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Event-triggered bumpless transfer control for switched systems with its application to switched RLC circuits

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Abstract This article concentrates on the event-triggered bumpless transfer control problem for switched linear systems. The goal is to reduce the control bumps induced by switchings and triggering, and to ensure the stability of the system. First, a novel description of the bumpless transfer performance is presented, quantifying the suppression level on the control bumps in both relative and absolute viewpoints. Then, an improved switching mechanism, an event-triggered scheme and a collection of event-driven controllers are jointly designed. Further, under the designed switching logic, event-triggered rule and controllers, a criterion is established to attain the goal. Besides, Zeno behavior is excluded. Finally, an application on a switched RLC circuit is offered, verifying the efficiency of the developed event-triggered bumpless transfer control strategy.

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1 Introduction

A switched system (SS) consists of a special rule called switching rule and a group of conventional systems called subsystems [1,2]. In the investigation of SSs, switchings play a dual role [3–6]. One is that switching logic design serves as an effective control approach, which adds the freedom of control design. For example, one can pursue a certain steady-state property, like the input-to-state stability, of a SS by design of a switching logic, even though the property is not shared by subsystems [7,8]. The other is that unsuitable switching behaviors usually lead to unfavorable transient responses as soon as a switching happens. The mainly concerned undesired transient behaviors produced by switchings are large and abrupt jumps in the control signal called control bumps [9]. What needs to be noticed is that almost all existing efforts on SSs are made on the obtainment of steady-state properties by means of switchings rather than the attenuation of unexpected control bumps generated by switchings, whereas undesirable control bumps may result in performance degradation and even instability of SSs [10]. Thus, it is indispensable to attenuate or alleviate those unexpected control bumps.

Bumpless transfer (BT) is known as an effective strategy to restrain the unexpected control bumps caused by switchings [11]. The attenuation level on the control bumps is described by the BT performance [12]. The original BT control schemes focus on the amendments of the controllers which are designed in prior. The modifications include the initial state value of an off-line dynamic feedback controller [9,11,13] or the structure of an off-line controller which may be dynamic or static [12,14–16]. Notice that a premise of the modification approaches is the partially or entirely pre-known information on switching signals. This renders the amendment schemes unapplicable to general SSs subjected to completely unknown switching logics in advance. Fortunately, an idea of limiting the amplitude of the control signal was introduced by [17], where the controllers and switching logic are simultaneously designed to rule out the control bumps in the whole state space. References [18,19] extended the result of [17] by restraining the magnitude of the control signal during the operation time interval of subsystems. In [17–19], although the benefit of switching scheme design is employed, the constraint on the amplitude variation of the control signal is relative to the amplitude of the system state. Additionally, the constraint is imposed on the active time interval of subsystems rather than only at the switching instants. This requires too many since the actual task of BT control is to reduce great fluctuations in the control signal only at switching instants. Thus, a question arises: Whether can we determine a switching mechanism and a collection of controllers to limit the control bumps only at switching instants or not?

Recently, network control systems, establishing the relationship among control components via network-based data transmission, have gained growing attention [20–23]. Usually, the data transmission time between different components in a network control system is decided by a time-triggered mechanism through which the signals in the sensor and controller are updated in a fixed time period [24]. Usually, a time-triggered strategy has superiority in simple design approaches and easy implementation procedures [25]. However, unnecessary waste in the communication resources and serious deterioration of system performance are often brought by a time-triggered scheme. In order to tackle this problem, event-triggered (ET) control strategies were provided by [26,27], in which the data are transmitted only when an ET criterion is satisfied. Therefore, a reduction in the communication resources is achieved. Under the ET mechanisms, the event-driven controllers were designed to obtain stability [28], robustness [29], synchronization [30], fast convergence [31] and so on of networked control systems.

Note that big oscillations in the control signal may also arise when an event is triggered. Such sudden chattering in the control signal can also be treated as control bumps and are undesirable transient behaviors needed to be suppressed [32]. For a non-SS, if the triggering frequency is big enough, the control bumps can be easily cut down. However, excessively frequent triggering violates the original intention of ET control. Therefore, a proper ET logic with less triggering frequencies is welcome. For a SS, the adaption of the ET control makes the attenuation of control bumps more difficult. This mainly lies in the lack of effective tools. When an ET rule is implemented on a SS, the SS may encounter larger control bumps at switching instants. This is because not only the controller gains may have great variation once a switching happens, but also the state value may have a big delay whenever a switching occurs. However, different from the traditional delay in the control signal, we do not know how long the asynchronous phenomenon in the control signal lasts. This makes the usual asynchronous switching techniques [33–35] frequently utilized to address the delay in the control signal unfeasible to deal with the control bumps generated by switchings and triggering.

Moreover, for a SS, when each subsystem does not share the BT performance, like [17–19], one can design a state-dependent switching law for the SS to achieve the BT performance. However, for a SS having an ET rule, if we attend to design a state-dependent switching logic to restrain the control bumps induced by triggering, the avoidance of Zeno phenomenon generated by the triggered mechanism is also challenging. This is due to the interaction between the switchings and triggering. In the existing researches on SSs with ET rules, the elimination of Zeno behavior is mainly realized in the framework of dwell-time-dependent switching rules [25,27], or sampling-based state-dependent switching law [36]. However, a state-dependent switching strategy usually does not share the time feature of a dwell-time dependent switching logic. Thus, another question is: Whether a state-dependent switching rule, an ET mechanism and a family of controllers can be jointly designed to restrain the control bumps caused by switchings and triggering while preventing Zeno behavior produced by switchings and triggering or not?

In addition, there is another difficulty that needs to be faced, even if the above question has a positive answer. The difficulty lies in the conflict in the requirements on the BT performance and the studied steady-state property. Usually, it is relatively easy to design a control scheme composed of a switching rule, an ET logic and a series of controllers to purse only a certain steady-state property. However, the designed control strategy may not be useful to suppress the control bumps and to avoid Zeno behavior. If a control approach is designed to achieve only the BT performance, the concerned steady-state property may be lost. In order to handle this conflict, we must establish a control method to ensure both the focused steady-state property and the less control bumps, while avoiding Zeno behavior.

For the study of practical control problems based on switched models, switched circuits are preferable test-beds [37,38]. What is verified by [39] is that a switched RLC circuit can usually conduct low-frequency signal processing in an integrated circuit. Lots of works on switched RLC circuits have been done, including the fault tolerance control [40], the output synchronization [41], the adaptive tracking [42] and so on. Note that almost all attention is paid to realizing steady-state properties instead of attenuating the unexpected control bumps frequently induced by switchings. However, those undesired control bumps usually cause performance degradation and, in the worse situation, serious accidents. Further, when communication resources saving from the sensor to the controller of a switched RLC circuit is pursued, an ET scheme is a pretty choice [43], whereas an unsuitably arranged ET rule may also induce unexpected control bumps at triggering instants, which may result in performance degradation and even serious accidents too. Therefore, it is essential and urgent to suppress those unwanted control bumps at not only switching instants but also triggering instants of a switched RLC circuit. Unfortunately, no results have been reported on this vital topic.

In this paper, we investigate the ETBT control problem for switched linear systems. The BT performance is described in a new way. By joint-design of an ET mechanism, a switching rule and a series of controllers, the control bumps brought by switchings as well as triggering are restrained, while the stability is ensured. Specifically, the features of this study are fourfold.

(i) A new definition of the BT performance for switched linear systems is proposed. The restrain level on control bumps caused by switchings and triggering is quantified in both relative and absolute ways. The magnitude jumps in the control signal are limited only at switching and triggering instants. These make the concept more general than that in [10,13,17–19].

(ii) By co-design of an ET scheme, an improved switching logic and a set of event-driven controllers, we solve the ETBT control issue of a switched linear system, even if no solution to the problem of subsystems exists. Unlike the existing ET rules of SSs [25], the ET strategy is mode-dependent and can rule out the control bumps resulting from triggering. The switching mechanism improves the traditional state-dependent switching logic subjected to a certain dwell-time limitation [8], allowing the Lyapunov functions to be increasing over the minimal dwell-time intervals of subsystems.

(iii) With the presented control strategy, we provide a sufficient condition by which the BT performance is satisfied, the stability is achieved, and no Zeno behavior occurs. In this study, Zeno behavior possibly caused by triggering is avoided by co-design of the ET rule and switching law, which is different from [25] and [36] in which the sampling mechanisms were employed to exclude Zeno behavior.

(iv) The established ETBT control scheme is applied to a switched RLC circuit to text the effectiveness of the established control scheme.

Structure. This study includes five Sections. In Sect. 2, the issue of ETBT control for the switched linear systems is formulated. Section 3 provides a solution to the ETBT control problem by design of an ET rule, a switching mechanism and a set of event-driven controllers. An example is offered in Sect. 4 to verify the effectiveness of the developed control scheme. Section 5 gives a conclusion of this study.

Notation. We denote $||\upsilon||$ as Euclidean norm of the vector υ, R^n as the $n-$dimensional real space, Q_0 as the set $Q_0 = \{0, 1, \ldots, q_0 - 1\}$ with q_0 being a positive integer, $S = \{1, 2, \ldots, s\}$ as the set of positive integers, N as the set of non-negative integers. $\lambda_{\min}(\Pi) and \lambda_{\max}(\Pi)$ indicate the minimal and maximal eigenvalues, respectively, of the square matrix Π. For a vector $\psi(t) \in R^n$, we define $\psi(t_k^+) = \lim\limits_{t\to t_k^+} \psi(t)$, $\psi(t_k^-) = \lim\limits_{t\to t_k^-} \psi(t)$.

2 Problem formulation

Consider a system

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \tag{1}$$

where $x(t) \in R^{n_x}$ stands for the state, $u(t) \in R^{n_u}$ and $\sigma(t)$ represent the control signal and switching law, respectively. Usually, the sequence

$$\{x_0; (i_0, t_0), (i_1, t_1), \ldots, (i_n, t_n), \ldots | i_n \in S, n \in N\}$$

is employed to express the switching signal $\sigma(t)$, where x_0, t_0, t_n and s stand for the initial state, initial time, nth switching instant and the number of subsystems, respectively. Also, the i_nth subsystem is in operation whenever $\sigma(t) = i_n$.

We define $\{t_m^i\}_{m=1}^{\infty}$ as the sequence of ET instants for the ith subsystem with $t_{m+1}^i > t_m^i$. We call the interval $[t_m^i,\ t_{m+1}^i)$ the ET interval. With the ET sequence, for the ith subsystem, we consider the controller described by

$$u(t) = K_i x(t_m^i), t \in [t_m^i,\ t_{m+1}^i), \tag{2}$$

where $x(t_m^i)$ denotes the last transmitted state value held by a zero-order holder till the next ET instant t_{m+1}^i, K_i indicates the controller gain to be selected.

Let $e_i(t) = K_i x(t) - K_i x(t_m^i), t \in [t_m^i,\ t_{m+1}^i)$ be the ET error of the ith subsystem. Replacing the control signal $u(t)$ in (1) by its specific form in (2), we infer the closed-loop system

$$\dot{x}(t) = E_{\sigma(t)}x(t) - B_{\sigma(t)}e_{\sigma(t)}(t), t \in [t_m^{\sigma(t)},\ t_{m+1}^{\sigma(t)}), \tag{3}$$

where $E_i = A_i + B_i K_i, i \in S$.

We now describe the BT performance of the system (3) at the triggering instant t_m^i.

Definition 1 System (3) has the BT performance with respect to (α, β) if for any instant t_m^i,

$$||u(t_m^{i^+}) - u(t_m^{i^-})|| \leq \alpha||x(t_m^i)|| \,(\text{or}\, \alpha\|x(t_{m-1}^i)\|) + \beta \tag{4}$$

is satisfied, where $\alpha \geq 0, \beta > 0$ are pre-specified scalars called the BT performance level.

Remark 1 The relation (4) characterizes the limitation on the amplitude chattering of the control signal $u(t)$ at triggering instants. Compared with [18,19], we do not introduce an additional reference signal. Again, the magnitude variations of the control signal $u(t)$ are not limited during the whole active time intervals of subsystems but at only triggering instants. This makes Definition 1 more suitable for the original goal of the BT control.

Remark 2 The terms $\alpha||x(t_m^i)||$ and β are the relative measure and absolute measure, respectively, for the control bump $||u(t_m^{i^+}) - u(t_m^{i^-})||$. First, the term $\alpha||x(t_m^i)||$ is relevant to the control bump $||u(t_m^{i^+}) - u(t_m^{i^-})||$ since the control signal $u(t)$ is expressed by $u(t) = K_{\sigma(t)}x(t_m^{\sigma(t)})$. Therefore, the term $\alpha||x(t_m^i)||$ quantifies the BT performance in a relative way. Second, the term β is irrelevant to the control bump $||u(t_m^{i^+}) - u(t_m^{i^-})||$. Therefore, the term β quantifies the BT performance in an absolute way.

Traditionally, the ET instant t_{m+1}^i is decided by the ET scheme

$$t_{m+1}^i = \inf\left\{t > t_m^i \;\middle|\; ||x(t) - x(t_m^i)|| > \tilde{\alpha}||x(t)||\right\} \tag{5}$$

with $\tilde{\alpha}$ being a constant (see [28]). Under the ET strategy (5), the communication resource from the system (1) to the original controller $u(t) = K_i x(t)$ is saved to a great extent. However, such an ET logic usually cannot ensure less control bumps at triggering instants t_m^i, which may result in bad performance of the system (3). To cope with this drawback, we propose a new ET mechanism.

The ET instant t_{m+1}^i of the ith subsystem is determined by

$$t_{m+1}^i = \begin{cases} t_n, & \text{if a switching happens,} \\ \inf\left\{t > t_m^i \;\middle|\; g_i(t) > 0\right\}, & \text{otherwise,} \end{cases} \tag{6}$$

where $g_i(t) = ||e_i(t)|| - \alpha_0||x(t)|| - \beta_0 e^{-\lambda_0 t}, \alpha_0 \geq 0, \beta_0 = \nu||x(t_0)||, \nu > 0, \lambda_0 > 0$ are pre-specified constants. Here, the triggering condition

$$t_{m+1}^i = \inf\left\{t > t_m^i \;\middle|\; g_i(t) > 0\right\} \tag{7}$$

is mode-dependent.

Notice that in the ET rule (6), $||e_i(t_m^{i^-})||$ is the control bump at the triggering instant t_m^i. For the ET rule (6) without switching considered, the control bump $||e_i(t_m^{i^-})||$ does not exceed $\alpha_0||x(t_{m-1}^i)|| + \beta_0 e^{-\lambda_0 t_{m-1}^i}$. Since $\beta_0 e^{-\lambda_0 t_{m-1}^i} \leq \beta_0$, if $\alpha \geq \alpha_0 \geq 0$ and $\beta \geq \beta_0 > 0$, then the control bump $||e_i(t_m^{i^-})||$ is suppressed. The terms $\alpha_0||x(t_{m-1}^i)||$ and $\beta_0 e^{-\lambda_0 t_{m-1}^i}$ quantify the control bump $||e_i(t_m^{i^-})||$ in an absolute way and a relative way, respectively.

Our control task is to design the event-driven control signal (2) and a switching rule $\sigma(t)$ to enforce the stability and the BT performance (4) of the system (3) under the ET mechanism (6). If there exists a control scheme to make the system (3) globally asymptotically stable and satisfy the BT performance (4), then we call the ETBT control problem of the system (3) is solvable.

3 Main result

This section focuses on the solution to the ETBT control problem of the system (3). Through design of the triggered strategy (6), switching logic $\sigma(t)$ and controllers (2), the control bumps are suppressed, and the stability is realized.

Theorem 1 *Consider the system* (3). *Suppose that it switches at the switching instant* t_n *to* $\sigma(t_n) = i$. *If there exist scalars* $l_{ij} \le 0, l'_{ij} \le 0, \lambda_{ip} \ge 0$, *matrices* K_i, *positive definite matrices* $Z_{i,q}, Z_{i,q_0}$, *negative definite matrices* W_i, *vector* η *such that for given positive constant* $\mathbb{T}_s$, *constants* $\alpha \ge \alpha_0 \ge 0, \beta \ge \beta_0 > 0, , \mu > 0, \xi > 0, \lambda_0 > 0$, *the following inequalities*

$$\Phi_{i,q} + \mu Z_{i,q} + 2\alpha_0^2 I + Z_{i,q} B_i B_i^{\mathrm{T}} Z_{i,q} < 0, \quad q = 0, 1, \ldots, l, \tag{8}$$

$$\Phi_{i,q+1} + \mu Z_{i,q+1} + 2\alpha_0^2 I + Z_{i,q+1} B_i B_i^{\mathrm{T}} Z_{i,q+1} < 0, \quad q = 0, 1, \ldots, l, \tag{9}$$

$$\bar{\Phi}_{i,q} - \xi Z_{i,q} + 2\alpha_0^2 I + Z_{i,q} B_i B_i^{\mathrm{T}} Z_{i,q} < 0, \quad q = l+1, l+2, \ldots, q_0 - 1, \tag{10}$$

$$\bar{\Phi}_{i,q+1} - \xi Z_{i,q+1} + 2\alpha_0^2 I + Z_{i,q+1} B_i B_i^{\mathrm{T}} Z_{i,q+1} < 0, \quad q = l+1, l+2, \ldots, q_0 - 1, \tag{11}$$

$$\Gamma_i + \mu Z_{i,q_0} + 2\alpha_0^2 I + \sum_{j=1, j\ne i}^{s} l_{ij}(Z_{i,q_0} - Z_{j,0}) < 0, \tag{12}$$

$$\bar{\Lambda}_{ip} - W_i \le 0, \tag{13}$$

$$\lambda(W_i \eta \eta^{\mathrm{T}} W_i) \le \alpha^2 \beta^2, \tag{14}$$

$$\eta^{\mathrm{T}} W_i \eta + \beta^2 \ge 0, \tag{15}$$

$$\frac{T_{us}[t_n, t_n + \mathbb{T}_s)}{T_{ss}[t_n, t_n + \mathbb{T}_s)} \le \frac{\mu - \mu^*}{\xi + \mu^*}, \tag{16}$$

$$\mu^* > 2\lambda_0 \tag{17}$$

hold for any $q \in Q_0, i, p \in S, i \ne p$, *where* $\mu^* \in (0, \mu)$, $T_{ss}[t_n, t_n + \mathbb{T}_s)$ *and* $T_{us}[t_n, t_n + \mathbb{T}_s)$ *denote the lengths of the time intervals over which the Lyapunov functions of subsystems must be decreasing and can be increasing within the time interval* $[t_n, t_n + \mathbb{T}_s)$, *respectively,*

$$\begin{aligned}
\Phi_{i,q} &= E_i^{\mathrm{T}} Z_{i,q} + Z_{i,q} E_i \\
&\quad + (Z_{i,q+1} - Z_{i,q}) q_0 / \mathbb{T}_s, \\
\Phi_{i,q+1} &= E_i^{\mathrm{T}} Z_{i,q+1} + Z_{i,q+1} E_i \\
&\quad + (Z_{i,q+1} - Z_{i,q}) q_0 / \mathbb{T}_s, \\
\bar{\Phi}_{i,q} &= E_i^{\mathrm{T}} Z_{i,q} + Z_{i,q} E_i + (Z_{i,q+1} - Z_{i,q}) q_0 / \mathbb{T}_s, \\
\bar{\Phi}_{i,q+1} &= E_i^{\mathrm{T}} Z_{i,q+1} + Z_{i,q+1} E_i \\
&\quad + (Z_{i,q+1} - Z_{i,q}) q_0 / \mathbb{T}_s, \\
\Lambda_{ip} &= (K_i - K_p)^{\mathrm{T}} (K_i - K_p) \\
&\quad + \lambda_{ip} (Z_{i,q_0} - Z_{p,0}), \\
\bar{\Lambda}_{ip} &= \Lambda_{ip} + \sum_{j=1, j\ne i}^{s} l'_{ij} (Z_{i,q_0} - Z_{j,0}) - \alpha^2 I, \\
\Gamma_i &= E_i^{\mathrm{T}} Z_{i,q_0} + Z_{i,q_0} E_i + Z_{i,q_0} B_i B_i^{\mathrm{T}} Z_{i,q_0},
\end{aligned}$$

then, the following logic for the next switching at $t = t_{n+1}$:

$$\begin{aligned}
&\sigma(t) = i, \forall t \in [t_n, t_n + \mathbb{T}_s), \\
&\sigma(t) = i, \forall t > t_n + \mathbb{T}_s, \\
&\quad \text{if } x^{\mathrm{T}}(t) Z_{i,q_0} x(t) \\
&\quad \le x^{\mathrm{T}}(t) Z_{j,0} x(t),\ j \in S, j \ne i, \\
&\sigma(t_{n+1}) = \arg\min_{j \in S} \{x^{\mathrm{T}}(t) Z_{j,0} x(t)\}, \ \text{otherwise},
\end{aligned} \tag{18}$$

the ET rule (6) *and the controller* (2) *can enforce the system* (3) *to be globally asymptotically stable and to satisfy the BT performance* (4). *Moreover, the switchings and triggering do not yield Zeno behavior.*

Proof The proof is threefold. First, let us show the stability of the system (3). Define $\theta_q(t) = 1 - (t - t_{n,q}) q_0 / \mathbb{T}_s$ with $t_{n,q} = t_n + q\mathbb{T}_s / q_0, q \in Q_0$. For $\sigma(t) = i$, we exploit the Lyapunov function $V_i(x, t) = x^{\mathrm{T}} Z_i(t) x$ with

$$Z_i(t) = \begin{cases} \theta_q(t) Z_{i,q} + [1 - \theta_q(t)] Z_{i,q+1}, & t \in [t_{n,q}, t_{n,q+1}), \\ Z_{i,q_0}, & t \in [t_{n,q_0}, t_{n+1,0}), \\ Z_{i0,q_0}, & t \in [0, t_1), \end{cases}$$

where $n = 1, 2, \ldots, t_{n,q_0} = t_n + \mathbb{T}_s$. □

Letting $\mathbb{T}_{ss}[t_{n,q}, t_{n,q+1}) = \bigcup_{q=0}^{l} [t_{n,q}, t_{n,q+1})$, $\mathbb{T}_{us}[t_{n,q}, t_{n,q+1}) = \bigcup_{q=l+1}^{q_0 - 1} [t_{n,q}, t_{n,q+1})$. For $\forall t \in \mathbb{T}_{ss}[t_{n,q}, t_{n,q+1})$, the minimal dwell-time interval of the ith subsystem in which the Lyapunov function must be decreasing, differentiating $V_i(x,t)$ associated with the trajectory of the system (3) generates

$$\begin{aligned}\dot V_i(x(t),t) &= 2\dot x^{\mathrm T}(t)Z_i(t)x(t) + x^{\mathrm T}(t)\dot Z_i(t)x(t)\\ &= x^{\mathrm T}(t)[E_i^{\mathrm T}Z_i(t) + Z_i(t)E_i + \dot Z_i(t)]x(t)\\ &\quad - 2x^{\mathrm T}(t)Z_i(t)B_ie_i(t)\\ &= \theta_q(t)\{x^{\mathrm T}(t)[E_i^{\mathrm T}Z_{i,q} + Z_{i,q}E_i + \dot Z_i(t)]x(t)\\ &\quad - 2x^{\mathrm T}(t)Z_{i,q}B_ie_i(t)\} + [1-\theta_q(t)]\{x^{\mathrm T}(t)\\ &\quad [E_i^{\mathrm T}Z_{i,q+1} + Z_{i,q+1}E_i + \dot Z_i(t)]x(t)\\ &\quad - 2x^{\mathrm T}(t)Z_{i,q+1}B_ie_i(t)\}\\ &= \theta_q(t)\{x^{\mathrm T}(t)\Phi_{i,q}x(t) - 2x^{\mathrm T}(t)Z_{i,q}B_ie_i(t)\}\\ &\quad + [1-\theta_q(t)]\{x^{\mathrm T}(t)\Phi_{i,q+1}x(t)\\ &\quad - 2x^{\mathrm T}(t)Z_{i,q+1}B_ie_i(t)\}.\end{aligned} \tag{19}$$

Following from (19) and the inequalities

$$\begin{aligned}&-2x^{\mathrm T}(t)Z_{i,q}B_ie_i(t)\\ &\quad\le x^{\mathrm T}(t)Z_{i,q}B_iB_i^{\mathrm T}Z_{i,q}x(t) + e_i^{\mathrm T}(t)e_i(t),\\ &\quad -2x^{\mathrm T}(t)Z_{i,q+1}B_ie_i(t)\\ &\quad\le x^{\mathrm T}(t)Z_{i,q+1}B_iB_i^{\mathrm T}Z_{i,q+1}x(t) + e_i^{\mathrm T}(t)e_i(t),\end{aligned}$$

one can infer

$$\begin{aligned}\dot V_i(x(t),t) &= \theta_q(t)\{x^{\mathrm T}(t)\Phi_{i,q}x(t) - 2x^{\mathrm T}(t)Z_{i,q}B_ie_i(t)\}\\ &\quad + [1-\theta_q(t)]\{x^{\mathrm T}(t)\Phi_{i,q+1}x(t)\\ &\quad - 2x^{\mathrm T}(t)Z_{i,q+1}B_ie_i(t)\}\\ &\le \theta_q(t)[x^{\mathrm T}(t)(\Phi_{i,q} + Z_{i,q}B_iB_i^{\mathrm T}Z_{i,q})x(t)]\\ &\quad + [1-\theta_q(t)][x^{\mathrm T}(t)(\Phi_{i,q+1}\\ &\quad + Z_{i,q+1}B_iB_i^{\mathrm T}Z_{i,q+1})x(t)] + e_i^{\mathrm T}(t)e_i(t).\end{aligned} \tag{20}$$

It is deduced from the triggered scheme (6) that as long as $t\in[t_m^i,\ t_{m+1}^i)$

$$\begin{aligned}||e_i(t)||^2 &\le [\alpha_0||x(t)|| + \beta_0e^{-\lambda_0t}]^2\\ &= \alpha_0^2||x(t)||^2 + 2\alpha_0\beta_0e^{-\lambda_0t}||x(t)|| + \rho(t)\\ &\le 2[\alpha_0^2||x(t)||^2 + \rho(t)],\end{aligned} \tag{21}$$

where $\rho(t) = \beta_0^2e^{-2\lambda_0t}$. Due to (20) and (21), we know that for any $t\in[t_m^i,\ t_{m+1}^i)$

$$\begin{aligned}\dot V_i(x(t),t) &\le \theta_q(t)[x^{\mathrm T}(t)(\Phi_{i,q} + Z_{i,q}B_iB_i^{\mathrm T}Z_{i,q})x(t)]\\ &\quad +[1-\theta_q(t)][x^{\mathrm T}(t)(\Phi_{i,q+1}\\ &\quad +Z_{i,q+1}B_iB_i^{\mathrm T}Z_{i,q+1})x(t)] + e_i^{\mathrm T}(t)e_i(t)\\ &\le \theta_q(t)[x^{\mathrm T}(t)(\Phi_{i,q}\\ &\quad +Z_{i,q}B_iB_i^{\mathrm T}Z_{i,q} + 2\alpha_0^2I)x(t)]\\ &\quad +[1-\theta_q(t)][x^{\mathrm T}(t)(\Phi_{i,q+1} + 2\alpha_0^2I\\ &\quad +Z_{i,q+1}B_iB_i^{\mathrm T}Z_{i,q+1})x(t)] + 2\rho(t).\end{aligned}$$

From (8) and (9), we deduce

$$\begin{aligned}\dot V_i(x(t),t) &\le \theta_q(t)[x^{\mathrm T}(t)(\Phi_{i,q} + Z_{i,q}B_iB_i^{\mathrm T}Z_{i,q}\\ &\quad + 2\alpha_0^2I)x(t)]\\ &\quad + [1-\theta_q(t)][x^{\mathrm T}(t)(\Phi_{i,q+1} + 2\alpha_0^2I\\ &\quad + Z_{i,q+1}B_iB_i^{\mathrm T}Z_{i,q+1})x(t)] + 2\rho(t)\\ &\le -\mu V_i(x(t),t) + \theta_q(t)[x^{\mathrm T}(t)(\Phi_{i,q}\\ &\quad + 2\alpha_0^2I\\ &\quad + \mu Z_{i,q} + Z_{i,q}B_iB_i^{\mathrm T}Z_{i,q})x(t)]\\ &\quad + [1-\theta_q(t)][x^{\mathrm T}(t)(\Phi_{i,q+1} + 2\alpha_0^2I\\ &\quad + \mu Z_{i,q+1}\\ &\quad + Z_{i,q+1}B_iB_i^{\mathrm T}Z_{i,q+1})x(t)] + 2\rho(t)\\ &\le -\mu V_i(x(t),t) + 2\beta_0^2e^{-2\lambda_0t}.\end{aligned}$$

Thus, we derive

$$\begin{aligned}&\dot V_i(x(t),t) \le -\mu V_i(x(t),t) + 2\rho(t),\\ &\forall t\in\mathbb{T}_{ss}[t_{n,q},t_{n,q+1}).\end{aligned} \tag{22}$$

Similarly, it can be inferred from (10) and (11) that

$$\begin{aligned}&\dot V_i(x(t),t) \le \xi V_i(x(t),t) + 2\rho(t),\\ &\forall t\in\mathbb{T}_{us}[t_{n,q},t_{n,q+1}).\end{aligned} \tag{23}$$

Further, for $t\in[t_{n,q_0},t_{n+1,0})$, that is, the time interval after the minimal dwell-time interval and before the next switching instant, along the solution of the system (3), differentiating $V_i(x,t)$ gives rise to

$$\begin{aligned}\dot V_i(x(t),t) &= 2\dot x^{\mathrm T}(t)Z_{i,q_0}x(t)\\ &= x^{\mathrm T}(t)(E_i^{\mathrm T}Z_{i,q_0} + Z_{i,q_0}E_i)x(t)\\ &\quad - 2e_i^{\mathrm T}(t)B_i^{\mathrm T}Z_{i,q_0}x(t).\end{aligned}$$

Because of

$$\begin{aligned}&-2e_i^{\mathrm T}(t)B_i^{\mathrm T}Z_{i,q_0}x(t)\\ &\quad\le x^{\mathrm T}(t)Z_{i,q_0}B_iB_i^{\mathrm T}Z_{i,q_0}x(t) + e_i^{\mathrm T}(t)e_i(t),\end{aligned}$$

we obtain

$$\begin{aligned}\dot V_i(x(t),t) &= x^{\mathrm T}(t)(E_i^{\mathrm T}Z_{i,q_0} + Z_{i,q_0}E_i)x(t)\\ &\quad - 2e_i^{\mathrm T}(t)B_i^{\mathrm T}Z_{i,q_0}x(t)\\ &\le x^{\mathrm T}(t)\Gamma_ix(t) + e_i^{\mathrm T}(t)e_i(t).\end{aligned}$$

In virtue of (21) again, one can claim that for $t\in[t_m^i,\ t_{m+1}^i)$

$$\begin{aligned}\dot V_i(x(t),t) &\le x^{\mathrm T}(t)\Gamma_ix(t) + e_i^{\mathrm T}(t)e_i(t)\\ &\le x^{\mathrm T}(t)\Gamma_ix(t) + 2(\alpha_0^2||x(t)||^2 + \rho(t))\\ &= x^{\mathrm T}(t)(\Gamma_i + 2\alpha_0^2I)x(t) + 2\rho(t)\end{aligned} \tag{24}$$

is true. According to the switching strategy (18), when $t \in [t_{n,q_0}, t_{n+1,0})$,

$$x^{\mathrm{T}}(t)\sum_{j=1}^{s} l_{ij}(Z_{i,q_0} - Z_{j,0})x(t) \geq 0 \tag{25}$$

holds. In conjunction with (24), for all $t \in [t_{n,q_0}, t_{n+1,0})$, it yields that

$$\begin{aligned}\dot{V}_i(x(t),t) &\leq x^{\mathrm{T}}(t)(\Gamma_i + 2\alpha_0^2 I)x(t) + 2\rho(t)\\ &\leq x^{\mathrm{T}}(t)\Bigg[\Gamma_i + 2\alpha_0^2 I\\ &\quad + \sum_{j=1}^{s} l_{ij}(Z_{i,q_0} - Z_{j,0})\Bigg]x(t)\\ &\quad + 2\rho(t).\end{aligned} \tag{26}$$

Through (12) and (26), one can further obtain

$$\begin{aligned}&\dot{V}_i(x(t),t) + \mu V_i(x(t),t)\\ &\leq x^{\mathrm{T}}(t)\Bigg[\Gamma_i + 2\alpha_0^2 I + \mu Z_{i,q_0}\\ &+ \sum_{j=1}^{s} l_{ij}(Z_{i,q_0} - Z_{j,0})\Bigg]x(t)\\ &+ 2\rho(t)\end{aligned}$$

over the time interval $[t_{n,q_0}, t_{n+1,0})$. This means

$$\begin{aligned}&\dot{V}_i(x(t),t) \leq -\mu V_i(x(t),t) + 2\rho(t),\\ &\forall t \in [t_{n,q_0}, t_{n+1,0}).\end{aligned} \tag{27}$$

Combing (22), (23) and (27) renders that

$$\begin{aligned}&\dot{V}_i(x(t),t)\\ &\leq \begin{cases} -\mu V_i(x(t),t) + 2\rho(t), \forall t \in \mathbb{T}_{ss}[t_n, t_{n+1}),\\ \xi V_i(x(t),t) + 2\rho(t), \quad \forall t \in \mathbb{T}_{us}[t_n, t_{n+1}).\end{cases}\end{aligned} \tag{28}$$

Defining $\Omega(r,v) = \xi T_{us}(r,v) - \mu T_{ss}(r,v)$. Recalling $\rho(t) = \beta_0^2 e^{-2\lambda_0 t}$ and noticing (28), one can deduce that for any $t \in [t_n + \mathbb{T}_s,\ t_{n+1})$,

$$\begin{aligned}V_{\sigma(t_n)}(x(t),t) &\leq e^{-\mu(t-t_n-\mathbb{T}_s)}V_{\sigma(t_n)}(x(t_n+\mathbb{T}_s), t_n+\mathbb{T}_s)\\ &\quad + 2\beta_0^2\int_{t_n+\mathbb{T}_s}^{t} e^{-\mu(t-\delta)}e^{-2\lambda_0\delta}d\delta\\ &\leq e^{-\mu(t-t_n-\mathbb{T}_s)}\Big[e^{\Omega(t_n,t_n+\mathbb{T}_s)}V_{\sigma(t_n)}(x(t_n),t_n)\\ &\quad + 2\beta_0^2\int_{t_n}^{t_n+\mathbb{T}_s} e^{\Omega(\delta,t_n+\mathbb{T}_s)}e^{-2\lambda_0\delta}d\delta\Big]\\ &\quad + 2\beta_0^2\int_{t_n+\mathbb{T}_s}^{t} e^{-\mu(t-\delta)}e^{-2\lambda_0\delta}d\delta\\ &= e^{\Omega(t_n,t)}V_{\sigma(t_n)}(x(t_n),t_n)\\ &\quad + 2\beta_0^2\int_{t_n}^{t} e^{\Omega(\delta,t)}e^{-2\lambda_0\delta}d\delta\\ &\leq e^{\Omega(t_n,t)}\Big[e^{\Omega(t_{n-1},t_n)}V_{\sigma(t_{n-1})}(x(t_{n-1}),t_{n-1})\\ &\quad + 2\beta_0^2\int_{t_{n-1}}^{t_n} e^{\Omega(\delta,t_n)}e^{-2\lambda_0\delta}d\delta\Big]\\ &\quad + 2\beta_0^2\int_{t_n}^{t} e^{\Omega(\delta,t)}e^{-2\lambda_0\delta}d\delta\\ &= e^{\Omega(t_{n-1},t)}V_{\sigma(t_{n-1})}(x(t_{n-1}),t_{n-1})\\ &\quad + 2\beta_0^2\int_{t_{n-1}}^{t} e^{\Omega(\delta,t)}e^{-2\lambda_0\delta}d\delta\\ &\leq \cdots\\ &\leq e^{\Omega(t_0,t)}V_{\sigma(t_0)}(x(t_0),t_0)\\ &\quad + 2\beta_0^2\int_{t_0}^{t} e^{\Omega(\delta,t)}e^{-2\lambda_0\delta}d\delta.\end{aligned}$$

It follows (16) that

$$\Omega(t_0,t) \leq -\mu^*(t-t_0), \tag{29}$$

which results in

$$\begin{aligned}V_{\sigma(t_n)}(x(t),t) &\leq e^{\Omega(t_0,t)}V_{\sigma(t_0)}(x(t_0),t_0)\\ &\quad + 2\beta_0^2\int_{t_0}^{t} e^{\Omega(\delta,t)}e^{-2\lambda_0\delta}d\delta\\ &\leq e^{\Omega(t_0,t)}V_{\sigma(t_0)}(x(t_0),t_0)\\ &\quad + 2\beta_0^2\int_{t_0}^{t} e^{-\mu^*(t-\delta)}e^{-2\lambda_0\delta}d\delta\\ &= e^{-\mu^*(t-t_0)}V_{\sigma(t_0)}(x(t_0),t_0)\\ &\quad + \tfrac{2\beta_0^2}{\mu^*-2\lambda_0}e^{-2\lambda_0 t}\\ &\quad - \tfrac{2\beta_0^2}{\mu^*-2\lambda_0}e^{(-2\lambda_0+\mu^*)t_0}e^{-\mu^* t}.\end{aligned} \tag{30}$$

Obviously, it is observed from the definition of $Z_i(t)$ that $Z_i(t)$ is bounded. Thus, keeping this point in mind and noting (30), we get for all $i \in S$ that

$$\begin{aligned}&\lambda_{\min}(Z_i(t))||x(t)||^2 \leq V_i(x(t),t)\\ &\quad \leq \lambda_{\max}(Z_i(t))||x(t)||^2,\end{aligned}$$

in conjunction with (17), it holds that

$$||x(t)|| \leq \sqrt{a}e^{-\frac{1}{2}\mu^*(t-t_0)}||x(t_0)|| + \frac{\sqrt{2}\beta_0}{\sqrt{\mu^*-2\lambda_0}}e^{-\lambda_0 t}.$$

where $a = \sup_t\left\{\min_{i\in S}\left\{\frac{\lambda_{\max}(Z_i(t))}{\lambda_{\min}(Z_i(t))}\right\}\right\}$. Letting

$c_1 = \sqrt{a}e^{\frac{1}{2}\mu^* t_0}||x(t_0)||$, $c_2 = \sqrt{2}\beta_0\Big/\sqrt{\mu^*-2\lambda_0}$, we have

$$||x(t)|| \leq c_1 e^{-\frac{1}{2}\mu^* t} + c_2 e^{-\lambda_0 t}. \tag{31}$$

Thus, the stability of the system (3) is realized.

Then, we show the exclusion of Zeno behavior, that is, infinite switchings or triggering does not occur in a finite time interval $[T_e, T_f]$ with $t_0 \leq T_e < T_f < \infty$. In what follows, two cases are considered.

Case (i) Adjacent triggering instants are induced by switchings or the event (7) only.

It is easily claimed that switchings do not bring Zeno behavior owing to the minimal dwell-time property of subsystems. Thus, we just need to show that if two consecutive triggering instants are only caused by events, Zeno behavior does not happen too. Based on the definition of the ET error, for $\sigma(t) = i$, we get the system

$$\dot{e}_i(t) = K_i \dot{x}(t) = K_i E_i x(t) - K_i B_i e_i(t). \tag{32}$$

For $\forall t \in [t^i_m,\ t^i_{m+1})$, the solution of the system (32) is solved by

$$e_i(t) = e^{-K_i B_i (t - t^i_m)} e_i(t^i_m) + \int_{t^i_m}^{t} e^{-K_i B_i (t-\tau)} K_i E_i x(\tau) \mathrm{d}\tau.$$

It is generated from the ET strategy (6) that for any triggered instant t^i_m, $e_i(t^i_m) = 0$. This leads to

$$e_i(t) = \int_{t^i_m}^{t} e^{-K_i B_i (t-\tau)} K_i E_i x(\tau) \mathrm{d}\tau.$$

Letting $b = c_1 + c_2$ and following from (31), one can infer

$$||x(t)|| \leq c_1 e^{-\frac{1}{2}\mu^* t} + c_2 e^{-\lambda_0 t} \leq b. \tag{33}$$

That is, $||x(t)|| \leq b$. Thereby, we know

$$\begin{aligned} ||e_i(t)|| &= \left\| \int_{t^i_m}^{t} e^{-K_i B_i (t-\tau)} K_i E_i x(\tau) \mathrm{d}\tau \right\| \\ &\leq \int_{t^i_m}^{t} e^{||-K_i B_i||(t-\tau)} ||K_i E_i|| ||x(\tau)|| \mathrm{d}\tau \\ &\leq \int_{t^i_m}^{t} b e^{||-K_i B_i||(t-\tau)} ||K_i E_i|| \mathrm{d}\tau \\ &= b||K_i E_i|| e^{||-K_i B_i|| t} \int_{t^i_m}^{t} e^{-||-K_i B_i||\tau} \mathrm{d}\tau. \end{aligned} \tag{34}$$

It is further deduced from the triggered logic (6) that for $\forall t \in [t^i_m,\ t^i_{m+1}]$, the relation

$$\alpha_0 ||x(t)|| + \beta_0 \leq b||K_i E_i|| e^{||-K_i B_i|| t} \int_{t^i_m}^{t} e^{-||-K_i B_i||\tau} \mathrm{d}\tau$$

holds, which means

$$\beta_0 \leq b||K_i E_i|| e^{||-K_i B_i|| t} \int_{t^i_m}^{t} e^{-||-K_i B_i||\tau} \mathrm{d}\tau.$$

Let us set $t = t^i_m + \mathbb{T}_e$ with $\mathbb{T}_e$ standing for the length of the triggered interval. If $|| - K_i B_i|| = 0$, then

$$\beta_0 \leq b||K_i E_i|| \mathbb{T}_e,$$

which results in a positive value of $\mathbb{T}_e$. If $|| - K_i B_i|| \neq 0$, then

$$\beta_0 \leq \frac{b||K_i E_i|| e^{||-K_i B_i||(t^i_m + \mathbb{T}_e)}}{-|| - K_i B_i||} \cdot \left[e^{-||-K_i B_i||(t^i_m + \mathbb{T}_e)} - e^{-||-K_i B_i|| t^i_m} \right],$$

which also implies that $\mathbb{T}_e > 0$.

Case (ii) Consecutive triggering instants result from the interactions between switchings and the event (7).

We first focus on the time interval $[t_{n+r}, t^i_{m+1})$ with t_{n+r} indicating a certain switching instant within $[T_e, T_f]$, and t^i_{m+1} indicating the latest triggering instant produced by the triggered mechanism (6) after t_{n+r}. It is inferred from the triggered logic (6) that switching instants are also triggering instants. This implies that t_{n+r} is a triggering instant. Consequently, this case becomes the same as Case (i), and thus, the positive inter-event length is ensured. Then, let us turn our attention to the time interval $[t^i_m, t_{n+r})$ with t^i_m representing the last triggering time brought by the triggered rule (6) before t_{n+r}. Assume that $t_{n+r}, t_{n+r+1}, \ldots, t_{n+q}, t^i_{m+1}$ are the next triggering instants, where k is a finite positive integer standing for the number of switchings and satisfying $r \leq q < k$. As discussed above, the interval $[t_{n+q}, t^i_{m+1})$ owns a positive lower bound. This further means that there does not exist Zeno behavior.

Next, we prove the BT performance (4) of the system (3). First, let us show the BT performance (4) at switching instants t_n. The switching logic (18) ensures

$$x^{\mathrm{T}}(t) \sum_{j=1, j\neq i}^{s} l'_{ij} (Z_{i,q_0} - Z_{j,0}) x(t) \geq 0, \quad \forall t \in [t_n + \mathbb{T}_s, t_{n+1}). \tag{35}$$

Combing (13) and (35) ensures that for $\forall t \in [t_n + \mathbb{T}_s, t_{n+1})$, $p \in S$,

$$\begin{aligned} &||K_i x(t) - K_p x(t)||^2 - (\alpha||x(t)|| + \beta)^2 \\ &\quad + \lambda_{ip} x^{\mathrm{T}}(t)(Z_{i,q_0} - Z_{p,0})x(t) \\ &= x^{\mathrm{T}}(t)\Lambda_{ip}x(t) - [\alpha^2 x^{\mathrm{T}}(t)x(t) \\ &\quad + 2\alpha\beta||x(t)|| + \beta^2] \\ &= x^{\mathrm{T}}(t)(\Lambda_{ip} - \alpha^2 I)x(t) - 2\alpha\beta||x(t)|| - \beta^2 \\ &\leq x^{\mathrm{T}}(t)\bar{\Lambda}_{ip}x(t) - 2\alpha\beta||x(t)|| - \beta^2. \end{aligned} \tag{36}$$

Note that if $x^{\mathrm{T}}(t)W_i\eta \geq 0$, then

$$-\alpha\beta||x(t)|| \leq x^{\mathrm{T}}(t)W_i\eta, \tag{37}$$

if $x^{\mathrm{T}}(t)W_i\eta < 0$, via (14), we have

$$x^{\mathrm{T}}(t)W_i\eta\eta^{\mathrm{T}}W_i x(t) \leq \alpha^2\beta^2 x^{\mathrm{T}}(t)x(t),$$

which also renders (37). By means of $W_i < 0$, (15), (36) and (37), one can know

$$\begin{aligned} &||K_i x(t) - K_p x(t)||^2 - (\alpha||x(t)|| + \beta)^2 \\ &\quad + \lambda_{ip} x^{\mathrm{T}}(t)(Z_{i,q_0} - Z_{j,0})x(t) \\ &\quad \leq x^{\mathrm{T}}(t)\bar{\Lambda}_{ip}x(t) - 2\alpha\beta||x(t)|| - \beta^2 \\ &\quad \leq x^{\mathrm{T}}(t)\bar{\Lambda}_{ip}x(t) + 2x^{\mathrm{T}}(t)W_i\eta - \beta^2 \\ &\quad \leq x^{\mathrm{T}}(t)\bar{\Lambda}_{ip}x(t) + 2x^{\mathrm{T}}(t)W_i\eta + \eta^{\mathrm{T}}W_i\eta \\ &\quad \leq [x(t) + \eta]^{\mathrm{T}}W_i[x(t) + \eta] \\ &\quad \leq 0 \end{aligned} \tag{38}$$

when $t \in [t_n + \mathbb{T}_s, t_{n+1}]$. Suppose that the system (3) switches from the ith subsystem to the pth subsystem at the switching instant t_{n+1}. Thus, it is known from $\lambda_{ip} \geq 0$ and the switching rule (18) that

$$x^{\mathrm{T}}(t_{n+1})(Z_{p,0} - Z_{i,q_0})x(t_{n+1}) = 0.$$

Incorporating with (38), we are in a position to declare that at the switching instant t_{n+1},

$$\begin{aligned} &||K_i x(t_{n+1}) - K_p x(t_{n+1})||^2 - (\alpha||x(t_{n+1})|| + \beta)^2 \\ &\quad \leq \lambda_{ip} x^{\mathrm{T}}(t_{n+1})(Z_{p,0} - Z_{i,q_0})x(t_{n+1}) = 0 \end{aligned}$$

which implies the BT performance (4). Further, we show how the BT performance (4) is ensured at the triggering instants t_m^i caused by the event (7). Recalling the triggered rule (6) and noticing $\alpha_0 \leq \alpha$, $\beta_0 \leq \beta$, we can easily infer the BT performance (4).

Remark 3 Theorem 1 develops a criterion under which the stability of the system (3) is obtained, the control bumps are restricted, and the communication resources

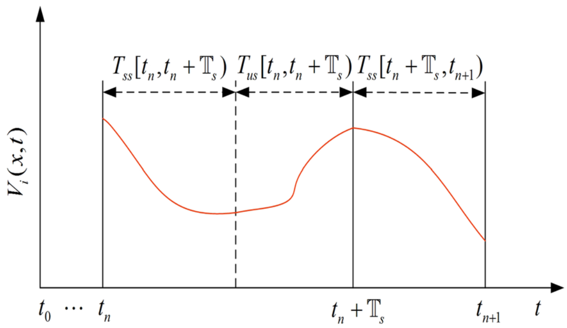

Fig. 1 Lyapunov function of the ith subsystem

are saved. Again, Zeno behavior generated by triggering and switchings is excluded. (8)–(12), (16) and (17) are utilized to force the stability. The BT performance (4) is ensured by (13)–(15). Further, we make some explanations on the switching law (18). The switching mechanism (18) is a state and time mixed switching rule. When $\mathbb{T}_s = 0$, the switching law (18) degrades into the classical state-dependent switching law of [1]. A switching law with the same construction way as (18) has been used by [8] to obtain only the steady-state property of the switched systems. Different from [8], in this manuscript, the switching law (18) is used to achieve both the steady-state property and the transient performance of the switched systems. Also, when only the steady-state property is considered, the condition is looser than that in [8]. This is due to the fact that the Lyapunov functions are allowed to be increasing during the ensured minimal working time intervals. The Lyapunov function of the ith subsystem is shown by Fig. 1. However, in [8], the Lyapunov functions must be decreasing during the ensured minimal working time intervals. This renders the switching strategy (18) more general than the traditional mixed switching rule of [8].

Remark 4 Now, we explain how to verify the condition (16). It is not difficult to know that if there exists a nonnegative constant υ satisfying $T_{us}[t_n, t_n + \mathbb{T}_s) = T_{ss}[t_n, t_n + \mathbb{T}_s) \cdot \frac{\mu-\mu^*}{\xi+\mu^*} - \upsilon$, then the condition (16) is ensured. Combining $\mathbb{T}_s = T_{us}[t_n, t_n + \mathbb{T}_s) + T_{ss}[t_n, t_n + \mathbb{T}_s)$ and $T_{us}[t_n, t_n+\mathbb{T}_s) = T_{ss}[t_n, t_n + \mathbb{T}_s) \cdot \frac{\mu-\mu^*}{\xi+\mu^*} - \upsilon$ yields $\mathbb{T}_s = T_{ss}[t_n, t_n + \mathbb{T}_s) \cdot (1 + \frac{\mu-\mu^*}{\xi+\mu^*}) - \upsilon$. Then, replacing $\mathbb{T}_s$ in the inequalities (8)–(11) by $T_{ss}[t_n, t_n + \mathbb{T}_s) \cdot (1 + \frac{\mu-\mu^*}{\xi+\mu^*}) - \upsilon$, we can obtain $T_{ss}[t_n, t_n + \mathbb{T}_s)$ and υ. Further, taking advantage of $T_{us}[t_n, t_n + \mathbb{T}_s) = T_{ss}[t_n, t_n + \mathbb{T}_s) \cdot \frac{\mu-\mu^*}{\xi+\mu^*} - \upsilon$, we can obtain $T_{us}[t_n, t_n + \mathbb{T}_s)$.

Remark 5 In practice, it is not difficult to realize the switching mechanism (18). This is because the minimal dwell time is pre-given and the switching signal value is kept unchanged during the minimal dwell-time interval. Also, for the time interval after the minimal dwell-time interval and before the next switching instant, the switching signal is determined by a well-known min logic rule which has been widely used in practical systems [1,44,45].

Remark 6 The condition of Theorem 1 is used to guarantee both the switching signal (18) and the triggering rule (6) instead of the switching signal or the triggering rule. Also, for each subsystem, the triggering rule (6) is not a stabilizing triggering rule.

Remark 7 In addition, it is not difficult to find that under the switching law (18), the event (6) only triggers during the switching interval $[t_n, t_{n+1}]$. If no event occurs during the switching interval $[t_n, t_{n+1}]$, the ET mechanism (6) is useless, that is, the control problem considered degrades into the BT control problem for the system (1).

Note that the constraints in Theorem 1 are nonlinear and thus usually difficult to be calculated. To overcome this drawback, we design an algorithm to solve a series of linear matrix inequalities. The main idea of the algorithm is twofold. First, taking advantage of the fact that the usual min-switching rule [1] may still have a minimal dwell time even though it cannot always guarantee the minimal dwell time, we can solve a part of parameters. Then, by substituting the derived parameters into the remaining constraints, we can check whether the minimal dwell time and the BT performance are ensured or not.

Before offering the algorithm, we simplify several matrix inequalities of Theorem 1.

First, let us consider (12). Recalling $E_i = A_i + B_i K_i$, setting $M_{i,q_0} = Z^{-1}_{i,q_0}, Z^{-1}_{j,0} = M_{j,0}, N_{i,q_0} = K_i M_{i,q_0}$ and multiplying the expression on the left side of (12) by M_{i,q_0} on the both sides generates

$$\begin{bmatrix} \Upsilon_{i,q_0} + \sum\limits_{j=1, j\neq i}^{s} l_{ij} M_{i,q_0} & M_{i,q_0} & \rho_{i,q_0} M_{i,q_0} \\ * & -\frac{1}{2}\alpha_0^{-2} I & 0 \\ * & * & -\Omega_{i,q_0} \end{bmatrix} < 0, \tag{39}$$

$$\begin{aligned} \Upsilon_{i,q_0} &= M_{i,q_0} A_i^{\mathrm{T}} + A_i M_{i,q_0} + N^{\mathrm{T}}_{i,q_0} B_i^{\mathrm{T}} + B_i N_{i,q_0} \\ &\quad + B_i B_i^{\mathrm{T}} + \mu M_{i,q_0}, \\ \Omega_{i,q_0} &= \mathrm{diag}\{M_{1,0}, \ldots, M_{i-1,0}, M_{i+1,0}, \ldots, M_{s,0}\}, \\ \rho_{i,q_0} &= \left[\sqrt{-l_{i1}} \cdots \sqrt{-l_{ii-1}}\ \sqrt{-l_{ii+1}} \cdots \sqrt{-l_{is}}\right]. \end{aligned}$$

Then, we turn to (14) and (15). When the matrices W_i are calculated, (14) and (15) can be ensured by

$$\begin{bmatrix} -\alpha^2\beta^2 I & W_i \eta \\ * & -I \end{bmatrix} \leq 0 \tag{40}$$

and

$$\begin{bmatrix} -\beta^2 I & \eta^{\mathrm{T}} \\ * & W_i^{-1} \end{bmatrix} \leq 0, \tag{41}$$

respectively.

Now, the algorithm is provided in detail.

Algorithm. Calculation of the controller gains K_i.

Step 1. Fix the scalars l_{ij} and then solve the matrices $K_i, M_{i,q_0}, M_{i,0}$ from (39).

Step 2. Substitute the derived matrices $K_i, M_{i,q_0}, M_{i,0}$ into (8)–(11) to test whether the time $\mathbb{T}_s$ exists or not. If $\mathbb{T}_s$ exists, then maximize $\mathbb{T}_s$ and turn to the next step. Otherwise, turn back to Step 1 and choose different constants l_{ij}.

Step 3. Substitute the derived matrices $K_i, M_{i,q_0}, M_{i,0}$ into (13) to calculate the matrices W_i and scalars l'_{ij}. If (13) has solutions, then turn to the next step. Otherwise, turn back to Step 1 and then update l_{ij}.

Step 4. Determine the vector η from (40), (41) with calculated matrices W_i. If η exists, then terminate the algorithm and export the solved parameters. Otherwise, adjust the constants l_{ij} and iterate Steps 1-4.

4 Application to switched RLC circuits

This section devotes to the demonstration of the superiority for the proposed ETBT control strategy.

We apply the developed control scheme to the switched RLC circuit in [40,43].

Figure 2 depicts the switched RLC circuit which is composed of a resistance R, an inductor L, an input voltage $u(t)$ and a collection of capacitors $C_i, i = 1, 2, \ldots, s$. The model of the switched RLC circuit is described by

$$\dot{x}(t) = A_{\sigma(t)} x(t) + B_{\sigma(t)} u(t), \tag{42}$$

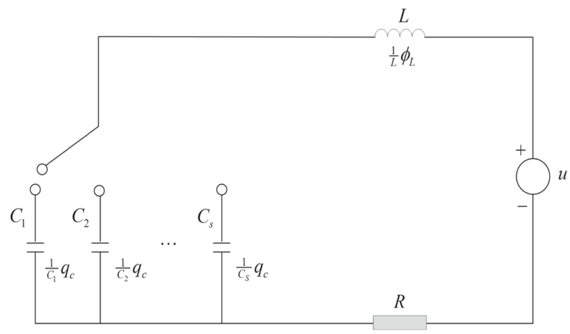

Fig. 2 Switched RLC circuit

where $x^{\mathrm{T}}(t) = [q_c\ \phi_L]$, $\sigma(t)$ takes its values in the set $S = \{1,\ 2\}$,

$$A_i = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C_i} & -\frac{R}{L} \end{bmatrix},\ B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ i = 1, 2.$$

Set the initial state values $x^{\mathrm{T}}(0) = [5\ \ -5]$, the parameters $L = 2.5,\ C_1 = 100,\ C_2 = 30,\ R = 2,\ \mu = 0.001,\ \xi = 0.01,\ \mathbb{T}_s = 0.1s$.

For the verification on the superiority of the presented control method, we make a comparison simulation. Two cases are considered.

Case (i) Both the BT performance and the stability are ensured.

In this case, we select $\alpha_0 = 10^{-5},\ \beta_0 = 0.008,\ \lambda_0 = 0.001,\ \alpha = 0.5,\ \beta = 1,\ l_{12} = -0.005,\ l_{21} = -1,\ \lambda_{12} = \lambda_{21} = 1$. By solving Algorithm 1, we obtain the controller gains

$$K_1 = [-0.3080\ \ -0.3781],$$
$$K_2 = [-0.3000\ \ -0.5374]$$

and the parameters

$$Z_{10} = 10^{-8} * \begin{bmatrix} 0.2465 & 0.0037 \\ 0.0037 & 0.2569 \end{bmatrix},$$

$$Z_{20} = 10^{-8} * \begin{bmatrix} 0.3965 & 0.0553 \\ 0.0553 & 0.3438 \end{bmatrix},$$

$$Z_{11} = 10^{-8} * \begin{bmatrix} 0.4199 & 0.1260 \\ 0.1260 & 0.3660 \end{bmatrix},$$

$$Z_{21} = 10^{-8} * \begin{bmatrix} 0.3965 & 0.0553 \\ 0.0553 & 0.3438 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -0.1250 & -0.0006 \\ -0.0006 & -0.1123 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} -0.1250 & -0.0006 \\ -0.0006 & -0.1123 \end{bmatrix},$$

$$\eta = [-0.1461\ \ -0.1465]^{\mathrm{T}},\ l'_{12} = l'_{21} = -1.0165.$$

Table 1 Comparison between Case (i) and Case (ii)

Case	$\lVert u(t_m^{i^+}) - u(t_m^{i^-})\rVert_{\max}$	Num_T	Num_S
(i)	0.0892	33	4
(ii)	0.8764	25	12

Case (ii) Only the stability is guaranteed.

In this case, we choose $\alpha_0 = 0.6,\ \beta_0 = 0.008,\ \lambda_0 = 0.001,\ l_{12} = -0.005,\ l_{21} = -1\ \lambda_{12} = \lambda_{21} = 1$. Here, we set $\alpha_0 > \alpha$ to indicate that the control bumps induced by triggering are not suppressed. Then, we work out the controller gains

$$K_1 = [-0.3182\ \ -0.3644],$$
$$K_2 = [-0.3022\ \ -0.5312]$$

and the variables

$$Z_{10} = 10^{-8} * \begin{bmatrix} 0.2454 & 0.0035 \\ 0.0035 & 0.2559 \end{bmatrix},$$

$$Z_{20} = 10^{-8} * \begin{bmatrix} 0.3968 & 0.0542 \\ 0.0542 & 0.3452 \end{bmatrix},$$

$$Z_{11} = 10^{-8} * \begin{bmatrix} 0.4146 & 0.1227 \\ 0.1227 & 0.3669 \end{bmatrix},$$

$$Z_{21} = 10^{-8} * \begin{bmatrix} 0.3968 & 0.0542 \\ 0.0542 & 0.3452 \end{bmatrix}.$$

In the following table, we make a quantitative analysis on the proposed ETBT control scheme by comparing the maximum value of the control bump $\lVert u(t_m^{i^+}) - u(t_m^{i^-})\rVert$, the number of triggering and the number of switchings in both cases.

In Table 1, the notations $\lVert u(t_m^{i^+}) - u(t_m^{i^-})\rVert_{\max}$, Num_T and Num_S denote the maximum value of $\lVert u(t_m^{i^+}) - u(t_m^{i^-})\rVert$, the number of triggering and the number of switchings, respectively.

Furthermore, several figures are provided. Figure 3 shows different switching logics. The differences of ET sequences are exhibited by Fig. 4. Figure 5 compares the different control signals. The trajectory comparisons are presented by Figs. 6 and 7. It is observed from Fig. 3 that the value of the switching signal $\sigma(t)$ of Case (ii) varies more frequently. Figure 4 tells that the ET frequency of Case (i) is greater than that of the Case (ii), which reveals that the cost of the better BT performance is frequently triggering. From Fig. 5, we can see that the variation of the magnitude for the control signal obtained in the Case (i) is much less than that in the other case. It is easily found from Figs. 6 and 7

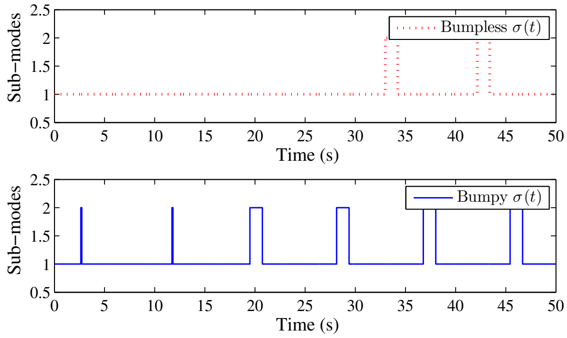

Fig. 3 Switching signals

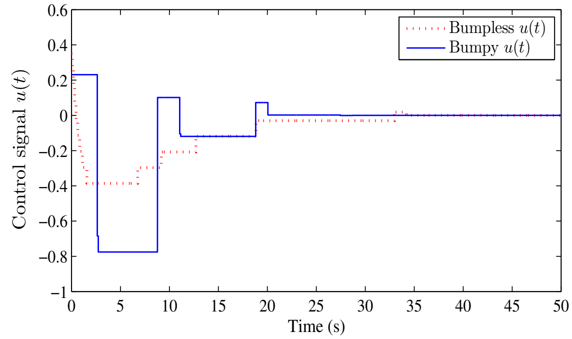

Fig. 5 Input voltage

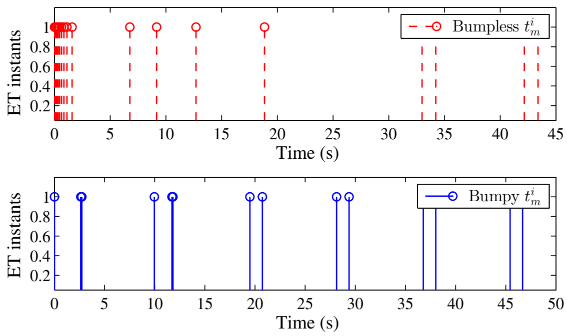

Fig. 4 Triggering instant sequences

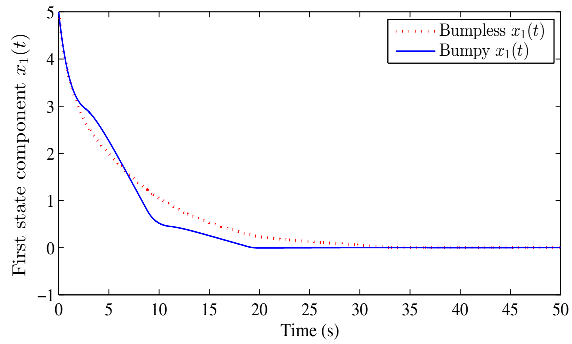

Fig. 6 Charge in the capacitor

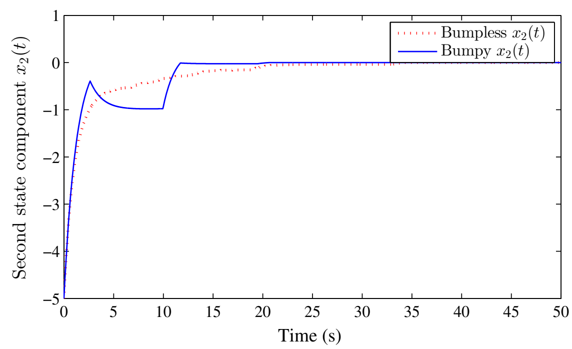

Fig. 7 Flux in the inductance

that each state component of the model (42) is stable in both situations. From Fig. 6, we can find that the trajectory of the charge in the capacitor in Case (i) varies more gently than that in Case (ii). It is observed from Fig. 7 that the trajectory of the flux in the inductance in Case (i) varies more gently than that in Case (ii). Figures 6 and 7 indicate that the presented ETBT control scheme can improve the trajectories of the system (42). Accordingly, the proposed ETBT control approach can effectively reduce the transmission resources, suppress the bumps in the input voltage and guarantee the steady-state operation of the switched RLC circuit (42).

5 Conclusions

In this article, we have studied the issue of ETBT control for a class of switched linear systems. In the first place, the suppression level on the control bumps induced by both switchings and triggering has been described in both relative and absolute manners. Then, by dual-design of a switching scheme, an ET logic and a family of controllers, the magnitude difference in the control signal at switching and triggering instants has been restrained, while the stability has been realized. The switching rule has generalized the usual state-dependent switching mechanism satisfying a certain dwell-time constraint, permitting the increase in the Lyapunov functions over the dwell-time intervals. Second, under the designed switching rule, triggered strategy and controllers, a criterion has been established, ensuring both the BT performance and the stability. Moreover, Zeno behavior has been excluded. At last, the developed control strategy has been effectively applied to a switched RLC circuit, showing the effectiveness of the presented control scheme.

Compliance with ethical standards

Conflicts of interest The authors declare that there is no conflict of interests regarding the publication of this paper, and the research does not involve Human Participants and/or Animals.

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