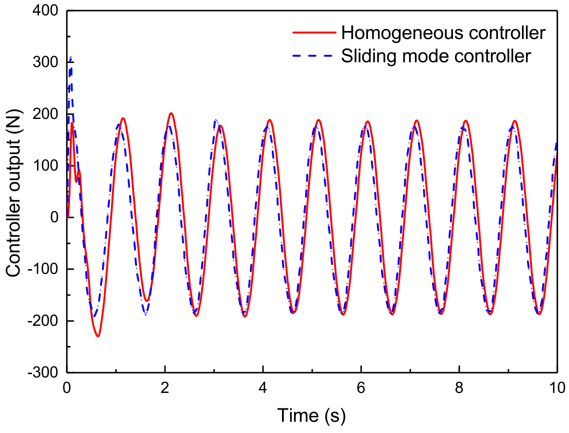

Fig. 8 The outputs of different controllers suffering sinusoidal road profile

troller via an active suspension testbed shown in Fig. 11. In the test, the parameters of the vehicle and the designed homogeneous controller are shown in Tables 4 and 5 respectively.

In the test, we use the vertical acceleration and dynamic travel of the spruang mass to verify the performance of the active suspension furtherly. The test is carried out with the three road profiles used in the Sect. 4 under 40 km/h. Figures 12, 13, 14 are the outputs of the homogeneous controller and sliding mode controller under sinusoidal, $0.01\sin(2\pi t)/(1+t^3)$m and step road profiles. Figures 15, 16, 17 are the vertical accelerations of the spruang mass under sinusoidal, $0.01\sin(2\pi t)/(1+t^3)$m and step road profiles with the homogeneous controller and sliding mode controller

Table 4 The parameters of an IEV-DFIM used in the test

Symbol	Value
Unsprung mass	44 kg
Sprung mass (All)	1370 kg
Axle distance	2776.22 mm
Height of center of mass	540 mm
Front axle tread	1550 mm
Rear axle tread	1550 mm
Yaw moment of inertia	4192 Nm^2
Pitching moment of inertia	4192 Nm^2
Wheel spring stiffness	56000 N/m
Distance from centroid to front axle	1110 mm

Table 5 The parameters of the designed homogeneous output feedback controller used in the test

Symbol	Value	Symbol	Value	Symbol	Value
r_1	1	r_2	1.018	r_3	1.036
r_4	1.054	k_1	1.9	k_2	2.5
k_3	2.7	τ	0.002	β_1	1.3
β_2	1.5	β_3	15	β_4	65
l_1	5	l_2	2.5	l_3	101

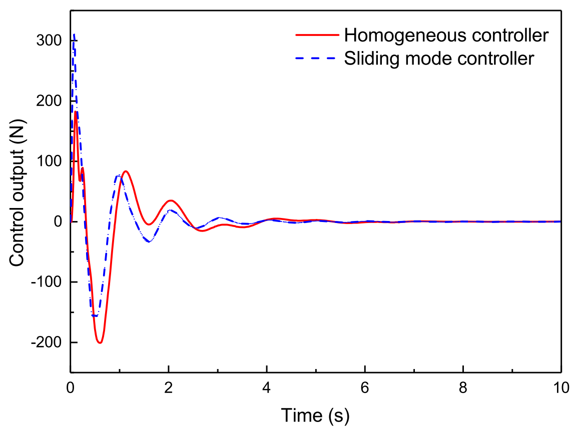

Fig. 9 The outputs of different controllers suffering $0.01\sin(2\pi t)/(1+t^3)$m road profile

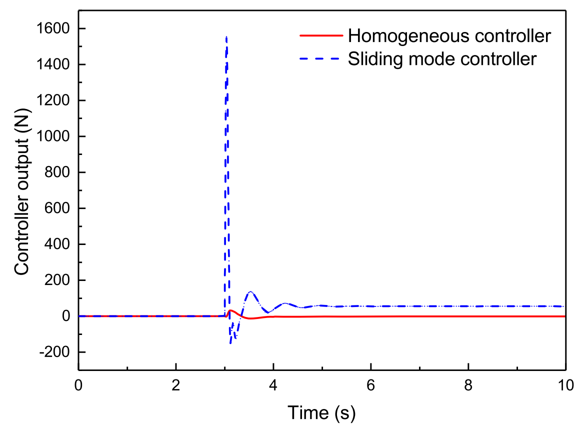

Fig. 10 The outputs of different controllers suffering step road profile

respectively. Figure 15 show that the homogeneous controller and sliding mode controller both decrease the vertical acceleration of the body to small values suffering $0.01\sin(2\pi t)$m road profile, but the homogeneous controller has smaller values. In Fig. 16, the homogeneous controller stabilizes the vertical acceleration to zero within 6 seconds which is faster than the sliding mode controller suffering $0.01\sin(2\pi t)/(1+t^3)$m road profile. As shown in Fig. 17, the two controllers stabilize the vertical acceleration to zero within almost the same time, but the homogeneous controller obtains smaller values. Figures 18, 19, 20 show the dynamic travels of the suspension have

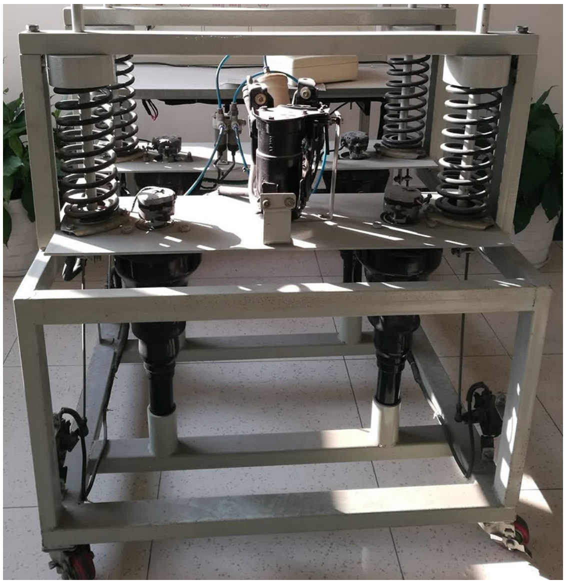

Fig. 11 The active suspension testbed

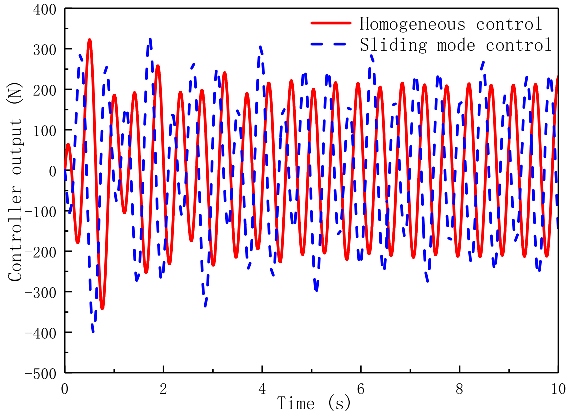

Fig. 12 The outputs of different controllers suffering $\sin(2\pi t)$m road profile in the test

the same trend with the vertical accelerations suffering three road profiles. In conclusion, the test results show that the designed homogeneous controller has better effectiveness than the sliding mode controller in practice.

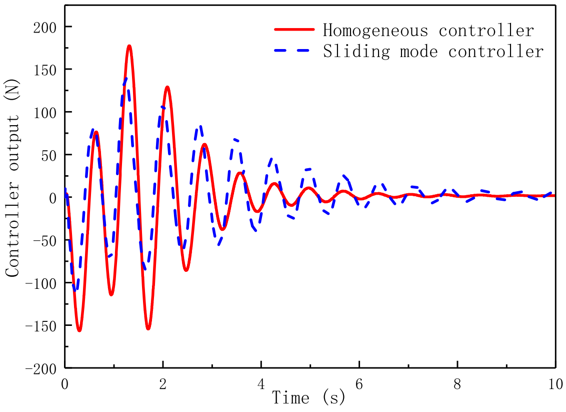

Fig. 13 The outputs of different controllers suffering $\sin(2\pi t)/(1+t^3)$ road profile in the test

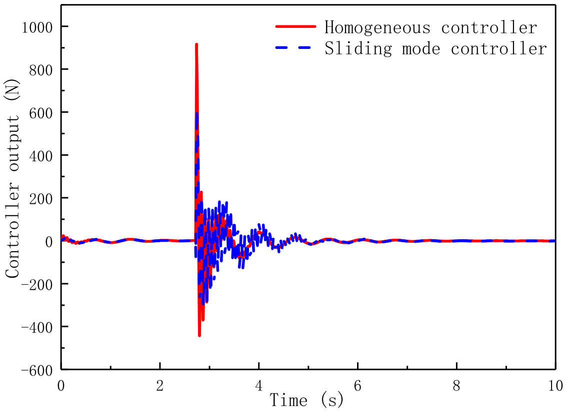

Fig. 14 The outputs of different controllers suffering step road profile in the test

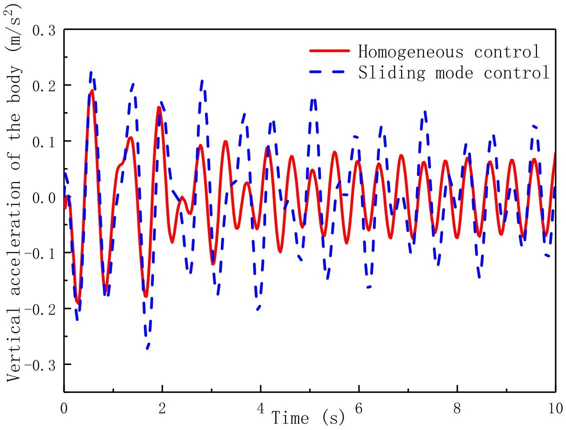

Fig. 15 The vertical acceleration of the body under different controllers suffering $\sin(2\pi t)$ road profile in the test

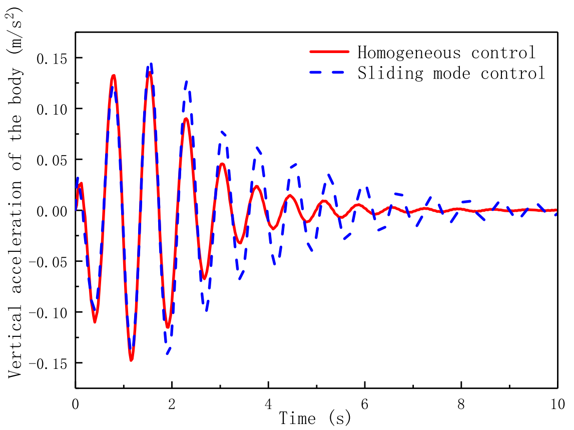

Fig. 16 The vertical acceleration of the body under different controllers suffering $\sin(2\pi t)/(1+t^3)$ road profile in the test

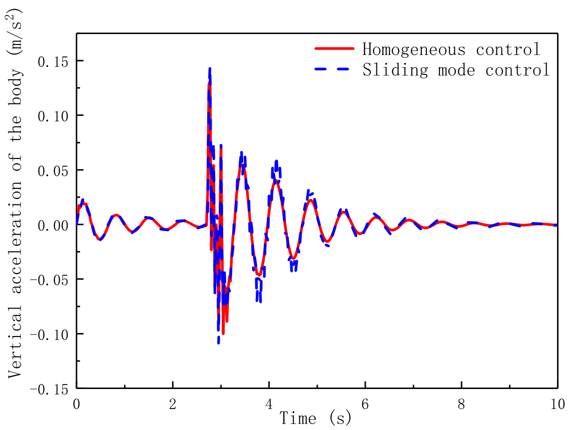

Fig. 17 The vertical acceleration of the body under different controllers suffering step road profile in the test

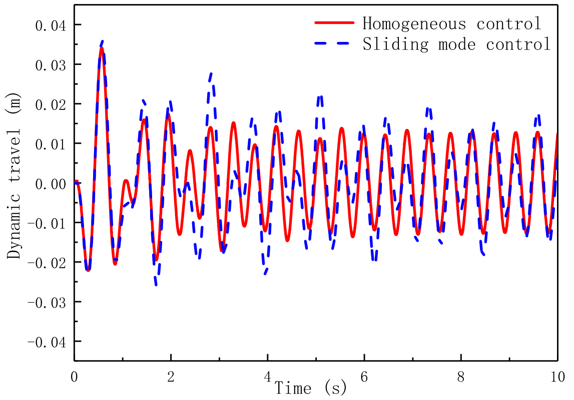

Fig. 18 The dynamic travel of the front-left suspension under different controllers suffering $\sin(2\pi t)$ road profile in the test

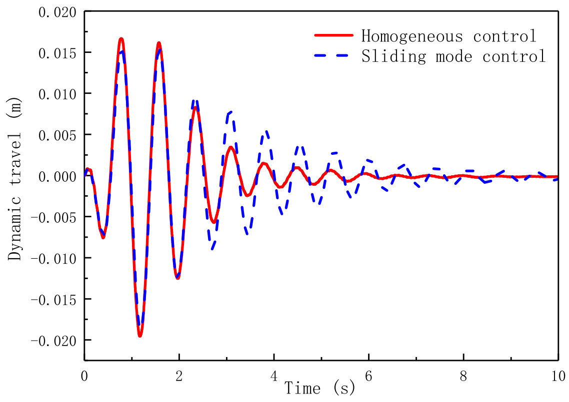

Fig. 19 The dynamic travel of the front-left suspension under different controllers suffering $\sin(2\pi t)/(1+t^3)$ road profile in the test

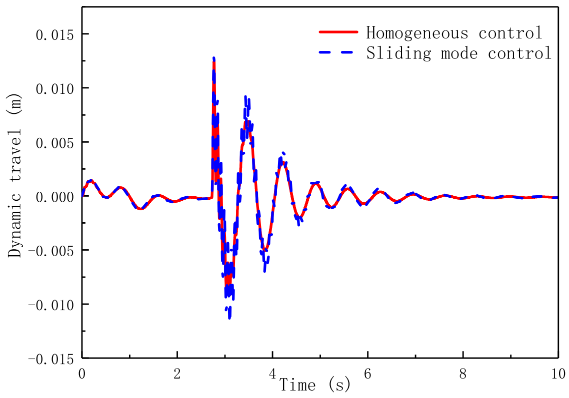

Fig. 20 The dynamic travel of the front-left suspension under different controllers suffering step road profile in the test

5 Conclusion

In this paper, we build an active suspension dynamic model of an IEV-DFIM which considers the coefficients' time-varying in practice. Based on the dynamic model, the active suspension control system is proposed for the IEV-DFIM. A homogeneous output feedback control method for the active suspension of the IEV-DFIM is presented. And we prove that the designed homogeneous controller can stabilize the system by Lyapunov method. The numerical simulation results show that the homogeneous controller is better than the sliding model controller to decrease the vertical displacement of the sprung mass under different road profiles. The test results verify that the designed homogeneous controller also has better effectiveness on vertical acceleration and dynamic travel than the sliding mode controller under different road profiles in practice.

Acknowledgements This research was supported by Zhejiang Provincial Natural Science Foundation of China under Grant No. LZ21E050002, National Natural Science Foundation of China under Grant No. 61773237.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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