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ORIGINAL PAPER

Distributed adaptive neural network consensus for a class of uncertain nonaffine nonlinear multi-agent systems

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Abstract This paper considers the distributed adaptive neural consensus tracking control problem for a class of uncertain nonaffine nonlinear multi-agent systems. By making use of the Taylor expansion technique, the nonaffine nonlinear control input of each subsystem is successfully separated under a weaker decoupling condition, and then, the distributed adaptive control is developed via neural networks (NNs) technique. By introducing the compensation adaptive laws with positive time-varying integrable functions to effectively handle the disturbances and the NN approximation errors in backstepping design process, a new distributed adaptive neural controller is constructed by means of the local output tracking error information of neighborhood agents. It can be proved that all the subsystem outputs asymptotically track to a desired reference trajectory. The efficiency of the established control strategy is demonstrated by the simulation experiment.

Keywords Consensus tracking · Multi-agent systems · Nonaffine nonlinear decoupling · Adaptive backstepping control · Neural networks

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1 Introduction

As we know, a large number of complicated mechanical systems, such as the formation of space spacecraft, underwater robots, multi-vehicle systems, networked autonomous teams, and so on, can be described by the nonlinear multi-agent structure using mathematical modeling method. Correspondingly, many significant control strategies have applied to several different types of MASs [1–9]. In [10], a distributed robust adaptive consensus tracking controller was designed for nonlinear multi-agent systems with bounded disturbances and parameter vectors. The adaptive consensus reliable fault compensation control of a linear system subject to actuator failure and intermittent communication constraint was addressed in [11]. Based on a reliable observer state feedback control method, a distributed fault-tolerant control algorithm [12] was proposed for a class of MASs subject to input time-delays and stochastic actuator faults. Recently, Wei and Xiao [13] solved the consensus control problem of a multi-agent system based on the nonlinear coupling. Furthermore, the scheme of event-triggered using leader-following consensus control approach was proposed in [14] for a fractional-order multi-agent systems.

As universal approximation tools, fuzzy logic systems (FLSs) [15–21] or neural network strategies [22–29] have been well investigated for handling dynamic nonlinear systems. Typically, the developed finite-time fuzzy adaptive controller [30] can be used for pure-feedback nonlinear systems using the backstepping technique. In [31], the authors studied a fuzzy adaptive tracking control for an uncertain nonlinear system subject to input delays. The problem of the adaptive cooperative control [32] was addressed for a nonlinear MAS with dead-zone input via a fuzzy approximation approach. Furthermore, by using the generalized fuzzy hyperbolic approximation method, the proposed fuzzy adaptive fault-tolerant controller with event-triggering weight updated law [33] ensured the closed-loop faulty system being the desired stability performance. Based on the NN technique, the novel adaptive state constraints design scheme [34] and the effective prescribed performance tracking control method [35] were successfully applied for the vehicle active suspension systems (ASSs), respectively.

All the time, the consideration on decoupling control design and analysis of nonaffine nonlinear systems is an interesting and challenging topic, and some significant results have been obtained [21,23,30,32]. In [36], the dynamic surface control (DSC) based on the adaptive finite-time technique of nonaffine nonlinear systems subject to dead-zone inputs was considered using a fuzzy approximation method. The authors of [37] considered an adaptive finite-time funnel control for nonaffine nonlinear systems by employing FLSs and backstepping procedure. Furthermore, Wu et al. [38] presented an adaptive FTC algorithm for nonaffine stochastic nonlinear systems subject to full state constraints and actuator faults. In [39], the consensus tracking control was taken into account for nonaffine nonlinear MASs subject to actuator faults. However, it should be pointed out that a strict decoupling condition $0 < f_l^* \leq \frac{\partial f(x,u)}{\partial u} \leq f_u^*$ for all $(x, u) \in \mathbb{R}^{n+1}$ is utilized by the above results about nonaffine nonlinear systems. Under this condition, the controller design can be realized for only a small number of practical nonaffine nonlinear systems. For instance, consider a nonaffine nonlinear function $f(x, u) = 3u + 3(1 + x^2) + 2.5\sin(ux)$, and then, it is not hard to verify that $\frac{\partial f(x,u)}{\partial u} = 3 + 2.5x\cos(ux)$ do not satisfy this strict decoupling condition.

In this paper, a novel framework of adaptive consensus tracking control-based distributed fuzzy approximation for nonaffine nonlinear MASs is provided. In contrast with the existing work, the distinct features of our results are as follows: (1) the approximation tracking scheme of adaptive fuzzy control is generalized to a class of nonaffine nonlinear MASs with unknown parameter uncertainties and external disturbances; (2) different from our previous work [32,38] and the relevant literature [21,23,36,37,39] using the strict decoupling condition, a weak decoupling condition $0 < f_{il}^* \leq \frac{\partial f_{i,n}(X_i,0)}{\partial u_i} \leq f_{iu}^*$ within a suitable compact set Ω_{X_i} is given, and thus, the Taylor expansion technique can be effectively applied to multiple input decoupling. Clearly, the considered strict-feedback nonlinear MASs in [10] can be reviewed as a special case of the nonaffine nonlinear MASs here; (3) by constructing the proper compensation adaptive laws with positive time-varying integrable function, the effects of the disturbances and the NN approximation errors in the backstepping procedure can be restrained effectively. Also, the designed distributed adaptive controller can ensure all the subsystem outputs asymptotically tracking to a preset reference trajectory.

2 Preliminaries and problem statement

2.1 Basic graph theory

In this paper, we consider an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ which consists of the set of N nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, a set of edges $\mathcal{E} = \mathcal{V} \times \mathcal{V}$ and an associated adjacency matrix $\mathcal{A} = [a_{ij}] \in R^{N\times N}$. An edge e_{ij} between nodes v_i and v_j is defined by (v_i, v_j), which means that the information can flow from v_i to v_j, and vice versa. Note that $e_{ij} \in \mathcal{E}$ if the weight of edge $a_{ij} = a_{ji} = 1$, otherwise $a_{ij} = a_{ji} = 0$. The pair of nodes v_i and v_j are neighbors if $e_{ij} \in \mathcal{E}$. The set of neighbors of node v_i is denoted by $\mathcal{N}_i = \{v_i \in \mathcal{V} | e_{ij} \in \mathcal{E}, j \neq i\}$. An in-degree matrix Δ and a Laplacian matrix $\mathcal{L}$ are both introduced such that $\Delta = \mathrm{diag}\{\Delta_1, \Delta_2, \ldots, \Delta_N\} \in R^{N\times N}$ with $\Delta_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ being the ith row sum of $\mathcal{A}$ and $\mathcal{L} = \Delta - \mathcal{A}$, respectively. In addition, $c_i = 1$ implies the case that the common desired reference signal x_r can be received from the subsystem ith directly; otherwise, it is set as $c_i = 0$.

2.2 Uncertain nonlinear multi-agent system

Consider the following MASs with N uncertain non-affine nonlinear subsystems

$$\begin{aligned}
\dot{x}_{i,1} &= x_{i,2} + f_{i,1}(X_{i,1}) + d_{i,1},\\
\dot{x}_{i,2} &= x_{i,3} + f_{i,2}(X_{i,2}) + d_{i,2},\\
&\vdots\\
\dot{x}_{i,n-1} &= x_{i,n} + f_{i,n-1}(X_{i,n-1}) + d_{i,n-1},\\
\dot{x}_{i,n} &= f_{i,n}(X_i, u_i) + d_{i,n_i},\\
y_i &= x_{i,1}
\end{aligned} \tag{1}$$

where $X_{i,j} = [x_{i,1}, x_{i,2}, \ldots, x_{i,j}]^{\mathrm{T}} \in \mathbb{R}^j$, $X_i = X_{i,n} = [x_{i,1}, x_{i,2}, \ldots, x_{i,n}]^{\mathrm{T}} \in \mathbb{R}^n$, $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ for $i = 1, 2, \ldots, N$; $j = 1, 2, \ldots, n$ are the system states, the control input and the control output, respectively; and $f_{i,j}(\cdot): \mathbb{R}^j \to \mathbb{R}$ and $d_{i,j}$, respectively, are unknown smooth functions and the external disturbances.

2.3 Basic assumptions and lemmas

The consensus tracking objective of this paper is to design a distributed adaptive NN controller, which can guarantee that all the closed-loop signals are uniformly bounded, and all the subsystem outputs y_i for $i = 1, 2, \ldots, N$ asymptotically track to a common desired reference trajectory x_r. For this, some necessary assumptions and lemmas are introduced in the subsequent developments.

Assumption 1 The graph $\mathcal{G}$ is undirected and connected. Also, the common desired reference trajectory x_r can be obtained from at least one subsystem of $\mathcal{G}$, which means $\sum_{i=1}^{N} c_i > 0$.

Assumption 2 For the external disturbance $d_{i,j}$, the reference trajectory x_r together with its jth-order derivative $x_r^{(j)}$ is continuous and bounded, i. e., there exist positive constants d_{ij}^*, x_r^* and x_{rj}^* such that $|d_{i,j}| \le d_{ij}^*$, $|x_r| \le x_r^*$ and $|x_r^{(j)}| \le x_{rj}^*$, $j = 1, 2, \ldots, n$, respectively.

Assumption 3 With a given compact set $\Omega_{X_i} \in \mathbb{R}^n$ and two positive constants f_{il}^* and f_{iu}^*, the following condition holds

$$0 < f_{il}^* \le \frac{\partial f_{i,n}(X_i, 0)}{\partial u_i} \le f_{iu}^* \tag{2}$$

for all $X_i \in \Omega_{X_i}$ and $i = 1, 2, \ldots, n$.

Remark 1 Obviously, it follows from Assumption 1 that the outputs of all subsystems can achieve the objective of consensus tracking. From Assumption 2, it is easy to see that both the external disturbance $d_{i,j}$ and the reference signal x_r together with its jth-order derivative $x_r^{(j)}$ are all bounded. Note that Assumption 3 is looser than the one such that

$$0 < f_{il}^* \le \frac{\partial f_{i,n}(X_i, u_i)}{\partial u_i} \le f_{iu}^* \tag{3}$$

for all $(X_i, u_i) \in \mathbb{R}^{n+1}$ as in [21,23,32,36–39].

Lemma 1 [22] *Given a compact set $\Omega_X \subset \mathbb{R}^m$, if $f(X)$ is a continuous nonlinear function on Ω_X, then there exists a radial basis function neural network (RBFNN) $g(X) = \theta^T \Psi(X)$ such that*

$$\sup_{X\in\Omega_X} |f(X) - \theta^T \Psi(X)| < \epsilon, \forall \epsilon > 0 \tag{4}$$

where $X = [X_1, X_2, \ldots, X_m]^{\mathrm{T}} \in \mathbb{R}^m$ and $\theta = [\theta_1, \theta_2, \ldots, \theta_L]^{\mathrm{T}} \in \mathbb{R}^L$ are state input and weight vectors. $\Psi(X) = [\Psi_1(X), \Psi_2(X), \ldots, \Psi_L(X)]^{\mathrm{T}}$ is basic function vector with $\Psi_i(X)$ being the form of Gaussian function as follows:

$$\Psi_i(X) = e^{-\frac{(X-V_i)^T(X-V_i)}{v_i}}, i = 1, 2, \ldots, L \tag{5}$$

where $v_i > 0$ is the width of Gaussian function and $V_i = [X_{i,1}, X_{i,2}, \ldots, X_{i,m}]^{\mathrm{T}}$ is the center vector.

From Lemma 1, there exists an optimal weight vector θ^* satisfying

$$\theta^* = \arg\min_{\theta\in\Omega_\theta} \left(\sup_{X\in\Omega_X} |f(X) - \theta^T \Psi(X)| \right), \quad \forall X \in \Omega_X, \theta^* \in \Omega_\theta \tag{6}$$

for the compact sets Ω_X and Ω_θ. Accordingly, the nonlinear function $f(X)$ can be expressed by

$$f(X) = \theta^{*T}\Psi(X) + \epsilon(X), \forall X \in \Omega_X, \theta^* \in \Omega_\theta \tag{7}$$

where $|\epsilon(X)| \le \epsilon^*$ and $\epsilon^* > 0$ is a approximation error accuracy.

Lemma 2 *Let Ω_X be a given compact set of $\mathbb{R}^n$, then the uncertain nonlinear multi-agent systems (1) can be transformed as*

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2} + f_{i,1}(X_{i,1}) + d_{i,1}, \\ \dot{x}_{i,2} &= x_{i,3} + f_{i,2}(X_{i,2}) + d_{i,2}, \\ &\vdots \\ \dot{x}_{i,n-1} &= x_{i,n} + f_{i,n-1}(X_{i,n-1}) + d_{i,n-1}, \\ \dot{x}_{i,n} &= g_{i,n}(X_i, 0)u_i + G_{i,n}(X_i) + d_{i,n}, \\ y_i &= x_{i,1} \end{aligned} \tag{8}$$

where

$$\begin{aligned} G_{i,n}(X_i) = f_{i,n}(X_i, 0) &+ \frac{\partial g_{i,n}(X_i, 0)}{\partial u_i} u_i^2 \\ &+ \frac{1}{2!}\frac{\partial^2 g_{i,n}(X_i, 0)}{\partial u_i^2} u_i^3 \\ &+ \cdots + \frac{1}{n!}\frac{\partial^n g_{i,n}(X_i, 0)}{\partial u_i^{n_i}} u_i^{n+1} \\ &+ R_{n+1}(X_i, u_i)u_i \end{aligned} \tag{9}$$

with

$$\begin{aligned} g_{i,n}(X_i, u_i) &= \frac{\partial f_{i,n}(X_i, u_{\lambda_i})}{\partial u_i}, \\ u_{\lambda_i} &= \lambda_i u_i, \lambda_i \in (0, 1) \end{aligned} \tag{10}$$

and

$$\begin{aligned} R_{n+1}(X_i, u_i) &= \frac{1}{(n+1)!}\frac{\partial^{n+1} g_{i,n}(X_i, u_{\mu_i})}{\partial u_i^{n+1}} u_i^{n+1}, \\ u_{\mu_i} &= \mu_i u_i, \mu_i \in (0, 1). \end{aligned} \tag{11}$$

Proof Using (2) and the mean-value theorem, it is true that the nonlinear term $f_{i,n}(X_i, u_i)$ of (1) can be decomposed into

$$f_{i,n}(X_i, u_i) = f_{i,n}(X_i, 0) + g_{i,n}(X_i, u_i)u_i \tag{12}$$

where

$$g_{i,n}(X_i, u_i) = \frac{\partial f_{i,n}(X_i, u_i)}{\partial u_i}\Big|_{u_i = u_{\lambda_i}} \tag{13}$$

with $u_{\lambda_i} = \lambda_i u_i, \lambda_i \in (0, 1)$. □

Clearly, we also see that $g_{i,n}(\cdot)$ is a smooth function for the variables u_i and X_i, respectively. Further, to separate u_i from $g_{i,n_i}(\cdot)$ and thus effectively design controller, the Taylor's theorem in [40] can be applied to decoupling analysis

$$\begin{aligned} g_{i,n}(X_i, u_i) = g_{i,n}(X_i, 0) &+ \frac{\partial g_{i,n}(X_i, 0)}{\partial u_i} u_i \\ &+ \frac{1}{2!}\frac{\partial^2 g_{i,n}(X_i, 0)}{\partial u_i^2} u_i^2 \\ &+ \cdots + \frac{1}{n!}\frac{\partial^n g_{i,n}(X_i, 0)}{\partial u_i^n} u_i^n \\ &+ R_{n+1}(X_i, u_i) \end{aligned} \tag{14}$$

where

$$R_{n+1}(X_i, u_i) = \frac{1}{(n+1)!}\frac{\partial^{n+1} g_{i,n}(X_i, u_{\mu_i})}{\partial u_i^{n+1}} u_i^{n+1} \tag{15}$$

with $u_{\mu_i} = \mu_i u_i, \mu_i \in (0, 1)$. Therefore, it is seen that

$$f_{i,n}(X_i, u_i) = g_{i,n}(X_i, 0)u_i + G_{i,n}(X_i) \tag{16}$$

where $G_{i,n}(X_i)$ is defined as in (9), this also shows that (8) holds.

Lemma 3 [1] *If a undirected graph $\mathcal{G}$ with the corresponding associated adjacency matrix $\mathcal{A} \in \mathbb{R}^{N\times N}$ is connective, then we can conclude that the matrix $\mathcal{L}+\mathcal{C}$ with $\mathcal{C} = \mathrm{diag}\{c_1, c_2, \ldots, c_N\}$ being at least one $c_i > 0$ is positive definite for $i = 1, 2, \ldots, N$.*

Lemma 4 [42] *With $\sigma > 0$, the inequality*

$$0 \le |x| - \frac{x^2}{\sqrt{x^2+\sigma^2}} \le \sigma \tag{17}$$

holds for any $x \in \mathbb{R}$.

Lemma 5 *For a given positive function $\delta(t) > 0, \forall t \ge 0$ and $x \in \mathbb{R}$, the following result is true*

$$0 \le |x| - x\tanh\left(\frac{x}{\delta(t)}\right) \le \kappa\delta(t), \forall t \ge 0 \tag{18}$$

where $\kappa = \sup_{t>0}\{\frac{t}{1+e^t}\} = 0.2785$.

Proof This conclusion can be straightforward obtained from [41], and we omit the details here for brevity.

Lemma 6 [43] *If a continuous function $q(t)$ such that $q \in \mathcal{L}^2[0, +\infty)$ and $\dot{q} \in \mathcal{L}_\infty$, then it holds*

$$\lim_{t\to\infty} q(t) = 0. \tag{19}$$

3 Distributed adaptive neural network control

This section will develop a distributed NN adaptive control scheme of nonlinear MASs (1) by using the backstepping method.

Step 1 The following two error variables are first introduced

$$e_{i,1} = \sum_{j=1}^{N} a_{ij}(y_i - y_j) + c_i(y_i - x_r), \qquad e_{i,2} = x_{i,2} - v_{i,1} \tag{20}$$

with $v_{i,1}$ being the corresponding virtual control. Setting $e_1 = [e_{1,1}, e_{2,1}, \ldots, e_{N,1}]^{\mathrm{T}}$ leads to

$$e_1 = (\mathcal{L} + \mathcal{C})e_y \tag{21}$$

where $e_y = y - \bar{x}_r = [e_{y1}, e_{y2}, \ldots, e_{yN}]^{\mathrm{T}}$ with $y = [y_1, y_2, \ldots, y_N]^{\mathrm{T}}$ and $\bar{x}_r = [x_r, x_r, \ldots, x_r]^{\mathrm{T}} \in \mathbb{R}^N$.

By using (1), (20) and (21), the derivative of e_1 is

$$\dot{e}_1 = (\mathcal{L} + \mathcal{C}) \begin{bmatrix} v_{1,1} + e_{1,2} + f_{1,1} + d_{1,1} - \dot{x}_r \\ v_{2,1} + e_{2,2} + f_{2,1} + d_{2,1} - \dot{x}_r \\ \vdots \\ v_{N,1} + e_{N,2} + f_{N,1} + d_{N,1} - \dot{x}_r \end{bmatrix}. \tag{22}$$

Based on Lemma 1, the following approximation scheme of FLS can be applied within a compact set $\Omega_{Z_{i,1}}$

$$F_{i,1}(Z_{i,1}) = f_{i,1}(Z_{i,1}) = \theta_{i,1}^{*T} \Psi_{i,1}(Z_{i,1}) + \epsilon_{i,1}(Z_{i,1}) \tag{23}$$

where $Z_{i,1} = x_{i,1} \in \Omega_{Z_{i,1}}$ and $|\epsilon_{i,1}| \le \epsilon_{i,1}^*$ with $\epsilon_{i,1}^* > 0$ being an error accuracy.

We can choose $v_{i,1}$ as

$$v_{i,1} = -\chi_{i,1}e_{i,1} - \hat{\theta}_{i,1}^T \Psi_{i,1} + c_i\dot{x}_r - \frac{e_{i,1}\hat{D}_{i,1}}{\sqrt{e_{i,1}^2 + \sigma_i^2}} \tag{24}$$

with $\chi_{i,1} > 0$ being a design parameter, and $\sigma_i(t) > 0$ being a positive function such that $\int_0^\infty \sigma_i(t) \le \bar{\sigma}_i < +\infty$. In addition, $\hat{D}_{i,1}$ is the estimate of $D_{i,1}^* = d_{i,1}^* + \epsilon_{i,1}^* + (1 - c_i)x_r^*$.

Invoking Assumption 1 and Lemma 3, the following Lyapunov function is defined by

$$V_1 = \frac{1}{2}e_y^T(\mathcal{L} + \mathcal{C})e_y + \frac{1}{2}\sum_{i=1}^{N}\left(\tilde{\theta}_{i,1}^T \Gamma_{i,1}^{-1}\tilde{\theta}_{i,1} + \gamma_{i,1}^{-1}\tilde{D}_{i,1}^2\right) \tag{25}$$

where $\tilde{\theta}_{i,1} = \theta_{i,1}^* - \hat{\theta}_{i,1}$ and $\tilde{D}_{i,1} = D_{i,1}^* - \hat{D}_{i,1}$ denote the parameter estimation errors, respectively. $\Gamma_{i,1} = \Gamma_{i,1}^T > 0$ and $\gamma_{i,1} > 0$ are some design parameters.

We take the time derivative of (25) and it holds

$$\begin{aligned}
\dot{V}_1 = {} & \sum_{i=1}^{N} e_{i,1}\Bigg(-\chi_{i,1}e_{i,1} + e_{i,2} + \tilde{\theta}_{i,1}^T\Psi_{i,1} + d_{i,1} \\
& + \epsilon_{i,1} - \dot{x}_r + c_i\dot{x}_r - \frac{e_{i,1}\hat{D}_{i,1}}{\sqrt{e_{i,1}^2 + \sigma_i^2}}\Bigg) \\
& - \sum_{i=1}^{N}\tilde{\theta}_{i,1}^T\Gamma_{i,1}^{-1}\dot{\hat{\theta}}_{i,1} - \gamma_{i,1}^{-1}\tilde{D}_{i,1}\dot{\hat{D}}_{i,1} \\
\le {} & \sum_{i=1}^{N}\Bigg(-\chi_{i,1}e_{i,1}^2 + e_{i,1}e_{i,2} + \tilde{\theta}_{i,1}^T\Psi_{i,1}e_{i,1} \\
& + |e_{i,1}|D_{i,1}^* - \frac{e_{i,1}^2\hat{D}_{i,1}}{\sqrt{e_{i,1}^2 + \sigma_i^2}}\Bigg) \\
& - \sum_{i=1}^{N}\tilde{\theta}_{i,1}^T\Gamma_{i,1}^{-1}\dot{\hat{\theta}}_{i,1} - \gamma_{i,1}^{-1}\tilde{D}_{i,1}\dot{\hat{D}}_{i,1} \\
\le {} & \sum_{i=1}^{N}\Bigg(-\chi_{i,1}e_{i,1}^2 + e_{i,1}e_{i,2} + \Bigg(|e_{i,1}| \\
& - \frac{e_{i,1}^2}{\sqrt{e_{i,1}^2 + \sigma_i^2}}\Bigg)D_{i,1}^* + \gamma_{i,1}^{-1}\tilde{D}_{i,1} \\
& \times \left(\gamma_{i,1}\frac{e_{i,1}^2}{\sqrt{e_{i,1}^2 + \sigma_i^2}} - \dot{\hat{D}}_{i,1}\right) + \tilde{\theta}_{i,1}^T\Gamma_{i,1}^{-1} \\
& \times \left(\Gamma_{i,1}\Psi_{i,1}e_{i,1} - \dot{\hat{\theta}}_{i,1}\right)\Bigg).
\end{aligned} \tag{26}$$

Introduce the adaptive control laws as

$$\begin{aligned}
\dot{\hat{D}}_{i,1} &= -\gamma_{i,1}\sigma_i\hat{D}_{i,1} + \gamma_{i,1}\frac{e_{i,1}^2}{\sqrt{e_{i,1}^2 + \sigma_i^2}} \\
\dot{\hat{\theta}}_{i,1} &= -\sigma_i\Gamma_{i,1}\hat{\theta}_{i,1} + \Gamma_{i,1}\Psi_{i,1}e_{i,1}.
\end{aligned} \tag{27}$$

Substituting (27) into (26) and applying Lemma 4, it can be verify that

$$\begin{aligned}
\dot{V}_1 \le {} & \sum_{i=1}^{N}\Big(-\chi_{i,1}e_{i,1}^2 + e_{i,1}e_{i,2} + \sigma_i\big(D_{i,1}^* \\
& + \tilde{D}_{i,1}\hat{D}_{i,1} + \tilde{\theta}_{i,1}^T\hat{\theta}_{i,1}\big)\Big).
\end{aligned} \tag{28}$$

Step 2 By introducing a new transformation $e_{i,3} = x_{i,3} - v_{i,2}$ and taking the derivative of $e_{i,2}$ in (20), we have

$$\begin{aligned}\dot{e}_{i,2} = & e_{i,3} + v_{i,2} + f_{i,2}(X_{i,2}) + d_{i,2} \\ & - \frac{\partial v_{i,1}}{\partial x_{i,1}}(x_{i,2} + f_{i,1} + d_{i,1}) - \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}}\dot{\hat{\theta}}_{i,1} \\ & - \frac{\partial v_{i,1}}{\partial \sigma_i}\dot{\sigma}_i - \frac{\partial v_{i,1}}{\partial \hat{D}_{i,1}}\dot{\hat{D}}_{i,1} - c_i \sum_{j=0}^{1} \frac{\partial v_{i,1}}{\partial x_r^{(j)}} x_r^{(j+1)} \\ & - \sum_{j=1}^{N} a_{ij} \frac{\partial v_{i,1}}{\partial x_{j,1}}(x_{j,2} + f_{j,1} + d_{j,1}). \end{aligned} \tag{29}$$

Invoking Lemma 1, we choose the following FLS to approximate the nonlinear compound function $F_{i,2}$ within a compact set $\Omega_{Z_{i,2}}$

$$\begin{aligned}F_{i,2}(Z_{i,2}) & = e_{i,1} + f_{i,2} - \frac{\partial v_{i,1}}{\partial x_{i,1}}(x_{i,2} + f_{i,1}) \\ & - \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}}\dot{\hat{\theta}}_{i,1} - \frac{\partial v_{i,1}}{\partial \sigma_i}\dot{\sigma}_i - \frac{\partial v_{i,1}}{\partial \hat{D}_{i,1}}\dot{\hat{D}}_{i,1} \\ & - c_i \sum_{j=0}^{1} \frac{\partial v_{i,1}}{\partial x_r^{(j)}} x_r^{(j+1)} - \sum_{j=1}^{N} a_{ij} \frac{\partial v_{i,1}}{\partial x_{j,1}} \\ & \times (x_{j,2} + f_{j,1}) \\ & = \theta_{i,2}^{*T} \Psi_{i,2}(Z_{i,2}) + \epsilon_{i,2}(Z_{i,2}) \end{aligned} \tag{30}$$

where $Z_{i,2} = [X_{i,2}, \hat{\theta}_{i,1}, \sigma_i, \dot{\sigma}_i, \hat{D}_{i,1}, x_{j,1}, x_{j,2}, x_r, \dot{x}_r, \ddot{x}_r]^{\mathrm{T}} \in \Omega_{Z_{i,2}}$ and $|\epsilon_{i,2}| \le \epsilon_{i,2}^*$ with $\epsilon_{i,2}^* > 0$ being an error accuracy.

The virtual controller $v_{i,2}$ can be chosen as

$$v_{i,2} = -\chi_{i,2} e_{i,2} - \hat{\theta}_{i,2}^T \Psi_{i,2} - \frac{(A_i + 2)\, e_{i,2} B_{i,1}^2 \hat{D}_{i,2}}{\sqrt{e_{i,2}^2 B_{i,1}^2 + \sigma_i^2}} \tag{31}$$

with $\chi_{i,2} > 0$ being a design parameter, and $A_i = \sum_{j=1}^{N} a_{ij}, B_{i,1} = \sqrt{4 + \left(\frac{\partial v_{i,1}}{\partial x_{i,1}}\right)^2 + \sum_{j=1}^{N} a_{ij}\left(\frac{\partial v_{i,1}}{\partial x_{j,1}}\right)^2}$. $\hat{\theta}_{i,2}$ and $\hat{D}_{i,2}$ are the estimates of $\theta_{i,2}^*$ and $D_{i,2}^* = \max\{d_{i,1}^*, d_{i,2}^*, d_{j,1}^*, \epsilon_{i,2}^*\}$, respectively.

The Lyapunov function can be chosen as

$$\begin{aligned}V_2 = V_1 + \frac{1}{2}\sum_{i=1}^{N}\Big(& e_{i,2}^2 + \gamma_{i,2}^{-1}(A_i + 2)\tilde{D}_{i,2}^2 \\ & + \tilde{\theta}_{i,2}^T \Gamma_{i,2}^{-1} \tilde{\theta}_{i,2}\Big) \end{aligned} \tag{32}$$

where $\tilde{\theta}_{i,2} = \theta_{i,2}^* - \hat{\theta}_{i,2}$ and $\tilde{D}_{i,2} = D_{i,2}^* - \hat{D}_{i,2}$ are the parameter estimation errors. $\Gamma_{i,2} = \Gamma_{i,2}^T > 0$ and $\gamma_{i,2} > 0$ are some design parameters.

Combining (28)–(31) and the following inequality

$$\sum_{k=1}^{n} \lambda_k \le n \sqrt{\sum_{k=1}^{n} \lambda_k^2} \tag{33}$$

for $\lambda_k \ge 0, k = 1, 2, \ldots, n$, the derivative of (32) is computed as

$$\begin{aligned}\dot{V}_2 \le \sum_{i=1}^{N}\Bigg(& -\sum_{l=1}^{2}\chi_{i,l} e_{i,l}^2 + e_{i,2}e_{i,3} + \tilde{\theta}_{i,2}^T \Psi_{i,2} e_{i,2} \\ & + \sigma_i\left(D_{i,1}^* + \tilde{D}_{i,1}\hat{D}_{i,1} + \tilde{\theta}_{i,1}^T \hat{\theta}_{i,1}\right) \\ & + (A_i + 2)\left(|e_{i,2}|B_{i,1} - \frac{e_{i,1}^2 B_{i,1}^2}{\sqrt{e_{i,1}^2 B_{i,1}^2 + \sigma_i^2}}\right) D_{i,2}^* \\ & + \gamma_{i,2}^{-1}(A_i + 2)\left(\gamma_{i,2}\frac{e_{i,2}^2 B_{i,1}^2}{\sqrt{e_{i,2}^2 B_{i,1}^2 + \sigma_i^2}} - \dot{\hat{D}}_{i,2}\right) \\ & \times \tilde{D}_{i,2} - \sum_{i=1}^{N} \tilde{\theta}_{i,2}^T \Gamma_{i,2}^{-1} \dot{\hat{\theta}}_{i,2}\Bigg). \end{aligned} \tag{34}$$

Choose the following adaptive control laws

$$\begin{aligned}\dot{\hat{D}}_{i,2} &= -\gamma_{i,2}\sigma_i \hat{D}_{i,2} + \gamma_{i,2}\frac{e_{i,2}^2 B_{i,1}^2}{\sqrt{e_{i,2}^2 B_{i,1}^2 + \sigma_i^2}}, \\ \dot{\hat{\theta}}_{i,2} &= -\sigma_i \Gamma_{i,2}\hat{\theta}_{i,2} + \Gamma_{i,2}\Psi_{i,2} e_{i,2}. \end{aligned} \tag{35}$$

Based on Lemma 4 and (35), (34) becomes

$$\begin{aligned}\dot{V}_2 \le \sum_{i=1}^{N}\Bigg(& -\sum_{l=1}^{2}\chi_{i,l} e_{i,l}^2 + e_{i,2}e_{i,3} + \sigma_i\Big(\big(D_{i,1}^* \\ & + (A_i + 2)\, D_{i,2}^*\big) + \Big(\tilde{D}_{i,1} D_{i,1}^* + (A_i + 2) \\ & \times \tilde{D}_{i,2} D_{i,2}^*\Big) + \sum_{l=1}^{2}\tilde{\theta}_{i,l}^T \hat{\theta}_{i,l}\Big)\Bigg). \end{aligned} \tag{36}$$

Step $k(3 \le k \le n-1)$ Similarly, by using the transformation $e_{i,k+1} = x_{i,k+1} - v_{i,k}$, its derivative is

$$\begin{aligned}\dot{e}_{i,k} &= e_{i,k+1} + v_{i,k} + f_{i,k}(X_{i,k}) + d_{i,k}\\ &- \sum_{l=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial x_{i,l}} \left(x_{i,l+1} + f_{i,l} + d_{i,l}\right)\\ &- \sum_{l=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,l}} \dot{\hat{\theta}}_{i,l} - \sum_{l=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \sigma_i^{(l-1)}} \sigma_i^{(l)}\\ &- \sum_{l=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{D}_{i,l}} \dot{\hat{D}}_{i,l} - c_i \sum_{l=0}^{k-1} \frac{\partial v_{i,k-1}}{\partial x_r^{(l)}} x_r^{(l+1)}\\ &- \sum_{j=1}^{N} a_{ij} \sum_{l=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial x_{j,l}} \left(x_{j,l+1} + f_{j,l} + d_{j,l}\right)\end{aligned} \tag{37}$$

Furthermore, by repeating the same process as in **Steps 1** and **2**, it is recursive to compute that

$$\begin{aligned}\dot{V}_{k-1} \le \sum_{i=1}^{N} \Bigg(&- \sum_{l=1}^{k-1} \chi_{i,l} e_{i,l}^2 + e_{i,k-1} e_{i,k}\\ &+ \sigma_i \Bigg(\left(D_{i,1}^* + \sum_{l=2}^{k-1} ((l-1)A_i + l) D_{i,l}^* \right)\\ &+ \Bigg(\tilde{D}_{i,1} D_{i,1}^* + \sum_{l=2}^{k-1} ((l-1)A_i + l) \tilde{D}_{i,l}\\ &\times D_{i,l}^* \Bigg) + \sum_{l=1}^{k-1} \tilde{\theta}_{i,l}^T \hat{\theta}_{i,l} \Bigg) \Bigg)\end{aligned} \tag{38}$$

where $\chi_{i,l} > 0$ are the design parameters for $l = 1, 2, \ldots, k-1$.

As in (30), we choose the following FLS to approximate the nonlinear compound function $F_{i,k}$ within a compact set $\Omega_{Z_{i,k}}$

$$\begin{aligned}F_{i,k}(Z_{i,k}) &= e_{i,k-1} + f_{i,k} - \sum_{l=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial x_{i,l}}\\ &\times \left(x_{i,l+1} + f_{i,l}\right) - \sum_{l=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,l}} \dot{\hat{\theta}}_{i,l}\\ &- \sum_{l=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \sigma_i^{(l-1)}} \sigma_i^{(l)} - \sum_{l=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{D}_{i,l}} \dot{\hat{D}}_{i,l}\\ &- c_i \sum_{l=0}^{k-1} \frac{\partial v_{i,k-1}}{\partial x_r^{(l)}} x_r^{(l+1)} - \sum_{j=1}^{N} a_{ij}\\ &\times \sum_{l=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial x_{j,l}} \left(x_{j,l+1} + f_{j,l} + d_{j,l}\right)\\ &= \theta_{i,k}^{*T} \Psi_{i,k}(Z_{i,k}) + \epsilon_{i,k}(Z_{i,k})\end{aligned} \tag{39}$$

where $Z_{i,k}=[X_{i,k}, \hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,k-1}, \sigma_i, \dot{\sigma}_i, \ldots, \dot{\sigma}_i^{(k-1)}, \hat{D}_{i,1}, \ldots, \hat{D}_{i,k-1}, x_{j,1}, \ldots, x_{j,k}, x_r, \dot{x}_r, \ldots, x_r^{(k)}]^{\mathrm{T}} \in \Omega_{Z_{i,k}}$ and $|\epsilon_{i,k}| \le \epsilon_{i,k}^*$ with $\epsilon_{i,k}^* > 0$ being an error accuracy.

The virtual control $v_{i,k}$ is then designed as

$$\begin{aligned}v_{i,k} &= -\chi_{i,k} e_{i,k} - \hat{\theta}_{i,k}^T \Psi_{i,k}\\ &- \frac{((k-1)A_i + k)\, e_{i,k} B_{i,k-1}^2 \hat{D}_{i,k}}{\sqrt{e_{i,k}^2 B_{i,k-1}^2 + \sigma_i^2}}\end{aligned} \tag{40}$$

where $\chi_{i,k} > 0$ is a design parameter, and $A_i = \sum_{j=1}^{N} a_{ij}$,

$$\begin{aligned}B_{i,k-1} = \Bigg(&4 + \sum_{l=1}^{k-1} \left(\frac{\partial v_{i,k-1}}{\partial x_{i,l}} \right)^2\\ &+ \sum_{j=1}^{N} a_{ij} \sum_{l=1}^{k-1} \left(\frac{\partial v_{i,k-l}}{\partial x_{j,l}} \right)^2 \Bigg)^{\frac{1}{2}}\end{aligned}$$

$\hat{\theta}_{i,k}$ and $\hat{D}_{i,k}$ are the estimates of $\theta_{i,k}^*$ and $D_{i,k}^* = \max\{d_{i,1}^*, \ldots, d_{i,k}^*, d_{j,1}^*, \ldots, d_{j,k-1}^*, \epsilon_{i,k}^*\}$, respectively.

As in (32), we set

$$\begin{aligned}V_k = V_{k-1} + \frac{1}{2} \sum_{i=1}^{N} \Big(&e_{i,k}^2 + \gamma_{i,k}^{-1} \left((k-1)\, A_i + k\right)\\ &\times \tilde{D}_{i,k}^2 + \tilde{\theta}_{i,k}^T \Gamma_{i,k}^{-1} \tilde{\theta}_{i,k} \Big)\end{aligned} \tag{41}$$

where $\tilde{\theta}_{i,k} = \theta_{i,k}^* - \hat{\theta}_{i,k}$ and $\tilde{D}_{i,k} = D_{i,k}^* - \hat{D}_{i,k}$ denote the parameter estimation errors. $\Gamma_{i,k} = \Gamma_{i,k}^T > 0$ and $\gamma_{i,k} > 0$ are some design constants.

From (37)–(40), the derivative of (41) is

$$\begin{aligned}\dot{V}_k \le \sum_{i=1}^{N} \Bigg(&- \sum_{l=1}^{k} \chi_{i,l} e_{i,l}^2 + e_{i,k} e_{i,k+1} + \sigma_i \Bigg(\Bigg(D_{i,1}^* + ((k\\ &- 2)A_i + (k-1)) \sum_{l=2}^{k-1} D_{i,l}^* \Bigg) + \Bigg(\tilde{D}_{i,1} D_{i,1}^* + ((k\\ &- 2)A_i + (k-1)) \sum_{l=2}^{k-1} \tilde{D}_{i,l} D_{i,l}^* \Bigg) + \sum_{l=1}^{k-1} \tilde{\theta}_{i,l}^T \hat{\theta}_{i,l} \Bigg) \Bigg)\\ &+ \sum_{i=1}^{N} \Bigg(((k-1)\, A_i + k) \Bigg(|e_{i,k}| B_{i,k-1}\\ &- \frac{e_{i,k}^2 B_{i,k-1}^2}{\sqrt{e_{i,k}^2 B_{i,k-1}^2 + \sigma_i^2}} \Bigg) D_{i,k}^* + ((k-1)\, A_i + k)\\ &\times \left(\gamma_{i,k} \frac{e_{i,k}^2 B_{i,k-1}^2}{\sqrt{e_{i,k}^2 B_{i,k-1}^2 + \sigma_i^2}} - \dot{\hat{D}}_{i,k} \right) \tilde{D}_{i,k} + \tilde{\theta}_{i,k}^T\\ &\times \Gamma_{i,k}^{-1} \times \left(\Gamma_{i,k} \Psi_{i,k}(Z_{i,k}) e_{i,k} - \dot{\hat{\theta}}_{i,k} \right) \Bigg).\end{aligned} \tag{42}$$

We choose the following adaptive control laws

$$\begin{aligned}\dot{\hat{D}}_{i,k} &= -\gamma_{i,k}\sigma_i \hat{D}_{i,k} + \gamma_{i,k}\frac{e_{i,k}^2 B_{i,k-1}^2}{\sqrt{e_{i,k}^2 B_{i,k-1}^2 + \sigma_i^2}} \\ \dot{\hat{\theta}}_{i,k} &= -\sigma_i \Gamma_{i,k}\hat{\theta}_{i,k} + \Gamma_{i,k}\Psi_{i,k}e_{i,k}. \end{aligned} \tag{43}$$

By using Lemma 4 and substituting (43) into (42), it follows that

$$\begin{aligned}\dot{V}_k \le \sum_{i=1}^{N}\Bigg(&-\sum_{l=1}^{k}\chi_{i,l}e_{i,l}^2 + e_{i,k}e_{i,k+1} + \sigma_i\Bigg(\Bigg(D_{i,1}^* \\ &+ ((k-1)A_i + k)\sum_{l=2}^{k-1}D_{i,l}^*\Bigg) + \Bigg(\tilde{D}_{i,1}D_{i,1}^* \\ &+ ((k-1)A_i + k)\sum_{l=2}^{k}\tilde{D}_{i,l}D_{i,l}^*\Bigg) \\ &+ \sum_{l=1}^{k}\tilde{\theta}_{i,l}^T\hat{\theta}_{i,l}\Bigg)\Bigg). \end{aligned} \tag{44}$$

Step n: Combining the transformation $e_{i,n} = x_{i,n} - v_{i,n-1}$ and Lemma 2, we can obtain

$$\begin{aligned}\dot{e}_{i,n} = {} & g_{i,n}(X_i,0)u_i + G_{i,n}(X_i) + d_{i,n} \\ &- \sum_{l=1}^{n-1}\frac{\partial v_{i,n-1}}{\partial \hat{\theta}_{i,l}}\dot{\hat{\theta}}_{i,l} - \sum_{l=1}^{n-1}\frac{\partial v_{i,n-1}}{\partial x_{i,l}}\big(x_{i,l+1} \\ &+ f_{i,l} + d_{i,l}\big) - \sum_{l=1}^{n-1}\frac{\partial v_{i,n-1}}{\partial \sigma_i^{(l-1)}}\sigma_i^{(l)} \\ &- \sum_{l=1}^{n-1}\frac{\partial v_{i,n-1}}{\partial \hat{D}_{i,l}}\dot{\hat{D}}_{i,l} - c_i\sum_{l=0}^{n-1}\frac{\partial v_{i,n-1}}{\partial x_r^{(l)}}x_r^{(l+1)} \\ &- \sum_{j=1}^{N}a_{ij}\sum_{l=1}^{n-1}\frac{\partial v_{i,n-1}}{\partial x_{j,l}}\big(x_{j,l+1} + f_{j,l} + d_{j,l}\big). \end{aligned} \tag{45}$$

Also, the nonlinear compound function $F_{i,n}$ can be approximated by the following FLS within a compact set $\Omega_{Z_{i,n}}$

$$\begin{aligned}F_{i,n}(Z_{i,n}) = {} & e_{i,n-1} + G_{i,n} - \sum_{l=1}^{n-1}\frac{\partial v_{i,n-1}}{\partial x_{i,l}} \\ &\times \big(x_{i,l+1} + f_{i,l}\big) - \sum_{l=1}^{n-1}\frac{\partial v_{i,n-1}}{\partial \hat{\theta}_{i,l}}\dot{\hat{\theta}}_{i,l} \\ &- \sum_{l=1}^{n-1}\frac{\partial v_{i,n-1}}{\partial \sigma_i^{(l-1)}}\sigma_i^{(l)} - \sum_{l=1}^{n-1}\frac{\partial v_{i,n-1}}{\partial \hat{D}_{i,l}}\dot{\hat{D}}_{i,l} \\ &- c_i\sum_{l=0}^{n-1}\frac{\partial v_{i,n-1}}{\partial x_r^{(l)}}x_r^{(l+1)} - \sum_{j=1}^{N}a_{ij} \\ &\times \sum_{l=1}^{n-1}\frac{\partial v_{i,n-1}}{\partial x_{j,l}}\big(x_{j,l+1} + f_{j,l} + d_{j,l}\big) \\ = {} & \theta_{i,n}^{*T}\Psi_{i,n}(Z_{i,n}) + \epsilon_{i,n}(Z_{i,n}) \end{aligned} \tag{46}$$

where $Z_{i,n}=[X_{i,n}, \hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,n-1}, \sigma_i, \dot{\sigma}_i, \ldots, \dot{\sigma}_i^{(n-1)}, \hat{D}_{i,1}, \ldots, \hat{D}_{i,n-1}, x_{j,1}, \ldots, x_{j,n}, x_r, \dot{x}_r, \ldots, x_r^{(n)}]^{\mathrm{T}} \in \Omega_{Z_{i,n}}$ and $|\epsilon_{i,n}| \le \epsilon_{i,n}^*$ with $\epsilon_{i,n}^* > 0$ being an error accuracy.

Correspondingly, similar to (40)–(43), we construct the following Lyapunov function

$$\begin{aligned}V_n = V_{n-1} + \frac{1}{2}\sum_{i=1}^{N}\Big(&e_{i,n}^2 + \gamma_{i,n}^{-1}\left((n-1)A_i + n\right) \\ &\times \tilde{D}_{i,n}^2 + \tilde{\theta}_{i,n}^T\Gamma_{i,n}^{-1}\tilde{\theta}_{i,n}\Big)\end{aligned} \tag{47}$$

and choose the continuous adaptive controller $v_{i,n}$ with parameter updated laws $\dot{\hat{\theta}}_{i,n}$ and $\dot{\hat{D}}_{i,n}$ as

$$\begin{aligned}v_{i,n} = {} & -\chi_{i,n}e_{i,n} - \hat{\theta}_{i,n}^T\Psi_{i,n} \\ &- \frac{\left((n-1)A_i + n\right)e_{i,n}B_{i,n-1}^2\hat{D}_{i,n}}{\sqrt{e_{i,n}^2B_{i,n-1}^2 + \sigma_i^2}}\end{aligned} \tag{48}$$

where $\chi_{i,n} > 0$ is a design parameter, and $A_i = \sum_{j=1}^{N}a_{ij}$,

$$\begin{aligned}B_{i,n-1} = \Bigg(&4 + \sum_{l=1}^{n-1}\left(\frac{\partial v_{i,n-1}}{\partial x_{i,l}}\right)^2 \\ &+ \sum_{j=1}^{N}a_{ij}\sum_{l=1}^{n-1}\left(\frac{\partial v_{i,n-l}}{\partial x_{j,l}}\right)^2\Bigg)^{\frac{1}{2}}\end{aligned}$$

$\tilde{\theta}_{i,n}=\theta_{i,n}^*-\hat{\theta}_{i,n}$, $\tilde{D}_{i,n} = D_{i,n}^* - \hat{D}_{i,n}$ and $D_{i,n}^*=\max\{d_{i,1}^*, \ldots, d_{i,n}^*, d_{j,1}^*, \ldots, d_{j,n-1}^*, \epsilon_{i,n}^*\}$, respectively.

Then, it leads to

$$\dot{V}_n \leq \sum_{i=1}^{N}\Bigg(-\sum_{l=1}^{n-1}\chi_{i,l}e_{i,l}^2+\sigma_i\Bigg(\Bigg(D_{i,1}^*+\sum_{l=2}^{n-1}((l-1)A_i +l)D_{i,l}^*\Bigg)+\Bigg(\tilde{D}_{i,1}D_{i,1}^*+\sum_{l=2}^{n-1}((l-1)A_i+l)\tilde{D}_{i,l} \times D_{i,l}^*\Bigg)+\sum_{l=1}^{n-1}\tilde{\theta}_{i,l}^T\hat{\theta}_{i,l}\Bigg)+e_{i,n}g_{i,n}(X_i,0)u_{i,n} + ((n-1)A_i+n)|e_{i,n}|B_{i,n-1}+\theta_{i,n}^{T*}\Psi_{i,n}(Z_{i,n})e_{i,n} - \tilde{\theta}_{i,n}^T\Gamma_{i,n}^{-1}\dot{\hat{\theta}}_{i,n} - \gamma_{i,n}^{-1}\left((n-1)\,A_i+n\right)\tilde{D}_{i,n}\dot{\hat{D}}_{i,n}\Bigg). \tag{49}$$

Next, the following adaptive controller with parameter updated laws can be described by

$$\begin{aligned} u_i &= -\zeta_{i,0}^{-1}v_{i,n}\tanh\left(\frac{v_{i,n}e_{i,n}}{\sigma_i}\right),\\ \dot{\hat{D}}_{i,n} &= -\gamma_{i,n}\sigma_i\hat{D}_{i,n}+\gamma_{i,n}\frac{e_{i,n}^2B_{i,n-1}^2}{\sqrt{e_{i,n}^2B_{i,n-1}^2+\sigma_i^2}},\\ \dot{\hat{\theta}}_{i,n} &= -\sigma_i\Gamma_{i,n}\hat{\theta}_{i,n}+\Gamma_{i,n}\Psi_{i,n}e_{i,n} \end{aligned} \tag{50}$$

where $\zeta_{i,0}>0$ is a positive design parameter such that $\zeta_{i,0}\leq f_{i,l}^*$.

Therefore, by invoking (2), (48)–(50), together with Lemma 5 and the fact $\tilde{D}_{i,l}\hat{D}_{i,l}=-\tilde{D}_{i,l}^2+D_{i,l}^*\tilde{D}_{i,l}\leq \frac{1}{4}D_{i,l}^{*2}$ and $\tilde{\theta}_{i,l}^T\hat{\theta}_{i,l}=-\|\tilde{\theta}_{i,l}\|^2+\theta_{i,l}^{*T}\tilde{\theta}_{i,l}\leq\frac{1}{4}\|\theta_{i,l}^*\|^2$ for $l=1,2,\ldots,n$, we conclude that

$$\begin{aligned} \dot{V}_n &\leq \sum_{i=1}^{N}\Bigg(-\sum_{l=1}^{n}\chi_{i,l}e_{i,l}^2+\sigma_i\Bigg(\Bigg(D_{i,1}^*+\sum_{l=2}^{n}((l-1)A_i+l)D_{i,l}^*\Bigg)+\frac{1}{4}\Bigg(D_{i,1}^{*2}+\sum_{l=2}^{n}((l-1)\times A_i+l)D_{i,l}^{*2}\Bigg)+\frac{1}{4}\sum_{l=1}^{n}\|\theta_{i,l}^*\|^2\Bigg)\Bigg)\\ &\leq -\sum_{l=1}^{n}\chi_l\|e_l\|^2+\sum_{l=1}^{n}\sigma_i\Bigg(\Bigg(D_{i,1}^*+\sum_{l=2}^{n}((l-1)A_i+l)D_{i,l}^*\Bigg)+\frac{1}{4}\Bigg(D_{i,1}^{*2}+\sum_{l=2}^{n}((l-1)\times A_i+l)D_{i,l}^{*2}\Bigg)+\frac{1}{4}\sum_{l=1}^{n}\|\theta_{i,l}^*\|^2\Bigg) \end{aligned} \tag{51}$$

where $e_l=[e_{1,l},e_{2,l},\ldots,e_{N,l}]^{\mathrm{T}}$. Integrating (51) yields

$$\begin{aligned} V_n(t)&+\sum_{l=1}^{n}\int_0^t\chi_l\|e_l(s)\|^2\mathrm{d}s\\ &\leq V_n(0)+\sum_{l=1}^{n}\Bigg(\Bigg(D_{i,1}^*+\sum_{l=2}^{n}((l-1)A_i+l)D_{i,l}^*\Bigg)+\frac{1}{4}\Bigg(D_{i,1}^{*2}+\sum_{l=2}^{n}((l-1)A_i+l)D_{i,l}^{*2}\Bigg)+\frac{1}{4}\sum_{l=1}^{n}\|\theta_{i,l}^*\|^2\Bigg)\bar{\sigma}_i. \end{aligned} \tag{52}$$

Remark 2 It is not hard to see that the considered nonlinear MASs in [10] are special cases of (1). In this paper, the mean-value theorem and the Taylor decoupling technique are applied to designing the distributed NN tracking controllers (24), (31), (40), (48) and (50). Concomitantly, the result of asymptotic consensus tracking of the outputs of all the subsystems can be obtained by introducing the corresponding parameter updated laws with positive time-varying integrable functions.

So far, the main closed-loop theorem of nonlinear multi-agent systems (1) for the consensus asymptotic tracking is shown as follows.

Theorem 1 *Consider uncertain nonlinear MASs (1) satisfying Assumptions 1–3. If the initial state $X_i(0)$ is subject to $X_i(0)\in\Omega_{Z_{i,n}}$ with $Z_{i,n}$ being defined in (46). Then, the designed virtual controllers (24), (31), (40), (48), the actual controller (50) together with corresponding to adaptive control laws guarantees all the closed-loop signals being locally uniformly bounded, and all the subsystem outputs $e_{yi}(t)$ for $i=1,2,\ldots,N$ asymptotically converging to zero, i. e.,* $\lim_{t\to\infty}e_{yi}(t)=0$.

Proof Noting the definition of V_n in (47) along with (25), (32) and (41), it can be seen that $e_{i,l}$, $\hat{\theta}_i$ and $\hat{D}_i$ for $i=1,2,\ldots,N;\ l=1,2,\ldots,n$ are bounded. Also, we conclude $e_{i,l}\in\mathcal{L}^2$ from (52). Based on (20) and Lemma 3, $x_{i,1}$ is bounded. From (5) and (24) with (27), the boundedness of $v_{i,1}$ can be obtained accordingly within the compact set $\Omega_{Z_{i,n}}$. It follows from $e_{i,2}=x_{i,2}-v_{i,1}$ that $x_{i,2}$ is also bounded on $\Omega_{Z_{i,n}}$. Similarly, the boundedness of $x_{i,l}$, $v_{i,j}$ and $u_{i,n}$ for $l=3,4,\ldots,n;\ j=2,3,\ldots,n$ can be verified on

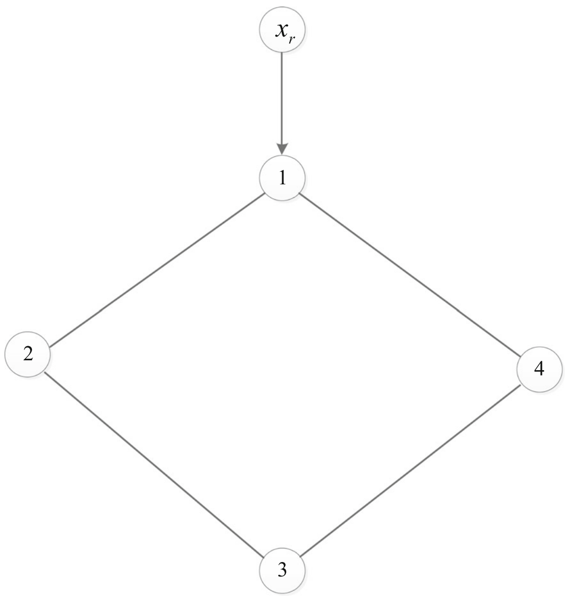

Fig. 1 Communication topology of a group of the 4 subsystems

$\Omega_{Z_{i,n}}$. Therefore, the boundedness of all the closed-loop signals is ensured and then $\dot{e}_{i,l}$ is bounded in the compact set $\Omega_{Z_{i,n}}$. Using Lemma 6, it is straightforward to deduce that $\lim_{t\to\infty} e_{i,l}(t) = 0$. By (22) and the positive definitiveness of $\mathcal{L}+\mathcal{C}$, we can get $\lim_{t\to\infty} e_{yi}(t) = 0$ for $i = 1, 2, \ldots, N$. This implies asymptotic consensus tracking of all the subsystem outputs can be achieved, and this completes the proof. □

Remark 3 From the above stability analysis, a novel distributed NN adaptive consensus tracking control design for nonaffine nonlinear MASs with nonlinear functions and external disturbances can be obtained. In particular, different from our previous work [32,38], a looser decoupling condition (2) in Assumption 3 is introduced. Correspondingly, the control objective of asymptotic output consensus tracking rather than uniformly ultimate bounded (UUB) as in [39] can be achieved by using the proposed adaptive neural controller (50). Also, the proposed NN adaptive consensus tracking control method here is more easily realized in practical applications.

4 Simulation studies

The following numerical example is used to verify the applicability of the presented NN consensus control scheme. Now a multi-agent system including four agents by communicating is considered as in Fig. 1. The nonaffine nonlinear dynamics of MAS is

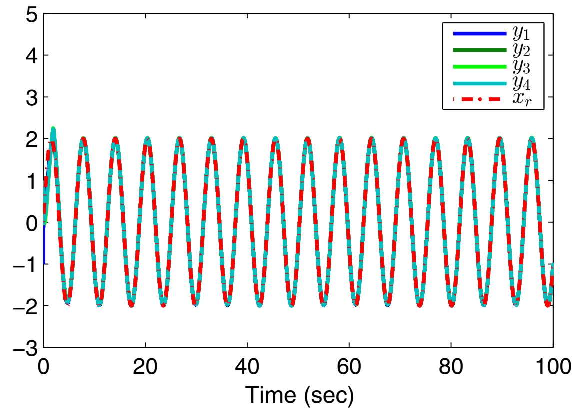

Fig. 2 Subsystem outputs $y_i, i = 1, 2, 3, 4$ and the reference signal x_r

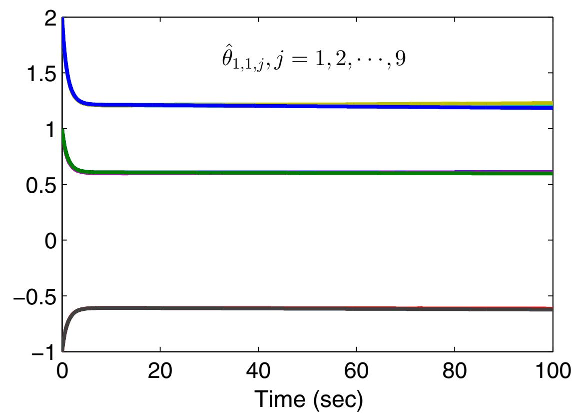

Fig. 3 Adaptive law $\hat{\theta}_{1,1}$

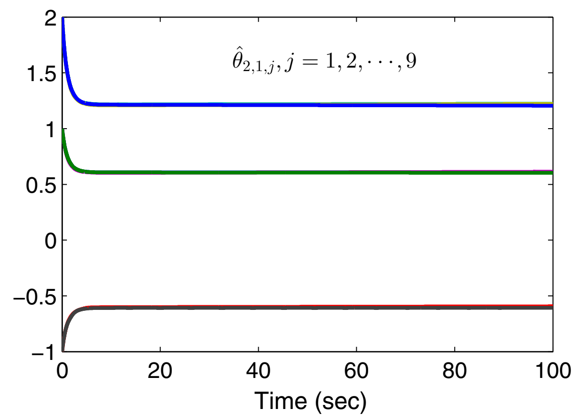

Fig. 4 Adaptive law $\hat{\theta}_{2,1}$

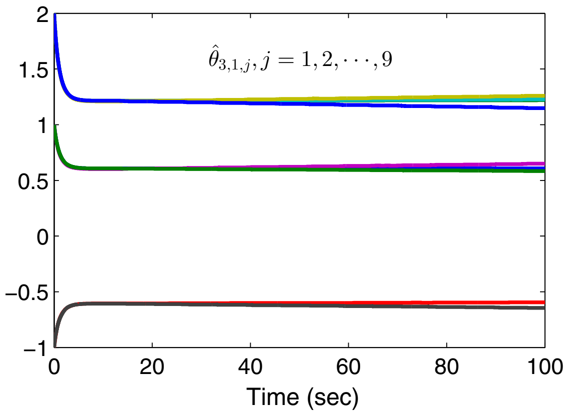

Fig. 5 Adaptive law $\hat{\theta}_{3,1}$

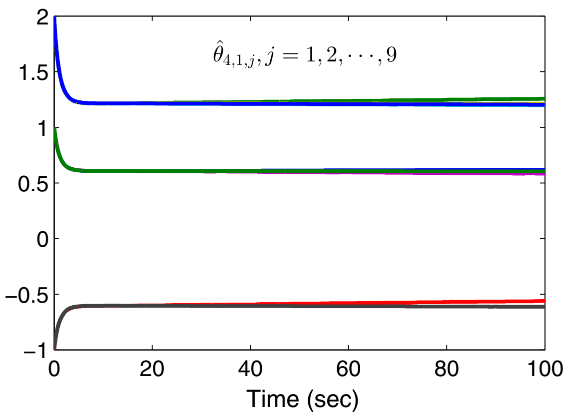

Fig. 6 Adaptive law $\hat{\theta}_{4,1}$

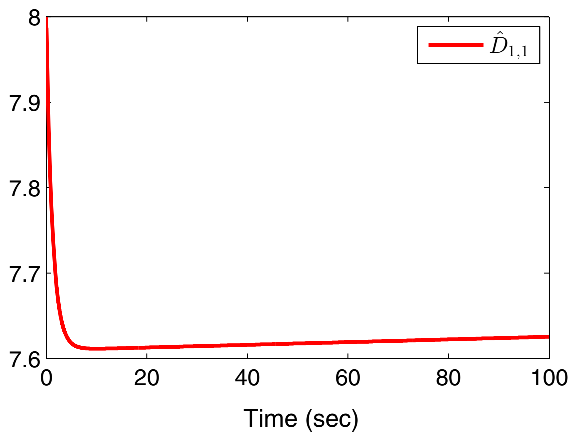

Fig. 7 Adaptive law $\hat{D}_{1,1}$

$$\begin{aligned}
\dot{x}_1 &= x_1 \sin(0.2x_1) + (1 + x_1^2)u_1 + \sin(x_1^2 u_1) + d_1, \\
\dot{x}_2 &= x_2^2 e^{-0.5x_2} + 3u_2 - 2\sin(x_2 u_2) + d_2, \\
\dot{x}_3 &= x_3^2 \cos(0.5x_3) + (3 + x_3)u_3 + d_3, \\
\dot{x}_4 &= \frac{1}{1 + x_4^2} + (5 - 2x_4)u_4 + d_4
\end{aligned} \tag{53}$$

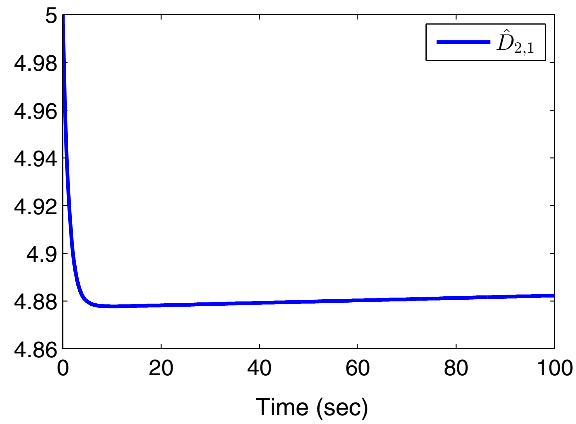

Fig. 8 Adaptive law $\hat{D}_{2,1}$

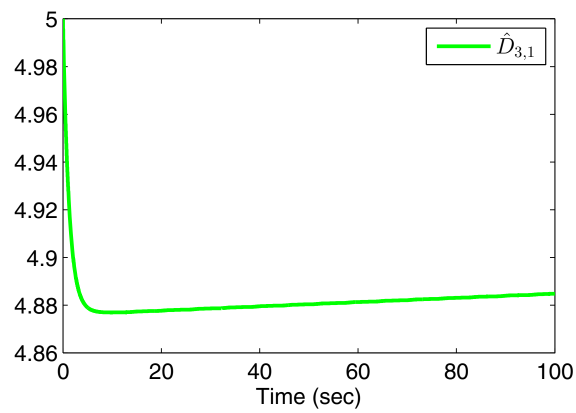

Fig. 9 Adaptive law $\hat{D}_{3,1}$

where the state x_i of each agent is confined in the compact set $\Omega_{x_i} = \{x_i||x_i| \leq 1\}$ for $i = 1, 2, 3, 4$. The disturbances are chosen as $d_1 = 0.2(1 - e^{-t})$, $d_2 = 0.1\cos(t)$, $d_3 = -0.5$ and $d_4 = \sin(0.8t)$, respectively, and the reference trajectory is taken as $x_r(t) = 2\sin(t)$. As before, using the adaptive controller (50) with corresponding to parameter updated laws, the simulation parameters are chosen as $\chi_{i,1} = 15, \gamma_{1,1} = \gamma_{4,1} = 0.02, \gamma_{2,1} = \gamma_{3,1} = 0.01, \Gamma_{i,1} = 0.2, \zeta_{1,0} = \zeta_{2,0} = 0.5, \zeta_{3,0} = 1, \zeta_{4,0} = 0.8$ and $\sigma_i(t) = 2e^{-0.8t}, i = 1, 2, 3, 4$. Meanwhile, the initial values are $x_1(0) = -1, x_2(0) = 0.8, x_3(0) = 0, x_4(0) = 1, \hat{D}_{1,1}(0) = \hat{D}_{4,1}(0) = 8, \hat{D}_{2,1}(0) = \hat{D}_{3,1}(0) = 5$ and $\hat{\theta}_{i,1}(0) = [1,2,-1,2,1,2,-1,2,1]^T, i=1,2,3,4$. Accordingly, the simulation curves are demonstrated by Figs. 2, 3, 4, 5, 6, 7, 8, 9 and 10. Figure 2 displays all the subsystems' outputs $y_i, i = 1, 2, 3, 4$ and the reference signal x_r. Figures 3, 4, 5, 6, 7, 8, 9 and 10

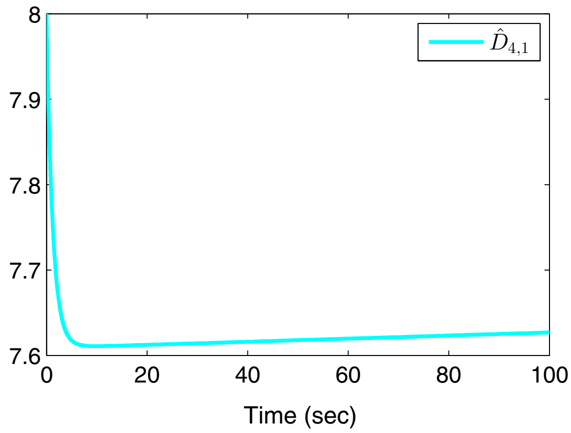

Fig. 10 Adaptive law $\hat{D}_{4,1}$

are given to demonstrate the adaptive estimation curves $\hat{\theta}_{i,1}$ and $\hat{D}_{i,1}$, $i = 1, 2, 3, 4$, respectively.

5 Conclusion

The distributed NN adaptive consensus control scheme for nonaffine nonlinear MASs is developed in this paper. By utilizing the Taylor expansion technique, the control input in the nonaffine nonlinear term is successfully decoupled. Then, the compensation adaptive laws with positive time-varying integrable functions are introduced to effectively handle the disturbances and the NN approximation errors. Moreover, on the basis of the local output tracking error information of neighborhood agents, a novel distributed adaptive control strategy is proposed and it is also shown that the output of each subsystem can asymptotically track to the desired reference trajectory. Finally, the simulation results verify the efficiency of the used approach.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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