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ORIGINAL PAPER

A novel two-stage ellipsoid filtering-based system modeling algorithm for a Hammerstein nonlinear model with an unknown noise term

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Abstract This paper considers the system modeling problem for the Hammerstein nonlinear model with unknown but bounded noise. A two-stage ellipsoid filtering-based modeling algorithm is proposed and the unknown noise term is wrapped in an ellipsoid during each recursive step. The normalized ellipsoid is varying and its center, as well as its volume, is updated by using the volume minimization criteria of the ellipsoid. Finally, the given simulations visually illustrate the feasible parameter set variation process and the motion trail of the ellipsoids, which shows the effectiveness and the accuracy of the proposed algorithm.

Keywords Ellipsoid space · Filtering method · Hammerstein system · Nonlinear model · Unknown noise term

1 Introduction

System modeling [1,2] is one of the most common methods to find the true value of the complex models and analyze the relation between the input and output signals. However, it is commonly infeasible to find the actual distribution law of the system disturbance, which yields that the probability distribution function of the noise term cannot be assumed facilely. The non-probabilistic noise term results in that the distribution law of the disturbance is hardly precise. Meanwhile, since the unknown feasible solutions of the uncertainty parameters are fitted in a convex set, it cannot directly analyze its variation law for the recursive irregular figures. In the research field of the system modeling, the unknown but bounded noise term are wrapped by some recursively computable spaces [3–5].

In recent years, the set membership modeling algorithm is studied for estimating the parameters of the systems with unknown but bounded noise term [6–8]. Some geometric spaces with regular structures are commonly used for describing the bounded noise terms, i.e., the ellipsoid space is usually adopted for the simplicity of its formulation [9,10]. However, the space sets listed in the previous works are commonly fitted for the linear system identification field, rather than in the system modeling for the nonlinear ones, i.e., the Hammerstein system. Considering the computational complexity and the estimation accuracy, in this paper, the ellipsoid space is employed for constructing the known boundary of the noise term.

Differing from the proposed algorithms in [9,10], in order to diminish the impact of unknown but bounded colored noise term, this paper considers the modeling problem of Hammerstein models and a two-stage

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ellipsoid filtering modeling algorithm is studied. Meanwhile, compared with the work in [11], this paper derives a two-stage algorithm for solving the nonlinear system identification problem under the disturbance of unknown but bounded noise term. The main contributions of this paper are listed as follows: (1) A two-stage filtering method is proposed for the Hammerstein system modeling by filtering the nonlinear model with unknown noises into two different subsystems, one contains the noise term and the other includes the system parameters; (2) the minimization criteria are adopted to determine the recursive step and find the minimum value of the ellipsoid volume; (3) the simulation results show the motion trail of the ellipsoid sets via the sample time, which can directly illustrate the parameter estimation process.

Briefly, the rest of this paper is organized as follows. Section 2 gives the Hammerstein system with an unknown but bounded noise term and its identification model. Section 3 presents a two-stage ellipsoid minimization volume-based filtering algorithm by adopting different ellipsoids to wrap the boundary of the feasible parameter sets. Section 4 provides the simulations to illustrate the accuracy and effectiveness of the proposed algorithm. Finally, the conclusions and some future works are offered in Sect. 5.

2 Problem statement

The following single input/single output nonlinear system is further considered:

$$y(t) = B(z)\sum_{i=1}^{n_c} c_i f_i(u(t)) + D(z)v(t),$$
$$\|v(t)\| \leqslant \delta,\ \delta > 0, \tag{1}$$

where $\{u(t), y(t)\}$ is the pair of the signal sequences at time t, $v(t)$ is the unknown bounded noise term with a priori bound δ. $B(z)$ and $D(z)$ are combinations of negative powers that are defined by $B(z) := 1 + \sum_{i=1}^{n_b} b_i z^{-i}$ and $D(z) := 1 + \sum_{j=1}^{n_d} d_j z^{-j}$ in the unit delay operator z^{-1} $[z^{-1}y(t) = y(t-1)]$. The nonlinear term in the model (1) is defined by $\bar{u}(t) := \sum_{i=1}^{n_c} c_i f_i(u(t))$.

The aim of identifying this nonlinear Hammerstein system is to propose a geometrical recursive algorithm to consistently estimate the unknown parameter vectors $\boldsymbol{b} := [b_1, b_2, \ldots, b_{n_b}]^{\mathrm{T}} \in \mathbb{R}^{n_b}$, $\boldsymbol{c} := [c_1, c_2, \ldots, c_{n_c}]^{\mathrm{T}} \in \mathbb{R}^{n_c}$, $\boldsymbol{d} := [d_1, d_2, \ldots, d_{n_d}]^{\mathrm{T}} \in \mathbb{R}^{n_d}$, from the measured data $\{u(t), y(t)\}_{t=1}^{L}$. From Eq. (1), the identification model can be rewritten as

$$\begin{aligned}
y(t) &= B(z)\bar{u}(t) + D(z)v(t)\\
&= \Big[1 + \sum_{i=1}^{n_b} b_i z^{-i}\Big]\bar{u}(t) + \Big[1 + \sum_{j=1}^{n_d} d_j z^{-j}\Big]v(t)\\
&= \Big[\sum_{i=1}^{n_b} b_i z^{-i}\Big]\bar{u}(t) + \bar{u}(t)\\
&\quad + \Big[\sum_{j=1}^{n_d} d_j z^{-j}\Big]v(t) + v(t)\\
&= \sum_{i=1}^{n_b} b_i \bar{u}(t-i) + \sum_{i=1}^{n_c} c_i f_i(u(t))\\
&\quad + \sum_{i=1}^{n_d} d_i v(t-i) + v(t)\\
&= \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta} + v(t),
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
\boldsymbol{\varphi}(t) &:= [\bar{u}(t-1), \bar{u}(t-2), \ldots, \bar{u}(t-n_b), f_1(u(t)),\\
&\quad f_2(u(t)), \ldots, f_{n_c}(u(t)), v(t-1),\\
&\quad v(t-2), \ldots, v(t-n_d)]^{\mathrm{T}} \in \mathbb{R}^{n_b+n_c+n_d},\\
\boldsymbol{\theta} &:= [\boldsymbol{b}^{\mathrm{T}}, \boldsymbol{c}^{\mathrm{T}}, \boldsymbol{d}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_b+n_c+n_d}.
\end{aligned}$$

From Eq. (2), since the error bound of the model is known, the parameters belong to the membership set $S(L) := \{\boldsymbol{\theta} | y(t) - \delta \leqslant \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta} \leqslant y(t) + \delta,\ t \in [1, L]\}$. In the geometry, the set $S(L)$ is delimited by L pairs of parallel hyperplanes, i.e., $H_1(t) := \{\boldsymbol{\theta} | \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta} = y(t) - \delta\}$ and $H_2(t) := \{\boldsymbol{\theta} | \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta} = y(t) + \delta\}$. In the whole parametric space, the hyperplanes are the boundaries of different subspaces. With the data length L increases, the parametric space will be divided into more blocks but only one narrower piece stands for the parameter uncertainty set $S(L)$.

The geometric space goes more flexible as the input data length getting bigger, which rises a gordian knot that it is hardly to form precisely the periphery of the parameter feasible set. The objective of this paper is to find a filtering-based recursive parameter feasible set $\Theta(t+1)$ that contains the solution $\boldsymbol{\theta}(t+1)$ at time $t+1$ when given the parameter feasible set $\Theta(t)$.

3 The ellipsoid volume minimization-based filtering algorithm

The unknown noise term $w(t) := D(z)v(t)$ in Eq. (1) at time t is determined by the input/output sampled data. When the polynomial $D(z)$ equals 1, the feasible parameters of the nonlinear system is easy to be filled in two parallel hyperplanes, as mentioned before. However, the polynomial $D(z)$ is commonly constructed by some unknown parameters d_i that satisfies $\prod_{i=1}^{n_d} d_i \neq 0, i \in [1, n_d]$, which means the noise term will not be a regular spatial graphic and it is difficult to use the traditional set estimation methods to solve this type of system identification problem.

To avoid the irregular geometric construction and reduce the computation complexity, the filtering idea is adopted to transfer the nonlinear system into two different parts in this section. Based on the input and output signals, the identification model in Eq. (2) can be changed into a controlled autoregressive model by adopting the unknown filter $D^{-1}(z)$. The filtered model can be written as

$$y_{\mathrm{f}}(t) = B(z)\bar{u}_{\mathrm{f}}(t) + v(t), \tag{3}$$

where $\bar{u}_{\mathrm{f}}(t) := \sum_{i=1}^{n_c} c_i U_i(t)$ and $y_{\mathrm{f}}(t) := \frac{1}{D(z)} y(t)$. The intermediate variable $U_i(t)$ is defined by $U_j(t) := \frac{1}{D(z)} f_j(u(t)),\ j = 1, 2, \ldots, n_c$. Then, Eq. (3) can be written as

$$\begin{aligned} y_{\mathrm{f}}(t) = &\sum_{i=1}^{n_c} c_i U_i(t) + \sum_{i=1}^{n_b} b_i \bar{u}_{\mathrm{f}}(t-i) \\ &+ v(t),\ \|v(t)\| \leqslant \delta,\ \delta > 0. \end{aligned} \tag{4}$$

Define the filtered information vector and two parameter vectors:

$$\begin{aligned} \boldsymbol{\varphi}_{\mathrm{f}}(t) := &[\bar{u}_{\mathrm{f}}(t-1), \bar{u}_{\mathrm{f}}(t-2), \ldots, \bar{u}_{\mathrm{f}}(t-n_b), \\ &U_1(t), U_2(t), \ldots, U_{n_c}(t)]^{\mathrm{T}} \in \mathbb{R}^{n_b+n_c}, \end{aligned} \tag{5}$$

$$\begin{aligned} \boldsymbol{\theta}_{\mathrm{s}} := &[\boldsymbol{b}^{\mathrm{T}}, \boldsymbol{c}^{\mathrm{T}}]^{\mathrm{T}} = [b_1, b_2, \ldots, b_{n_b}, c_1, c_2, \\ &\ldots, c_{n_c}]^{\mathrm{T}} \in \mathbb{R}^{n_b+n_c}, \end{aligned} \tag{6}$$

$$\boldsymbol{\theta}_{\mathrm{n}} := \boldsymbol{d} = [d_1, d_2, \ldots, d_{n_d}]^{\mathrm{T}} \in \mathbb{R}^{n_d}. \tag{7}$$

From Eq. (4), the parameters c_i and b_j are determined by a feasible set and the two parallel hyperplanes $H_{1,\mathrm{f}}(t)$, $H_{2,\mathrm{f}}(t)$ that divide the n-dimensional space, are listed as follows:

$$\begin{aligned} H_{1,\mathrm{f}}(t) = &\left\{ \boldsymbol{\theta}_{\mathrm{s}} | \sum_{i=1}^{n_c} c_i U_i(t) \right. \\ &\left. + \sum_{i=1}^{n_b} b_i \bar{u}_{\mathrm{f}}(t-i) = y_{\mathrm{f}}(t) + \delta \right\}, \\ H_{2,\mathrm{f}}(t) = &\left\{ \boldsymbol{\theta}_{\mathrm{s}} | \sum_{i=1}^{n_c} c_i U_i(t) \right. \\ &\left. + \sum_{i=1}^{n_b} b_i \bar{u}_{\mathrm{f}}(t-i) = y_{\mathrm{f}}(t) - \delta \right\}. \end{aligned}$$

However, the true values of the parameters are all in part of the space instead of the whole space, i.e.,

$$H_{\mathrm{f}}^{+}(t) = H_{1,\mathrm{f}}^{+}(t) \cap H_{2,\mathrm{f}}^{+}(t), \tag{8}$$

$$\begin{aligned} H_{1,\mathrm{f}}^{+}(t) = &\left\{ \boldsymbol{\theta}_{\mathrm{s}} | \sum_{i=1}^{n_c} c_i U_i(t) \right. \\ &\left. + \sum_{i=1}^{n_b} b_i \bar{u}_{\mathrm{f}}(t-i) \leqslant y_{\mathrm{f}}(t) + \delta \right\}, \end{aligned} \tag{9}$$

$$\begin{aligned} H_{2,\mathrm{f}}^{+}(t) = &\left\{ \boldsymbol{\theta}_{\mathrm{s}} | \sum_{i=1}^{n_c} c_i U_i(t) \right. \\ &\left. + \sum_{i=1}^{n_b} b_i \bar{u}_{\mathrm{f}}(t-i) \geqslant y_{\mathrm{f}}(t) - \delta \right\}. \end{aligned} \tag{10}$$

Since the unknown but bounded noise term $v(t) = y_{\mathrm{f}}(t) - \boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}(t)\boldsymbol{\theta}_{\mathrm{s}}$ determines the spatial distance between the two parallel hyperplanes $H_{1,\mathrm{f}}(t)$, $H_{2,\mathrm{f}}(t)$, the ellipsoid set membership idea can be adopted in estimating $\hat{\boldsymbol{\theta}}(t)$ as the first stage in solving all the parameter estimates. The filtered model in Eq. (4) can be rewritten in a vector form:

$$y_{\mathrm{f}}(t) = \boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}(t)\boldsymbol{\theta}_{\mathrm{s}} + v(t), \tag{11}$$

or

$$v(t) = y_{\mathrm{f}}(t) - \boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}(t)\boldsymbol{\theta}_{\mathrm{s}}, \tag{12}$$

where $y_{\mathrm{f}}(t) = y(t) - \sum_{i=1}^{n_d} d_i y_{\mathrm{f}}(t-i)$. Because of the unknown parameters d_i, it is impossible to use $\bar{u}_{\mathrm{f}}(t)$ to construct the known parameter vector $\boldsymbol{\varphi}_{\mathrm{f}}(t)$ in Eq. (5). The solution here is to use their estimates to derive the following filtering-based ellipsoid recursive algorithm for the Hammerstein models.

For the filtered model in Eq. (11), the normalized ellipsoidal set is

$$\begin{aligned} E(P_{\mathrm{s}}^{-1}, \hat{\boldsymbol{\theta}}_{\mathrm{s}}) = \{&\boldsymbol{\theta}_{\mathrm{s}} \in \mathbb{R}^{n_{\mathrm{s}}} : (\boldsymbol{\theta}_{\mathrm{s}} - \hat{\boldsymbol{\theta}}_{\mathrm{s}})^{\mathrm{T}} \\ &P_{\mathrm{s}}^{-1}(\boldsymbol{\theta}_{\mathrm{s}} - \hat{\boldsymbol{\theta}}_{\mathrm{s}}) \leqslant \sigma_{\mathrm{s}}^2\}, \end{aligned} \tag{13}$$

where P_{s} is the symmetric, positive definite shape matrix for ellipsoidal set $E(P_{\mathrm{s}}^{-1}, \hat{\boldsymbol{\theta}}_{\mathrm{s}})$. The priori assumed noise bound σ_{s} also represents the radius of the ellipsoid $E(P^{-1}, \hat{\boldsymbol{\theta}}_{\mathrm{s}})$ and the error bound of feasible parameter $\boldsymbol{\theta}_{\mathrm{s}}$ is lower than the $\boldsymbol{\theta}$, that is, $\sigma_{\mathrm{s}} \leqslant \sigma$. The estimates $\hat{B}(t,z)$ and $\hat{D}(t,z)$ are constructed by $\hat{B}(t,z) := 1 + \sum_{i=1}^{n_b} \hat{b}_i(t) z^{-i}$ and $\hat{D}(t,z) := 1 + \sum_{j=1}^{n_d} \hat{d}_j(t) z^{-j}$, respectively.

In the first stage, the intermediate variable estimates are $\hat{w}(t) := \hat{\boldsymbol{\varphi}}_{\mathrm{n}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_{\mathrm{n}}(t-1) + \hat{v}(t)$, where the $\hat{\boldsymbol{\varphi}}_{\mathrm{n}}(t)$ is determined by the estimates $\hat{v}(t-i), i = 1, 2, \ldots, n_d$. Similarly to the procedure of forming the normalized ellipsoid in Eq. (13), the set of feasible parameter vector $\boldsymbol{\theta}_{\mathrm{n}}$ is defined as follows:

$$E(P_{\mathrm{n}}^{-1}, \hat{\boldsymbol{\theta}}_{\mathrm{n}}) = \{\boldsymbol{\theta}_{\mathrm{n}} \in \mathbb{R}^{n_{\mathrm{n}}} : (\boldsymbol{\theta}_{\mathrm{n}} - \hat{\boldsymbol{\theta}}_{\mathrm{n}})^{\mathrm{T}} P_{\mathrm{n}}^{-1}(\boldsymbol{\theta}_{\mathrm{n}} - \hat{\boldsymbol{\theta}}_{\mathrm{n}}) \leqslant \sigma_{\mathrm{n}}^2\}, \tag{14}$$

where P_{n} is also a symmetric, positive definite shape matrix that determines the position of the normalized ellipsoid. Using the ellipsoid volume minimization principle, we list the ellipsoid volume minimization-based filtering (EVMF for short) algorithm in the first stage to compute $\hat{\boldsymbol{\theta}}_{\mathrm{n}}(t)$:

$$\begin{aligned}
\hat{\boldsymbol{\theta}}_{\mathrm{n}}(t) &= \hat{\boldsymbol{\theta}}_{\mathrm{n}}(t-1) + \beta_{\mathrm{n}}(t) P_{\mathrm{n}}(t) \hat{\boldsymbol{\varphi}}_{\mathrm{n}}(t) r_{\mathrm{n}}(t),\\
r_{\mathrm{n}}(t) &= \hat{w}(t) - \hat{\boldsymbol{\varphi}}_{\mathrm{n}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_{\mathrm{n}}(t-1),\\
P_{\mathrm{n}}(t) &= \frac{1}{\alpha_{\mathrm{n}}(t)}\Big[P_{\mathrm{n}}(t-1) - \frac{\beta_{\mathrm{n}}(t) P_{\mathrm{n}}(t-1)\hat{\boldsymbol{\varphi}}_{\mathrm{n}}(t)\hat{\boldsymbol{\varphi}}_{\mathrm{n}}^{\mathrm{T}}(t) P_{\mathrm{n}}(t-1)}{\alpha_{\mathrm{n}}(t) + \beta_{\mathrm{n}}(t) g_{\mathrm{n}}(t)}\Big],\\
\sigma_{\mathrm{n}}^2(t) &= 1 + q_{\mathrm{n}}(t) - \frac{\alpha_{\mathrm{n}}(t)\beta_{\mathrm{n}}(t) r_{\mathrm{n}}^2(t)}{\alpha_{\mathrm{n}}(t) + \beta_{\mathrm{n}}(t) g_{\mathrm{n}}(t)},\\
g_{\mathrm{n}}(t) &= \hat{\boldsymbol{\varphi}}_{\mathrm{n}}^{\mathrm{T}}(t) P_{\mathrm{n}}(t-1)\hat{\boldsymbol{\varphi}}_{\mathrm{n}}(t),\\
\alpha_{\mathrm{n}}(t) &= \frac{1}{\sigma_{\mathrm{n}}^2(t-1)},\\
\beta_{\mathrm{n}}(t) &= \frac{q_{\mathrm{n}}(t)}{\delta^2(t)}.
\end{aligned}$$

The intermediate variable $q_{\mathrm{n}}(t)$ is the positive real root of the equation

$$\lambda_{\mathrm{n},2}(t) q_{\mathrm{n}}^2(t) + \lambda_{\mathrm{n},1}(t) q_{\mathrm{n}}(t) + \lambda_{\mathrm{n},0}(t) = 0, \tag{15}$$

where $\lambda_{\mathrm{n},2}(t) := (n_d - 1)\sigma_{\mathrm{n}}^4(t-1) g_{\mathrm{n}}^2(t)$, $\lambda_{\mathrm{n},1}(t) := [(2n_d - 1)\delta^2(t) - \sigma_{\mathrm{n}}^2(t-1) g_{\mathrm{n}}(t) + r_{\mathrm{n}}^2(t)]\sigma_{\mathrm{n}}^2(t-1) g_{\mathrm{n}}(t)$, $\lambda_{\mathrm{n},0}(t) := [n_d(\delta^2(t) - r_{\mathrm{n}}^2(t)) - \sigma_{\mathrm{n}}^2(t-1) g_{\mathrm{n}}(t)]\delta^2(t)$. If Eq. (15) does not have any positive real root, i.e., the sampling data at time t do not update the ellipsoid $E(P_{\mathrm{n}}^{-1}, \hat{\boldsymbol{\theta}}_{\mathrm{n}})$, let $q_{\mathrm{n}}(t) = 0$ at time t.

In the second stage, the other ellipsoidal set in Eq. (13) is to be formed. The filter $\hat{D}^{-1}(t,z)$ is obtained after running the first stage of the EVMF algorithm and it is easy to compute the estimates, such as $\hat{\tilde{u}}_{\mathrm{f}}(t) = \sum_{i=1}^{n_c} \hat{c}_i(t)\hat{U}_i(t)$, $\hat{y}_{\mathrm{f}}(t) = -\sum_{j=1}^{n_d} \hat{d}_j(t)\hat{y}_{\mathrm{f}}(t-j) + y(t)$. The intermediate term $\hat{U}_k(t)$ can be computed by $\hat{U}_k(t) := \frac{1}{D(z)} f_k(u(t)) = -\sum_{l=1}^{n_d} \hat{d}_l(t)\hat{U}_k(t-l) + f_k(u(t))$. Construct the estimate of $\boldsymbol{\varphi}_{\mathrm{f}}(t)$ with $\hat{\tilde{u}}_{\mathrm{f}}(t)$ and $\hat{U}_j(t)$:

$$\begin{aligned}
\hat{\boldsymbol{\varphi}}_{\mathrm{f}}(t) = [&\hat{\tilde{u}}_{\mathrm{f}}(t-1), \hat{\tilde{u}}_{\mathrm{f}}(t-2),\\
&\ldots, \hat{\tilde{u}}_{\mathrm{f}}(t-n_b), \hat{U}_1(t), \hat{U}_2(t), \ldots, \hat{U}_{n_c}(t)]^{\mathrm{T}}.
\end{aligned}$$

Similarly to the first stage of EVMF algorithm, by using the ellipsoid volume minimization principle, the estimation of $\hat{\boldsymbol{\theta}}_{\mathrm{s}}(t)$ can be obtained in the second stage. By replacing $v(t)$, $y_{\mathrm{f}}(t)$, $\boldsymbol{\varphi}_{\mathrm{f}}(t)$ and $\boldsymbol{\theta}_{\mathrm{s}}$ in Eq. (12) with their estimates $\hat{v}(t)$, $\hat{y}_{\mathrm{f}}(t)$, $\hat{\boldsymbol{\varphi}}_{\mathrm{f}}(t)$ and $\hat{\boldsymbol{\theta}}_{\mathrm{s}}(t)$ at time t, the filtered noise vector can be computed as

$$\hat{v}(t) = \hat{y}_{\mathrm{f}}(t) - \hat{\boldsymbol{\varphi}}_{\mathrm{f}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_{\mathrm{s}}(t).$$

In conclusion, the two-stage ellipsoid filtering-based system modeling algorithm for Hammerstein models can be summarized.

Algorithm 1 Framework of the two-stage ellipsoid filtering-based system modelling algorithm.

Input: The initial values of the parameter estimation variables and covariance matrices, such as $\hat{\boldsymbol{\theta}}_{\mathrm{s}}(i) = \mathbf{1}_{n_b+n_c}/p_0$, $\hat{\boldsymbol{\theta}}_{\mathrm{n}}(i) = \mathbf{1}_{n_d}/p_0$, $\hat{y}_{\mathrm{f}}(i) = 1/p_0$, $\hat{w}(i) = 1/p_0$, $\hat{\tilde{u}}_{\mathrm{f}}(i) = 1/p_0$, $\hat{\tilde{u}}(i) = 1/p_0$, $\hat{v}(i) = 1/p_0$ for $i \leqslant 0$, $\hat{U}_j(i) = 1/p_0$ for $i \leqslant 0$ and $j = 1, 2, \ldots, n_c$, $p_0 = 10^6$, and give the basis functions $f_j(\cdot)$.

Output: The parameter estimates, $\hat{\boldsymbol{\theta}} := \{\hat{\boldsymbol{\theta}}_{\mathrm{s}}; \hat{\boldsymbol{\theta}}_{\mathrm{n}}\}$;

1: $k \leftarrow 0$;
2: $L \leftarrow$ Constant;
3: **for** $k = 1 : L$
4: Collect the sampling data $\{u(k), y(k)\}$ at time k;
5: Construct the information vectors $\hat{\boldsymbol{\varphi}}_{\mathrm{s}}(k)$, $\hat{\boldsymbol{\varphi}}_{\mathrm{n}}(k)$. Compute $\hat{w}(t)$;
6: Compute $\rho_{\mathrm{n}}(k)$, $\alpha_{\mathrm{n}}(k)$, find the first ellipsoid via the $P_{\mathrm{n}}(k)$ and $\sigma_{\mathrm{n}}^2(k)$. Update parameter vector $\hat{\boldsymbol{\theta}}_{\mathrm{n}}(k)$;
7: Construct the filtered information factor $\hat{\boldsymbol{\varphi}}_{\mathrm{f}}(k)$ and calculate the undetermined coefficients;
8: Find the second ellipsoid via the $P_{\mathrm{s}}(k)$ and $\sigma_{\mathrm{s}}^2(k)$ and and get the parameter vector $\hat{\boldsymbol{\theta}}_{\mathrm{s}}(k)$;
9: Compute the filtered input $\hat{\tilde{u}}_{\mathrm{f}}(k)$ and the filtered noise term $\hat{v}(k)$;
10: **endfor**
11: **return** $\hat{\boldsymbol{\theta}}$;

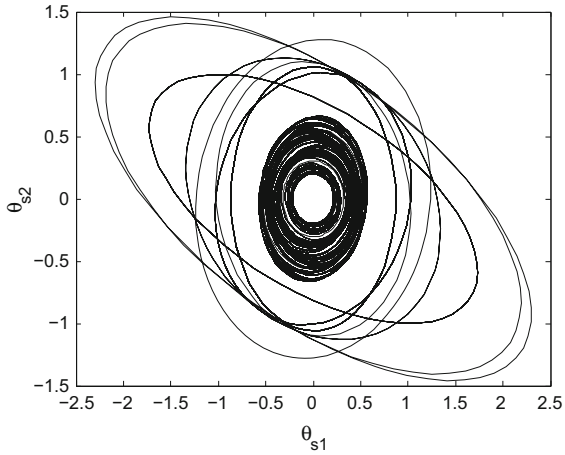

Fig. 1 Variation of the feasible parameter ellipsoid sets by the EVMF algorithm

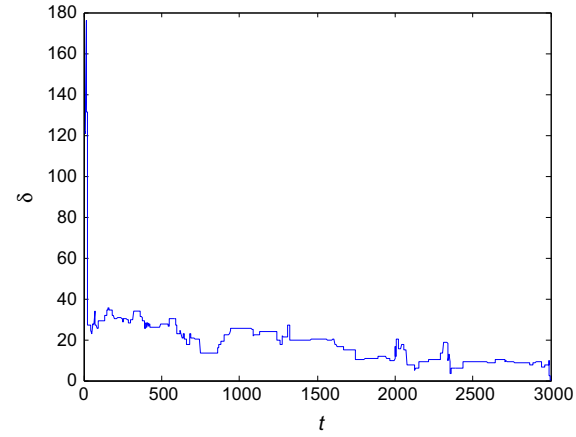

Fig. 2 The estimation errors via sampling time by the EVMF algorithm

4 Examples

Example 1 In order to verify the feasibility of the proposed method in this paper, a finite impulse response system is studied. The mathematical model can be written as:

$$y(t) = [1 - 0.6786z^{-1}][-0.0176u(t)] + [1 + 0.0286z^{-1} + 0.0202z^{-2}]v(t),$$

where $\boldsymbol{\theta}_{\mathrm{s}} = [-0.6786, -0.0176]^{\mathrm{T}}$ and $\boldsymbol{\theta}_{\mathrm{n}} = [0.0286, 0.0202]^{\mathrm{T}}$ are, respectively, the system term parameter vector and the noise term parameter vector. The unknown but bounded noise term $\|v(t)\| \leqslant 1$. In this simulation, the input and noise signals are uniform random numbers that are randomly distributed in the interval $[-1, 1]$. By using the proposed EVMF algorithm, the ellipsoid set of noise term in stage one and the other ellipsoid set of the system noise term parameters are changing their positions during the recursive steps. The noise term ellipsoid sets and estimation errors via sampling time are shown in Figs. 1 and 2, respectively.

From Figs. 1 and 2, it can be seen that

1. The volumes of the ellipsoids reduce increasing slower as the sampling time goes, which shows the good convergence of the proposed algorithm;
2. The unknown parameter vectors can be directly obtained from the centers of the ellipsoids;
3. The elliptical track illustrates the feasible set at each time instant, and the proposed ellipsoids are hardly changing their positions, which means the parameter estimates are converging to the true values.

Example 2 Consider the mathematical model:

$$y(t) = [1 + bz^{-1}][cu(t)] + [1 + d_1 z^{-1} + d_2^{-2}]v(t),$$

where $\boldsymbol{\theta}_{\mathrm{s}} = [b, c]^{\mathrm{T}} = [0.2582, -0.8415]^{\mathrm{T}}$, and $\boldsymbol{\theta}_{\mathrm{n}} = [d_1, d_2]^{\mathrm{T}} = [0.0865, 0.0427]^{\mathrm{T}}$ are the system term parameter vector and the noise term parameter vector, respectively. The noise term for this example is narrow-band random noise $v(t)$, which defined by a unified model [12]:

$$v(t) = h\cos[\Omega t + \gamma W(t)] \tag{16}$$

where h is the intensity of the random excitation, Ω is the center frequency of the random excitation, $W(t)$ is the standard Wiener process, and $\gamma \geqslant 0$ is the bandwidth of the random excitation. When the bandwidth γ is small, $v(t)$ is a narrow-band random noise. The random process $v(t)$ defined by Eq. (16) can be rewritten as follows:

$$\begin{aligned} v(t) &= h\cos\varphi(t) \\ \dot{\varphi}(t) &= \Omega + \gamma\zeta(t) \\ \zeta(t) &= \dot{W}(t) \end{aligned}$$

where the formal derivative $\zeta(t)$ of the unit Wiener process is a Gaussian white noise. Similarly to the works in [12], let the spectral density $S_v(w)$ of noise term $v(t)$ be 1 when $0 < w \leqslant 2w_0$, and 0 at the rest. Then, $\zeta(t)$ can be simulated by:

$$\zeta(t) = \sqrt{\frac{4w_0}{N}} \sum_{k=1}^{N} \cos\left[\frac{w_0}{N}(2k-1)t + \varphi_k\right]$$

where φ_k is a sequence of independent and identically distributed random variables that uniformly distributed

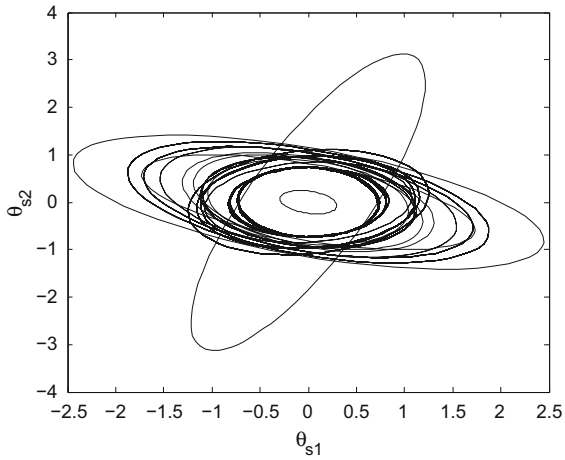

Fig. 3 Variation of the feasible parameter ellipsoid sets by the EVMF algorithm under narrow-band random noise disturbance

over $(0, 2\pi]$, where N is a larger integer. Take $\Omega = 2.0$, $w_0 = 1.0$, $\gamma = 0.01$, $h = 1.0$, $N = 3100$, and set the input signal be uniform random numbers that are randomly distributed in the interval $[-1, 1]$. By using the proposed EVMF algorithm, the noise term ellipsoid sets via sampling time are shown in Fig. 3. From Fig. 3, it can be concluded that the EVMF algorithm is also effective in estimating the system parameters under the narrow-band random noise disturbance.

5 Conclusions

This paper presents a two-stage ellipsoid estimation algorithm for the Hammerstein nonlinear system with unknown noises. The probability distribution of the noise term is unknown, and ellipsoidal sets are formed to contain the feasible noise parameters. This work can be also extended to deal with other types of nonlinear system modeling problems, such as the Wiener nonlinear system modeling and parameter estimation for the error-in-variables systems.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This manuscript is original and has not been published and will not be submitted elsewhere for publication while being considered by Nonlinear Dynamics. The study is not split up into several parts to increase the quantity of submissions and submitted to various journals or to one journal over time. No data have been fabricated or manipulated to support the conclusions. No data, text, or theories by others are presented as if they were our own. The submission has been received explicitly from all co-authors. The authors whose names appear on the submission have contributed sufficiently to the scientific work and therefore share collective responsibility and accountability for the results.

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